Money, Growth, and Welfare in a Schumpeterian Model with the Spirit of Capitalism

Qichun He, Yulei Luo, Jun Nie, and Heng-fu Zou
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Money, Growth, and Welfare in a Schumpeterian Model with the Spirit of Capitalism*

Qichun He† Yulei Luo‡ Jun Nie§ Heng-fu Zou¶

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Abstract

According to Schumpeter (1934), entrepreneurs are driven to innovate not for the fruits of success but for success itself. This description of entrepreneurship echoes Weber’s (1958) description of the “spirit of capitalism,” which states that people enjoy the accumulation of wealth irrespective of its effect on smoothing consumption. This paper explores the implications of the spirit of capitalism on monetary policy, growth, and welfare in a Schumpeterian growth model. Different from the existing literature, we show that money is not superneutral in the long run and it could promote economic growth when the spirit of capitalism is strong. Furthermore, we show the optimal nominal interest rate decreases with the strength of the spirit of capitalism, potentially supporting a negative interest rate. Finally, our calibrated model suggests that the spirit of capitalism explains an important share (about one-third) of long-run growth in the United States.

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Keywords: spirit of capitalism; cash-in-advance; Schumpeterian model; monetary policy; growth and welfare

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† He: China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. Email: qichunhe@gmail.com.

‡ Luo: Faculty of Business and Economics, University of Hong Kong, Hong Kong. Email: yulei.luo@gmail.com.

§ Nie: Research Department, Federal Reserve Bank of Kansas City, U.S.A. E-mail: jun.nie@kc.frb.org.

¶ Zou: China Economics and Management Academy, Central University of Finance and Economics, Beijing, China. Email: hzoucema@gmail.com.
1 Introduction

Since the pioneering work by Aghion and Howitt (1992), Schumpeterian growth theory has developed into a key framework for understanding long-run economic growth. This theory formulates Schumpeter’s 1942 notion of “creative destruction”—the process by which new innovations replace old technologies—and shows that innovations resulting from entrepreneurial investments are crucial to long-run economic growth.

But what drives entrepreneurs to innovate? Schumpeter (1934) refutes the traditional and hedonistic assumption that defines entrepreneurs’ utility on consumption and instead emphasizes the “psychology of entrepreneurs”: the entrepreneur is strongly motivated by the “dream and the will to found a private kingdom, usually, though not necessarily, also a dynasty.”


dThen there is the will to conquer, the impulse to fight, to prove oneself superior to others, to succeed for the sake, not of the fruits of success, but of success itself, from this aspect, economic action becomes akin to sport.... The financial result is a secondary consideration, or, at all events, mainly valued as an index of success and as a symptom of victory, the displaying of which very often is more important as a motive of large expenditure than the wish for the consumers’ goods themselves.” (p.93)

This description of entrepreneurship is very similar to Max Weber’s description of the “spirit of capitalism”:

“In fact, the summum bonum of his ethic, the earning of more and more money, combined with the strict avoidance of all spontaneous enjoyment of life, is above all completely devoid of any eudaemonistic, not to say hedonistic, admixture. It is thought of so purely as an end in itself, that from the point of view of the happiness of, or utility to, the single individual, it appears entirely transcendental and absolutely irrational. Man is dominated by the making of money, by acquisition as the ultimate purpose of his life. Economic acquisition is no longer subordinated to man as the means for the satisfaction of his material needs. This reversal of what we should call the natural relationship, so irrational from a naive point of view, is evidently as definitely a leading principle of capitalism as it is foreign to all peoples not under capitalistic influence.” (p.53)

The literature on the spirit of capitalism—enjoying the accumulation of wealth irrespective of its effect on consumption smoothing—shows that it has important implications on economic

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1See Weber (1958).
growth and asset pricing in non-Schumpetarian growth frameworks (Zou 1994; Bakshi and Chen 1996). However, little research has explored how the spirit of capitalism would influence innovation and long-run growth in a Schumpeterian growth model.

Incorporating the spirit of capitalism into such a model could also have significant implications for the conduct of monetary policy. In a Schumpeterian model with money, Chu and Cozzi (2014) show that an increase in the nominal interest rate reduces both R&D and economic growth, and that the optimal monetary policy features a positive nominal interest rate (thereby violating the Friedman rule (Friedman 1969), which says the optimal non-negative nominal interest rate is zero). The key intuition behind their results is that a higher interest rate raises the borrowing cost of entrepreneurial investment that is subject to a cash-in-advance (CIA) constraint, which helps mitigate the possible overinvestment issue in Schumpeterian models (see Aghion and Howitt 1992, 1998). However, the spirit of capitalism introduces a new channel for monetary policy to influence R&D investment, labor allocation, and consumption-leisure decisions; therefore, whether these new findings on monetary policy still hold requires a careful analysis.

To fill these gaps in the literature, we formalize Schumpeter’s idea of the “psychology of entrepreneurs” by introducing the spirit of capitalism into a stylized Schumpeterian model with money based on Aghion et al. (2013) and Chu and Cozzi (2014). Our model differs from the models in these papers by introducing the spirit of capitalism. Moreover, our model differs from Chu and Cozzi (2014) by including separate parameters for the taste for leisure and the elasticity of the labor supply. This separation allows us to examine if these two important structural parameters play different roles in driving the key results. To the best of our knowledge, this is the first paper to provide a unified framework to investigate (both theoretically and quantitatively) the implications of the spirit of capitalism for the effects of monetary policy on long-run growth and welfare.

Our analysis delivers three main findings. First, we show that introducing the spirit of capitalism into the Schumpeterian growth framework yields novel insights into the effects of monetary policy on long-run growth and welfare. Specifically, we theoretically prove that money is not superneutral in this framework and that the effect of higher nominal interest rates on long-run growth depends on the strength of the spirit of capitalism in an economy. When the spirit of capitalism is small or absent, a higher interest rate reduces growth. However, when the spirit of capitalism is strong enough (relative to the elasticity of labor and taste for leisure), higher

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2 As explained in Aghion and Howitt (1992), due to the “business-stealing” effect (i.e., the private business does not internalize the loss to the previous monopolist caused by an innovation), economic growth could be excessive relative to the optimal level (in which case the social planner takes into account the loss to the previous monopolist caused by an innovation).
nominal interest rates could promote growth.

The key intuitions are as follows. Without the spirit of capitalism, an increase in the nominal interest rate reduces long-run growth through two channels. The first channel is that, given an elastic labor supply and the CIA constraint on consumption, higher nominal interest rates reduce the labor supply by encouraging households to choose leisure over consumption, leading to lower R&D labor and therefore lower long-run growth (this is the market size effect highlighted in Aghion et al. [2013]). The second channel is that, with a CIA constraint on R&D, higher nominal interest rates also increase borrowing costs for entrepreneurs, shifting labor away from R&D toward manufacturing and reducing long-run growth (a negative labor reallocation effect).

In contrast, with the spirit of capitalism, an increase in the nominal interest rate could promote long-run growth. Specifically, when the spirit of capitalism is strong, a higher nominal interest rate induces consumers (who have a direct preference for wealth) to increase their savings, thereby lowering the real interest rate and borrowing costs for entrepreneurs. Lower borrowing costs, in turn, increase R&D labor. We show that higher nominal interest rates could amplify the spirit of capitalism effect. Therefore, when the spirit of capitalism is strong enough, the positive labor reallocation effect could dominate the two negative effects described above, causing growth to increase in response to an increase in the nominal interest rate.

Second, we quantitatively show that the optimal nominal interest rate decreases with the degree of the spirit of capitalism, suggesting the Friedman rule— that the optimal non-negative interest rate is zero—is likely to be valid. This finding is in direct contrast to the finding in Chu and Cozzi (2014) which shows the optimal nominal interest rate is positive. It also provides theoretical support for negative interest rates, which have been discussed intensively in recent years.

Third, we calibrate our model to the U.S. economy and quantify the contribution of the spirit of capitalism to long-run growth. Our analysis suggests that the spirit of capitalism explains one-third of long-run growth in the United States (0.6 percent out of 1.8 percent annually), which is substantial. To the best of our knowledge, this is the first study to quantify the growth contribution of the spirit of capitalism. In this sense, our analysis enriches Aghion and Howitt (1992) and provides a more complete view of Schumpeterian growth models in understanding long-run growth.

Our paper contributes to a large literature studying the implications of the spirit of capitalism or the quest for status on economic growth (Zou 1994; Futagami and Shibata 1998; Smith 1999; Corneo and Jeanne 2001), savings (Cole, Malaith and Postlewaite 1992; Zou 1995; Carroll 2000; Luo et al. 2009), asset pricing (Bakshi and Chen 1996; Smith 2001; Gong and Zou 2002), wealth distribution (Luo and Young, 2009; Corneo and Jeanne, 2001), business cycles (Karnizova 2010;
Michaillat and Saez 2015), money (Gong and Zou 2001; Michaillat and Saez 2019), taxation (Saez and Stantcheva 2018), comparison with recursive utility (Alaoui and Sandroni 2018), patent protection (Pan et al. 2018), expansion of variety (Hof and Prettl 2019), and industrialization (Chu et al. 2020). Different from the existing literature, our paper emphasizes the role of the spirit of capitalism on monetary policy and long-run growth in a R&D-based Schumpeterian growth model. In addition, we carefully calibrate the model and quantify the growth and welfare implications of the spirit of capitalism.

Our paper also adds to the literature on the effect of monetary policy on growth and welfare (for example, Sidrauski 1967; Stockman 1981; Gomme 1993; Marquis and Reffett 1994; Jones and Manuelli 1995; Dotsey and Sarte 2000; Akyol 2004; Funk and Kromen 2010; Chu and Cozzi 2014; Brunnermeire and Sannikov 2016; Arawatari et al. 2018; Chu et al. 2019). The novel contribution of our paper is an examination of how, in a Schumpeterian model, the effects of monetary policy on growth and welfare crucially depend on the strength of the spirit of capitalism.

The remainder of this paper is organized as follows. Section 2 presents the monetary Schumpeterian model with the spirit of capitalism. Section 3 presents theoretical results. Section 4 presents quantitative results. Section 5 provides further discussions. Section 6 concludes.

2 A Monetary Schumpeterian Model with the Spirit of Capitalism

We introduce the spirit of capitalism (henceforth the SOC)—a direct preference for wealth—into a Schumpeterian growth model with money. In particular, we introduce money demand via CIA constraints on both consumption (Clower, 1967; Lucas, 1980) and R&D (Chu and Cozzi, 2014). Our Schumpeterian quality-ladder model is quite standard which combines Aghion et al. (2013) and Chu and Cozzi (2014) and provides a general framework to evaluate the SOC effects.

2.1 Households

We assume there is a unit continuum of identical households in the economy. The population size of each household is fixed at $L$. Each household has a lifetime utility function as follows:

$$U = \int_0^\infty e^{-\rho t} \left[ \ln (c_t) + \theta \ln (a_t) - \eta \frac{l_t^{1+1/\sigma}}{1 + 1/\sigma} \right] dt,$$

where $c_t$ is per capita real consumption of final goods; $a_t$ is per capita wealth, and $l_t$ is per capita labor supply at time $t$. $\rho$ is the discount rate; $\eta$ measures the disutility of labor (Aghion et al.
2013) or the taste for leisure (Doepke and Zilibotti, 2008); \( \sigma > 0 \) is the Frisch elasticity of labor supply.

\( \theta \geq 0 \) captures the preference for wealth or the strength of the SOC. A larger \( \theta \) means a stronger preference on wealth or stronger SOC. Notice that in a representative-agent framework, introducing the SOC in the household sector can be considered as a general way to model the “psychology of entrepreneurs” described by Schumpeter’s entrepreneurship. Similar to Angeletos and Panousi (2009), this can be rationalized by assuming internal division of labor within households—some household members specialize in making consumption/savings decisions, while the rest are responsible for entrepreneurial decisions.

Each household maximizes its lifetime utility given in equation (1) subject to the following asset-accumulation equation:

\[
\dot{e}_t + \dot{m}_t = r_t e_t + w_t l_t - c_t - \pi_t m_t + i_t b_t + \tau_t, \tag{2}
\]

where \( e_t \) is the real value of assets (i.e., the real value of equity shares in monopolistic intermediate-goods firms); \( r_t \) and \( w_t \) are the rate of real interest and the wage, respectively; \( m_t \) is the real money balance holding; \( \pi_t \) is the cost of holding money (or the inflation rate); \( b_t \) refers to the cash borrowed by entrepreneurs to finance the wage bill of R&D workers which incurs an interest payment, \( i_t b_t \). Following the standard literature, we assume households receive a lump-sum transfer of seigniorage revenue, \( \tau_t \) (which will be defined below).

Given the budget constraint, we define wealth \( a_t \) as the sum of the real equity value and the real money balance: \( a_t \equiv e_t + m_t \). The CIA constraint is given by \( c_t + b_t \leq m_t \). Using the Hamiltonian function, we have \( i_t = \pi_t + r_t \) (the Fisher equation) and the optimality condition for consumption is

\[
\frac{1}{c_t} = \mu_t (1 + i_t), \tag{3}
\]

where \( \mu_t \) is the Hamiltonian co-state variable on (2). (See Appendix 7.1 for the derivation) The optimal condition for labor supply is

\[
\eta^{1/\sigma} = w_t \mu_t. \tag{4}
\]

Combining (3) and (4) leads to the optimal condition for labor supply:

\[
l_t = \left( \frac{w_t}{\eta c_t (1 + i_t)} \right)^{\sigma}, \tag{5}
\]
which shows that a higher nominal interest rate reduces labor supply, all else equal; so does a higher taste for leisure.

The Euler equation for real assets \((e_t)\) is

\[
- \frac{\dot{\mu}_t}{\mu_t} = r_t - \rho + \frac{\theta}{\mu_t a_t}.
\]

Combining (4) and (5) yields the following consumption Euler equation:

\[
\frac{\dot{c}_t}{c_t} = r_t - \rho + \theta (1 + i_t) \frac{c_t}{a_t}.
\]

where the consumption-wealth ratio, \(c_t/a_t\), is the marginal propensity of consumption out of total wealth (MPC). Note that \(\theta = 0\) gives the monetary Schumpeterian model without the SOC, which says that a higher real interest rate would lead to more savings and less consumption (i.e., leading to higher marginal utility of consumption); in contrast, when the discount rate \((\rho)\) is higher, it encourages more consumption (i.e., leading to lower marginal utility of consumption). When \(\theta > 0\), the preference for wealth accumulation tends to lower consumption by introducing a third term on the right side of equation (7). We will use this equation to discuss more the interaction between the nominal interest rate and the spirit of capitalism in section 3.

2.2 Monetary Authority

Let the nominal money stock at the beginning of time \(t\) be \(M_t\). For convenience, we assume that the monetary authority controls the nominal interest rate \(i_t\) (through controlling the growth rate of money stock \(\dot{M}_t/M_t\)) and rebates the seigniorage revenue back to households. The per capita seigniorage revenue is \(\tau_t = M_t/(P_t L)\). The per capita real money balance is \(m_t = M_t/(P_t L)\), where \(P_t\) is the price level of the final goods and the inflation rate is given by \(P_t/P_t = \pi_t\). Thus, \(\dot{m}_t/m_t = M_t/M_t - \pi_t\) and \(\tau_t = (\dot{m}_t/m_t + \pi_t) m_t = \dot{m}_t + \pi_t m_t\).

If we substitute the Fisher equation \(i_t = \pi_t + r_t\), the equilibrium result \(\dot{m}_t/m_t = \dot{c}_t/c_t\), and \(\dot{M}_t/M_t = \dot{m}_t/m_t + \pi_t\) into the consumption Euler equation (7), we have

\[
\frac{\dot{M}_t}{M_t} = i_t - \rho + \theta (1 + i_t) \frac{c_t}{a_t}.
\]

The existence of the SOC slightly complicates the relationship between the nominal interest rate \(i_t\) and the money supply growth rate \(\dot{M}_t/M_t\) by introducing an extra term on the right side of (8). Fixing the consumption-to-wealth ratio \(c_t/a_t\), it is easy to see higher nominal interest rates also lead to higher money supply growth. However, as we will derive later, the consumption-to-wealth ratio is also a function of nominal interest rate. In Online Appendix C, we numerically confirm that the positive relationship is well maintained in the presence of the SOC.
2.3 Production Side

We follow Chu and Cozzi (2014) who rewrite the Schumpeterian model of Aghion and Howitt (1992) to remove the scale effect (explained later). First, there is a perfectly competitive final goods sector in which the final-goods firms have the following production function:

\[ y_t = \exp \left( \int_0^1 \ln x_t(\epsilon) \, d\epsilon \right), \quad (9) \]

where \( x_t(\epsilon) \) denotes differentiated intermediate goods \( \epsilon \in [0,1] \). Perfect competition means the final goods firms take the price of each intermediate good \( \epsilon, p_t(\epsilon) \), as given. Firms’ profit maximization of final goods firms leads to the demand function for \( x_t(\epsilon) \):

\[ x_t(\epsilon) = \frac{y_t}{p_t(\epsilon)}. \quad (10) \]

Second, there is a monopolistic intermediate goods sector, with a unit continuum of manufacturing industries producing differentiated intermediate goods. Each industry produces one intermediate good for the final goods sector. Within each monopolistic industry \( \epsilon \), an industry leader with the highest level of productivity dominates and produces intermediate good \( \epsilon \) with labor as the only input:

\[ x_t(\epsilon) = \gamma^{N_t(\epsilon)} L_{x,t}(\epsilon), \quad (11) \]

where \( L_{x,t}(\epsilon) \) is the production labor in industry \( \epsilon \); \( \gamma > 1 \) is the step size of innovation; \( N_t(\epsilon) \) is the number of innovations that have occurred in industry \( \epsilon \) as of time \( t \).

The marginal cost of production for the industry leader in industry \( \epsilon \) is \( mc_t(\epsilon) = w_t / \gamma^{N_t(\epsilon)} \). Therefore, as \( \gamma > 1 \), each vertical innovation allows a worker to produce one unit of intermediate good with less time. When the next innovation arrives, its owner has a lower marginal cost: \( mc_t(\epsilon) / \gamma \). Bertrand price competition leads to a profit-maximizing price \( p_t(\epsilon) \) equal to the marginal cost of the previous innovation \( w_t / \gamma^{N_t(\epsilon)-1} \), which is a markup \( \gamma \) over its own marginal cost. Therefore, the labor income from the manufacturing sector is

\[ w_t L_{x,t}(\epsilon) = \left( \frac{1}{\gamma} \right) p_t(\epsilon) x_t(\epsilon) = \left( \frac{1}{\gamma} \right) y_t, \quad (12) \]

and the monopolistic profit of each industry’s leader is given by

\[ \Pi_t(\epsilon) = \left( 1 - \frac{1}{\gamma} \right) p_t(\epsilon) x_t(\epsilon) = \left( \frac{\gamma - 1}{\gamma} \right) y_t. \quad (13) \]
2.4 Innovation Process

Within each industry \( \epsilon \), there is a unit continuum of R&D firms \( j \in [0, 1] \) that are driven to innovate to capture the positive monopolistic profit given in (13). The value of each innovation (i.e., the value of the monopolistic firm in industry \( \epsilon \)) is denoted \( v_t(\epsilon) \). In a symmetric equilibrium, \( v_t(\epsilon) = v_t[1] \). There is free entry into R&D (i.e., free labor mobility, which means the cost of innovation equals the foregone wage rate).

To capture the strength of the CIA on R&D, we assume that entrepreneurs borrow cash to finance \( \beta \in [0, 1] \) fraction of the wage bills of R&D workers. So the total cost of R&D for firm \( j \) would be \( (1 - \beta) w_t L_{r,t}(j) + \beta (1 + i_t) w_t L_{r,t}(j) = (1 + \beta i_t) w_t L_{r,t}(j) \), and thus the zero-expected-profit condition of R&D firm \( j \) in each industry is

\[
\lambda_t(j) v_t = (1 + \beta i_t) w_t L_{r,t}(j),
\]

(14)

where \( L_{r,t}(j) \) is the amount of labor hired by R&D firm \( j \), and \( \lambda_t(j) \) is the firm-level innovation rate per unit time: \( \lambda_t(j) = \varphi (L_{r,t}(j)/L) \), where \( \varphi \) is a constant. Note that \( \lambda_t(j) \) is scaled by total population \( L \), which eliminates the scale effects to yield the scale-invariant model as the labor share rather than total population matter for growth (see discussions of scale effects in Laincz and Peretto, 2006; Segerstrom, 1998).

The aggregate arrival rate of innovation is

\[
\lambda_t = \int_0^1 \lambda_t(j) \, dj = \varphi \frac{L}{L} L_{r,t} = \varphi l_{r,t},
\]

(15)

where \( l_{r,t} \) is the share of labor employed in the R&D sector. Similarly, the share of labor in production/manufacturing is \( l_{x,t} = L_{x,t}/L \).

The no-arbitrage condition for \( v_t \) is

\[
 r_t v_t = \Pi_t + \dot{v}_t - \lambda_t v_t.
\]

(16)

Equation (16) says that the opportunity cost of holding an innovation \( (r_t v_t) \) equals the sum of the flow profit of innovation \( (\Pi_t) \) and potential capital gain \( (\dot{v}_t) \) less the expected capital loss \( (\lambda_t v_t) \), where \( \lambda_t \) is the arrival rate of the next innovation.

2.5 Labor Market and the Final Goods Market Clearing Condition

Labor is used in both manufacturing (producing intermediate goods) and R&D activities. The labor market clearing condition is

\[
L_{x,t} + L_{r,t} = l_t L,
\]

(17)

See the justifications provided in Cozzi et al. (2007) for this equilibrium.
where \( L_{x,t} \) and \( L_{r,t} \) are the total employment in manufacturing and R&D, respectively.

To derive the final goods market clearing condition, we first plug \( \tau_t = (\pi_t + \pi_t m_t) \) into (2) which yields the resource constraint of the economy: \( \dot{c}_t = r_t e_t + w_t l_t - c_t + \beta_i b_t \). Then, using \( v_t = c_t L \) and \( l_t = l_{x,t} + l_{r,t} \), we can rewrite the resource constraint as \( \dot{v}_t / L = r_t v_t / L + w_t (l_{x,t} + l_{r,t}) - c_t + \beta_i b_t \).

Since the amount of cash borrowed by entrepreneurs satisfies \( \dot{b}_t L = \beta w_t L_{r,t} \), we have \( \dot{v}_t / L = \dot{r}_t v_t / L + w_t l_{x,t} + (1 + \beta_i) w_t l_{r,t} - c_t \). Finally, using (12) to replace \( w_t l_{x,t} \) and substituting (13), (14), and (16) into the previous equation to replace \( (1 + \beta_i) w_t l_{r,t} \), we get

\[
\frac{\dot{v}_t}{L} = \frac{\dot{r}_t v_t}{L} + \frac{y_t}{\gamma L} + \left( \frac{2 - 1}{\gamma} \right) y_t + \dot{v}_t - \frac{r_t v_t}{L} - c_t,
\]

which suggests that the final goods market clearing condition is \( c_t = y_t / L \).

### 2.6 Equilibrium Balanced Growth

Plugging equation (11) into (9), we have

\[
y_t = \exp \left( \int_0^1 N_t(\epsilon) \, d\epsilon \ln \gamma \right) L_x = \exp \left( \int_0^t \lambda_t \, dv \ln \gamma \right) L_x \equiv Z_t L_x,
\]

(18)

where the law of large numbers has been applied and \( Z_t \equiv \exp \left( \int_0^t \lambda_t \, dv \ln \gamma \right) \) is the level of aggregate technology. The growth rate of \( Z_t \) is

\[
g_t = \lambda_t \ln \gamma = \varphi l_{r,t} \ln \gamma.
\]

(19)

We can now define the general equilibrium for our model:

**Definition** Given a nominal interest rate \( i_t \) and an initial condition \( Z_0 \), a dynamic equilibrium for the model is a time path of prices \( \{ p_t(\epsilon), r_t, w_t, \pi_t, v_t \} \) and allocations \( \{ c_t, e_t, m_t, y_t, x_t(\epsilon), L_{x,t}(\epsilon), L_{r,t}(j) \} \) such that given prices, the households maximize utility, competitive final-goods firms maximize profit, monopolistic intermediate-goods firms choose \( L_{x,t}(\epsilon), p_t(\epsilon) \) to maximize profit, and R&D firms choose \( L_{r,t}(j) \) to maximize expected profit; all markets clear, that is, the labor market clears \( (L_{x,t} + L_{r,t} = L_t L) \); and the final goods market clears \( (y_t = c_t L) \); the CIA constraint binds: \( c_t + b_t = m_t \), and the amount of cash borrowed by entrepreneurs satisfies \( b_t L = \beta w_t L_{r,t} \); the value of monopolistic firms adds up to the value of households’ assets (i.e., \( v_t = c_t L \)).

As shown in Kurz (1968), multiple steady-state equilibria will emerge when capital enters the utility function. In our model, we find that the dynamics of the model remains similar to that in Chu and Cozzi (2014)\(^5\). That is, the economy immediately jumps to a unique and saddle-point

\(^5\)Hof and Prettmann (2019) also show that there exists a unique saddle-point stable balanced growth path in a Romer-type expanding varieties model (Romer, 1990) with capital accumulation.
stable balanced growth path on which each variable grows at a constant rate, as will be shown in Proposition 1 below.

**Proposition 1** Given a fixed nominal interest rate (i.e., \( i_t = i \)), the dynamics of the economy is that the economy immediately jumps to unique and saddle-point stable balanced growth path on which each variable grows at a constant rate.

**Proof.** See Appendix 7.2 for the derivation.

On the balanced growth path, the output market clearing condition implies that \( c_t \) and \( y_t \) must grow at the same rate: \( g_c = g_y \) (we use \( g \) to denote the growth rate). Equation (18) yields \( g_y = g_z \). The binding CIA constraint delivers \( g_c = g_m \). Therefore, the balanced growth rate \( g_t \) (the growth rate of per capita consumption or output) is the one given in (19).

Because the balanced growth rate is uniquely pinned down by the share of labor employed by R&D firms, we solve for the equilibrium labor allocation for a fixed nominal interest rate \( i_t = i \). Using equations (7) and (12)-(15), we can derive the following two equations regarding labor allocation and consumption to wealth ratio (see Appendix 7.3 for details):

\[
(\gamma - 1) l_{x,t} = (1 + \beta i) \left( l_{r,t} + \rho/\varphi - \frac{\theta (1 + i) c_t}{\varphi a_t} \right). 
\]

\[
\frac{c_t}{a_t} = \frac{\gamma w_t L_{x,t}}{(1 + \beta i) w_t L_{r,t}/\lambda + \gamma w_t L_{x,t} + \beta w_t L_{r,t}} = \frac{\gamma l_{x,t}}{(1 + \beta i) / \varphi + \gamma l_{x,t} + \beta l_{r,t}}. 
\]

Combining \( c_t = y_t/L \) and (12), we have \( c_t = \gamma w_t l_{x,t} \). Now the labor market clearing condition becomes

\[
l_{r,t} + l_{x,t} = l_t = \left( \frac{1}{\eta \gamma l_{x,t} (1 + i)} \right)^\sigma. 
\]

Using (21) to substitute out the consumption wealth ratio in (20), we have

\[
(\gamma - 1) l_{x,t} = (1 + \beta i) \left( l_{r,t} + \rho/\varphi - \frac{\theta (1 + i) \gamma l_{x,t}}{(1 + \beta i) + \varphi \gamma l_{x,t} + \varphi \beta l_{r,t}} \right). 
\]

For a given \( i \), the labor market clearing condition (22) and the free labor mobility condition (23) solve for the stationary equilibrium labor allocation \( \{l_r, l_x\} \) on a balanced growth path.

### 3 Theoretical Results and Implications

In this section, we first derive theoretical results regarding the effects of monetary policy on growth and welfare under the assumption that the CIA constraint only applies to consumption. This part helps understand the intuition on why the presence of the SOC generates a new channel
for monetary policy to impact growth and welfare. We then describe the results when the CIA constraint applies to both consumption and R&D.

3.1 CIA on Consumption Only

3.1.1 Growth

**Proposition 2** With the CIA constraint on consumption and elastic labor supply (i.e., \( \eta > 0 \)), if \( \left[ \eta \gamma (1 + i) \right]^{-\sigma/(1+\sigma)} \leq 1 \), whether a higher nominal interest rate leads to higher long-run economic growth depends on the relative sizes of the elasticity of labor supply (\( \sigma \)) and the taste for leisure (\( \eta \)) versus the degree of the SOC (\( \theta \)). In particular, when the degree of the SOC is strong enough, higher nominal interest rates promote growth; when the elasticity of labor supply or the taste for leisure is large enough, higher nominal interest rates reduce growth.

**Proof.** See Appendix 7.4 for the derivation.

As Proposition 2 shows the key results in our benchmark model, we provide some explanations to help understand the results. First, like in Chu and Cozzi(2014), on the balanced growth path, the growth rate of the economy in our model is a linear function of the R&D labor share given in (19). The optimal labor allocation between the production sector and the R&D sector plays a crucial role in understanding Proposition 2. Two conditions pin down the equilibrium labor allocation in our model: (i) the labor market clearing condition as described by (22) which gives the total amount of labor resource, and (ii) the free labor mobility condition as described by (23) which determines the optimal allocation of the labor resource between production and R&D by equating the wage rates between the two sectors. Putting manufacturing labor \( l_{x,t} \) on the horizontal axis, (22) is a downward-sloping curve (the \( L \) curve in Figure 1), while (23) is an upward-sloping curve (the \( M \) curve in Figure 1). The intersection of the two lines shows the equilibrium labor allocation.

Second, let’s discuss the key mechanisms for monetary policy to affect growth with and without the SOC. Without the SOC, an increase in the nominal interest rate reduces total labor supply \( l_t \) and therefore lowers both manufacturing and R&D labor, leading to lower long-run growth (we can name it the consumption-leisure choice effect). Graphically, an increase in the nominal rate leads the \( L \) curve to shift downward but does not change the \( M \) curve. Notice that a larger Frisch elasticity of labor supply (\( \sigma \)) or a stronger taste for leisure (\( \eta \)) would strengthen the negative consumption-leisure choice effect, as shown by equation (22).

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6In Online Appendix A, we show that our results hold under inelastic labor supply and CIA on R&D.

7This can be seen by equation (23) when \( \beta = 0 \) and \( \theta = 0 \).
In contrast, the presence of the SOC creates a new channel—the labor reallocation channel—for monetary policy to affect the optimal R&D labor input and thereby long-run growth. This channel operates through altering the saving behavior of households which impacts the real interest rate $r_t$. We can see this in two ways. The first way is to compare the Euler equations with and without the SOC:

\[
\begin{align*}
\frac{c_t}{c_{t-1}} &= r_t - \rho + \theta (1 + i_t) \frac{a_t}{a_{t-1}}, \text{ with the SOC,} \\
\frac{c_t}{c_{t-1}} &= r_t - \rho, \text{ without the SOC,}
\end{align*}
\]

An increase in the nominal interest rate in (24) will lead to an immediate (downward) jump in consumption, followed by a gradual increase in wealth ($a_t$) and a decline in the real interest rate ($r_t$). This lowers entrepreneurs' borrowing costs and raises the value of innovations, which attracts more labor in the R&D sector (we therefore call this effect the labor reallocation effect). Thus, with the SOC, an increase in the nominal interest rate generates a positive effect on R&D labor and long-run growth through the labor reallocation effect. Without the SOC, this effect does not exist as shown by equation (25).

We can also use Figure 1 to provide a graphical explanation. An increase in the nominal interest rate shifts the $M$ curve upward, and a stronger SOC tends to amplify the upward shift in the $M$ curve from an increase in the nominal interest rate. Note that, without the SOC, an increase in the nominal interest rate cannot shift the $M$ curve (also seen from the free labor mobility condition (23) under $\beta = 0$ and $\theta = 0$). That is, the existence of the SOC creates a new channel (i.e., the positive labor reallocation channel) for monetary policy to affect R&D labor (and thereby long-run growth). In particular, when the degree of the SOC is large enough, the positive labor reallocation effect may outweigh the negative consumption-leisure choice effect, leading the R&D labor to increase in response to a rise in the nominal interest rate. In contrast, without the SOC, the positive labor reallocation channel does not exist and R&D labor will always fall when the nominal interest rate rises.

It is worth noting that the results in Proposition 2 regarding the effects of money policy on long-run growth are in contrast to those in canonical models with capital accumulation. For example, in Ramsey-Cass-Koopmans capital accumulation models with a CIA constraint on consumption, monetary policy has no effect on long-run growth. We will further compare our model with the capital accumulation models that use the SOC in Section 5.2.

**Corollary 1** Under the assumptions of an elastic labor supply and a CIA constraint on consumption, the consumption-wealth ratio is a decreasing function of the nominal interest rate and the strength of the SOC.
Proof. The consumption to wealth ratio in (21) shows that it is increasing in the manufacturing labor $l_x$ when the CIA on R&D is absent (i.e., $\beta = 0$). Based on Proposition 2 and its proof, we have

$$\frac{\partial (c_t/a_t)}{\partial i} = \frac{\partial (c_t/a_t)}{\partial l_x} \frac{\partial l_x}{\partial i} < 0 \quad \text{and} \quad \frac{\partial (c_t/a_t)}{\partial \theta} = \frac{\partial (c_t/a_t)}{\partial l_x} \frac{\partial l_x}{\partial \theta} < 0.$$

3.1.2 Welfare

As Proposition 1 shows, given an increase in the nominal interest rate, the economy will jump immediately to a new balanced growth path. This makes welfare comparison easier as we only need to focus on the balanced growth path. We can explicitly derive a welfare function based on (1) on the balanced growth path:

$$U = \frac{1}{\rho} \left[ \ln (c_0) + \theta \ln (a_0) + \frac{(1 + \theta) g}{\rho} - \eta \frac{l_0^{1+1/\sigma}}{1 + 1/\sigma} \right]$$

$$= \frac{1}{\rho} \left[ (1 + \theta) \ln (c_0) - \theta \ln \left( \frac{c_0}{a_0} \right) + \frac{(1 + \theta) g}{\rho} - \eta \frac{l_0^{1+1/\sigma}}{1 + 1/\sigma} \right]$$

$$= \frac{1}{\rho} \left[ (1 + \theta) \ln (Z_0 l_x, 0) - \theta \ln \left( \frac{\gamma l_x, 0}{1/\varphi + \gamma l_x, 0} \right) + \frac{(1 + \theta) g}{\rho} - \eta \frac{l_0^{1+1/\sigma}}{1 + 1/\sigma} \right]$$

$$= \frac{1}{\rho} \left[ \ln (l_x, 0) + \theta \ln \left( \frac{1}{\varphi} + \gamma l_x, 0 \right) + \frac{(1 + \theta) g}{\rho} - \eta \frac{l_0^{1+1/\sigma}}{1 + 1/\sigma} - \theta \ln \gamma \right],$$

where $l_x, 0$ is the share of production labor at time 0, $l_0 = l_x, 0 + l_r, 0$, and $g = \varphi l_r, 0 \ln \gamma$. The second to last equality uses the final goods market clearing condition to replace $c_0$ and the consumption-wealth ratio in (21) (imposing $\beta = 0$) to replace $c_0/a_0$; the last equality normalizes $Z_0$ (the aggregate technology at time 0) to 1.

From (28), we can see that an increase in the nominal interest rate affects welfare via several channels. Specifically, an increase in the nominal interest rate reduces welfare by decreasing manufacturing labor $l_x, 0$ (thereby initial consumption), while it increases welfare through reducing labor supply $l_0$. It could also affect the growth rate $g$. As a result, the overall effect on welfare depends on which effect dominates and we will further discuss it using our calibrated model in Section 4. The following proposition provides a general characterization of the effect of the SOC on optimal monetary policy.

**Proposition 3** Suppose welfare is an inverted-U function of the nominal interest rate with and without the SOC. When an increase in the nominal interest rate reduces growth, then the optimal nominal interest rate is a decreasing function of the degree of the SOC.
Proof. See Appendix 7.5 for the derivation. ■

The intuition behind Proposition 3 can be explained as follows. On the one hand, a higher nominal interest rate reduces labor supply and increases leisure, which raises welfare. This positive effect is captured by changes in the fourth term in the square bracket on the right-hand-side (RHS) of (28) and can be deemed as the marginal benefit of the nominal interest rate on welfare. On the other hand, the increase in the nominal interest rate also reduces manufacturing labor and in turn current consumption and wealth, hence decreasing welfare (the first two terms in the square bracket on the RHS of (28)). When an increase in the nominal interest rate reduces long-run growth, it also decreases future consumption and thereby welfare (the third term in the square bracket on the RHS of (28)). These two negative effects on welfare are captured by changes in the first three terms on the RHS of (28) and can be deemed the marginal cost of the nominal interest rate on welfare.

If welfare is an inverted-U function of the nominal interest rate, it means that starting from a very low nominal interest rate, the marginal benefit dominates the marginal cost as the nominal interest rate increases; thus, welfare increases. However, beyond a threshold, the marginal benefit is dominated by the marginal cost as the nominal interest rate increases, and thus welfare declines. The optimal nominal interest rate equates the marginal cost with the marginal benefit. As an increase in the SOC pushes up the marginal cost but leaves the marginal benefit unchanged, the optimal nominal interest rate decreases with the SOC. One implication of Proposition 3 is that a larger SOC tends to drive the optimal nominal interest rate toward a level closer to zero or even negative.

3.2 The Full-fledged Model: CIA Constraints on Both Consumption and R&D

In the full-fledged model, in addition to the CIA constraint on consumption, we follow Chu and Cozzi (2014) to introduce a CIA constraint on R&D investment. Compared to the results in the previous section, the existence of the CIA constraint on R&D raises the borrowing cost of entrepreneurial R&D investment and brings a new channel through which monetary policy influences long-run growth. Due to this new channel, it is more complicated to theoretically prove a proposition similar to proposition 2. However, we will describe the key mechanisms on how monetary policy influences growth and welfare in this section and calibrate this full-fledged model in the next section.
3.2.1 Effects of Monetary Policy on Growth

When the nominal interest rate increases, there are three effects on growth. First, it leads to a lower labor supply which also lowers R&D labor and long-run growth (the negative consumption-leisure choice effect). Second, it leads to higher saving which lowers the real interest rate and borrowing costs, and in turn increases R&D labor and growth (the positive labor reallocation effect due to the SOC). Third, a higher nominal interest rate raises the borrowing cost due to the CIA constraint on R&D investment which reduces R&D labor and growth. Among these three effects, the first two are the same as in section 3.1 (when there is no CIA constraint on R&D), while the third effect is new and driven by the CIA constraint on R&D.

In general, the presence of the CIA constraint on R&D generates a new negative effect on R&D labor, and therefore an increase in the nominal interest rate is more likely to reduce long-run growth. More precisely, if the positive labor reallocation effect due to the SOC dominates the other two effects, an increase in the nominal rate leads to higher growth; otherwise it lowers long-run growth.

We can use Figure 1 to provide further explanations on how key parameters influence these three effects and therefore the final outcome on growth. On the one hand, the elasticity of labor supply ($\sigma$) and the taste for leisure ($\eta$) affect equilibrium labor allocations via the labor supply channel (i.e., shifting the $L$ curve). Given the nominal interest rate level, a higher $\eta$ or $\sigma$ leads to a larger leftward shift in the downward-sloping $L$ curve (i.e., reducing labor supply), which lowers both R&D labor and manufacturing labor. On the other hand, both the SOC and the CIA constraint on R&D affect equilibrium labor allocations via the free labor mobility condition (the $M$ curve). Given the nominal interest rate, a stronger SOC ($\theta$) leads to a larger leftward shift in the upward-sloping $M$ curve, while a tighter CIA constraint on R&D ($\beta$) leads to a larger rightward shift in the $M$ curve. Thus, to which direction the $M$ curve shifts depends on the relative sizes of $\theta$ versus $\beta$. In the next section we will quantitatively analyze these effects using our calibrated model.

3.2.2 Effects of Monetary Policy on Welfare

The key channels for monetary policy to impact welfare are similar to those in section 3.1.2. The only difference is the CIA constraint on R&D which also influences the labor allocation. Specifically, an increase in the nominal interest rate generates several effects on welfare. It could decrease/increase welfare through decreasing/increasing manufacturing labor ($l_{x,0}$). Additionally, it

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8In our model, total labor supply is decreasing in the elasticity of labor supply, all else equal. Therefore, in terms of promoting growth, the lower elasticity of labor supply, the better.
improves welfare through reducing total labor supply \((l_0)\). Furthermore, it may decrease/increase balanced growth \((g)\) and thereby welfare. The magnitudes of these effects depend on the structural parameters. We conjecture a counterpart of Proposition 3 may still hold: if welfare is an inverted-U function of the nominal interest rate with and without the SOC, then the optimal nominal interest rate is a decreasing function of the degree of the SOC. We will discuss welfare and optimal monetary policy more in calibration.

4 Quantitative Results

In this section, we calibrate our model to the US economy and quantitatively explore how monetary policy’s effect on long-run growth and welfare could crucially depend on the degree of the SOC. In addition, we also quantify the growth effects of the SOC based on our calibrated model.

4.1 Calibration

We calibrate the model in two steps. In the first step, we pin down a few parameters’ values directly from the data or the values commonly used in the literature. In the second step, we jointly calibrate other key parameters (which are difficult to directly assign values) to match a few key moments.

Our first group consists of 3 parameters. We set the labor elasticity parameter \(\sigma = 0.5\) according to the macro literature (see Aghion et al. 2013). We set the discount rate \(\rho = 0.04\), a standard value used in the literature. We set the parameter on the CIA constraint \(\beta = 0.05\) which means 5% of R&D investment faces the CIA constraint.

Our second group consists of 4 parameters: the degree of the SOC \((\theta)\), the taste of leisure parameter \(\eta\), and the two innovation parameters \((\varphi, \gamma)\). As these parameters are difficult to directly measure, we jointly calibrate them to four data moments: (1) U.S. long-run economic growth (from the FOMC’s published long-run growth projection); (2) the consumption to wealth ratio; (3) the real interest rate; and (4) total labor supply \(l = 1/3\) (i.e., 8 hours out of 24)

\footnote{We provide sensitive analysis in Section 4.2.4 and show this parameter has little effects on our calibration results.}

\footnote{The aggregate consumption to wealth ratio is constructed based on its formula \(\frac{c}{y} = \frac{c}{y} + \frac{m}{y} = \frac{c}{y} + \frac{c}{y}\) where we used \(m = c\) as in the model. \(\frac{c}{y}\) is measured by the U.S. consumption-GDP ratio and \(\frac{c}{y}\) is measured by the U.S. market-cap to GDP ratio \(\text{https://www.longtermtrends.net/market-cap-to-gdp/}\).}

\footnote{The real interest rate is calculated by subtracting inflation from the nominal interest rate. The normal interest rate is based on the average level of 1-year Treasury yield in 1960-2019, which is 5.04%. We use core PCE inflation (the measure used by the Federal Reserve to set monetary policy) as our measure of inflation and set \(\pi = 3.25\%\) which is the average level in 1960 – 2019. As report in Table 1, the resulting real 1.79%.}
hours). Parameter values are reported in Table 1. It is worth noting that our calibrated value of the degree of the SOC (0.09) is slightly smaller but comparable to that obtained in Bakshi and Chen (1996). In a model with the SOC and CRRA preferences, they find that when the relative weight of the degree of the SOC compared to the degree of risk aversion to consumption is about 1/3 − 1/2, the model can explain the joint behavior of aggregate consumption and the equity return observed in the U.S. economy. As we will show in the next section, using a larger value of \( \theta \) will strengthen our new results.

4.2 Quantitative Implications

In this section, we present three sets of results on implications of the SOC on monetary policy, growth, and welfare based on our calibrated model.

4.2.1 Implications of the SOC on the Growth Effect of Monetary Policy

We revisit Proposition 2 by quantitatively showing a higher nominal interest rate may lead to different growth outcomes depending on the degree of the SOC. Our Proposition 2 shows that, in a simplified model which assumes that the CIA constraint only applies to consumption, the effect of an increase in the nominal interest rate on long-run growth could switch from a negative to a positive when the SOC is stronger. With our calibrated model, we can quantitatively show this in a more general case which assumes the CIA constraint applies to both consumption and R&D.

Figure 3 shows how growth changes with the nominal interest rate at different degrees of the SOC. The solid line is based on our calibrated value of \( \theta \). It shows that monetary policy has very little effect on long-run growth as the flatness of the curve suggests (the slope is only slightly negative). When the SOC is absent, this curve (the blue dashed line) is more downward sloping, meaning that an increase in the nominal interest rate reduces growth. This is consistent with the finding in the existing literature (Chu and Cozzi, 2014). However, when the degree of the SOC is stronger than the benchmark value, the slope could become positive, as the green dashed line shows. This suggests that an increase in the nominal interest rate could promote long-run economic growth when the degree of the SOC is strong enough.

This finding is important as the degree of the SOC could differ significantly across countries. Weber (1958) uses the emergence of the SOC in western countries to explain why capitalism developed in western countries rather than in China. Weber refutes the fact that only westerners love accumulating wealth by arguing that people in ancient China also loved accumulating wealth. The difference, according to our understanding, is that the Protestant Reformation tries to instill the SOC into every secular person such as through the intergenerational transmission and shaping
of Children’s religious preference by their parents (as highlighted in Doepke and Zilibotti, 2008). Therefore, unlike ancient China where the preference for wealth was not spread to the majority of the population, western countries have the SOC in secular Protestants who account for a large fraction of total population. From this point of view, the influences of different religion/culture may lead to the possible differences in the SOC across countries.

The different effects of monetary policy on growth is mainly through the labor-allocation channel. As the top left panel in Figure 4 shows, the response of R&D labor \( l_r \) to the nominal interest rate shows the same pattern as growth: when there is no SOC, it is a downward sloping curve; but, when SOC is strong enough it becomes an upward sloping curve. As explained in earlier sections, this is a result of three competing forces: two negative effects due to elastic labor supply and the CIA constraint on R&D and one positive effect due to the existence of the SOC. Specifically, higher nominal interest rates reduce total labor supply via the consumption-leisure choice, which reduces both R&D labor and manufacturing labor. In addition, higher nominal interest rates also increase the borrowing cost of entrepreneurs through the CIA on R&D, thereby reducing demand for R&D labor. These two together reduce \( l_r \) when the nominal interest rate increases.

On the positive effect, in the presence of the SOC, an increase in the nominal interest rate increases savings (see household’s Euler equation \( (7) \)) and thus lowers the real interest rate \( (r) \), which means lower borrowing costs for entrepreneurs. This raises the value of innovations, as seen from the no-arbitrage condition, \( (16) \), and thus attracts more labor into the R&D sector (which can be seen in \( (14) \)). Overall, the positive effect gets larger when the SOC is stronger and could dominate the negative effect, which leads the total effect to be positive on R&D labor. That is exactly what we see in the top left panel in Figure 4.

### 4.2.2 Implications of the SOC on Growth

Figure 3 shows that a stronger SOC leads to higher long-run growth, which is consistent with the existing findings in the literature (Zou 1994; Futagami and Shibata, 1998; Smith, 1999; Corneo and Jeanne, 2001; Hof and Prettner, 2019). Most of these papers are using an AK growth model which we will discuss the differences with our Schumpeterian growth model in the next section. Further adding to this literature, we use our calibrated model to quantify the contribution of the SOC to long-run growth. Table 2 reports growth rates and other key variables in our model economy (which is calibrated to the US economy) and in an economy without the SOC. Notice that the optimal labor allocation between the R&D sector and the production sector will change when we assume that the SOC is absent, which also explains why growth is different. In particular,
as the first column of Table 2 shows, the long-run growth would be only 1.2 percent if the SOC is absent. In other words, the presence of the SOC helps explain about one-third of US long-run growth.

The decline in growth can be explained by the decline in R&D labor as the second column of Table 2 shows. In particular, $l_r$ is about 33.5% lower in the economy without the SOC than in the calibrated economy. Though labor in the production sector ($l_x$) increases 3.7%, total labor supply still drops 1.8%. To help understand why R&D labor ($l_r$) has declined in an economy without the SOC, Figure 5 provides a graphic view on the determination of $l_r$. Remember $l_r$ is jointly determined by the L curve (which represents the labor-market clearing condition in equation (22)) and the $M$ curve (which represents the free labor mobility condition (23)). As the $L$ curve is not influenced by the SOC, a decline in the degree of the SOC shifts the $M$ curve rightward, leading to a lower value of equilibrium $l_r$.

### 4.2.3 Implications of the SOC on Welfare

Figure 6 shows that the SOC also has important implications for optimal monetary policy. Specifically, as the figure shows, the optimal nominal interest rate (represented by circles on each line) shifts to the left as the degree of the SOC increases. For example, the optimal nominal interest rate is positive in a model without the SOC, confirming the findings in Chu and Cozzi (2014) and Akyol (2004). However, in our benchmark economy, the optimal nominal interest rate is either zero (if we set zero as a lower bound for the nominal rate) or negative (if we allow the nominal rate to fall into the negative territory).

These findings generate two implications. First, it suggests that the existence of the SOC could make the Friedman rule more likely as the optimal. That is, a zero inflation rate could maximize welfare due to the existence of the SOC. Second, if we allow the nominal interest rate to fall into the negative territory, such as what several advanced countries are using now, the existence of the SOC provides a possible justification on why a negative interest rate could be the optimal one.

### 4.2.4 Sensitivity Analysis

In the calibration, we have assumed $\beta = 0.05$ for the CIA constraint on R&D. Here we provide robustness checks to show our calibration results are not sensitive to this value. In particular, we set two alternative values $\beta = 0.1$ and $\beta = 0.2$ and recalibrate our model. We find the calibration results are very close to our benchmark economy. In addition, the key patterns in Figures 8-10 do not change, which are shown in Figures 7-11. These comparisons also uncover one finding that,
as CIA constraint on R&D becomes tighter (i.e., $\beta$ increases), the optimal nominal interest rate shifts to the right and therefore is more likely to be positive.

5 Further Discussions

So far we have explored how the SOC affects the growth and welfare implications of monetary policy in a Schumpeterian model with CIA constraints. In this section, we investigate the implications of the SOC on growth and monetary policy in two alternative models and examine whether the main results we obtained in the benchmark model are robust to alternative model specifications. Specifically, in the first model, we take out money and study the role of the SOC on growth in a non-monetary Schumpeterian model. In the second model, we replace the Schumpeterian growth model with the AK growth model and study the implications of the SOC on monetary policy in this alternative growth framework. We find that the main results regarding the SOC obtained in our benchmark model still hold in the two alternative models.

5.1 A Non-monetary Schumpeterian Model with the SOC

Without money, the new asset-accumulation equation becomes

$$\dot{e}_t = r_t e_t + w_t l_t - c_t. \quad (29)$$

Per capita wealth $a_t$ becomes $a_t = e_t$. The optimal condition for labor supply can thus be written as:

$$l_t = \left( \frac{w_t}{\eta e_t} \right)^{\sigma}. \quad (30)$$

The Euler equation is

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho + \frac{\theta}{\mu_t a_t} = r_t - \rho + \frac{\theta c_t}{a_t}. \quad (31)$$

The dynamics of the model remain the same: it immediately jumps to the (saddle-point stable) balanced growth path on which the labor allocations are stationary and unique.

Now the labor market clearing condition becomes

$$l_{r,t} = \left( \frac{1}{\eta \gamma} \right)^{\sigma} l_{x,t} - l_{x,t}. \quad (32)$$

The final goods market clearing condition gives $c_t = y_t / L$. We have $e_t = v_t / L$. Therefore, the consumption wealth ratio is

$$\frac{c_t}{a_t} = \frac{y_t}{v_t} = \frac{\gamma w_t L_{x,t}}{w_t L_{r,t}/\lambda} = \varphi \gamma l_{x,t}. \quad (33)$$
The free labor mobility condition is \((\gamma - 1) l_{x,t} = l_{r,t} + \rho/\varphi - \theta c_t/(\varphi a_t)\). Combining with \((33)\) yields
\[
l_{r,t} = (\gamma - 1) l_{x,t} - \rho/\varphi + \theta \gamma l_{x,t}. \tag{34}
\]

**Proposition 4** Given \((\eta \gamma)^{-\sigma/(1+\sigma)} \leq 1\), the steady state growth increases with \(\theta\) (the degree of the SOC), all else equal.

**Proof.** See Appendix 7.6 for the proof. ■

The intuition behind Proposition 4 can be explained in a similar way. With the SOC, the savings decision will depend on \(\theta c_t/a_t + r_t - \rho\). An increase in \(\theta\) raises the marginal benefit of saving through the direct preference for wealth. This would lead people to substitute more consumption with savings, which drives down the real interest rate and the borrowing cost of entrepreneurs. This increases the return to entrepreneurial investment and attracts more labor into R&D (causing labor reallocation from manufacturing to R&D). However, an increase in \(\theta\) would not affect the labor supply choice, leaving total labor supply unchanged (i.e., the consumption-leisure choice effect is absent). The larger labor reallocation effect due to a higher \(\theta\) results in larger R&D labor and thereby higher long-run growth.

This result echoes the finding in the literature that the existence of the SOC promotes growth in the Ramsey-Cass-Koopmans capital accumulation models, the AK model (see Zou, 1994), the Romer’s expanding-varieties model (Pan et al., 2018), and Romer’s expanding-varieties model with capital accumulation (Hof and Prettner, 2019).

### 5.2 Comparison with A Monetary AK Model with the SOC

In this section, we replace the Schumpeterian growth model with an AK growth model. To introduce money into the growth model, we continue to assume a CIA constraint on consumption. The utility function remains the same. Each individual is endowed with one unit of labor. There is no population growth and total population is fixed at \(L\). Each household maximizes its lifetime utility given in equation \((1)\) subject to the asset-accumulation equation:
\[
\dot{k}_t + m_t = Ak - c_t - \pi t m_t + \tau_t, \tag{35}
\]
where \(k_t\) is the real value of capital stock held by each person, and \(A\) is the level of technology; the other variables remain the same as before. Here the per capita wealth is \(a_t = k_t + m_t\). The CIA constraint is still \(c_t \leq m_t\).

**Proposition 5** In the AK model with the SOC and a CIA constraint on consumption, growth in the steady state increases with the nominal interest rate (i.e., money is not superneutral). Without the SOC, money is superneutral.
Proof. See Online Appendix [7.7] for the proof.

The intuition behind this proposition is similar to that provided in Proposition 2. It is worth noting that Sidrauski (1967) proves the superneutrality of money in a qualitatively equivalent approach which assumes money enters the utility function. As explained by Sidrauski (1967, p. 544), “In the short run, an increase in the rate of monetary expansion is equivalent to a rise in government transfers to the private sector. It therefore results in an increase in consumption and a fall in the rate of capital accumulation.” In addition, monetary expansion pushes people to substitute real balance with capital holdings (the Mundell-Tobin portfolio-shift effect that refers to shifts between assets related to inflation changes, see Mundell, 1965; Tobin, 1965), which tends to reduce consumption under the CIA constraint on consumption. The two opposing effects (the positive Sidrauski government transfer effect and the negative Mundell-Tobin portfolio-shift effect) offset each other, leaving the steady state consumption, savings and capital stock unchanged in Ramsey models and the saving rate and thereby long-run growth unchanged in the AK model (see e.g., Dotsey and Sarte, 2000). In other words, with a CIA constraint on consumption, long-run growth in capital accumulation models depends solely on the real return to capital accumulation \( r_t \) (i.e., not on labor supply \( l_t \)) that is independent of monetary policy because the nominal interest rate does not alter the saving behavior of households nor the marginal productivity of capital.

To recap, in the AK model with the CIA constraint on consumption, money is superneutral without the SOC because the aforementioned two opposing effects (the positive Sidrauski government transfer effect and the negative Mundell-Tobin portfolio-shift effect) on consumption still offset each other, leaving the saving rate and thereby long-run growth unchanged. By contrast, in the presence of the SOC, a higher nominal interest rate yields a larger consumption-portfolio effect that decreases the marginal propensity to consume due to stronger saving motivation. In the AK model, labor supply has no effect on long-run growth (i.e., the negative consumption-leisure choice effect is absent). Taken together, long-run growth is an increasing function of the nominal interest rate.

There is one important difference between our Schumpeterian model and the AK model. That is, a higher nominal interest rate always increases growth in the AK model, while it possibly decreases growth in our Schumpeterian model. The reason is as follows. In the Schumpeterian model with the SOC and elastic labor supply, there exists the consumption-leisure choice channel from a higher nominal interest rate. This consumption-leisure choice effect through decreasing market size tends to reduce growth when the nominal interest rate increases. When this effect dominates the labor reallocation effect, a higher nominal interest rate reduces long-run growth.

\[ \text{This is an equivalent way, as using a CIA constraint, to introduce money into growth models.} \]
However, this consumption-leisure choice effect is absent in the AK model and thus growth always increases with the nominal interest rate in the AK model.

6 Conclusion

In this paper we formalize Schumpeter's idea of “psychology of entrepreneurs” —entrepreneurs innovate for the sake of success itself, not for the fruits of success—by introducing the spirit of capitalism (or a direct preference on wealth) into a Schumpeterian growth model with money. We show the existence of the spirit of capitalism creates a new channel for monetary policy to impact growth and welfare and generates new implications for monetary policy. In particular, unlike recent research which shows that higher nominal interest rates always lead to lower long-run growth, we show a higher nominal interest rate could promote long-run growth when the spirit of capitalism is strong. We theoretically show the key mechanism through which the SOC alters the growth implications of monetary policy.

In addition, we show a stronger SOC also pushes the optimal nominal interest rate lower, making the Friedman rule (the optimal non-negative nominal interest rate is zero) more likely to hold, contrasting the finding in a recent paper study by Chu and Cozzi (2014) which studies the optimal monetary policy in a similar Schumpeterian model. This finding also provides support to the adoption of a negative interest rate which was intensively discussed by policymakers in recent years. Finally, when calibrating the model to the US economy, we show the SOC can help explain a substantial part (about one-third) of long-run growth in the US.

Our model can also be extended to study interactions between fiscal policy and monetary policy. In particular, our model suggests that the effect of nominal interest rate on long-run economic growth depends on the strength of the spirit of capitalism and the tightness of the CIA constraint on the R&D sector. Suppose some fiscal policy subsidizes the R&D borrowing costs, it would influence the effects of nominal interest rate on long-run growth, and the impact of the fiscal policy would also depend on the strength of the spirit of capitalism. We will leave this for our future research.

7 Appendix

7.1 Solving Household’s Optimization Problem

Household’s Hamiltonian function is

\[
H_t = \ln c_t + \theta \ln (a_t) - \eta \frac{1 + 1/\sigma}{1 + 1/\sigma} + \mu_t (r_t e_t + w_t l_t - c_t - \pi_t m_t + i_t b_t + \tau_l) + \xi_t (m_t - c_t - b_t).
\]
where $\mu_t$ is the co-state variable on (2); $\xi_t$ is the Lagrangian multiplier for the CIA constraint; 

\[ a_t = e_t + m_t. \]

The first-order conditions include:

\[
\frac{\partial H}{\partial c_t} = \frac{1}{c_t} - \mu_t - \xi_t = 0, \quad (36)
\]

\[
\frac{\partial H}{\partial l_t} = -\eta_l^{1/\sigma} + \mu_t w_t = 0, \quad (37)
\]

\[
\frac{\partial H}{\partial b_t} = \mu_t i_t - \xi_t = 0, \quad (38)
\]

\[
\frac{\partial H}{\partial e_t} = \theta a_t + \mu_t r_t = \rho \mu_t - \mu_t, \quad (39)
\]

\[
\frac{\partial H}{\partial m_t} = \theta a_t - \mu_t \pi_t + \xi_t = \rho \mu_t - \mu_t. \quad (40)
\]

Combining (39) and (40) yields 

\[
\xi_t = \mu_t (r_t + \pi_t). \quad (41)
\]

Further combining with (38) yields  

\[
i_t = r_t + \pi_t. \quad (42)
\]

Plugging this condition into (36) yields 

\[
\frac{1}{c_t} = \mu_t (1 + i_t), \quad (41)
\]

which is (3) in the main text. Rewriting (37) we get the optimal condition for labor supply 

\[
\eta_l^{1/\sigma} = w_t \mu_t, \quad (42)
\]

which is (4) in the main text. Rewriting (39) as 

\[
-\frac{\mu_t}{\mu_t} = r_t - \rho + \theta \frac{1 + i}{\mu_t a_t} \quad (43)
\]

yields the intertemporal optimality condition (6) in the main text.

### 7.2 Proof of Proposition 1

For convenience, we define the ratio of final output to the value of the monopolistic firms $\frac{y_t}{v_t}$ as a new variable $\Theta_t$ (i.e., $\Theta_t = \frac{y_t}{v_t}$). The law of motion of $\Theta_t$ is 

\[
\frac{\Theta_t}{\Theta_t} = \frac{y_t}{y_t} - \frac{v_t}{v_t}. \quad (44)
\]

Given a fixed nominal interest rate $i$, combining (3) and (6) yields $\dot{c}_t = c_t - \rho + \theta (1 + i) c_t / a_t$. From the final goods market clearing condition $c_t = y_t / L$, we have $y_t / y_t = \dot{c}_t / c_t$.

Equation (15) gives $\dot{v}_t / v_t = r_t + \lambda_t - \Pi_t / v_t$. Equation (15) delivers $\lambda_t = \varphi l_{r_t}$. Using (13), we have $\Pi_t / v_t = \frac{(\gamma - 1) / \gamma}{\Theta_t}$. Plugging these into (14) we have: 

\[
\frac{\Theta_t}{\Theta_t} = \frac{(\gamma - 1)}{\gamma} \Theta_t - \varphi l_{r_t} - \rho + \theta \frac{(1 + i) c_t}{a_t}. \quad (45)
\]
Combining (12), (14) and (15) yields \( l_{x,t} = \frac{(1+\beta i)\Theta_t}{\varphi\gamma} \). From (22) we have \( l_{r,t} + l_{x,t} = l_t = [\eta \gamma l_{x,t} (1+i)]^{-\sigma} \). Therefore, \( l_{r,t} = \frac{\varphi}{\eta \Theta_t (1+\beta i)(1+i)} - \frac{(1+\beta i)\Theta_t}{\varphi\gamma} \).

Plugging \( l_{x,t} = \frac{(1+\beta i)\Theta_t}{\varphi\gamma} \) into (21) produces

\[
\frac{c_t}{a_t} = \frac{(1+\beta i)\Theta_t}{(1+\beta i)(1+\Theta_t) + \varphi \beta \left[ \frac{\varphi}{\eta \Theta_t (1+\beta i)(1+i)} \right]^{-\sigma} - \frac{(1+\beta i)\Theta_t}{\varphi\gamma}}.
\]

Plugging these results into (45), we have

\[
\frac{\dot{\Theta}_t}{\Theta_t} = \left[ 1 - \left( \frac{1}{\eta (1+\beta i)(1+i)} \right)^\sigma \left( \frac{\varphi}{\Theta_t} \right)^{1+\sigma} + \frac{\theta (1+i) (1+\beta i)}{(1+\beta i)(1+\Theta_t) + \varphi \beta \left[ \frac{\varphi}{\eta \Theta_t (1+\beta i)(1+i)} \right]^{-\sigma} - \frac{(1+\beta i)\Theta_t}{\varphi\gamma}} \right] \Theta_t - \rho. \tag{46}
\]

Because \( \Theta_t > 0 \), equation (46) shows that \( \Theta_t \) is saddle-point stable, meaning that \( \Theta_t \) jumps immediately to its interior steady state given by the univariate equation:

\[
\frac{\rho}{\Theta_t} = 1 + \frac{\beta i}{\varphi\gamma} - \left( \frac{1}{\eta (1+\beta i)(1+i)} \right)^\sigma \left( \frac{\varphi}{\Theta_t} \right)^{1+\sigma} + \frac{\theta (1+i) (1+\beta i)}{(1+\beta i)(1+\Theta_t) + \varphi \beta \left[ \frac{\varphi}{\eta \Theta_t (1+\beta i)(1+i)} \right]^{-\sigma} - \frac{(1+\beta i)\Theta_t}{\varphi\gamma}}.
\]

This can be rewritten as:

\[
\left( \frac{1}{\eta (1+\beta i)(1+i)} \right)^\sigma \left( \frac{\varphi}{\Theta_t} \right)^{1+\sigma} = \frac{\varphi \gamma + \beta i}{\varphi\gamma} + \frac{\theta (1+i) (1+\beta i)}{(1+\beta i)(1+\gamma \Theta_t) + \frac{\beta \varphi^{1+\sigma} \Theta_t^{-\sigma}}{\eta (1+\beta i)(1+i)}} - \frac{\rho}{\Theta_t}. \tag{47}
\]

With \( \Theta_t > 0 \) and \( \Theta_t \) on the horizontal axis, the LHS (left-hand-side) of (47) is a downward-sloping curve with the vertical and horizontal axes as the asymptotes. The RHS is a hyperbola with the vertical asymptote as the vertical axis, and the horizontal asymptote is \( y = \frac{\varphi \gamma + \beta i}{\varphi\gamma} \) when \( \theta \) is small (we are considering reasonable values of \( \theta \); it is clear this holds when \( \theta = 0 \)). Therefore, the two lines intersect once at \( \Theta_t^* \). When \( \Theta_t > \Theta_t^* \), \( \dot{\Theta}_t > 0 \); when \( \Theta_t < \Theta_t^* \), \( \dot{\Theta}_t < 0 \). Therefore, given a fixed nominal interest rate \( i \), the dynamics of \( \Theta_t \) show saddle-point stability: \( \Theta_t \) jumps immediately to its interior steady state that is stationary and unique. Given \( l_{x,t} = \frac{(1+\beta i)\Theta_t}{\varphi\gamma} \) and \( l_{r,t} = \frac{\varphi}{\eta \Theta_t (1+\beta i)(1+i)} - \frac{(1+\beta i)\Theta_t}{\varphi\gamma} \), we know that \( l_r, l_x \) and \( l \) must be stationary and unique as well.

### 7.3 Derivation of Equations (20) and (21)

Dividing both sides of equation (16) by \( v_t \), we have \( r_t = \frac{\Pi_t}{v_t} + g_t - \lambda_t \), where we have used \( v_t/v_t = g_t \).

We rewrite this equation as \( \Pi_t = (r_t + \lambda_t - g_t) v_t \), which is equivalent to

\[
\lambda_t \Pi_t = (r_t + \lambda_t - g_t) \lambda_t v_t. \tag{48}
\]
Using the Euler equation (7), we have \( g_t = r_t - \rho + \theta (1 + i_t) \frac{c_t}{a_t} \), which gives \( r_t - g_t = \rho - \theta (1 + i_t) \frac{c_t}{a_t} \). The free entry condition (14) can be written as \( \lambda_t v_t = (1 + \beta i_t) w_t L_{r,t} \). Plugging these two conditions into (48) to replace \((r_t - g_t) \) and \( \lambda_t v_t \), we have

\[
\lambda_t \Pi_t = [\rho + \lambda_t - \theta (1 + i) c_t/a_t] (1 + \beta i_t) w_t L_{r,t},
\]

(49)

Using (13) and (12), we have \( \Pi_t = (\gamma - 1) w_t L_{x,t} \). Plugging this condition into (49) to replace \( \Pi_t \) yields

\[
\lambda_t (\gamma - 1) l_{x,t} = [\rho + \lambda_t - \theta (1 + i) c_t/a_t] (1 + \beta i_t) l_{r,t},
\]

(50)

where we have used \( l_{x,t} = L_{x,t}/L \) and \( l_{r,t} = L_{r,t}/L \).

Using (15), (50) becomes \( (\gamma - 1) l_{x,t} = \left( \frac{\phi}{\varphi} + \frac{\lambda_t}{\varphi} - \frac{\theta (1 + i) c_t}{\varphi a_t} \right) (1 + \beta i_t) \). Using \( \lambda_t = \varphi l_{r,t} \), we have \( (\gamma - 1) l_{x,t} = (1 + \beta i_t) \left( l_{r,t} + \rho/\varphi - \frac{\theta (1 + i) c_t}{\varphi a_t} \right) \), which is equation (20) in the main text.

The derivation for equation (21) is as follows.

\[
\frac{c_t}{a_t} = \frac{c_t L}{a_t L} = \frac{y_t}{c_t L + m_t L},
\]

(51)

where we have used the output market clearing condition \( c_t = y_t/L \), and the definition of \( a_t = e_t + m_t \).

The CIA constraint binds: \( c_t + b_t = m_t \), where \( b_t L = \beta w_t L_{r,t} \). We also have \( e_t = v_t/L \). Therefore, (51) can be written as

\[
\frac{c_t}{a_t} = \frac{y_t}{v_t + c_t L + b_t L} = \frac{y_t}{v_t + y_t + \beta w_t L_{r,t}}.
\]

(52)

Using (12), we have \( y_t = \gamma w_t L_{x,t} \). Combining (14) and (15), we have \( v_t = (1 + \beta i_t) w_t L_{r,t}/\varphi L_{r,t} = (1 + \beta i_t) w_t L/\varphi \), where we have used \( l_{r,t} = L_{r,t}/L \). Therefore, (52) can be written as

\[
\frac{c_t}{a_t} = \frac{\gamma w_t L_{x,t}}{(1 + \beta i_t) w_t L/\varphi + \gamma w_t L_{x,t} + \beta w_t L_{r,t}} = \frac{\gamma l_{x,t}}{(1 + \beta i_t)/\varphi + \gamma l_{x,t} + \beta l_{r,t}},
\]

(53)

which is equation (21) in the main text.

### 7.4 Proof of Proposition 2

We resort to the graphical proof presented in Aghion and Howitt (1998, ch. 2). Two conditions pin down the equilibrium labor allocation: the labor market clearing condition in (22) and the free labor mobility condition in (23). We rewrite the labor market clearing condition (22) as

\[
l_{r,t} = [\eta \gamma (1 + i_t)]^{-\sigma} l_{x,t}^{-\sigma} - l_{x,t}.
\]

(54)

We have the constraint \( l_{r,t} + l_{x,t} = l_t \leq 1 \). We have \( \lim_{t \to 0} l_{r,t} = \infty \). Therefore, the labor market clearing condition becomes \( l_{r,t} + l_{x,t} = 1 \) when \( l_{x,t} \) is smaller than a threshold \( l_{x} \). We also have
\[ l_{x,t} = \left[ \eta \gamma (1 + i) \right]^{-\sigma / (1 + \sigma)} \] when \( l_{r,t} = 0 \). Therefore, we impose the restriction on the parameters: 
\[ [\eta \gamma (1 + i)]^{-\sigma / (1 + \sigma)} \leq 1. \] Using (54), we have

\[
\frac{\partial l_{r,t}}{\partial l_{x,t}} = -\sigma \left( \frac{1}{\eta \gamma (1 + i)} \right)^{\sigma - 1} - 1 < 0,
\]

(55)

\[
\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} = \sigma (\sigma + 1) \left( \frac{1}{\eta \gamma (1 + i)} \right)^{\sigma - 2} > 0.
\]

(56)

Therefore, the labor market clearing condition shows that, with manufacturing labor \( l_{x,t} \) on the horizontal axis, R&D labor share \( l_{r,t} \) is a downward-sloping convex function of \( l_{x,t} \) when \( l_{x,t} \in [l_x, 1] \). We denote this curve as the \( L \) curve in Figure 1. Imposing \( \beta = 0 \), we rewrite the free labor mobility (23) as

\[ l_r = (\gamma - 1) l_x - \rho / \varphi + \frac{\theta (1 + i) \gamma l_x}{1 + \varphi \gamma l_x}. \]

(57)

We have \( l_{r,t} = -\rho / \varphi \) when \( l_{x,t} = 0 \). Taking derivatives, we have

\[
\frac{\partial l_{r,t}}{\partial l_{x,t}} = (\gamma - 1) + \frac{\theta (1 + i) \gamma}{(1 + \varphi \gamma l_x)^2} > 0,
\]

(58)

\[
\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} = -\frac{2 \theta \varphi (1 + i) \gamma^2 l_x}{(1 + \varphi \gamma l_x)^3} < 0.
\]

(59)

Thus, the free labor mobility condition shows that \( l_{r,t} \) is an upward-sloping concave function of manufacturing labor \( l_{x,t} \) (with \( l_{x,t} \) on the horizontal axis). We denote this curve as the \( M \) curve in Figure 1.

The two lines intersect once, meaning there is a unique solution (a unique balanced growth path). Two properties follow. First, the SOC only shifts the \( M \) curve, with an increase in \( \theta \) rotating the \( M \) curve counter-clockwise around \((0, -\rho / \varphi)\), thereby leading to higher R&D labor \( l_r \) and lower manufacturing labor \( l_x \), all else equal. Thus, a stronger degree of the SOC raises the balanced growth rate, all else equal (i.e., the equilibrium would move from \( O \) to \( A \)). Second, both the taste for leisure \( \eta \) and the Frisch elasticity of labor supply \( \sigma \) only shift the \( L \) curve, with a larger \( \eta \) or \( \sigma \) shifting the \( L \) curve leftward, thereby decreasing both R&D labor \( l_r \) and manufacturing labor \( l_x \), all else equal. So, either the increase in the taste for leisure \( \eta \) or that in the Frisch elasticity of labor supply \( \sigma \) would decrease the balanced growth rate (i.e., the equilibrium would move from \( O \) to \( B \)), all else equal.

Using (54) and (57), an increase in the nominal interest rate shifts both curves: it rotates the \( M \) curve counter-clockwise around \((0, -\rho / \varphi)\), and the degree of rotation depends on the size of \( \theta \); Additionally, it shifts the \( L \) curve to the left with the size of the shift depending on the magnitudes of both \( \eta \) and \( \sigma \) (i.e., the equilibrium would move from \( O \) to \( E \)). Therefore, as the
nominal interest rate increases, when growth-enhancing effect of the SOC dominates (i.e., with a relatively large \( \theta \)), the \( M \) curve shifts more than the \( L \) curve does, yielding a larger R&D labor \( l_r \) and thereby higher long-run growth; when the growth-reducing effects of the taste for leisure \( \eta \) or the Frisch elasticity of labor supply \( \sigma \) dominate (it is more likely with a larger \( \eta \) or \( \sigma \)), the \( L \) curve shifts more than the \( M \) curve does, producing a smaller R&D labor \( l_r \) and thereby lower long-run growth.

By contrast, without the SOC (\( \theta = 0 \)), an increase in the nominal interest rate shifts the \( L \) curve to the left. The new equilibrium will move along the \( M \) curve (i.e., the equilibrium would move from \( O \) to \( B \)), yielding a smaller R&D labor \( l_r \) and thereby lower long-run growth.

7.5 Proof of Proposition 3

Using Figure 1 and the proof of Proposition 2, we have \( \frac{\partial l_r}{\partial i} < 0 \), \( \frac{\partial \theta}{\partial i} < 0 \), and \( \frac{\partial x_0}{\partial i} \geq 0 \iff \frac{\partial g}{\partial i} \geq 0 \).

When \( \theta > 0 \), taking the derivative of \( U \) in (28) with respect to \( i \), we have
\[
\frac{\partial U}{\partial i} = \frac{1}{\rho} \left[ \frac{\varphi \gamma \theta}{1 + \varphi \gamma l_{x,0}} \frac{\partial l_{x,0}}{\partial i} + \frac{(1 + \theta) \partial g}{\rho} \frac{\partial i}{\partial i} - \frac{\eta l_{x,0}^{1/\sigma}}{\partial l_{x,0}} \frac{\partial l_{x,0}}{\partial i} \right].
\] (60)

When \( \theta = 0 \), we have
\[
\left. \frac{\partial U}{\partial i} \right|_{\theta=0} = \frac{1}{\rho} \left[ \frac{1}{l_{x,0}} \frac{\partial l_{x,0}}{\partial i} + \frac{1}{\rho} \frac{\partial g}{\partial i} - \frac{\eta l_{x,0}^{1/\sigma}}{\partial l_{x,0}} \frac{\partial l_{x,0}}{\partial i} \right].
\] (61)

Given welfare is an inverted-U function of the nominal interest rate without the SOC, there is a unique nominal interest rate \( i^* \) that satisfies \( \frac{\partial U}{\partial i} \big|_{\theta=0} = 0 \). Using (60) and (61), we have
\[
\left. \frac{\partial U}{\partial i} \right|_{i=i^*} = \frac{1}{\rho} \left[ \frac{\varphi \gamma \theta}{1 + \varphi \gamma l_{x,0}} \frac{\partial l_{x,0}}{\partial i} + \frac{(1 + \theta) \partial g}{\rho} \frac{\partial i}{\partial i} - \frac{\eta l_{x,0}^{1/\sigma}}{\partial l_{x,0}} \frac{\partial l_{x,0}}{\partial i} \right] < 0.
\] (62)

Given welfare is an inverted-U function of the nominal interest rate with the SOC, there exists a unique nominal interest rate \( i^{**} < i^* \) that satisfies \( \frac{\partial U}{\partial i} \big|_{\theta>0} = 0 \).

7.6 Proof of Proposition 4

For the labor market clearing condition, (58) and (59) still hold: \( \frac{\partial l_{r,t}}{\partial x_{x,t}} < 0 \) and \( \frac{\partial^2 l_{r,t}}{\partial l_{x,t}} > 0 \). Given \( \lim_{l_{x,t} \to 0} l_{r,t} = \infty \), the labor market clearing condition becomes \( l_{r,t} + l_{x,t} = 1 \) when \( l_{x,t} \) is below a threshold. We assume \( l_{x,t}|_{l_r=0} = \left( \frac{1}{\eta} \right)^{\sigma/(1+\sigma)} \leq 1 \). Therefore, the labor market clearing condition is described by the \( L \) curve in Figure 2. Using the free labor mobility condition, we
have \( \frac{\partial l_{r,t}}{\partial x,t} = (\gamma - 1) + \theta \gamma > 0 \) and \( \frac{\partial^2 l_{r,t}}{\partial x^2,t} = 0 \). Therefore, the \( M \) curve for the free labor mobility condition is a straight upward-sloping line. Using Figure 2, an increase in \( \theta \) does not shift the \( L \) curve, but it does rotate the \( M \) curve counter-clockwise around \((0, -\rho/\varphi)\) (i.e., the equilibrium would move from \( O \) to \( E \)), thereby increasing R&D labor \( l_r \) and decreasing manufacturing labor \( l_x \), all else equal.

### 7.7 Proof of Proposition 5

Household’s Hamiltonian function in this AK model is given by

\[
H_t = \ln c_t + \theta \ln (a_t) + \mu_t (Ak_t - c_t - \pi_t m_t + \tau_t) + \xi_t (m_t - c_t),
\]

where \( \mu_t \) is the co-state variable; \( \xi_t \) is the Lagrangian multiplier for the CIA constraint; \( a_t = k_t + m_t \).

The corresponding first-order conditions are given by:

\[
\begin{align*}
\frac{\partial H_t}{\partial c_t} &= \frac{1}{c_t} - \mu_t - \xi_t = 0, \\
\frac{\partial H_t}{\partial k_t} &= \frac{\theta}{a_t} + A\mu_t = \rho\mu_t - \dot{\mu}_t, \\
\frac{\partial H_t}{\partial m_t} &= \frac{\theta}{a_t} - \mu_t \pi_t + \xi_t = \rho\mu_t - \dot{\mu}_t.
\end{align*}
\]

On the balanced growth path, we have \( \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = g_{ak} \). Using the resource constraint, we have \( \dot{k}_t = Ak_t - c_t \), which gives

\[
\frac{c_t}{k_t} = A - g_{ak}. 
\]

Combining (64) and (65) yields \( \xi_t = (A + \pi_t) \mu_t \). We have \( \dot{m}_t = M_t/(P_t L) \), which yields \( \dot{m}_t/m_t = \dot{M}_t/M_t - \pi_t \). The binding CIA constraint yields \( \dot{c}_t/c_t = \dot{m}_t/m_t = g_{ak} \). Taken together, we have

\[
\xi_t = \left( A + \frac{\dot{M}_t}{M_t} - g_{ak} \right) \mu_t.
\]

Using the Fisher equation \( i_t = \pi_t + r_t \), we have \( \dot{i}_t = \dot{M}_t/M_t - \dot{m}_t/m_t + A \), where we have used \( r_t = A \) in the AK model. Therefore, we have \( \dot{i}_t = \dot{M}_t/M_t - g_{ak} + A \), and combining with (67) yields \( \xi_t = i_t \mu_t \). Plugging this condition into (63) yields

\[
\frac{1}{c_t} = \mu_t (1 + i_t),
\]

which shows that, given \( i_t = i \), on the balanced growth path, \( \dot{c}_t/c_t = -\dot{\mu}_t/\mu_t = g_{ak} \).
Using (64), we have

$$\frac{\dot{\mu}_t}{\mu_t} = A - \rho + \frac{\theta}{\mu_t a_t}, \quad (69)$$

Plugging (68) into (69), we have the Euler equation

$$-\frac{\dot{\mu}_t}{\mu_t} = g_{a_k} = A - \rho + \frac{\theta c_t (1+i)}{a_t}. \quad (70)$$

In equilibrium, the CIA constraint binds, which gives $c_t = m_t$. Using (66), we can rewrite (70) as

$$g_{a_k} = A - \rho + \frac{\theta (1+i)}{1 + \frac{1}{A-g_{a_k}}}. \quad (71)$$

Rewriting (71), we have the univariate quadratic equation for $g_{a_k}$:

$$g_{a_k}^2 - \Omega_1 g_{a_k} + \Omega_2 = 0, \quad (72)$$

where $\Omega_1 = \theta (1+i) + (A+1) + (A-\rho)$ and $\Omega_2 = \theta A (1+i) + (A+1) (A-\rho)$.

When $i \geq -1$, we have $\Omega_1 > 0$ and $\Omega_2 > 0$. Therefore, the quadratic equation (72) has two positive roots when $\Delta = \Omega_1^2 - 4\Omega_2 > 0$ and no solution (in real numbers) when $\Delta < 0$. We assume $\Delta > 0$ because $\Delta|_{\theta=0} > 0$.

When $\theta = 0$ (i.e., there is no SOC), we have

$$g_{a_k}^2 - [(A+1) + (A-\rho)] g_{a_k} + (A+1) (A-\rho) = 0, \quad (73)$$

where the two roots are $g_{a_k}|_{\theta=0} = A + 1$ and $g_{a_k}|_{\theta=0} = A - \rho$. The only admissible solution is the smaller root $g_{a_k}|_{\theta=0} = A - \rho$. That is, money is superneutral in the AK model with a CIA constraint on consumption (see also Dotsey and Sarte, 2000).

Therefore, when $\theta > 0$, focusing on the unique perfect-foresight equilibrium, the only admissible solution is the smaller root of (72). Thus, the balanced growth rate is

$$g_{a_k} = \frac{\Omega_1 - \sqrt{\Omega_1^2 - 4\Omega_2}}{2}. \quad (74)$$

When the nominal interest rate $i$ increases, the y-intercept $\Omega_2$ of the quadratic function increases, and its axis of symmetry $\frac{\Omega_1}{2}$ shifts to the right. Therefore, the smaller positive root of the quadratic function (i.e., the balanced growth rate) increases.

**References**


### Table 1: Parameters and Moments in the Joint Calibration

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<tr>
<th>Parameter</th>
<th>Value</th>
<th>Joint Targets</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOC</td>
<td>$\vartheta$</td>
<td>0.09</td>
<td>FOMC long-run projection</td>
<td>1.8%</td>
</tr>
<tr>
<td>Taste of leisure</td>
<td>$\eta$</td>
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<td>total labor supply</td>
<td>$\frac{1}{3}$</td>
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<tr>
<td>Innovation param.</td>
<td>$\varphi$</td>
<td>2.25</td>
<td>real interest rate</td>
<td>1.8%</td>
</tr>
<tr>
<td>Innovation param.</td>
<td>$\gamma$</td>
<td>1.17</td>
<td>consumption to wealth ratio</td>
<td>0.43</td>
</tr>
</tbody>
</table>

### Table 2: Effects of SOC on Growth and Labor Allocations

<table>
<thead>
<tr>
<th></th>
<th>long-run growth</th>
<th>labor in R&amp;D</th>
<th>labor in production</th>
<th>total labor supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark Economy</td>
<td>1.8%</td>
<td>0.050</td>
<td>0.287</td>
<td>0.337</td>
</tr>
<tr>
<td>Economy without SOC</td>
<td>1.2%</td>
<td>0.033</td>
<td>0.297</td>
<td>0.331</td>
</tr>
<tr>
<td>Change</td>
<td>−0.6 (ppt.)</td>
<td>−33.5%</td>
<td>3.7%</td>
<td>−1.8%</td>
</tr>
</tbody>
</table>
Figure 1: **Equilibrium Labor Allocation under a CIA Constraint on Consumption**

This graph shows the determination of equilibrium labor allocation in the R&D sector and the production sector. The $L$ is the labor-market clearing condition (equation (22)) and the $M$ curve is the labor mobility condition (equation (23)).
Figure 2: Effect of the SOC on Labor Allocation in a Non-monetary Schumpeterian Model

This graph shows the determination of equilibrium labor allocation in the R&D sector and the production sector in a Schumpeterian model without money. The $L$ is the labor-market clearing condition (equation (32)) and the $M$ curve is the labor mobility condition (equation (34)).
Figure 3: Effect of Nominal Interest Rate on Growth at Different Degrees of SOC ($\beta = 0.05$) The three lines show how long-run growth changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 4: **Effect of Nominal Interest Rate on Labor Allocation at Different Degrees of SOC ($\beta = 0.05$)** The three lines show how labor input in each sector changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 5: **Effect of SOC on Labor Allocation in R&D** This figure shows a decline in the SOC from the calibrated value to zero shifts the $M$ curve rightward, leading to a lower equilibrium R&D labor input ($l_r$) while labor input in the production sector ($l_x$) increases.
Figure 6: Optimal Nominal Interest Rate at Different Degrees of SOC ($\beta = 0.05$) The three lines show how welfare changes with the nominal interest rate at different degrees of the spirit of capitalism. The circle on each line represents the maximum. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 7: Robustness Check ($\beta = 0.1$): Effect of Nominal Interest Rate on Growth at Different Degrees of SOC

The three lines show how long-run growth changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 8: Robustness Check ($\beta = 0.1$): Optimal Nominal Interest Rate at Different Degrees of SOC The three lines show how welfare changes with the nominal interest rate at different degrees of the spirit of capitalism. The circle on each line represents the maximum. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 9: **Robustness Check ($\beta = 0.1$): Effect of Nominal Interest Rate on Labor Allocation at Different Degrees of SOC** The three lines show how labor input in each sector changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 10: Robustness Check ($\beta = 0.2$): Effect of Nominal Interest Rate on Growth at Different Degrees of SOC The three lines show how long-run growth changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 11: **Robustness Check ($\beta = 0.2$): Optimal Nominal Interest Rate at Different Degrees of SOC** The three lines show how welfare changes with the nominal interest rate at different degrees of the spirit of capitalism. The circle on each line represents the maximum. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.
Figure 12: **Robustness Check ($\beta = 0.2$): Effect of Nominal Interest Rate on Labor Allocation at Different Degrees of SOC** The three lines show how labor input in each sector changes with the nominal interest rate at different degrees of the spirit of capitalism. The solid line is based on the U.S. calibration. The green dashed line represents a country with a stronger spirit of capitalism, while the blue dashed line shows the case without the spirit of capitalism.