A New Approach to Integrating Expectations into VAR Models

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A New Approach to Integrating Expectations into VAR Models∗

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Abstract
Expectations about future economic conditions play a central role in macroeconomic theory. These expectations are empirically measured from surveys or financial markets and then are frequently analyzed in Vector autoregression (VAR) models alongside realized data of the same variable. However, jointly analyzing realized data and external forecasts in a VAR leads to the simultaneous existence of two different expectations of the same variable: the VAR-based forecast and the survey or market forecast. This paper proposes a Bayesian prior over the VAR parameters which allows the econometrician to impose the desired degree of consistency between these two forecasts. Our approach leverages the existence of multiple forecasts to aid in structural VAR identification and enhance VAR forecasts. We illustrate the usefulness of our approach in two applications exploring the identification of forward guidance shocks and the role that inflation expectations played in shaping inflation tail-risks during and after the Great Recession.

Keywords: Bayesian Vector Autoregression (VAR), Sign Restrictions, Information Frictions, Monetary Policy, Forward Guidance, Inflation Expectations
JEL classification codes: C11, C32, E52, E31
1 Introduction

The conduct of monetary policy has shifted from solely managing reserves and overnight interest rates to managing expectations, often through central bank communication. Therefore, analyzing expectations is necessary to study monetary policies which, for instance, aim to steer interest rate expectations through forward guidance or anchor inflation expectations. While Vector autoregression (VAR) models remain one of the most flexible time-series tools for conducting such empirical macroeconomic research, there is no agreed upon way to integrate expectations into VAR models. At one extreme, including survey or financial market-implied expectations in a VAR model in a “model consistent” manner, in the sense that the survey or market forecast and the VAR-implied forecast align at every horizon, seems to contradict mounting empirical evidence against full information rational expectations (Coibion, Gorodnichenko and Kamdar, 2018). Alternatively, at the other extreme, including expectations without imposing any connection between the VAR forecast and the survey or market forecast fails to fully leverage the rich information that expectations contain for VAR estimation, identification, and inference.

In this paper, we propose a Bayesian approach to integrating forecasts from surveys or from financial markets in a VAR that includes realized values of the same or a closely related variable. We leverage the fact that VAR’s themselves have the ability to generate a forecast for any variable included in the model. Then, using this VAR-implied forecast together with the survey forecast, we argue for constructing a non-degenerate prior for VAR coefficients that places greater mass on areas of the parameter space where the two forecasts align. We call this the forecast consistent prior. Our prior allows for dynamic correlation between the realized data and its’ survey or market forecast, unlike the popular Minnesota prior, without dogmatically restricting the discrepancy between the two forecasts.\footnote{The Minnesota prior ignores dynamic cross correlations by assuming each series follows a random walk.} From an implementation standpoint, our method relies on a computationally efficient importance sampling technique which simply re-weights the posterior draws. Intuitively, under the forecast consistent prior, a draw which is closer to satisfying forecast consistency receives a greater weight. These weights are informed by a hyperparameter which governs the precision of our prior. By setting this parameter very small or very large, our approach can accommodate varying degrees of information rigidity ranging from survey expectations which are formed independently of the VAR model all the way towards full information rational expectations.
A novel aspect of our prior is its applicability to structural VAR models to aid in the identification of structural shocks. For some intuition, note that structural impulse responses are simply conditional forecasts from the VAR. When the forecast consistent prior is placed on unconditional forecasts — in other words, forecasts that are not conditional on the realization of a structural shock and instead are based only on the observed data — then our prior shrinks the reduced-form VAR coefficients in the direction of satisfying forecast consistency. However, when the forecast consistent prior is placed on impulse response functions, or conditional forecasts, then our prior can also inform the structural VAR coefficients. One natural application which we explore in this paper is the addition of our prior to a structural VAR identified using sign-restrictions. Structural VAR (SVAR) models with only sign-restrictions on impulse responses lead to partial identification in which the model parameters are identified only up to a set as discussed by Moon and Schorfheide (2012) and Uhlig (2017). Within this set, any two alternative SVAR models are equally probable. Forecast consistency restrictions on impulse responses provide easily motivated probabilistic restrictions based on economic theory that can break this equality and distinguish two alternative SVAR models based on a variable’s impulse response and the impulse response of its survey or market forecast.

We illustrate the usefulness of our approach in different settings to shed light on important issues for monetary policy, including: the effects of forward guidance shocks on output and the role that inflation expectations played in shaping inflation tail-risks during the Great Recession and its aftermath. The forward guidance application is particularly illustrative. In this application we add Blue Chip forecasts of one-year ahead short-term interest rates to the Uhlig (2005) VAR model. We apply similar sign restrictions that Uhlig (2005) proposes and we find that, similar to Uhlig’s finding for conventional monetary policy shocks, forward guidance shocks which reduce survey expectations of future short-term interest rates have an ambiguous effect on output. However, we also show that these sign-restrictions admit a wide range of VAR-implied forecasts of the federal funds rate. And many of these VAR-implied forecasts deviate significantly from the path of rates predicted by forecasters following the forward guidance shock. We then layer the forecast consistent prior on top of sign-restrictions to identify forward guidance shocks which better align these two forecasts. We find that such shocks lead output to rise and, moreover, the output effects increase monotonically with the degree of forecast consistency. These empirical results suggest that clear communication which effectively synchronizes interest rate expectations is a salient factor that shapes the real effects of forward guidance shocks, as suggested by theoretical models of forward guidance with information frictions (e.g. Angeletos and Lian, 2018; Campbell et al., 2019).
2 The Forecast Consistent Prior

In this section we introduce our VAR notation, next we use a simple example with a bivariate VAR to introduce the forecast consistent prior, and then we show how to apply our prior in a general VAR setting.

2.1 VAR Preliminaries

A reduced-form VAR($l$) is given by:

$$Y_t = A(1)Y_{t-1} + A(2)Y_{t-2} + \ldots + A(l)Y_{t-l} + u_t,$$

where $Y_t$ is an $m \times 1$ vector of data at date $t = 1 - l, \ldots, T$, $A(i)$ are coefficient matrices of size $m \times m$, and $u_t$ is the one-step ahead prediction error, or reduced-form residuals, which are assumed to be distributed normally with mean 0 and covariance matrix $\Sigma$. To simplify notation, we suppress constants and other deterministic components of the VAR model. We assume that this reduced-from VAR is derived from an underlying structural VAR model:

$$BY_t = B(1)Y_{t-1} + B(2)Y_{t-2} + \ldots + B(l)Y_{t-l} + \varepsilon_t,$$

in which $B$ is an $m \times m$ non-singular coefficient matrix which governs the contemporaneous interactions between the variables $Y_t$, $B(i)$ are coefficient matrices of size $m \times m$, and $\varepsilon_t$ are the structural shocks which are independent of one another, mean zero, and have a standard deviation of one.

Combining equations (1) and (2) reveals that the reduced-form lag coefficient matrices $A(i)$ are related to the structural coefficient matrices $B(i)$ by the mapping $A(i) = B^{-1}B(i)$. And, similarly, the reduced form residuals $u_t$ are related to the structural shocks by the mapping $u_t = B^{-1}\varepsilon_t$. Therefore, knowledge of $B$ allows one to uncover the structural VAR model from the estimated reduced form VAR model. Following much of the structural VAR literature, we parameterize $B$ as follows:

$$B^{-1} = CQ,$$

where $C$ is the lower-triangular Cholesky factor of $\Sigma$ and $Q$ is a square $m \times m$ orthogonal rotation matrix such that $Q'Q = QQ' = I_m$. This parameterization is able to encompass multiple identification strategies including sign-restrictions which consider a set of alternative
rotation matrices $Q$ and recursive short-run restrictions which achieve point identification by assuming that $Q = I_m$.

We take a Bayesian approach to estimation and inference throughout the paper. In principle, our forecast consistent prior could be joined with any prior over the reduced-form VAR parameters. However, as we discuss later, popular priors such as the Minnesota prior may be undesirable in our setting where survey forecasts and realized data on the same variable are jointly included in $Y_t$. We describe the prior in terms of the stacked version of the reduced-form VAR in equation (1):

$$Y = XA + u,$$

where $X_t = [Y_{t-1}', Y_{t-2}', \ldots, Y_{t-l}]'$, $Y = [Y_1, \ldots, Y_T]'$, $X = [X_1, \ldots, X_T]'$, $u = [u_1, \ldots, u_T]'$ and $A = [A(1), \ldots, A(l)]'$. Within this notation, the OLS estimates of $A$, $\Sigma$, and $C$ are given by:

$$\hat{A} = (X'X)^{-1}X'Y$$
$$\hat{\Sigma} = \frac{1}{T}(Y - X\hat{A})'(Y - X\hat{A})$$
$$\hat{C} = \text{Cholesky}(\hat{\Sigma}).$$

We assume the following Normal and Inverse-Wishart conjugate priors for $\alpha = \text{vec}(A)$ and $\Sigma$, parameterized by $\nu_0, V_0, \alpha_0 = \text{vec}(A_0)$, and $S_0$:

$$\Sigma \sim IW(S_0, \nu_0),$$
$$\alpha|\Sigma \sim N(\alpha_0, \Sigma \otimes V_0).$$

Given these priors, the posterior distributions of $\alpha$ and $\Sigma$ become:

$$\Sigma|Y \sim IW(S_T, \nu_T),$$
$$\alpha|\Sigma, Y \sim N(\alpha_T, \Sigma \otimes V_T),$$

where:

$$\nu_T = \nu_0 + T,$$
$$V_T = [V_0^{-1} + X'X]^{-1},$$
$$A_T = V_T[V_0^{-1}A_0 + X'X\hat{A}],$$
$$S_T = (Y - X\hat{A})'(Y - X\hat{A}) + \hat{A}'(X'X)\hat{A} + A_0'V_0^{-1}A_0 - A_T'(V_0^{-1} + X'X)A_T,$$
with $\alpha_T = \text{vec}(A_T)$. We assume weak or non-informative priors throughout the empirical applications in this paper by setting $V_0$ to the zero matrix and $\nu_0 = 0$. Using our earlier parameterization of $\Sigma = B^{-1}B^{-1'} = CQQ'C'$, these Normal and Inverse-Wishart priors can be described by $p(\alpha, C, Q) = p(\alpha, \Sigma) = p(\alpha|\Sigma)p(\Sigma)$.

### 2.2 Introducing the Forecast Consistent Prior

We now introduce the forecast consistent prior. For illustrative purposes, assume for a moment that we are interested in a bi-variate VAR(1) given by:

\[
\begin{bmatrix}
\pi_t

\end{bmatrix}
\begin{bmatrix}
E^S_t(\pi_{t+1})
\end{bmatrix}
= A
\begin{bmatrix}
\pi_{t-1}

\end{bmatrix}
+ u_t
= \begin{bmatrix}
a_{11} & a_{12}
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1}

\end{bmatrix}
+ \begin{bmatrix}
u_{1,t}
u_{2,t}
\end{bmatrix},
\]

where $\pi_t$ is a variable of interest such as inflation and $E^S_t(\pi_{t+1})$ is the one-period ahead forecast of $\pi_t$ obtained from survey data.\(^2\) When left unconstrained, this model implies two forecasts for the same variable: the VAR-based forecast and the survey forecast. From the first equation of the VAR, we can generate the time $t$ VAR-based forecast for $\pi_{t+1}$:

\[
E^\text{VAR}_t(\pi_{t+1}) = a_{11}\pi_t + a_{12}E^S_t(\pi_{t+1}).
\]

The only way for the VAR and survey forecasts to always be consistent with one another, so that $E^\text{VAR}_t(\pi_{t+1}) = E^S_t(\pi_{t+1})$ for all possible realizations of $u_t = [u_{1,t}, u_{2,t}]'$, is to impose the following restrictions on the VAR coefficients:

\[
g(A) = e'_1A - e'_2 = [0, 0] \Leftrightarrow [a_{11}, a_{12} - 1] = [0, 0],
\]

where $e_i$ is an $m \times 1$ selection vector with a 1 in the $i$'th position and 0's elsewhere. However, imposing strict consistency at all times assumes that the two forecasts are conceptually identical, an assumption which may be difficult to defend. Consider for a moment the case of inflation and inflation expectations, an area of great interest to macroeconomists. It is common for researchers to splice together multiple surveys of professional forecasters and even interpolate semi-annual surveys into a quarterly or monthly series (Clark and Davig, 2011). Therefore, survey measures of inflation forecasts that researchers include in their empirical analysis almost certainly contain measurement error. Furthermore, the survey

\(^2\)Throughout the paper we analyze survey-based expectations. However, the same methodology applies to expectations obtained from financial markets or other sources.
forecasts from professional forecasters are, at best, an imperfect proxy for the expectations of actual price setters (Coibion, Gorodnichenko and Kumar, 2018). While acknowledging these issues, completely ignoring the relevance of the survey forecasts for the VAR forecasts neglects the potentially useful information the econometrician possesses about the a-priori relationships between the variables in the VAR.

By adopting a Bayesian approach, we can vary the tightness of these cross-equation restrictions to allow for the possibility that survey forecasts and VAR-based forecasts are related to one another, albeit imperfectly. We can consider the following three cases which vary based on the degree of forecast consistency imposed:

1. **Strict Forecast Consistency**: $g(A) = [0, 0]$. Therefore, $a_{11} = 0$ and $a_{12} = 1$ is dogmatically imposed.

2. **Forecast Consistent Prior**: $g(A) \sim \mathcal{N}(0, (\lambda W)^{-1})$. The forecast consistent prior is centered on zero so that, on average, VAR-based forecast and survey forecasts are consistent with one another.\(^3\) However, the two forecasts may deviate from each other from time to time. We specify the forecast consistent prior as a normal distribution which is the maximum entropy prior for $g(A)$ under the constraint that the first moment of $g(A)$ is zero and the second moment of $g(A)$ is $(\lambda W)^{-1}$.\(^4\) Therefore, the size of the potential forecast deviations are governed by the weighting matrix $W$ and the tuning parameter $\lambda$.

3. **No Forecast Consistency**: $g(A)$ is left unrestricted which is equivalent to a diffuse prior over $g(A)$.

Strict forecast consistency and no forecast consistency can be regarded as limiting cases when $\lambda$ approaches $\infty$ and 0, respectively. In terms of $A$, the prior density function of $g(A)$ can be treated as the likelihood function for $A$ using observations satisfying these restrictions.\(^5\)

\(^3\)Note, this assumption is not necessary. For instance, if one wanted to model the fact that, over say a training sample, survey forecasts are somehow biased then a non-zero mean could be included in the forecast consistent prior.

\(^4\)Our choice of the maximum entropy prior is motivated by the fact that, in the information-theoretic sense, it minimizes the amount of prior information on non-targeted higher-order moments of $g(A)$. See, for example, Robert (2007); Cover and Thomas (2012).

\(^5\)The maximum entropy prior for $g(A)$ is similar to the limited information likelihood for $A$ in Kim (2002) if first and second moment restrictions for $g(A)$ are interpreted as nonlinear moment restrictions for $A$. We replace sample moment conditions for artificial observations satisfying weak consistency by population moment conditions, as in Del Negro and Schorfheide (2004).
We can calculate the posterior density of $\alpha$ under the forecast consistent prior at low computational cost by importance sampling. First, define the posterior density of $\alpha$ obtained without imposing forecasting consistency by $p(\alpha|Y, C)$ and define the forecast consistent prior density kernel for $\alpha$ by $h(\alpha) = p(g(\text{vec}(A)))$. Second, obtain the forecast consistent posterior density of $\alpha$:

$$p(\alpha|Y, C, g(A)) = \frac{h(\alpha)p(\alpha|Y, C)}{\int h(\alpha)p(\alpha|Y, C)d\alpha}.$$  

(13)

Using the above definition, we can simulate posterior draws of $\alpha$ from $p(\alpha|Y, C, g(A))$ simply by re-weighting the posterior draws from $p(\alpha|Y, C)$. Define the following importance weight for $\alpha^d$ that is randomly drawn from $p(\alpha|Y, C)$, by:

$$w(\alpha^d) = \frac{h(\alpha^d)}{\sum_{j=1}^{M} h(\alpha^j)}, \quad \alpha^j \sim p(\alpha|Y, C).$$  

(14)

We then re-sample these draws according to the weights $[w(\alpha^1), \ldots, w(\alpha^M)]$ to simulate the posterior density $p(\alpha|Y, C, g(A))$.

The forecast consistent prior can be used on top of any fully specified prior distribution for the VAR coefficients. One widely used prior for $\alpha$ is the Minnesota prior in which each variable is centered around a univariate random-walk process so that, in the above example, $\alpha_0$ is set to $[1, 0, 0, 1]'$. Therefore, the Minnesota prior ignores the cross-equation linkages between variables by setting prior means of off-diagonal terms of $A$ to 0. Although this prior might be a good benchmark when there is no a-priori obvious dynamic correlation between variables, the Minnesota prior is not ideal when there is a clear dynamic linkage between VAR variables, as is likely to be the case when survey forecasts are included alongside realized data.\(^6\)

### 2.3 The Forecast Consistent Prior in a Structural VAR Model

The previous section motivates our forecast consistent prior in the setting of a reduced-form VAR. In this environment, the forecast consistent prior offers a theoretically grounded approach to parameter shrinkage. However, the full conceptual appeal of this prior is best illustrated in the context of a structural VAR model. In a structural VAR, the forecast

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\(^6\)In the above bi-variate VAR(1) example, imposing the weak consistency prior on top of the Minnesota prior will induce multiple modes for the prior distribution of $[a_{11}, a_{12}]$, with the one from the Minnesota prior $(1, 0)$ and the other from the forecast consistency prior $(0, 1)$. The practice would be equivalent to imposing a mixture of two prior distributions, in which mixing weights are determined by the relative magnitude of the prior tightness of each prior.
consistent prior informs the structural VAR parameters for which, even with infinite observations, the data is uninformative. Therefore, the benefits of the forecast consistent prior can extend beyond shrinkage when the econometrician is attempting to identify structural innovations.

As discussed above, the mapping between the reduced form and structural VAR models is defined by the coefficient matrix $B$ which we parameterize by $B^{-1} = CQ$ where $C$ is the lower-triangular Cholesky factor of $\Sigma$ and $Q$ is a square $m \times m$ orthogonal rotation matrix. For illustrative purposes, consider for a moment the VAR(1), which can generalized to a VAR($l$) by writing the VAR in companion form:

$$Y_t = AY_{t-1} + u_t = AY_{t-1} + CQ\varepsilon_t,$$

where $\varepsilon_t$ are the structural innovations of interest. Suppose that we are interested in the effects of one particular structural shock which, without loss of generality, we label $\varepsilon_{1,t}$. Then the initial or impact-effect of a 1 unit realization of $\varepsilon_{1,t}$ is given by the first column of $CQ$ which is simply the 1-step ahead forecast of $Y_t$ conditional on $\varepsilon_{1,t} = 1$ in period $t$ and 0 in all other periods:

$$E^{VAR}_{t-1}(Y_t|\varepsilon_{1,t} = 1) = CQ\varepsilon_1. \quad (16)$$

The $h$-step ahead impulse response to this structural shock can be written as:

$$IRF(A, C, Q, h|\varepsilon_{1,t} = 1) = E^{VAR}_{t-1}(Y_{t+h}|\varepsilon_{1,t} = 1) = AE^{VAR}_{t-1}(Y_{t+h-1}|\varepsilon_{1,t} = 1) = A^hCQ\varepsilon_1, \quad (17)$$

for $h = 0, 1, \ldots, H$. The elements $A$ and $C$ of the $h$-step ahead impulse response can be informed by the observed data contained in $Y$. However, the matrix $Q$ must be identified from prior economic reasoning. In other words, the likelihood function of the VAR is invariant to alternative choices of $Q$.

When the VAR contains both realized and survey data, forecast consistency provides theoretically grounded restrictions on the matrix $Q$ which, together with $C$, provides the mapping between reduced form VAR residuals and structural shocks. For example, if we suppose once again that $y_t = [\pi_t, ES_t(\pi_{t+1})]^\prime$, where $ES_t(\pi_{t+1})$ is the 1-step ahead forecast of $\pi_t$ obtained from survey data, then survey consistency suggests the restrictions:

$$e_1^\prime IRF(A, C, Q; h|\varepsilon_{1,t} = 1) = e_2^\prime IRF(A, C, Q; h-1|\varepsilon_{1,t} = 1) \Leftrightarrow$$

\[9\]
should hold for \( h = 2, \ldots, H \). Equation (18) reveals that, in general, the forecast consistent prior will shape the posterior distribution of \( A, C, \) and \( Q \). More formally, the forecast consistent prior can be expressed as 

\[
g(A, C, Q|H) \sim \mathcal{N}(0_{H-1}, (\lambda W)^{-1})
\]

where \( W \) is a \( H - 1 \times H - 1 \) weighting matrix and \( \lambda \) calibrates the overall tightness the econometrician wishes to impose on these forecast consistent restrictions.

Structural VAR identification in the presence of survey data can benefit from the use of the forecast consistent prior by weighting posterior draws of \( A, C \) and \( Q \) by the degree to which VAR-based and survey forecasts align. While this is true regardless of the identification strategy pursued, a growing structural VAR literature aims to identify structural shocks of interest by sign restrictions which restrict the shapes of the impulse responses to identify parameters in \( Q \). VAR models identified by sign restrictions typically find a large set of \( Q \) matrices that are compatible with these restrictions (Moon and Schorfheide, 2012; Uhlig, 2005, 2017). In this setting, forecast consistent priors may be especially of interest. In particular, the sign-restricted VAR literature has been criticized on the grounds that sign restrictions do not rule out structural VAR models implying impulse responses which are inconsistent with plausible empirical specifications of equilibrium models (Arias, Caldara and Rubio-Ramirez, 2018; Wolf, 2020). Our approach provides theoretically motivated restrictions to narrow the set of potential structural VAR models based on the dynamic linkages between two different forecasts of the same variable.

We briefly illustrate the potential for our conditional forecast consistent prior to sharpen the identification of structural VAR parameters in the context of sign restrictions. First, we define the identified set of orthonormal matrices \( Q \) that satisfy the sign restrictions, denoted by \( Q \), and defined as follows:

\[
Q = \{Q|B^{-1} = CQ, e'_i \cdot IRF(\hat{A}, \hat{C}, Q, h|\varepsilon_{1,t} = 1) \geq (\leq)0, \forall r = 1, \ldots, R\},
\]

where, we assume for a moment that, \( A \) and \( C \) are fixed at their OLS estimates, \( e'_i \cdot IRF(\hat{A}, \hat{C}, Q, h|\varepsilon_{1,t} = 1) \geq (\leq)0 \) denotes the \( r \)-th restriction on the impulse response of VAR variables and \( R \) is the total number of restrictions on impulse-responses, and where \( e_i \) is a selection vector with a 1 in the \( i \)'th location and zeros elsewhere. Since the data do

\[\text{At the extreme of strict consistency, } e_1A = e_2I_m, \text{ implying no restrictions on } CQ. \text{ However, weak forecast consistency induces probabilistic restrictions on } Q \text{ because the term in Equation (18) becomes a random quantity.}\]
not provide additional information to distinguish different $Q$ matrices in the set of $Q$, the conditional posterior of $Q$ is same as the conditional prior, which is uniform over $Q$. In other words, without further restrictions, each matrix from $Q$ is equally probable, meaning $p(Q_i) = p(Q_j)$ for any $Q_i$ and $Q_j$ belonging to $Q$. Our forecast consistent prior breaks this symmetry by penalizing structural coefficients that generate the greater divergence between the VAR-based forecast and the survey forecast. Hence, our new prior for $Q$ induces a new prior for the structural VAR matrix $B$ given $C$, which is $p(B) \propto w(Q)p(B) \propto w(Q)$, where $w(Q)$ can be constructed based on $g(Q|A, C, H)$.

To make matters concrete, consider the simple VAR(1) model:

$$
\begin{bmatrix}
\pi_t \\
E_t^S(\pi_{t+1})
\end{bmatrix} = 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
E_{t-1}^s(\pi_t)
\end{bmatrix} + 
\begin{bmatrix}
1 & 0 \\
0.5 & 1
\end{bmatrix}
\begin{bmatrix}
q_{11} & q_{12} \\
q_{21} & q_{22}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix},
$$

where $E_t^S(\pi_{t+1})$ is the 1-step ahead forecast of $\pi_t$ obtained from survey data. We aim to identify a “news” shock for which survey forecasts increase. In addition to this sign normalization, we impose the sign restriction that the impact effect of this news shock on the realized data in $\pi_t$ is positive. Identifying this shock requires identifying a column of $Q$. Without loss of generality, we label the news shock $\varepsilon_{1,t}$ and therefore aim to identify the elements of the first column of $Q$, $[q_{11}, q_{21}]'$. Without identifying restrictions, the identified set of $q_{11}$ and $q_{21}$ is the entire unit circle. We impose the following restrictions to further narrow the identified set of $q_{11}$ and $q_{21}$.

1. Normalization Restriction: The “news” shock increases survey forecasts in period $t$.

$$R1(q_{11}, q_{21}) = \left\{ (q_{11}, q_{21}) \bigg| \frac{\partial E_t^S(\pi_{t+1})}{\partial \varepsilon_{1,t}} = 0.5q_{11} + q_{21} > 0 \right\}.$$

2. Sign Restriction: Realized data increases alongside survey forecasts in period $t$.

$$R2(q_{11}, q_{21}) = \left\{ (q_{11}, q_{21}) \bigg| \frac{\partial E_t^{VAR}(\pi_{t+1})}{\partial \varepsilon_{1,t}} = q_{11} > 0 \right\}.$$

3. Forecast Consistency Restriction: The response of realized data in period $t + 1$ is consistent with the increase in survey forecasts in period $t$.

$$R3(q_{11}, q_{21}) = \left\{ (q_{11}, q_{21}) \bigg| \frac{\partial E_t^S(\pi_{t+1})}{\partial \varepsilon_{1,t}} - \frac{\partial E_t^{VAR}(\pi_{t+1})}{\partial \varepsilon_{1,t}} = q_{21} - 0.5q_{11} = 0 \right\}.$$

$p(B)$ is flat because our forecast consistency prior is imposed through $Q$. 

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8 $p(B)$ is flat because our forecast consistency prior is imposed through $Q$. 

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11
Under the forecast consistent prior, the forecast consistency restrictions is loosely imposed by assuming that $q_{21} - 0.5q_{11}$ follows a Normal distribution centered around 0. We denote this prior by $g(Q|A, C, H) = q_{21} - 0.5q_{11} \sim \mathcal{N}(0, \lambda^{-1})$.

We can graphically illustrate how the forecast consistent prior shapes the posterior set of the structural VAR parameters. Panel (a) of Figure 1 highlights the identified set of $(q_{11}, q_{21})$ when we impose only the normalization and sign restrictions ($R_1$ and $R_2$). The blue line on the unit circle outlines the identified set under these two restrictions. We cannot discriminate different locations in the blue line without additional identifying restrictions. When we augment $R_1$ and $R_2$ with $g(Q|A, C, H)$, the forecast consistent prior, we can discriminate different points on the blue line in panel (a) in a probabilistic way. Panel (b) of Figure 1 illustrates the forecast consistent prior’s impact on the posterior distribution of $(q_{11}, q_{21})$ by shading with darker colors the region of the parameter space that has a higher probability mass under our forecast consistent prior. The forecast consistent prior adds curvature to the posterior distribution of the space of structural parameters which enables the econometrician to discriminate between alternative structural VAR models that are all equally likely according to the sign-restrictions. In our forward guidance application, which we turn to next, we illustrate in a much richer structural VAR model — identified by sign restrictions — how the forecast consistent prior can enhance structural identification.

### 3 Identifying Forward Guidance Shocks

The textbook view in macroeconomics is that what matters for consumption and investment decisions is the entire path of expected future interest rates, not just interest rates today (Woodford, 2003). This notion underpins the theoretical result that, through its ability to communicate a path of future short-term interest rates, a central bank retains powerful ammunition to combat economic downturns in the face of constraints on the current policy rate (Eggertsson and Woodford, 2003). However, more recent research has highlighted a number of frictions that could dampen the effects of forward guidance, including, among others, limited information and bounded rationality. These frictions can lead to dispersion

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9 Notice the fact that our approach adds curvature to the posterior distribution of $Q$ addresses another shortcoming of the sign-restriction literature which is the tendency for researchers to report point-wise median impulse responses across the draws from $Q$ as a measure of the central tendency of the posterior distribution of impulse responses. As Kilian and Lütkepohl (2017) point out, the standard approach of reporting the point-wise median impulse response not correspond to any particular structural VAR model. In our approach, the posterior mode of $g(Q|A, C, H)$ is one particular structural model and provides an alternative measure of central tendency around which to perform inference.
across various forecasts for the path of interest rates from a given piece of central bank communication. The potential for this forecast dispersion to arise from forward guidance calls into question how effective Federal Reserve communication has actually been in shaping macroeconomic outcomes.

Given the range of macroeconomic outcomes that might be associated with forward guidance, our central question surrounds the response of output following a forward guidance shock. Forward guidance shocks are inherently shocks to expectations for future short-term interest rates; therefore we augment a standard monetary VAR model with survey forecasts of future interest rates. While it is crucial to include expectations of future policy rates in the VAR to identify shocks to interest-rate expectations, as Ramey (2016) stresses, embedding survey forecasts of future interest rates alongside the federal funds rate generates two simultaneous forecasts for future interest rates: the VAR forecast and the survey forecast.

Our novel contribution to the forward guidance literature is to leverage the existence of these two interest rate forecasts together with our forecast consistent prior to distinguish forward guidance which better synchronized interest-rate expectations and those that allowed for dispersion across interest rate interest-rate expectations. To illustrate the role that our forecast consistent prior plays in shaping the estimated response of output following a forward guidance shock, we first estimate the structural VAR model using a pure sign restrictions approach following Uhlig (2005). We find that forward guidance shocks identified solely with sign restrictions have an ambiguous effect on output. However, when we add the forecast consistent prior to the sign restrictions, we find expansionary effects on output for large enough values of $\lambda$ which governs the dispersion across interest-rate forecasts.

Our key result is that the effectiveness of forward guidance in synchronizing interest rate expectations is a salient factor shaping the estimated effects of forward guidance shocks. Forward guidance which synchronizes various interest rate expectations on a lower path of rates leads to higher levels of economic activity. Alternatively, forward rate guidance which shifts some interest rate expectations but not others may have an ambiguous or even perverse effect on output. These findings are consistent with the recent theoretical work of Angeletos and Lian (2018) which underscores the role that dispersed expectations can play in dampening the effects of forward guidance. Interpreting our results through the lens of their model, we could imagine that some agents are using a VAR to forecast future policy rates while Blue Chip forecasters are using other information. In the model of Angeletos and Lian (2018), these two forecasts, the VAR-implied forecast and the survey forecast, may not align due to a lack of common knowledge leading to reduced output effects from forward guidance. However, as the degree of common knowledge increases, interest rate forecasts become better
aligned and the output effects from forward guidance increase. In our forecast consistency framework, the parameter $\lambda$ governs the degree of forecast consistency. And, as the model of Angeletos and Lian (2018) predicts, larger values of $\lambda$ imply more synchronized interest-rate forecasts and result in larger estimated output effects from forward guidance.

### 3.1 Data and VAR Model

To estimate the effects of forward guidance shocks on output, we augment the model specified in Uhlig (2005). In particular, we use monthly GDP as produced by Macroeconomic Advisers, the consumer price index, an index of commodity prices, the effective federal funds rate, non-borrowed reserves, total reserves, and 1 year ahead Blue Chip consensus economic forecasts for the 3-month Treasury bill rate. We take 100 times the natural log of all variables except for the federal funds rate and Blue Chip forecast for the 3-month Treasury bill rate. Unlike Uhlig (2005), we use the CPI directly as opposed to using the CPI and PPI to interpolate the quarterly GDP deflater to a monthly frequency. Also key is the addition of survey forecasts of short-term interest rates. The inclusion of forecasted interest rates is central to our analysis of forward guidance shocks.

We model these time series as a VAR(3) where the number of lags is selected based on standard information criteria over the sample January 1994 to December 2007.\(^{10}\) We begin our estimation in 1994 due to the chronology of communication from the Federal Open Market Committee (FOMC). Our interest lies in understanding the output effects of forward guidance, which has often been primarily issued through press statements at the conclusion of FOMC meetings. Prior to 1994, the FOMC did not issue press statements, limiting the starting point of our sample. We end our sample in 2007 due to the financial crisis and zero lower bound dynamics of non borrowed reserves. In particular, beginning in January of 2008, this series begins to take on negative values which precludes using the natural log of this series after 2007. However, in a robustness check, we “correct” the non-borrowed reserves series to offset the fact some of the borrowing through the Fed’s liquidity facilities appear to have been counted in borrowed reserves but not total reserves and extend our estimation sample through the 2008-2015 zero lower bound period.

\(^{10}\)We use pre-sample data beginning in October of 1993 to account for the 3 lags included in the VAR.
3.2 Sign Restrictions & The Forecast Consistency Prior

We identify a forward guidance shock by restricting the sign of the impulse response of commodity prices, the price level, and forecasts of future interest rates for the first 6 months after a forward guidance shock.\textsuperscript{11} These restrictions follow Uhlig (2005) who uses similar restrictions, though without expected interest rates, to distinguish conventional monetary policy shocks from other demand and supply disturbances.\textsuperscript{12} The notion that forward guidance shocks cause nominal interest rates and prices to move in opposite directions is shared by standard sticky-price models (Eggertsson and Woodford, 2003), models which attribute a large role to a “Fed information effect” (Nakamura and Steinsson, 2018), and models which dampen the output effects of forward guidance through limited information or other frictions (Kiley, 2016; McKay, Nakamura and Steinsson, 2016; Angeletos and Lian, 2018). Therefore, we restrict inflation and future interest rates to move persistently in opposing directions to distinguish the monetary policy innovation from other demand shocks.

Depending on the reaction of the central bank, supply shocks, including productivity and commodity price shocks, could also cause inflation and nominal interest rates to move in opposite directions. However, according to the estimates in Barsky, Basu and Lee (2015) using U.S. data, both TFP shocks and TFP news shocks have historically caused inflation and short-term interest rates to comove. Kilian and Lewis (2011) similarly find no evidence over our sample of a systematic monetary policy responses to oil price shocks. Moreover, using an agnostic identification strategy, Angeletos, Collard and Dellas (2020) find that inflation and interest rates tend to co-move across the business cycle. While this suggests that we are on fairly solid ground by assuming that monetary policy shocks are the only innovations that result in inflation and nominal interest rates persistently moving in opposite directions, Wolf (2020) argues that linear combinations of supply and demand shocks could contaminate our identification strategy by masquerading as monetary policy shocks. He argues for including other restrictions or incorporating external information to better isolate monetary policy shocks. We do both. We employ our forecast consistent prior which, to

\textsuperscript{11}Baumeister and Hamilton (2015) propose an alternative approach of restricting the signs of the parameters in the matrix $B$ which governs the contemporaneous relationships between variables in the VAR, including the monetary policy rule. However, Kilian and Lütkepohl (2017) argue that there may be little to gain by this approach as it can induce unintentionally informative priors on $B^{-1}$. Uhlig (2017) suggests that whether one imposes restrictions on $B$ or $B^{-1}$ likely depends on the application at hand. Given the difficulty in specifying a policy rule for forward guidance shocks, direct restrictions on $B^{-1}$ which governs impulse responses seems most suitable for our application.

\textsuperscript{12}We make no explicit orthogonalization to distinguish conventional monetary policy shocks from forward guidance shocks. However, we note that by restricting our focus to policy shocks which result in persistent movements of expected future interest rates as implied by one year ahead survey forecasts, we rule out many short-lived monetary policy shocks.
the extent that forward guidance shocks better align interest rate expectations than other macroeconomic shocks, helps to distinguish among competing shocks. In addition, we also use high-frequency financial market measures of forward guidance shocks to calibrate the degree of forecast consistency.\textsuperscript{13}

In addition to sign restrictions, we also employ the forecast consistent prior to identify forward guidance shocks. In particular, the VAR contains both the effective federal funds rate and survey forecasts of the 3-month Treasury Bill rate, one year ahead.\textsuperscript{14} Forward guidance which is uniformly understood should reduce dispersion across measures of interest-rate expectations and lead to larger output effects. On the other hand, forward guidance which is interpreted differently by different agents could lead to larger dispersion across forecasts and smaller output effects. This relationship between the real effects of forward guidance and interest-rate forecast dispersion can be naturally explored in the context of our forecast consistent prior. Specifically, we specify the forecast consistent prior over the survey forecast for the 3-month Treasury bill rate and the VAR forecast of the federal funds rate as follows:

\begin{equation}
g(Q|A,C,h) = e'_{BC}IRF(A,C,Q,h - 12|\varepsilon_{fg,1} = 1) \\
- \frac{1}{3} \sum_{j=0}^{2} [e'_{FF}IRF(A,C,Q,h - 1 + j|\varepsilon_{fg,1} = 1)] \\
= \frac{\partial}{\partial \varepsilon_{fg,1}} E^{BC}_{h-12} R_{t+12}^{T-bill} - \frac{1}{3} \sum_{j=0}^{2} \left[ \frac{\partial}{\partial \varepsilon_{fg,1}} FF_{h-1+j} \right],
\end{equation}

where $e_{BC}$ and $e_{FF}$ are selection vectors which respectively extract the responses of BlueChip forecasts and the federal funds rate and $\frac{\partial y}{\partial \varepsilon_{fg,1}}$ denotes the period $h$ impulse response of variable $y$ to a forward guidance shock that occurred in period 1. $E^{BC}_{t} R_{t+12}^{T-bill}$ denotes the Blue Chip consensus economic forecast for the 3-month Treasury bill 4-quarters from now. In the monthly Blue Chip survey, forecasters report what they expect the 3-month T-bill to average over in the three months ending 4 quarters ahead. So, in December, forecasters report what they expect the yield on the 3-month Treasury bill to average in the three months of October, November, and December of the following year.\textsuperscript{15} To the extent that

\textsuperscript{15}We stress however that external information, in this case high-frequency financial market data, are not essential for implementing our forecast consistent prior. As we illustrate in a robustness check, we can instead calibrate $\lambda$ using only the data in the VAR and we obtain qualitatively similar results on the shape of impulse responses. In addition, Giacomini, Kitagawa and Uhlig (2019) note that the choice of a hyperparameter like $\lambda$ must be subjective to a certain degree and therefore it is reassuring that we find a robust pattern when employing multiple strategies to calibrate $\lambda$.

\textsuperscript{14}While these financial instruments are conceptually distinct, arbitrage activities typically result in a small gap between the two interest rates.

\textsuperscript{15}Depending on whether the month is at the end or beginning of the quarter, the horizon of the forecast varies between 12 and 14 months. However, we must fix the horizon when implementing forecast consistency.
the yield on the 3-month Treasury bill rate closely tracks the federal funds rate, as has been the case historically, then this forecast should be linked with what the average federal funds rate over the three months ending in December. According to the VAR forecast, the average federal funds rate over the three months ending 4-quarters from now, conditional on a forward guidance shock in period 1, is given by

\[ \frac{1}{3} \sum_{j=0}^{2} \left[ \frac{\partial}{\partial \varepsilon_{fg,1}} FF_{h-1+j} \right] \] \[ ]_{16}

In the context of forward guidance, higher values of \( \lambda \) imply greater synchronocity of these two interest rate forecasts since it will tend to drive the posterior distribution of the VAR and survey forecasts of interest rates together. However, alternative values of \( \lambda \) result in structural VAR models that fit the data equally well. To be clear, this issue arises in all structural VAR models that are not over-identified. For example, in the case of recursive short-run restrictions, there is no statistical criterion on which to select one candidate Cholesky ordering over another. Analogously in this application, there is no obvious statistical reason to prefer small versus large values of \( \lambda \). Instead, we first highlight how alternative values of \( \lambda \) affect the structural response of output. We then argue that given the effect that different values of \( \lambda \) have on the impulse response of output, the effectiveness of FOMC communication in synchronizing interest rate expectations is a salient factor shaping the estimated effects of forward guidance shocks. Finally, we use external information from high-frequency event studies of FOMC announcements to tune \( \lambda \) and find that, tuning the forecast consistent prior based on this external information, output has modestly increased following FOMC communication which signals a lower path of future interest rates.

### 3.3 Implementation of Sign Restrictions & The Forecast Consistency Prior

We specify our reduced form VAR as:

\[ y_t = A_0 + A_1 y_{t-1} + A_2 y_{t-2} + A_3 y_{t-3} + u_t, u_t \sim \mathcal{N}(0, \Sigma), A = [A_0, A_1, A_2, A_3]' \] \[ (22) \]

We specify a non-informative prior, as described in Section 2.1, by setting \( V_0 \) to the zero matrix and \( \nu_0 = 0 \). Therefore, the posterior distributions for \( \alpha = vec(A) \) and \( \Sigma \), the VAR lag coefficients and the covariance matrix, are centered at the OLS estimates. Using our earlier

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16 We can stack the forecast consistency restriction in equation (21) for \( h = 13, 14, \ldots, H \) to form the forecast consistent prior \( g(Q|A, C, H) \sim \mathcal{N}(0_{H-12, I_{H-12} \lambda^{-1}}) \) where \( \lambda \) tunes the precision over the forecast consistent prior. In practice, we set \( H = 60 = 48+12 \) to encompass the 48 period impulse responses we plot.  

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This suggest that even under common knowledge forecast consistency should be expected to only weakly hold.
parameterization of $\Sigma$ from Section 2.1, $\Sigma = CQQ' C'$, where $C$ is the lower-triangular Cholesky factor of $\Sigma$ and $Q$ is a square $m \times m$ orthogonal rotation matrix. The mapping between the reduced form VAR residuals $u_t$ and structural VAR shocks $\varepsilon_t$, is governed by the linear mapping $u_t = CQ\varepsilon_t$.

To focus on the issue of identification, rather than inference, we initially keep $A = \hat{A}$ and $\Sigma = (Y - X\hat{A})'(Y - X\hat{A})/T$ and consider only random draws of $Q$. However, in a robustness check, we sequentially draw from the posterior of $A$ and $\Sigma$ as well as $Q$. We draw random orthogonal rotation matrices $Q$ by drawing a $6 \times 6$ random square matrix denoted by $\chi$, with each element of $\chi$ independently drawn from a standard normal distribution, and then we take the QR decomposition of $\chi$ using MATLAB’s $[Q,R]=qr(\chi)$ function. Each draw of $Q$ represents a candidate structural VAR model. Structural VAR models are kept if they satisfy the sign restrictions and are otherwise discarded.

After accumulating $M$ structural VAR models, or equivalently, $M$ orthogonal rotation matrices $Q$, which satisfy the sign restrictions, then we calculate the posterior weight under the forecast consistent prior according to:

$$w(Q(m)) = \frac{\exp(-0.5\lambda g(Q(m)|A, C, H)' g(Q(m)|A, C, H))}{\sum_{k=1}^{M} \exp(-0.5\lambda g(Q(k)|A, C, H)' g(Q(k)|A, C, H))}.$$  

(23)

These weights form the importance sampling weights we use to simulate the posterior set of impulse responses. In practice, we set $M = 5000$ and find that for our calibrated values of $\lambda$, the effective sample size – defined as $\hat{M} = (\sum_{k=1}^{M} w(Q(k))^2)^{-1}$ – suggests the posterior weight is distributed across thousands of draws. Finally, we scale the size of each draw to have the same initial effect on forecasted interest rates before calculating the importance sampling weights. This prevents larger shocks from being penalized simply due to the size of the forward guidance shock. This is similar to the approach taken in Uhlig (2005, pg. 413).

### 3.4 Impulse Response Functions

Figure 2 shows the median and 68 percent error bands of impulse response functions across the draws that satisfy the forward guidance sign restrictions. In this figure we set $\lambda = 0$. Per the imposed restrictions, forecasted interest rates decline and commodity prices along with the overall price level rise for the first 6 months. In the months that follow the restricted periods, survey forecasts of interest rates remain low and commodity prices remain elevated. The price level continues to gradually climb throughout the impulse response horizon. Recall that our sign restrictions leave non-borrowed reserves, total reserves, and the path of the
actual federal funds rate unconstrained. However, 12 months after the forward guidance shock, measures of bank reserves increase which precipitates a decline in the federal funds rate. After reaching a trough around the 12-month horizon, the federal funds rate begins rising back to its pre-shock level and thereafter continues to rise, resulting in a persistent interest rate overshoot.

How does the VAR-implied path of the federal funds rate compare to forecasters’ expectations of short-term interest rates? The red-dashed line in the bottom-left panel shows the VAR-implied forecast for the one-year ahead short-term interest rate based on the impulse response of the federal funds rate. One year after the forward guidance shock, the VAR-forecast of short-term rates fall by an amount similar to what forecasters anticipated. However, in subsequent months, the VAR path of short-term interest rates exceeds the path anticipated by forecasters and remains above the survey forecast for interest rate for several years.

The identified response of output using only sign restrictions suggests that forward guidance may not be effective in stimulating economic activity as output initially declines after the forward guidance shock and then gradually increases towards its pre-shock path. However, in this sign-restriction identified VAR, the VAR-implied path of interest rates diverges from the path of interest rates anticipated by professional forecasters. This raises the question of whether the apparent inability of forward guidance to stimulate output stems from the dispersion between interest rate expectations, as models of information frictions along the lines of Angeletos and Lian (2018) predict.

To understand the effects of FOMC forward guidance which better synchronizes interest rate forecasts, we now impose the forecast consistent prior in conjunction with sign restrictions. That is to say that we now consider $\lambda > 0$ whereas in Figure 2 we set $\lambda = 0$. Of course, this requires choosing a value for $\lambda$. Before picking a particular value for $\lambda$, we show how the response of output is influenced by the choice of $\lambda$. To first gain some intuition for how alternative values of $\lambda$ will influence the posterior distribution of the impulse responses, it is instructive to examine two particular candidate SVAR models. Among the 5000 draws that satisfy the sign restriction, we isolate two particular SVAR model draws: the SVAR draw that comes the closest to satisfying the forecast consistency restriction in equation (21) and the SVAR draw that is the furthest from satisfying the forecast consistency restriction in equation (21). In other words, these are the two SVAR draws that register the highest
and lowest values in our forecast consistent prior density. We respectively refer to these as the “best” and “worst” draws from a forecast consistency standpoint.\footnote{The “best” draw corresponds closely with the posterior mode over the set $Q$.}

Figure 3 plots the best and worst SVAR draws. The green-dashed-dotted line represents the best draw and the red-dashed line represents the worst draw. To visually understand what is behind these rankings, the top-right panel of Figure 3 shows the cumulative forecast deviation. By construction, the period 48 cumulative deviation is closer to zero for the best draw than for the worst draw. In other words, the best draw results in a VAR-implied forecast of the federal funds rate which more closely mirrors the path of rates that professional forecasters expected. For the worst draw, the VAR-implied forecast of the federal funds rate meaningfully diverges from the path expected by forecasters, consistent with a forward guidance announcement which is not uniformly understood. Although the response of output played no role in our selection, the best draw implies a persistent expansion in output while the worst draw suggests that output persistently declines following a reduction in the path of short-term interest rates. The diverging output responses in Figure 3 offer some evidence that forward guidance has the potential to be effective when it is able to synchronize interest-rate expectations.

The impulse response of output is heavily influenced by the choice of $\lambda$. A tighter forecast consistent prior, achieved by selecting higher values of $\lambda$, will place greater weight on draws like the “best” one under which output expands and lower weight on draws like the “worst” one under which output contracts. Figure 4 specifically focuses on the effect $\lambda$ has on the response of output 18 months following the forward guidance shock. For smaller values of $\lambda$ output fails to meaningfully rise — and may actually decline — following what ought to be an expansionary forward guidance shock. However, as $\lambda$ increases so too does the response of output at 18 months. Beyond a certain threshold, the output response at 18 months ceases to increase with further increases in $\lambda$. Interpreting $\lambda$ as governing the degree to which forward guidance announcements synchronize interest rate expectations, then Figure 4 provides empirical evidence that forecast agreement regarding FOMC communications is a salient factor shaping the estimated effects of forward guidance shocks.

Which of these responses best characterizes the U.S. experience with forward guidance? To answer this, we must select a particular value of $\lambda$. As previously discussed, the likelihood function cannot distinguish between alternative values of $\lambda$. Therefore, we bring external information to bear to calibrate $\lambda$. In particular, we select $\lambda$ to maximize the correlation between our structural VAR forward guidance shocks and high-frequency financial market
measures of forward guidance shocks as similarly constructed by Gurkaynak, Sack and Swanson (2005). The path factor constructed by these authors is a measure of forward guidance shocks which are orthogonal to unexpected changes in the target federal funds rate. $\lambda$ is selected from a grid of values across which we compare the correlation between the structural shocks from the SVAR with the updated path factor series from Bundick and Smith (2020). This approach to calibrating $\lambda$ is most natural in our setting as it effectively extends the notion of synchronizing interest-rate expectations across VAR-based forecasts and survey forecasts to include some degree of synchronicity with the expectations embedded in financial markets.

By incorporating high-frequency financial market measures of monetary policy shocks into the identification of forward guidance shocks in our monthly SVAR model, we can relate our approach to the burgeoning external instruments approach to VAR identification of monetary policy shocks, as employed by Gertler and Karadi (2015). However, our approach is less exposed to the critiques in Ramey (2016) since we directly incorporate forward-looking survey measures of interest rate expectations into our VAR model which lessens concerns around VAR invertibility. Our approach is also related to the narrative sign restrictions approach proposed by Antolín-Díaz and Rubio-Ramírez (2018) in that it allows the econometrician to incorporate narrative information to select among alternative SVAR models. However, as we show with an alternative calibration in Section 3.5, narrative or external information is not strictly required to implement our approach.

Figure 5 shows the median and 68 percent error bands of impulse response functions across the draws that satisfy the sign restrictions when reweighted with $\lambda > 0$. The top-left panel of Figure 5 shows that output rises in a gradual but persistent manner following an expansionary forward guidance shock for the value of $\lambda$ which maximizes the correlation between the SVAR shocks and the high-frequency forward guidance shock series. The bottom left panel of Figure 5 illustrates that this setting of the forecast consistent prior reduces the deviation between the VAR-implied path of the federal funds rate and the path of rates anticipated by professional forecasters following a forward guidance shock. More precisely, the cumulative deviation between the VAR-based forecast and the Blue Chip forecast is reduced by roughly 25 basis points under under our calibration of $\lambda$ compared to when $\lambda = 0$. 
3.5 Impulse Response Functions for Alternative Specifications

The finding that forward guidance shocks which better synchronize interest rate expectations, as calibrated through larger values of λ, lead to more expansionary output effects is a common finding across several alternative VAR specifications. We consider three variants of our baseline VAR model: one which extends the estimation sample to include the 2008-2015 zero lower bound period, one which calibrates λ without the use of high-frequency measures, and one which sequentially draws from the posterior of A and Σ as well as Q. We discuss each in turn.18

The 2008-2015 zero lower bound period included more extensive use of forward guidance with the FOMC’s conventional policy tool constrained. However, our one-year ahead survey forecasts became truncated around 2010, according to Swanson and Williams (2014), therefore we leave this full sample estimation as a robustness check. The first column of Figure 6 shows the impulse responses when we extend the estimation sample through December 2015. Over this extended sample, we continue to find expansionary effects for the value of λ which best correlates the SVAR forward guidance shocks with high-frequency forward guidance shocks and contractionary effects when λ = 0.

The use of high-frequency measures of forward guidance shocks provides a natural approach to calibrating λ in our setting since it effectively extends the notion of forecast synchronicity from survey and VAR-based interest-rate forecasts to include some degree of synchronicity with financial-market measures of expected future interest rates. However, this approach to tuning λ may be restrictive since our monthly VAR model can, in principle, capture forward guidance shocks outside of regularly scheduled FOMC meetings which comprise the event set of the high-frequency forward guidance surprises. Therefore, in the second column of Figure 6, we show impulse responses when we calibrate λ to match the discrepancy between the VAR and survey forecasts for future interest rates over the range of sign-restricted impulse responses. This discrepancy then serves as the basis for the prior variance we expect between the VAR and survey forecasts without referencing high-frequency shock measures. Under this alternative calibration of λ we continue to find that output expands following a downward revision to forecasts of interest rates.

Finally, we can extend our forecast consistent prior to shape the posterior of the full set of VAR parameters, including A and Σ, not just the set of rotation matrices Q. More concretely, we sequentially draw from the posterior distributions of A, C where Σ = CC′, and Q. A given joint draw d of the triplet (A, C, Q)d is kept if the associated impulse responses

18We include the full impulse responses for these alternative specifications in the appendix.
satisfy the sign restrictions and otherwise discarded. Then we re-weight this posterior set according to the forecast consistent prior \( g(A, C, Q|H) \), as extended from equation (21).

The third column of Figure 6 shows the impulse responses jointly drawn from the posterior of the full set of VAR parameters. The range of responses is understandably wider compared to the baseline VAR model. However, the ranking of the median responses remains with larger values of \( \lambda \) implying more expansionary effects from forward guidance.

4 Inflation Tail Risks

According to former Federal Reserve Chairman, Alan Greenspan, “the conduct of monetary policy in the United States has come to involve, at its core, crucial elements of risk management.” One element of this policy strategy involves managing risks around the Federal Reserve’s price stability mandate. While during much of the 1980’s and 1990’s the FOMC was primarily concerned with defending its inflation mandate from above, in more recent decades it has confronted the risk of too low inflation. Concerns of deflation were especially elevated following the Global Financial Crisis and its aftermath, sparking broad interests in analyzing inflation tail risks over this period.

Several recent papers have measured inflation tail risks using financial market data and identify an elevated risk of deflation in the aftermath of the Global Financial Crisis. Fleckenstein, Longstaff and Lustig (2017) estimate a model of time-varying deflation risks identified from inflation swaps and options. Similarly, Anene and D’Amico (2017) and Hattori, Schrimpf and Sushko (2016) use inflation derivatives to study the impact of the FOMC’s unconventional policy actions on stemming the risk of deflation. We contribute to the literature by studying inflation tail risks implied by a time-varying parameter (TVP-)VAR model of inflation that includes both near-term and longer-term survey forecasts of inflation. Relative to this previous literature, our approach has the appeal that, unlike financial market data, survey forecasts are not contaminated by time-varying risk premiums.\(^{20}\)

\(^{19}\)In the terminology laid out in Uhlig (2005), this is similar to the “pure sign-restrictions approach” whereas our baseline implementation is more closely related with his “penalty-function approach.”

\(^{20}\)Our approach is comparable to Kozicki and Tinsley (2012) who estimate a time-varying parameter TVP-AR(p) model for inflation together with survey forecasts. However, they allow only constant terms (“inflation end-point”) to drift over time while other coefficients in the AR(p) model remain time-invariant. Keeping AR coefficients time-invariant has some limitations in modeling changes in the relationship between inflation and inflation expectations For example, the FOMC’s 2012 adoption of an explicit inflation target may have made inflation expectations less responsive to temporary shocks. For this reason, we consider a more general TVP-VAR model in which constant terms and lag coefficients drift over time.
This application illustrates several novel features of our forecast consistent prior that we have yet to demonstrate. First, in this application, the forecast consistent prior is applied in a setting with multiple survey measures at multiple horizons. Second, the setting for this application is a TVP-VAR model which highlights the computational ease of implementing the forecast consistent prior. Third, in this application, by simulating the predictive density of inflation from a VAR model with survey forecasts we demonstrate the appeal of augmenting VAR models with survey data to elicit moments of the forecast density that may not be explicitly provided by surveys. Finally, we apply the prior to unconditional forecasts from the VAR rather than impose restrictions on impulse response dynamics. Therefore, the tightness of the forecast consistent prior can be chosen to maximize the marginal likelihood.

Our results suggest that inflation expectations as measured from survey forecasts play an important role in shaping inflation tail risks. In particular, estimates from our forecast consistent TVP-VAR suggest that the risk of deflation during the Global Financial Crisis and its aftermath were generally lower than unconstrained VAR models and financial market estimates might suggest. The role of inflation expectations in influencing tail risks is especially vivid when survey forecasts for near-term inflation expectations hit all-time lows in 2009:Q1 which led deflation risks from the TVP-VAR model to sharply rise. However, as survey forecasts rebounded in subsequent quarters deflation probabilities quickly fell. In contrast, deflation risks from the TVP-VAR model without forecast consistency remain persistently elevated after 2009. The marginal likelihood criterion favors the TVP-VAR model with some degree of forecast consistency imposed, implying that the data prefers a model which tightly links deflation risks to variation in survey forecasts for inflation.

4.1 Data and TVP-VAR Model

The specification of our VAR follows Clark and Davig (2011) closely by including long-term and near-term survey forecasts of inflation alongside realized inflation, a measure of real economic activity, and a measure of the policy rate. We use both 1 year and 10-year ahead forecasts for Consumer Price Index (CPI) inflation from the Survey of Professional Forecasters (SPF) as well as realized CPI inflation. We include the Chicago Fed National Activity Index to broadly measure real activity and the effective federal funds rate to account for inflation.

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21 The 10-year ahead forecasts for CPI inflation from SPF are available beginning in 1991. Prior to 1991, we use long-run inflation forecasts obtained from the public release of the Federal Reserve Board of Governors’s FRB/SU econometric model which is constructed using alternative surveys and econometric estimates. We use realized inflation and inflation nowcasts to construct our inflation expectations measures to prevent overlap between long-term survey forecasts, near-term survey forecasts, and realized inflation.
for the stance of monetary policy. Our formal estimation sample is 1982-2015 which includes the
zero lower bound period. Therefore, to better account for the full spectrum of the
FOMC’s policy actions from 2009-2015 we splice the federal funds rate together with Wu
and Xia (2016) shadow federal funds rate.

We model these five variables as a TVP-VAR(4) with stochastic volatility:

\[ y_t = A_{0,t} + \sum_{j=1}^{4} A_{j,t} y_{t-j} + u_t, \quad u_t \sim \mathcal{N}(0, B^{-1} \Sigma_{u,t} B^{-1'}) , \]

\[ \Sigma_{u,t} = \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{5,t}^2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ B_{21} & 1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ B_{51} & \cdots & B_{54} & 1 \end{pmatrix}. \] (24)

We detail the construction of our priors and the algorithm used to estimate and simulate
the TVP-VAR model in the appendix.

4.2 The Forecast Consistency Prior

The TVP-VAR model contains both near-term and long-term survey forecasts of inflation as
well as realized inflation. Therefore, we impose our forecast consistent prior over the survey
and VAR-based forecasts for both forecast horizons. To ease the notation of writing out the
forecast consistency restrictions, we express the TVP-VAR model in companion form:

\[ \tilde{y}_t = \tilde{A}_{0,t} + \tilde{A}_{1,t} \tilde{y}_{t-1} + \tilde{u}_t. \] (25)

Let \( \pi_t \) denote realized CPI quarterly annualized inflation and let \( \pi_{t}^{e,L} \) and \( \pi_{t}^{e,S} \) denote
the long-term forward and short-term weighted averages of expected inflation at different
horizons under the expectation operator \( E^e \):

\[ \pi_{t}^{e,L} = \frac{\sum_{j=5}^{40} E_t^e(\pi_{t+j})}{36}, \]
\[ \pi_{t}^{e,S} = \frac{\sum_{j=1}^{4} E_t^e(\pi_{t+j})}{4}. \] (26)
where $E_t^e$ is the survey expectation when $e = S$ and the VAR-based expectation when $e = VAR$.$^{22}$

Forecast consistency at both forecast horizons requires the following restrictions:

$$
\pi_t^{S,L} - \pi_t^{VAR,L} = e'_\pi, e'_\pi, S, L, \tilde{y}_t - \frac{\left[\sum_{h=5}^{40} \sum_{j=0}^{h-1} \tilde{A}_{1,t}^{j} \tilde{A}_{0,t} + \sum_{h=5}^{40} \tilde{A}_{1,t}^{h} \tilde{y}_t\right]}{36},
$$

$$
g(\tilde{A})_L = [-e'_\pi, e'_\pi, S, L, \tilde{y}_t - \frac{\left[\sum_{h=5}^{40} \tilde{A}_{1,t}^{h}\right]}{36}]',
$$

$$
\pi_t^{S,S} - \pi_t^{VAR,S} = e'_\pi, e'_\pi, S, S, \tilde{y}_t - \frac{\left[\sum_{h=1}^{4} \sum_{j=0}^{h-1} \tilde{A}_{1,t}^{j} \tilde{A}_{0,t} + \sum_{h=1}^{4} \tilde{A}_{1,t}^{h} \tilde{y}_t\right]}{4},
$$

$$
g(\tilde{A})_S = [-e'_\pi, e'_\pi, S, S, \tilde{y}_t - \frac{\left[\sum_{h=1}^{4} \tilde{A}_{1,t}^{h}\right]}{4}]',
$$

$$
g(\tilde{A}) = [g(\tilde{A})_L, g(\tilde{A})_S]'^T,
$$

where $e_i$ is a selection vector whose $i$th element is 1 while all the other elements are zeros. In the above calculations of the VAR-implied forecasts, we assume no future parameter drift:

$$
E_t^{VAR}(\prod_{k=1}^{h} \tilde{A}_{1,t+k} | \mathcal{F}_t) = \tilde{A}_{1,t}^h.
$$

In this application we calibrate the hyperparameter $\lambda$ that controls the tightness of forecast consistency prior restrictions by calculating the following marginal data density for different values of $\lambda$.$^{23}$

$$
p(y^T | \lambda) = \int p(y^T | A^T, \Sigma_u^T, B)p(A^T, \Sigma_u^T, B) d(A^T, \Sigma_u^T, B).
$$

The log marginal likelihood is maximized at $\lambda = 1.42$.$^{24}$ Therefore, imposing a modest degree of forecast consistency improves the time series fit of the TVP-VAR model.

$^{22}$The end-point of the SPF 10-year inflation forecasts changes only in the first quarter of each year. Therefore, the number of quarterly inflation forecasts contained in the 10-year forecast varies depending on the quarter of year. We deal with this in the TVP-VAR by adjusting the lower limit of the sum and the denominator in $\pi_t^e$ depending on the quarter of the year. Therefore, the notation below is illustrative and applies to the first quarter of the year.

$^{23}$We calculate the inverse of the marginal likelihood using the harmonic mean of the likelihood implied by posterior draws. The calculations and details are explained in the appendix.

$^{24}$We truncate the value of $\lambda$ at 1.5 because the effective sample size becomes too small (less than 2 percent of posterior draws) above that value.
4.3 Time-Varying Inflation Tail Risks

While survey forecasts are available for mean inflation outcomes, our interest in this application lies in assessing potential tail outcomes for inflation for which survey forecasts are not consistently available. Therefore, the forecast consistent prior is particularly useful as it tilts the mean of the predictive distribution of inflation in the TVP-VAR towards the available survey forecasts and then relies on the VAR model to generate other moments of the predictive distribution of inflation. We simulate the predictive distribution of inflation outcomes using posterior draws of parameters and shocks up to time $t$ from the TVP-VAR model. In practice, we achieve this by generating a full trajectory of inflation for the $i$-th posterior draw of parameters and shocks, denoted by $\pi_{t+h|t}(i)$, for each draw $i = 1, \cdots, M$ from the posterior. We then compute the probability that inflation will be less than 0 percent on average over the next $H$-periods according to:

$$P(\frac{1}{H}\sum_{h=1}^{H} \pi_{t+h|t} < 0) = \frac{\sum_{i=1}^{M} \mathbb{I}(\frac{1}{H}\sum_{h=1}^{H} \pi_{t+h|t}(i) < 0)}{M}.$$  

(30)

We can perform a similar calculation to assess the likelihood of high inflation which, following Fleckenstein, Longstaff and Lustig (2017), we define as inflation above 4 percent.

Table 1 compares estimated deflation probabilities from the TVP-VAR with those from Fleckenstein, Longstaff and Lustig (2017) and the cross-sectional distribution of individual expectations from the University of Michigan consumer survey over the period of 2009:Q4-2015:Q4. Although the underlying source data are completely different, the mean and median probabilities of deflation are quite comparable given the magnitude of the standard deviation of each measure. However, the TVP-VAR model with consistency restrictions implies uniformly lower deflation probabilities across all three quantile estimates (minimum, median, maximum) at both the 1- and 2-year horizons. This is also true for the estimated probability of high inflation (inflation greater than 4 percent) in Table 2.

The lower levels of inflation tail risks from our forecast consistent TVP-VAR model suggests that both financial market measures as well as unconstrained VAR models tend to overstate tail risks to inflation. Moreover, simply including mean survey forecasts of inflation in the VAR fails to fully capture the potential role of survey expectations in driving inflation tail risks. The relative stability of professional forecasters’ inflation expectations over the 2009-2015 sample reduces the overall threat of high inflation and deflation. More

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25 For the University of Michigan survey, we calculate the percentage of respondents who anticipated prices would go down among all the respondents who provided answers on expected prices.
generally, the forecast consistency restrictions more tightly link inflation tail risks to survey forecasts of inflation. For example, in early 2009 as oil prices fell by more than $100 per barrel, one-year ahead survey forecasts for inflation hit all time lows. The bottom row of Figure 7 shows that the risk of deflation sharply increased at this time. Then, as near-term inflation expectations rebounded, the risk of deflation subsided according to the forecast consistent TVP-VAR but remained elevated in the VAR model without the forecast consistency restrictions.

5 Conclusion

A growing literature has incorporated survey measures of expectations into VAR models to capture the importance of forward-looking behavior. In this paper, we have proposed the forecast consistent prior as a computationally efficient Bayesian framework for estimation and inference of VAR models with survey forecasts and realized data of a closely related variable. We highlight a range of possible applications of our framework in the context of both structural VAR shock identification as well as VAR forecasting. The applications shed light on the importance of synchronizing interest rate expectations to generate the desired effects from forward guidance and the role that professional forecasters’ inflation expectations play in shaping inflation tail risks. However, many applications remain given the growing interest in identifying monetary and non-monetary news shocks as well as the increased interest in macroeconomics of understanding the formation and evolution of expectations. Therefore, as these literatures continue to advance, there appears to be a growing need to develop frameworks which can flexibly and efficiently incorporate expectations into VAR models.

\[26\text{In principle, the ordering of variables may affect the predictive density of inflation because a linear transformation of log-normal stochastic volatility through the } B^{-1} \text{ matrix is not a log-normal random variable. Since the stochastic volatility of the first variable in the VAR must follow the log-normal distribution, this creates the issue that the predictive density of inflation may not be invariant to the ordering. However, since forecast consistency restrictions are imposed onto the time-varying VAR coefficients not stochastic volatilities, this issue is unlikely to be a big concern when we compare inflation risks with or without forecast consistency restrictions although it may change the overall magnitude of tail risks for both cases depending on the ordering of inflation in the VAR. Primiceri (2005) acknowledges this issue in his estimation of a time-varying structural VAR model using the U.S. data and notes that the effect was quantitatively small in his application.}\]
References


Table 1: Summary Statistics for Deflation Probabilities: 2009:Q4 - 2015:Q4

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Source</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>Fleckenstein et al. (2017)</td>
<td>18.759</td>
<td>10.155</td>
<td>2.003</td>
<td>15.739</td>
<td>47.757</td>
</tr>
<tr>
<td>1 year</td>
<td>University of Michigan Survey</td>
<td>15.906</td>
<td>6.1249</td>
<td>6.06</td>
<td>14.1414</td>
<td>32.6531</td>
</tr>
<tr>
<td>1 year</td>
<td>TVP-VAR (λ = 0)</td>
<td>13.74</td>
<td>9.62</td>
<td>2.72</td>
<td>10.54</td>
<td>45.98</td>
</tr>
<tr>
<td>1 year</td>
<td>TVP-VAR (λ = 1.4)</td>
<td>9.03</td>
<td>8.22</td>
<td>1.66</td>
<td>7.32</td>
<td>42.78</td>
</tr>
<tr>
<td>2 years</td>
<td>Fleckenstein et al. (2017)</td>
<td>13.799</td>
<td>7.261</td>
<td>1.801</td>
<td>11.621</td>
<td>35.421</td>
</tr>
<tr>
<td>2 years</td>
<td>TVP-VAR (λ = 0)</td>
<td>16.34</td>
<td>6.27</td>
<td>8.38</td>
<td>15.14</td>
<td>32.06</td>
</tr>
<tr>
<td>2 years</td>
<td>TVP-VAR (λ = 1.4)</td>
<td>9.54</td>
<td>4.59</td>
<td>4.1</td>
<td>7.98</td>
<td>23.96</td>
</tr>
</tbody>
</table>

Notes: Deflation probabilities from Fleckenstein et al. (2017) are based on daily observations on inflation swaps and options from October 5, 2009 to October 28, 2015 while those from the University of Michigan survey are quarterly average values of monthly observations from October, 2009 to October, 2015.

Table 2: Summary Statistics for High Inflation (>4 percent) Probabilities: 2009:Q4 - 2015:Q4

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Source</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Median</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>Fleckenstein et al. (2017)</td>
<td>2.615</td>
<td>2.46</td>
<td>0.139</td>
<td>2.086</td>
<td>16.268</td>
</tr>
<tr>
<td>1 year</td>
<td>TVP-VAR (λ = 0)</td>
<td>1.34</td>
<td>2.3</td>
<td>0.04</td>
<td>0.4</td>
<td>10.92</td>
</tr>
<tr>
<td>1 year</td>
<td>TVP-VAR (λ = 1.4)</td>
<td>0.64</td>
<td>1.29</td>
<td>0</td>
<td>0.16</td>
<td>6.06</td>
</tr>
<tr>
<td>2 years</td>
<td>Fleckenstein et al. (2017)</td>
<td>3.089</td>
<td>2.394</td>
<td>0.295</td>
<td>2.702</td>
<td>15.315</td>
</tr>
<tr>
<td>2 years</td>
<td>TVP-VAR (λ = 0)</td>
<td>4.10</td>
<td>2.86</td>
<td>0.88</td>
<td>3.14</td>
<td>14.02</td>
</tr>
<tr>
<td>2 years</td>
<td>TVP-VAR (λ = 1.4)</td>
<td>2.3</td>
<td>2.17</td>
<td>0.32</td>
<td>1.32</td>
<td>9.64</td>
</tr>
</tbody>
</table>

Notes: Deflation probabilities from Fleckenstein et al. (2017) are based on daily observations on inflation swaps and options from October 5, 2009 to October 28, 2015.
(a) Identified Set: Normalization and Sign Restriction

(b) Identified Set: Normalization, Sign Restriction, and Forecast Consistency
Figure 2: **Forward Guidance Shock: Sign Restrictions Only**

Notes: This figure shows the impulse responses to an identified forward guidance shock using only sign restrictions. The solid blue line is the median response and the shaded region is the 68% interval among structural VAR models. The red-dashed line shows the VAR-implied response of future short-term interest rates. The estimation sample period is 1994-2007.
Figure 3: **Forward Guidance Shocks: “Best” and “Worst” Fitting Draws**

Notes: This figure shows the impulse responses to an identified forward guidance shock for the “best” and “worst” fitting models. The “best” fitting model is the draw that comes the closest to satisfying the forecast consistency restrictions and the “worst” fitting model is the draw that is the furthest from satisfying the forecast consistency restrictions. The estimation sample period is 1994-2007.
Figure 4: Output Effects of Forward Guidance: The Role of the Forecast Consistent Prior

Notes: This figure shows the median output response and the corresponding 68% intervals after 18 months for alternative values of $\lambda$, which governs the tightness of the forecast consistent prior. The vertical red line denotes the value of $\lambda$ selected to maximize the correlation between the structural VAR series of forward guidance innovations and high-frequency financial market measures of forward guidance shocks. The estimation sample period is 1994-2007.
Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our forecast consistent prior. The solid blue line is the median response and the shaded region is the 68% error band. The red line shows the VAR-implied response of future short-term interest rates. The estimation sample period is 1994-2007.
Figure 6: **Forward Guidance Shock: Alternative Specifications**

Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our forecast consistent prior. The solid blue line is the median response and the shaded region is the 68% error band. The green-dashed line shows the median impulse response to an identified forward guidance shock using only sign restrictions. Each column shows impulse responses from an alternative VAR model. For details, see Section 3.5.
Notes: This figure shows the time-varying probabilities of high inflation (inflation > 4 percent) in the top row and the time-varying probability of deflation (inflation < 0 percent) from our TVP-VAR model. The left column shows these probabilities over the 1 year horizon and the right column shows these probabilities over the 2 year horizon. The solid black lines show these probabilities from the TVP-VAR without the forecast consistent prior, labeled “No Resampling”, which corresponds to $\lambda = 0$ and the dotted blue lines show these probabilities from the TVP-VAR with the forecast consistent prior, labeled “Resampling”, which corresponds to $\lambda = 1.42$. 