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Communicating Monetary Policy Rules*

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Abstract

Sixty-two countries around the world use some form of inflation targeting as their monetary policy framework, though none of these countries express explicit policy rules. In contrast, models of monetary policy typically assume policy is set through a rule such as a Taylor rule or optimal monetary policy formulation. Central banks often connect theory with their practice by publishing inflation forecasts that can, in principle, implicitly convey their reaction function. We return to this central idea to show how a central bank can achieve the gains of a rule-based policy without publicly stating a specific rule. The approach requires central banks to specify an inflation target, tolerance bands, and provide economic projections. Thus, when inflation moves outside the band, inflation forecasts provide a time frame over which inflation will return to within the band. We show how this approach replicates and provides the same information as a rule-based policy.

Keywords: monetary policy, inflation targeting, Taylor rule, communication

JEL Codes: E10, E52, E58, E61

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1 Introduction

Since its advent more than 25 years ago, inflation targeting has become the dominant paradigm for how central banks conduct monetary policy. It would be natural to assume, given experience and the ubiquity of inflation targeting across the globe, questions about how central banks communicate their frameworks and monetary policy decisions would largely be resolved. However, given that no central bank mechanically follows an instrument-based monetary policy rule, such as a Taylor rule, central banks have adopted a variety of approaches in how they communicate their strategy. Still, inflation targeting central banks generally seek to follow a systematic monetary policy guided by some underlying rule or optimal monetary policy framework, but are reluctant about turning policy over to a simple rule that may not be appropriate in all circumstances. Consequently, the challenge is how to convey a monetary policy strategy that is systematic and accountable for achieving some stated objectives, such as an inflation target or full employment, without having to specify a specific interest rate rule.

In this paper, we develop a clear link between rules-based policy and communication. Three ingredients are essential to an effective communication strategy and if combined properly, can replicate the information provided by a policy rule. Specifically, we show that specifying a point inflation target, coupled with tolerance bands and economic projections, provide the same information as if a specific policy rule was revealed to the public.

To illustrate the connection, we use two simple models - a Fisherian model of inflation and a New Keynesian model. Within these frameworks, a tolerance band, inflation target and inflation forecast convey the underlying reaction function. For example, in states when inflation is outside of the band, a wide band combined with a forecast showing a slow return of inflation to a rate inside the band signals an underlying policy rule with a weak response to inflation. A tight band with a forecast showing a rapid return of inflation to within the band conveys a rule with a stronger response to inflation.¹ Importantly, the reaction function responds to inflation in a uniform way whether inflation is inside or outside the band. That is, there is no ‘kink’ in the underlying rule if inflation deviates outside of the band. Instead, the reason for specifying a band, along with an inflation forecast, is to convey the underlying policy rule.²

Alternatively, one straightforward approach to communication would be for a central bank

¹Central banks are well aware of these trade-offs. For example, Banco Central Do Brasil (2016) notes that the band cannot be too wide, since it could signal a lack of commitment to the inflation target, which implies an instrument rule that responds weakly when inflation deviates from target.

²See Smets (2000) for some of the issues regarding the appropriate time frame for returning inflation to target, as well as Orphanides and Weiland (2000)

to simply specify and follow the prescriptions from a policy rule. Indeed, a pervasive approach to modeling monetary policy is to assume the central bank specifies an inflation target that it achieves by following an instrument-based policy rule or optimal monetary policy formulation.³ While widely accepted in modeling monetary policy, in practice no central bank strictly follows a rule. A common rationale for this omission is that mechanical adherence to a rule may misguide practical policy decisions due to the multitude of shocks that commonly impinge on actual economies.⁴ Some central banks have addressed this gap between theory and practice by issuing monetary policy reports that include inflation, output, and interest rate projections. Such projections can implicitly convey the central bank’s reaction function, which is one of the key points of the inflation-forecast targeting literature as developed by, for example, Svensson (1999), Svensson (2002), Svensson and Woodford (2005) and Woodford (2005).

We build on the insights from inflation-forecast targeting, but emphasize the importance of a tolerance band. Failure to specify a band may leave the public with a perception that hitting the target is always far into the future or conveys precision that is not achievable in practice. Even in standard NK models after a shock, a central bank following a standard Taylor rule hits its inflation point objective only asymptotically, implying that inflation is missing its target with near certainty at all points in time. The failure to hit an exact target can lead to some central banks persistently facing questions about their strategy and how to assess whether they are achieving their objectives.⁵ Thus, one of the beneficial by-products of our approach is that specifying tolerance bands and a horizon for moving back into the band, as indicated by forecasts, provides a clear performance metric with which to assess whether a central bank is meeting its objectives.⁶

The remainder of the paper proceeds as follows. In Section 2 we review inflation targeting and communication strategies around the world, focusing on whether countries use tolerance bands and inflation forecasts. In Section 3 we present a simple Fisherian model of inflation determination, and show how a tolerance band and forecasts for inflation can be used to communicate rules. Section 4 extends the results developed in the Fisherian model to a New Keynesian model of inflation and output, and Section 5 concludes.

³Rule-based policy has been shown to have a number of advantages in terms of stabilizing economic activity and inflation, for example in closely approximating a fully optimal policy framework in a relatively simple and transparent manner (Schmitt-Grohe and Uribe, 2007).

⁴For example, setting a single rule could generate inferior outcomes in the presence of structural change in the economy (Choi and Foerster, 2016) or if the rule is improperly specified given the economic environment (Ikeda and Kurozumi, 2014).

⁵In the U.S., for example, Cochrane and Taylor (2016) and Warsh (2017).

⁶Walsh (2015) discusses the importance of such metrics and how they can affect monetary policy actions.

2 Inflation Targeting Around the World

In this section, we review the conduct of inflation targeting around the world. First, we highlight that while a large number of countries have targets for inflation, they differ in whether they have tolerance bands or not, as well as if they produce explicit inflation forecasts. Second, we discuss the experience of the Sveriges Riksbank with inflation tolerance bands as an illustrative example of the issues surrounding tolerance bands.

2.1 Inflation Targeting: Tolerance Bands and Inflation Forecasts

Table 1 lists the 62 countries with some form of an explicitly stated inflation target. Among these, only 22 have single point targets, although these include some major economies such as the U.K. and U.S. The remaining countries have some sort of tolerance band: 27 specify a band around a specific midpoint (e.g. Canada's target of $2\% \pm 1\%$), 11 specify a band without an explicit midpoint (e.g. Australia's target of $2\% - 3\%$), while 2 have one-sided bands with an inflation target that acts as an upper bound (e.g. Switzerland and the Euro area have targets of $< 2\%$).

The table therefore highlights disagreement about how to set inflation targets across the world, and prompts some concerns about how specifying point targets in addition to, or instead of, tolerance bands helps the performance and communication of monetary policy. In the cases without a band, there may be difficulty achieving a level of inflation that exactly hits the target, which might translate to difficulty communicating an implicit policy rule. In the cases of a band without a midpoint, the center of the band may be the implicit midpoint, but is still a gap in providing the information set needed to replicate a rules-based policy. In addition, for those countries that set a band of some sort, the widths of the bands varies across central banks, and this variation may signal something about the implicit policy rule. Lastly, these inflation targets and tolerance bands convey objectives to be hit, but without reference to a time frame. However, economic projections about inflation or other objectives provide this information.

Table 1 lists the set of countries that provide explicit inflation forecasts, and how they relate to countries that set tolerance bands around their inflation targets. Of the 62 countries that provide inflation targets, 25 provide inflation bands and forecasts, while 15 use only forecasts and just a point target, 15 use a band with no forecasts, and 7 use only point targets with no forecasts. The wide range of practices highlights the disparity about how to communicate inflation objectives. In Sections 3 and 4, we develop a framework where effective communication can take the place of explicitly declaring a monetary policy rule. The keys to our framework

Table 1: List of Inflation Targets Across Countries

F-cast?	No Band		Band w/ Midpoint		Band w/o Midpoint		Upper Bound	
No	Bangladesh	6.2%	Albania	3%+/-1%	Argentina*	12%-17%	Euro	<2%
	Belarus	12%	Armenia	4%+/-1.5%	Azerbaijan	5%-6%	Switz.	<2%
	China	4%	Colombia	3%+/-1%	Israel	1%-3%		
	Kyrgyzstan	7%	Costa Rica	4%+/-1%	Sri Lanka	3%-5%		
	Pakistan*	6%	Domin. Rep.	4%+/-1%	Uruguay	3%-7%		
	Vietnam	7%	Guatemala	4%+/-1%				
	Zambia	7%	Indonesia	4%+/-1%				
			Kenya	5%+/-2.5%				
		Paraguay	4.5%+/-2%					
Yes	Georgia*	5%	Brazil	4.5%+/-2%	Australia	2%-3%		
	Iceland	2.5%	Canada	2%+/-1%	Botswana	3%-6%		
	India	8%	Chile	3%+/-1%	Jamaica	5.5%-7.5%		
	Japan	2%	Czech Rep.	2%+/-1%	Kazak.*	6%-8%		
	Malawi*	14.2%	Ghana	8%+/-2%	Nigeria	6%-9%		
	Mongolia	7%	Hungary	3%+/-1%	S. Africa	3%-6%		
	Mozambique	5.6%	Mexico	3%+/-1%				
	Norway	2.5%	Moldova	5%+/-1.5%				
	Russia	4%	New Zealand	2%+/-1%				
	Samoa	3%	Peru	2%+/-1%				
	S. Korea*	2%	Philippines	3%+/-1%				
	Sweden	2%	Poland	2.5%+/-1%				
	Ukraine*	12%	Romania	2.5%+/-1%				
	U.K.	2%	Serbia	4%+/-1.5%				
	U.S.	2%	Thailand	2.5%+/-1.5%				
			Turkey	5%+/-2%				
			Uganda	5%+/-2%				
			W African S	2% +/-1%				

Note: List of inflation targets is from Central Bank News (2017). Countries marked with an asterisk (*) have varying targets over time.

are a point target for inflation, a band around that objective, and forecasts. As a result, the countries listed in as having “Bands with Midpoints” and “Yes” forecasts in Table 1 come closest to following the communication strategy that replicates a rule-based policy.

Despite the fact that the Sweden does not appear on our list of countries that provide both forecasts and tolerance bands, the Sveriges Riksbank has changed how it communicates policy several times, and these changes illustrate some of the issues with such bands and forecasts. So before presenting our theoretical Fisherian and New Keynesian models, we first examine the

Riksbank's experience in detail.

2.2 The Sveriges Riksbank's Experience with Tolerance Bands

The Riksbank is a useful example for understanding some of the issues central banks encounter regarding tolerance bands. In January 1993, a 2% inflation target was set that was expected to be achieved in 1995 and then remain in effect going forward. A tolerance band of $\pm 1\%$ was also set around this target. According to Heikensten (1999), the purpose of the band was to convey that deviations from the target are probable, but that the Riksbank had an intention of limiting such deviations. In May 2010, the tolerance bands were removed for a few reasons. First, as explained in Riksbank (2010), there was a view that the public had sufficient understanding that monetary policy persistently faces uncertainty and unexpected events will cause inflation to deviate from its target. Second, the Riksbank communicated that deviations from target can be part of a deliberate strategy under a flexible inflation targeting framework, which places weight on achieving other objectives than only hitting the inflation target. These deviations can, at times, exceed the tolerance interval. Third, inflation expectations were viewed as well anchored, so deviations from the target, or even outside the tolerance interval, were not seen as having a tangible effect on longer-term inflation expectations. Since 1995 to the time the tolerance band was eliminated, inflation was outside of the band about half of the time, but was not viewed as having any effect on the Riksbank credibility. In sum, the Riksbank viewed the tolerance band as “obsolete,” as deviations outside of the tolerance band were viewed as a “natural part of monetary policy.” Dropping the bands were also viewed as having no consequences for the “way in which monetary policy is conducted and communicated.”

Riksbank (2016) revisits some of the costs and benefits of specifying a tolerance band. In terms of benefits, a band provides a signal that some variation around the target should be expected, though monetary policy will aim to limit such deviations. This is the same rationale supporting the original specification of the tolerance band in 1993. One question is whether detailed inflation forecasts, as given in the regular Monetary Policy Reports, provide these same benefits. In addition, however, Sveriges Riksbank (2016) notes a tolerance band may provide a “clearer alternative” of illustrating uncertainty around inflation, so could complement the inflation forecasts. In addition, a band may aid in deflecting public criticism about the level of inflation, as long as it was running within the interval. If so, then a band may support the credibility around monetary policy and support anchoring of longer-term inflation expectations. The size of the band, in practice, is also important, not only from the standpoint of supporting

central bank credibility, but also from a communication perspective, as we illustrate in the next section.

3 A Fisherian Model of Inflation

In this section, we present a simple Fisherian model of inflation and monetary policy in order to highlight how central bank communication about bands and horizons can pin down a rule.

3.1 Model Setup and Basic Results

The model is a simple Fisherian model of inflation determination. The log-linearized equation for pricing a bond that costs \$1 at time t and pays out at a net interest rate of i_t in $t + 1$ is

$$i_t = \mathbb{E}_t [\pi_{t+1}] + r_t, \tag{1}$$

where $\mathbb{E}_t [\pi_{t+1}]$ denotes the time t expectation of inflation in the subsequent period, and r_t denotes the equilibrium ex ante real interest rate. This real interest rate is taken to be an exogenous process given by

$$r_t = \rho r_{t-1} + \varepsilon_t \tag{2}$$

with $0 \leq \rho < 1$ and ε_t is iid with mean zero. Monetary policy follows a simple Taylor rule given by

$$i_t = \alpha \pi_t, \tag{3}$$

where α governs how responsive the nominal interest rate is to inflation. Note that, by log-linearizing around the appropriate steady state, we have already required a point inflation target for monetary policy.

In this simple setup, a monetary authority could communicate its policy rule by stating a value of α that would guide interest rate policy in response to shocks to the real interest rate that subsequently altered inflation. However, the discussion in Section 2 highlights that most central banks prefer language about hitting an inflation target or moving inflation within some band, possibly within a specified time frame. In this case, effective communication will uniquely pin down α , but vague communication will not.

The unique solution to the Fisherian economy in equations (1), (2), and (3), given the Taylor

principle ($\alpha \geq 1$) holds, is

$$\pi_t = \frac{1}{\alpha - \rho} r_t, \quad (4)$$

which relates realized inflation to the current real interest rate r_t , the monetary policy rule parameter α , and features of the structural economy, which in this simple example are captured by the persistence of the real rate ρ . Given that the structural shocks ε_t are iid, expected inflation is given by

$$\mathbb{E}_t [\pi_{t+j}] = \mathbb{E}_t \left[\frac{r_{t+j}}{\alpha - \rho} \right] = \frac{\rho^j}{\alpha - \rho} r_t. \quad (5)$$

Using this equation that characterizes the path of expected inflation given the current real interest rate, the monetary authority can give guidance about the policy parameter α by communicating how fast policy will bring inflation back to target. One way to accomplish this guidance is to give the entire expected path of inflation $\{\mathbb{E}_t [\pi_{t+j}]\}_{j=0}^{\infty}$. Communicating the path provides more than enough information for households and firms to back out the policy parameter α .

An additional way of communicating policy instead of producing an entire path for expected inflation is to give a specific horizon and inflation objective. If, given a current value for the real rate r_t , the central bank states that it expects “inflation will be μ from the inflation target in N_t periods,” this statement implies

$$\mu = \mathbb{E}_t [\pi_{t+N_t}] = \frac{\rho^{N_t}}{\alpha - \rho} r_t. \quad (6)$$

In this case, the choice of tolerance μ and horizon N_t are not necessarily pinned down as there is a continuum of (μ, N_t) that are implied by a policy parameter α . Two important results, however, key on how to use this communication strategy and why it works to reveal the rule.

First, the inflation target cannot be hit with precision in finite time, so $\mu = 0$ is impossible, as this implies $N_t \rightarrow \infty$ for all values of α . Since the inflation target is a point target, communication about returning inflation to target in any time frame is infeasible. In other words, specifying a degree of tolerance μ around the inflation target is imperative.

Second, the reason a statement about a tolerance μ and a horizon N_t is effective in communicating the policy rule is that equation (6) is invertible. Mathematically, after a shock r_t and given μ and N_t , the private sector can recover the policy parameter through

$$\alpha = \frac{\rho^{N_t} r_t}{\mu} + \rho. \quad (7)$$

Further, the expected path of inflation in equation (5) can be put in terms of the tolerance μ and the horizon N_t , which now satisfies

$$\mathbb{E}_t [\pi_{t+j}] = \rho^{j-N_t} \mu, \quad (8)$$

and hence the communication uniquely pins down the expected path for inflation. In this simple Fisherian model, directly communicating the policy parameter is equivalent to communicating the tolerance μ and a horizon N_t due to the unique mapping, but in more complex environments such as the New Keynesian model of Section 4, invertibility of communicated targets might be a problem.

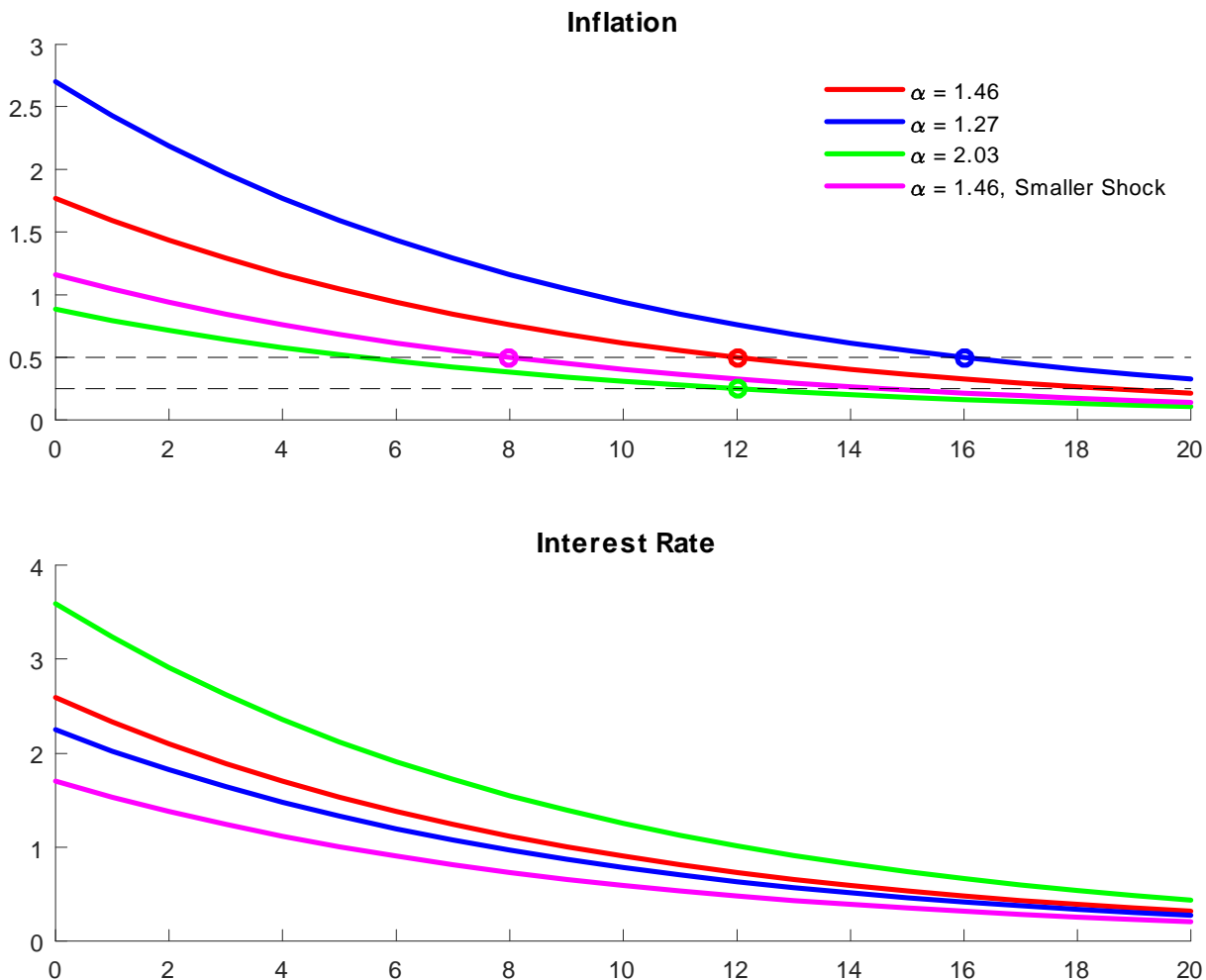
3.2 Examples

The mapping in equation (6) has intuitive implications for how changes in the policy parameter α produce different tolerances μ and horizons N_t . In particular, a higher α implies either a shorter horizon N_t in order to hit a given tolerance μ , or a lower tolerance μ associated with a given horizon N_t . Likewise, the inverse mapping in equation (6) also has straightforward implications for how changes in stated tolerances μ and horizons N_t affect the implied policy parameter. Given a specified tolerance band μ , then a desire to hit that band in a shorter time (lower N_t) necessarily implies a higher value of the policy parameter α . Likewise, given a specific horizon N_t , then a desire to hit a smaller tolerance band (smaller μ) requires a higher value of α as well.

Figure 1 shows how, given a calibrated value of $\rho = 0.9$, for different policy parameters α , the impulse response function of inflation to a real rate shock can be used to pin down the band μ and the horizon N_t . For example, after a 1pp shock to the real rate ($r_t = 1$), a policy parameter of $\alpha = 1.46$ can be communicated through a range of (μ, N_t) combinations given by the red line, but if the monetary authority has an inflation tolerance of $\mu = 0.5$, then they can communicate the rule by saying “inflation will be $\mu = 0.5$ from the inflation target in $N_t = 12$ periods.” Similarly, the central bank with a policy rule of $\alpha = 1.27$ could state “inflation will be $\mu = 0.5$ from the inflation target in $N_t = 16$ periods,” while one with $\alpha = 2.03$ and a lower tolerance μ could state “inflation will be $\mu = 0.25$ from the inflation target in $N_t = 12$ periods.”

Using tolerance bands and horizons also provides flexibility to vary communication after shocks of various sizes. As noted, Figure 1 depicts a band of $\mu = 0.5$ and horizon $N_t = 12$ can be used to convey a policy parameter of $\alpha = 1.46$ after a 1pp shock to the real rate. If instead, there is a 0.66pp shock, then the same rule can be communicated by “inflation will be $\mu = 0.5$

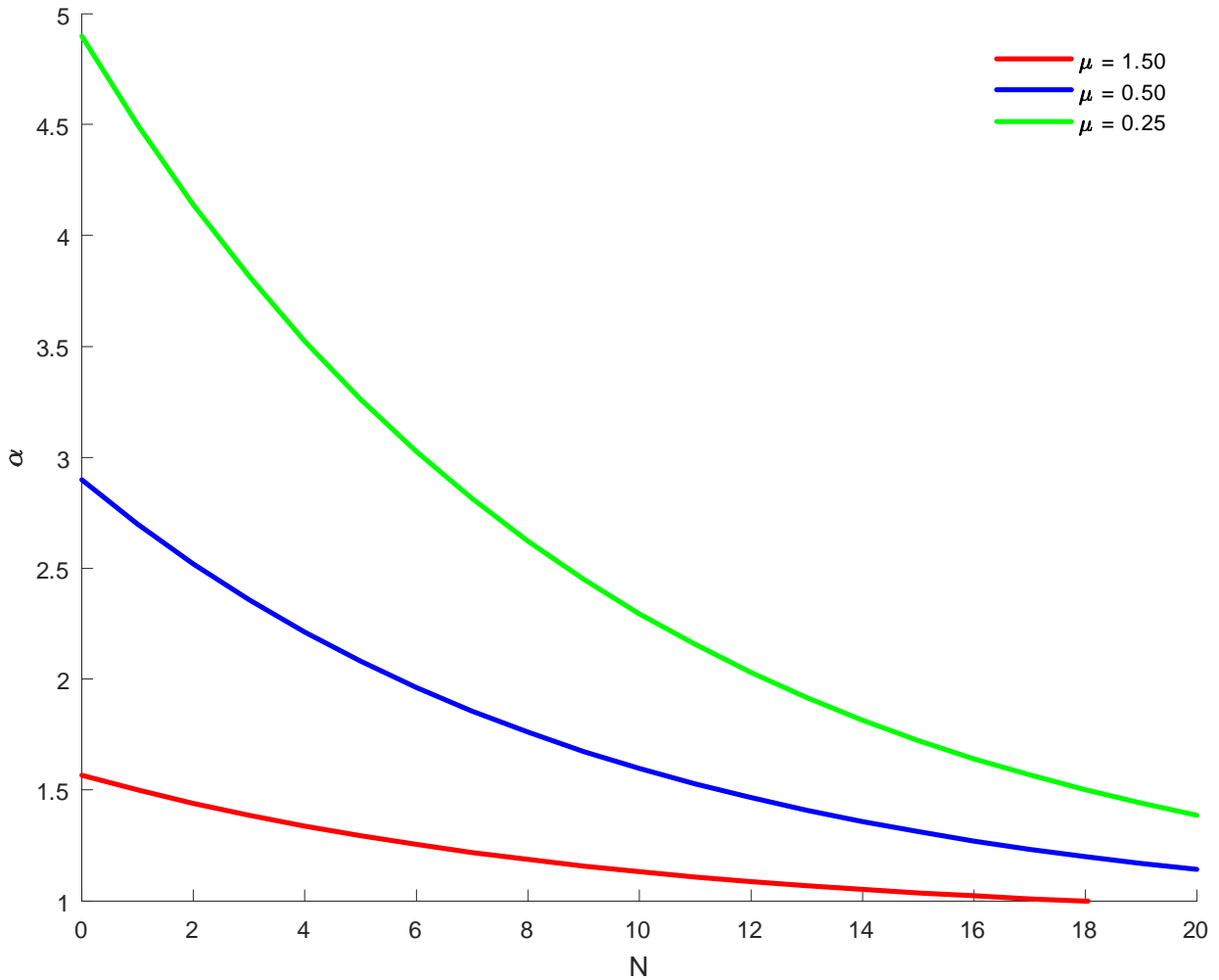
Figure 1: Impulse Responses in the Fisherian Model



from the inflation target in $N_t = 8$ periods.” Thus, smaller shocks end up meeting the tolerance band in shorter time frames, while larger shocks extend the horizon.

The implications of Figure 1 for communication also highlight that given (μ, N_t) , the communication can pin down the policy parameter α . Figure 2 builds upon this calibrated example to show the inverse mapping in equation (7) by showing how a tolerance band μ and horizon N_t imply a unique policy parameter α . The curves show, given (μ, N_t) , the implied α that achieves those objectives. The mapping is unique, and given μ a larger N_t implies a lower α as monetary policy does not have to react as strongly to meet the horizon objective. Similarly, given a horizon N_t , if the band is smaller, meaning μ is lower, then α must be larger in order to bring inflation within the band in the given time frame. Importantly, given a band μ , there are values of N_t that imply $\alpha < 1$, which generates indeterminacy; for example if $\mu = 1.5$ then the

Figure 2: Given (μ, N) , the Policy Parameter α is Pinned Down in the Fisher Model



maximum N_t is 18 periods, and if the monetary authority communicates a longer horizon the implied equilibrium is non-unique.

This simple Fisherian example thus highlights how a clearly articulated band for inflation and a horizon can be used in place of specifying an exact policy rule. Given these simple results, we now turn to a discussion of several implications for communication.

3.3 Implications for Communication

There are several important implications for how the inverse mapping between (μ, N_t) and α shown in equation (7) matters for communication of policy. These results highlight how vague communication or being imprecise about the objectives of (μ, N_t) can lead to an rule with

undetermined parameters.

First, if the communication is vague by stating “inflation will be *close* to the inflation target in N_t periods,” rather than specifying an exact tolerance, a range of values for α are possible. In this case, if *close* is interpreted as possibly a range of tolerances $\mu \in [\underline{\mu}, \bar{\mu}]$, then the range of possible policy parameters is given by

$$\alpha \in \left[\frac{\rho^{N_t} r_t}{\bar{\mu}} + \rho, \frac{\rho^{N_t} r_t}{\underline{\mu}} + \rho \right]. \quad (9)$$

For example, when $\rho = 0.9$, if $N_t = 12$, then Figure 1 shows that if $\mu \in [0.25, 0.50]$, then $\alpha \in [1.46, 2.03]$.

Second, as a range of tolerances produces a range of possible policy parameters, so does a range of horizons. Even if a band around the target is specified, if a range of horizons is vague by stating “inflation will be μ of the inflation target over the *medium term*” and the phrase *medium term* is interpreted as a range of horizons $N_t \in [\underline{N}, \bar{N}]$, then the range of possible policy parameters is

$$\alpha \in \left[\frac{\rho^{\bar{N}_t} r_t}{\mu} + \rho, \frac{\rho^{\underline{N}_t} r_t}{\mu} + \rho \right] \quad (10)$$

For example, when $\rho = 0.9$, if $\mu = 0.50$, then the top panel of Figure 1 shows that if $N_t \in [12, 16]$, then $\alpha \in [1.27, 1.46]$.

Third, given a fixed band μ and horizon N_t , the mapping to policy parameters α is still dependent on ρ . As a result, if there is structural change in the economy—incorporated by possibly a different value of ρ in this economy—then the mapping from (μ, N_t) to α will change as well. From this perspective, given a constant statement about the band and horizon, the policy parameter must adjust given any changes in ρ .

This simple Fisherian model has therefore provided some clear implications for communicating monetary policy rules. However, the model lacks features that play an important role in the real world, namely the joint determination of inflation and output, and how the monetary authority reacts to each of these. The next section therefore uses a New Keynesian model to further develop how communication about bands and horizons can be used to convey a policy rule.

4 A New Keynesian Model of Inflation and Output

In this section, we extend the intuition built in the previous section for a model in which inflation and output are jointly determined, and show a similar mapping from bands and horizons to a policy parameter exists, but also document in which cases it does not. We start with a simple forward looking model, and discuss communication in cases with only demand and supply shocks before turning to the case with both shocks; we then extend the simple model to one in which interest rates are set inertially.

4.1 Model Setup

The model is a simple New Keynesian model where inflation and output are jointly determined by supply and demand shocks. The two equations describing the private economy are the log-linearized equations derived from a consumption Euler equation

$$x_t = \mathbb{E}_t [x_{t+1}] - \sigma^{-1} (i_t - \mathbb{E}_t [\pi_{t+1}]) + g_t, \quad (11)$$

and an aggregate supply condition

$$\pi_t = \beta \mathbb{E}_t [\pi_{t+1}] + \kappa x_t + u_t, \quad (12)$$

where x_t denotes the output gap, g_t is an aggregate demand shock, u_t is an aggregate supply shock. The shocks follow autoregressive processes given by

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}, \quad (13)$$

and

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \quad (14)$$

with, for $j = g, u$, $0 \leq \rho_j < 1$, and $\varepsilon_{j,t}$ is iid with mean zero. Monetary policy is given by a Taylor rule of the form

$$i_t = \alpha \pi_t + \gamma x_t. \quad (15)$$

Again, by log-linearizing around the appropriate steady state, we have required a point inflation target for the monetary authority.

As in the Fisherian example, a monetary authority could communicate its policy rule by stating values of α and γ , but as noted in Section 2, this communication strategy is not embraced

by central banks around the world. Instead, they tend to prefer giving bands around the inflation target as well as forecasts.

Following the same logic as used in the Fisherian economy, provided the Taylor principle ($\alpha \geq 1$) holds, a unique solution is given by

$$\pi_t = \frac{\kappa}{\Phi_g(\alpha, \gamma)} g_t + \frac{\sigma^{-1}\gamma + 1 - \rho_u}{\Phi_u(\alpha, \gamma)} u_t, \quad (16)$$

for inflation, and

$$x_t = \frac{1 - \beta\rho_g}{\Phi_g(\alpha, \gamma)} g_t - \frac{\sigma^{-1}(\alpha - \rho_u)}{\Phi_u(\alpha, \gamma)} u_t \quad (17)$$

for output, where

$$\Phi_g(\alpha, \gamma) = 1 + \sigma^{-1}(\alpha\kappa + \gamma) - \rho_g(1 + \sigma^{-1}(\kappa + \beta\gamma) + \beta(1 - \rho_g)), \quad (18)$$

and

$$\Phi_u(\alpha, \gamma) = 1 + \sigma^{-1}(\alpha\kappa + \gamma) - \rho_u(1 + \sigma^{-1}(\kappa + \beta\gamma) + \beta(1 - \rho_u)). \quad (19)$$

In this solution, we have used notation to explicitly stress that $\Phi_g(\alpha, \gamma)$ and $\Phi_u(\alpha, \gamma)$ are functions of the policy parameters α and γ .

Given that the structural shocks $\varepsilon_{g,t}$, and $\varepsilon_{u,t}$ are iid, expected inflation is given by

$$\mathbb{E}_t[\pi_{t+j}] = \frac{\kappa}{\Phi_g(\alpha, \gamma)} \rho_g^j g_t + \frac{\sigma^{-1}\gamma + 1 - \rho_u}{\Phi_u(\alpha, \gamma)} \rho_u^j u_t \quad (20)$$

and the expected output gap equals

$$\mathbb{E}_t[x_{t+j}] = \frac{1 - \beta\rho_g}{\Phi_g(\alpha, \gamma)} \rho_g^j g_t - \frac{\sigma^{-1}(\alpha - \rho_u)}{\Phi_u(\alpha, \gamma)} \rho_u^j u_t. \quad (21)$$

Note that, similar to equation (5), that these two equations are functions of the structural parameters governing the economy, as well as the policy parameters α and γ .

While we will consider analytic derivations in the results below where possible, a set of baseline parameters presented in Table 2 will be used for numerical results as well. We first consider the environment with only demand shocks, and then only supply shocks, before returning to the case with both types of shocks.⁷

⁷For simplicity, throughout this section we focus on the cases with positive shocks that raise inflation.

Table 2: Baseline Parameters in the New Keynesian Model

Parameter	Description	Value
β	Discount Factor	0.99
σ^{-1}	Intertemporal Elasticity of Substitution	1
κ	Slope of the Phillips Curve	0.17
ρ_g	Persistence of Demand Shock	0.90
ρ_u	Persistence of Supply Shock	0.85

4.2 Demand Shocks Only

We first consider a monetary authority that only faces demand shocks, so $u_t = 0$ for all t . As in the Fisherian model, the monetary authority can communicate that, given a current value for the demand shock g_t , that it expects “inflation will be μ_π from the inflation target in $N_{\pi,t}$ periods,” and from equation (20) this communication satisfies

$$\mu_\pi = \mathbb{E}_t [\pi_{t+N_{\pi,t}}] = \frac{\kappa}{\Phi_g(\alpha, \gamma)} \rho_g^{N_{\pi,t}} g_t. \quad (22)$$

Conditional on policy rule parameters α and γ , there are a continuum of choices for guidance on the tolerance μ_π and the horizon $N_{\pi,t}$. The inflation target cannot be hit in finite time, so $\mu_\pi = 0$ remains impossible in this case. However, in contrast with the Fisherian model, the communication of a tolerance μ_π and horizon $N_{\pi,t}$ is not invertible in the sense that the mapping for α is a linear function of γ given by

$$\alpha = \frac{\kappa \left(\mu_\pi \rho_g + \sigma \rho_g^{N_{\pi,t}} g_t \right) - \mu_\pi (1 - \rho_g) (1 - \beta \rho_g) \sigma}{\kappa \mu_\pi} - \frac{(1 - \beta \rho_g)}{\kappa} \gamma. \quad (23)$$

In the case of demand shocks only, the fact that under active policy ($\alpha \geq 1$) inflation and the output gap move in the same direction means no guidance can pin down the exact policy coefficients α and γ , but since these policy parameters are effectively substitutes, there is no issue with pinning down dynamics. To see this fact, suppose the monetary authority announces “inflation will be μ_π from the inflation target in $N_{\pi,t}$ periods,” which implies that α and γ must satisfy the restriction in equation (23); using the restriction with the definition of $\Phi_g(\alpha, \gamma)$ in this case, the expected paths for inflation, the output gap, and interest rates are

$$\mathbb{E}_t [\pi_{t+j}] = \rho_g^{j-N_{\pi,t}} \mu_\pi, \quad (24)$$

$$\mathbb{E}_t [x_{t+j}] = \frac{1 - \beta\rho_g}{\kappa} \rho_g^{j-N_{\pi,t}} \mu_{\pi}, \quad (25)$$

and

$$\mathbb{E}_t [i_{t+j}] = \left(\frac{\kappa\mu_x\rho_g + \sigma\rho_g^{N_{x,t}} g_t - \mu_x (1 - \rho_g) (1 - \beta\rho_g) \sigma}{\kappa\mu_x} \right) \rho_g^{j-N_{\pi,t}} \mu_{\pi}, \quad (26)$$

respectively. The key result in these equations is that providing communication about inflation directly pins down the expected paths for inflation, the output gap, and the interest rate since they comove in direct fashion, and hence how the economy evolves does not depend on the exact specification of α or γ .

Figure 3 provides a numerical example based upon the calibration in Table 2. In this case, if there is a 1pp demand shock ($g_t = 1$) and the monetary authority expresses its rule by stating “inflation will be $\mu_{\pi} = 0.25$ from the inflation target in $N_{\pi,t} = 16$ periods,” this communication produces a restriction on α and γ given by $\alpha = 1.5731 - 0.68125\gamma$. Given a rule that satisfies this restriction, the impulse responses for inflation, the output gap, and interest rates are all identical.

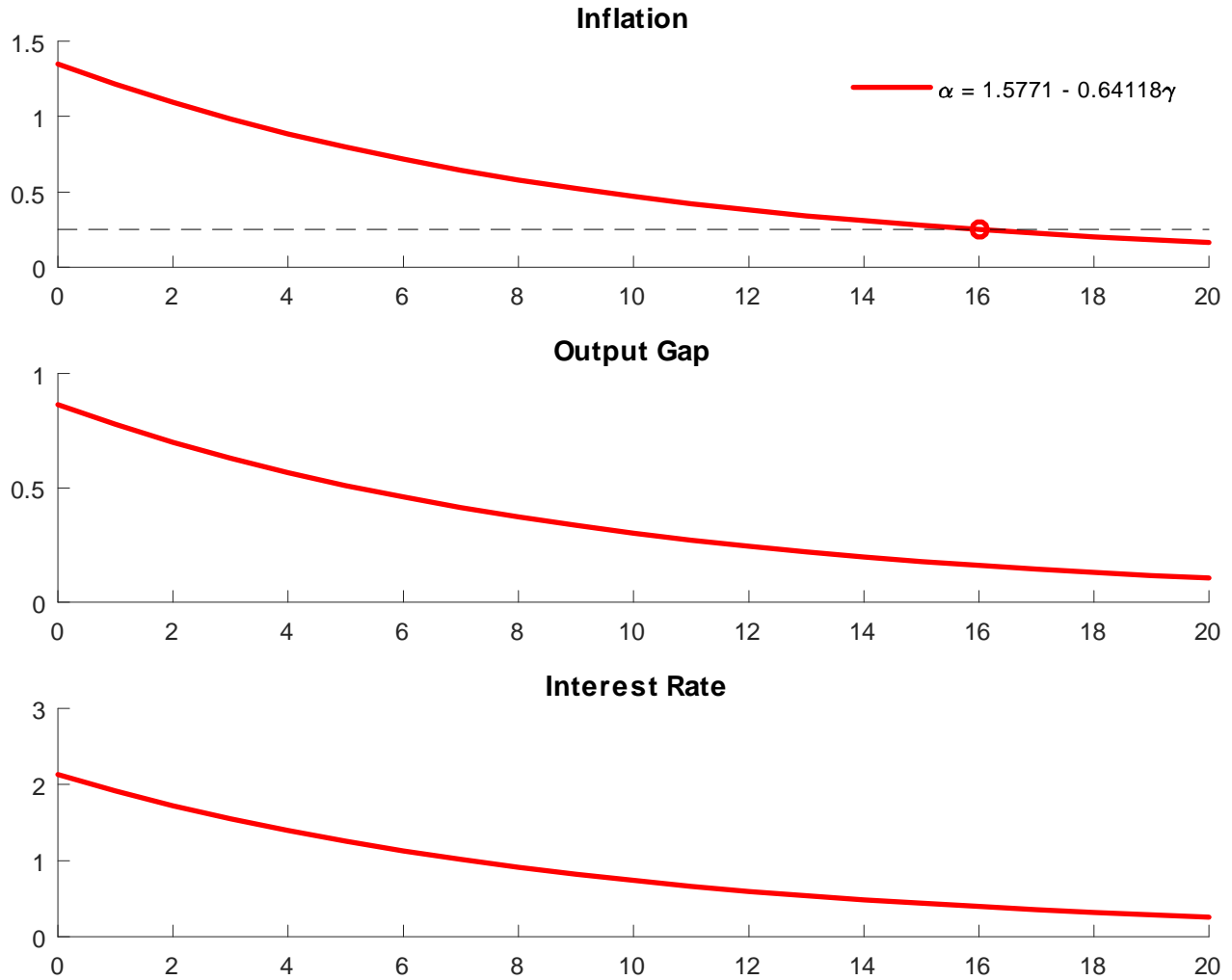
Since giving communication about inflation implies a set of rules under which dynamics for inflation, the output gap, and the interest rate are all identical, any further communication ends up being irrelevant. Suppose the monetary authority gives guidance about the output gap, specifically that “the size of the output gap will be μ_x in $N_{x,t}$ periods,” where from equation (21)

$$\mu_x = \mathbb{E}_t [x_{t+N_{x,t}}] = \frac{1 - \beta\rho_g}{\Phi_g(\alpha, \gamma)} \rho_g^{N_{x,t}} g_t. \quad (27)$$

However, as noted, given communication about inflation, then the path of the output gap is already pinned down by μ_{π} and $N_{\pi,t}$, and hence any communication about the output gap is redundant.

Returning to considering only communication about inflation, even though equation (23) shows a non-unique mapping from the band μ_{π} and horizon $N_{\pi,t}$ to the policy parameters α and γ , changes in the communication produce intuitive changes in policy. Figure 4 shows the set of (α, γ) given different values of μ_{π} and $N_{\pi,t}$. The sets of parameters that correspond to a given communication is a downward sloping line, as increases in γ go along with decreases in α , since demand shocks move inflation and the output gap in the same direction and hence reacting more strongly to one means reacting less strongly to the other in order to hit the same band and horizon. The top panel shows that, for $\mu_{\pi} = 0.25$, increasing the horizon implies lower values of (α, γ) , while the bottom panel shows that given $N_{\pi,t} = 16$, increasing the size of the

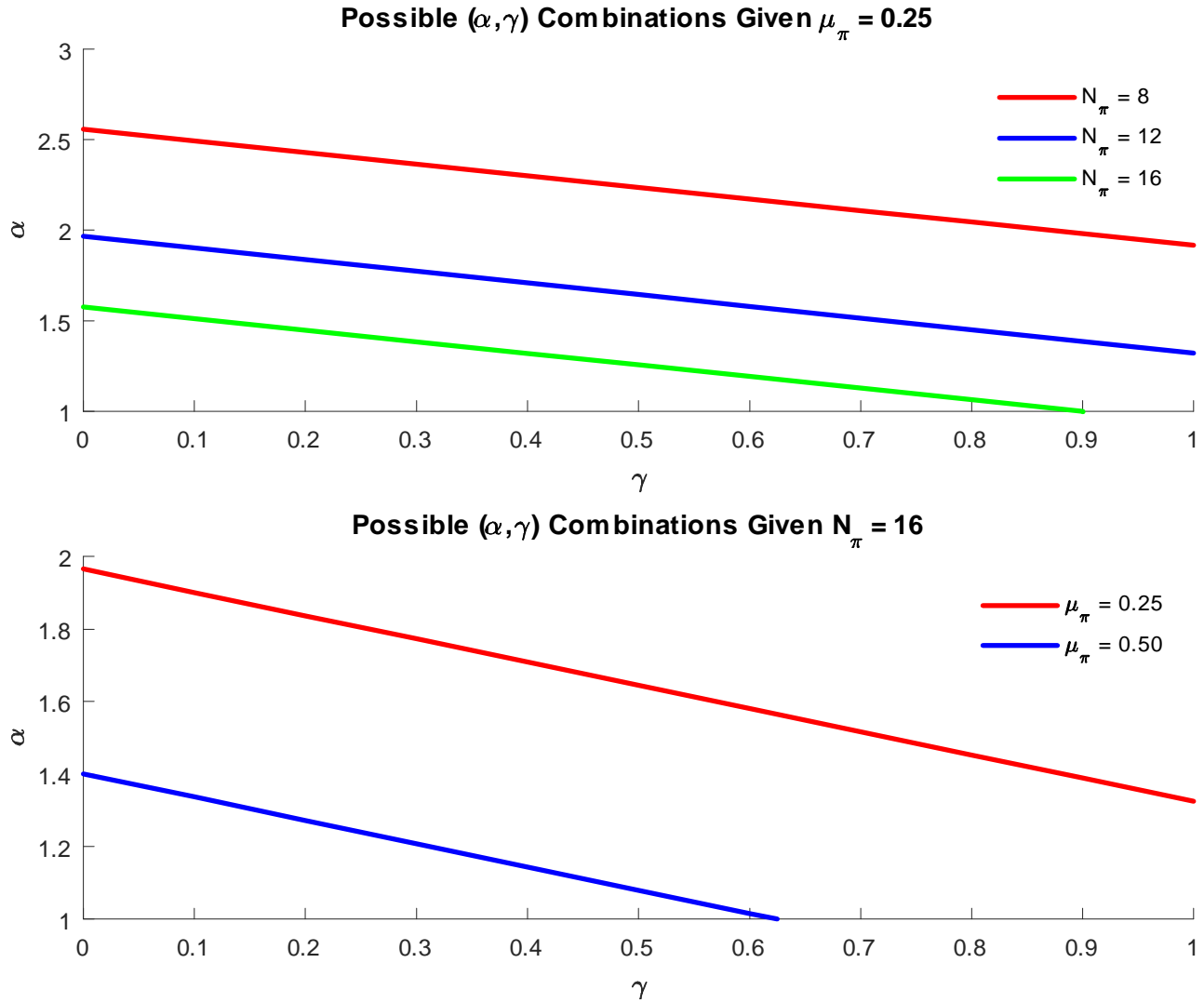
Figure 3: Impulse Responses to a Demand Shock in the NK Model



band implies lower values as well. These results mimic the Fisherian results shown in Figure 2, as longer horizons or wider bands are associated with less reaction to inflation, or in this New Keynesian demand shock case, less reaction to the output gap. In addition, these results show that too long a horizon given a band width or too wide a band given a horizon can imply an indeterminate equilibrium with $\alpha < 1$.

There are three implications from these results on demand shocks only. First, a monetary authority in an economy that faces only demand shocks will never need to specify an exact value for α and γ , because if it produces μ_π and $N_{\pi,t}$ it generates a class of rules that produce identical paths for inflation and the output gap. Second, if the monetary authority is a strict inflation targeter with $\gamma = 0$ so that it does not react to the output gap, then inflation communication

Figure 4: Given (μ_π, N_π) , the Policy Parameters (α, γ) are not Pinned Down in the NK Model after Demand Shocks



uniquely pins down the rule. Third, in this context any communication about the output gap in the form of μ_x and $N_{x,t}$ is redundant, since communication about μ_π and $N_{\pi,t}$ pins down the path for both inflation and the output gap. Of course, the opposite is true as well, implying the monetary authority only needs to provide one of $(\mu_\pi, N_{\pi,t})$ or $(\mu_x, N_{x,t})$ to describe its rule.

4.3 Supply Shocks Only

Now we consider a monetary authority that faces only supply shocks, so $g_t = 0$ for all t . In this case, the monetary authority could communicate that after a supply shock of u_t , “inflation will

be μ_π from the inflation target in $N_{\pi,t}$ periods,” and from equation (20) this communication satisfies

$$\mu_\pi = \mathbb{E}_t [\pi_{t+N_{\pi,t}}] = \frac{\sigma^{-1}\gamma + 1 - \rho_u}{\Phi_u(\alpha, \gamma)} \rho_u^{N_{\pi,t}} u_t. \quad (28)$$

Once again, there are a continuum of choices for guidance on the tolerance μ_π and the horizon $N_{\pi,t}$ conditional on policy rule parameters α and γ , although $\mu_\pi = 0$ is not feasible. As in the case with demand shocks only, the communication of a tolerance μ_π and horizon $N_{\pi,t}$ is not invertible in the sense that the mapping for α is a linear function of γ given by

$$\alpha = \frac{\kappa\mu_\pi\rho_u - (1 - \rho_u) \left(\mu_\pi (1 - \beta\rho_u) - \rho_u^{N_{\pi,t}} u_t \right) \sigma}{\kappa\mu_\pi} + \frac{\rho_u^{N_{\pi,t}} u_t - \mu_\pi (1 - \beta\rho_u)}{\kappa\mu_\pi} \gamma. \quad (29)$$

Similar to the demand shock only case, communication about inflation is sufficient to pin down the expected paths of inflation and the output gap, and the exact policy parameters α and γ do not matter for dynamics. Again, suppose the monetary authority announces “inflation will be μ_π from the inflation target in $N_{\pi,t}$ periods,” which implies that α and γ must satisfy the restriction in equation (29); using the restriction with the definition of $\Phi_u(\alpha, \gamma)$ in this case, the expected paths for inflation, the output gap, and the interest rate are

$$\mathbb{E}_t [\pi_{t+j}] = \rho_u^{j-N_{\pi,t}} \mu_\pi, \quad (30)$$

$$\mathbb{E}_t [x_{t+j}] = - \frac{\rho_u^{j-N_{\pi,t}} \left(\rho_u^{N_{\pi,t}} u_t - \mu_\pi (1 - \beta\rho_u) \right)}{\kappa}, \quad (31)$$

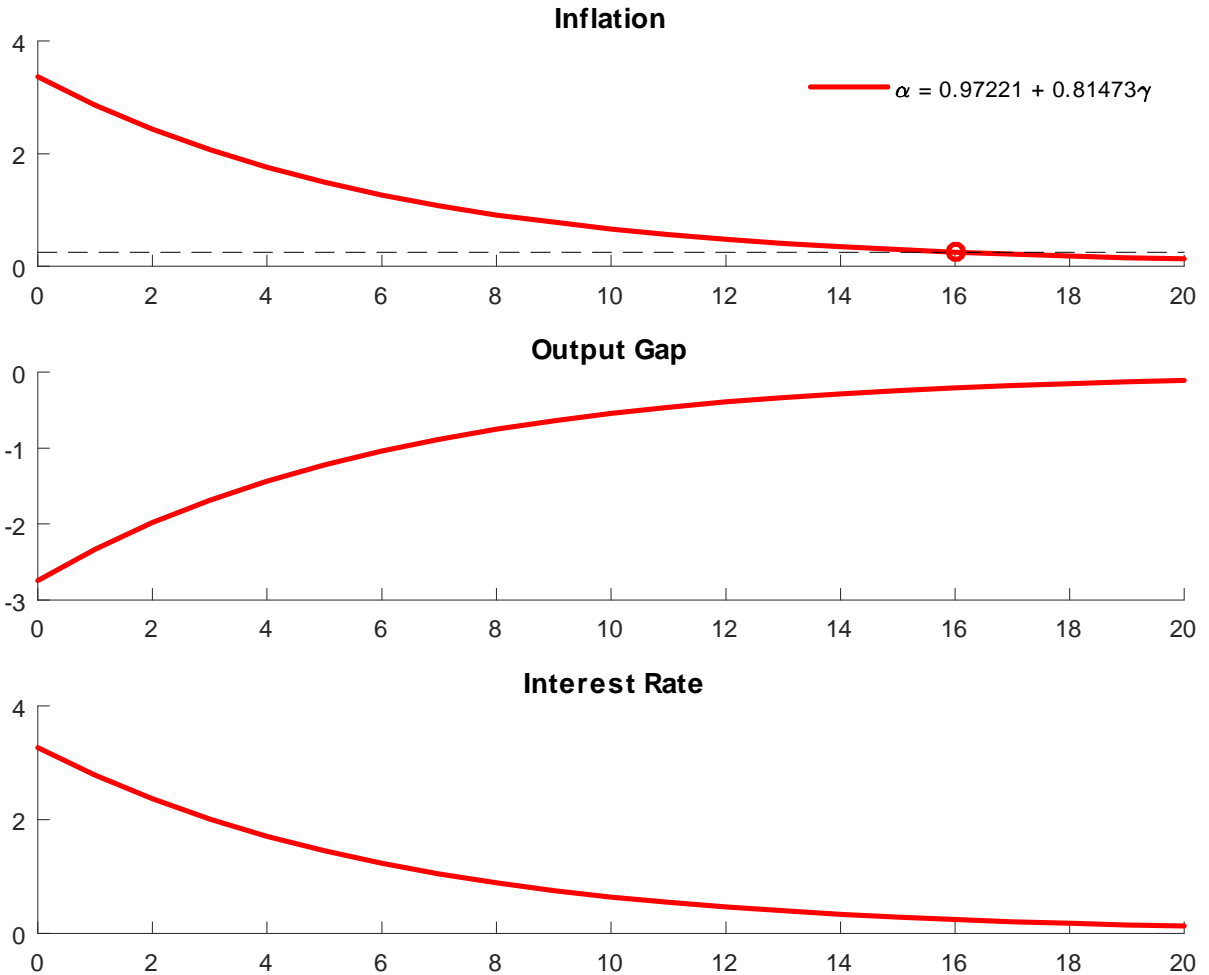
and

$$\mathbb{E}_t [i_{t+j}] = \frac{\kappa\mu_\pi\rho_u - (1 - \rho_u) \left(\mu_\pi (1 - \beta\rho_u) - \rho_u^{N_{\pi,t}} u_t \right) \sigma}{\kappa} \rho_u^{j-N_{\pi,t}}, \quad (32)$$

respectively. The key result from these equations is the independence from α and γ , which highlights that providing communication about inflation directly pins down the expected paths for inflation, the output gap, and the interest rate without the need to specify the exact policy rule.

Figure 5 provides a numerical example based upon the calibration in Table 2. In this case, if there is a 1pp supply shock ($u_t = 1$) and the monetary authority expresses its rule by stating “inflation will be $\mu_\pi = 0.25$ from the inflation target in $N_{\pi,t} = 16$ periods,” this communication produces a restriction on α and γ given by $\alpha = 0.97221 + 0.81473\gamma$. Given a rule that satisfies this restriction, the impulse responses for inflation, the output gap, and interest rates are all

Figure 5: Impulse Responses to a Supply Shock in the NK Model



identical.

Mirroring the case with demand shocks only, in the presence of only supply shocks any additional communication about the output gap ends up being redundant. If the monetary authority states “the size of the output gap will be μ_x in $N_{x,t}$ periods,” where from equation (21)

$$\mu_x = \mathbb{E}_t [x_{t+N_{x,t}}] = \left| -\frac{\sigma^{-1}(\alpha - \rho_u)}{\Phi_u(\alpha, \gamma)} \rho_u^{N_{x,t}} u_t \right|, \quad (33)$$

this restriction adds no information to the inflation communication, as the path of the output gap is pinned down by a statement of μ_π and $N_{\pi,t}$.

Returning to considering only communication about inflation, Figure 6 shows the set of (α, γ) given different values of μ_π and $N_{\pi,t}$, and how these change with different communication. As

opposed to the demand shock only case show in Figure 4, the set of (α, γ) that achieve a given communication is an upward sloping line under supply shocks; this feature is due to the fact that supply shocks move inflation and output in opposite directions and hence increasing the responsiveness to inflation requires an offsetting stronger response to the output gap in order to hit the same communicated band and horizon. At the same time, the differences in these lines mirrors those in the demand shock case; increasing the horizon with a fixed band size implies lower values of (α, γ) as in the top panel, while increasing the size of the band with a fixed horizon implies lower values as well as in the bottom panel. Communication can imply an indeterminate equilibrium with $\alpha < 1$ when the horizon is too long given a band or if the band is too wide given a horizon.

The implications for communication from supply shocks only are similar to those with demand shocks only. A monetary authority facing only supply shocks does not need to specify α and γ exactly, as stating μ_π and $N_{\pi,t}$ generates a class of rules with identical paths for inflation and the output gap, but a strict inflation targeter with $\gamma = 0$ has its rule pinned down by inflation communication. In addition, communication about the output gap μ_x and $N_{x,t}$ ends up being redundant, since inflation communication pins down the dynamics. With these results in mind, we now turn to the case with both demand and supply shocks, which generate the need for communication about the output gap.

4.4 Demand and Supply Shocks

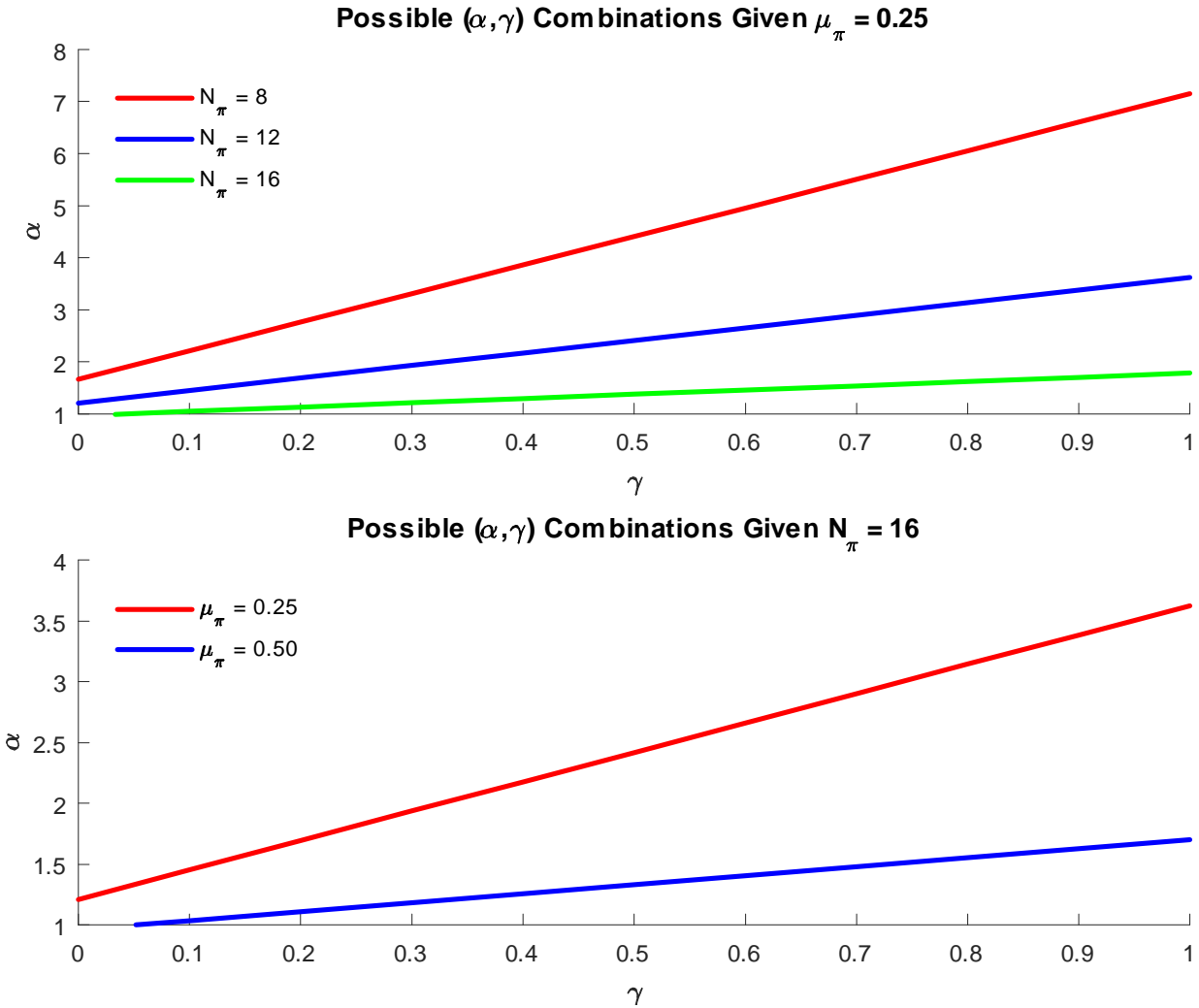
We now return to the case when with both supply and demand shocks. In this case, guidance that “inflation will be μ_π from the inflation target in $N_{\pi,t}$ periods,” requires

$$\mu_\pi = \mathbb{E}_t [\pi_{t+N_{\pi,t}}] = \frac{\kappa}{\Phi_g(\alpha, \gamma)} \rho_g^{N_{\pi,t}} g_t + \frac{\sigma^{-1}\gamma + 1 - \rho_u}{\Phi_u(\alpha, \gamma)} \rho_u^{N_{\pi,t}} u_t, \quad (34)$$

given current demand and supply shocks. As in the cases already considered, a continuum of choices for communicating a tolerance μ_π and horizon $N_{\pi,t}$ conditional on policy rule parameters α and γ exist, although $\mu_\pi = 0$ is not feasible. As in the case with demand shocks only, the communication of a tolerance μ_π and horizon $N_{\pi,t}$ is not invertible. However, rather than being a linear function of γ as under supply or demand shocks only, the mapping is instead a complicated nonlinear function of γ .⁸

⁸At this point, general analytic expressions, while feasible, become too cumbersome to provide any insight, so we focus solely on our numerical example.

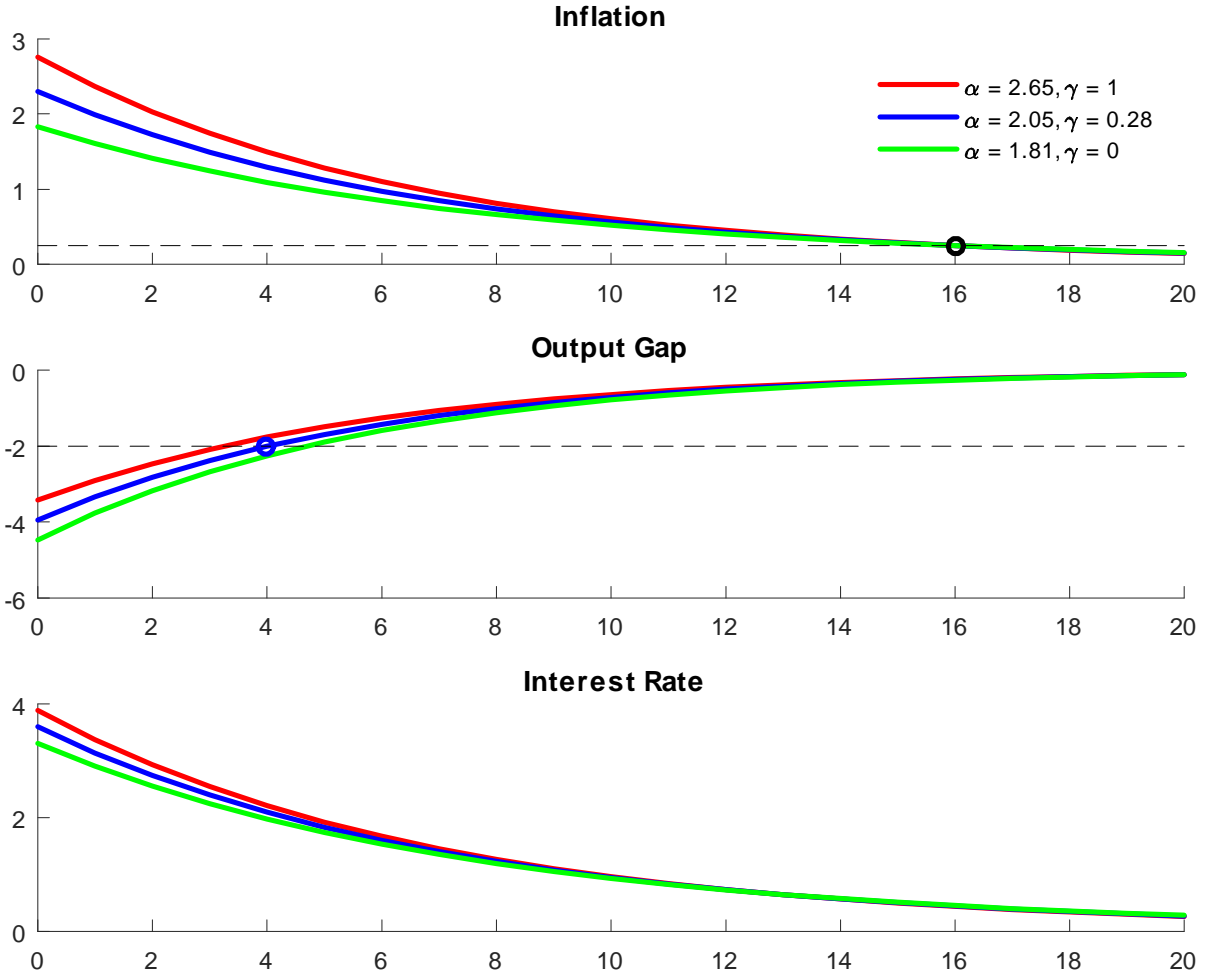
Figure 6: Given (μ_π, N_π) , the Policy Parameters (α, γ) are not Pinned Down in the NK Model after Supply Shocks



Unlike the cases with only demand or supply shocks, the dynamics associated with the set of policy coefficients implied by communication of μ_π and $N_{\pi,t}$ vary by rule. Figure 7 shows three sets of impulse responses associated with 1pp supply and demand shocks; each response varies depending on the exact rule, but as the top panel shows, all of them are consistent with communication about inflation returning to $\mu_\pi = 0.25$ in $N_{\pi,t} = 16$ periods. As a result, communication only about inflation is will neither pin down the exact rule nor lead to unique dynamics.

In the case with only supply or demand shocks, output communication was redundant, as communication about inflation pinned down equilibrium dynamics. In the case with both types

Figure 7: Impulse Responses to a Supply and Demand Shock in the NK Model



of shocks, however, communication about output can be used to help pin down the rule. If, in addition to communication about inflation, the monetary authority gives communication about the output gap, then the rule can be pinned down. If the monetary authority states “the size of the output gap will be μ_x in $N_{x,t}$ periods,” where equation (21) implies

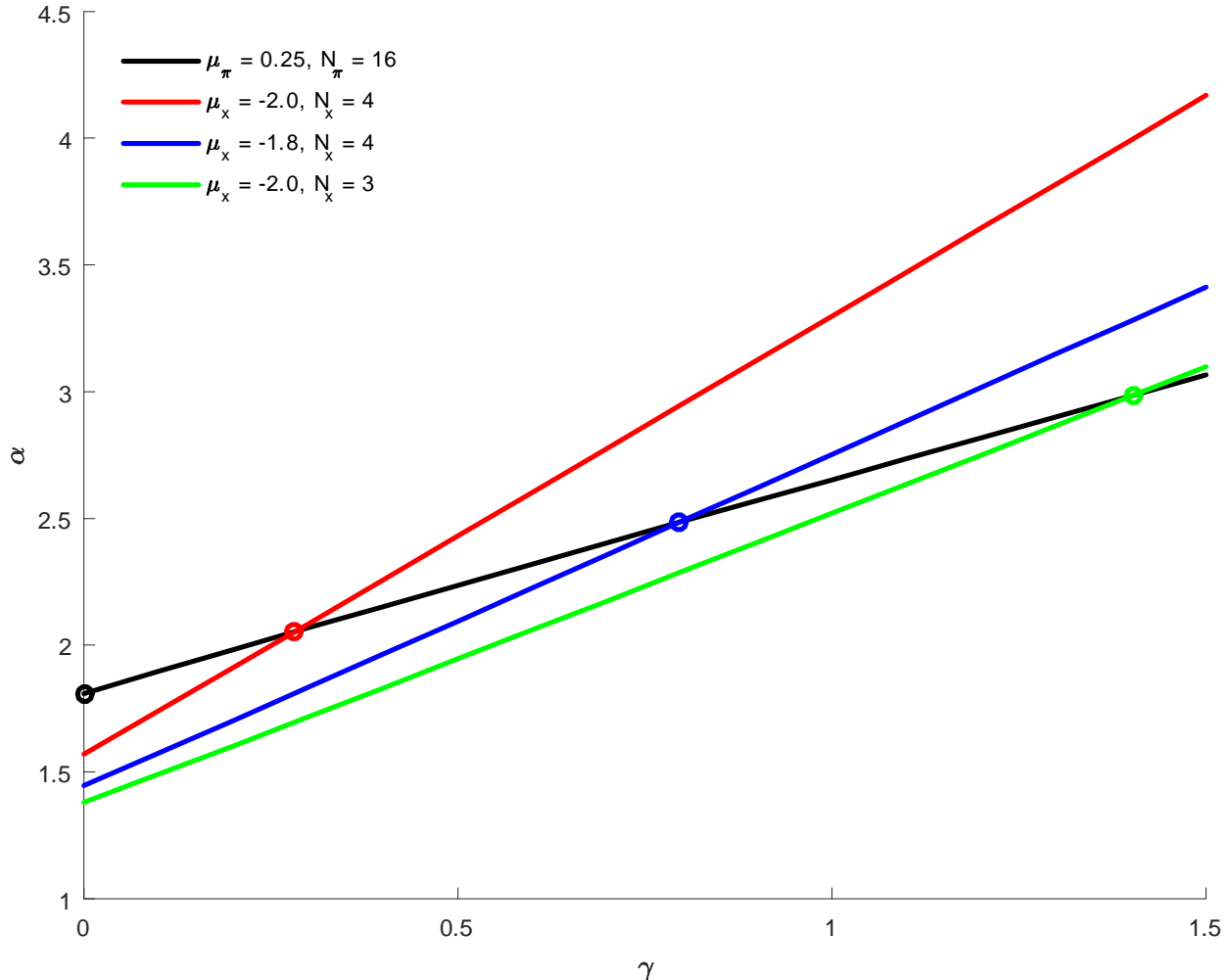
$$\mu_x = \left| \mathbb{E}_t [x_{t+N_{x,t}}] \right| = \left| \frac{1 - \beta\rho_g}{\Phi_g(\alpha, \gamma)} \rho_g^{N_{x,t}} g_t - \frac{\sigma^{-1}(\alpha - \rho_u)}{\Phi_u(\alpha, \gamma)} \rho_u^{N_{x,t}} u_t \right|, \quad (35)$$

and the absolute value shows that the output gap may be negative.

Figure 8 shows mappings from the two types of communication to the parameter space of α and γ .⁹ If the monetary authority communicates $\mu_\pi = 0.25$ and $N_{\pi,t}$, the set of policy parameters

⁹We restrict consideration to rules that have $\alpha \geq 1$ and $\gamma \geq 0$, which imply unique equilibria conditional on

Figure 8: Communication about (μ_π, N_π) and (μ_x, N_x) Pins Down the Policy Parameters (α, γ) in the NK Model



implied is given by the black line. Communication about $\mu_x = -2$ and $N_{x,t} = 4$ likewise leads to a set of parameters given by the red line. The intersection of these two lines is the value of α and γ that satisfies both sets of communication, and hence is the rule that is exactly pinned down. In this case, the values are $\alpha = 2.05$ and $\gamma = 0.28$, and Figure 7 shows the impulse responses under this parameterization satisfy the stated targets.

Figure 8 also illustrates how changing the communication about the output gap, given a statement about inflation, changes the implied parameters. A smaller output gap band of $\mu_x = -1.8$ in the blue line or a shorter horizon of $N_{x,t} = 3$ in the green line both imply stronger reactions to the output gap, which therefore implies a larger γ ; at the same time, α must increase a rule and a non-negative response to the output gap.

to continue to satisfy the inflation guidance, as seen in the discussion of supply shocks.

As alluded to in the discussion on demand and supply shocks only, an additional way to uniquely pin down the rule using only inflation communication is for monetary policy to be a strict inflation targeter with $\gamma = 0$. In this case, the restriction on γ effectively replaces the communication of the output gap, and Figure 8 shows that communication that “inflation will be $\mu_\pi = 0.25$ from the inflation target in $N_{\pi,t} = 16$ periods,” conveys a unique value $\alpha = 1.81$, while Figure 7 shows the unique path of inflation, the output gap, and the interest rate in this case.

The implications for the model with both types of shocks differ from the implications when just considering supply or demand shocks in isolation. In the presence of both shocks, communication about an inflation band and horizon neither pin down the exact rule nor pin down a class of rules with identical dynamics; instead, inflation communication is insufficient for the public to understand the rule. If a tolerance band and horizon are given for output as well, however, then the exact values of the policy parameter become revealed and unique dynamics of inflation and the output gap result. On the other hand, if the monetary authority is a strict inflation targeter that does not react to the output gap, then simple inflation communication pins down the exact rule and dynamics.

4.5 Interest Rate Smoothing

We lastly turn to a model where monetary policy is given by an inertial Taylor rule of the form

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\alpha \pi_t + \gamma x_t). \quad (36)$$

In this case, the existence of an endogenous predetermined variable i_{t-1} implies there is no longer a simple closed form solution for π_t and x_t , but instead there are solutions of the form

$$\begin{bmatrix} \pi_t \\ x_t \\ i_t \end{bmatrix} = \Gamma(\rho_i, \alpha, \gamma) \begin{bmatrix} i_{t-1} \\ g_t \\ u_t \end{bmatrix} = \begin{bmatrix} \Gamma_{\pi i} & \Gamma_{\pi g} & \Gamma_{\pi u} \\ \Gamma_{xi} & \Gamma_{xg} & \Gamma_{xu} \\ \Gamma_{ii} & \Gamma_{ig} & \Gamma_{iu} \end{bmatrix} \begin{bmatrix} i_{t-1} \\ g_t \\ u_t \end{bmatrix} \quad (37)$$

where $\Gamma(\rho_i, \alpha, \gamma)$ is a function of the parameters, and where we have used notation to explicitly stress that the solution depends on the policy parameters ρ_i , α , and γ . The expected paths of

inflation, the output gap, and the interest rate are given by

$$\mathbb{E}_t [\pi_{t+j}] = \Gamma_{\pi i} \left(\Gamma_{ii}^j i_{t-1} + \sum_{k=0}^{j-1} \Gamma_{ii}^k \Gamma_{ig} \rho_g^{j-1-k} g_t + \sum_{k=0}^{j-1} \Gamma_{ii}^k \Gamma_{iu} \rho_u^{j-k} u_t \right) + \Gamma_{\pi g} \rho_g^j g_t + \Gamma_{\pi u} \rho_u^j u_t, \quad (38)$$

$$\mathbb{E}_t [x_{t+j}] = \Gamma_{xi} \left(\Gamma_{ii}^j i_{t-1} + \sum_{k=0}^{j-1} \Gamma_{ii}^k \Gamma_{ig} \rho_g^{j-1-k} g_t + \sum_{k=0}^{j-1} \Gamma_{ii}^k \Gamma_{iu} \rho_u^{j-k} u_t \right) + \Gamma_{xg} \rho_g^j g_t + \Gamma_{xu} \rho_u^j u_t, \quad (39)$$

and

$$\mathbb{E}_t [i_{t+j}] = \Gamma_{ii}^{j+1} i_{t-1} + \sum_{k=0}^j \Gamma_{ii}^k \Gamma_{ig} \rho_g^{j-k} g_t + \sum_{k=0}^j \Gamma_{ii}^k \Gamma_{iu} \rho_u^{j-k} u_t, \quad (40)$$

respectively. As a result, communication that “inflation will be μ_π from the inflation target in $N_{\pi,t}$ periods,” requires

$$\mu_\pi = \mathbb{E}_t [\pi_{t+N_{\pi,t}}], \quad (41)$$

while “the size of the output gap will be μ_x in $N_{x,t}$ periods,” implies

$$\mu_x = \left| \mathbb{E}_t [x_{t+N_{x,t}}] \right|. \quad (42)$$

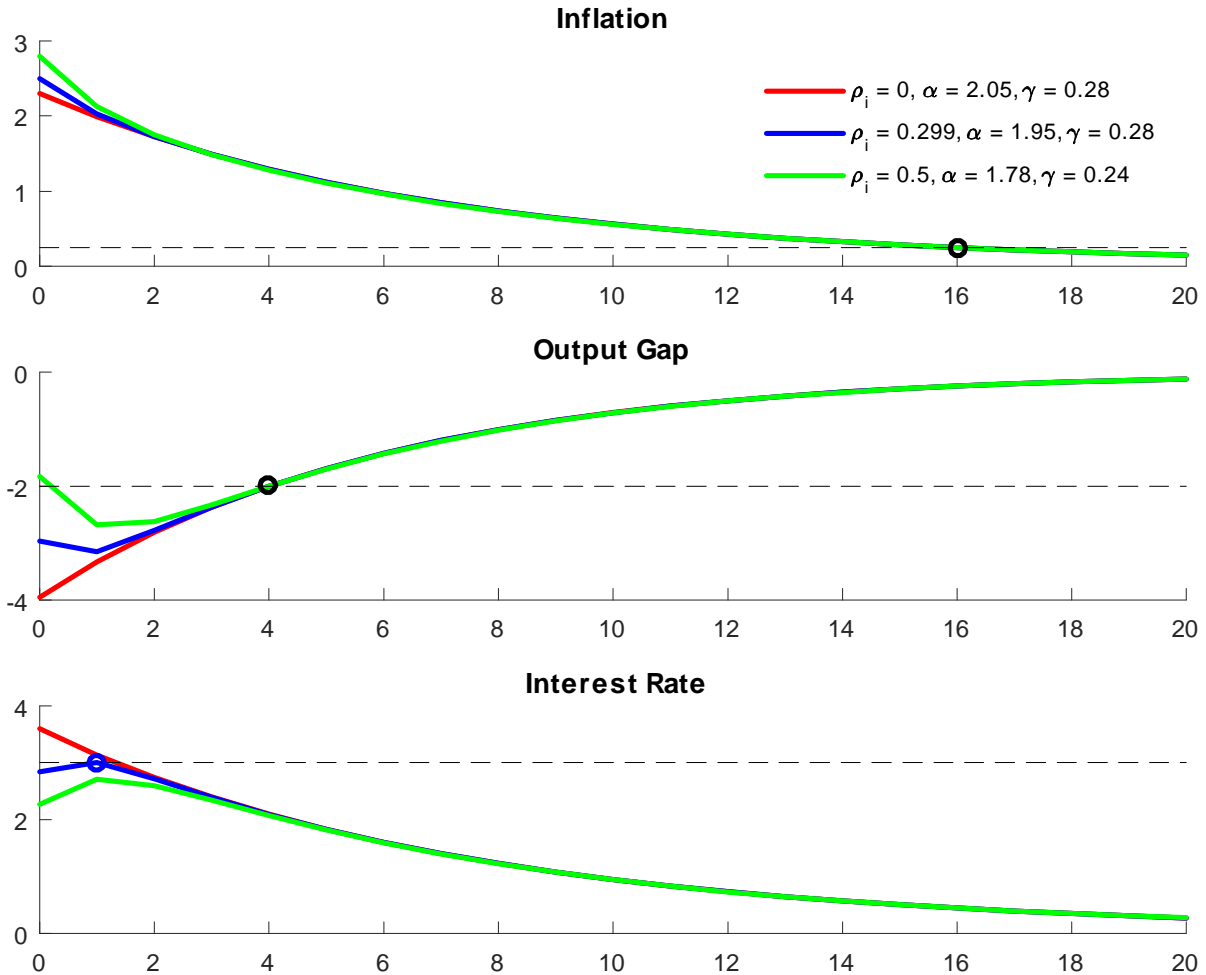
Building on the numerical example using the calibration in Table 2 and presented in Figure 7 for the case without interest rate inertia ($\rho_i = 0$), communication after a 1pp demand and supply shock that “inflation will be $\mu_\pi = 0.25$ from the inflation target in $N_{\pi,t} = 16$ periods, and the size of the output gap will be $\mu_x = 2$ in $N_{x,t} = 4$ periods” will pin down the parameters of the rule α and γ . However, as Figure 9 shows, when $\rho_i \geq 0$ this communication is not in fact enough to pin down the rule. In this case, there are infinitely many combinations of the full set of policy parameters (ρ_i, α, γ) that achieve $(\mu_\pi, N_{\pi,t}) = (0.25, 15)$ and $(\mu_x, N_{x,t}) = (2, 4)$.

Consequently, if in addition to communication about inflation and the output gap, the monetary authority gives communication about the interest rate, then the rule can be pinned down. If the monetary authority states “the level of the interest rate will be μ_i in $N_{i,t}$ periods,” then

$$\mu_i = \mathbb{E}_t [i_{t+N_{i,t}}], \quad (43)$$

and this statement generates an additional restriction on (ρ_i, α, γ) that fully pins down the rule. As shown in Figure 9, given the inflation and output gap communication, there are many different possible rules, with the three in particular being shown. If the monetary authority states “the level of the interest rate will be $\mu_i = 3$ in $N_{i,t} = 1$ periods,” then this statement pins

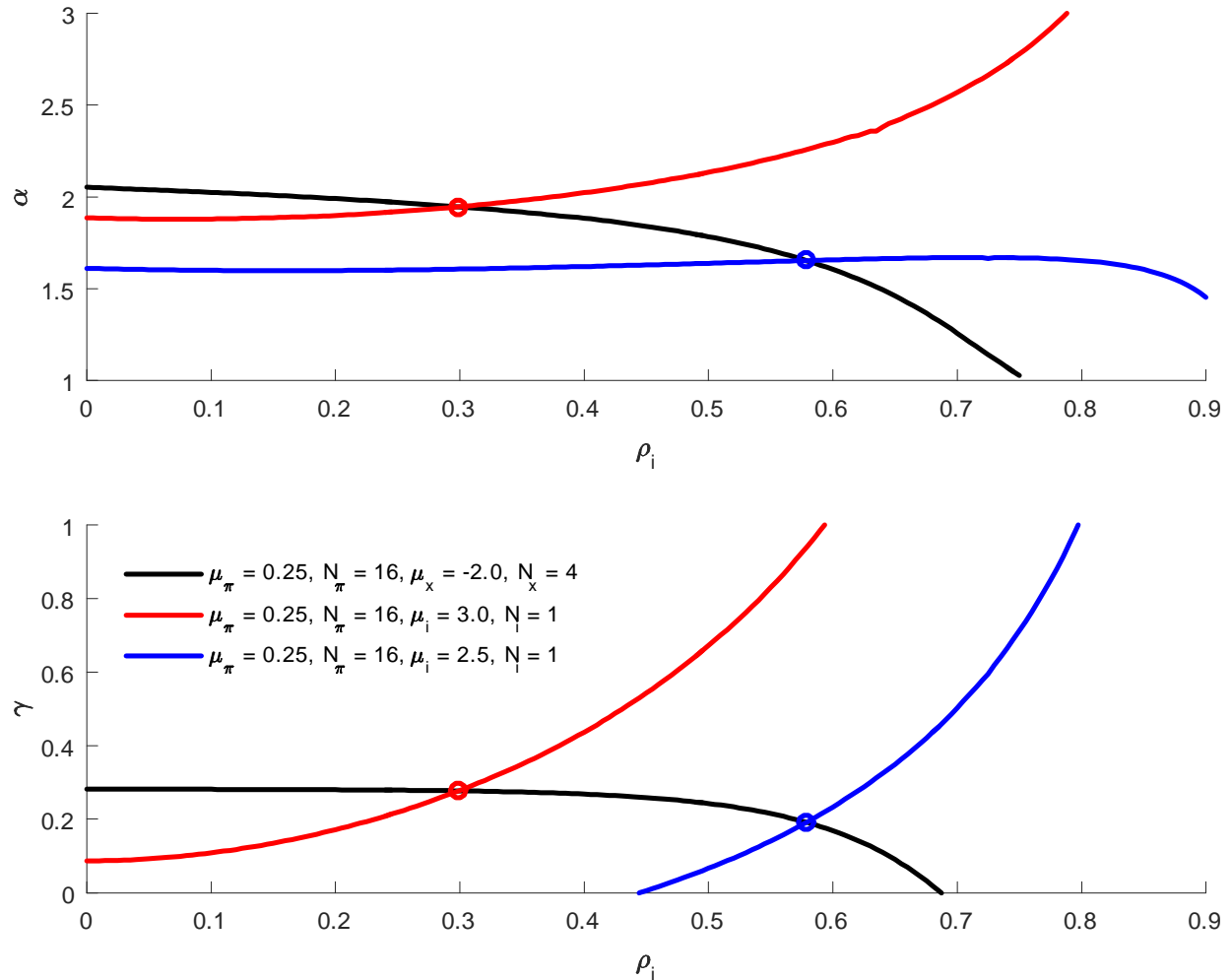
Figure 9: Impulse Responses to a Supply and Demand Shock in the NK Model with Smoothing



down the rule as $\rho_i = 0.299$, $\alpha = 1.96$, and $\gamma = 0.3$.

Figure 10 builds upon the results shown in Figure 9 by showing the restriction given by different communication. If the monetary authority states “inflation will be $\mu_\pi = 0.25$ from the inflation target in $N_{\pi,t} = 16$ periods, and the size of the output gap will be $\mu_x = -2$ in $N_{x,t} = 4$ periods,” this communication gives exact values of α and γ for each ρ_i , as shown by the black line; in particular, the values when $\rho_i = 0$ are those in the case considered above without interest rate inertia. Similarly, if the monetary authority replaces the output communication with “the level of the interest rate will be $\mu_i = 3$ in $N_{i,t} = 1$ periods,” then α and γ are pinned down for each value of ρ_i as shown by the red line. The intersection when $\rho_i = 0.299$, $\alpha = 1.96$, and $\gamma = 0.3$ is when the inflation, output gap, and interest rate communication are all met. If the interest rate communication is changed so that $\mu_i = 2.5$, a higher value of ρ_i and lower values

Figure 10: Communication about (μ_π, N_π) , (μ_x, N_x) , and (μ_i, N_i) Pins Down the Policy Parameters (α, γ) in the NK Model with Inertia



of α and γ meet the new set of targets.

It is worth noting that while in the version of the model without interest rate inertia, a strict inflation targeter with $\gamma = 0$ could uniquely pin down α using inflation guidance. With interest rate inertia, this result no longer holds, as communication only about μ_π and $N_{\pi,t}$ leads to a set of possible (ρ_i, α) rather than pinning down the rule. If interest rate communication is added, then the rule is pinned down by a strict inflation targeter. For example, the restriction $\gamma = 0$ along with $\mu_i = 2.5$ and $N_{i,t} = 1$ on top of $\mu_\pi = 0.25$, $N_{\pi,t} = 16$ uniquely pins down $\rho_i = 0.445$ and $\alpha = 1.628$, as shown in Figure 10.

To conclude, the model with inertia has implications that extend from those when considering a simpler monetary policy rule. In the case with inertia, communication about an inflation band

and horizon, coupled with similar communication about the output gap, will not pin down a unique rule. In this case, additional communication about interest rates can pin down the unique rule from the public’s perspective. Of course, if the monetary authority is a strict inflation targeter that does not react to the output gap, then communication about output can be dropped while maintaining a rule that is pinned down by communication.

5 Conclusion

Motivated by the wide-ranging communication strategies of the 62 central banks around the world that use some form of inflation targeting, we develop a link between rules-based monetary policy and effective communication. Our framework relies on three key features: the specification of a point inflation target, tolerance bands around the point, and economic projections. In simple Fisherian and New Keynesian models, we show that effective communication about the band and a horizon for achieving that band given shocks can implicitly pin down a rule without having to explicitly express one. In addition, our results suggest that central banks that react to the output gap or set rates inertially need to add bands and horizons about the output gap and the interest rate to their projections. Central banks that use this communication can then reap the benefits of rules-based policy without necessarily having to codify their rule.

References

- Banco Central Do Brasil. 2016. “Inflation Targeting Regime in Brazil.” Frequently Asked Questions Series.
- Central Bank News. 2017. “Inflation Targets Table for 2017.” <http://www.centralbanknews.info/p/inflation-targets.html>. Accessed January 13, 2017.
- Choi, Jason and Andrew Foerster. 2016. Optimal Monetary Policy Regime Switches. Research Working Paper 16-7 Federal Reserve Bank of Kansas City.
- Cochrane, John and John Taylor, eds. 2016. *Central Bank Governance and Oversight Reform*. The Hoover Institution.
- Heikensten, Lars. 1999. “The Riksbank’s Inflation Target - Clarification and Evaluation.” *Sveriges Riksbank Quarterly Review* 1999(1):5–17.

- Ikeda, Daisuke and Takushi Kurozumi. 2014. Post-Crisis Slow Recovery and Monetary Policy. IMES Discussion Paper Series 14-E-16 Institute for Monetary and Economic Studies, Bank of Japan.
- Orphanides, Athanasios and Volker Weiland. 2000. "Inflation Zone Targeting." *European Economic Review* 44(7):1351–1387.
- Riksbank. 2010. "The Riksbank Removes the Tolerance Interval from its Specified Monetary Policy Target." Monetary Policy Department. Decision - 31 May 2010.
- Riksbank. 2016. "The Riksbank's Inflation Target - Target Variable and Interval." Riksbank Studies.
- Schmitt-Grohe, Stephanie and Martin Uribe. 2007. "Optimal Simple and Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics* 54(6):1702–1725.
- Smets, Frank. 2000. "What Horizon for Price Stability." European Central Bank Working Paper No. 24.
- Svensson, Lars E. O. 1999. "Inflation Targeting as a Monetary Policy Rule." *Journal of Monetary Economics* 43(3):607–654.
- Svensson, Lars E.O. 2002. "Inflation Targeting: Should it be Modeled as an Instrument Rule or a Targeting Rule?" *European Economic Review* 46(4-5):771–780.
- Svensson, Lars E.O. and Michael Woodford. 2005. Limits to Inflation Targeting. In *Implementing Optimal Policy through Inflation-Forecast Targeting*, ed. Ben S. Bernanke and Michael Woodford. Chicago: The University of Chicago Press pp. 19–91.
- Walsh, Carl. 2015. "Goals and Rules in Central Bank Design." *International Journal of Central Banking* 11(S1):295–352.
- Warsh, Kevin. 2017. "America Needs a Steady, Strategic Fed." *Wall Street Journal*. January 30.
- Woodford, Michael. 2005. Central-Bank Communication and Policy Effectiveness. In *The Greenspan Era: Lessons for the Future*. Jackson Hole, Wyoming: Federal Reserve Bank of Kansas City pp. 399–474.