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Abstract

We use a large set of economic and financial indicators to assess tail risks of the three macroeconomic variables: real GDP, unemployment, and inflation. When applied to the U.S. data, we find evidence that a dense model using principal components (PC) as predictors might be misspecified by imposing the “common slope,” assumption on the set of predictors across multiple quantiles. The common slope assumption ignores the heterogeneous informativeness of individual predictors on different quantiles. However, the parsimony of the PC-based approach improves the accuracy of out-of-sample forecasts when combined with a sparse model using the dynamic model averaging method. Out-of-sample analysis for the U.S. data suggests that the downside risk for real macro variables spiked to a highly elevated level by the end of the Great Recession but subsequently declined to a negligible level. On the other hand, the downside tail risk for inflation fluctuated around a non-negligible level even after the end of the Great Recession. The disconnect between the downside risk of inflation and that of real activities can be in line with the evidence for the reduced role of output gap for inflation during the recent period.

Keywords: Quantile Regressions; Tail Risks; Variable Selection; Dynamic Model Averaging.

JEL classification codes: C22, C55, E27, E37.

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1 Introduction

Monetary policymakers have increasingly communicated to the public that policy decisions are “data dependent,” meaning they take into account a variety of indicators to judge economic conditions and the outlook. At the same time, they also emphasize that policy can respond to not just changes in the most likely outcome but also the uncertainty surrounding the baseline forecast. For example, [Powell \(2019\)](#) mentions that “the risk of a less favorable outlook,” is a key factor in policy decision even without “a major shift in the baseline outlook for the economy.” Hence, policymakers would benefit from quantifying the uncertainty about their economic outlook while leveraging a large number of predictors.

In this paper, we investigate the use of quantile regression to create forecasts of uncertainty from in a data-rich environment. This method builds on a rich literature that uses quantile predictions to characterize the full distribution of future macroeconomic outcomes. We extend this literature by incorporating information from a large set of economic and financial indicators. Using quantile regressions of future real GDP growth on the National Financial Conditions Index (NFCI) created by the Federal Reserve Bank of Chicago, [Adrian, Boyarchenko and Giannone \(2019\)](#) show that the lower quantiles of GDP growth is sensitive to changes in the NFCI while the upper quantiles are stable. Similarly, [Giglio, Kelly and Pruitt \(2016\)](#) find that the systemic risk measure constructed from a variety of financial market variables is informative about the left tail of shocks to measures of real economic activities such as industrial production and the Chicago Fed National Activity Index (CFNAI). However, these papers do not include non-financial macroeconomic indicators that can be potentially useful for macroeconomic forecasting (for example, [Stock and Watson \(2002\)](#)). [Adrian, Boyarchenko and Giannone \(2019\)](#) use the current-quarter GDP growth as the only non-financial predictor while [Giglio, Kelly and Pruitt \(2016\)](#) do not use any non-financial macroeconomic predictor. We include a large set of non-financial macroeconomic indicators ([Elliott, Gargano and Timmermann, 2015](#); [Kotchoni, Leroux and Stevanovic, 2019](#)) as well as financial indicators but focus on conditional quantile predictions instead of conditional mean predictions.

Quantile regression models, estimated directly on a large predictor space, can generate a substantial estimation uncertainty that can offset the value of the additional predictive power from more predictors. This estimation uncertainty can be reduced by either selecting predictors with non-negligible coefficients (sparse model) or using a few common factors that largely explain comovements of multiple indicators (dense model). To reduce the dimension of predictors, we first investigate quantile regression on principal components of the data.

We refer to this model as “No-Selection Principal Components Analysis (NSPCA).” However, the principal component used in the NSPCA approach determines weights on individual indicators regardless of the target quantile level, imposing the “common slope,” assumption on indicators across quantiles. For example, if the predictor $x_{1,t}$ gets a larger weight than the predictor $x_{2,t}$ in the principal component, $x_{1,t}$ is more informative about any quantile level of the target variable y_t than $x_{2,t}$. This generates a misspecification issue that is likely to bias tail risk predictions if certain indicators are more informative about the lower (upper) tail part of the distribution than other parts.

To correct this issue, we examine a variable selection approach to quantile regression using a Least Absolute Shrinkage and Selection Operator (LASSO) penalty. This penalized quantile regression step selects variables with non-negligible coefficients. Then, we run the ordinary quantile regression only with selected indicators as in [Belloni and Chernozhukov \(2011\)](#). We call this model “Post-Selection Quantile Regression (PSQR).” Post-Selection Quantile Regression may not sufficiently reduce the number of predictors. To further reduce the dimension of the data, we present a hybrid method that uses variable selection to extract a subset of predictors and then extracts principal components from only those selected variables. It is a direct extension of [Bai and Ng \(2008\)](#). We call this model “Post-Selection Principal Component Analysis (PSPCA).”¹

How significant would this misspecification error be in assessing macroeconomic tail risks? In our empirical application that looks into conditional distributions of three key macroeconomic variables (Real GDP growth, unemployment, PCE inflation), we first quantify the degree of the potential misspecification by comparing the in-sample model fit across various specifications. We find evidence for the misspecification because sparse models like the PSQR and PSPCA tend to have superior fit over the NSPCA. Although in-sample analysis is a useful starting point to diagnose the degree of the potential misspecification by reducing the role of estimation uncertainty, out-of-sample analysis might be better for evaluating tail risks in real-time. In this case, evidence is mixed and the NSPCA often has superior model fit because the reduced uncertainty about variable selection improves the accuracy of recursive out-of-sample forecasts.

The difference between in-sample analysis and out-of-sample analysis does not necessarily mean that the misspecification error can be ignored. When regression coefficients and

¹[Giglio, Kelly and Pruitt \(2016\)](#) propose a partial quantile regression that iteratively constructs principal component weights on individual indicators based on univariate quantile regressions on each individual indicator. We considered this specification but found that it generated similar results as the PSPCA approach for our empirical application. For the interest of space, we skip reporting the results from the partial quantile regression approach but detailed results are available upon request.

variable selection are unstable over time, a misspecified but parsimonious model can be competitive with an overparameterized model without the misspecification. It is known that pooling predictions from diverse models can generate more stable and accurate forecasts on average in such an environment by reducing the reliance on any single model (Elliott, Gargano and Timmermann, 2013, 2015; Geweke and Amisano, 2011; Koop and Korobilis, 2012; Katchoni, Leroux and Stevanovic, 2019). In empirical analysis, we combine forecasts from the above three different specifications using the dynamic model averaging framework used in Koop and Korobilis (2012). We find that the combined weights of sparse models like PSQR and PSPCA are typically higher than a dense model like NSPCA, suggesting that sparse models contain additional predictive values not shown by dense models especially when we are interested in the tail part of the conditional distribution. Our results refine the conclusion in Giannone, Lenza and Primiceri (2018), who argue that a single sparse model rarely shows dominant out-of-sample predictive performances in six empirical applications with a large set of economic data. While they suggest that a dense model using principal components that take into account every indicator is generally preferable, we argue that combining forecasts from both sparse models and dense models can be better because sparse models generate more accurate forecasts when the data provide clear signals on variable selection. The dynamic model averaging framework can increase and decrease weights on sparse models based on the degree of the signal from the data about variable selection.

Our main empirical findings on time-varying tail risks of real activities and inflation from out-of-sample analysis can be summarized as follows. First, a dynamic model averaging of all the three different specifications with a moderate degree of forgetting factor (discount for the predictive performance in the more distant past) typically performs better than any single specification in terms of forecast accuracy. Koop and Korobilis (2012) calibrate a fairly slow decay rate for the past predictive performance to target the conditional mean or the conditional median forecasts but we find that a relatively fast decay rate provides better predictions for the tail part of the conditional distribution in some cases. However, the results are not that sensitive to the choice of the forgetting factor parameter. Second, based on predictive densities from the recursive out-of-sample estimation, we find that the downside risk for real macro variables spiked to a highly elevated level by the end of the Great Recession but subsequently declined. On the other hand, the downside tail risk for inflation declined but did not disappear to a negligible level even after the end of the Great Recession. We also find that the realized PCE inflation data indicate that quantile regression estimates might have overpredicted intermediate lower quantiles of inflation rather frequently. This disconnect between the downside risk of real activities and that of inflation might seem to be puzzling in light of Fleckenstein, Longstaff and Lustig (2017) who show

that deflation risks inferred from inflation-related derivatives are strongly correlated with other financial and macro tail risks exemplified by the stock market return and consumer confidence. However, [Fleckenstein, Longstaff and Lustig \(2017\)](#) do not find any significant relationship between deflation risks and hard data on real activities such as industrial production and unemployment. In addition, policymakers noted the flattening of the Phillips curve during the recent period in which “movements in inflation do not reflect the cyclical position of the economy.”([Powell \(2018\)](#)). Our finding is in line with this observation and suggests that the recent movements in inflation might have been driven by factors different from what determines domestic growth.

The rest of our paper is organized as follows. Section 2 describes the details of the econometric framework that we use to assess macroeconomic tail risks. Section 3 will apply the framework to the U.S. data on GDP, unemployment, and inflation. Section 4 concludes.

2 Econometric Framework

We consider the following linear quantile regression pioneered by [Koenker and Bassett Jr \(1978\)](#) to predict tail outcomes of a macroeconomic variable (y_{t+h}) based on a set of predictors x_t .

$$Q_\tau(y_{t+h}|x_t) = \inf\{y_{t+h} : F(y_{t+h}|x_t) \geq \tau\} = \beta_{0,h}(\tau) + \sum_{k=1}^J \beta_{k,h}(\tau)x_{k,t} , t = 1, \dots, T - h. \quad (1)$$

x_t can include the current value of the target response variable (y_t) as well as potentially irrelevant predictors ($x_{k,t}$ where $\beta_{k,h}(\tau) = 0$). We can stack all the regression coefficients into a large vector $\beta_h(\tau) = [\beta_{0,h}(\tau), \dots, \beta_{J,h}(\tau)]$. The quantile regression has been increasingly used to assess economic and financial tail risks (([Adrian, Boyarchenko and Giannone, 2019](#); [Boyarchenko, Giannone and Shachar, 2018](#); [Giglio, Kelly and Pruitt, 2016](#))). The main attraction of the quantile regression framework is that it provides a flexible but yet highly tractable way of modelling the departure from normality in which tail risks are small. Although the estimation method for the quantile regression is well-established, the presence of a potentially large set of predictors can raise some challenges in precisely estimating regression coefficients. We address them using dimension reduction techniques.

2.1 Reducing the Dimension of Predictors

2.1.1 Principal Component Analysis and the Potential Misspecification in Quantile Predictions

One method for dimension reduction is to estimate a small number of factors who have non-zero weights on every indicator. For example, the first principal component of x_t ($f_{1,t}$) is a factor that explains the most common variations in x_t . We can select the number of principal components based on the information criterion suggested by [Ahn and Horenstein \(2013\)](#).² Suppose that only one factor ($f_t = \sum_{k=1}^J w_{k,f} x_{k,t}$) is used in quantile regressions. This reduces the dimension of the estimated regression coefficients to 1 from J . This method imposes the common slope for each indicator across multiple quantiles because individual weights in the factor ($w_{k,f}$) do not change depending on quantiles. In our empirical analysis, NSPCA adopts this method. The method would work well when y_{t+h} is sensitive to the average signal from a large set of indicators in which idiosyncratic variations in individual indicators are stripped out. It is a dense model in the terminology of [Giannone, Lenza and Primiceri \(2018\)](#) because every indicator has a non-zero weight and no indicator is excluded.

While convenient, using principal components as predictors raises a potential misspecification issue in conditional quantile regression, which may not be severe in conditional mean regression. The difference is that in conditional mean regression, the misspecification from the common slope restriction at different quantile levels can be averaged out but this does not hold in conditional quantile prediction. Below, we illustrate this point using results in [Angrist, Chernozhukov and Fernández-Val \(2006\)](#) on misspecified quantile regressions.

First, we establish the result connecting the conditional mean and a series of conditional quantiles. Under mild assumptions on the conditional distribution of Y given X , we show that the conditional mean can be expressed as the integral of conditional quantiles.

Proposition 1. *Suppose that*

1. *the conditional density $f_Y(y|X)$ exists a.s.*
2. *$E[Q_\tau(Y|X)]$ is finite for every $\tau \in [0, 1]$.*

Then, $E(Y|X) = \int_0^1 Q_\tau(Y|X) d\tau$.

²The method selects the number of factors to maximize the ratio of adjacent eigenvalues of $E(x_t x_t')$. In practice, the method can select too few factors that explain even less than a half the variation in x_t . In the subsequent empirical analysis, we add the restriction that requires selected factors to explain at least more than one half of cross-sectional variations in x_t .

Proof. We can show the equivalence of the two using the change of variables.

$$\begin{aligned}
E(Y|X) &= \int Y f_Y(y|X) dy = \int Q_{\tau(y)}(Y|X) f_Y(y|X) dy, \\
&= \int Q_{\tau}(Y|X) f_Y(y|X) \left| \frac{dy}{d\tau} \right| d\tau, \\
&= \int_0^1 Q_{\tau}(Y|X) d\tau, \quad (\because \tau(y) = F_Y(y|x), \quad \frac{dy}{d\tau} = \left[\frac{d\tau}{dy} \right]^{-1} = [f_Y(y|X)]^{-1}).
\end{aligned} \tag{2}$$

□

Second, we derive the quantile-specific misspecification error from the common slope assumption implicit in using the principal component as a predictor. Suppose we run the τ -th quantile regression of y_{t+h} on x_t . The regression coefficient can be obtained by minimizing the following check loss function $\rho_{\tau}(y_{t+h} - x_t \beta_h(\tau))$.

$$\begin{aligned}
\beta_h(\tau) &= \underset{\beta_h(\tau) \in \mathbb{R}^J}{\operatorname{argmin}} E[\rho_{\tau}(y_{t+h} - x_t \beta_h(\tau))], \\
\rho_{\tau}(y_{t+h} - x_t \beta_h(\tau)) &= (\tau - \mathbb{I}(y_{t+h} - x_t \beta_h(\tau) \leq 0))(y_{t+h} - x_t \beta_h(\tau)), \\
&= \tau \mathbb{I}(y_{t+h} - x_t \beta_h(\tau) < 0) |y_{t+h} - x_t \beta_h(\tau)| + (1 - \tau) \mathbb{I}(y_{t+h} - x_t \beta_h(\tau) > 0) |y_{t+h} - x_t \beta_h(\tau)|,
\end{aligned} \tag{3}$$

where $\mathbb{I}(\cdot)$ is an indicator function that equals to 1 when the condition inside the parenthesis is met.

Proposition 2. *Following Angrist, Chernozhukov and Fernández-Val (2006), we define the quantile-specific misspecification error as $\Delta_{\tau}(X, \beta) = X'(w\beta_F(\tau)) - Q_{\tau}(Y|X)$ where w is a vector of weights such that $F_t = w'X_t$. In addition to assumptions made in the **Proposition 1**, suppose that i) $\beta_F(\tau)$ is the unique solution for the following check loss function.*

$$\beta_F(\tau) = \underset{\beta_F(\tau) \in \mathbb{R}}{\operatorname{argmin}} E[\rho_{\tau}(Y - F\beta_F(\tau))]. \tag{4}$$

and ii) $E(Y), E \| X \|$ are finite. Under these assumptions, the conditional quantile regression coefficient ($\beta_F(\tau)$) and the conditional mean regression coefficient (β_F) with respect to

F_t minimize the following loss functions, respectively.

$$\begin{aligned}\beta_F(\tau) &= \operatorname{argmin}_{\beta_F(\tau) \in \mathbb{R}} E_X[\omega_\tau(X, \beta)(\Delta_\tau(X, \beta))^2], \\ \omega_\tau(X, \beta) &= \int_0^1 (1-u) f_Y(uF\beta_F(\tau) + (1-u)Q_\tau(Y|X)|X) du, \\ \beta_F &= \operatorname{argmin}_{\beta_F \in \mathbb{R}} E_X[(\int_0^1 \Delta_\tau(X, \beta) d\tau)^2].\end{aligned}\tag{5}$$

Proof. The first part regarding $\beta_F(\tau)$ is a direct application of **Theorem 1** in [Angrist, Chernozhukov and Fernández-Val \(2006\)](#) that shows $\beta_F(\tau)$ minimizes the expected weighted mean-squared approximation error. We can prove the second part by using the independence between $Y - \int_0^1 Q_\tau(Y|X) d\tau$ and X given the fact $\int_0^1 Q_\tau(Y|X) d\tau$ is the conditional mean of Y given X . This means $Y - \int_0^1 Q_\tau(Y|X) d\tau$ is orthogonal to any function of X including $\int_0^1 Q_\tau(Y|X) d\tau - F\beta_F$. By definition,

$$\begin{aligned}\beta_F &= \operatorname{argmin}_{\beta_F \in \mathbb{R}} E_X[(Y - F\beta_F)^2], \\ &= \operatorname{argmin}_{\beta_F \in \mathbb{R}} E_X[(Y - \int_0^1 Q_\tau(Y|X) d\tau + \int_0^1 Q_\tau(Y|X) d\tau - F\beta_F)^2].\end{aligned}\tag{6}$$

If we eliminate the part that does not depend on β_F and use the above-mentioned orthogonality, we can show that

$$\beta_F = \operatorname{argmin}_{\beta_F \in \mathbb{R}} E_X[(\int_0^1 [Q_\tau(Y|X) - F\beta_F] d\tau)^2] = \operatorname{argmin}_{\beta_F \in \mathbb{R}} E_X[(\int_0^1 \Delta_\tau(X, \beta) d\tau)^2].\tag{7}$$

□

The following corollary clarifies the implication of **Proposition 2** for quantile regressions.

Corollary 2.1. *Suppose that $Q_\tau(Y|X) = X'\beta_X(\tau)$ and $\beta_X(\tau)$ is $w\beta_F(\tau)$ except for countably many quantile point $[\tau_1, \dots, \tau_k]$. The misspecification in conditional quantiles does not affect the estimation of the conditional mean regression coefficient.*

Proof. Countably many points in the unit interval is a measure-zero set. Since the misspecification error $\Delta_\tau(X, \beta)$ is non-zero except for only countably many points, the loss function of the conditional mean regression coefficient $E_X[(\int_0^1 \Delta_\tau(X, \beta) d\tau)^2]$ is zero. Therefore, the estimation of the conditional mean regression coefficient is unaffected by the misspecification error at a particular quantile point. □

Hence, if the quantile-specific misspecification error from the common slope assumption is concentrated only at the tail part of the conditional distribution, this may not change the conditional mean prediction much but can create a significant bias at the conditional quantile prediction near the tail part. In particular, risk assessment from the conditional distribution of the target macro variable to fit a few selected quantile regression predictions as in [Adrian, Boyarchenko and Giannone \(2019\)](#) can be sensitive to the type of misspecification error from imposing the common slope assumption. To address the misspecification issue, we turn to a different dimension reduction method.

2.1.2 Variable Selection

An alternative method is to directly selecting only a subset of indicators to use. Suppose that the total number of available predictors (J) is greater than or equal to the total number of observations (T). Without dimension reduction, uncertainty about the estimated $\beta_h(\tau)$ would dominate the prediction uncertainty for y_{t+h} , resulting in less accurate forecasts. One way of achieving dimension reduction is to select individual predictors with non-zero coefficients by running the quantile regression with the LASSO penalty ([Belloni and Chernozhukov \(2011\)](#)).

The LASSO penalty consists of the sum of absolute values of quantile regression coefficients. For some predictors that have only weak signals for y_{t+h} , the estimated coefficients from the solution of the following minimization problem are zeros.³

$$\min_{\beta \in \mathbb{R}^J} \sum_{t=1}^{T-h} \rho_{\tau}(Y_{t+h} - X'_t \beta_h(\tau)) + \lambda \sum_{k=1}^J |\beta_{k,h}(\tau)| \quad (8)$$

Denote the number of selected indicators by $J^*(\tau) \leq J$ and their indices by $k_i(\tau)$ where $i = 1, \dots, J^*(\tau)$. Although we can directly use predictions from this single step penalized quantile regression, [Belloni and Chernozhukov \(2011\)](#) recommend to run the ordinary quantile regression with only selected predictors to obtain predictions. The rationale for the two-step PSQR is that the LASSO penalty shrinks the coefficients of very informative predictors toward zero too although this might result in less accurate forecasts. The second step in the PSQR basically corrects this shrinkage toward zeros for selected predictors. This method is very effective when the true data generating process is close to a high-dimensional

³The LASSO penalty function requires us to set the value of λ , which is a hyperparameter that determines the magnitude of the LASSO penalty. We calibrate this hyperparameter using the ten-fold cross-validation for the estimation sample.

sparse model (HDSM) in which y_{t+h} can be well predictable by a relatively small group of indicators out of a potentially large group of indicators.

We can also mix these two methods by constructing the principal components from the selected indicators in the first step of PSQR. This PSPCA method is essentially the targeted predictor approach proposed in [Bai and Ng \(2008\)](#) extended to quantile regressions instead of conditional mean regressions of different variables.

2.2 Recovering the Full Conditional Distribution

While quantile regression estimates provide local information on the conditional distribution of the target macro variable, a finite number of quantiles cannot precisely characterize tail risks such as the probability of the extreme outcomes for the target macro variable. To do this, we fit the skewed- t -distribution as done in [Adrian, Boyarchenko and Giannone \(2019\)](#) to infer the full conditional distribution of the target macro variable. Following [Adrian, Boyarchenko and Giannone \(2019\)](#), we fit skewed- t -distribution determined by four parameters in the following probability function to match four (5%, 25%, 75%, 95%) different quantile predictions from regression models ($\widehat{Q_{y_{t+h}|x_t}(\tau|x_t)}$) with the implied quantiles from the skewed- t -distribution at each time t ($F^{-1}(\tau; \mu_t, \sigma_t, \alpha_t, \nu_t)$).

$$f(y_{t+h}; \mu_t, \sigma_t, \alpha_t, \nu_t) = \frac{2}{\sigma_t} t\left(\frac{y_{t+h} - \mu_t}{\sigma_t}; \nu_t\right) T\left(\alpha_t \frac{y_{t+h} - \mu_t}{\sigma_t} \sqrt{\frac{\nu_t + 1}{\nu_t + \frac{y_{t+h} - \mu_t}{\sigma_t}}}; \nu_t + 1\right), \quad (9)$$

where $t(\cdot)$ and $T(\cdot)$ respectively denote the probability density function and cumulative density function of the Student t -distribution. The distribution is pinned down by the four parameters known as the location (μ_t), scale (σ_t), shape (α_t), and fatness (ν_t). The above distribution captures the departure from the symmetric normal distribution, which is the special case when $\alpha_t = 0$ and $\nu_t = \infty$.

With the fitted full conditional density function, we can determine the probability of tail events at both extremes as follows: BI_NG represents variables used in [Bai and Ng \(2008\)](#). The horizontal axis represents 167 predictors. The selected variable is colored at each point of time. We plot results for five quantiles (0.05, 0.25, 0.5, 0.75, 0.95).

$$P(y_{t+h} \geq \bar{y}) = 1 - F(\bar{y}|\mu_t, \sigma_t, \alpha_t, \nu_t), \quad P(y_{t+h} \leq \underline{y}) = F(\underline{y}|\mu_t, \sigma_t, \alpha_t, \nu_t). \quad (10)$$

These time-varying probabilities will provide relevant metrics for risk analysis. We can define tail events \bar{y} and \underline{y} in multiple ways. One way is to determine tail events from the unconditional 5% and 95% quantiles of y_t estimated from the sample data. Alternatively, we can define tail events with pre-set constants. For instance, we can set \underline{y} to zero to determine deflation as the negative tail event for inflation regardless of the past realization of inflation.

2.3 Dynamic Model Averaging

The potential misspecification from quantile regressions with principal components does not necessarily mean that we should exclude them in out-of-sample forecasts. [Geweke and Amisano \(2011\)](#) show that pooling predictions from a large set of misspecified models can perform better than relying on the prediction from any single model when we do not assume that the true data generating process is one of predictive models that we use. Since the best predictive model that we consider is likely to be the approximation to the true data generating process, the finding implies that we should not necessarily ignore forecasts from models with inferior in-sample fit. Similarly, combining forecasts from a variety of models with equal weights ([Elliott, Gargano and Timmermann, 2013, 2015](#); [Kotchoni, Leroux and Stevanovic, 2019](#)) or dynamic weights ([Koop and Korobilis \(2012\)](#)) has been found to generate superior forecasts from any single benchmark model in terms of the accuracy of out-of-sample forecasts. In the subsequent empirical analysis, we take the dynamic model averaging approach in ([Raftery, Kárný and Ettler, 2010](#); [Koop and Korobilis, 2012](#)) rather than the complete subset regression approach in ([Elliott, Gargano and Timmermann, 2013, 2015](#); [Kotchoni, Leroux and Stevanovic, 2019](#)) because it is less computationally demanding especially when the total number of relevant predictors exceeds the range of 10 to 20 as in the case of quantiles near the median.⁴

The dynamic model averaging is using a sequence of the past predictive performance to determine weights of alternative model for the current forecast. However, the predictive performance in the more distant past decays at an exponential rate at α to reflect a potential instability in the data generating process over time. More specifically, when we combine

⁴Another method to combine forecasts using time-varying weights is suggested by [Del Negro, Hasegawa and Schorfheide \(2016\)](#) who extend the [Geweke and Amisano \(2011\)](#) to time-varying weights by adding a latent variable that determines dynamic weights of alternative models based on their predictive densities. Given the reliance on the latent variable, calculating time-varying weights is computationally challenging when the number of alternative models is large or the computation of the exact predictive density is not tractable because we have to use a nonlinear filter for multiple latent variables.

forecasts from M different models, the i -th model’s weight at time t ($w_{i,t}$) is determined as follows:

$$w_{i,t} = \frac{P(y_{t+h}|I_{t-1}^i, \mathcal{M}_i)w_{i,t-1}^\alpha}{\sum_{j=1}^M P(y_{t+h}|I_{t-1}^j, \mathcal{M}_j)w_{j,t-1}^\alpha}, \quad \alpha \in [0, 1], \quad (11)$$

where $P(y_{t+h}|I_{t-1}^i, \mathcal{M}_i)$ is the predictive likelihood for y_{t+h} conditional on the i -th model and the relevant information set at $t - 1$ under the model.⁵ α is called the “forgetting factor” parameter and determines the discount for the predictive performance in the far distant past. When α is 1, the past performance is never forgotten (discounted) while is it rapidly forgotten (highly discounted) when α is close to 0.

3 Empirical Analysis

We apply the econometric method outlined in the previous section to assessing the tail risks of U.S. data on real GDP growth, the unemployment rate, and the PCE (Personal Consumption Expenditures) inflation rate with a large set of economic and financial indicators. These three variables are key macro variables that the Federal Open Market Committee (FOMC) provides forecast updates through the “Summary of Economic Projections,” at the quarterly frequency.

3.1 Data

Our sample period is from 1992:Q1 to 2019:Q1 and we aggregate weekly and monthly indicators into the quarterly frequency by the arithmetic average. Our choice of indicators is motivated by inflation predictors used in [Bai and Ng \(2008\)](#), financial market indicators used to construct the NFCI as described in [Brave, Kelly et al. \(2017\)](#), and labor market indicators used to construct the Labor Market Conditions Index (LMCI) published by the Federal Reserve Bank of Kansas City as explained in [Willis and Hakkio \(2014\)](#). Some of original indicators used in the above three references were unavailable for us either for the proprietary nature of the data or the updates by the statistical agency. We further trimmed down the number of indicators by eliminating those with missing observations for entire

⁵We allow the information set to vary across different models because the set of relevant predictors can vary by models.

sample period. Finally, we have used 167 different indicators. Table 1 provides the summary information on predictors that we have used in empirical analysis.

3.2 In-sample Analysis

3.2.1 GDP Growth Risk

We start our analysis by estimating quantile regressions for real GDP growth averaged over h -periods from the current quarter. We focus the one-year ahead ($h = 4$) prediction, which is the main target horizon in [Adrian, Boyarchenko and Giannone \(2019\)](#). In addition to the three specifications (PSQR, PSPCA, NSPCA) discussed in the previous section, we also consider the specification in [Adrian, Boyarchenko and Giannone \(2019\)](#) that use the current-quarter real GDP growth and the published NFCI number as predictors. We call this specification “NFCI.” One commonly used goodness-of-fit criterion is the estimated mean value of the check loss function ($E[\rho_\tau(y_{t+h} - x_t\beta_h(\tau))]$) as used in [Koenker and Machado \(1999\)](#). Following [Koenker and Machado \(1999\)](#) and [Giglio, Kelly and Pruitt \(2016\)](#), we evaluate the model fit via the following quantile $R^2(\tau)$:

$$R^2(\tau) = 1 - \frac{\frac{1}{T} \sum_t [\rho_\tau(y_{t+h} - x_t' \widehat{\beta}(\tau))]}{\frac{1}{T} \sum_t [\rho_\tau(y_{t+h} - q_y(\tau))]}, \quad (12)$$

where $q_y(\tau)$ is the unconditional τ -th quantile of y . Quantile R^2 numbers in Table 2 show that the PSQR specification dominates all the other specification based on this metric at every five quantiles that we consider (0.05, 0.25, 0.5, 0.75, 0.95). Predictive distributions of all the four specification shown in Figure 1 suggest the PSQR specification not only captures the median prediction close to the realized value but also generates much sharper quantile intervals that cover most of the realized values. In contrast, realized outcomes deviate from the median prediction significantly on occasion and the data lie beside even the 5%-95% interval in some cases.

One of main findings in [Adrian, Boyarchenko and Giannone \(2019\)](#) that the NFCI provides much more information on the downside risk (lower quantile) of the GDP growth but little information on the upside risk (upper quantile) above and beyond what is already known by the current-quarter GDP growth. Although we find that financial market variables improve quantile R^2 similar to [Adrian, Boyarchenko and Giannone \(2019\)](#), the effect is less skewed toward the downside in our case. Quantile R^2 numbers in Table 3 suggest

that the improvement comes at both upper and lower quantiles. In addition, the PSQR specification without financial market variables included in the NFCI still dominates the NFCI specification, suggesting that omitting a large dataset of macro variables can distort the evaluation of the predictive performance of financial market indicators used in the NFCI for future GDP growth.⁶

To fully investigate GDP growth risk, we calculate time-varying probabilities of tail events using the fitted skewed- t distribution. We define the downside (upside) tail event as GDP growth below (above) the 5-th (95-th) unconditional quantile value during the sample period and denote it by \underline{y} (\bar{y}). Figure 2 describes time-varying tail risks for the PSQR specification and the NFCI specification. The PSQR appears to capture the upside risk of GDP growth during the late 1990s reflecting the productivity boom while the NFCI specification completely misses it. While both specifications correctly capture the spike in the downside risk in the second half of 2007, the PSQR detects it one quarter ahead in 2007Q3 while the NFCI does in 2007Q4. Overall, our analysis suggests that selecting relevant predictors targeted for quantiles of interest improves the model fit and can be more important for assessing GDP growth tail risks than simply adding financial market indicators into the set of predictors. One concern for our analysis is that the in-sample fit may prefer the overparameterized PSQR specification instead of a more parsimonious PCA-based approach. The Bayesian Information Criterion (BIC) for linear quantile regressions in [Lee, Noh and Park \(2014\)](#) penalizes the overparameterization. While the BIC still prefers the PSQR specification when predictors for the NFCI are not excluded as shown in Table 2, it prefers a more parsimonious specification such as PSPCA or NSPCA at upper quantiles when predictors used for the NFCI are excluded as shown in Table 3. The finding indicates that the PSQR specification may overfit the in-sample data and that reducing the dimension of predictors by the PCA can be competitive in terms of the out-of-sample forecast accuracy despite potential concerns for misspecifications. We will examine this issue in the subsequent out-of-sample analysis after performing similar in-sample analysis for the unemployment rate and the PCE inflation rate.

⁶One caveat in our analysis is that indicators from [Bai and Ng \(2008\)](#) include 19 financial market variables although none of them is directly used in the NFCI. Our finding simply points out that the asymmetric predictive power of the NFCI for future GDP growth may be sensitive to the presence of a large dataset for macroeconomic indicators.

3.2.2 Unemployment Risk

Understanding the risk of a large increase in unemployment is important for policymakers who are concerned about risks to achieving their objectives (Kiley (2018)). We run quantile regressions on the change in the unemployment rate over the next four quarters using the large set of economic and financial indicators to evaluate risks associated with the outlook for the unemployment rate. To preserve the boundedness of the unemployment rate in the unit interval, we transform the measured unemployment rate (u_t) into a variable with an unbounded support ($z_t = \ln(\frac{u_t}{1-u_t})$).

Table 4 compares the in-sample model fit for the four different specifications. In the last one, we use two principal components constructed from indicators used in the LMCI. The PSQR again dominates all the other ones. Figure 3 also confirms that the PSQR specification covers the realized value in the narrow quantile range throughout the sample period including the Great Recession during which the unemployment rate spiked in a short amount of time. But when we exclude financial market indicators used in the NFCI from the set of predictors, the most parsimonious specification that use only two LMCI factors is dominant in terms of the BIC at the median and lower (5%, 25%) quantiles. Excluding financial market indicators in the NFCI deteriorates the model fit across all the quantiles but the impact is slightly more pronounced at lower quantiles when we consider the penalty for overparameterization through the BIC. The finding is somewhat in contrast to the literature emphasizing the asymmetric predictive power of financial market variables for the downside risk of real economic activities which means the upside risk of the unemployment rate (Adrian, Boyarchenko and Giannone, 2019; Giglio, Kelly and Pruitt, 2016). However, our finding is in line with Kiley (2018) who shows that once we include more cyclical macro variables as predictors, the asymmetric predictive power of financial market variables for economic downturn weakens.

Time-varying tail risks of the unemployment rate from the PSQR specification in Figure 4 suggest that the spike in the unemployment rate during the Great Recession is largely predictable by the end of 2008 with the deterioration in economic and financial conditions. It also indicates that the unemployment rate was expected to be below 4 percent from 2018.

3.2.3 Inflation Risk

Bianchi, Melosi and Rottner (2019) argue that inflation has not stabilized at the Federal Reserve’s 2 percent target even after it was formally adopted in 2012. Instead, they empha-

size that inflation persistently undershoot the desired target, resulting in the “deflationary bias.” They attribute the bias to the increased chance of hitting the effective lower bound on the nominal interest rate due to the decline in the equilibrium real interest rate. However, drawing such a conclusion presumes that the actual deflation risk also has persistently risen over the same time period, which is not obvious from looking at only the realized inflation rate. To address this issue, we use the quantile regression framework and characterize the risk associated with inflation outlook both in the near-term (one-year ahead) and the medium-term (three-year ahead). We run quantile regressions on the future change in the PCE inflation rate using the large set of economic and financial indicators. As before, we estimate four different specifications with or without variable selection and compare the model fit. In the last one, we use principal component factors extracted from the set of predictors included in [Bai and Ng \(2008\)](#).

Table 6 suggests that the PSQR specification again dominates in terms of quantile R^2 . However, the PSPCA specification fares better at the 50-th and the 75-th quantiles when we take into account the penalty of overparameterization using the BIC. Figure 5 shows that the realized PCE inflation rate is close to median predictions from the PSQR specification and the PSPCA specification most of the time with the main difference that the range between the upper quantile and the lower quantile is much wider in the PSPCA.

Figure 6 shows time-varying tail risks of the PCE inflation rate from the PSQR specification. We define the downside risk as the probability that the one-year forward quarterly PCE inflation rate is below 0 percent. Similarly, we define the upside risk as the probability that the one-year forward quarterly PCE inflation rate is above 4 percent.⁷ Before the financial crisis of 2008, an uptick in the downside risk is subsequently followed by a period of low downside risk. However, the downside risk has remained elevated above the pre-crisis average since 2010. In contrast, the upside risk nearly vanished during the same period except for an uptick to the pre-crisis average value at the end of the sample period (2018Q1). A similar picture emerges even when we consider risks at a longer forecast horizon (three-year) in Figure 7. While quarter-to-quarter fluctuations in the downside risk even during the post-crisis period do not fully support the permanent shift in the downside risk due to the deflationary bias, managing the elevated downside risk relative to the pre-crisis period can be an important consideration in the inflation targeting regime.

⁷We take this definition for the tail event rather than the one based on the unconditional distribution. The unconditional 5% quantile of inflation for the sample period is close to 0% but the 95% quantile is only slightly above 3%, showing the asymmetry around the 2% target.

3.3 Out-of-sample Analysis

In-sample analysis in the previous section has limitations in guiding real-time policy decisions because the best-performing model can overfit the in-sample data but do poorly for the out-of-sample data. [Adrian, Boyarchenko and Giannone \(2019\)](#) suggests that their conclusions on the predictive power of the NFCI for future GDP growth from in-sample analysis are largely valid in the pseudo out-of-sample analysis. However, the NFCI is constructed from the full-sample data and unless you use the real-time data for the NFCI, even pseudo out-of-sample analysis can introduce the “look-ahead bias.” In our case, this is less problematic because we use underlying indicators not the principal component based on the full-sample data.⁸ To generate pseudo out-of-sample forecasts, we run quantile regressions recursively using the data from 2008Q1 to 2019Q1 as out-of-sample forecast target. For example, for the one-year ahead quantile prediction, we use data from 1992Q1 to 2007Q1 to generate forecast for 2008Q1.

3.3.1 Forecast Comparison

In out-of-sample forecast exercises, pooling forecasts from different specifications can improve forecasts from individual models under certain conditions ([Elliott, Gargano and Timmermann, 2013, 2015](#); [Geweke and Amisano, 2011](#); [Del Negro, Hasegawa and Schorfheide, 2016](#); [Koop and Korobilis, 2012](#); [Kotchoni, Leroux and Stevanovic, 2019](#); [Raftery, Kárný and Ettler, 2010](#)) though it is difficult to know ex-ante which method is the best way of combining different forecasts. In this paper, we use the dynamic model averaging framework with alternative values of forgetting factor ([Kotchoni, Leroux and Stevanovic, 2019](#); [Raftery, Kárný and Ettler, 2010](#)). We use four different values for the forgetting factor parameter (α in equation (11)): [0 0.5 0.9 0.99]. $\alpha = 0$ ignores the past predictive performance of each model other than the immediate past. By contrast, the past predictive performance decays very slowly in determining weights when $\alpha = 0.99$.

Table 7 ~ 9 show quantile-specific R^2 for pseudo out-of-sample forecasts from various specifications. We test the statistical significance of quantile-specific R^2 using the [Diebold and Mariano \(1995\)](#) test-statistic. Unlike in-sample analysis, the PSQR specification rarely fares better than other more parsimonious specifications like the PSPCA or the NSPCA. The stark difference between in-sample results and out-of-sample results suggests that the

⁸Strictly speaking, we should use real-time vintage data to properly replicate out-of-sample forecasting environments. We do not pursue that route here because we cannot find real-time vintage data for some predictors.

forecasting environment is unstable. Indeed, we find that selected variables are time-varying as shown in Figure 8 ~ 10. However, the finding does not necessarily imply that we should ignore forecasts from the PSQR specification. In fact, combining forecasts from the PSQR specification with those from the PSPCA and the NSPCA improves the forecast accuracy especially for the tail part of the conditional distribution of real GDP and the unemployment rate. Forecast combination also improves the forecast accuracy for the median of the PCE inflation rate. The predictive accuracy of forecast combination methods does not appear to be highly sensitive to the calibration of the forgetting factor parameter. Forecasts from specifications with alternative forgetting parameter values are generally competitive with the best forecasts from any single model.

Predictive distributions of real GDP growth out of quantile estimates in Figure 11 show that the Great Recession was not predicted by any specification with the realized growth rate below 5% quantile level. But realized outcomes after 2009 generally lie within the range between the 5% quantile and the 95% quantile except for the PSQR specification whose narrow quantile range does not capture realized outcomes between 2015 and 2017. The spike in the unemployment rate during the Great Recession was similarly missed by all the specifications while the predictive performance after 2009 appears to be similar across alternative specifications as suggested by Figure 12. A similar picture emerges for the out-of-sample predictive distribution of the PCE inflation rate from quantile estimates as shown in Figure 13. The similar predictability pattern across multiple variables during the Great Recession suggests that it was driven by a large unexpected demand shock which increased the downside risk for real GDP and inflation and the upside risk for unemployment.

3.3.2 Time-varying Tail Risk

By fitting skewed- t distribution recursively, we can compute the probability of tail events recursively. While recursive forecasts missed the magnitude of the decline in real GDP during the Great Recession, it predicted at least the timing of the decline relatively well as shown in Figure 14. To reflect this, when we test the correct specification for the fitted skewed- t density using the test statistic from Rossi and Sekhposyan (2019), lower quantiles are within the confidence band of the correct specification while upper quantiles slightly lie outside the confidence band as shown in Figure 15.⁹ Similarly, the tail risk for the unemployment rate

⁹The test statistic in Rossi and Sekhposyan (2019) is based on the idea that the skewed- t cumulative probability density value of the realized outcome must follow the uniform distribution if the skewed- t distribution is the correct specification. Since the cumulative distribution function of a uniform random variable is a 45-degree line in the unit interval, the confidence band of the test statistic is centered around the 45-degree

in Figure 16 shows that the upside risk of the unemployment rate spiked during the Great Recession while the downside risk spiked in 2018 in line with the continued decline of the unemployment rate below the downside cutoff value of about 4 percent. In this case, the skewed- t density missed upper quantiles of the unemployment rate as shown in Figure 17. This finding is consistent with what we find for real GDP. In both cases, the tail probability spiked during a relatively short-period of time and subsequently declined fast.

Regarding the PCE inflation rate, we find a somewhat different pattern. Unlike tail risks for real variables, we find that tail risks for the PCE inflation rate in Figure 18 exhibit more frequent fluctuations, meaning that extreme outcomes are not completely ruled out even during a more tranquil period. Reflecting this pattern, the test statistic in Figure 19 shows that models generally miss the intermediate lower quantiles (0.2th - 0.4th). The finding is not surprising that realized outcomes stayed below the median forecast rather persistently since 2011 as shown in Figure 13. However, this disconnect between the downside risk of real macro variables and that of inflation might seem to be inconsistent with [Fleckenstein, Longstaff and Lustig \(2017\)](#) who show that deflation risks inferred from inflation-related derivatives are strongly correlated with tail risks for the return on real assets (stock market return) and consumer confidence. We do not see our finding as contradictory to theirs. In our analysis, stock market return is selected as a relevant predictor for the lower (5%) quantile of inflation but never selected as a relevant predictor for the lower (5%) quantile of real GDP growth. Like us, [Fleckenstein, Longstaff and Lustig \(2017\)](#) do not find any significant relationship between deflation risks and real macro data such as industrial production and unemployment. In addition, policymakers frequently mentioned the flattening of the Phillips curve during the recent period in which inflation did not respond much to the cyclical movement in economic activities. The disconnect between the tail risk of inflation and that of a real macro variable is consistent with the hypothesis that some of recent time-variations in inflation might have been driven by factors that do not have a tight relationship with domestic growth.

4 Conclusion

In this paper, we assess macroeconomic tail risks using a large set of economic and financial indicators. By combining quantile regressions with dimension reduction techniques for predictors, we propose a method to address potential misspecifications in ignoring heterogeneous informative values of predictors across quantiles of the target macro variable. We

line. We compute the confidence band using the bootstrapping as in [Adrian, Boyarchenko and Giannone \(2019\)](#).

show that a parsimonious model using the principal components of predictors regardless of the target predicted variable can result in significant misspecification errors in conditional quantile predictions although that can be mitigated by the averaging across different quantiles in the case of the conditional mean prediction. The impact of the misspecification is present even after considering the penalty for overparameterization in some versions of the sparse model. In pseudo out-of-sample analysis, this kind of misspecification error can be masked by an additional uncertainty related to variable selection. In this environment, we find that combining forecasts from sparse models with those from a dense model without variable selection typically improves the predictive accuracy for target macro variables.

Out-of-sample analysis for the U.S. data suggests that the downside risk for real macro variables spiked to a highly elevated level by the end of the Great Recession but subsequently declined. On the other hand, the downside tail risk for inflation exhibited non-negligible quarter-to-quarter fluctuations even after the end of the Great Recession. In fact, the realized PCE inflation data indicate that quantile regression estimates might have overpredicted intermediate lower quantiles of inflation rather frequently. The disconnect between the tail risk of inflation and that of a real macro variable appears to be in line with the recent flattening of the Phillips curve relationship and suggests that inflation tail risk during the recent period might have been driven by factors different from those determining domestic growth.

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Table 1: SUMMARY STATISTICS FOR PREDICTORS:1992Q1-2019Q1

Group	Total	Retrieved	Incomplete	Used
Bai and Ng (2008)	135	96	7	89
Brave, Kelly et al. (2017)	105	77	22	55
Willis and Hakkio (2014)	23	23	0	23
Total	263	196	29	167

Notes: We could not retrieve some proprietary indicators in Brave, Kelly et al. (2017) and missed some series used in Bai and Ng (2008) because recent updates made by statistical agencies made some series originally used in Bai and Ng (2008) unavailable. In addition, some series in Bai and Ng (2008) were overlapped with those in (Brave, Kelly et al., 2017) and we counted as belonging to Bai and Ng (2008). We subtracted the unemployment rate from the number of indicators in Willis and Hakkio (2014) because the unemployment rate is one of target macro variables in our analysis and we always include the current-quarter target variable to predict the future value of the target macro variable in quantile regressions.

Table 2: IN-SAMPLE MODEL FIT FOR THE ONE-YEAR AVERAGE REAL GDP GROWTH:1992Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA	NFCI
0.05	0.841 (2.033)	0.523 (2.6)	0.459 (2.748)	0.482 (2.68)
0.25	0.764 (3.5)	0.252 (3.767)	0.236 (3.788)	0.151 (3.872)
0.5	0.668 (3.841)	0.189 (3.981)	0.145 (4.033)	0.090 (4.074)
0.75	0.697 (3.383)	0.151 (3.77)	0.227 (3.675)	0.099 (3.807)
0.95	0.549 (2.119)	0.132 (2.418)	0.271 (2.266)	0.08 (2.476)

Notes: Numbers in the parenthesis are the Bayesian Information Criterion (BIC) for linear quantile regressions discussed in [Lee, Noh and Park \(2014\)](#). The lower BIC number means better fit.

Table 3: IN-SAMPLE MODEL FIT FOR THE ONE-YEAR AVERAGE REAL GDP GROWTH:1992Q1-2019Q1

Quantile R^2	PSQR (ex-NFCI)	PSPCA (ex-NFCI)	NSPCA (ex-NFCI)	NFCI
0.05	0.614 (2.609)	0.295 (2.989)	0.363 (2.91)	0.482 (2.68)
0.25	0.621 (3.685)	0.088 (3.944)	0.167 (3.875)	0.151 (3.872)
0.5	0.680 (4.025)	0.142 (4.037)	0.125 (4.057)	0.090 (4.074)
0.75	0.362 (3.728)	0.180 (3.713)	0.194 (3.718)	0.099 (3.807)
0.95	0.245 (2.389)	0.193 (2.345)	0.253 (2.289)	0.08 (2.476)

*Notes:*We exclude financial market indicators coming out of the NFCI for the three specifications; PSQR, PSPCA, and NSPCA. Numbers in the parenthesis are the Bayesian Information Criterion (BIC) for linear quantile regressions discussed in [Lee, Noh and Park \(2014\)](#). The lower BIC number means better fit.

Table 4: IN-SAMPLE MODEL FIT FOR THE ONE-YEAR CHANGE IN THE UNEMPLOYMENT RATE:1992Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA	LMCI
0.05	0.663 (-0.657)	0.349 (-0.465)	0.349 (-0.442)	0.337 (-0.447)
0.25	0.764 (0.718)	0.259 (0.953)	0.298 (0.898)	0.22 (0.982)
0.5	0.763 (1.066)	0.313 (1.331)	0.321 (1.319)	0.246 (1.402)
0.75	0.839 (0.874)	0.442 (1.253)	0.452 (1.236)	0.353 (1.379)
0.95	0.886 (-0.591)	0.646 (0.032)	0.614 (0.117)	0.587 (0.164)

Notes: Numbers in the parenthesis are the Bayesian Information Criterion (BIC) for linear quantile regressions discussed in [Lee, Noh and Park \(2014\)](#). The lower BIC number means better fit.

Table 5: IN-SAMPLE MODEL FIT FOR THE ONE-YEAR CHANGE IN THE UNEMPLOYMENT RATE:1992Q1-2019Q1

Quantile R^2	PSQR (ex-NFCI)	PSPCA (ex-NFCI)	NSPCA (ex-NFCI)	LMCI
0.05	0.512 (-0.442)	0.221 (-0.285)	0.302 (-0.373)	0.337 (-0.447)
0.25	0.426 (1.029)	0.161 (1.055)	0.23 (1)	0.22 (0.982)
0.5	0.465 (1.458)	0.206 (1.453)	0.248 (1.421)	0.246 (1.402)
0.75	0.658 (1.251)	0.344 (1.416)	0.355 (1.399)	0.353 (1.379)
0.95	0.776 (-0.1366)	0.374 (0.579)	0.488 (0.4)	0.587 (0.164)

*Notes:*We exclude financial market indicators coming out of the NFCI for the three specifications; PSQR, PSPCA, and NSPCA. Numbers in the parenthesis are the Bayesian Information Criterion (BIC) for linear quantile regressions discussed in [Lee, Noh and Park \(2014\)](#). The lower BIC number means better fit.

Table 6: IN-SAMPLE MODEL FIT FOR THE ONE-YEAR CHANGE IN THE PCE INFLATION RATE:1992Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA
0.05	0.587 (2.874)	0.293 (3.145)	0.033 (3.48)
0.25	0.454 (4.186)	0.002 (4.279)	0.005 (4.298)
0.5	0.523 (4.281)	0.19 (4.255)	0.014 (4.452)
0.75	0.581 (4.005)	0.262 (3.973)	0.063 (4.212)
0.95	0.717 (2.534)	0.419 (2.925)	0.276 (3.168)

Notes: Numbers in the parenthesis are the Bayesian Information Criterion (BIC) for linear quantile regressions discussed in [Lee, Noh and Park \(2014\)](#). The lower BIC number means better fit.

Table 7: OUT-OF-SAMPLE MODEL FIT FOR THE ONE-YEAR AVERAGE REAL GDP GROWTH:2008Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA	DMA($\alpha = 0$)	DMA ($\alpha = 0.5$)	DMA ($\alpha = 0.9$)	DMA ($\alpha = 0.99$)
0.05	-0.55	0.24**	0.214**	0.342*	0.286**	0.285**	0.284**
0.25	0.041	-0.194	0.12	0.224	0.249	0.216	0.147
0.5	0.105	-0.018	-0.007	0.161	0.229	0.188	0.064
0.75	0.078	-0.032	0.204**	0.279***	0.3***	0.277**	0.237*
0.95	-0.489	0.078*	0.223***	0.186**	0.194**	0.194**	0.197**

*Notes:**** (**, *) indicates the statistical significance that conditional quantile forecasts are more accurate than forecasts based on the recursive estimation of unconditional quantiles at the 1% (5%, 10%) level.

Table 8: OUT-OF-SAMPLE MODEL FIT FOR THE ONE-YEAR CHANGE IN THE UNEMPLOYMENT RATE:2008Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA	DMA($\alpha = 0$)	DMA ($\alpha = 0.5$)	DMA ($\alpha = 0.9$)	DMA ($\alpha = 0.99$)
0.05	-1.559	0.081	-0.007	-0.059	-0.059	-0.051	-0.044
0.25	-0.017	0.17	0.099	0.109	0.101	0.094	0.089
0.5	0.012	0.168	0.156	0.385	0.382	0.374	0.374
0.75	0.326	0.395*	0.37*	0.688**	0.686**	0.683**	0.683**
0.95	0.095	0.409	0.492***	0.826**	0.826**	0.826**	0.817**

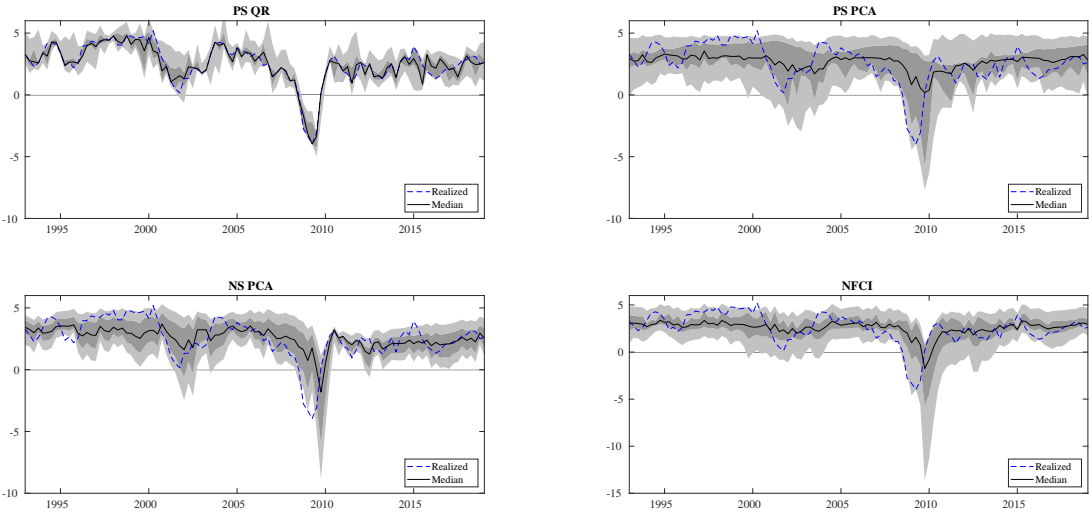
*Notes:**** (** , *) indicates the statistical significance that conditional quantile forecasts are more accurate than forecasts based on the recursive estimation of unconditional quantiles at the 1% (5%, 10%) level.

Table 9: OUT-OF-SAMPLE MODEL FIT FOR THE ONE-YEAR CHANGE IN THE PCE INFLATION RATE:2008Q1-2019Q1

Quantile R^2	PSQR	PSPCA	NSPCA	DMA($\alpha = 0$)	DMA ($\alpha = 0.5$)	DMA ($\alpha = 0.9$)	DMA ($\alpha = 0.99$)
0.05	-1.102	-0.149	-0.227	-0.071	-0.083	-0.084	-0.084
0.25	-0.413	0.079	-0.029	0.084*	0.057	0.052	0.054
0.5	-0.061	0.119*	-0.04	0.169**	0.17**	0.177**	0.181**
0.75	0.05	0.008	-0.083	0.143	0.13	0.102	0.059
0.95	-0.367	-0.451	-0.424	0.345	0.281	0.274	0.276*

Notes:*** (** , *) indicates the statistical significance that conditional quantile forecasts are more accurate than forecasts based on the recursive estimation of unconditional quantiles at the 1% (5%, 10%) level.

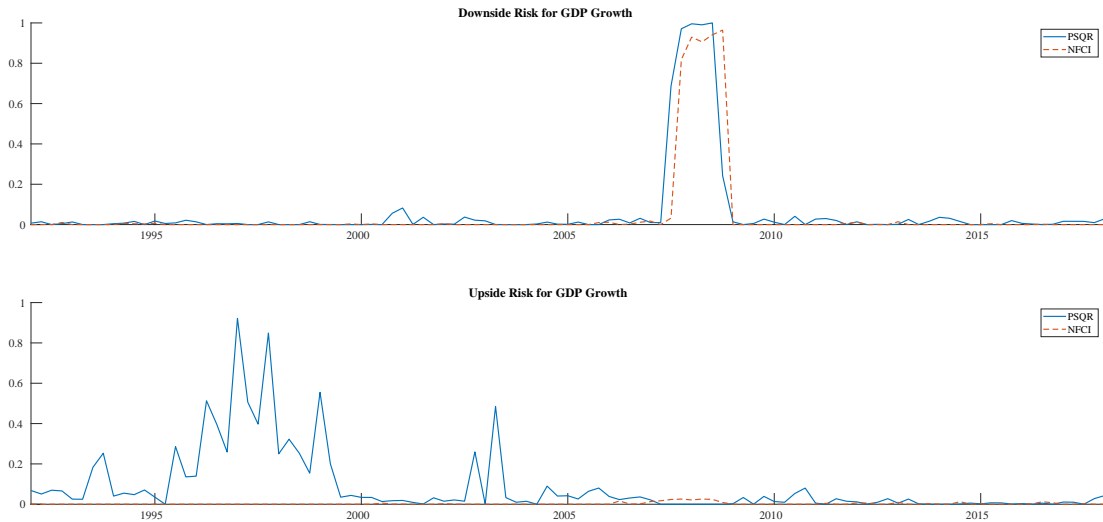
Figure 1: IN-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR AHEAD REAL GDP GROWTH



Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

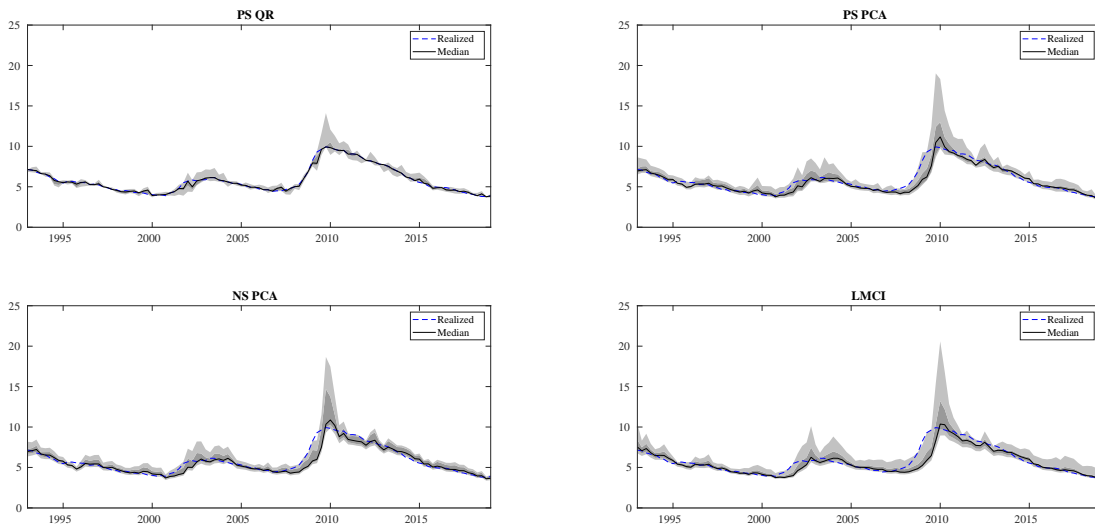
8 ~ 10

Figure 2: IN-SAMPLE TAIL RISKS FOR THE ONE-YEAR AHEAD REAL GDP GROWTH:PSQR



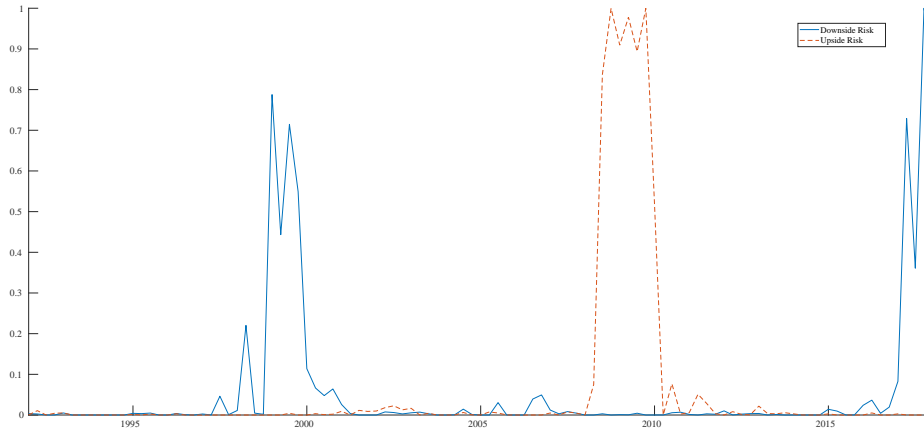
Notes: Downside risk is the probability that the one-year ahead average growth rate is below -0.128 percent, which is the 5-th unconditional quantile during the sample period. Similarly, upside risk is defined as the probability that the one-year ahead average growth rate is above 4.523 percent, which is the 95-th unconditional quantile during the sample period.

Figure 3: IN-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR FORWARD UNEMPLOYMENT RATE



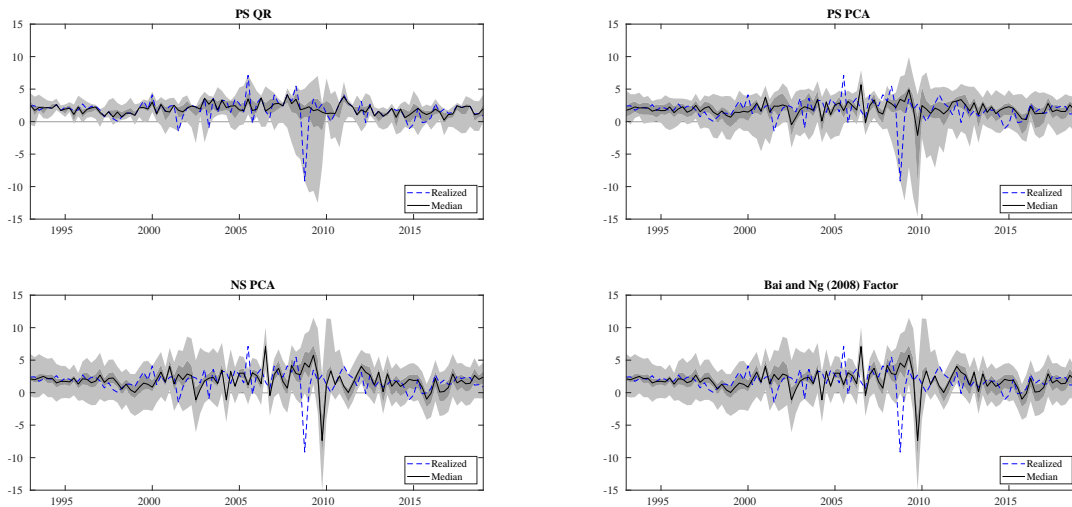
Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

Figure 4: IN-SAMPLE TAIL RISKS FOR THE ONE-YEAR FORWARD UNEMPLOYMENT RATE: PSQR



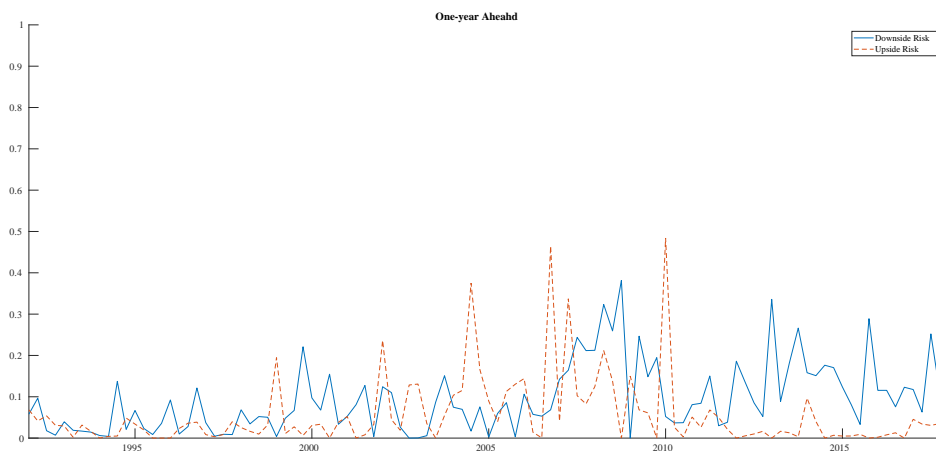
Notes: Downside risk is the probability that the one-year forward quarterly unemployment rate is below 4.017 percent, which is the 5-th unconditional quantile during the sample period. Similarly, upside risk is defined as the probability that the one-year forward quarterly unemployment rate is above 9.283 percent, which is the 95-th unconditional quantile during the sample period.

Figure 5: IN-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR FORWARD PCE INFLATION RATE



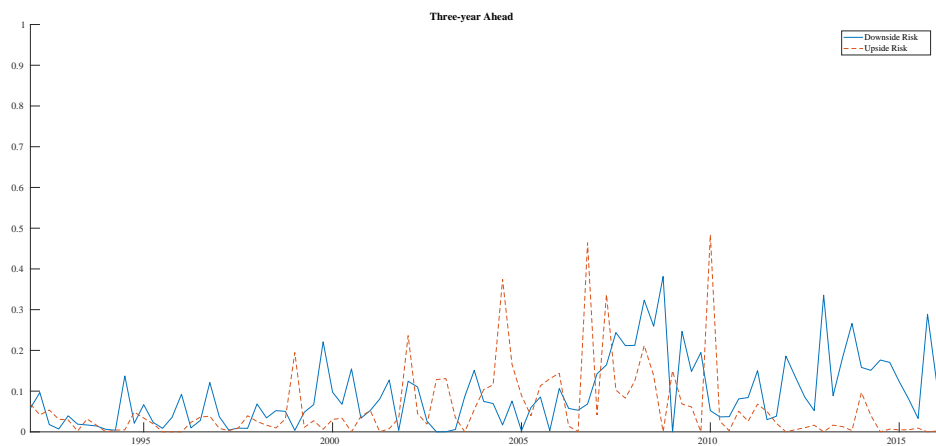
Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

Figure 6: IN-SAMPLE TAIL RISKS FOR THE ONE-YEAR FORWARD PCE INFLATION RATE:PSQR



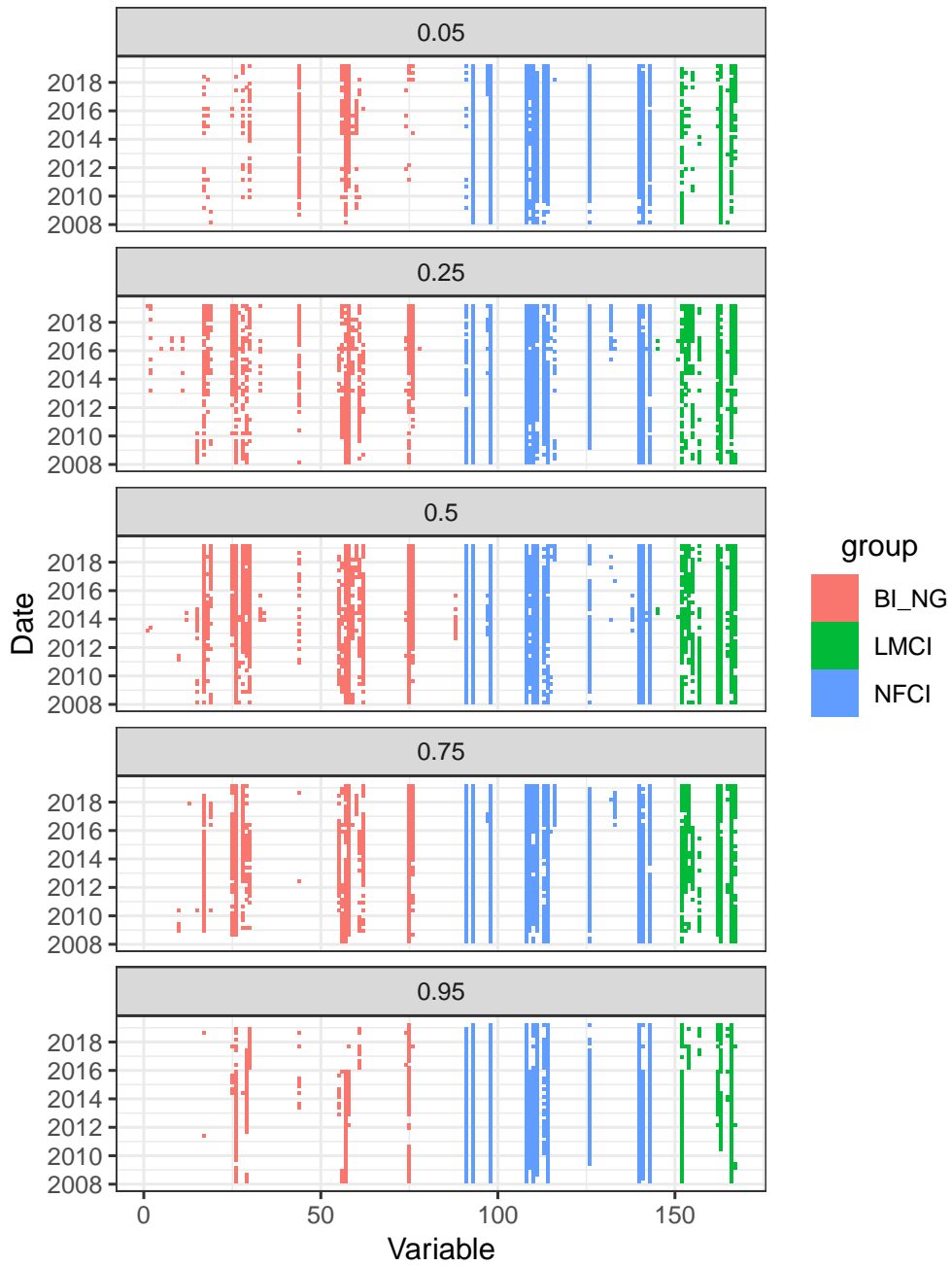
Notes: Downside risk is the probability that the one-year forward quarterly PCE inflation rate is below 0 percent. The upside risk is defined as the probability that the one-year forward quarterly PCE inflation rate is above 4 percent.

Figure 7: IN-SAMPLE TAIL RISKS FOR THE ONE-YEAR FORWARD PCE INFLATION RATE:PSQR



Notes: Downside risk is the probability that the one-year forward quarterly PCE inflation rate is below 0 percent. The upside risk is defined as the probability that the one-year forward quarterly PCE inflation rate is above 4 percent.

Figure 8: OUT-OF-SAMPLE VARIABLE SELECTION: REAL GDP GROWTH

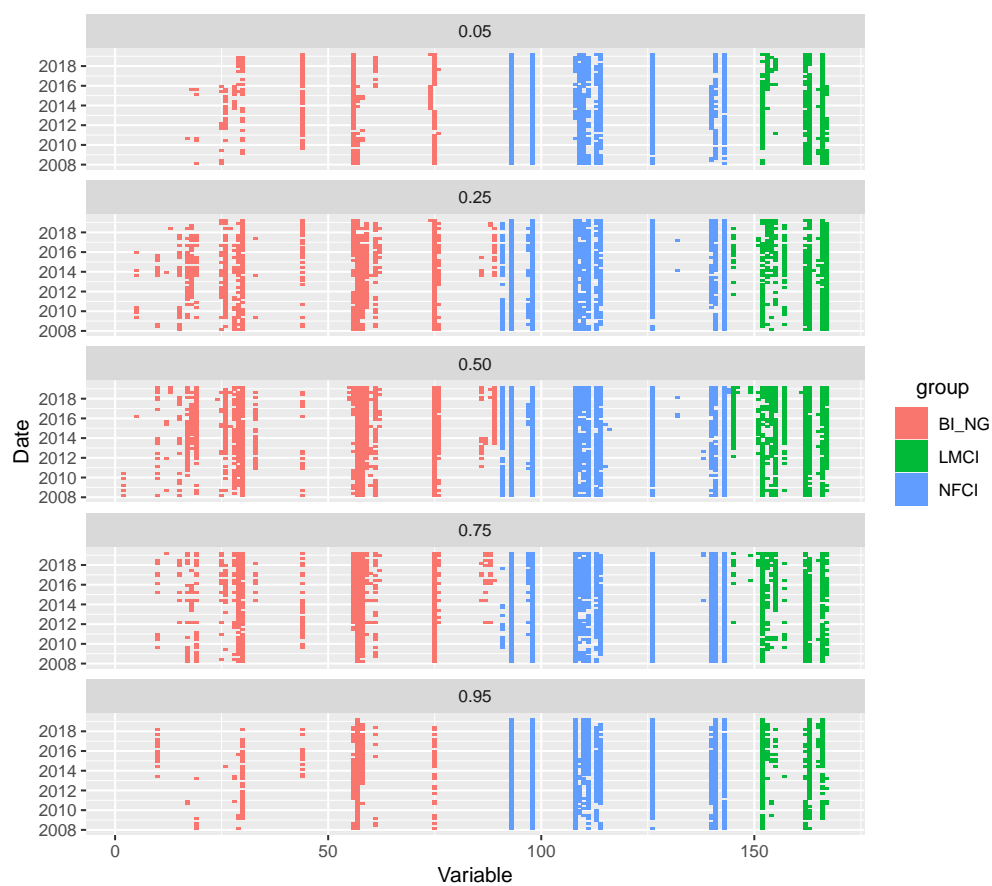


Notes:

BI_NG

represents variables used in [Bai and Ng \(2008\)](#). The horizontal axis represents 167 predictors. The selected variable is colored at each point of time. We plot results for five quantiles (0.05, 0.25, 0.5, 0.75, 0.95).

Figure 9: OUT-OF-SAMPLE VARIABLE SELECTION: UNEMPLOYMENT RATE

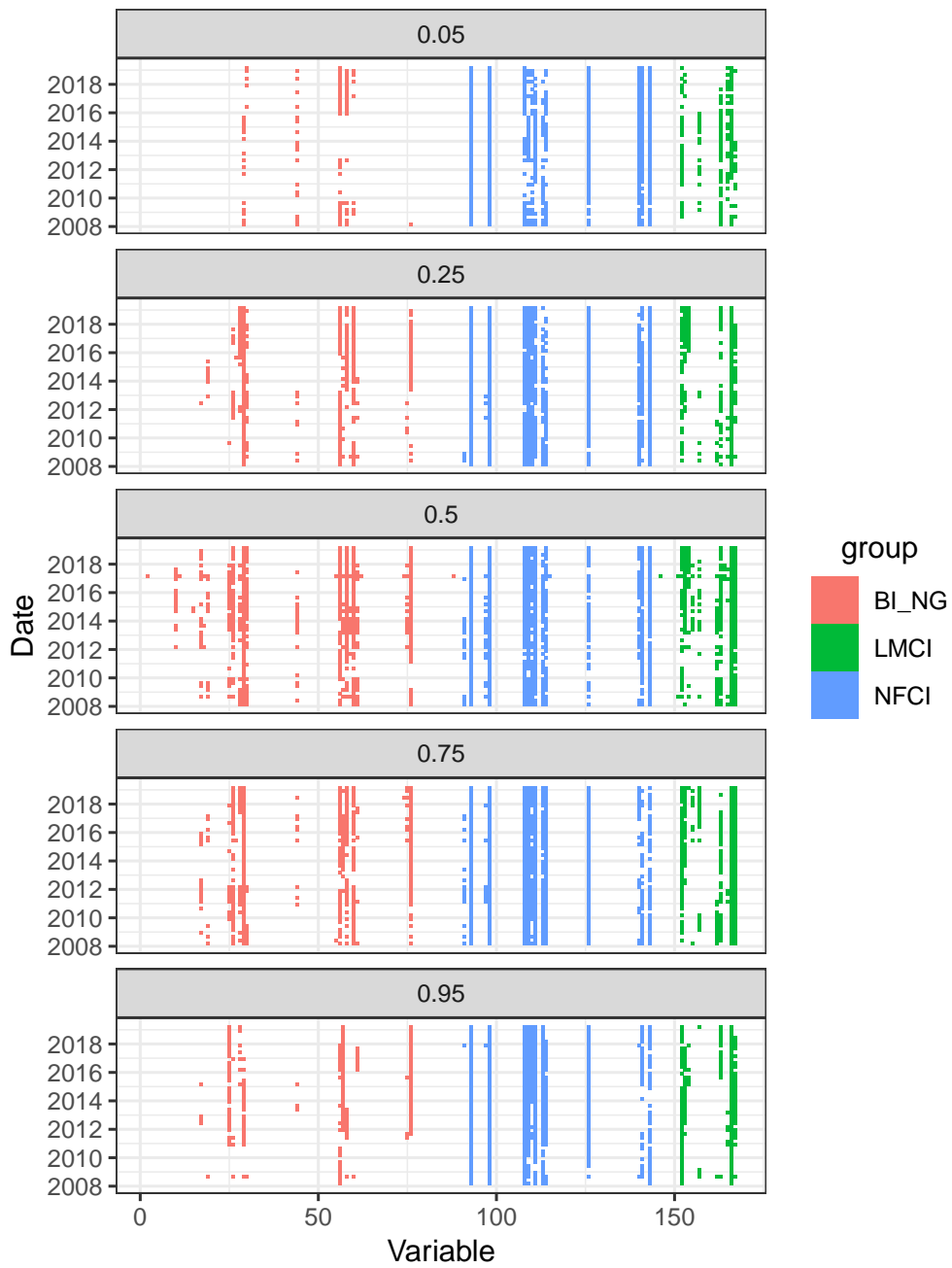


Notes:

BI_NG

represents variables used in [Bai and Ng \(2008\)](#). The horizontal axis represents 167 predictors. The selected variable is colored at each point of time. We plot results for five quantiles (0.05, 0.25, 0.5, 0.75, 0.95).

Figure 10: OUT-OF-SAMPLE VARIABLE SELECTION: PCE INFLATION

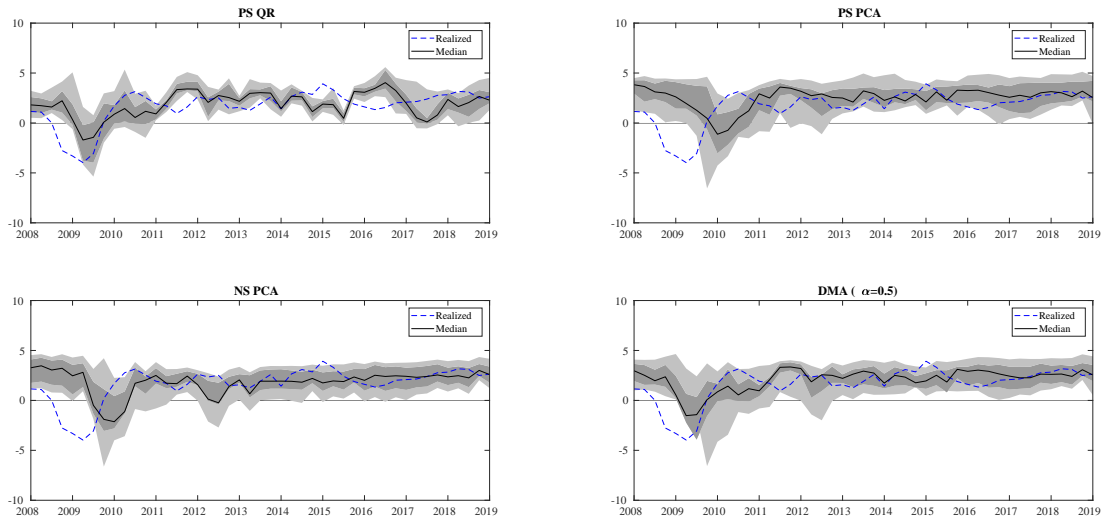


Notes:

BI_NG

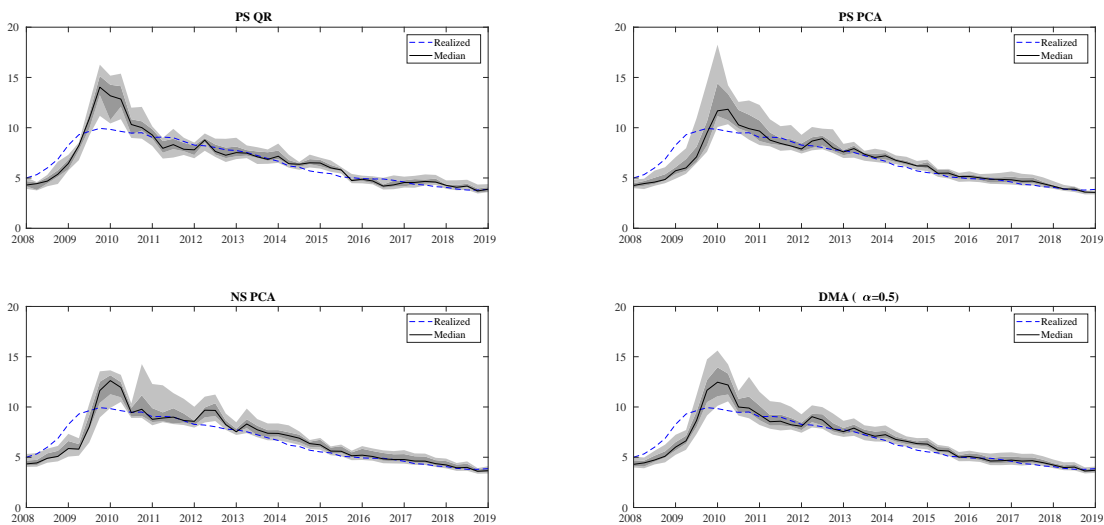
represents variables used in [Bai and Ng \(2008\)](#). The horizontal axis represents 167 predictors. The selected variable is colored at each point of time. We plot results for five quantiles (0.05, 0.25, 0.5, 0.75, 0.95).

Figure 11: OUT-OF-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR AHEAD REAL GDP GROWTH



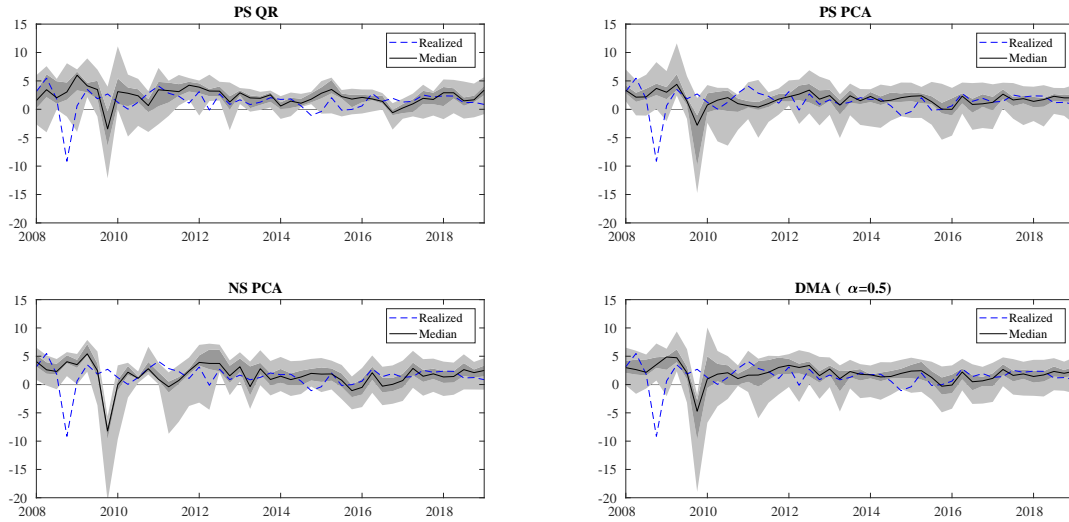
Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

Figure 12: OUT-OF-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR FORWARD QUARTERLY UNEMPLOYMENT RATE



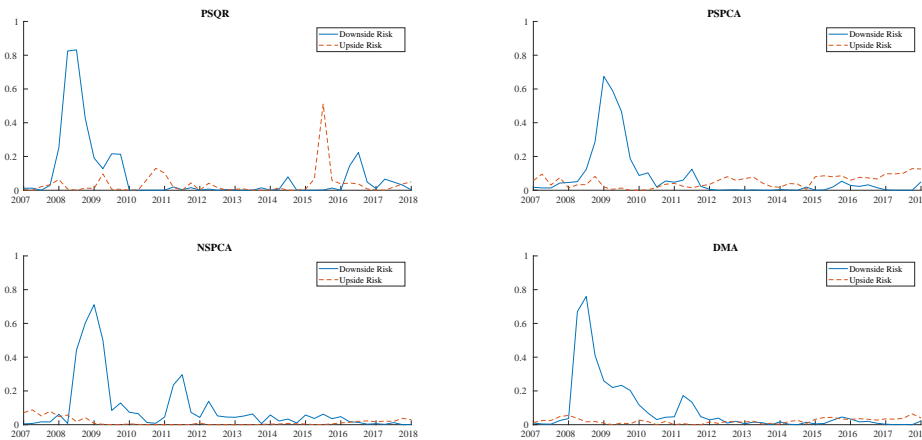
Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

Figure 13: OUT-OF-SAMPLE PREDICTIVE DISTRIBUTIONS FOR THE ONE-YEAR FORWARD PCE INFLATION RATE



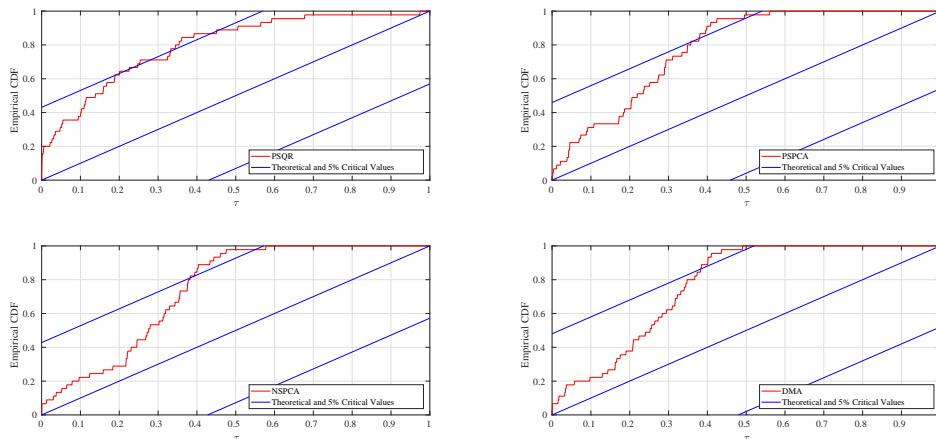
Notes: The dark shaded area represents the 0.25-0.75 quantile interval while the light shaded area describes the 0.05-0.95 quantile interval.

Figure 14: OUT-OF-SAMPLE TAIL RISKS FOR THE ONE-YEAR AHEAD REAL GDP GROWTH



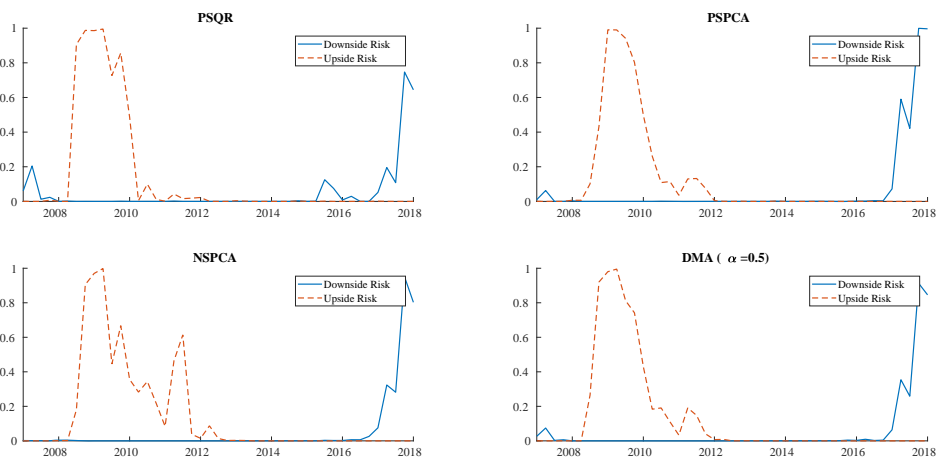
Notes: Downside risk is the probability that the one-year ahead average growth rate is below -0.128 percent, which is the 5-th unconditional quantile during the sample period. Similarly, upside risk is defined as the probability that the one-year ahead average growth rate is above 4.523 percent, which is the 95-th unconditional quantile during the sample period.

Figure 15: SPECIFICATION TEST FOR THE OUT-OF-SAMPLE PREDICTIVE DENSITY:REAL GDP GROWTH



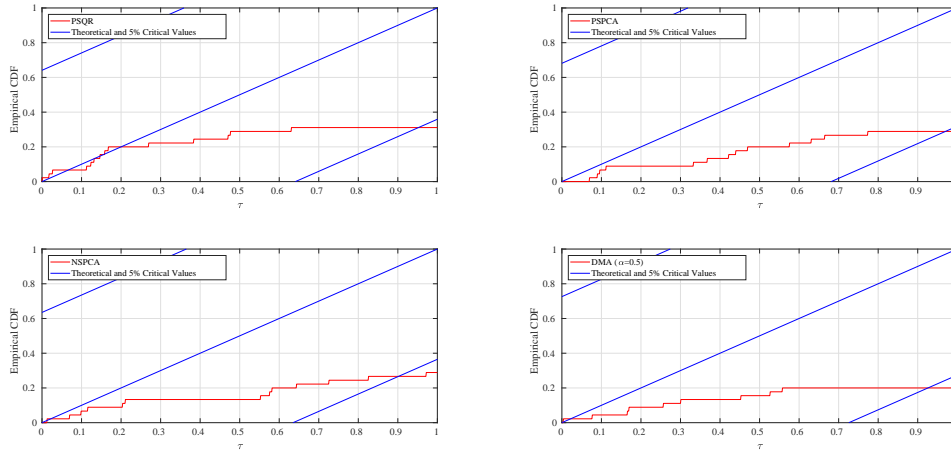
Notes: The red line describes the empirical CDF from pseudo out-of-sample forecasts and lines represent the theoretical and 5% critical values for the test of the correct specification of the conditional predictive density in Rossi and Sekhposyan (2019).

Figure 16: OUT-OF-SAMPLE TAIL RISKS FOR THE ONE-YEAR FORWARD UNEMPLOYMENT RATE



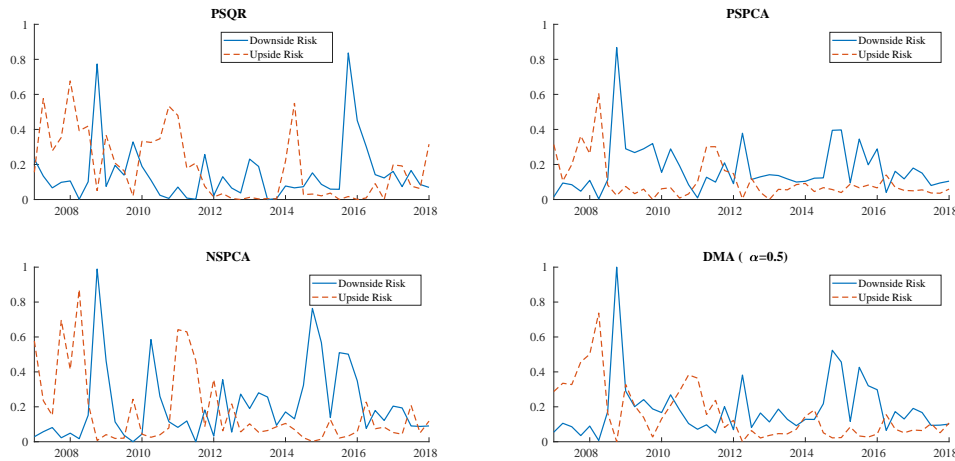
Notes: Downside risk is the probability that the one-year forward unemployment rate is below 4.017 percent. The upside risk is defined as the probability that the one-year forward quarterly PCE inflation rate is above 9.283 percent.

Figure 17: SPECIFICATION TEST FOR THE OUT-OF-SAMPLE PREDICTIVE DENSITY: UNEMPLOYMENT RATE



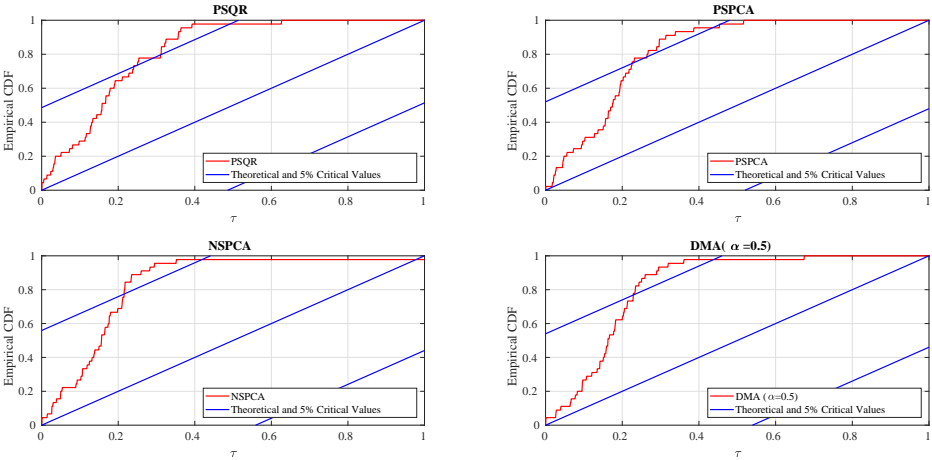
Notes: The red line describes the empirical CDF from pseudo out-of-sample forecasts and lines represent the theoretical and 5% critical values for the test of the correct specification of the conditional predictive density in Rossi and Sekhposyan (2019).

Figure 18: OUT-OF-SAMPLE TAIL RISKS FOR THE ONE-YEAR FORWARD PCE INFLATION RATE



Notes: Downside risk is the probability that the one-year forward quarterly PCE inflation rate is below 0 percent. The upside risk is defined as the probability that the one-year forward quarterly PCE inflation rate is above 4 percent.

Figure 19: SPECIFICATION TEST FOR THE OUT-OF-SAMPLE PREDICTIVE DENSITY:PCE INFLATION RATE



Notes: The red line describes the empirical CDF from pseudo out-of-sample forecasts and lines represent the theoretical and 5% critical values for the test of the correct specification of the conditional predictive density in Rossi and Sekhposyan (2019).