Online Appendix for
A Model of Monetary Policy Shocks for Financial Crises
and Normal Conditions*

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This section describes the DSGE model used in the paper in detail. Our approach to modeling the household’s portfolio of monetary assets follows closely from Belongia and Ireland (2014). The production side of the economy is a standard New-Keynesian model.

**A.1 The Household**

The representative household enters any period \( t = 0, 1, 2, \ldots \) with a portfolio consisting of maturing bonds \( B_{t-1} \) and monetary assets totaling \( A_{t-1} \). The household faces a sequence of budget constraints in any given period. In the first sub-period the household buys and sells bonds, receives real wages \( W_t \) for hours worked \( H_t \) during the period, is paid dividends \( F_t \) from intermediate goods firms, purchases consumption goods \( C_t \), pays lump-sum taxes \( T_t \), and allocates its monetary assets \( A_{t-1}/\Pi_t \) net of central bank transfers \( \tau_t \) between currency \( N_t \) and deposits \( D_t \). Any loans \( L_t \) needed to finance these transactions are made at this time. This is summarized in the constraint below:

\[
N_t + D_t = \frac{A_{t-1}}{\Pi_t} + \frac{B_{t-1}}{\Pi_t} - \frac{B_t}{R_t} + W_t H_t + F_t - C_t - T_t + L_t + \tau_t, \tag{A.1}
\]

where \( \Pi_t = P_t/P_{t-1} \). In the second sub-period, the household receives deposits with interest \( R_t^D D_t \) and receives any residual assets of the bank \( F^b_t \). The household then combines this income with currency to repay loans with interest \( R_t^L L_t \). Any remaining funds are carried over into the next period in the form of monetary assets \( A_t \), as summarized below:

\[
A_t = N_t + F^b_t + R_t^D D_t - R_t^L L_t. \tag{A.2}
\]

The household seeks to maximize their lifetime utility, discounted at rate \( \beta \), subject to these constraints. The period flow utility of the household takes the following form:

\[
U_t = \left[ \ln(C_t - hY_{t-1}) - \xi H_t - H_t^s \right].
\]

The household receives utility from consumption relative to last periods aggregate demand (i.e. the household has external habits) and disutility from working and shopping. Time spent shopping increases with aggregate demand \( Y_t \) (i.e. long lines) but is reduced with higher liquidity services. Therefore the time spent shopping takes the following form:

\[
H_t^s = \frac{1}{\chi} \left( \frac{Y_t}{M_t} \right)^{\chi}. \tag{A.3}
\]
The monetary aggregate, $M_t$, which enters the shopping-time function takes a rather general CES form:

$$M_t = \left[ \nu \left( N_t \right)^{\frac{\omega - 1}{\omega}} + \left( 1 - \nu \right) \left( D_t \right)^{\frac{\omega - 1}{\omega}} \right]^{\frac{1}{\omega - 1}},$$  

(A.4)

where $\nu$ calibrates the relative expenditure shares on currency and deposits and $\omega$ calibrates the elasticity of substitution between the two monetary assets. Given these parameters, $\chi$ is left free to calibrate the interest semi-elasticity of money demand.

The representative household faces the problem of maximizing its lifetime utility subject to its budget constraints. Letting $C_t = [C_t, H_t, M_t, N_t, D_t, L_t, B_t, A_t]$ denote the vector of choice variables, the household’s problem can be recursively defined using Bellman’s method:

$$V_t(B_{t-1}, A_{t-1}) = \max_{C_t} \left\{ \ln(C_t - hY_{t-1}) - \xi H_t - \frac{1}{\chi} \left( \frac{Y_t}{M_t} \right)^\chi \right\}$$

$$-\lambda_1^{1}\left( N_t + D_t - \frac{A_{t-1}}{\Pi_t} - \frac{B_{t-1}}{R_t} + \frac{B_t}{R_t} - W_t H_t - F_t + C_t + T_t - L_t - \tau_t \right)$$

$$-\lambda_2^{1}\left( M_t - \left[ \nu \left( N_t \right)^{\frac{\omega - 1}{\omega}} + \left( 1 - \nu \right) \left( D_t \right)^{\frac{\omega - 1}{\omega}} \right]^{\frac{1}{\omega - 1}} \right)$$

$$-\lambda_3^{1}\left( A_t - N_t - F^b_t - R^D_t D_t + R^L_t L_t \right) + \beta \mathbb{E}_t \left[ V_{t+1}(B_{t+1}, A_{t+1}) \right].$$

The first order necessary conditions can be expressed by the following equations:

$$\frac{1}{(C_t - hY_{t-1})} = \beta \mathbb{E}_t \left[ \frac{1}{(C_{t+1} - hY_{t+1}) \Pi_{t+1}} \frac{R_t}{\Pi_t} \right]$$

(A.5)

$$W_t = \xi (C_t - hY_{t-1})$$

(A.6)

$$\frac{C_t - hY_{t-1}}{M_t} = \frac{\lambda_2^{1}}{\lambda_1^{1}} \left( \frac{Y_t}{M_t} \right)^{-\chi}$$

(A.7)

$$N_t = \nu M_t \left[ \frac{\lambda_2^{1}/\lambda_1^{1}}{(R_t - 1)/R_t} \right]^{\omega}$$

(A.8)

$$D_t = (1 - \nu) M_t \left[ \frac{\lambda_2^{1}/\lambda_1^{1}}{(R_t - R^D_t)/R_t} \right]^{\omega}$$

(A.9)

$$M_t = \left[ \nu \left( N_t \right)^{\frac{\omega - 1}{\omega}} + \left( 1 - \nu \right) \left( D_t \right)^{\frac{\omega - 1}{\omega}} \right]^{\frac{1}{\omega - 1}}.$$  

(A.10)

### A.2 The Goods Producing Sector

The goods producing sector features a final goods firm and an intermediate goods firm. There is a unit measure of intermediate goods producing firms indexed by $i \in [0, 1]$ who produce a differentiated product. The final goods firm produces $Y_t$ combining inputs $Y_{it}$. 

using the production technology:

\[ Y_t = \left( \int_0^1 Y_{i,t}^{\theta-1} \, di \right)^{\theta/\theta-1}, \]

in which \( \theta > 1 \) governs the elasticity of substitution between inputs. The final goods producing firm sells its product in a perfectly competitive market, hence solving the profit maximization problem:

\[
\max_{Y_{i,t} \in [0,1]} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} \, di,
\]

subject to the above constant returns to scale technology. The resulting first order condition defines the demand curve for each intermediate goods producing firm’s product:

\[
Y_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} Y_t. \tag{A.11}
\]

Given the downward sloping demand for its product in (A.11), the intermediate goods producing firm has the ability to set the price of its product above marginal cost. To permit aggregation and allow for the consideration of a representative firm, we assume all such firms have the same constant returns to scale technology:

\[
Y_{i,t} = H_{i,t}. \tag{A.12}
\]

The term \( H_{i,t} \) in the production function denotes the level of employment chosen by the intermediate goods firm. Given the linear production function, the intermediate goods producing firm’s real marginal cost takes the same functional form:

\[
MC_t = (1 - S) W_t.
\]

A production subsidy, \( S \), is introduced to make the steady state price of goods equal to the marginal cost of production.

The price setting ability of each firm is constrained as in Calvo (1983). In this staggered price-setting framework, the price level \( P_t \) is determined in each period as a weighted average of the fraction of firms \( 1 - \alpha \) that are able to re-optimize their price and the fraction \( \alpha \) that leave their prices unchanged. Therefore, each firm maximizes the present value of its current and future discounted profits, taking into account the probability that the firm will not be able to re-optimize:
\[
\max_{P_{i,t}} \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{\lambda_{t+j}^1}{\lambda_t^1} \left[ \Pi_{t+j-1,t-1} P_{i,t}^* Y_{i,t+j} - MC_{t+j} P_{t+j} Y_{i,t+j} \right]
\]
subject to
\[
Y_{i,t+j} = \left( \frac{P_{t+j-1}^*}{P_{t+j}} \right)^{-\theta} Y_{t+j},
\]
where \(\Pi_{t+j-1,t-1} \equiv P_{t+j-1}/P_{t-1}\) captures the indexation of prices to lagged inflation. The firm’s first order condition is given by:
\[
\mathbb{E}_t \sum_{j=0}^{\infty} (\beta \alpha)^j \frac{\lambda_{t+j}^1}{\lambda_t^1} \left( \frac{P_{i,t}^*}{P_{t+j}} \Pi_{t+j-1,t-1} - \Pi_{t+j,t-1} \frac{\theta}{\theta - 1} MC_{t+j} \right) = 0. \tag{A.13}
\]

Finally, in equilibrium, the aggregate price dynamics are determined by the following price aggregate:
\[
\Pi_{t}^{1-\theta} = \alpha \Pi_{t-1}^{1-\theta} + (1 - \alpha)(\Pi_t^*)^{1-\theta}, \tag{A.14}
\]
where \(\Pi_{t}^* = P_{t}^*/P_{t-1}\) and \(P_{t}^*\) is the optimal price firms choose who re-optimize in period \(t\).

A.3 The Financial Firm

The financial firm performs the intermediation process of accepting household’s deposits and making loans. The financial firm must satisfy the accounting identity which specifies assets (loans to firms plus reserves) equal liabilities (deposits):
\[
L_t + rr D_t = D_t. \tag{A.15}
\]

Although changes in banking regulation have effectively eliminated reserve requirements, banks may often choose to hold reserves in lieu of making loans. Therefore, instead of assuming the central bank controls the reserve ratio \(rr\), we assume it is exogenously fixed and represents the average ratio of deposits banks hold for regulatory and liquidity purposes.

The financial firm chooses \(L_t\) and \(D_t\) in order to maximize period profits:
\[
\max_{L_t,D_t} R_t^L L_t - R_t^D D_t - L_t + D_t - xL_t,
\]
subject to the balance sheet constraint (A.15). The term \(xL_t\) denotes the real resource costs banks bear in making loans. We assume for simplicity that these resources are not destroyed in the loan production process, but instead are rented and remitted back to the household.
as dividends $F_t^b$. Since the loan and deposits markets are perfectly competitive, substituting the balance-sheet constraint into the profit function and imposing zero profits results in the loan-deposit spread:

$$R_t^L - R_t^D = (R_t^L - 1)rr + x(1 - rr). \quad (A.16)$$

This expression describes the loan deposit spread as a weighted average of the (opportunity) cost of accepting one unit of deposits. The fraction $rr$ are held as reserves which bears the foregone revenue of making loans while the remaining fraction $(1 - rr)$ are loaned out which bears the real resource cost of making a new loan.

### A.4 Equilibrium And The Output Gap

Here we define the equilibrium conditions which close the model. Equilibrium in the final goods market requires that the accounting identity

$$Y_t = C_t \quad (A.17)$$

holds. Equilibrium in the money market and bond market requires that at all times: $A_t = A_{t-1}/\Pi_t + \tau_t$ and $B_t = B_{t-1} = 0$ respectively. Market clearing in the labor market requires that labor supply equals labor demand:

$$H_t = \int_0^1 H_{i,t} di = \int_0^1 Y_{i,t} di = \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} di Y_t$$

where the second equality uses the firm’s production function (A.12) and the third equality uses the demand for the intermediate goods product (A.11). Therefore, aggregate output is related to price dispersion and aggregate labor supply by:

$$Y_t = \left[ \int_0^1 \left( \frac{P_{i,t}}{P_t} \right)^{-\theta} di \right]^{-1} H_t. \quad (A.18)$$

The production subsidy $(1 - S)$ to the intermediate goods producers is set so that in steady state the subsidy offsets the steady state markup of the monopolistically competitive firm implying $1 - S = (\theta - 1)/\theta$. Finally, the government funds this subsidy with lump-sum taxes from the household implying the following government budget constraint:

$$T_t = SW_t H_t. \quad (A.19)$$
A.5 Monetary Aggregates

In addition to the monetary aggregate $M_t$, three other monetary aggregates are defined in the DSGE model. Since we consider policy rules that stabilize the inflation rate, the price level will inherit a unit root. Therefore, we define all three aggregates in terms of their nominal growth rates. The first aggregate we define is the monetary base, which we denote by $M_t^B$:

$$\mu_t^B = \ln \left( \frac{M_t^B}{M_{t-1}^B} \Pi_t \right) = \ln \left( \frac{N_t + rrD_t}{N_{t-1} + rrD_{t-1}} \right) + \ln(\Pi_t), \quad (A.20)$$

so that $M_t^B$ is equal to the sum of currency and reserves. The second aggregate is a weighted nonparametric Divisia aggregate $M_t^W$ used to approximate the parametric aggregate $M_t$ as defined by Barnett (1980):

$$\mu_t^W = \ln \left( \frac{M_t^W}{M_{t-1}^W} \Pi_t \right) = \frac{S_t^N + S_{t-1}^N}{2} \ln \left( \frac{N_t}{N_{t-1}} \right) + \frac{S_t^D + S_{t-1}^D}{2} \ln \left( \frac{D_t}{D_{t-1}} \right) + \ln(\Pi_t), \quad (A.21)$$

where $S_t^N = (R_t - 1)N_t / ((R_t - 1)N_t + (R_t - R_t^D)D_t)$ is the share of total implicit spending on monetary assets allocated to currency and $S_t^D = 1 - S_t^N$ is the complimentary share spent on deposits. The third aggregate is an unweighted aggregate $M_t^U$ used to approximate the parametric aggregate $M_t$:

$$\mu_t^U = \ln \left( \frac{M_t^U}{M_{t-1}^U} \Pi_t \right) = \ln \left( \frac{N_t + D_t}{N_{t-1} + D_{t-1}} \right) + \ln(\Pi_t). \quad (A.22)$$
A.6 The Non-Linear Model

\[
\frac{1}{C_t - hY_{t-1}} = \beta E_t \left[ \frac{1}{C_{t+1} - hY_t \Pi_{t+1}} \right] \quad (A.23)
\]

\[
W_t = \xi (C_t - hY_{t-1}) \quad (A.24)
\]

\[
M_t = Y_t^{\lambda_t} (C_t - hY_{t-1})^{\lambda_t^{-1}} \left( \frac{\lambda_t^2}{\lambda_t^1} \right)^{-\lambda_t} \quad (A.25)
\]

\[
N_t = \nu M_t \left[ \frac{\lambda_t^2 / \lambda_t^1}{(R_t - 1)/R_t} \right]^\omega \quad (A.26)
\]

\[
D_t = (1 - \nu) M_t \left[ \frac{\lambda_t^2 / \lambda_t^1}{(R_t - R_t^D)/R_t} \right]^\omega \quad (A.27)
\]

\[
M_t = \left[ \nu^{\lambda_t/\lambda_t^1} (N_t)^{\lambda_t/\lambda_t^1} + (1 - \nu)^{\lambda_t/\lambda_t^1} (D_t)^{\lambda_t/\lambda_t^1} \right]^{-1} \quad (A.28)
\]

\[
Y_t = \left[ \int_0^1 \left( \frac{P_{t,t}}{P_t} \right)^{-\theta} \, dt \right]^{-1} H_t \quad (A.29)
\]

\[
0 = E_t \sum_{j=0}^\infty (\beta \alpha)^j \frac{\lambda_t^1 + j}{\lambda_t^1} Y_{t,t+j} \left( \Pi_{t+j-1,t-1} \Pi_t^* - \Pi_{t+j,t-1} \frac{\theta}{\theta - 1} MC_{t+j} \right) \quad (A.30)
\]

\[
\Pi_t^{-\theta} = \alpha \Pi_{t-1}^{-\theta} + (1 - \alpha)(\Pi_t^*)^{1-\theta} \quad (A.31)
\]

\[
MC_t = \frac{\theta - 1}{\theta} W_t \quad (A.32)
\]

\[
R_t - R_t^D = (R_t - 1)rr + x(1 - rr) \quad (A.33)
\]

\[
Y_t = C_t \quad (A.34)
\]

\[
\mu_t^B = \ln \left( \frac{N_t + rrD_t}{N_{t-1} + rrD_{t-1}} \right) + \ln(\Pi_t) \quad (A.35)
\]

\[
\mu_t^W = \frac{S_t^N + S_t^N/2}{\ln(\Pi_t)} + \frac{S_t^D + S_t^D/2}{\ln(D_t/D_{t-1})} + \ln(\Pi_t) \quad (A.36)
\]

\[
S_t^N = \frac{(R_t - 1)N_t}{(R_t - 1)N_t + (R_t - R_t^D)D_t} \quad (A.37)
\]

\[
S_t^D = \frac{(R_t - R_t^D)D_t}{(R_t - 1)N_t + (R_t - R_t^D)D_t} \quad (A.38)
\]

\[
\mu_t^d = \ln \left( \frac{N_t + D_t}{N_{t-1} + D_{t-1}} \right) + \ln(\Pi_t) \quad (A.39)
\]
A.7 The Log-Linear Model

In this section we provide a linear representation of the model by taking a first order Taylor expansion of the relevant equations around the steady state. Lower case variables denote log deviations from the steady-state: \( g_t = \ln(G_t) - \ln(G) \), where \( G \) is the steady state value of \( G_t \).

The Euler equation can be derived from combining (A.23) and (A.34):

\[
y_t = \frac{1}{1 + h} E_t y_{t+1} + \frac{h}{1 + h} y_{t-1} - \frac{1 - h}{1 + h} \left( r_t - E_t \pi_{t+1} \right) \tag{A.40}
\]

The money demand equation can be derived in two steps. First, we combine (A.26), (A.27) and (A.28) to show that:

\[
\lambda_t^2 / \lambda_t^1 = \left[ \nu \left( \frac{R_t - 1}{R_t} \right)^{1-\omega} + (1 - \nu) \left( \frac{R_t - R^D_t}{R_t} \right)^{1-\omega} \right]^{\frac{1}{1-\omega}}.
\]

Then we use this expression for \( \lambda_t^2 / \lambda_t^1 \) in (A.25) along with equations (A.33) and (A.34) to arrive at the following log-linear expression for real money balances:

\[
m_t = \frac{1 + \chi (1 - h)}{(1 + \chi)(1 - h)} y_t - \frac{h}{(1 + \chi)(1 - h)} y_{t-1} - \eta r_t, \tag{A.41}
\]

so that in equation (2) in the main text, \( \eta = \frac{1}{1 + \chi} \left( \frac{\nu(R-1)/R)^{-\omega} + (1 - \nu)((R-R^D)/R)^{-\omega}(1-rr)}{\nu(R-1)/R)^{\omega} + (1 - \nu)((R-R^D)/R)^{\omega-1} \right) \frac{1}{1 + \chi} \).

The Phillips Curve can be derived in two steps. First, log-linearizing (A.30):

\[
\pi_t^* - \pi_{t-1} = (1 - \beta \alpha) \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \alpha)^j (mc_{t+j} + \pi_{t+j,t-1} - \pi_{t+j-1,t-1}) - \pi_{t-1}
= (1 - \beta \alpha) \mathbb{E}_t \sum_{j=0}^{\infty} (\beta \alpha)^j (mc_{t+j} + \sum_{k=0}^{j} \pi_{t+j} - \sum_{k=0}^{j} \pi_{t+j-1})
= \beta \alpha \mathbb{E}_t (\pi_{t+1}^* - \pi_t) + (1 - \beta \alpha) mc_t + (\pi_t - \pi_{t-1}).
\]

The first relationship linearizes the firm pricing decision. The second equality uses \( \pi_{t-1} = (1 - \beta \alpha) \sum_{j=0}^{\infty} (\beta \alpha)^j \pi_{t-1} \) and the third equality rewrites the infinite sum recursively. Next, linearize (A.31) and use the resulting expression \( \pi_t - \pi_{t-1} = (1 - \alpha)(\pi_t^* - \pi_{t-1}) \) to eliminate \( \pi_t^* - \pi_{t-1} \) above:

\[
\pi_t = \pi_{t-1} + \frac{(1 - \alpha)(1 - \beta \alpha)}{\alpha} \left( \frac{1}{1 - h} y_t - \frac{h}{1 - h} y_{t-1} \right) + \beta \mathbb{E}_t (\pi_{t+1} - \pi_t). \tag{A.42}
\]
where we have also written real marginal cost in terms of output by combining (A.24), (A.32), and (A.34) and linearizing.

A linear expression for the nominal growth in the monetary base is obtained by log-linearizing equation (A.35):

$$\mu_B^t = \gamma_B^t (n_t - n_{t-1}) + (1 - \gamma_B^t)(d_t - d_{t-1}) + \pi_t,$$

where $$\gamma_B^t = \frac{\nu((R-1)/R)^{-\omega}}{\nu((R-1)/R)^{-\omega} + \nu((R-R^D)/R)^{-\omega}}$$. Log-linearizing (A.36) reveals that: $$\mu_W^t = m_t - m_{t-1} + \pi_t$$. Log-linearizing (A.39) provides an expression for the nominal growth rate of the unweighted monetary aggregate:

$$\mu_U^t = \gamma_U^t (n_t - n_{t-1}) + (1 - \gamma_U^t)(d_t - d_{t-1}) + \pi_t,$$

where $$\gamma_U^t = \frac{\nu((R-1)/R)^{-\omega}}{\nu((R-1)/R)^{-\omega} + (1-\nu)((R-R^D)/R)^{-\omega}}$$. Log-linear expressions for $$N_t$$ and $$D_t$$ are obtained from equations (A.26) and (A.27) as follows:

$$n_t = m_t + \omega \left[ (1 + \chi)\eta - \frac{1}{R-1} \right] r_t$$

$$d_t = m_t + \omega \left[ (1 + \chi)\eta - \frac{rr - x(1-rr)}{R - R^D} \right] r_t$$

A.8 DSGE Calibration

We set $$\beta = 0.99$$ which implies an annualized nominal bond rate of 4% in this quarterly model. We set $$\alpha = 0.75$$ so the average duration of prices is about 1 year, as found by Nakamura and Steinsson (2008). The degree of habit persistence $$h = 0.65$$ as estimated in Christiano, Eichenbaum, and Evans (2005). The parameters governing the CES aggregate of monetary assets are calibrated as in Ireland (2014) who uses component level data on $$N_t$$ and $$M_t$$ to estimate equation (A.26) so that $$\omega = 0.5$$ and $$\nu = 0.2$$. We set $$rr = 0.02$$ which is the average ratio of reserve to non-currency components of M2 from 1967 to 2007 using data from the Federal Reserve Bank of St. Louis. Using data from the Center for Financial Stability we find that setting $$x = 0.0067$$ implies the annualized steady state spread between $$R^L$$ and $$R^D$$ of 2.7% which is the average interest rate differential between the benchmark interest rate used to measure $$R^L$$ and the own-rate on the non-currency components of M2. The interest semi-elasticity of money demand is set to $$\eta = 1.9$$ as estimated in Ireland (2009). This value of $$\eta$$ is achieved by setting $$\chi = 12$$. 

9
REFERENCES


