ISSN 1936-5330

kcFED Research Working Papers

Bank Competition and Risk-Taking under Market Integration

Kaniska Dam and Rajdeep Sengupta

December 2020; updated January 2022
RWP 20-21
http://doi.org/10.18651/RWP2020-21
This paper supersedes the old version:
"Market Integration and Bank Risk-Taking"

Federal Reserve Bank of Kansas City

Bank Competition and Risk-Taking under Market Integration*

Kaniṣka Dam† Rajdeep Sengupta‡

January 12, 2022

Abstract

As banking markets integrate, bank competition increases with the expansion in market size because deposit supply and loan demand become more elastic. At the same time, the expansion in the measure of depositors is not always equal to the expansion in the measure of borrowers. This unequal expansion is sufficient to generate a hitherto unexplored risk-incentive mechanism that operates through loan rates, which we term the "bank-customer effect". A sufficiently strong bank-customer effect of market integration can reverse any relation between competition and risk-taking that prevails when markets are segmented.

JEL codes: D82, G21, L13.
Keywords: Market integration; loan rate; risk-shifting.

*We are grateful to Eric Bond for insightful discussions that helped improve the paper. We also thank Panos Avramidis, Albert Banal-EstaÃ±*ol*, *PhilipBond*, *SubhayuBandyopadhyay*, *ClaudeD′Aspremont*, *GeorgeFortier*, *SayantanGhosal*, *FaustoHernndez*, *KaraM*

†Center for Research and Teaching in Economics (CIDE), Carretera Mexico-Toluca 3655, Lomas de Santa Fe, 01210 Mexico City, Mexico. E-mail: kaniska.dam@cide.edu

‡Corresponding author. Research Division, Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, Missouri 64198. E-mail: rajdeep.sengupta@kc.frb.org

1. Introduction

The Financial Crisis of 2008-2009 has rekindled interest in the linkages between bank competition and financial stability. After the Great Depression, bank competition has evolved in two distinct phases. Prior to the 1970s, U.S. states suppressed competition by restricting banking activity both within and across state borders.[1] Since the early 1970s, U.S. banks have been permitted to expand beyond a single local market into new areas, initially within state boundaries and then across multiple states (Kroszner and Strahan, 1999). As financial deregulation over the years removed constraints on competition, banking markets became increasingly "integrated".[2] Over the years, *market integration* has been the key driver of increased competition in the banking industry.[3] Understanding this evolution of competition and its impact on risk-taking is critical to achieving financial stability goals.

Recent studies reviewing the linkages between bank competition and financial stability have arrived at different conclusions. For example, Beck, Coyle, Dewatripont, Freixas, and Seabright (2010) and Vives (2019) find that competition is important for financial stability. In contrast, the OECD (2011, p. 10) report on bank competition and financial stability argues that "[T]he pre-crisis regulatory landscape has set in motion changes in business models and activities in response to competition that proved not to be conducive to financial stability." Recently, Corbae and Levine (2018) has re-emphasized this tradeoff between competition and stability. By some accounts, the intensification of bank competition under market integration has also led to greater instability (Jiang, Levine, and Lin, 2017). Notwithstanding the lack of consensus on empirical evidence, theory has largely defined an increase in bank competition as an increase in the number of rival banks. Paradoxically, the impact of the evolution of bank competition through geographic expansion and the integration of banking markets on risk-taking has received less attention in banking theory.

The purpose of this paper is to study how increased competition through the geographic expan-

[1] Although the primary motivation here is the evolution of the banking industry in the United States (Berger, Kashyap, and Scalise, 1995; Jones and Critchfield, 2008; DeYoung, 2008; Kroszner and Strahan, 2014), the European experience has followed a similar trajectory (Gaspar, Hartmann, and Sleijpen, 2003; Carletti and Vives, 2009).

[2] In particular, state-by-state banking deregulation in the United States occurred through bilateral, regional, and even national reciprocal arrangements (Amel, 1993). Reciprocal agreements helped the geographic expansion of bank operations and enabled bank competition to span multiple local markets (Radecki, 1998).

[3] The term "market integration" is used as a shorthand to refer to the removal of entry barriers and other geographic restrictions that prevailed around the early 1970s. The process involved branching deregulation, initially across counties within a state (for example, in the case of unit banking that existed in 17 states), and subsequently across states on a reciprocal basis, culminating in the Riegle-Neal Interstate Banking and Branching Efficiency Act of 1994.

sion of banking markets affects risk-taking. Increased competition under the integration of previously segmented banking markets not only increases the number of rival banks—as modeled in traditional theory—but also expands the customer base (depositors and borrowers) available to each bank. This additional effect—unmodeled in previous studies—is shown to also increase competition and influence risk-taking behavior through its effect on loan rates. As markets integrate, the effect of increased competition on risk-taking comprises both effects, and consequently, goes beyond a simple increase in the number of competing banks.

Any examination of how increased competition affects risk-taking can be viewed as comprising two important but distinct relations: First, the relation between competition and interest rates, and second, the relation between interest rates and risk-taking incentives of entrepreneurs. Conventional wisdom has concluded that increased competition lowers loan rates and recent studies have focused on how interest rates affect risk-taking (Dell'Ariccia and Marquez, 2013; Martinez-Miera and Repullo, 2019). In this paper, we examine both relations under market integration using a model of oligopolistic competition wherein banks face entrepreneurial moral hazard. The interest rate we focus on is the loan rate banks charge borrowers (Boyd and De Nicoló, 2005; Martinez-Miera and Repullo, 2010).[4]

This paper reveals a risk-incentive mechanism from increased competition under market integration that operates through loan rates. Increased competition, traditionally defined as an increase in the number of competing banks, reduces loan rates. We term this negative relationship the *bank-competitor effect* of market integration. However, when markets integrate, the potential expansion of banks' customer base (borrowers and depositors) generates an additional mechanism of increased competition. As more markets integrate, market expansion makes loan demand and deposit supply more elastic and the resulting increase in competition induces banks to behave like price-takers. Unlike an increase in the number of banks, this *bank-customer effect*—a previously unexplored effect of increased competition—is not unambiguously negative and can even increase loan rates. Moreover, when the bank-customer effect is positive and sufficiently large, it dominates the negative bank-competitor effect. As a result, this customer effect of market integration can reverse the competitor effect of an increase in the number of rival banks. In reversing the effect of competition on loan rates, increased

[4] In alternative settings, where banks solve a portfolio choice problem taking asset prices and return distributions as given, models have focused on the interest rate banks pay on deposits (Allen and Gale, 2004).

competition under market integration can also reverse the effect of competition on risk-taking.

Our results underscore the importance of incorporating market integration in any examination of the linkages between competition and risk-taking. Even in our tractable setting, we demonstrate that this broader interpretation of increased competition increases the range of possible outcomes. As a result, we can show that predictions of prior theory prevail under specific conditions but are not the only outcomes of the model. Moreover, the richer set of results can also help understand as to why the weight of empirical evidence regarding bank market structure and risk-taking is mixed, with no clear consensus.

Main results. We begin with an examination of the association between loan rates and risk-shifting.[5] Banks lend their deposits to entrepreneurs (borrowers) who invest in risky projects but have limited liability. Banks face entrepreneurial moral hazard because the borrowers' choice of project risk is unverifiable and cannot be contracted upon. Typically, models of moral hazard in banking yield a positive relation between loan rates and risk-taking. Raising the loan rate decreases the net return on successful projects, incentivizing borrowers to seek projects less likely to succeed but with higher returns when successful (Stiglitz and Weiss, 1981). In a generalized version of this basic model, where the project output exhibits diminishing marginal productivity of investment, we find that the relation between loan rates and risk-taking can also be negative. Under decreasing marginal productivity, we show that risk-taking increases (decreases) with loan rates according as the output elasticity decreases (increases) with investment.

Prior research has shown that the bank-competitor effect is negative (Boyd and De Nicoló, 2005). Although formulated in the context of individual (segmented) markets, we show that this result extends to market integration. As more markets integrate and each bank faces more competitors, the increased competition lowers loan rates charged by banks.

Combining the effect of competition on loan rates with that of loan rates on risk-taking generates interesting implications for risk-taking. Throughout, we assume that project risks are perfectly corre-

[5] We use the terms risk-taking and risk-shifting interchangeably. In debt-financed firms, owners and managers have incentives to take excessive risks because they benefit from the upside potential while debt-holders bear the downside risks. This well-known risk-shifting problem is particularly acute in banks where a substantial share of liabilities is insured deposits.

lated so that risk-taking by borrowers coincides with the risk-taking by banks (Allen and Gale, 2000; Boyd and De Nicoló, 2005). In any *segmented market equilibrium* (SME), the negative bank-competitor effect combines with the effect of loan rates on risk-taking to yield the effect of increased competition on risk-taking. And unlike previous studies that have shown this effect to be unambiguously negative, we find that increased competition in an SME can both increase or decrease risk-taking. In an environment where risk-taking increases with loan rates, the bank-competitor effect of increased competition decreases risk-taking. The opposite is true when risk-taking decreases with loan rates.

Market integration not only increases the number of banks, but also the number of potential customers (depositors and borrowers) available to each bank—thereby expanding deposit supply and loan demand. Competition increases with this expansion in market size because both deposit supply and loan demand become more elastic and individual banks become small relative to the market. In expanding the customer base, this bank-customer effect of market integration induces banks to behave more like price takers. We show that, if integrating markets are heterogenous in size (measure of depositors and borrowers), increased competition under market integration can raise or lower loan rates. When markets of different sizes integrate, the rate of expansion in the measure of depositors is not always equal to that of borrowers. This unequal rate of expansion between deposit supply and loan demand is sufficient to generate a nonzero bank-customer effect. If the expansion in deposit supply is greater than the expansion in loan demand, the increase in the supply of loanable funds, relative to loan demand, tends to lower loan rates. Conversely, when integration increases the measure of borrowers more than that of depositors, the relative increase in loan demand tends to increase the loan rate. Through its effect on the loan rate, the bank-customer effect can increase or decrease risk-taking depending on the relative magnitudes of changes in the deposit supply and loan demand schedules. The overall effect of market integration depends on the relative strength of both the bank-competitor and the bank-customer effects. A negative bank-customer effect reinforces the bank-competitor effect and the aggregate effect of market integration on loan rates is unambiguously negative. In contrast, when the bank-customer effect is positive and sufficiently large, it dominates the bank-competitor effect, increasing loan rates offered by banks.

We show that any association between competition and risk-taking in the SME can be reversed by a sufficiently strong and positive bank-customer effect in the *integrated market equilibrium* (IME).

First, consider the case where risk-taking increases with the loan rate. With the bank-competitor effect always negative, risk-taking decreases with increased competition. However, a sufficiently strong and positive bank-customer effect in the IME increases risk-taking if it can outweigh the negative bank-competitor effect. Next, consider the case where risk-taking decreases with the loan rate. Here, risk-taking tends to increase with competition because of the negative bank-competitor effect. This relationship can also change with a sufficiently strong bank-customer effect in the IME. With a sufficiently large bank-customer effect, increased competition in the IME increases loan rates and thereby reduces risk-taking. To summarize, this paper shows how increased competition under market integration affects loan rates in ways beyond a simple increase in the number of rival banks.

Extensions. In a first extension, we shows that results of the baseline model are robust to the inclusion of interbank lending (Section 5.1). Increasing the number of banks decreases individual banks' market power, which tends to raise deposit rates and lower loan rates. Ceteris paribus, increases in deposit costs pass-through to loan rates.[6] The bank-competitor effect on loan rates comprises a negative direct effect and a positive indirect effect that operates through deposit rates. Increasing the interbank rate dampens the negative direct effect but strengthens the positive indirect effect. At low interbank rates, the negative direct effect dominates and the overall bank-competitor effect remains negative. On the other hand, the positive indirect effect dominates at high interbank rates and the bank-competitor effect is positive. As with the bank-customer effect discussed above, the sign on the bank-competitor effect is no longer unambiguous in the presence of interbank lending. Moreover, the result of the baseline model still holds, that is, the risk-incentive mechanism of increased competition in the SME can be reversed in the IME.

A second extension allows for bank consolidation under market integration as has been the trend in the banking industry.[7] Market segmentation and the ensuing lack of competitive pressures has been viewed as the source of inefficiencies among banks. As a result, regulatory barriers to competition can generate heterogeneity so that banks across different segmented markets operate at different efficien-

[6]With interbank lending, aggregate loan and deposit volumes—and by extension, loan and deposit rates are independent of each other only if the probability of default is exogenous. Because the probability of default is endogenous in our framework, loan and deposit rates are no longer independent even with an interbank market.

[7]We relax the assumption of no entry and exit of banks. This assumption in the baseline model is intended to ensure that the number of competitor banks increases with the number of integrating markets.

cies.[8] In Section 5.2, we model this heterogeneity in terms of differences in bank-specific operating costs (non-interest expenses) across different markets. Upon integration, the presence of banks with heterogeneous costs in the IME creates incentives for bank mergers. The ensuing consolidation yields efficiency gains as described in numerous empirical studies (Berger, Demsetz, and Strahan, 1999; DeYoung, Evanoff, and Molyneux, 2009). The negative bank-competitor effect implies that bank mergers increase loan rates and risk-taking by reducing the number of competitor banks. To establish the robustness of our baseline results, we show that a positive bank-customer effect in the post-merger IME can reinforce the effects of consolidation and increase loan rates and risk-taking relative to the SME. While this analysis is intended to examine risk-taking effects of market integration, it also captures some of the broad patterns of merger activity in U.S. banking. In particular, we model how efficient banks can target relatively inefficient, less profitable banks that operate within the same integrated market.[9] Moreover, we model scenarios wherein bank consolidation can yield greater risk-taking even though the unilateral effect of mergers is pro-competitive and welfare enhancing.[10]

Related literature. The theoretical literature on the effect of increased competition on risk-taking can be viewed as comprising two segments. The first set of studies emphasize the effect of increased competition on interest rates (Allen and Gale, 2004; Repullo, 2004; Boyd and De Nicoló, 2005), while the second set of studies examine the effect of interest rates on risk-taking incentives of banks and borrowers (Stiglitz and Weiss, 1981; Martinez-Miera and Repullo, 2010; Wagner, 2010; Dell'Ariccia and Marquez, 2013; González-Aguado and Suárez, 2015). This paper contributes to both segments of this literature. In relation to the first segment, we identify an additional risk-incentive mechanism—namely, the bank-customer effect—that differs from the bank-competitor effect covered in these stud-

[8]For example, Kroszner (2001, p. 38) argues that, "...branching restrictions tend to reduce the efficiency and consumer convenience of the banking system, and small banks tend to be particularly inefficient in states where branching restrictions offer them the most protection."

[9]Berger et al. (1999, p. 150) presents empirical evidence in support of such merger activity: "The prior geographic restrictions on competition may have allowed some inefficient banks to survive. The removal of these constraints allowed some previously prohibited M&As to occur, which may have forced inefficient banks to become more efficient by acquiring other institutions, by being acquired, or by improving management practices internally."

[10]Regulatory reviews of bank merger applications in the United States have prevented consolidation where excessive increases in risks would be expected. This established regulatory practice has finally been formalized: The 'financial stability' factor is included in Section 604(d) of the Dodd-Frank Wall Street Reform and Consumer Protection Act, which amended Section 3(c) of the Bank Holding Company Act of 1956. "[T]he addition of a financial stability factor ... contrasts with an antitrust pre-merger review, in which the focus is solely on whether the transaction would substantially lessen competition" (Tarullo, 2012). When evaluating a proposed bank acquisition or merger, the Federal Reserve Board is now required to consider "the extent to which [the] proposed acquisition, merger, or consolidation would result in greater or more concentrated risks to the stability of the United States banking or financial system".

ies (Boyd and De Nicoló, 2005; Martinez-Miera and Repullo, 2010). We show that any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME. With respect to the second group, we present a general model of entrepreneurial moral hazard wherein risk-taking can increase or decrease with loan rates. Models of borrower moral hazard and limited liability have shown that risk-taking increases with loan rates (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). However, risk-taking can decrease with loan rates in moral hazard settings where banks monitor borrower actions (Besanko and Kanatas, 1993; Dell'Ariccia and Marquez, 2013; Martinez-Miera and Repullo, 2017). We show that both effects are possible, even in the absence of bank monitoring as long as borrowers' project returns exhibit diminishing marginal productivity.

While our model reveals that the overall effect of market integration on risk-taking is not unambiguous, this feature does mirror the lack of consensus in empirical work on the topic.[11] It is important to mention that our results do not contradict previous findings in this literature. As with Boyd and De Nicoló (2005), our model also predicts scenarios under which the negative bank-competitor effect lowers risk-taking. Nevertheless, we present a generalized framework in which these predictions emerge under specific conditions but are not the only outcomes of the model. Despite the negative bank-competitor effect, we find that increased competition can also increase risk-taking when the relation between loan rates and risk-taking is negative. Although this result departs from predictions in Boyd and De Nicoló (2005), recent empirical studies on segmented markets in the National Banking Era find that risk-taking increases with competition (Carlson, Correia, and Luck, 2021).

Despite a large volume of empirical studies on the effects of bank competition on loan and deposit rates (see Degryse and Ongena, 2008, for a survey), there are no direct tests of the impact of market integration on these rates. Still, as Park and Pennacchi (2008) observe in the case of U.S. deregulation, "a greater presence of large multi-market banks tends to promote competition in retail loan markets

[11]Research on bank competition and risk-taking has often been framed as a debate between the 'competition-fragility' and the 'competition-stability' views (Berger, Klapper, and Turk-Ariss, 2009; Beck, De Jonghe, and Schepens, 2013; Akins, Li, Ng, and Rusticus, 2016; Corbae and Levine, 2018). The competition-fragility view argues that an increase in competition increases risk-taking incentives by reducing bank profit margins and lowering franchise values (Keeley, 1990; Hellmann, Murdock, and Stiglitz, 2000; Matutes and Vives, 2000; Repullo, 2004). In contrast, the competition-stability view argues that competition is critical to financial stability and increased concentration has been shown to increase risk-taking using different empirical measures (Jayaratne and Strahan, 1998; Barth, Lin, Lin, and Song, 2009; Houston, Lin, Lin, and Ma, 2010; Akins, Li, Ng, and Rusticus, 2016).

but also tends to harm competition in retail deposit markets".[12] In terms of our theoretical model, these conditions are necessary but not sufficient for increased risk-taking by banks.[13]

Our work builds on the traditional definition of competition because market integration presents us with seemingly *broader* meanings of "increased competition". The first is an increase in the number of banks, and the second is a convergence towards banks' price-taking behavior in deposit and loan markets. The notion that increased competition goes beyond a simple increase in the number of banks (firms) has its roots in Novshek (1980). Hermalin (1994, p. 526) summarizes the argument as follows:

> "[t]his requires defining the phrase 'more competitive'. As Novshek (1980) points out, this is not a straightforward task, because more competitive has at least two meanings. It suggests (1) more firms and (2) a closer approximation to perfect competition. This second meaning is not fully captured by the first—simply adding firms does not yield a closer approximation of perfect competition because the firms do not become price takers in the limit. Following Novshek, a definition that is consistent with this second meaning is one in which the slope of the inverse demand curve tends to zero with the increase in the number of firms in such a way that, in the limit, each firm produces zero, earns zero profits, and faces a flat demand curve."

2. Microfoundations

We begin with the model presented in Allen and Gale (2004) and Boyd and De Nicoló (2005) which forms the basic framework of our analysis. There are $J = \{1, 2, \ldots\}$ segmented banking markets.[14] Each market $j \in J$ consists of three distinct groups of risk-neutral agents—a continuum of depositors of measure $a_j > 0$, a continuum of borrowers or entrepreneurs of measure $b_j > 0$, and n_j (≥ 1) banks. Within each group, all agents are identical—we assume that borrowers, depositors, and banks are

[12] In related work, some accounts trace the increased risk-taking in residential real estate lending at banks prior to the Financial Crisis to the competitive pressures induced by bank deregulation—a strong loan supply effect. For example, Favara and Imbs (2015) showed that deregulation affected the supply of mortgage loans, and via its effect on credit, the price of housing.

[13] Needless to say, these results have strong implications for the role of banking deregulation and subsequent episodes of financial turmoil (Mian and Sufi, 2018). Mian, Sufi, and Verner (2020) trace the credit expansion in early deregulation states to a boost in household loan demand.

[14] We use the terms markets, regions, and economies interchangeably. Market can be segmented due to institutional, non-economic barriers such as geography, legislation, or regulation.

homogenous both within and across markets. However, markets vary in size so that no two markets necessarily have the same number of agents in each group. We refer to depositors and borrowers collectively as *customers* of banks.

2.1. Supply of deposits

Let $R_j \geq 1$ be the deposit rate offered by banks in market j. Then, depositor i in market j solves

$$\max_{d_{ij}} \; R_j d_{ij} - V(d_{ij}), \tag{1}$$

where $V(\cdot)$ is the strictly increasing and strictly convex foregone utility associated with making deposits d_{ij}. Dropping individual subscripts for ease of notation, we obtain from (1) that individual deposit supply is strictly increasing in the deposit rate because $V''(\cdot) > 0$:

$$d^* = V'^{-1}(R_j) \equiv d(R_j) \quad \text{for all } \; i.$$

Given identical depositors, aggregate deposit supply in market j will be $D_j = a_j d(R_j)$ and the inverse deposit supply is written as

$$R_j = V'(D_j/a_j) \equiv R(D_j/a_j) \quad \text{with } \; R'(D_j/a_j) > 0.$$

2.2. Demand for loans under moral hazard and limited liability

We generate loan demand in market j from a simple model of loan contracts under moral hazard and limited liability. We assume a contractual environment where entrepreneurs have access to a set of risky projects indexed by θ whose returns are random and perfectly correlated.[15] Entrepreneurs have

[15] We assume that entrepreneurs' returns are perfectly correlated as in Boyd and De Nicoló (2005). This is equivalent to the assumption of bank portfolios comprising perfectly correlated risks (Allen and Gale, 2004). The risk associated with each project can in general be decomposed into a systemic and idiosyncratic component. With a large number of projects, the idiosyncratic component can be perfectly diversified away. This assumption helps focus the analysis on the common component representing systemic risks. Martinez-Miera and Repullo (2010) present a model with imperfectly correlated project returns, whereas Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015) consider a model wherein banks' risk choices are both idiosyncratic and systemic.

no assets and must borrow to invest in the project. If k dollars are invested in a given project, it yields

$$\tilde{y}(\theta, k) = \begin{cases} y(\theta)f(k) & \text{with probability } p(\theta), \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

We assume that (i) the return, $y(\theta)$, is strictly increasing and strictly concave on $[0, \bar{\theta}]$, (ii) the probability of success, $p(\theta)$, is strictly decreasing and strictly concave on $[0, \bar{\theta}]$ with $p(0) = 1$ and $p(\bar{\theta}) = 0$, and (iii) the production function, $f(k)$, is strictly increasing and strictly concave on $[0, \bar{k}]$ with $f(0) \geq 0$. The variable θ represents the "riskiness" of the project—for a given k, the higher the θ, the higher is the return $y(\theta)$, but the lower is the probability of success, $p(\theta)$. Borrowers' choice of risk is not publicly verifiable, and therefore, not contractible.

There are two decision stages. In stage 1, the borrower chooses the level of investment, k, which generates individual demand for loans as a function of the loan rate $r_j \geq 1$.[16] In stage 2, taking the stage 1 borrowing decision as given, the borrower privately chooses project riskiness θ after which, project returns are realized, and payoffs are made. In granting the loan, banks cannot write loan contracts that are contingent on project riskiness θ as this is private information of the borrower. However, banks correctly anticipate the risk-shifting incentives of borrowers. This imposes a sequential rationality constraint on the equilibrium as in Brander and Spencer (1989). In stage 2, given k and r_j, each borrower in market j solves

$$\theta(k; r_j) = \underset{\theta}{\operatorname{argmax}}\ p(\theta) \left\{ y(\theta)f(k) - r_j k \right\}. \tag{3}$$

The first-order condition of this maximization problem can be written as

$$h(\theta) = \frac{r_j}{f(k)/k}, \tag{4}$$

where $h(\theta) \equiv y(\theta) + y'(\theta)(p(\theta)/p'(\theta))$ is strictly increasing in θ. In (4), borrower's risk-shifting choice, θ, depends on the average product of capital, $f(k)/k$, and the loan rate, r_j . We find that $\theta_r(k; r_j) > 0$ because $h'(\theta) > 0$, so that risk-shifting is increasing in the loan rate r_j for a given k. We also find that

[16] More formally, borrower i in market j chooses k_{ij}. Again, we drop individual subscripts for ease of notation.

$\theta_k(k; r_j) > 0$ because $f(k)/k$ decreases with k as $f(\cdot)$ is concave and $f(0) \geq 0$, so that risk-shifting is also increasing in investment k for a given r. Accordingly, when investment k increases, capital is less productive, and this incentivizes the borrower to take on more risk. In short, those who invest more are more liable to moral hazard (Banerjee, 2003).

Individual loan demand is determined at stage 1 where the borrower chooses investment k so that

$$k(r_j) = \underset{k}{\operatorname{argmax}}\ p(\theta(k; r_j)) \left\{ y(\theta(k; r_j)) f(k) - r_j k \right\}. \tag{5}$$

It follows that $k'(r_j) \leq 0$, so that each borrower's loan demand, $k(r_j)$, is declining in the market loan rate, r_j . Given identical borrowers, aggregate loan demand in market j will be $L_j(r_j) = b_j k(r_j)$, and the inverse loan demand is written as

$$r_j = k^{-1}(L_j/b_j) \equiv r(L_j/b_j) \quad \text{with} \ \ r'(L_j/b_j) < 0.$$

2.3. Loan rates and risk-taking

There are two ways in which loan rates affect risk-taking in this framework. In addition to the direct effect on entrepreneurs' optimal choice of risk, loan rates also affect risk-taking through its effect on borrower's loan demand, so that $\theta^*(r_j) \equiv \theta(k(r_j); r_j)$. Therefore,

$$\frac{d\theta^*}{dr_j} = \theta_r(k; r_j) + \theta_k(k; r_j) k'(r_j) \tag{6}$$

From (4), the direct effect, denoted by the first term on the right-hand side of (6), is positive. However, with $\theta_k(k; r_j) > 0$ from (4) and $k'(r_j) \leq 0$ from (5), the indirect effect of the loan rate on risk-taking—denoted by the second term on the right-hand side of (6)—is negative.

Because they point in opposite directions, the effect of loan rate changes on risk-taking depends on the relative magnitudes of the direct and indirect effects. It turns out that the sign of $d\theta^*/dr_j$ depends on the *output elasticity of investment*, $\varepsilon(k) \equiv kf'(k)/f(k)$.[17] An increase in the loan rate increases

[17] In particular, $kf'(k)/f(k)$ is the ratio of the marginal product to the average product of capital and is often defined as the *output elasticity of input* for any given factor of production (see Jehle and Reny, 2011, p. 133). It has also been termed as the *scale elasticity* or the *elasticity of production*.

	Functional form of $f(k)$	$\varepsilon'(k)$	Risk-taking ___ with loan rate
Example 1	$f(k) = k(1-k);\ 0 \leq k \leq 1/2$	negative	increases
Example 2	$f(k) = \sqrt{k_0 + k};\ \ k, k_0 > 0$	positive	decreases
Example 3	$f(k) = k^{\delta};\ k > 0, 0 < \delta < 1$	zero	does not change

Table 1: *The relationship between loan rate and the optimal risk-taking under different functional forms with $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$.*

(decreases) the choice of risk depending on whether the output elasticity of investment is decreasing (increasing) in the level of investment (i.e., according as $\varepsilon'(k) < (>) \ 0$).[18]

Table 1 illustrates how different functional forms of $f(k)$ yield differences in the relation between loan rates and risk-taking. Taken together, the relation between loan rates and risk-taking is summarized by the following proposition:

Proposition 1 *Borrowers' optimal risk choice in market j, $\theta^*(r_j) \equiv \theta(k(r_j); r_j)$, depends on the loan rate, r_j, and borrower investment, $k(r_j)$, which is a decreasing function of the loan rate. Loan rates affect risk-taking through two channels—a positive direct effect, $\theta_k(k; r_j)$, and a negative indirect effect, $\theta_k(k; r_j)k'(r_j)$, which works through changes in optimal investment, $k(r_j)$. Optimal risk-taking $\theta^*(r_j)$ is increasing (decreasing) in r_j according as the elasticity of investment, $\varepsilon(k)$, is decreasing (increasing) in k.*

The possibility of a negative relation between loan rates and risk-taking deviates from conventional models of moral hazard with limited liability which obtain an unambiguously positive relation between loan rates and risk-taking (Stiglitz and Weiss, 1981). We show that risk-taking can increase or decrease with loan rates in settings where the production technology exhibits decreasing marginal productivity of investment. However, if the technology exhibits constant returns to scale, say $f(k) = k$, we obtain from (4) that $\theta_k(k; r_j) = 0$. In terms of (6), the effect of loan rates on risk-taking is positive and comprised entirely of the direct effect $\theta_r(k; r_j)$ —the positive relation in Stiglitz and Weiss (1981).[19]

[18]The sign of $\varepsilon'(k)$ is related to the degree of concavity of $f(k)$. Formally,

$$\text{sign}[\varepsilon'(k)] = \text{sign}[1 - \varepsilon(k) - \sigma(k)]$$

where $\sigma(k) \equiv -kf''(k)/f'(k) > 0$ is a measure of the degree of concavity of $f(k)$. Note that $\varepsilon(k) < 1$ for all k as $f(k)$ is strictly concave. Thus, a low degree of concavity (low $\sigma(k)$) means that marginal product decreases at a slower rate than average product so that their ratio, $\varepsilon(k)$, is increasing in k. Likewise, a high degree of concavity (high $\sigma(k)$) means that marginal product decreases faster than the average product, or equivalently, $\varepsilon'(k) < 0$. Accordingly, risk-shifting and loan rate are positively associated whenever $f(k)$ is sufficiently concave.

[19]In Section 4, we present a linear model with $f(k) = k$ wherein the effect of loan rates is positive and comprised entirely of the direct effect, and hence, optimal risk-shifting is monotonically increasing in the loan rate.

Before concluding this section, it is important to point out that, in our framework with perfectly correlated risks, entrepreneurial risk-taking is synonymous with risk-taking by banks. In this context, banks' optimal asset allocation is determined by an optimal contracting problem as discussed in Boyd and De Nicoló (2005) instead of a portfolio choice problem as modeled in Allen and Gale (2004).

3. Market Equilibrium

We begin by characterizing equilibrium loan rates and risk-shifting for each segmented market. We refer to this equilibrium as the *segmented market equilibrium* (SME) and show how increasing competition affects loan rates and risk-taking in the SME. Next, we solve for the equilibrium when $m(\geq 2)$ segmented markets are integrated into a single banking market. We refer to this equilibrium as the *integrated market equilibrium* (IME). We analyze the effects of increased competition on loan rates and risk-shifting in the IME. Finally, we compare the effect of increased competition on loan rates and risk-shifting between the SME and the IME.

In solving the model, we make several simplifying assumptions. First, we assume that banks compete in a Cournot fashion. Second, we assume that banks are funded entirely by customer deposits—banks have no equity and there is no interbank market for deposits. We relax this assumption when we introduce interbank markets in Section 5.1. Third, there is no exit or entry of banks. Again, we relax this assumption in Section 5.2 where we allow for bank mergers. Fourth, all bank deposits are insured for which each bank pays a flat premium that is normalized to zero. Lastly, all customers (borrowers and depositors) can switch between banks at no costs. Given our assumptions that agents are homogenous within each group, this is a fairly innocuous (zero transaction cost) assumption.[20]

[20] As banks obtain information on borrower quality during the course of a lending relationship, switching between banks is beset with problems of information asymmetry. A large literature examines how competition in credit markets affects the screening problem banks face in granting loans (Broecker, 1990; Sharpe, 1990). It is important to mention that such models of switching costs are generally bank-specific (i.e., they apply to customers switching from one bank to another, even in the absence of market integration) and not necessarily market-specific (i.e., they do not generally apply to customers switching from local to non-local banks when markets integrate). We abstract from these considerations both for the SME and the IME.

3.1. The segmented market equilibrium

3.1.1. *Equilibrium*

Bank i in market j chooses the volume of deposits, D_{ij}, to maximize expected profits, taking into account choices made by its competitors and the entrepreneurs' choice of risk. In the SME, each bank in market j solves

$$\max_{D_{ij}} P(D_j/b_j)[r(D_j/b_j) - R(D_j/a_j)]D_{ij}, \tag{7}$$

where $P(L_j/b_j) \equiv p(\theta^*(r(L_j/b_j)))$ and $R(0) \geq 0$, $R''(D_j/a_j) \geq 0$, $r(0) \geq R(0)$ and $r''(L_j/b_j) \leq 0$. With no equity and no interbank market, the balance sheet identity of bank i in market j implies $L_{ij} = D_{ij}$. Consequently, aggregate deposit supply equals aggregate loan demand in market j so that $D_j = \sum_{i=1}^{n_j} D_{ij} = \sum_{i=1}^{n_j} L_{ij} = L_j$.

Given that all banks are identical, and they face the same aggregate deposit supply and loan demand schedules, in the segmented market, there are no asymmetric equilibria (see the proof of Lemma 2 in the Appendix). In a symmetric Cournot equilibrium, we have $D_{ij} = D_j/n_j$ for all i. The following lemma characterizes the (symmetric) SME in market j.

Lemma 1 *An SME is characterized by loan rate, $r_j = r(D_j/b_j)$, risk-shifting, $\theta_j = \theta^*(r(D_j/b_j))$, and intermediation margin*

$$r(D_j/b_j) - R(D_j/a_j) = \frac{\left[b_j R'(D_j/a_j) - a_j r'(D_j/b_j)\right] P(D_j/b_j)D_j}{a_j\left[n_j b_j P(D_j/b_j) + P'(D_j/b_j)D_j\right]}, \tag{8}$$

where D_j denotes the aggregate deposits in market j.

3.1.2. *Effect of increased competition in the SME*

Increased competition in the SME is defined as an increase in the number of banks. For example, competition increases in any of the segmented markets in if local banking authorities lower fixed set-up costs as in Mankiw and Whinston (1986). Such costs include charter fees or capital requirements at the founding of the bank as was required during the National Banking Era (Carlson et al., 2021). From Lemma 1, comparative statics reveal that the SME loan rate, r_j, decreases as the number of banks, n_j, increases. It follows from Proposition 1 that entrepreneurial risk-shifting in the SME may increase or decrease with n_j.

Proposition 2 *In the SME of each market j, loan rate r_j is strictly decreasing in n. As a result, risk-shifting θ_j decreases (increases) according as the output elasticity of investment, $\varepsilon(k)$ is increasing (decreasing) in k.*

While increased competition in the SME unambiguously decreases loan rates, the effect on risk-taking is not unambiguous. Increased competition from an increase in the number of banks decreases risk-taking if and only if risk-taking increases with loan rates. Proposition 2 asserts that this risk-incentive mechanism, first shown in Boyd and De Nicoló (2005), is obtained in the SME when the production technology exhibits decreasing elasticity of investment. However, in situations where the production technology exhibits increasing elasticity of investment, we find that increased competition can increase risk-taking by borrowers. Carlson et al. (2021) find that banks operating in markets with lower entry barriers in the National Banking Era increased riskiness in lending and were more likely to default. This empirical finding lends support to our result that increased competition in the SME can also lead to more risk-taking.

3.2. The integrated market equilibrium

3.2.1. The deposit supply and loan demand in the integrated market

Now, suppose that a subset of segmented markets, $J_m \subset J$, are integrated to form a single banking market, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Given the assumption that there is no entry or exit of banks, the number of banks in the integrated market is equal to the total number of banks across the m markets combined, so that

$$n(m) = \sum_{j=1}^{m} n_j.$$

Although the total number of banks remains unaltered even after market integration, each bank faces new rivals in the integrated market. In a similar vein, the measure of depositors and borrowers in the integrated market equals the aggregate of the measures of depositors and borrowers in each of the m

individual markets, respectively. Therefore

$$a(m) \equiv \sum_{j=1}^{m} a_j \quad \text{and} \quad b(m) \equiv \sum_{j=1}^{m} b_j. \tag{9}$$

Under market integration, depositors and borrowers solve the same maximization problems as given in (1) and (5), respectively. This follows directly from our assumption that depositors and borrowers are homogenous across the m integrating markets. Customer homogeneity implies that, for a given loan (deposit) rate, individual loan demand (deposit supply) in the segmented and the integrated markets are the same. Accordingly, given deposit rate, R, and loan rate, r, individual deposit supply and loan (investment) demand in the integrated market are $d_i^* = d(R)$ and $k_i^* = k(r)$ for all i respectively.

From (9), aggregate deposit supply and loan demand in the integrated market are $D = a(m)d(R)$ and $L = b(m)k(r)$, respectively. It follows that the inverse deposit supply and loan demand functions in the integrated market are

$$R = R(D/a(m)) \quad \text{where} \quad R'(D/a(m)) > 0 \tag{10}$$

$$\text{and} \quad r = r(L/b(m)) \quad \text{where} \quad r'(L/b(m)) < 0 \tag{11}$$

respectively. The inverse deposit supply of the integrated market, $R(D/a(m))$, is the horizontal sum of the m individual inverse deposit supply schedules, and therefore, more elastic than $R_j(D_j)$ for any $j = 1, \ldots, m$. Likewise, the inverse loan demand function, $r(L/b(m))$, which is the horizontal sum of the m individual inverse loan demand schedules, is more elastic than $r_j(L_j)$ for any $j = 1, \ldots, m$. Being the horizontal sum of convex deposit supply (concave loan demand) schedules, aggregate deposit supply (loan demand) is also convex (concave), that is, $R''(D/a(m)) \geq 0$ ($r''(L/b(m)) \leq 0$). Figure 1 shows the inverse loan demand function for $m = 2$ and $b_1 < b_2$.

3.2.2. *The integrated market equilibrium*

In the integrated market, all $n(m)$ banks face the same inverse deposit supply (10) and inverse loan demand (11). Given that banks are exclusively deposit financed, $L_i = D_i$ holds for all i. This yields the

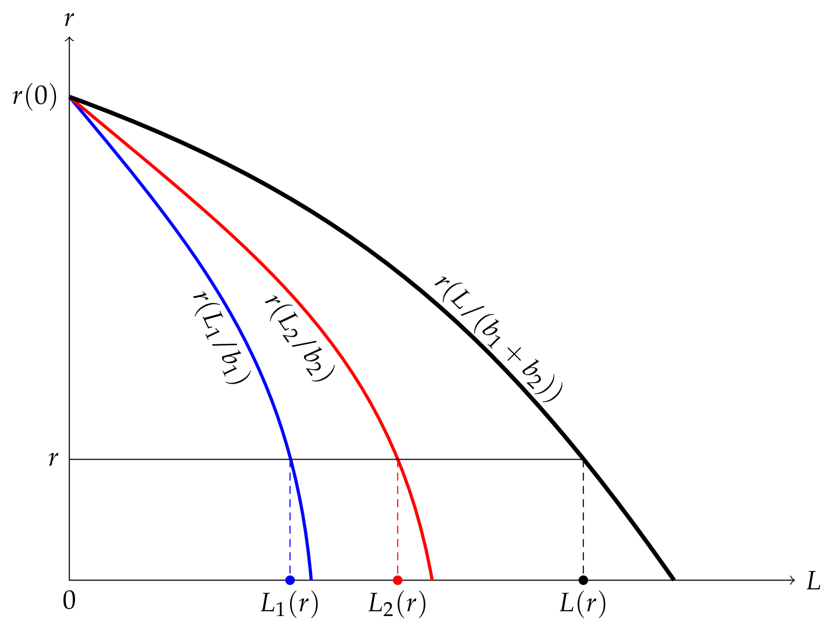

Figure 1: *Loan demand when two markets integrate. Loan demand in the segmented market i is $L_j(r) = b_j k(r)$, $j = 1, 2$. If $b_1 < b_2$, $r_1(L_1)$ lies below $r_2(L_2)$. Loan demand in the integrated market, $L(r) = L_1(r) + L_2(r) = (b_1 + b_2)k(r)$, is more elastic than that in any of the segmented markets.*

market clearing condition, $L = D$. In any IME, bank i solves

$$\max_{D_i} P(D/b(m))[r(D/b(m)) - R(D/a(m))]D_i. \tag{12}$$

In the integrated market, J_m, all banks are identical, and they face the same aggregate inverse deposit supply and loan demand functions. Therefore, like the SME, all equilibria of the integrated market are symmetric, that is, $D_i = D/n(m)$ for all i, which is described in the following proposition.

Lemma 2 *The IME is characterized by loan rate, $r = r(D/b(m))$, optimal risk-shifting $\theta = \theta^*(r(D/b(m)))$, and the intermediation margin*

$$r(D/b(m)) - R(D/a(m)) = \frac{[b(m)R'(D/a(m)) - a(m)r'(D/b(m))]\, P(D/b(m))D}{a(m)\,[n(m)b(m)P(D/b(m)) + P'(D/b(m))D]}, \tag{13}$$

where D denotes the aggregate deposits the integrated market and $a(m)$ and $b(m)$ are given by (9).

Evidently, the expressions for equilibrium loan rate and risk-taking for the IME in Lemma 2 are isomorphic to those obtained for the SME in Lemma 1. In spite of this similarity, we show below how the integration of heterogenous markets can alter the relation between competition and loan rates, and consequently, competition and risk-taking in the SME.

3.2.3. *Effect of increased competition on loan rates*

Increased competition in the IME is defined as an increase in the number of integrated markets. Given that integrating markets are heterogenous in our setting, we take an increase in m to imply that additional markets are integrated. Formally, if the set of integrated markets expands from J_m to $J_{m'}$, where $|J_m| = m$, $|J_{m'}| = m'$, then $J_m \subseteq J_{m'} \subset J$. In other words, we compare loan rates and risk-shifting between the IME for the (smaller) set of J_m markets and the IME for the expanded set of $J_{m'}$ markets. Across the two sets of markets, agents (banks and customers) are identical but deposit supply and loan demand schedules are more elastic in the expanded market.[21]

Market integration intensifies bank competition in two ways. First, an increase in m increases the number of competitor banks so that each bank faces competition from more rivals in $J_{m'}$ relative to J_m . Second, integration of additional markets increases competition by expanding the size of deposit and loan markets. The deposit supply schedules for J_m and $J_{m'}$ are given by $D(R, m) = a(m)d(R)$ and $D(R, m') = a(m')d(R)$, respectively. With $a(m) < a(m')$, the inverse supply function for deposits in the larger market, $J_{m'}$, is more elastic than that in J_m. In this way, market integration makes the deposit market, and by the same logic, the loan market, more competitive. When additional markets are integrated, the resulting market expansion increases competition because the inverse deposit supply and inverse loan demand functions become more elastic.[22]

The bank-competitor and the bank-customer effects. We explore the comparative static properties of the IME analyzed in Proposition 2 to disentangle the effects of $a(m)$, $b(m)$, and $n(m)$ on loan rates.

[21]Going forward, we treat m as a continuous variable. Clearly, $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing functions of m.

[22]An increase in the measure of consumers also occurs with the integration of national markets in models of *intra-industry trade* (Krugman, 1979). Although each consumer can potentially transact with more firms, products are horizontally differentiated in these models. Given the expanded choice under integration, consumers spend less on each variety prompting some firms to exit the market. Such exits do not occur in the IME in our setting because we consider homogeneous deposit and loan services.

Formally,

$$\frac{dr}{dm} = \underbrace{\frac{\partial r}{\partial n} \cdot n'(m)}_{\text{bank-competitor effect}} + \underbrace{\frac{\partial r}{\partial a} \cdot a'(m) + \frac{\partial r}{\partial b} \cdot b'(m)}_{\text{bank-customer effect}}. \tag{14}$$

Increasing m increases the number of competitor banks in the integrated market $n(m)$ and so, the first term on the right-hand-side of (14) denotes the *bank-competitor effect*. The second and third terms together constitute the *bank-customer effect*—the effect of an increase in the measure of customers (depositors and borrowers) under market integration. Because $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing in m, the sign of each term on the right-hand side of (14) is determined by the sign of the partial derivative in each term. We summarize our findings in terms of the following proposition.

Proposition 3 *In the IME, the effect on an increase in m on the loan rate r is*

$$\frac{dr}{dm} = Z_n \cdot \hat{n}(m) + Z_c \cdot \left\{ \hat{b}(m) - \hat{a}(m) \right\}, \tag{15}$$

where $Z_n < 0$, $Z_c > 0$, *and* $\hat{n}(m) \equiv n'(m)/n(m)$, $\hat{a}(m) \equiv a'(m)/a(m)$ *and* $\hat{b}(m) \equiv b'(m)/b(m)$ *are the* expansion rates *of the banks, the depositors and the borrowers, respectively. A negative bank-customer effect, that is,* $\hat{b}(m) < \hat{a}(m)$ *is a* sufficient condition *for a negative association between competition and loan rates. A positive bank-customer effect, that is,* $\hat{b}(m) > \hat{a}(m)$, *on the other hand, is a* necessary condition *for a positive association between competition and loan rates.*

Equation (15) is isomorphic to (14). Just as the first term on the right-hand-side of (14), that of (15) captures the bank-competitor effect. Moreover, as in the case of the SME, the bank-competitor effect is negative (i.e., $Z_n < 0$) in the IME as well. Holding the measures of depositors and entrepreneurs fixed, an increase in the number of competitor banks due to more integration reduces the market power of each bank in both the deposit and loan markets, which in turn lowers the loan rate.

In addition to increasing the number of banks, increased competition under market integration also increases the measures of depositors and borrowers. All else equal, increases in $a(m)$ tends to lower the deposit rate as $R'(D/a) > 0$. This tends to reduce loan rates by lowering the cost of loanable funds for banks and allowing them to price loans more competitively. On the other hand, an increase in $b(m)$ tends to increase loan rates, other things being equal, as $r'(L/b) < 0$. In short, the bank-customer effect comprises two terms: a negative effect associated with deposit supply and a positive effect associated with loan demand. Because they point in opposite directions, the sign of the bank-customer effect depends on the relative magnitudes of the expansions of deposit supply and loan demand. In (15), $Z_c > 0$, and therefore, the bank-customer effect is positive, zero, or negative according as $\hat{b}(m) \gtreqless \hat{a}(m)$. When $\hat{b}(m) > \hat{a}(m)$, an increase in m expands the loan market more than the deposit market and this tends to increase loan rates. Conversely, when $\hat{b}(m) < \hat{a}(m)$, increasing m expands the deposit market more than the loan market, which tends to reduce loan rates.

The overall effect of market integration on the equilibrium loan rate comprises the bank-competitor and bank-customer effects. A negative bank-customer effect that lowers the equilibrium loan rate reinforces the negative bank-competitor effect. Therefore, a negative bank-customer effect is a *sufficient condition* for a negative association between competition and loan rates. In contrast, a positive bank-customer effect tends to increase the equilibrium loan rate. Therefore, a positive bank-customer effect is a *necessary condition* for a positive association between competition and loan rates. Moreover, if this positive effect is sufficiently strong, it can outweigh the negative bank-competitor effect. Taken together, the overall effect of market integration on loan rates is not unambiguous.

The contrast between the two effects of competition under market integration is noteworthy. While we can make unambiguous predictions about the bank-competitor effect, we are unable to do so for the bank-customer effect. The intuition is straightforward. As banks operate in both deposit and loan markets, an increase in $n(m)$ affects both the deposit and loan markets symmetrically. Therefore, in scenarios where the effect of increased competition is comprised only of the bank-competitor effect, loan rates decrease as m increases. However, integrating dissimilar markets implies that increases in $a(m)$ and $b(m)$ affect deposit and loan markets asymmetrically. Therefore, in scenarios where the effect of increased competition is comprised only of the bank-customer effect, the effect on loan rates is indeterminate and depends on the relative expansion rates of each market.

3.2.4. Competition and risk-taking in the IME

The effect of increased competition on risk-taking in the IME comprises two effects: first, the effect of increased competition on interest rates on loans as shown above and second, the effect of interest rates on risk-taking incentives of borrowers as analyzed in Section 2.3. Taken together, the effect on increased competition on risk-taking in any IME is non-trivial.

First, consider the case where risk-taking increases with the loan rate, that is, $\varepsilon'(k) < 0$. As described in Section 2.3, this is a standard result in most models of borrower moral hazard and limited liability (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). With the bank-competitor effect always negative, we have shown above that risk-taking decreases with competition in the SME. However, increased competition under market integration can reverse the relation between competition and risk-taking. A sufficiently strong and positive bank-customer effect can lead to increased risk-taking in the IME, through its effect of increasing loan rates.

Next, consider the case where risk-taking decreases with the loan rate. In Section 2.3, this result is obtained when the production technology exhibits increasing elasticity of investment, that is, $\varepsilon'(k) > 0$. In this case, risk-taking increases with competition in the SME because of the negative bank-competitor effect—an increase in the number of banks lowers loan rates, and that leads to increased risk-taking. This relationship can also change with a sufficiently strong and positive bank-customer effect in the IME. With a sufficiently large bank-customer effect, increased competition in the IME increases loan rates and thereby reduces risk-taking. Overall, these results can be summarized as follows.

Proposition 4 *Any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME.*

Proposition 4 summarizes the impact of the bank-customer effect on risk-taking. As the generalized model here shows, the bank-customer effect exists even if bank customers (borrowers and depositors) are homogenous across all markets. As long as markets vary in terms of size (measures of the customers in each market), the bank-customer effect has the potential to alter the relationship between competition and risk-taking as more markets integrate.

3.2.5. Free entry and market replication

We have defined increased competition in the SME somewhat differently from that in the IME. This was done in the interest of clarity of exposition. It is important to mention that the bank-competitor effect is obtained from *any* increase in the number of competitor banks whether it be through an increase in the number of *de novo* banks or the entry of banks operating in other markets. Likewise, the bank-customer effect is obtained whenever there is an unequal expansion of depositors and borrowers either because individual markets have expanded in an unbalanced fashion or because previously segmented and heterogeneous markets have been integrated.

The definition of the term "increased competition" makes a material difference in the context of Cournot models. Novshek (1980) argues that one must exercise caution before concluding that the Cournot equilibria approach the perfectly competitive equilibrium as the number of firms (banks, in our context) grow without bound. Simply adding more firms (e.g. through new charters given to *de novo* banks as mentioned in Section 3.1.2) lowers profit margins but does not yield price-taking behavior. In order for firms to become price-takers, one would have to consider a sequence of markets in which firms become arbitrarily small relative to the market. One way in which this is achieved is when the increase in the number of firms is accompanied by a proportionate increase in the number of consumers—also known as *market replication*. Allen and Gale (2004, p. 463) describe the effect of replicating the market to analyze the effect of increased deposit market competition on bank risk-taking.

Understanding the implications of market replication in our setting underscores the significance of market heterogeneity in the IME. In Proposition 1, if deposit supply and loan demand expand at the same rate (i.e., $\hat{a}(m) = \hat{b}(m)$ for all m), the bank-customer effect from increased competition is null. A special case here is that of homogenous markets, where $n_j = \bar{n}$, $a_j = \bar{a}$ and $b_j = \bar{b}$, for all $j \in J$, and the integration of homogenous markets is equivalent to replicating the same deposit and loan markets many times over. In all such cases of market replication, or equivalently, the integration of homogenous markets, the bank-customer effect is null and the overall effect of an increase in m is comprised entirely of the bank-competitor effect and is unambiguously negative. Therefore, heterogeneity of the segmented markets is a necessary condition for a non-null bank-customer effect in the IME.

4. A Linear Model

In this section, we show that the results described in Sections 2 and 3 can be obtained using a simple linear model. In the next section, we use the linear model described here to demonstrate the robustness of our results to alternative assumptions.

Obtaining a linear loan demand function using the borrowers' maximization problem in (5) is somewhat involved. It is a challenge to pin down the exact functional forms of $p(\theta)$, $y(\theta)$ and $f(k)$ in (3) that would yield a linear loan demand function. Nevertheless, we prove the following existence result for the model presented in Section 2.[23]

Lemma 3 *Let $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$. Then, there exists a unique $f(k)$ on $(0, \bar{k}) \subset (0, \lambda)$ such that $k(r)$ is a linear function of the form $k(r) = \lambda - r$.*

The above result motivates the use of linear deposit supply and loan demand functions for the general model. In this section, we present an alternative formulation (microfoundation) that yields a model with linear functions. This alternative formulation differs from the generalized model presented in Sections 2 and 3 in that bank customers are heterogenous in terms of their reservation utilities, as in Martinez-Miera and Repullo (2010). This exercise demonstrates that the model results are also applicable to case where borrowers are heterogenous. We describe the model below and demonstrate how the results of the general model are obtained in this tractable parsimonious setting.

4.1. Heterogeneous customers and linear deposit supply and loan demand functions

There are m distinct segmented markets. Each risk-neutral depositor has an endowment, normalized to \$1, to deposit in a bank. Depositors are heterogeneous in their reservation utility v. Let $G_j(v)$ denote the measure of depositors in market j that have reservation utility less than or equal to v, with $G_j'(v) > 0$ for all v. We assume that the reservation utilities of depositors are distributed uniformly over the interval $[0, (R-1)/a_j R]$. A depositor would deposit \$1 only if $R - 1 \geq v$, where R is the deposit rate. Therefore, the deposit supply in market j is given by $D_j(R) = G_j(R-1) = a_j R$. Markets are heterogeneous in their measure of depositors, so that $a_j \neq a_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the supply of deposits is $D(R, m) = a(m)R$ where $a(m) \equiv \sum_{j=1}^{m} a_j$ and the inverse supply function is

$$R(D, m) = \frac{D}{a(m)}. \tag{16}$$

[23]The result is obtained from purely technical conditions related to the existence and uniqueness of a solution to an ordinary differential equation (e.g. Picard-Lindelöf theorem).

In loan market j, each entrepreneur requires an investment $k \in [0, 1]$ which yields θk in case the project succeeds with probability $p(\theta) = 1 - \theta/\lambda$ with $\lambda > 1$, and yields zero when it fails with probability $1 - p(\theta)$. Given loan rate r in market j, each entrepreneur in stage 2 solves

$$\theta(k; r) = \underset{\theta}{\operatorname{argmax}} \ (1 - \theta/\lambda)\{\theta k - rk\}. \tag{17}$$

The optimal risk-shifting is given by $\theta = (\lambda + r)/2$. Unlike the expression for θ obtained in (4), optimal risk does not depend on the level of investment because the production function is linear (i.e., $f(k) = k$). So, we denote the optimal risk-taking by $\theta(r)$ instead of $\theta(k; r)$. In stage 1, each entrepreneur solves

$$k(r) = \underset{k \in [0,1]}{\operatorname{argmax}} \ (1 - \theta(r)/\lambda)\{\theta(r)k - rk\}.$$

The objective function is increasing in k and the first-order condition of the above maximization problem yields $k(r) = 1$. In this simplified setting, the relation between loan rates and risk-taking is always positive, that is, $d\theta^*/dr > 0$.[24]

Let $u(r)$ denote the value function of the entrepreneurs' maximization problem (17). Also, let $H_j(u)$ denote the measure of entrepreneurs in market j that have reservation utility less than or equal to u, with $H_j'(u) > 0$ for all u. An entrepreneur would participate in the loan market only if $u(r) \geq u$. Therefore, loan demand in market j is given by $L_j(r) = H_j(u(r))$. We assume $H_j(u) = 2b_j\sqrt{\lambda u}$ defined on the support $[0, 1/4\lambda b_j^2]$, so that $L_j(r) = H_j(u(r)) = b_j(\lambda - r)$.

Markets are heterogeneous in their measure of borrowers, so that $b_j \neq b_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the demand for loans is $L(r, m) = b(m)(\lambda - r)$ where $b(m) \equiv \sum_{j=1}^{m} b_j$ and the inverse demand function is

$$r(L, m) = \lambda - \frac{L}{b(m)}. \tag{18}$$

Optimal risk-shifting is $\theta(r(L/b(m))) = \lambda - L/2b(m)$ so that the probability of success is given by $P(L/b(m)) \equiv 1 - \theta(L/b(m))/\lambda = L/2\lambda b(m)$.

[24] In terms of (6), $d\theta^*/dr = \theta_r$ because $\theta_k = 0$.

4.2. Effect of competition on risk-taking in the linear model

The solution to this linear model is made simpler by the fact that the association between loan rates and risk-shifting is always positive. We obtain this result in (17) because equilibrium risk-taking is a function of the loan rate and not the level of investment. In what follows, we describe the equilibrium in terms of the effect of increased competition under market integration on equilibrium risk-taking. Although omitted here for the sake of brevity, the effect of increased competition on the loan rate is analogous. In fact, the effect of competition on risk-taking operates strictly through the loan rate in our setting.

We assume that $a(m) = m^{\alpha}$ and $b(m) = m^{\beta}$ with $\alpha, \beta \in [0, 1]$. When $a_j = b_j = 1/m$ for all j, it follows that $\alpha = \beta = 0$. On the other hand, when $a_j = b_j = 1$ for all j, $\alpha = \beta = 1$. For any other distribution of $\{a_j\}_{j=1}^{m}$ and $\{b_j\}_{j=1}^{m}$, we have $\alpha, \beta \in (0, 1)$. Note that $\hat{a}(m) = \alpha/m$ and $\hat{b}(m) = \beta/m$, and therefore, the bank-customer effect is positive, zero, or negative according as $\alpha \gtreqless \beta$. We also assume that $n_j = j$, so that

$$n(m) = \sum_{j=1}^{m} j = \frac{m(m+1)}{2}.$$

Moreover, using (13) and $\theta(L/b(m)) = \lambda - L/2b(m)$, risk-shifting in the IME is obtained as

$$\theta(m; \alpha, \beta) = \lambda \left\{1 - \frac{1}{2} \cdot \frac{a(m)[n(m)+1]}{[a(m)+b(m)][n(m)+2]}\right\} = \lambda \left\{1 - \frac{1}{2} \cdot \frac{m(m+1)+2}{m(m+1)+4} \cdot \frac{1}{1+m^{\beta-\alpha}}\right\}. \quad (19)$$

The following result describes the IME in the linear model.

Proposition 5 *In the IME with linear deposit supply and loan demand schedules, where risk-shifting is given by (19), we obtain the following relation between competition and risk-shifting.*

(i) *If $\beta \leq \alpha$, the bank-customer effect is negative and risk-shifting is monotonically decreasing in m.*

(iii) *If $\beta \geq \alpha + x$, the bank-customer effect is positive and sufficiently strong so that risk-shifting is monotonically increasing in m.*

(ii) *If $\alpha < \beta < \alpha + x$, $x \approx 0.435$, the bank-customer effect is positive but not sufficiently strong, and risk-shifting is U-shaped with respect to m. In particular, there is a unique $\widehat{m} > 2$ such that risk-shifting is monotonically decreasing (increasing) in m for all $m < (>)\widehat{m}$.*

Figure 2 describes the effect of increased competition on risk-shifting for the IME in the α-β space.

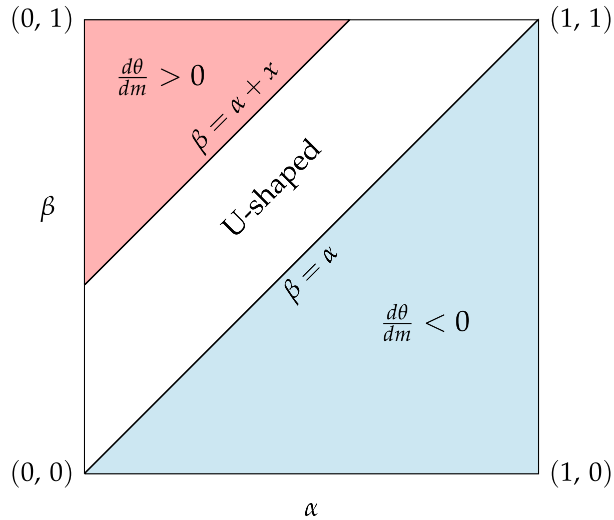

Figure 2: *Equilibrium association between risk-shifting and the number of integrated markets when* $n(m) = \frac{1}{2}m(m+1)$, $a(m) = m^{\alpha}$ *and* $b(m) = m^{\beta}$.

The simple linear model allows us to describe the differences in the relationship between competition, as measured by m, and risk-taking in terms of the parameters α and β. If $\beta < \alpha$, the negative bank-customer effect is a sufficient condition for the negative association between competition and risk-shifting. On the other hand, if $\beta > \alpha$, the bank-customer effect is strictly positive and leans against the negative bank-competitor effect. We show that when the difference $\beta - \alpha$ is sufficiently large, that is, $\beta - \alpha \geq x$, the positive bank-customer effect dominates the negative bank-competitor effect for all values of m and the association between risk-shifting and competition is positive. However, if $0 < \beta - \alpha < x$, the positive bank-customer effect is not strong enough to outweigh the negative bank-competitor effect for all m. If fewer markets are integrated, (i.e., m is small), the bank-competitor effect dominates the bank-customer effect, and consequently, risk-shifting is decreasing in m. In contrast, when a greater number of markets are integrated, (i.e., m is sufficiently large), the bank-customer effect dominates the bank-competitor effect, and risk-shifting is increasing in m. Put differently, equilibrium risk-shifting first decreases and then increases with the number of integrated markets generating a non-monotonic (U-shaped) relationship between risk-shifting and market integration.

If the number of banks is invariant to market integration, that is, $n(m) = \bar{n}$, then the effect of increased competition on risk-taking in the IME is composed entirely of the bank-customer effect.[25] In this case, Proposition 5 implies that entrepreneurial risk-taking is decreasing (increasing) under market integration according as $\alpha > (<) \beta$. In other words, the region in Figure 2 over which risk-shifting is U-shaped with respect to m would disappear.

5. Extensions

In this section, we show that the results of the linear model are robust to different modeling choices. In doing so, we point to other factors that affect loan rates and risk-taking when markets integrate. For the IME described above, we assumed that (1) banks do not transact in the interbank market and (2) there is no exit and entry of banks. In this section, we relax each of these assumptions in turn to demonstrate that the baseline results are robust to different settings.[26]

5.1. Interbank lending

We extend the linear model described Section 4 to include an interbank market. In particular, we assume that banks trade funds at the interbank rate, ρ, which is a policy variable chosen by the central bank (as in Freixas and Rochet, 2008, p. 79). The key assumption here is that all banks take ρ as given irrespective of whether markets are segmented or integrated. Under this assumption, we show below that banks' maximization problems for the SME and the IME are isomorphic.

The purpose of the present analysis is to show that the bank-customer effect of market integration is significant in determining the loan rate and risk-taking even when banks have alternative funding sources such as interbank lending. Moreover, there are additional implications of interbank transactions for the bank-competitor effect of market integration as we describe below.

[25]This stylized setting has an interesting parallel in U.S. banking history. When bank branching was restricted by law, the formation of multi-bank holding companies provided banks with an alternative means to expand beyond their local banking market. With the removal of branching barriers, these subsidiary banks were often consolidated as branches under a single charter. In the extreme case where integrating markets are populated exclusively by the same holding companies, the post-integration consolidation of subsidiaries would leave the number of rival banks invariant to market integration.

[26]Fecht, Inderst, and Pfeil (2017) consider interbank lending and bank mergers alternatives in the process of market integration. We abstract from situations where interbank lending and bank mergers are substitutes and model them separately.

5.1.1. *The SME and interbank market*

The model setup is similar to that outlined in Section 4.1. Although indiviudal banks operate in only one of m segmented markets, they face the same interbank rate, ρ, chosen by the central bank, where $\rho \in [0, \bar{\rho}]$ where $\bar{\rho} \leq \lambda$. Let the probability of success be given by $p(\theta) = 1 - (\theta/\lambda)$. Bank i in market j solves

$$\max_{\{L_{ij}, D_{ij}\}} P(L_j/b_j)[(r(L_j/b_j) - \rho)L_{ij} + (\rho - R(D_j/a_j)D_{ij}] \tag{20}$$

where $L_j = \sum_{i=1}^{n_j} L_{ij}$ and $D_j = \sum_{i=1}^{n_j} D_{ij}$. We show below that this problem is isomorphic to that for the IME. It follows that their solutions are similar as described below.

5.1.2. *The IME and interbank market*

As in Section 3.2, we consider the integration of a subset of segmented markets, J_m, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Banks are allowed to transact with each other at the interbank rate ρ set by the central bank. With (inverse) loan demand and deposit supply given by (16) and (18), respectively, bank i solves

$$\max_{\{L_i, D_i\}} P(L/b(m))[(r(L/b(m)) - \rho)L_i + (\rho - R(D/a(m))D_i], \tag{21}$$

where $L = \sum_{i=1}^{n(m)} L_i$ denotes aggregate loans and $D = \sum_{i=1}^{n(m)} D_i$ denotes aggregate deposits in the integrated market, J_m. However, with an interbank market $L_i = D_i$, as argued in (12), no longer holds.

Because we follow the setup in Section 4, risk-taking is always increasing in the loan rate. Therefore, in all comparative static analysis that follows, any effect on the loan rate, r, translates directly to risk-shifting, θ. The following proposition summarizes the effect of increased competition under the IME on loan rates and risk-shifting in the presence of interbank lending.

Proposition 6 *The following hold for the IME with an interbank market:*

(a) *If $n(m)$ is invariant to m (i.e., the bank-competitor effect is set to zero), the bank-customer effect is positive (negative) according as $\hat{b}(m) > (<) \hat{a}(m)$.*

(b) *If $a(m)$ and $b(m)$ are invariant to m (i.e., the bank-customer effect is set to zero), there is a unique threshold*

$\rho^* \in [0, \bar{\rho}]$ such that the bank-competitor effect is negative (positive) according as $\rho < (>) \rho^*$. Therefore, r and θ are decreasing (increasing) in m according as the interbank rate ρ is low (high).

The bank-competitor and the bank-customer effects comprise the aggregate effect of market integration on the equilibrium loan rate and risk-shifting. It follows that the overall effect of market integration on the loan rate, and consequently, risk-taking is indeterminate.

Proposition 6 extends the results in Proposition 3 to the IME with an interbank market. Proposition 6(a) shows that a non-zero bank-customer effect exists even with interbank lending. Moreover, loan rate and risk-taking can increase under market integration if this bank-customer effect is positive.

Proposition 6(b) shows that the bank-competitor effect is no longer unambiguously negative in the presence of interbank lending. To demonstrate this, we simplify by setting the bank-customer effect to zero so that any changes in m yield changes only in the number of banks, n. From the first-order conditions of the maximization problem in (21), we obtain the following relation between loan and deposit rates in the symmetric equilibrium (see Appendix for details):

$$R = \frac{\rho n}{n+1} \tag{22}$$

$$b(n+2)(\lambda - r)^2 - b(\lambda - \rho)(n+1)(\lambda - r) = aR(\rho - R). \tag{23}$$

While (22) helps pin down the deposit rate as a function of the the number of banks, n, and the interbank rate, ρ, (23) reveals that the equilibrium loan rate, r, depends additionally on the deposit rate, R.

Because the probability of default is endogenous in our framework, the equilibrium loan rate depends on deposit rate as in Dermine (1986).[27] If we denote the equilibrium loan rate function as $r(n, R; \rho)$, it follows that

$$\frac{dr}{dn} = \frac{\partial r}{\partial n} + \frac{\partial r}{\partial R} \cdot \frac{dR}{dn}. \tag{24}$$

Using (22) and (23), we obtain $\partial r/\partial n < 0$, $\partial r/\partial R > 0$ and $dR/dn > 0$. Increasing the number of

[27] Loan and deposit rates are independent of each other in the presence of interbank lending as long as the probability of default is exogenous. Setting $p(\theta) = p^0$ in each bank's maximization problem yields the equilibrium loan rate, $r^* = (\lambda + \rho n)/(n+1)$, and the equilibrium deposit rate, $R^* = \rho n/(n+1)$. Notably, the loan rate is monotonically decreasing and the deposit rate is monotonically increasing in the number of banks, n.

banks decreases individual banks' market power, which tends to raise deposit rates and lower loan rates. Ceteris paribus, increases in deposit costs pass-through to loan rates. The bank-competitor effect on loan rates comprises a negative direct effect—the first term on the right-hand side of (24), and a positive indirect effect that operates through deposit rates—the second term on the right-hand side of (24). Increasing the interbank rate dampens the negative direct effect but strengthens the positive indirect effect.[28] Thus, at low interbank rates, the negative direct effect dominates and the overall bank-competitor effect is negative. At high interbank rates, on the other hand, the positive indirect effect dominates and the overall bank-competitor effect is positive.

The full effect of market integration comprises the bank-competitor and bank-customer effects. Unlike the results in Sections 3.2 and 4, the bank-competitor effect is no longer unambiguously negative. With interbank lending, both bank-competitor and bank-customer effects can be positive or negative depending on the parameters of the model. It follows that the effect of market integration on loan rates and risk-taking can be positive or negative. Moreover, any existing relationship between competition and risk-taking in the SME can be reversed in the IME with a countervailing and sufficiently strong bank-customer effect. We illustrate this result in terms of the numerical example below. Example 1 confirms the robustness of our baseline findings in Propositions 4 and 5 in the presence of interbank lending.

Example 1 Let $a(m) = m^{\alpha}$, $b(m) = m^{\beta}$ and $n(m) = m^{\nu}$ with α, b, $\nu \in [0, 1]$. We allow the number of integrated markets, m, to vary between 2 and 10. We set $\alpha = \nu = 0.3$, $\lambda = 4$ and $\rho = 2.4$. Figure 3 depicts the equilibrium loan rate $r(m)$ as a function of the number of integrated markets. In the left panel, $r(m)$ is drawn for $\beta = 0.3$, thereby setting the bank-customer effect to zero. In this case, the equilibrium loan rate decreases under market integration because of the negative bank-competitor effect. The right panel depicts the reversal because of a positive and sufficiently strong bank-customer effect by setting $\beta = 0.6 > 0.3 = \alpha$. As a result, $r(m)$ is now increasing in m. ■

[28] More formally, we obtain that $\partial |(\partial r/\partial n)|/\partial \rho < 0$ and $\partial((\partial r/\partial R)(dR/dn))/\partial \rho > 0$.

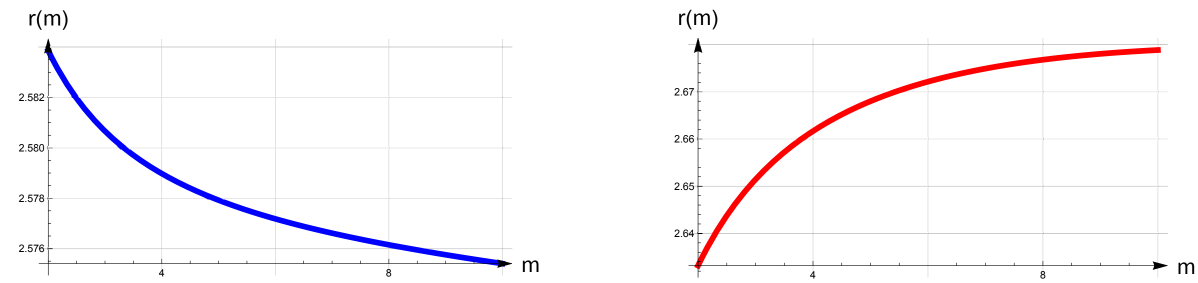

Figure 3: In the presence of interbank lending, the negative association between loan rates and market integration in the left panel is driven by negative bank-competitor effect and a bank-customer effect set to zero. A strong and countervailing (positive) bank-customer effect reverses this association as shown in the right panel.

5.2. Bank mergers

In this extension to the model, we present a parsimonious setting that allows for entry and exit of banks so that bank market power is determined endogenously. In particular, we show that market integration creates incentives for bank mergers. Mergers increase the market power of surviving banks thereby increasing equilibrium loan rates and risk-taking due to the (negative) bank-competitor effect. Moreover, consolidation in the integrated market would only reinforce any positive bank-customer effect.

5.2.1. *The SME with heterogenous bank operating costs*

We follow the linear model presented in Section 4. To this model, we introduce market-specific operating costs for the banks. In particular, operating costs of bank i in market j is given by

$$C_{ij}(D_{ij}) = c_j D_{ij},$$

where $c_j \geq 0$ is the constant marginal operating cost of funds.[29] We assume that all banks operating in market j have identical (marginal) operating cost of funds. However, markets are heterogeneous in terms of bank operating costs, so that $c_j \neq c_{j'}$ for any $j \neq j'$. We denote market-j banks with cost of operations c_j as a type-j banks. The assumption of heterogeneity in operating costs (non-interest expenses) captures differences in efficiencies, with more efficient banks having lower operating costs.

[29]The cost function described here is similar to the Monti-Klein model of bank competition (e.g. Klein, 1971) where costs are typically a function of both deposits and loans. However, given that banks in our model do not hold equity, it is fairly innocuous to assume that the cost function depends only on the volume of deposits.

The source of differences in efficiency has been attributed to the lack of competition due to restrictions on geographic expansion that provided banks with local market power. Kroszner (2001) argues that, prior to the relaxation of branching restrictions in the United States, geographic variation in banks' cost efficiencies could be linked to variations in the degree of protection across the different banking jurisdictions (cf. footnote 8).

From the linear model, we retain the assumption that each bank is financed entirely by deposits—there is no equity-financing and no interbank market. The probability of success is given by $p(\theta) = 1 - (\theta/\lambda)$. Using the results of the linear model, we obtain D_j/a_j and $\lambda - (L_j/b_j)$ as the (linear) inverse deposit supply and inverse loan demand schedules, where D_j and L_j denote the aggregate deposits and loans in market j, respectively.

We further simplify with the following additional assumptions: (i) we consider the integration of only two markets, market 1 and market 2, (ii) the banks in market 1 are more efficient than those in market 2 in that $c_1 = 0 < c = c_2$, (iii) both markets are otherwise identical so that $n_j = n \geq 1$, $a_j = a > 0$ and $b_j = b > 0$ for $j = 1, 2$. By Proposition 3, this sets the bank-customer effect to zero and increases in m translate to increases only in n. We shall relax assumption (iii) in Section 5.2.4.

We denote the SME loan rate and risk-shifting in market $j = 1, 2$ by $r_j^0(n, c)$ and $\theta_j^0(n, c)$, respectively. The SME loan rate and risk-taking are lower in markets where banks have lower operating costs, that is, $r_2^0(n, c) > r_1^0(n, c)$ and $\theta_2^0(n, c) > \theta_1^0(n, c)$. Put differently, equilibrium loan rates and risk-taking are higher when banks are less efficient.

5.2.2. *The IME with heterogenous bank operating costs*

The integrated market has n type-1 (low-cost) and n type-2 (high-cost) banks. Using the linear model in Section 4, each bank faces the following deposit supply and loan demand schedules:

$$R(D) = \frac{D}{2a} \quad \text{where } D = D_1 + D_2, \tag{25}$$

$$r(L) = \lambda - \frac{L}{2b} \quad \text{where } L = L_1 + L_2. \tag{26}$$

For the IME, we have $L = D$, and a type-j bank maximizes expected profit

$$P(D)\left(r(D) - R(D) - c_j\right) D_{ij},$$

where D_{ij} denotes the deposits of each type-j bank, and $P(D) = D/4b\lambda$. The result of the maximization problem is summarized in the following Lemma.

Lemma 4 *Let $D_j^* \equiv D_j^*(n, c)$ be the aggregate deposits of the type-j banks in the IME. If $c < \bar{c} \equiv \frac{\lambda}{n+2}$, all banks make loans so that $D_1^* > 0$, $D_2^* > 0$, and aggregate deposits, $D^* = D_1^* + D_2^*$ are given by*

$$D^*(n, c) = \frac{ab\left\{(3n+2)(2\lambda - c) + \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda - c)}\right\}}{4(a+b)(n+1)}.$$

However, if $c \geq \bar{c}$, the difference in costs between the two bank types is sufficiently large so that high-cost banks do not lend in the IME, and hence, $D_2^* = 0$. As a result, the IME comprises only low-cost banks. Given this result, the following proposition characterizes risk-shifting in the IME with heterogenous costs of funding.[30]

Proposition 7 *In the IME where both bank types compete, that is, $D_1^* > 0$ and $D_2^* > 0$, both high- and low-cost banks take the same level of risk which is given by*

$$\theta^*(n, c) = \lambda - \frac{D^*(n, c)}{4b} = \lambda - \frac{a\left\{(3n+2)(2\lambda - c) + \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda - c)}\right\}}{16(a+b)(n+1)}.$$

Equilibrium risk-shifting in the IME is less than that in the SME, that is, $\theta^*(n, c) < \theta_j^0(n, c)$ for $j = 1, 2$.

With the bank-customer effect set to zero by construction, the effect of increased competition in the IME is comprised exclusively of the bank-competitor effect. With an increase in the number of rival banks, the loan rate in the IME is lower than the loan rates for both markets in the SME.

[30]This result and the ones that follow are stated in terms of equilibrium risk-taking. With the bank-customer effect set to zero and the positive relation between loan rates and risk-shifting, it is important to keep in mind that the effect on risk-taking operates exclusively through loan rates.

5.2.3. The IME and bank mergers

We demonstrate that incentives for mergers exist in the IME with heterogenous banking costs. Therefore, if mergers are allowed, it is profitable for a high-cost bank to merge with a low-cost bank.[31] When the differences in operating costs are small (i.e., $c < \bar{c}$) both bank types make positive profits. We show that, under the same regularity condition, each merger between a low-cost and a high-cost bank is profitable. Given that the source of cost heterogeneity lies in differences between operating conditions in segmented markets, merger activity in this setting would not be possible prior to market integration. Given that the source of cost heterogeneity lies in differences between operating conditions in segmented markets, merger activity in this setting would not be possible prior to market integration. A large body of empirical works has highlighted the efficiency-enhancing role of mergers post integration (Berger et al., 1999; DeYoung et al., 2009). In particular, this model demonstrates how efficient banks can target relatively inefficient, less profitable banks that operate within the same integrated market (cf. footnote 9).

We solve the model for the equilibrium wherein each low-cost bank acquires exactly one high-cost bank and the merged entity takes the (zero) operating cost of the low-cost bank. In doing so, we assume that the merged entity captures the entire efficiency gains from the merger. As a result, the post-merger integrated market comprises only n low-cost banks. We refer to this equilibrium as the post-merger IME to distinguish it from the pre-merger IME in Proposition 7.

Proposition 8 *If $c < \bar{c}$, then in the post-merger IME where each low-cost bank acquires exactly one high-cost bank, and the merged entity operates at zero cost, the following results hold:*

(a) *mergers are profitable, that is, the profits of the merged entity is greater than the sum of the profits of individual banks prior to the merger;*

(b) *risk-shifting in the post-merger IME, $\theta_M(n)$, is greater than risk-shifting in the pre-merger IME, $\theta^*(n, c)$, and equals risk-shifting in the low-cost market in the SME. Formally,*

$$\theta^*(n, c) < \theta_M(n) = \theta_1^0(n, c).$$

[31] We adopt a tractable, reduced-form approach wherein the decision to merge is not modeled and all possible of bank mergers (e.g. mergers between banks with the same costs) are not considered. We discuss such possibilities later in this section.

Each low-cost bank acquires exactly one high-cost bank so that there are only n low-cost banks in the post-merger IME. The supply of deposits and demand for loans are given by (25) and (26), respectively. With only n type-1 banks in the integrated market, each bank chooses deposits D_i to maximize

$$\pi_i = P(D)\,(r(D) - R(D))\,D_i.$$

We denote the expected profits for each merged entity as $\pi_M(n)$. Solving the maximization problem, we find that mergers are profitable, that is,

$$\pi_M(n) > \pi_1^*(n, c) + \pi_2^*(n, c),$$

where π_j^* denotes type-j bank's expected profits in the pre-merger IME. Importantly, mergers are profitable under the same regularity conditions as those required for the interior equilibrium (cf. Lemma 4). It follows that risk-taking in the post-merger IME is the same as that in the low-cost market in the SME, $\theta_M(n) = \theta_1^0(n, c)$.

In the absence of the bank-customer effect, the effect of integration on risk-taking is comprised entirely of the bank-competitor effect. In this tractable setting, both loan rates and risk-taking are determined by the number of competitor banks. Market integration increases the number of banks thereby reducing loan rates and risk-taking. However, mergers reduce the number of banks, thereby raising loan rates and risk-taking. In the end, risk-taking in the post-merger IME is the same as that in the low-cost market in the SME because both equilibria comprise n low-cost banks.

Unilateral effects of bank mergers in the integrated market. From the standpoint of social welfare, the results from the merger analysis offer an important insight. Social welfare, W, is defined as the sum of the surplus obtained by depositors, borrowers, and banks. In the IME, $n(\pi_1^*(n, c) + \pi_2^*(n, c))$ is the pre-merger bank surplus and $n\pi_M(n)$ is the post-merger bank surplus. Profitable mergers capture efficiency gains and unambiguously increase bank surplus in the post-merger IME because $\pi_M(n) > \pi_1^*(n, c) + \pi_2^*(n, c)$. However, mergers also have countervailing effects on bank customers (borrowers and depositors). The increase in market power from bank mergers tends to increase the loan rate, thereby reducing borrower surplus. Increased market power also tends to decrease the deposit rate, reducing depositor surplus. With deposit insurance, the net payoff for each depositor is $R-1$. Because there are $G_1(R)+G_2(R)=2aR$ such depositors, the (expected) depositor surplus is

$$DS = 2aR(R-1) = D\left(\frac{D}{2a}-1\right).$$

Likewise, borrower expected payoff is $u(r)=(\lambda-r)^2/4$. Because there are $H_1(u(r))+H_2(u(r)) = 4b\sqrt{\lambda u(r)} = 2b(\lambda-r)$ such borrowers, the aggregate borrower surplus is

$$BS = 2b(\lambda-r)\cdot\frac{(\lambda-r)^2}{4} = \frac{D^3}{16\lambda b^2}.$$

Together, aggregate customer surplus is

$$CS = DS + BS = \frac{D^2}{2a} - D + \frac{D^3}{16\lambda b^2}.$$

We denote customer surplus and social welfare in the pre-merger IME by CS^* and W^* and those in the post-merger IME by CS_M and W_M, respectively. Although the post-merger IME yields a relatively larger surplus for banks, we find that it yields a relatively smaller surplus for customers, so that $CS^* > CS_M$. Therefore, bank mergers are not always welfare enhancing. Even when the merged entity captures the entire gains from the merger (between a low-cost and a high-cost bank) thereby making it profitable, the unilateral effect of mergers may be anti-competitive. However, when efficiency gains from mergers are sufficiently large (i.e., $c > c^*$ for some $c^* \in (0, \bar{c})$), we find that the increase in bank surplus exceeds the decrease in customer surplus. Consequently, social welfare is greater in the post-merger IME.[32] The following example illustrates this result.

Example 2 Let $\Delta(c;\lambda,a,b,n) \equiv W^*(c;\lambda,a,b,n) - W_M(\lambda,a,b,n)$ denote the difference between social welfare in the pre-merger IME, W^*, and social welfare in the post-merger IME, W_M. While there are only n low-cost banks in the post-merger IME, the pre-merger IME comprises both low-cost and high-cost banks. For this reason, W_M does not depend on c. We set $\lambda = 1.3$, $a=b=2$, $n=4$. Figure 4 depicts the function $\Delta(c; 1.3, 2, 2, 4)$, which is convex in c, where $c \in (0,\bar{c}) = (0, 0.217)$. The function intersects the horizontal axis at $c^* \approx 0.08535$. Accordingly, if $c < c^*$, efficiency gains are sufficiently

[32] A formal proof of this assertion is available upon request.

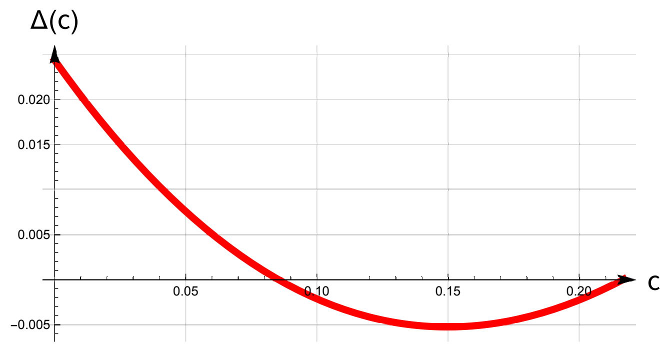

Figure 4: The unilateral effects of bank mergers. For high (low) efficiency gains, welfare under the post-merger IME is higher than that under the pre-merger IME.

low so that welfare under the post-merger IME is lower than that under the pre-merger IME. On the contrary, if $c \geq c^*$ then efficiency gains are sufficiently large so that welfare under the post-merger IME is greater than that under the pre-merger IME. Efficiency gains from mergers between low- and high-cost banks yield higher social welfare. ■

Example 2 reveals an important insight for regulatory reviews of bank mergers. Mergers may be approved because the unilateral effect of mergers is pro-competitive. However, our results above show that such efficiency-enhancing mergers can also increase risk-taking. With the inclusion of the financial stability factor in merger reviews under the Dodd Frank Act of 2010, welfare considerations are no longer the sole factor in approving bank mergers. Our results present scenarios where the unilateral effect and the risk-taking effect provide conflicting recommendations for merger approvals.

Pro-collusive effects of bank mergers. Market integration can also support bank mergers by incentivizing collusion in the post-merger equilibrium. We have modeled mergers between banks with dissimilar marginal costs, wherein the takeover of all the high-cost banks yields a market comprising only low-cost (symmetric) banks in the post-merger IME. Previous research has shown that collusive outcomes are more easily sustained when firms (banks) are symmetric.[33] This *pro-collusive effect* of mergers can potentially raise loan rates and increase risk-taking beyond that obtained in the post-merger IME. Boyd and Graham (1998) present an account of mergers between similar banks throughout the 1980s and 1990s. However, for such mergers to be profitable in a quantity competition setting, it is

[33] Bernheim and Whinston (1990) show that tacit collusion is easier when firms are symmetric with respect to production costs and market shares. See Motta (2004, Chapter 4) for a detailed analysis of how symmetry among firms facilitates collusive agreements.

required that a significantly large fraction of banks merge horizontally (Salant, Switzer, and Reynolds, 1983) or there are significant merger-related synergies (Farrell and Shapiro, 1990).

5.2.4. Bank mergers and a positive bank-customer effect

Finally, we confirm the robustness of our baseline results to cases where bank market power is determined endogenously. In particular, we show that a positive bank-customer effect reinforces the negative bank-competitor effect as banks merge in the integrated market. For a nonzero bank-customer effect, we relax the assumption that $a_j = a$ and $b_j = b$ for $j = 1, 2$. Instead, we return to our assumption of heterogenous market size ($a_1 \neq a_2$ and $b_1 \neq b_2$). Note that the bank-customer effect is positive if and only if

$$\frac{b_2}{b_1} > \frac{a_2}{a_1}. \tag{27}$$

Using the results of the linear model in Section 4, a positive bank-customer effect yields the following proposition[34]

Proposition 9 *With a positive bank-customer effect, risk-shifting in the post-merger IME is higher than risk-shifting in the low-cost market in the SME, that is,* $\theta_M(n) > \theta_1^0(n, c)$.

In the absence of the bank-customer effect, equilibrium risk-shifting decreases in the pre-merger IME but increases (back to its level in the low-cost market in the SME) in the post-merger IME because surviving low-cost banks gain market power. A positive bank-customer effect reinforces this increase in risk-taking in the post-merger IME. In other words, a positive bank-customer effect can increase risk-shifting in the post-merger IME above the level obtained prior to market integration and bank mergers.

6. Conclusion

The lack of consensus among studies examining the linkages between bank competition and risk-taking poses a significant challenge for policymakers and academics. The model presented in this pa-

[34] With the integration of markets 1 and 2, incentives for mergers exist between low- and high-cost banks when $a_1 \neq a_2$ and $b_1 \neq b_2$. We take the low-cost market, namely market 1, as the benchmark market because only low-cost banks survive in the post-merger equilibrium. The bank-customer effect is positive if $(b_1 + b_2 - b_1)/b_1 > (a_1 + a_2 - a_1)/a_1$, which is equivalent to condition (27) for the continuous case.

per provides one possible reason behind the mixed empirical evidence on this question. In the context of integrating banking markets, we find that the effect of increased competition extends beyond that explained by a simple increase in the number of rival banks. Importantly, we point to a risk-incentive mechanism, namely the bank-customer effect of market integration, that can potentially reverse *any* observed association between bank competition and risk-taking prior to integration. Moreover, while the predictions of existing theory are shown to prevail under specific conditions within our generalized framework, they are not the only outcomes of the model.

To the best of our knowledge, this paper is the first to directly examine the effect of market integration on risk-taking. While future modeling efforts can include richer settings that improve our understanding about the linkages between competition and stability in banking, we have opted for a tractable approach. Even in this parsimonious setting, the richer set of results demonstrate the need for a broader definition of increased competition that captures the evolution of competition. Most theory on bank competition tends to focus on changes in the number of competitor banks within a limited geographic area. Our results suggest the need for reexamining the idea that bank competition is confined to local markets.

Appendix: Proofs

Proof of Proposition 1

Let $\phi(k) \equiv f(k)/k$ be the average product of investment. The first-order condition of (3) is given by:

$$\underbrace{\left\{y(\theta) + y'(\theta) \cdot \frac{p(\theta)}{p'(\theta)}\right\}}_{h(\theta)} f(k) - rk = 0 \quad \Longleftrightarrow \quad h(\theta) = \frac{r}{\phi(k)}, \tag{28}$$

which defines $\theta = \theta(k; r)$. Note that, because $y''(\theta) \leq 0 < y'(\theta)$, $p'(\theta) < 0$ and $p''(\theta) \leq 0$,

$$h'(\theta) = 2y'(\theta) + \frac{p(\theta)}{p'(\theta)} \left\{y''(\theta) - \frac{y'(\theta)p''(\theta)}{p'(\theta)}\right\} \geq 2y'(\theta) > 0.$$

Differentiating (28) with respect to k and r, respectively we obtain

$$\theta_k(k;r) = \frac{h(\theta)[1-\varepsilon(k)]}{h'(\theta)k} = \underbrace{\frac{rk}{f(k)}}_{h(\theta)\text{ from (28)}} \cdot \frac{1-\varepsilon(k)}{h'(\theta)k} > 0, \quad \text{and} \quad \theta_r(k;r) = \frac{k}{h'(\theta)f(k)} > 0, \tag{29}$$

where $\varepsilon(k) \equiv kf'(k)/f(k)$ is the output elasticity of investment. Because $f(k)$ is strictly concave and $f(0) \geq 0$, the average product of investment, $\phi(k)$ is strictly decreasing in k which is equivalent to $\varepsilon(k) < 1$.[35] Therefore, $\theta_k(k;r) > 0$. Note that the objective function of the maximization problem (3) is strictly concave in θ because $p(\theta) \geq 0$, $p'(\theta) < 0$, $p''(\theta) < 0$, $y(\theta) \geq 0$, $y'(\theta) > 0$ and $y''(\theta) < 0$ for all $\theta \in [0, \bar{\theta}]$, and hence, $\theta(k;r)$ is unique. Let $U(k;r)$ be the value function of the maximization problem (3). Then, by the Envelope theorem, we have $U_k(k;r) = p(\theta)\{y(\theta)f'(k) - r\}$, and hence, the first-order condition of (5) is given by:

$$U_k(k;r) = 0 \quad \Longrightarrow \quad y(\theta)f'(k) = r. \tag{30}$$

The second-order necessary condition is given by:

$$\begin{aligned} & p(\theta)\{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)\} + p'(\theta)\theta_k\{\underbrace{y(\theta)f'(k) - r}_{=0\text{ by (30)}}\} \leq 0 \\ \Longrightarrow \quad & \underbrace{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)}_{\Omega(k,r)} \leq 0. \end{aligned} \tag{31}$$

Differentiating (30) with respect to r we obtain

$$k'(r) = \frac{1 - (y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)}.$$

[35] To see this, take a twice differentiable function $f(k)$ that is strictly concave with $f(0) \geq 0$. Note that $\phi'(k) < 0$ is equivalent to $\phi(k) > f'(k) \iff \varepsilon(k) < 1$. Take any point $(k_0, f(k_0))$ on the graph of $f(k)$. Then, there is $\kappa \in (0, k_0)$ such that

$$\phi(k_0) \equiv \frac{f(k_0)}{k_0} \geq \frac{f(k_0) - f(0)}{k_0} = f'(\kappa) > f'(k_0).$$

The first (weak) inequality follows from the fact that $f(0) \geq 0$, the second equality holds for some $\kappa \in (0, k_0)$ which follows from the Mean Value theorem, and the last (strict) inequality is implied by $f''(k) < 0$ and $\kappa < k_0$. This proves that $\phi'(k) < 0$ as k_0 has been chosen arbitrary.

Observe that $h'(\theta) \geq 2y'(\theta)$ implies that $y'(\theta)/h'(\theta) \leq 1/2$. Therefore, the numerator of the last expression is strictly positive because $\varepsilon(k) < 1$. On the other hand, the denominator is negative by the second-order condition. Consequently, $k'(r) \leq 0$. Because $d\theta^*/dr = \theta_k \cdot k'(r) + \theta_r$ with $\theta_k, \theta_r > 0$ and $k'(r) \leq 0$, the sign of $d\theta^*/dr$ is indeterminate.

We now prove the final part of Proposition 1 that $d\theta^*/dr > (<)\, 0$ according as $\varepsilon'(k) < (>)\, 0$. Note that

$$\varepsilon'(k) = \frac{d}{dk}\left(\frac{kf'(k)}{f(k)}\right) = \frac{kf''(k) + f'(k)[1-\varepsilon(k)]}{f(k)} \tag{32}$$

Now,

$$\begin{aligned} \frac{d\theta^*}{dr} &= \underbrace{\frac{h(\theta)[1-\varepsilon(k)]}{h'(\theta)k}}_{\theta_k} \cdot \underbrace{\frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)}}_{k'(r)} + \underbrace{\frac{k}{h'(\theta)f(k)}}_{\theta_r} \\ &= \underbrace{\frac{rk}{f(k)}}_{h(\theta)\text{ from (28)}} \cdot \frac{1-\varepsilon(k)}{h'(\theta)k} \cdot \frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)} + \frac{k}{h'(\theta)f(k)} \\ \iff \frac{d\theta^*}{dr} &= \frac{h'(\theta)k\Omega(k,r) + r[1-\varepsilon(k)][h'(\theta) - y'(\theta)\varepsilon(k)]}{[h'(\theta)]^2 f(k)\Omega(k,r)} \equiv \frac{Q(k,r)}{[h'(\theta)]^2 f(k)\Omega(k,r)}. \end{aligned} \tag{33}$$

Using the expression of θ_k from (29) and that of $\Omega(k,r)$ from (31), we obtain

$$Q(k,r) = \frac{rh'(\theta)}{f'(k)} \cdot \underbrace{\left\{kf''(k) + f'(k)[1-\varepsilon(k)]\right\}}_{f(k)\varepsilon'(k)\text{ from (32)}}.$$

Therefore,

$$\frac{d\theta^*}{dr} = \frac{r\varepsilon'(k)}{h'(\theta)f'(k)\Omega(k,r)},$$

which implies that $\text{sign}[d\theta^*/dr] = -\text{sign}[\varepsilon'(k)]$ because $r, h'(\theta), f'(k) > 0$ and $\Omega(k,r) \leq 0$. This completes the proof of the proposition.

Examples in Table 1

For all the examples below, we assume that $p(\theta) = 1-\theta$, $y(\theta) = \theta$.

1. Consider $f(k) = k(1-k)$ defined on $[0, 1/2]$ so that $f'(k) > 0$. For this functional form, $\varepsilon(k)$

decreases with k. In this case, equilibrium risk-shifting is strictly increasing in r_j and is given by:

$$\theta^*(r_j) = \frac{1}{4}\left(1 + \sqrt{1 + 8r_j}\right).$$

2. Consider $f(k) = \sqrt{k_0 + k}$ with $k_0 > 0$ an $k \geq 0$. In this case, the elasticity of investment is increasing in k, i.e., $\varepsilon'(k) > 0$. The equilibrium risk-shifting is given by:

$$\theta^*(r_j) = \frac{1}{2} + \frac{\sqrt{2}\left(1 - 24k_0 r_j^2 + \sqrt{1 - 12k_0 r_j^2}\right)}{12\left(1 - 6k_0 r_j^2 + \sqrt{1 - 12k_0 r_j^2}\right)^{1/2}}.$$

   It is easy to show that the above expression is decreasing in r_j.

3. Finally, let $f(k) = k^\delta$ with $\delta \in (0, 1)$. In this case, $\varepsilon(k) = \delta$ for all k. The optimal risk-taking is given by $\theta^*(r_j) = (2 - \delta)^{-1}$, which is independent of the loan rate.

Proof of Lemma 1

The proof of this Lemma is identical to that of Lemma 2 below. Replace a, b, n and D_i in the proof of Lemma 2 respectively by a_j, b_j, n_j and D_{ij} to obtain the result.

Proof of Proposition 2

This proposition can be obtained as a special case of Proposition 3. Replace $a(m)$, $b(m)$, $n(m)$ and D_i respectively by a_j, b_j, n_j and D_{ij}, and set $\hat{a}(m) = \hat{b}(m) = 0$ in the proof of Proposition 2 to obtain the result.

Proof of Lemma 2

We suppress for the time being the argument m from $a(m)$, $b(m)$ and $n(m)$. Let $P(L/b) \equiv p(\theta^*(r(L/b)))$. With $L = \sum_{i=1}^n D_i = D$, the first-order condition of the maximization problem of bank i in the integrated market J_m is given by:

$$\begin{aligned}
&P(D/b)[r(D/b) - R(D/a)] + D_i[r(D/b) - R(D/a)]P'(D/b) \cdot \frac{1}{b} \\
&= P(D/b)D_i \left(R'(D/a) \cdot \frac{1}{a} - r'(D/b) \cdot \frac{1}{b} \right) \\
\iff D_i = &\frac{P(D/b)[r(D/b) - R(D/a)]}{P(D/b) \left(R'(D/a) \cdot \frac{1}{a} - r'(D/b) \cdot \frac{1}{b} \right) - [r(D/b) - R(D/a)]P'(D/b) \cdot \frac{1}{b}} \quad \text{for all } i.
\end{aligned}$$

Because the right-hand side of the above condition depends only on the aggregate deposits, D, it immediately follows that $D_i = D_{i'}$ for any $i \neq i'$. Therefore, there are no asymmetric equilibria.[36] In the (symmetric) IME, $D_i = D/n$ for all i. Thus, the above optimality condition boils down to:

$$\mu(D, a, b) - F(D, a, b, n) = 0, \tag{34}$$

where $\mu(D, a, b) \equiv r(D/b) - R(D/a)$ is the equilibrium intermediation margin, and

$$F(D, a, b, n) \equiv \frac{[bR'(D/a) - ar'(D/b)]\, P(D/b)D}{a\,[nbP(D/b) + P'(D/b)D]}.$$

The second order necessary condition is given by:

$$\mu_D(D, a, b) - F_D(D, a, b, n) < 0. \tag{35}$$

The equilibrium loan rate is given by $r(D/b)$, and the equilibrium risk-shifting is given by $\theta^*(r(D/b))$.

Proof of Proposition 3

The first-order condition (34) can be written as

$$\mu(D, a, b) = F(D, a, b, n) \equiv \frac{\zeta(D, a, b)}{\xi(D, b, n)},$$

where $\zeta(D, a, b) \equiv (b/a)R'(D/a) - r'(D/b) > 0$, and $\xi(D, b, n) \equiv \frac{bn}{D} + \eta(D/b)$ with $\eta(D/b) \equiv P'(D/b)/P(D/b)$. Because the optimal risk-shifting of the entrepreneurs may be increasing or de-

[36] Because the first-order conditions are non-linear in deposits of the banks, there may be multiple symmetric equilibria. However, none of our results hinges on the unicity of equilibrium.

creasing in the loan rate, the sign of $P'(D/b)$ is indeterminate. If $P'(D/b) > 0$, then $\xi(D, b, n)$ is also positive. Therefore, we would assume that $\xi(D, b, n) \geq 0$ if $P'(D/b) < 0$. Note that

$$\mu_D = r'(D/b) \cdot \frac{1}{b} - R'(D/a) \cdot \frac{1}{a} < 0\,, \quad \mu_a = R'(D/a) \cdot \frac{D}{a^2} > 0, \;\text{ and }\; \mu_b = -r'(D/a) \cdot \frac{D}{b^2} > 0.$$

On the other hand, from the expression of $F(D, a, b, n)$, we obtain

$$\begin{aligned}
F_D &= \frac{1}{\xi}\left[\frac{bR''(D/a)}{a^2} - \frac{r''(D/b)}{b} + F(D, a, b, n)\left(\frac{bn}{D^2} - \frac{\eta'(D/b)}{b}\right)\right], \\
F_a &= -\frac{b}{\xi a^2}\left(R''(D/a)(D/a) + R'(D/a)\right), \\
F_b &= \frac{1}{\xi}\left[\frac{R'(D/a)}{a} + \frac{Dr''(D/b)}{b^2} + F(D, a, b, n)\left(\frac{D\eta'(D/b)}{b^2} - \frac{n}{D}\right)\right], \\
F_n &= -\frac{\zeta b}{\xi^2 D} < 0.
\end{aligned}$$

Note that $F_D - \mu_D > 0$ by (35). Moreover, and $F_a < 0$ because $R''(D/a) \geq 0$, and hence, $\mu_a - F_a > 0$. Lastly, it is immediate to verify that

$$1 - \frac{b(\mu_b - F_b)}{D(F_D - \mu_D)} = \frac{a(\mu_a - F_a)}{D(F_D - \mu_D)}\,.$$

Totally differentiating the first-order condition (34), we obtain

$$\frac{dD}{D} = -\frac{nF_n}{D(F_D - \mu_D)} \cdot \frac{dn}{n} + \frac{a(\mu_a - F_a)}{D(F_D - \mu_D)} \cdot \frac{da}{a} + \frac{b(\mu_b - F_b)}{D(F_D - \mu_D)} \cdot \frac{db}{b}.$$

Because $r = r(D/b)$, we have

$$\begin{aligned}
dr &= -\,r'(D/b)(D/b)\left(\frac{db}{b} - \frac{dD}{D}\right) \\
&= -\,r'(D/b)(D/b)\left\{\frac{nF_n}{D(F_D - \mu_D)} \cdot \frac{dn}{n} + \underbrace{\left(1 - \frac{b(\mu_b - F_b)}{D(F_D - \mu_D)}\right)}_{=\frac{a(\mu_a - F_a)}{D(F_D - \mu_D)}}\frac{db}{b} - \frac{a(\mu_a - F_a)}{D(F_D - \mu_D)} \cdot \frac{da}{a}\right\}. \qquad (36)
\end{aligned}$$

Because $\hat{n}(m) \equiv n'(m)/n(m)$, $\hat{a}(m) \equiv a'(m)/a(m)$ and $\hat{b}(m) \equiv b'(m)/b(m)$, it follows from (36) that

$$\frac{dr}{dm} = -r'(D/b)(D/b)\left[\frac{nF_n}{D(F_D - \mu_D)} \cdot \hat{n}(m) + \frac{a(\mu_a - F_a)}{D(F_D - \mu_D)}\left(\hat{b}(m) - \hat{a}(m)\right)\right].$$

Define

$$Z_n \equiv -r'(D/b)(D/b) \cdot \frac{nF_n}{D(F_D - \mu_D)} \quad \text{and} \quad Z_c \equiv -r'(D/b)(D/b) \cdot \frac{a(\mu_a - F_a)}{D(F_D - \mu_D)}.$$

Because $r'(\cdot) < 0$, $F_n < 0$ and $F_D - \mu_D > 0$, we have $Z_n < 0$. On the other hand, $Z_c > 0$ as $r'(\cdot) < 0$, $F_D - \mu_D > 0$ and $\mu_a - F_a > 0$. This completes the proof of the Proposition.

Proof of Lemma 3

If $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$, then $h(\theta) = 2\theta - 1$. The first-order condition (4) of the maximization problem (3) implies that

$$\theta(k; r) = \frac{1}{2}\left\{1 + \frac{rk}{f(k)}\right\}.$$

Then, (30) reduces to

$$\frac{1}{2}\left\{1 + \frac{rk}{f(k)}\right\} f'(k) = r \iff r = \frac{f(k)f'(k)}{2f(k) - kf'(k)}.$$

Note that $k = \lambda - r \iff r = \lambda - k$ is equivalent to

$$\underbrace{\frac{f(k)f'(k)}{2f(k) - kf'(k)}}_{r} = \lambda - k \iff f'(k) = \frac{2f(k)(\lambda - k)}{f(k) + k(\lambda - k)} \equiv \Phi(k, f(k)). \tag{37}$$

Let $f(k_0) = z_0$ such that $(k_0, z_0) \in [0, \bar{k}] \times \mathbb{R}$. Observe that both $\Phi(k, z)$ and $\Phi_z(k, z)$ are continuous functions. The second assertion is true because

$$\Phi_z(k, z) = \frac{2z(\lambda - k)^2}{\{z + k(\lambda - k)\}^2}$$

whose denominator is different from zero if $z_0 > 0$ which implies that $\Phi_z(k, z)$ is a bounded function, and hence, $\Phi(k, z)$ is Lipschitz continuous in z. Therefore, there exists a unique $f(k)$ on $(0, \bar{k})$ which satisfies (37) (see Simmons, 2017, Theorem B, chapter 13, p. 634).

Proof of Proposition 5

With $R = D/a$ and $r = \lambda - (L/b)$, we have $\theta(L/b) = \lambda - (L/2b)$ and $P(L/b) \equiv 1 - \theta(L/b) = L/(2\lambda b)$. Therefore in the integrated market, bank i's objective function, using $L = D$, becomes

$$\frac{D}{2\lambda b}\left\{\lambda - \frac{D}{a} + \frac{D}{b}\right\} D_i.$$

The first-order condition of the above maximization problem with respect to D_i is given by:

$$\lambda D - \left(\frac{1}{a} + \frac{1}{b}\right) D^2 + D_i \left\{\lambda - 2\left(\frac{1}{a} + \frac{1}{b}\right) D\right\} = 0. \tag{38}$$

In the symmetric equilibrium, we have $D_i = D/n$ for all i. Substituting this into (38), and solving for $D > 0$ yields:[37]

$$D = \frac{\lambda ab(n+1)}{(a+b)(n+2)}.$$

Thus, the equilibrium risk-shifting of the integrated market is given by:

$$\theta(D/b) = \lambda - \frac{D}{2b} = \lambda\left\{1 - \frac{a(n+1)}{2(a+b)(n+2)}\right\}.$$

Substituting $n \equiv n(m) = \frac{1}{2}m(m+1)$, $a \equiv a(m) = m^{\alpha}$ and $b \equiv b(m) = m^{\beta}$ into the above expression, we obtain (19). Let $\gamma \equiv \beta - \alpha$. Because $0 \leq \alpha, \beta \leq 1$, we have $-1 \leq \gamma \leq 1$. Differentiating the expression in (19) with respect to m we obtain

$$\frac{d\theta}{dm} = \frac{\lambda m^{\gamma-1}(2+m+m^2)}{2(1+m^{\gamma})^2(4+m+m^2)} \underbrace{\left(\gamma - \frac{2m(1+2m)(1+m^{-\gamma})}{(2+m+m^2)(4+m+m^2)}\right)}_{h(m;\gamma)}. \tag{39}$$

Therefore, $\text{sign}[d\theta/dm] = \text{sign}[h(m;\gamma)]$. Note first that $\gamma \leq 0$ or $\beta \leq \alpha$ implies that $h(m;\gamma) < 0$ for all $m \geq 2$, and hence, $d\theta/dm < 0$. Next, consider $\gamma \in (0,1]$. In this case, the bank-customer effect is

[37] The other symmetric solution is $D = 0$ which is obviously discarded.

positive. Note that

$$h_\gamma(m;\,\gamma) = 1 + \frac{2m^{1-\gamma}(1+2m)(\log m)}{(2+m+m^2)(4+m+m^2)} > 0 \quad \text{for all} \;\; m \geq 2,$$

$h(m;\,\gamma)$ is strictly increasing in θ for all $m \geq 2$. Write $h(m;\,\gamma) = \gamma - l(m)g(m;\,\gamma)$, where

$$l(m) \equiv \frac{2m(1+2m)}{(2+m+m^2)(4+m+m^2)} \quad \text{and} \quad g(m;\,\gamma) \equiv 1 + m^{-\gamma}\,.$$

Note that $l(m) > 0$ and $g(m;\,\gamma) > 0$ for all $m \geq 2$ and $\gamma > 0$. Clearly, $l(m)$ is strictly decreasing in m, that is, $l'(m) < 0$ for all $m \geq 2$, and $g(m;\,\gamma)$ is strictly decreasing in m for all $\gamma > 0$ as $g_m(m;\,\gamma) = -\gamma m^{-(1+\gamma)} < 0$. Therefore, $h_m(m;\,\gamma) = -(l'(m)g(m;\,\gamma) + l(m)g_m(m;\,\gamma)) > 0$. It can also be shown that $h(m;\,\gamma)$ is strictly concave in m. Next, note that

$$\lim_{m\to\infty} h(m;\,\gamma) = \gamma.$$

The above expression is equal to 0 for $\gamma = 0$, that is, $\beta = \alpha$, and is strictly positive for any $\gamma > 0$,that is, $\beta > \alpha$. Finally,

$$h(2;\,\gamma) = \gamma - \frac{1}{4}\left(1 + 2^{-\gamma}\right).$$

Because $h(2;\,\gamma)$ is strictly increasing in γ, $h(2;\,0) = -0.5$ and $h(2;\,0.5) \approx 0.073$, by the Intermediate Value theorem, there is a unique $x \in (0,\,0.5)$ such that $h(2;\,x) = 0$. It turns out that $x \approx 0.435$. Therefore, for any $\gamma \in (0,\,x)$, $h(m;\,\gamma) < 0$ if and only if $m < \widehat{m}(\gamma)$. Note that, for any $\gamma \in (0,\,x)$, we have $h(2;\,\gamma) < 0$ and $\lim_{m\to\infty} h(m;\,\gamma) = \gamma > 0$, and hence, the Intermediate Value theorem guarantees the existence of $\widehat{m}(\gamma)$. Because $h(m;\,\gamma)$ is strictly increasing in m for all $\gamma > 0$, $\widehat{m}(\gamma)$ is unique. Finally, $\widehat{m}'(\gamma) < 0$ as $h(m;\,\gamma)$ is strictly increasing in γ. Finally, for any $\gamma > x$, we have $h(m;\,\gamma) > 0$ for all $m \geq 2$. Figure 5 summarizes the above discussion.

Proof of Proposition 6

To save on notations, write $a = a(m)$, $b = b(m)$ and $n = n(m)$. Given that $R(D/a) = D/a$ and $r(L/b) = \lambda - (L/b)$, the first-order conditions associated with the maximization problem (21) with

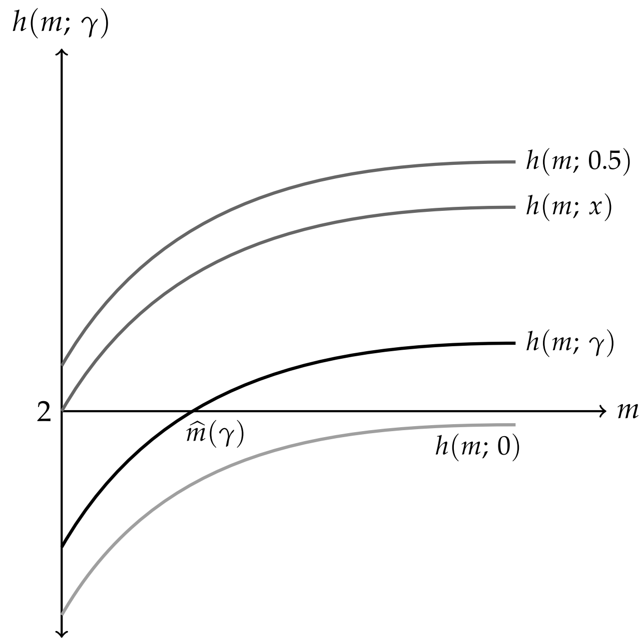

Figure 5: *For $\theta = 0$, $h(m;\gamma) < 0$, and hence, $d\theta/dm < 0$. For any $\gamma \in (0, x)$, $h(m;\gamma)$ intersects the horizontal axis at a unique point $\widehat{m}(\gamma)$, that is, $h(m;\gamma) < (>)\, 0$ if $m < (>)\, \widehat{m}(\gamma)$, and hence, $\theta(m)$ is U-shaped with respect to m. For any $\gamma \in (x, 1]$, $h(m;\gamma) > 0$, and hence, $d\theta/dm > 0$.*

respect to L_i and D_i, under symmetry, i.e., $L_i = L/n$ and $D_i = D/n$ for all i, are given by

$$a(n+2)L^2 - ab(\lambda-\rho)(n+1)L = b(a\rho - D)D, \tag{40}$$

$$a\rho n = (n+1)D, \tag{41}$$

respectively. Solving the above system, we obtain the aggregate loans, L and deposits, D in equilibrium. Using $L = b(\lambda - r)$ and $D = aR$, we get (22) and (23). The equilibrium loan rate is thus given by:

$$r(a, b, n, \rho) = \lambda - \frac{L}{b} = \lambda - \frac{b(\lambda-\rho)(n+1)^2 + \sqrt{b^2(\lambda-\rho)^2(n+1)^4 + 4ab\rho^2 n(n+2)}}{2b(n+1)(n+2)}. \tag{42}$$

The equilibrium deposit rate, on the other hand, is given by:

$$R(n, \rho) = \frac{D}{a} = \frac{\rho n}{n+1}.$$

Note first that our assumption of $m \geq 2$ implies that $\min\{n(m)\} = n(2) \geq 2$. We require that $R(n, \rho) \leq \rho \leq r(a, b, n, \rho)$. The first inequality holds because $n \geq 2$. The second inequality is equivalent to

$$g(\rho) \equiv \frac{b(\lambda - \rho)^2}{a\rho^2} \geq \frac{n}{(n+1)^2}.$$

The last inequality is equivalent to

$$\rho \leq \frac{\lambda b(n+1)}{b(n+1) + \sqrt{an}} \equiv \bar{\rho} \leq \lambda.$$

Note that $\bar{\rho}$ is increasing in n and $\lim_{n \to \infty} \bar{\rho} = \lambda$.

We first prove part (a). Fix $n(m) = \bar{n}$ so that $n'(m) = 0$, i.e., the bank-competitor effect is zero. Note that

$$\frac{\partial r}{\partial a} = -\frac{n\rho^2}{(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2 + 4abn(n+2)\rho^2}} < 0,$$
$$\frac{\partial r}{\partial b} = \frac{an\rho^2}{b(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2 + 4abn(n+2)\rho^2}} = -\frac{a}{b} \cdot \frac{\partial r}{\partial a} > 0.$$

Then, differentiating r with respect to m, we obtain

$$\frac{dr}{dm} = \frac{\partial r}{\partial a} \cdot a'(m) + \frac{\partial r}{\partial b} \cdot b'(m) = \left\{ b'(m) - \frac{b(m)}{a(m)} \cdot a'(m) \right\} \frac{\partial r}{\partial b}$$

The above expression is positive (negative) according as $\hat{b}(m) \gtrless \hat{a}(m)$, i.e., the bank-customer effect is positive (negative).

We next prove part (b). By implicitly differentiating (22), we obtain

$$\frac{\partial r}{\partial R} = \frac{a(2R-\rho)}{b\{2(\lambda-r)(n+2) - (\lambda-\rho)(n+1)\}} > 0 \quad \text{and} \quad \frac{\partial r}{\partial n} = -\frac{(\lambda-r)(r-\rho)}{2(\lambda-r)(n+2) - (\lambda-\rho)(n+1)} < 0.$$

Using the expression of r in (42), it is easy to show that the denominator of each of the above two expressions is positive. On the other hand, from (23), it follows that

$$\frac{dR}{dn} = \frac{\rho}{(n+1)^2} = \frac{R}{n(n+1)} > 0.$$

Note that the sign of the bank-competitor effect is completely determined by that of dr/dn, which is given by:

$$\frac{dr}{dn} = \underbrace{\frac{\partial r}{\partial n}}_{\text{direct effect}} + \underbrace{\frac{\partial r}{\partial R} \cdot \frac{dR}{dn}}_{\text{indirect effect}} .$$

We first show that both the terms in the right-hand-side of the above expression is increasing in ρ. Substituting the equilibrium values of r and R into the expression of $\partial r/\partial n$, and differentiating with respect to ρ we obtain the following:[38]

$$\begin{aligned} &\text{sign}\left[\frac{\partial}{\partial \rho}\left(\frac{\partial r}{\partial n}\right)\right] \\ &= \text{sign}\left[1 + \frac{b^2(\lambda-\rho)^3(n+1)^7 + 8a^2\rho^3 n^2(n+2)^2 + 2ab\rho(\lambda-\rho)n(n+1)^3(n+2)(\lambda(n+5)+(\lambda-\rho)(n-1))}{b^{\frac{1}{2}}(b(\lambda-\rho)^2(n+1)^4 + 4a\rho^2 n(n+2))^{\frac{3}{2}}}\right], \end{aligned}$$

which is strictly positive. Clearly, $\partial|(\partial r/\partial n)|/\partial\rho < 0$, i.e., the negative direct effect dampens as ρ increases. As far as the indirect effect is concerned, using $R = \rho n/(n+1)$ in $\partial r/\partial R$ and dR/dn, and differentiating the indirect effect with respect to ρ, we obtain

$$\text{sign}\left[\frac{\partial}{\partial \rho}\left(\frac{\partial r}{\partial R} \cdot \frac{dR}{dn}\right)\right] = \text{sign}\left[b(\lambda-\rho)(2\lambda-\rho)(n+1)^4 - 2a\rho^2 n(n+2)(n-3)\right],$$

which is also positive for any $a > 0$, $b > 0$, $\lambda > 0$ and $0 < \rho \leq \lambda$. Therefore, dr/dn is strictly increasing in ρ. Let $f(\rho)$ denote dr/dn evaluated at ρ. Note that

$$f(0) = \frac{\lambda(b+1)\{b(n+1)-(n+3)\}}{4b(n+2)^2} \quad \text{and} \quad f(\lambda) = \frac{\lambda^4 a^2 bn\{n(n+1)+b(n^2+n-2)\}}{2(n+1)^2[\lambda^2 abn(n+2)]^{3/2}} > 0.$$

The sign of $f(0)$ ambiguous. There are the following possible cases. First, if $f(0) \geq 0$, i.e., $b \geq \frac{n+3}{n+1}$, then $f(\rho) \geq 0$ for all $\rho \geq 0$. Then, set $\rho^* = 0$. Thus, the equilibrium loan rate is increasing for all possible values of the interbank rate. Next, consider the case when $f(0) < 0$ i.e., $b < \frac{n+3}{n+1}$. Because $f(\lambda) > 0$ and $f'(\rho) > 0$, it follows from the Intermediate Value theorem, there is a unique $\rho^* \in (0, \lambda)$ such that $dr/dn < 0$ if and only if $\rho < \rho^*$. If $\rho^* \geq \bar{\rho}$, then set $\rho^* = \bar{\rho}$, i.e., $dr/dn < 0$ for all $\rho \in [0, \bar{\rho}]$. Otherwise, $\rho^* < \bar{\rho}$, and hence, the bank-competitor effect is positive if and only if $\rho > \rho^*$. This case is more likely for large $n(m)$ because $\bar{\rho} \to \lambda$ and $n \to \infty$ and $f(\lambda) > 0$. This completes the proof of part

[38]We omit the unmanageable algebraic expressions. The *Mathematica* codes are available upon request.

(b).

Proof of Lemma 4

Note first that $p(\theta) = 1 - \theta/\lambda$ and $r(L) = \lambda - L/2b$ together imply that the probability of success as a function of the aggregate loan volume is given by $P(L) = L/4b\lambda$. Moreover the equilibrium must respect $L = D_1 + D_2 = D$. The IME is characterized by within-group symmetry, i.e., $D_{ij} = D_j/n$ because there are n each type of banks. Therefore, the first-order condition of each type-j bank i yields

$$(D_1 + D_2)\left(\lambda - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)(D_1 + D_2)\right) + \frac{D_1}{n}\left(\lambda - \left(\frac{1}{a} + \frac{1}{b}\right)(D_1 + D_2)\right) = 0, \tag{43}$$

$$(D_1 + D_2)\left(\lambda - c - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)(D_1 + D_2)\right) + \frac{D_2}{n}\left(\lambda - c - \left(\frac{1}{a} + \frac{1}{b}\right)(D_1 + D_2)\right) = 0. \tag{44}$$

The above system of non-linear equations have three sets of solutions in (D_1, D_2). The first set is discarded because it has $D_1 = D_2 = 0$. The final expressions for D_1 and D_2 in the other two sets of solutions are very cumbersome, so we omit them. The second set of solutions (D_1, D_2) yields

$$D = D_1 + D_2 = \frac{ab\left\{(2\lambda - c)(3n + 2) - \sqrt{c^2(3n + 2)^2 + 4\lambda(\lambda - c)n^2}\right\}}{4(a + b)(n + 1)}.$$

Note that, at $c = 0$, the above expression becomes $\lambda ab/(a + b)$ which is independent of n, and hence, is discarded. The last set of solutions (D_1, D_2) yields

$$D^*(n, c) = \frac{ab\left\{(2\lambda - c)(3n + 2) + \sqrt{c^2(3n + 2)^2 + 4\lambda(\lambda - c)n^2}\right\}}{4(a + b)(n + 1)}.$$

It turns out that $D_2^* > 0$ if $c < \lambda/(n + 2) \equiv \bar{c}$ because $n \geq 2$.

Proof of Proposition 7

The equilibrium risk-shifting is given by:

$$\theta^*(n, c) = \lambda - \frac{D^*(n, c)}{4b} = \lambda - \frac{a\left\{(2\lambda - c)(3n + 2) + \sqrt{c^2(3n + 2)^2 + 4\lambda(\lambda - c)n^2}\right\}}{16(a + b)(n + 1)}.$$

It is immediate to show that $\theta^*(n, c) < \theta_1^0(n, c)$ if Lemma 4 holds, and hence, $\theta^*(n, c) < \theta_j^0(n, c)$ for $j = 1, 2$.

Proof of Proposition 8

Within-group symmetry and $D_1 + D_2 = D$ together imply that the expected profits of each type j is given by:

$$\pi_j = P(D)\left(r(D) - R(D) - c_j\right) \cdot \frac{D_j}{n} = \frac{D}{4b\lambda n}\left(\lambda - c_j - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right) D\right) D_j.$$

Therefore in the pre-merger IME, each type-1 and type-2 banks earn

$$\pi_1^*(n, c) = \frac{D^*}{4b\lambda n}\left(\lambda - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right) D^*\right) D_1^*,$$
$$\pi_2^*(n, c) = \frac{D^*}{4b\lambda n}\left(\lambda - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right) D^*\right) D_2^* - \frac{cD^*D_2^*}{4b\lambda n}.$$

In the post-merger IME, on the other hand, there are only n identical low-cost banks, and the expected profit of each of them is given by:

$$\pi_M(n) = \frac{(D_M)^2}{4b\lambda n}\left(\lambda - \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right) D_M\right),$$

where $D_M = 2\lambda ab(n + 1)/(a + b)(n + 2)$ is the aggregate deposit in the post-merger IME. It is easy to show that $\pi_M(n) > \pi_1^*(n, c) + \pi_2^*(n, c)$ for $n \geq 2$ and $c \in (0, \bar{c})$. Therefore, $\pi_M(n)$ can be written as $\pi_M(n) = \pi_1^*(n, c) + \pi_2^*(n, c) + \varepsilon$ where $\varepsilon > 0$. In the post-merger equilibrium thus the aggregate profit from the merger between low- and high-cost banks, $\pi_M(n)$ can be divided between them in a way such that the low-cost bank receives $\pi_1^*(n, c) + \varepsilon/2 > \pi_1^*(n, c)$, and the high-cost bank receives $\pi_2^*(n, c) + \varepsilon/2 > \pi_2^*(n, c)$. Therefore, these two banks would agree on the merger which generates strictly higher profits for both of them.

If the deposit supply and loan demand in market j (with cost parameter c_j) are respectively given by $R_j = D_j/a_j$ and $r_j = \lambda - (L_j/b_j)$, then following similar steps as in the proof of Proposition 5, it is east to show that the equilibrium risk-shifting is given by:

$$\theta_j^0(n, c) = \lambda - \frac{a_j(\lambda - c_j)(n + 1)}{2(a_j + b_j)(n + 2)}.$$

Because we have assumed that $a_1 = a_2 = a$, $b_1 = b_2 = b$, $c_1 = 0$ and $c_2 = c$, in the SME we have

$$\theta_1^0(n, c) = \lambda - \frac{\lambda a(n+1)}{2(a+b)(n+2)},$$
$$\theta_2^0(n, c) = \lambda - \frac{a(\lambda - c)(n+1)}{2(a+b)(n+2)}.$$

On the other hand, the deposit supply and loan demand schedules are respectively given by $R = D/2a$ and $r = \lambda - (L/2b)$. Therefore, the equilibrium risk-shifting in the post-merger IME is given by:

$$\theta_M^0(n) = \lambda - \frac{\lambda(2a)(n+1)}{2(2a+2b)(n+2)} = \lambda - \frac{\lambda a(n+1)}{2(a+b)(n+2)} = \theta_1^0(n, c).$$

Proof of Proposition 9

Let the deposit supply and loan demand functions in the integrated market are given by

$$R\,\underbrace{(D_1 + D_2)}_{D} = \frac{D}{a}, \quad \text{where } a \equiv a_1 + a_2,$$
$$r\,\underbrace{(L_1 + L_2)}_{L} = \lambda - \frac{L}{b}, \quad \text{where } b \equiv b_1 + b_2.$$

The post-merger risk-shifting in the integrated market is given by

$$\theta_M(n) = \lambda - \frac{\lambda a(n+1)}{2(a+b)(n+2)} = \lambda - \frac{\lambda(a_1 + a_2)(n+1)}{2(a_1 + a_2 + b_1 + b_2)(n+2)}.$$

Note that

$$\begin{aligned} & \theta_M(n) > \theta_1^0(n, c) \\ \Longleftrightarrow \quad & \lambda - \frac{\lambda(a_1 + a_2)(n+1)}{2(a_1 + a_2 + b_1 + b_2)(n+2)} > \lambda - \frac{\lambda a_1(n+1)}{2(a_1 + b_1)(n+2)} \\ \Longleftrightarrow \quad & \frac{b_2}{b_1} > \frac{a_2}{a_1}. \end{aligned}$$

Recall that $\theta^*(n, c)$ is the equilibrium risk-shifting in the integrated market before any merger takes place. Because the bank-customer effect is positive, it is not clear whether $\theta^*(n, c) < \theta_1^0(n, c)$. However, our concern is whether the risk-taking in the post-merger equilibrium of the integrated market has increased relative to the autarkic equilibrium in market 1, the low-cost market, which we have shown.

References

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2015), "Systemic Risk and Stability in Financial Networks." *The American Economic Review*, 105, 564–608.

Akins, Brian, Lynn Li, Jeffrey Ng, and Tjomme Rusticus (2016), "Bank Competition and Financial Stability: Evidence from the Financial Crisis." *Journal of Financial and Quantitative Analysis*, 51, 1–28.

Allen, Franklin and Douglas Gale (2000), *Comparing Financial Systems*. MIT Press, Cambridge, MA.

Allen, Franklin and Douglas Gale (2004), "Competition and Financial Stability." *Journal of Money, Credit and Banking*, 36, 453–480.

Amel, Dean (1993), "State Laws Affecting the Geographic Expansion of Commercial Banks." Board of Governors of the Federal Reserve System.

Banerjee, Abhijit (2003), "Contracting Constraints, Credit Markets, and Economic Development." In *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress, Volume III* (Mathias Dewatripont, Lars Peter Hansen, and Stephen Turnovsky, eds.), Cambridge University Press, Cambridge.

Barth, James, Chen Lin, Ping Lin, and Frank M. Song (2009), "Corruption in Bank Lending to Firms: Cross-country Micro Evidence on the Beneficial Role of Competition and Information Sharing." *Journal of Financial Economics*, 91, 361–388.

Beck, Thorsten, Diane Coyle, Mathias Dewatripont, Xavier Freixas, and Paul Seabright (2010), "Bailing out the Banks: Reconciling Stability and Competition." Centre for Economic Policy Research.

Beck, Thorsten, Olivier De Jonghe, and Glenn Schepens (2013), "Bank Competition and Stability: Cross-country Heterogeneity." *Journal of Financial Intermediation*, 22, 218–244.

Berger, Allen, Rebecca Demsetz, and Phillip Strahan (1999), "The Consolidation of the Financial Services Industry: Causes, Consequences, and Implications for the Future." *Journal of Banking and Finance*, 23, 135–194.

Berger, Allen, Anil Kashyap, and Joseph Scalise (1995), "The Transformation of the U.S. Banking Industry: What a Long, Strange Trips It's Been." *Brookings Papers on Economic Activity*, 26, 55–218.

Berger, Allen, Leora Klapper, and Rima Turk-Ariss (2009), "Bank Competition and Financial Stability." *Journal of Financial Services Research*, 35, 99–118.

Bernheim, Douglas and Michael Whinston (1990), "Multimarket Contact and Collusive Behavior." *The RAND Journal of Economics*, 21, 1–26.

Besanko, George and David Kanatas (1993), "Credit Market Equilibrium with Bank Monitoring and Moral Hazard." *The Review of Financial Studies*, 6, 213–232.

Boyd, John and Gianni De Nicoló (2005), "The Theory of Bank Risk Taking and Competition Revisited." *The Journal of Finance*, 60, 1329–1343.

Boyd, John and Stanley Graham (1998), "Consolidation in U.S. Banking: Implications for Efficiency and Risk." In *Banking Mergers and Acquisitions* (Yakov Amidhud and Geoffrey Miller, eds.), Klewer Publishing Co., New York.

Brander, James and Barbara Spencer (1989), "Moral Hazard and Limited Liability: Implications for the Theory of the Firm." *International Economic Review*, 30, 833–849.

Broecker, Thorsten (1990), "Credit-Worthiness Tests and Interbank Competition." *Econometrica*, 58, 429–452.

Carletti, Elena and Xavier Vives (2009), "Regulation and Competition Policy in the Banking Sector." In *Competition Policy in the EU: Fifty Years on from the Treaty of Rome* (Xavier Vives, ed.), 260–282, Oxford University Press.

Carlson, Mark, Sergio Correia, and Stephan Luck (2021), "The Effects of Banking Competition on Growth and Financial Stability: Evidence from the National Banking Era." Forthcoming, *Journal of Political Economy*.

Corbae, Dean and Ross Levine (2018), "Competition, Stability, and Efficiency in Financial Markets." Jackson Hole Symposium: Changing Market Structure and Implications for Monetary Policy.

Degryse, Hans and Steven Ongena (2008), "Competition and Regulation in the Banking Sector: A Review of the Empirical Evidence on the Sources of Bank Rents." In *Handbook of Financial Intermediation and Banking* (Anjan Thakor and Arnoud Boot, eds.), chapter 15, 483–554, Elsevier, Amsterdam.

Dell'Ariccia, Giovanni and Robert Marquez (2013), "Interest Rates and the Bank Risk-Taking Channel." *Annual Review of Financial Economics*, 5, 123–141.

Dermine, Jean (1986), "Deposit Rates, Credit Rates and Bank Capital: The Klein-Monti Model Revisited." *Journal of Banking and Finance*, 10, 99–114.

DeYoung, Robert (2008), "Safety, Soundness, and the Evolution of the U.S. Banking Industry." In *Handbook of Financial Intermediation and Banking* (Anjan Thakor and Arnoud Boot, eds.), chapter 11, 347–373, Elsevier, Amsterdam.

DeYoung, Robert, Douglas Evanoff, and Philip Molyneux (2009), "Mergers and Acquisitions of Financial Institutions: A Review of the Post-2000 Literature." *Journal of Financial Services Research*, 36, 87–110.

Farrell, Joseph and Carl Shapiro (1990), "Horizontal Mergers: An Equilibrium Analysis." *The American Economic Review*, 80, 107–126.

Favara, Giovanni and Jean Imbs (2015), "Credit Supply and the Price of Housing." *The American Economic Review*, 105, 958–992.

Fecht, Falco, Roman Inderst, and Sebastian Pfeil (2017), "A Theory of the Boundaries of Banks with Implications for Financial Integration and Regulation." IMFS Working Paper Series 87.

Freixas, Xavier and Jean-Charles Rochet (2008), *Microeconomics of Banking*, second edition. The MIT Press, Cambridge, MA.

Gaspar, Vítor, Philipp Hartmann, and Olaf Sleijpen (2003), *The Transformation of the European Financial System: Second ECB Central Banking Conference, October 2002, Frankfurt, Germany*. European Central Bank.

González-Aguado, Carlos and Javier Suárez (2015), "Interest Rates and Credit Risk." *Journal of Money, Credit and Banking*, 47, 445–480.

Hellmann, Thomas, Kevin Murdock, and Joseph Stiglitz (2000), "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" *The American Economic Review*, 90, 147–165.

Hermalin, Benjamin (1994), "Heterogeneity in Organizational Form: Why Otherwise Identical Firms Choose Different Incentives for their Managers." *The RAND Journal of Economics*, 25, 518–537.

Houston, Joel, Chen Lin, Ping Lin, and Yue Ma (2010), "Creditor Rights, Information Sharing, and Bank Risk Taking." *Journal of Financial Economics*, 96, 485–512.

Jayaratne, Jith and Phillip Strahan (1998), "Entry Restrictions, Industry Evolution, and Dynamic Efficiency: Evidence from Commercial Banking." *Journal of Law and Economics*, 41, 239–73.

Jehle, Geoffrey and Philip Reny (2011), *Advanced Microeconomic Theory*, third edition. Pearson, Harlow, England.

Jiang, Liangliang, Ross Levine, and Chen Lin (2017), "Does Competition Affect Bank Risk?" NBER Working paper 23080.

Jones, Kenneth and Tim Critchfield (2008), "Consolidation in the U.S. Banking Industry: Is the "Long, Strange Trip" About to End?" In *Handbook of Financial Intermediation and Banking* (Anjan Thakor and Arnoud Boot, eds.), chapter 10, 309–346, Elsevier, Amsterdam.

Keeley, Michael (1990), "Deposit Insurance, Risk, and Market Power in Banking." *The American Economic Review*, 80, 1183–1200.

Klein, Michael (1971), "A Theory of the Banking Firm." *Journal of Money, Credit and Banking*, 3, 205–218.

Kroszner, Randall (2001), "The Motivations Behind Banking Reform: Why Do Lawmakers Pursue Deregulation?" *Regulation*, Summer, 36–41.

Kroszner, Randall and Philip Strahan (1999), "What Drives Deregulation? Economics and Politics of the Relaxation of Bank Branching Restrictions." *The Quarterly Journal of Economics*, 114, 1437–1467.

Kroszner, Randall and Phillip Strahan (2014), "Regulation and Deregulation of the U.S. Banking Industry: Causes, Consequences, and Implications for the Future." In *Economic Regulation and Its Reform: What Have We Learned?* (Nancy Rose, ed.), 485–543, National Bureau of Economic Research.

Krugman, Paul (1979), "Increasing Returns, Monopolistic Competition and International Trade." *Journal of International Economics*, 9, 469–479.

Mankiw, Gregory and Michael Whinston (1986), "Free Entry and Social Inefficiency." *RAND Journal of Economics*, 17, 48–58.

Martinez-Miera, David and Rafael Repullo (2010), "Does Competition Reduce the Risk of Bank Failure?" *The Review of Financial Studies*, 23, 347–3664.

Martinez-Miera, David and Rafael Repullo (2017), "Search for Yield." *Econometrica*, 85, 351–378.

Martinez-Miera, David and Rafael Repullo (2019), "Monetary Policy, Macroprudential Policy, and Financial Stability." *Annual Review of Economics*, 11, 809–832.

Matutes, Carmen and Xavier Vives (2000), "Imperfect Competition, Risk, and Regulation in Banking." *European Economic Review*, 44, 1–34.

Mian, Atif and Amir Sufi (2018), "Finance and Business Cycles: The Credit-Driven Household Demand Channel." *Journal of Economic Perspectives*, 32, 31–58.

Mian, Atif, Amir Sufi, and Emil Verner (2020), "How Does Credit Supply Expansion Affect the Real Economy? The Productive Capacity and Household Demand Channels." *The Journal of Finance*, 75, 949–994.

Motta, Massimo (2004), *Competition Policy: Theory and Practice*. Cambridge University Press, Cambridge.

Novshek, William (1980), "Cournot Equilibrium with Free Entry." *The Review of Economic Studies*, 47, 473–486.

OECD (2011), "OECD Report on Bank Competition and Financial Stability."

Park, Kwangwoo and George Pennacchi (2008), "Harming Depositors and Helping Borrowers: The Disparate Impact of Bank Consolidation." *The Review of Financial Studies*, 22, 1–40.

Radecki, Lawrence (1998), "The Expanding Geographic Reach of Retail Banking Markets." *Economic Policy Review, Federal Reserve Bank of New York*, 4, 15–34.

Repullo, Rafael (2004), "Capital Requirements, Market Power, and Risk-Taking in Banking." *Journal of Financial Intermediation*, 13, 156–182.

Salant, Stephen, Sheldon Switzer, and Robert Reynolds (1983), "Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium." *The Quarterly Journal of Economics*, 98, 185–199.

Sharpe, Steven (1990), "Asymmetric Information, Bank Lending, and Implicit Contracts: A Stylized Model of Customer Relationships." *The Journal of Finance*, 45, 1069–1087.

Simmons, George (2017), *Differential Equations with Applications and Historical Notes*, third edition. CRC Press, Boca Raton, FL.

Stiglitz, Joseph and Andrew Weiss (1981), "Credit Rationing in Markets with Imperfect Information." *The American Economic Review*, 71, 393–410.

Tarullo, Daniel (2012), "Financial Stability Regulation." Distinguished Jurist Lecture, University of Pennsylvania Law School, Philadelphia.

Vives, Xavier (2019), "Competition and Stability in Modern Banking: A Post-crisis Perspective." *International Journal of Industrial Organization*, 64, 55–69.

Wagner, Wolf (2010), "Loan Market Competition and Bank Risk-Taking." *Journal Financial Services Research*, 37, 71–81.