THE SOCIAL VALUE OF RISK-FREE GOVERNMENT DEBT

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Abstract

This paper considers whether eliminating the stock of government debt outstanding would reduce welfare. It models an economy with three assets—currency, government bonds, and storage, a transactions role for money, and a demand for liquidity and thus a role for banks. The Friedman rule is not optimal in this economy, so there is potentially a role for interest-bearing, risk-free government bonds. Because the government must raise enough revenue to meet its interest obligations on any bonds outstanding, the social value of government debt hinges on whether the benefits from greater portfolio diversification outweigh the costs associated with the necessary revenue-raising efforts. The paper shows that a positive stock of government debt is optimal only if interest payments on the debt are financed via money creation, agents are not too risk averse, there is a primary government budget deficit, and the economy is operating on the bad side of the Laffer curve. But under these conditions, welfare would be even higher if monetary policy were conducted to put the economy on the good side of the Laffer curve and there were no government bonds outstanding. Thus, there is little support for keeping a stock of interest-bearing, risk-free government debt outstanding.

Keywords: Government debt, fiscal policy, monetary policy, portfolio allocation

JEL Codes: E0, E4, E5, E6, H6, G1

1. Introduction

Until relatively late in 2001, many projections had the stock of U.S. government debt outstanding shrinking dramatically—possibly going to zero—over the ensuing decade. The projected debt reductions were, of course, a consequence of sustained projected government budget surpluses.

The possibility that the stock of government debt outstanding might become extremely small raised concerns on Wall Street and within the government. The concern was that an absence of government debt would deprive savers of an essentially safe asset. According to the argument, the complete, or near complete, elimination of government debt would force at least some economic actors to bear increased portfolio risk, thereby making them worse off.

While this argument received considerable attention in informal discussions, there appears to have been no attempt to evaluate it in a formal economic model. This paper aims to fill this gap. It addresses the optimal level of a stock of risk-free government bonds in a monetary economy, described in Sections 2 through 4. In that economy, currency coexists with a risky real asset that dominates it in rate of return. The economy is also one in which the Friedman rule is not optimal, as shown in Section 5, because it raises the rate of return on currency, resulting in insufficient investment in the real asset. By implication, if it is optimal for the government to issue bonds, then it is optimal for those bonds to pay a positive rate of interest.

With government bonds paying interest, a primary factor in the analysis is how the government finances its interest obligations on the debt. If the government could issue debt at no cost, clearly there would be a social benefit to its doing so because the debt would provide superior access to insurance. However, the real return on government debt is an endogenous variable, and if the government issues debt, it must also raise additional revenue to cover its interest obligations. This paper considers two scenarios with respect to how the government raises the necessary revenue. In the first, presented in Section 6, the government pays interest on the debt with seigniorage revenue. In the second, analyzed in Section 7, the government raises the necessary revenue through nondistortionary (i.e., lump-sum) taxation.

The analysis indicates that there is little case to be made for issuing government debt solely to provide agents with a risk-free asset. In particular, when interest payments on the debt are monetized, there is no benefit in terms of steady-state welfare to having a positive stock of government debt if there is a primary government budget surplus. In fact, issuing positive amounts of debt reduces steady-state utility. This result is important in the appropriate context because projections of a shrinking stock of U.S. government debt were based on projections that the budget would be in surplus. The findings suggest that, in such a scenario, there would be no argument in favor of having a stock of government debt outstanding.

If, on the other hand, there is a primary government budget deficit, then having safe government debt outstanding can raise steady-state welfare if agents are not too risk averse, and if the economy is on the wrong side of the Laffer curve. In other words, it is optimal to have a positive stock of safe government debt outstanding if and only if increases in the bond-to-money ratio reduce the equilibrium inflation rate. This is the case on the wrong side of the Laffer curve because unpleasant monetarist arithmetic (Sargent and Wallace, 1981) does not apply there. But in that case, agents would be better off at the feasible equilibrium with a lower inflation rate.

Situations are also considered in which interest payments on government debt can be financed through nondistortionary taxation. It is shown that positive stocks of government debt are never optimal if the real return on government debt exceeds the economy's real rate of growth.

The framework for examining these issues is an economy with three assets: money, government bonds, and a risky storage technology. In this economy, as in Townsend (1980, 1987), spatial separation and limited communication create a transactions role for currency, despite its relatively low rate of return. Additionally, random movements of agents between locations create the analog of a liquidity-preference shock. Agents will want to be insured against such shocks. As in Diamond and Dybvig (1983), such insurance can be provided through intermediaries. The presence of intermediaries seems a reasonable feature of any analysis of the potential benefits of government debt since most holdings of government bonds are intermediated.

It is worth noting that this analysis of the potential benefits of having a stock of government debt outstanding is very different from that in the existing literature. For example, Aiyagari and McGratten (1998) and Woodford (1990) analyze the optimal level of government debt in an economy with capital and government debt. Government debt has social value in their models precisely because some agents are borrowing constrained. In effect, the government's borrowing on behalf of private agents relaxes such constraints. In a different context, Kocherlakota (2001) considers the optimality of risk-free government debt in a model where money and government bonds are the only assets, and where again some agents may face borrowing constraints. Finally, Holmstrom and Tirole (1998) demonstrate a potential role for government debt in a nonmonetary economy where the only assets are private debt. In their economy, government debt may again permit the relaxation of borrowing constraints that arise because of moral hazard problems.

This paper, in constrast, differs from the existing literature in two fundamental ways. First, it allows for a richer set of assets than in most of the literature. Here, money, bonds, and a real asset—storage—can all coexist. Second, agents face no borrowing constraints. In fact, no agents other than the government can conceivably want to borrow under any circumstances. Indeed, under appropriate assumptions, markets are complete, although they are characterized by frictions that motivate monetary exchange. Thus, compared to most of the other literature on the optimal stock of government debt, the analysis here relies neither on the presence of borrowing constraints nor on market incompleteness.

Romer (1993) is also an exception to the literature on the social value of government debt in that the economy he models has money, government bonds, and capital, but no borrowing constraints. Instead, he assumes that using bonds or capital involves a positive, finite transaction cost, while money is costless to use. The assumption of differing transaction costs is comparable to the assumptions here that give rise to differences in liquidity. In contrast to this paper, however, Romer does not allow for randomness in any of his asset returns. Hence, his model is silent on the value of government debt as a risk-free asset, which is the subject of this paper.

2. The Environment

The economy exists at discrete dates $t = 1, 2, \ldots$ and consists of two islands. Each island is populated by an identical infinite sequence of two-period-lived overlapping generations. At each date, a new young generation, consisting of a continuum of ex ante identical agents with unit mass, appears on each island. For simplicity, there is no population growth.

There is a single consumption good in the economy. Each agent born at dates $t \geq 1$ is endowed with $\omega > 0$ units of this good when young and nothing when old. Agents are assumed to consume only when old.[1] Letting c_t denote the old-age consumption of agents born at t, these agents have lifetime utility of $u(c_t) = c_t^{1-\rho}/(1-\rho)$, $\rho > 0$.

There are three primary assets in the economy. One is a linear storage technology. A unit of consumption invested in this technology at t yields x units of consumption at $t + 1$. The variable x is random and realized at $t + 1$. Each period it is drawn from a distribution F, with probability density function f. The function f has support $[\underline{x}, \overline{x}]$. Since all investments in storage at t yield the same random return, there is aggregate randomness governing the return on storage investments. Once x is realized, it is observable by all agents. The mean of x is $\hat{x} > 1$, so in an expected value sense, investments in the storage technology are socially efficient.

In addition to storage, there is a stock of one-period government bonds. This stock may be positive, negative, or zero. Government bonds pay a certain gross real return of R_t from t to $t + 1$. Thus, in keeping with the discussion of the importance of government debt in providing a safe, real return, there is no randomness to the payoff on bonds. B_t denotes the nominal per capita stock of government debt outstanding at t and is a choice variable of the government.

The third primary asset in the economy is fiat money. M_t denotes the per capita stock of

[1] This assumption is not important to the analysis. It merely simplifies the consumption-saving decision of the young.

money outstanding at time *t*. p_t denotes the price level at time *t*. As will become apparent, these variables display no randomness in a perfect-foresight equilibrium. In what follows, the per capita stock of real money balances is denoted $z_t \equiv M_t / p_t$, while the real value of debt outstanding is denoted $b_t \equiv B_t / p_t$.

2.1 The Government

The government is assumed throughout to have a constant nonnegative per capita real expenditure level of *g*. In addition, it levies a real, time-independent lump-sum tax of τ on all young agents at all dates. The government then faces the following budget constraint for $t > 1$:

$$g - \tau + R_{t-1} b_{t-1} = b_t + z_t - z_{t-1}\left(\frac{p_{t-1}}{p_t}\right) \tag{2.1}$$

Government policy consists of a choice of two variables. First, following standard specifications in a variety of monetary models (e.g., Schreft and Smith 2001, 2002), the government sets once and for all a value for the bond-to-money ratio. That value is $\mu_t \equiv b_t / z_t$. This specification is not central to the analysis since the primary concern here is simply to establish when it is optimal to have a positive stock of government debt. Clearly, the stock of government debt outstanding is zero if $\mu_t = 0$.

In addition to setting a value for μ, the government confronts some choices about monetary and or fiscal policy . Two scenarios are considered below. In the first scenario, the money supply grows according to $M_{t+1} = \sigma M_t$, with σ set once and for all at *t* = 1 and M_0, the initial money supply, exogenously given and held by the old at *t* = 0. This scenario leaves tax collections τ to be determined endogenously. In the second scenario, τ is exogenously given. This renders the money growth rate an endogenous variable.

The reason for considering two alternative scenarios for government policy is as follows. As noted in the introduction, when government debt with a safe real return is introduced, interest on this debt must be paid in some way. With money (tax) financing, an increase in government interest obligations associated with a change in μ_t must be financed by changes in seigniorage revenue (taxation). The desirability of having the government issue a positive stock of debt with a safe real return potentially depends on how interest obligations on the debt are financed. Thus, each scenario is considered below.

2.2 Spatial Separation and Limited Communication

The reason for introducing two islands is to create a role for money in transactions and a role for banks. This section describes the nature of the economy's spatial separation and limited communication.

In the first period of life, each agent is assigned to one of the two locations. Trade and communication can occur only among agents who are in the same location at the same time. In particular, cross-location trade or communication is not possible.

When young, agents must allocate their savings, either directly or through intermediaries, among the economy's three primary assets. After this portfolio allocation occurs, a fraction π of young agents is randomly selected to move to the other location, with π being common across locations. The value π is known at the beginning of each period, but the specific identities of the agents to be relocated are realized only after portfolio allocation and consumption have occurred at t.

The significance of stochastic relocation is as follows. First, storage investments made at t do not mature until $t + 1$, and goods invested in storage cannot be transported across locations. In addition, bonds are assumed to not be usable in interlocation exchange. This could occur because government debt is issued in denominations that are too large to be used by individuals in transactions. In any event, the objective of the analysis is to determine whether it is optimal for the government to issue liabilities with a safe real return that exceeds the rate of return on money. Doing so is only possible if the government limits the potential for bonds to be used in transactions, as is assumed to be the case.

The observations of the previous paragraph imply that agents who are relocated do not carry bonds or storage investments with them. In addition, the assumption about the lack of interlocation communication implies that relocated agents cannot trade using checks, credit cards, or any liabilities that are claims against assets held in the other location. Thus, the event of being relocated forces agents to liquidate bonds and storage to obtain cash, which can be used in transactions in the other location. As in Townsend (1980, 1987), spatial separation and limited communication create a role for currency as a transactions medium for agents who are relocated.

This need to liquidate assets if an agent is relocated represents a type of liquidity preference shock against which agents will wish to be insured. This insurance can be provided by banks, as in Diamond and Dybvig (1983). The behavior of these banks is described in the next section.

Finally, after agents are relocated, period t ends. At $t + 1$ storage returns are realized, and agents who are not relocated can consume out of the returns on any asset. Agents who are relocated use their cash balances to purchase consumption. The timing of events is depicted in Figure 1.

3. Bank Behavior

As in Diamond and Dybvig (1983), the structure of the model makes it natural to think of banks at t as coalitions of ex ante identical young agents. These agents deposit their entire after-tax endowment, $\omega - \tau$, with a bank.[2] The bank then chooses a gross real return, d_t^m, to pay to agents who are relocated between t and $t + 1$, and a return schedule $d_t\left(x_{t+1}\right)$ to be paid at $t + 1$ to agents who did not relocate. The latter repayment necessarily depends on the returns on the bank's storage investments. However, since these returns are not realized until $t + 1$, the payments made to relocated agents cannot depend on the realization of x. In addition, the bank chooses values m_t, $\tilde{b}_t$, and s_t for the real value of money balances, government bonds, and storage investments that it holds. These choices must obviously satisfy the bank's balance sheet

[2] The assumptions on the timing of exchange imply that banks cannot usefully transfer resources between members of different generations.

constraint:

$$m_t + \tilde{b}_t + s_t \le \omega - \tau. \tag{3.1}$$

In making these choices, a bank views itself as unable to affect the returns on storage investments. In addition, the bank is assumed to act competitively in asset markets. That is, it takes the return on real balances, p_t/p_{t+1}, and the return on bonds, R_t, as unaffected by its own choices. Finally, each bank views itself as being able to trade bonds with a safe real return, even if the outstanding supply of government debt is zero.

Throughout the analysis, it is assumed that $R_t \ge p_t/p_{t+1}$. If I_t denotes the gross nominal rate of interest on debt, then $I_t \equiv R\left(p_{t+1}/p_t\right)$ and this assumption is equivalent to the assumption that $I_t \ge 1$, so that the nominal interest rate on bonds is positive.[3] This assumption implies that when $I > 1$ banks will hold cash reserves only to make payments to relocated agents, given that withdrawal demand is predictable. They will carry no cash reserves across periods. In contrast, when $I = 1$, some reserves may be carried across periods. In specifying the bank's problem, then, it is useful to let $\tilde{m}_t$ be money held by banks to pay movers and $\hat{m}_t$ be money held across periods to pay nonmovers, where $m_t = \tilde{m}_t + \hat{m}_t$.

Given this observation, a bank faces the following additional constraints. First, it must hold enough cash to honor its promised payments to agents who are relocated:

$$\pi d_t^m \left(\omega - \tau\right) \le \tilde{m}_t \frac{p_t}{p_{t+1}}. \tag{3.2}$$

In addition, promised repayments to nonrelocated agents cannot exceed the value of the bank's other assets. That is,

$$\left(1-\pi\right) d_t\left(x_{t+1}\right)\left(\omega - \tau\right) \le \hat{m}_t \frac{p_t}{p_{t+1}} + R_t \tilde{b}_t + x_{t+1} s_t. \tag{3.3}$$

The bank chooses its returns and its portfolio to maximize the expected utility of a representative depositor,

$$\left[\frac{\left(\omega-\tau\right)^{1-\rho}}{1-\rho}\right]\left\{\pi\left(d_t^m\right)^{1-\rho} + \left(1-\pi\right)\left[\int_{\underline{x}}^{\bar{x}} \left(d_t\left(x\right)\right)^{1-\rho} f\left(x\right)dx\right]\right\},$$

[3] As will be shown in Section 5 below, it is optimal to have $I > 1$.

subject to (3.1)-(3.3).

Alternatively, since (3.1)-(3.3) must hold with equality in an optimum, the bank's problem can be written slightly differently if $\tilde{\gamma}_t^m \equiv \tilde{m}_t/(\omega-\tau)$ denotes the fraction of deposits held as cash to pay movers, $\hat{\gamma}_t^m \equiv \hat{m}_t/(\omega-\tau)$ denotes the fraction of deposits held across periods as cash to pay nonmovers, and $\gamma_t^b \equiv \tilde{b}/(\omega-\tau)$ denotes the fraction of deposits held in the form of government debt. Then the bank can be viewed as choosing $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and γ_t^b to maximize

$$\left[\frac{(\omega-\tau)^{1-\rho}}{1-\rho}\right]\left\{\pi\left(\frac{\tilde{\gamma}_t^m(p_t/p_{t+1})}{\pi}\right)^{1-\rho}+(1-\pi)\int_{\underline{x}}^{\bar{x}}\left[\frac{\hat{\gamma}_t^m(p_t/p_{t+1})+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)}{1-\pi}\right]^{1-\rho}f(x)\,dx\right\}.$$

In this optimization problem, the bank can choose either positive or negative debt levels. The bank's optimal behavior is then described by the following first-order conditions for $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and γ_t^b, respectively:

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\tilde{\gamma}_t^m\right)^{-\rho}=(1-\pi)^{\rho}\int_{\underline{x}}^{\bar{x}}x\left[\hat{\gamma}_t^m\frac{p_t}{p_{t+1}}+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho}f(x)\,dx, \quad (3.4)$$

$$0=\int_{\underline{x}}^{\bar{x}}\left(\frac{p_t}{p_{t+1}}-x\right)\left[\hat{\gamma}_t^m+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho}f(x)\,dx, \quad (3.5)$$

$$0=\int_{\underline{x}}^{\bar{x}}(R_t-x)\left[\hat{\gamma}_t^m+R_t\gamma_t^b+x\left(1-\tilde{\gamma}_t^m-\hat{\gamma}_t^m-\gamma_t^b\right)\right]^{-\rho}f(x)\,dx. \quad (3.6)$$

4. General Equilibrium

In any general equilibrium, the money and bond markets must clear. Since all beginning-of-period demand for money balances derives from banks' demands for cash reserves, the money market clears at *t* if

$$(\omega-\tau)\left(\tilde{\gamma}_t^m+\hat{\gamma}_t^m\right)=z_t\equiv\frac{M_t}{p_t}. \quad (4.1)$$

Because $\tilde{\gamma}_t^m$, $\hat{\gamma}_t^m$, and τ_t are chosen before the relevant return on storage is realized, equation (4.1) implies that p_t is not stochastic. In addition, since the demand for and the supply of government debt must be equal,

$$(\omega-\tau)\gamma_t^b=b_t\equiv\frac{B_t}{p_t}. \quad (4.2)$$

It follows that the stance of monetary policy, μ_t, is $b/z = \gamma^b / (\tilde{\gamma}^m + \hat{\gamma}^m)$. Hereafter, attention is confined to steady-state equilibria so time subscripts are omitted.

Using (4.1) and (4.2), the government budget constraint (2.1) in a steady state becomes

$$(R-1)\gamma^b = \left(1 - \frac{p_t}{p_{t+1}}\right)(\tilde{\gamma}^m + \hat{\gamma}^m) - \frac{g-\tau}{\omega-\tau}. \tag{4.3}$$

In addition, the first-order conditions (equations (3.4) - (3.6)) become

$$\pi^\rho \left(\frac{p_t}{p_{t+1}}\right)^{1-\rho} (\tilde{\gamma}^m)^{-\rho} = (1-\pi)^\rho \int_{\underline{x}}^{\overline{x}} x \left[\hat{\gamma}^m \frac{p_t}{p_{t+1}} + R\gamma^b + x(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b)\right]^{-\rho} f(x)\,dx, \tag{4.4}$$

$$0 = \int_{\underline{x}}^{\overline{x}} \left(\frac{p_t}{p_{t+1}} - x\right)\left[\hat{\gamma}^m + R\gamma^b + x(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b)\right]^{-\rho} f(x)\,dx, \tag{4.5}$$

$$0 = \int_{\underline{x}}^{\overline{x}} (R-x)\left[\hat{\gamma}^m + R\gamma^b + x(1 - \tilde{\gamma}^m - \hat{\gamma}^m - \gamma^b)\right]^{-\rho} f(x)\,dx. \tag{4.6}$$

Given a specification of government policy ($b = \mu z$ and a choice of either money or tax financing), equations (4.3) through (4.6) are the steady-state equilibrium conditions.

The remainder of this paper analyzes the steady states of this economy. The analysis begins by considering the optimality of the Friedman rule. If the Friedman rule is optimal, as it is in most monetary models, then the government cannot improve welfare by introducing a stock of risk-free, interest-bearing government debt. That is, any government debt issued should be indistinguishable from currency in its rate of return. As the next section shows, in the economy studied here, the Friedman rule does not hold.

5. Steady States with *I* = 1

The Friedman rule recommends conducting monetary policy to achieve an equilibrium with $I = 1$. This can be achieved in this economy by setting $M_{t+1} = \sigma M_t$, with $\sigma = 1/R$. Government bonds will then play no role in the economy for two reasons. First, bonds pay the same rate of return as currency but are less liquid because they cannot be carried across islands. Second, storage is as illiquid as government bonds, but dominates bonds in rate of return. Thus, the optimality of the Friedman rule can be considered by setting $\mu = 0$.

Following the discussion in section 3, steady-state welfare is given by the expression

$$W(\mu, I) \equiv \frac{(\omega-\tau)^{1-\rho}}{1-\rho}\left\{\pi\left[\left(\frac{\tilde{\gamma}^m(I)}{\pi}\right)\left(\frac{p_t}{p_{t+1}}\right)\right]^{1-\rho}\right.$$

$$\left.+(1-\pi)\int_{\underline{x}}^{\overline{x}}\left[\frac{\hat{\gamma}^m(I)\frac{p_t}{p_{t+1}}+R\gamma^b+x\left(1-\tilde{\gamma}^m(I)-\hat{\gamma}^m(I)-\gamma^b\right)}{1-\pi}\right]^{1-\rho} f(x)\,dx\right\}, \tag{5.1}$$

where R, $\tilde{\gamma}^m$, and $\hat{\gamma}^m$ are functions of μ and $\gamma^b = \mu\left(\tilde{\gamma}^m + \hat{\gamma}^m\right)$. With $\mu = 0$ and $R = p_t/p_{t+1}$, W is only a function of I, and the Friedman rule is not optimal if $W'(I)|_{I=1} > 0$.

Equilibrium is indeterminate with $I = 1$ because $\hat{\gamma}^m$ is indeterminate. But certainly a possible equilibrium is the equilibrium of the economy with $I > 1$ in the limit as $I \to 1$. Any other equilibrium with $I = 1$ involves banks holding more excess reserves, fewer government bonds, and less storage. In such equilibria, any benefit from having risk-free government bonds that pay a positive nominal rate of interest is even lower, so nothing is lost by looking at the limit of the $I > 1$ economy.

In this limit economy, $\hat{\gamma}^m = 0$. And with $\mu = 0$, which implies $\gamma^b = 0$, (4.6) reduces to

$$R = \frac{\int_{\underline{x}}^{\overline{x}} x^{1-\rho} f(x)\,dx}{\int_{\underline{x}}^{\overline{x}} x^{-\rho} f(x)\,dx} < \hat{x}. \tag{5.2}$$

In what follows, it is assumed that

$$\int_{\underline{x}}^{\overline{x}} x^{1-\rho} f(x)\,dx > \int_{\underline{x}}^{\overline{x}} x^{-\rho} f(x)\,dx \tag{5.3}$$

so that $R > 1$ holds when there is no government debt outstanding. Thus (5.3) implies that the safe real rate of return exceeds the economy's growth rate. While there is some dispute about whether this is true empirically, $R > 1$ is a common feature of many theoretical models, and having $R > 1$ simplifies the conditions under which $I > 1$ in a steady state.[4] Thus, for simplicity, the focus below is on economies that satisfy (5.3).

Since $\hat{\gamma}^m = 0$, $\tilde{\gamma}^m$ will be denoted without the tilde henceforth for notational simplicity. The characterization of steady states with $\mu = 0$ can then be completed by setting $\hat{\gamma}^m = \gamma^b = 0$ in

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[4] See, for instance Abel et al (1989), Caporale and Grier (2000), and Prescott (1986).

(4.4) and solving for γ^m to obtain

$$\gamma^m = \left\{ 1 + \frac{1-\pi}{\pi} \left[\int_{\underline{x}}^{\bar{x}} \left(x \frac{p_{t+1}}{p_t} \right)^{1-\rho} f(x)\,dx \right]^{\frac{1}{\rho}} \right\}^{-1}. \tag{5.4}$$

This can be rewritten as

$$\gamma^m(I) = \left\{ 1 + \frac{1-\pi}{\pi} I^{\frac{1-\rho}{\rho}} \left[\int_{\underline{x}}^{\bar{x}} \left(\frac{R}{x} \right)^{\rho} f(x)\,dx \right]^{\frac{1}{\rho}} \right\}^{-1} \tag{5.5}$$

so that γ^m is explicitly a function of *I*.

In addition, the government budget constraint (4.3) reduces to

$$\left(1 - \frac{p_t}{p_{t+1}} \right) \gamma^m(I) = \frac{g-\tau}{\omega-\tau}, \tag{5.6}$$

indicating that any deficit must be financed through seigniorage and that there are no debt obligations to honor since there are no bonds. Solving (5.6) for $\omega-\tau$ yields

$$\omega - \tau = \frac{\omega - g}{1 - \left(\frac{I-R}{I} \right) \gamma^m(I)} \equiv \Omega(I).$$

This allows steady-state welfare (5.1) to be written as

$$W(I) \equiv \frac{\Omega(I)^{1-\rho}}{1-\rho} \Gamma(I),$$

where

$$\Gamma(I) \equiv \pi \left[\frac{\gamma^m(I)}{\pi} \frac{R}{I} \right]^{1-\rho} + (1-\pi) \int_{\underline{x}}^{\bar{x}} \left(\frac{x\left[1-\gamma^m(I)\right]}{1-\pi} \right)^{1-\rho} f(x)\,dx.$$

Given $\rho < 1$, the condition for the nonoptimality of the Friedman rule, $W'(I)|_{I=1} > 0$, can be written

$$\frac{I\Omega'}{\Omega} > -\frac{1}{1-\rho} \frac{\Gamma' I}{\Gamma}. \tag{5.7}$$

Equation (5.7) indicates that the optimality of the Friedman rule depends essentially on the relative magnitudes of the interest elasticities of Ω and Γ. The elasticity on the left-hand side is

$$\left[\frac{\gamma^m(1)}{1+(R-1)\gamma^m(1)} \right] \left(R + \left(\frac{1-\rho}{\rho} \right) (R-1) \left[1 - \gamma^m(1) \right] \right),$$

and the one on the right is

$$\frac{\pi^{\rho}\left[\gamma^{m}(1)R\right]^{1-\rho}}{\pi^{\rho}\left[\gamma^{m}(1)R\right]^{1-\rho}+(1-\pi)^{\rho}\int_{\underline{x}}^{\overline{x}}x\left[1-\gamma^{m}(1)\right]^{1-\rho}f(x)\,dx}.$$

After some algebra, (5.7) reduces to

$$\gamma^{m}(1)\left\{\frac{R\gamma^{m}(1)+\left[1-\gamma^{m}(1)\right]\left[1+\frac{R-1}{\rho}\right]}{R\gamma^{m}(1)+\left[1-\gamma^{m}(1)\right]}\right\}>\gamma^{m}(1),$$

which always holds since $R>1$. Thus, the Friedman rule is suboptimal.

Here, as in Smith (2002) and Paal and Smith (2000), the Friedman rule is not optimal in this economy because it makes currency too good an asset. In an equilibrium with $I = 1$, banks hold too small a share of their portfolios in storage and too large a share in currency compared to what is optimal. Since it is not optimal for all government liabilities to bear a zero rate of return, the remainder of this paper considers only equilibria in which all government bonds bear interest.

6. Steady States with *I* > 1 and Money Financing

This section and the next consider how the size of the stock of government debt outstanding affects steady-state welfare when $I > 1$. As noted in the introduction, a central issue concerns how the government raises the revenue needed to cover its interest obligations on its debt. This section considers the scenario in which taxation remains fixed as μ is varied. By implication, the government must finance its interest payments with seigniorage revenue. This section starts with a local analysis, considering the welfare implications of introducing a stock of risk-free government bonds into an economy that has no government bonds outstanding (that is, one with $\mu = 0$). General results are derived from the local analysis. The section proceeds to consider the implications of introducing government bonds into an economy in which some bonds already existing. Because of the complexity of this problem, numerical methods are used to solve it.

6.1 Local Analysis

As already stated, with $I > 1$, $\hat{\gamma}^m = 0$. Substituting (5.5) into (5.6) yields the government budget constraint with money financing of interest obligations $\mu = 0$:

$$\left(1-\left(\frac{p_{t+1}}{p_t}\right)^{-1}\right)\left(1+\left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\,dx\right]^{\frac{1}{\rho}}\right)^{-1} = \frac{g-\tau}{\omega-\tau}. \tag{6.1}$$

The left side of (6.1) is seigniorage revenue. If $\rho < 1$, then (6.1) describes a relatively conventional seigniorage Laffer curve, as depicted in Figure 2.

Seigniorage revenue is maximized at the inflation rate, denoted $\left(p_{t+1}/p_t\right)^*$, that satisfies

$$1+\left(\frac{1}{\rho}\right)\left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\,dx\right]^{\frac{1}{\rho}} - \left(\frac{1-\rho}{\rho}\right)\left(\frac{1-\pi}{\pi}\right)\left(\frac{p_{t+1}}{p_t}\right)^{\frac{1}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\,dx\right]^{\frac{1}{\rho}} = 0. \tag{6.2}$$

It follows that a steady-state equilibrium exists (that is, (6.1) has nonnegative solutions for p_{t+1}/p_t, given $(g-\tau)/(\omega-\tau)$) iff the government's primary budget deficit does not exceed the maximum revenue that can be raised from seigniorage:

$$\left(1-\left(\left(\frac{p_{t+1}}{p_t}\right)^*\right)^{-1}\right)\left(1+\left(\frac{1-\pi}{\pi}\right)\left(\left(\frac{p_{t+1}}{p_t}\right)^*\right)^{\frac{1-\rho}{\rho}}\left[\int_{\underline{x}}^{\bar{x}} x^{1-\rho} f(x)\,dx\right]^{\frac{1}{\rho}}\right)^{-1} \geq \frac{g-\tau}{\omega-\tau}. \tag{6.3}$$

Equation (6.3) is assumed to be satisfied. It follows that (6.1) has at least one solution. Moreover, if (6.3) is satisfied as a strict inequality, and if $\rho < 1$, then (6.1) has two solutions. The solution with a low (high) inflation rate lies on the "good" ("bad") side of the Laffer curve. As Figure 2 indicates, higher values of g, given τ, lead to higher (lower) steady-state rates of inflation on the good (bad) side of the Laffer curve.

Following the discussion in Section 5, steady-state welfare is given by (5.1). With money financing, not only are R, $\tilde{\gamma}^m$, and $\hat{\gamma}^m$ functions of μ, but p_t/p_{t+1} is as well. These functions are implicitly defined by equations (4.3) through (4.6). It remains to evaluate $W'(\mu)|_{\mu=0}$. When this derivative is positive, steady-state welfare will be increased if the government introduces some (though perhaps a small amount) safe government debt.

With τ fixed, the envelope theorem implies that

$$\frac{W'(\mu)}{(\omega-\tau)^{1-\rho}}=\pi^{\rho}\left(\gamma^{m}\right)^{1-\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\frac{\partial\left(\frac{p_t}{p_{t+1}}\right)}{\partial\mu}+(1-\pi)^{\rho}\gamma^{b}\left(\frac{\partial R}{\partial\mu}\right)\int_{\underline{x}}^{\bar{x}}\left[\frac{R\gamma^{b}+x\left(1-\gamma^{m}-\gamma^{b}\right)}{1-\pi}\right]^{-\rho}f(x)\,dx.$$

Thus, since $\gamma^{b}=\mu\gamma^{m}$,

$$\frac{W'(0)}{(\omega-\tau)^{1-\rho}}=\pi^{\rho}\left(\gamma^{m}\right)^{1-\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}, \tag{6.4}$$

with γ^{m} given by (5.4) and p_t/p_{t+1} given by (6.1). Moreover, as is clear from (6.4), $W'(0)$ has the same sign as $\left.\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\right|_{\mu=0}$. Thus, as μ is increased slightly above zero, the effect on steady-state welfare depends on the effect of higher values of μ on p_t/p_{t+1}. To analyze the social value of risk-free government debt when interest obligations on that debt are monetized, it remains, then, to determine the sign of $\left.\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\right|_{\mu=0}$.

Differentiating the government budget constraint (equation (4.3)) with respect to μ yields

$$\gamma^{b}\frac{\partial R}{\partial\mu}+(R-1)\frac{\partial\gamma^{b}}{\partial\mu}=\left(1-\frac{p_t}{p_{t+1}}\right)\frac{\partial\gamma^{m}}{\partial\mu}-\gamma^{m}\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}.$$

Evaluating this expression at $\mu=0=\gamma^{b}$ gives

$$\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}=-(R-1)+\left(1-\frac{p_t}{p_{t+1}}\right)\left(\frac{1}{\gamma^{m}}\right)\left.\frac{\partial\gamma^{m}}{\partial\mu}\right|_{\mu=0}. \tag{6.5}$$

Using (4.6), (4.4) can be rewritten as

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^{m}\right)^{-\rho}=(1-\pi)^{\rho}R\int_{\underline{x}}^{\bar{x}}\left[R\gamma^{b}+x\left(1-\gamma^{m}-\gamma^{b}\right)\right]^{-\rho}f(x)\,dx. \tag{6.6}$$

Differentiating (4.6) and (6.6) with respect to μ yields

$$\begin{aligned}&\frac{\partial}{\partial\mu}\left\{R\int_{\underline{x}}^{\bar{x}}\left[R\gamma^{b}+x\left(1-\gamma^{m}-\gamma^{b}\right)\right]^{-\rho}f(x)\,dx\right\}=\\&\qquad-\rho\int_{\underline{x}}^{\bar{x}}\left\{x\left[(R-x)\frac{\partial\gamma^{b}}{\partial\mu}+\gamma^{b}\frac{\partial R}{\partial\mu}-x\frac{\partial\gamma^{m}}{\partial\mu}\right]\left[R\gamma^{b}+x\left(1-\gamma^{m}-\gamma^{b}\right)\right]^{-(1+\rho)}\right\}f(x)\,dx\end{aligned} \tag{6.7}$$

and

$$(1-\rho)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left(\gamma^m\right)^{-\rho}\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}-\rho\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\frac{\partial\gamma^m}{\partial\mu}$$
$$=(1-\pi)^{\rho}\frac{\partial}{\partial\mu}\left\{R\int_{\underline{x}}^{\overline{x}}\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho}f(x)\,dx\right\}. \tag{6.8}$$

Substituting (6.7) into (6.8) and evaluating the result at $\mu=0=\gamma^b$ gives

$$(1-\rho)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\left(\gamma^m\right)^{-\rho}\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}-\rho\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}=$$
$$-(1-\pi)^{\rho}\rho\int_{\underline{x}}^{\overline{x}}\left\{x^{-\rho}\left(1-\gamma^m\right)^{-(1+\rho)}\left[(R-x)\gamma^m-x\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\right]\right\}f(x)\,dx. \tag{6.9}$$

Using the fact that, with $\mu=0=\gamma^b$, (4.6) becomes

$$\int_{\underline{x}}^{\overline{x}}(R-x)x^{-\rho}f(x)\,dx=0,$$

equation (6.9) can be written (after rearranging terms) as

$$\left(\frac{1-\rho}{\rho}\right)\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{-\rho}\gamma^m\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}=$$
$$\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left\{\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}+(1-\pi)^{\rho}\left(\frac{\gamma^m}{1-\gamma^m}\right)^{1+\rho}\int_{\underline{x}}^{\overline{x}}x^{1-\rho}f(x)\,dx\right\}. \tag{6.10}$$

Since (5.4) implies that

$$\left[\frac{\gamma^m}{1-\gamma^m}\right]^{1+\rho}=\pi^{1+\rho}\left\{(1-\pi)^{\rho}\left[\int_{\underline{x}}^{\overline{x}}\left[x\left(\frac{p_{t+1}}{p_t}\right)\right]^{1-\rho}f(x)\,dx\right]\right\}^{-\left(\frac{1+\rho}{\rho}\right)}$$

when $\mu=0$, (6.10) can be written as

$$\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_t}{p_t+1}\right)^{-1}\gamma^m\left.\frac{\partial\left(p_t/p_{t+1}\right)}{\partial\mu}\right|_{\mu=0}=$$
$$\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left\{1+\pi\left(\frac{p_t}{p_{t+1}}\right)^{\frac{1-\rho}{\rho}}\left[(1-\pi)^{\rho}\int_{\underline{x}}^{\overline{x}}x^{1-\rho}f(x)\,dx\right]^{\frac{-1}{\rho}}\right\}, \tag{6.11}$$

providing a second equation in $\left.\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\right|_{\mu=0}$ and $\left.\left(\partial\gamma^m/\partial\mu\right)\right|_{\mu=0}$.

Solving (6.5) and (6.11) for $\left.\left(\partial\gamma^m/\partial\mu\right)\right|_{\mu=0}$ yields

$$-\left(\frac{1-\rho}{\rho}\right)\left(\frac{p_{t+1}}{p_t}\right)\gamma^m(R-1)=\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\left(\frac{1}{\rho}\right)\left\{1-H\left(\frac{p_{t+1}}{p_t}\right)\right\}, \tag{6.12}$$

where

$$H\left(\frac{p_{t+1}}{p_t}\right) \equiv (1-\rho)\left(\frac{p_{t+1}}{p_t}\right) - \rho\pi\left(\frac{p_{t+1}}{p_t}\right)^{\frac{\rho-1}{\rho}}\left[(1-\pi)^{\rho}\int_{\underline{x}}^{\overline{x}} x^{1-\rho} f(x)\,dx\right]^{\frac{-1}{\rho}}.$$

Some properties of the function H will prove useful in the subsequent analysis. These are stated in Lemma 1.

Lemma 1. (a) $H(1) < 1$. (b) When $\rho < 1$, $H'(p_{t+1}/p_t) > 0$. (c) $H\left[(p_{t+1}/p_t)^*\right] = 1$.

Lemma 1 follows directly from inspection of $H(p_{t+1}/p_t)$ and from the observation that the condition $H(p_{t+1}/p_t) = 1$ is exactly condition (6.2) and therefore satisfied for $p_{t+1}/p_t = (p_{t+1}/p_t)^*$.

From (6.12) it is clear that the sign of $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0}$ depends on the magnitude of ρ and of $H(p_{t+1}/p_t)$. It is instructive to consider two cases: $\rho < 1$ and $\rho > 1$.

Case 1: $\rho < 1$. If $\rho < 1$, then (6.11) implies that $\left.\left(\partial(p_t/p_{t+1})/\partial\mu\right)\right|_{\mu=0}$ has the same sign as $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0}$. Thus, $W'(0) > 0$ iff $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0} > 0$. Moreover, since $R > 1 > \rho$, it is evident from (6.12) that $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0} > 0$ iff $H(p_{t+1}/p_t) > 1$. The effect of μ on welfare depends, then, on whether $H(p_{t+1}/p_t) > 1$. Lemma 1 asserts that $H(p_{t+1}/p_t) > 1$ iff $p_{t+1}/p_t > (p_{t+1}/p_t)^*$. This can transpire only if $g > \tau$ (that is, if there is a primary government budget deficit), and if the economy's equilibrium lies on the wrong side of the Laffer curve. These observations are summarized in the following propositions.

Case 2: $\rho > 1$. When $\rho > 1$, (6.11) implies that the derivatives $\left.\left(\partial(p_t/p_{t+1})/\partial\mu\right)\right|_{\mu=0}$ and $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0}$ are opposite in sign. In addition, (6.12) implies that $\left.(\partial\gamma^m/\partial\mu)\right|_{\mu=0} > 0$ because $H(p_{t+1}/p_t) < 0$ necessarily. Thus, $\left.\left(\partial(p_t/p_{t+1})/\partial\mu\right)\right|_{\mu=0} < 0$ and consequently $W'(0) < 0$. That is, when agents are relatively risk averse, there are no welfare gains from introducing a small stock of risk-free government debt into an economy.

Proposition 1. (a) If $\rho < 1$ and $g \le \tau$, then given $R > 1$, $\left.\partial\gamma^m/\partial\mu\right|_{\mu=0} < 0$, $\left.\left(\partial(p_t/p_{t+1})/\partial\mu\right)\right|_{\mu=0} < 0$, and thus $W'(0) < 0$. (b) If $\rho < 1$ and $g > \tau$, given $R > 1$, then

$\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0} < 0$, $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0} < 0$, and thus $W'(0) < 0$ in equilibria on the good side of the Laffer curve. On the bad side of the Laffer, $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0} > 0$, $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0} > 0$, and $W'(0) > 0$.

Proposition 2. If $\rho > 1$, then given $R > 1$, $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0} > 0$, $\left(\partial\left(p_t/p_{t+1}\right)/\partial\mu\right)\big|_{\mu=0} < 0$, and thus $W'(0) < 0$.

Proposition 1(a) states a strong result. When $R > 1 > \rho$, introducing a small stock of interest-bearing, risk-free government bonds into an economy without bonds necessarily reduces steady-state welfare unless there is a primary budget deficit. Thus, with budget surpluses, which imply that any debt issued will ultimately vanish, agents will not benefit from actions taken to maintain a small stock of debt if the interest payments on the debt will be financed with seigniorage.

The existence of a primary budget deficit, however, does not necessarily imply that introducing a small stock of government debt will increase welfare. As Proposition 1(b) indicates, only in equilibria on the bad side of the Laffer curve does an increase in the bond-to-money ratio raise the rate of return on currency and thus welfare. There unpleasant monetarist arithmetic (Sargent and Wallace 1981 or Bhattacharya, Guzman, and Smith 1998) does not hold because the introduction of government debt has two beneficial consequences. It introduces a risk-free asset, providing insurance against the rate-of-return risk associated with the economy's real asset. It also reduces the inflation tax rate, lowering the cost of holding currency as insurance against the risk of relocation. Both consequences are welfare improving. In contrast, on the good side of the Laffer curve, unpleasant monetarist arithmetic does hold. The introduction of government debt gives agents beneficial insurance against rate-of-return risk in that region as well, but it also has the negative—and offsetting—effect of raising the inflation tax rate. The net effect is a reduction in welfare.

Proposition 2 simply reflects the fact that reserve demand is increasing in the nominal interest rate because the income effect dominates the substitution effect when $\rho > 1$ and $\mu = 0$. In this case, the Laffer curve is strictly upward sloping, so there is no bad side of the curve. Introducing a small stock of bonds into the economy thus reduces welfare.

6.2 Global Analysis

The impact on W of increasing μ from any level, given $I > 1$ and thus $\hat{\gamma}^m = 0$, can be determined by fixing $(g-\tau)/(\omega-\tau)$ and solving (3.4) through (3.6) and (4.3) for γ^m, R, and p_{t+1}/p_t as functions of μ and using $\gamma^b = \mu\gamma^m$ to obtain γ^b. Since closed-form solutions for this system cannot be derived, numerical methods are used. As discussed in section 6.1, not all values of $(g-\tau)/(\omega-\tau)$ can be financed in equilibrium. Consequently, the first step in solving the model numerically is to derive the Laffer curve under a particular parameterization. Attention can then be restricted to the effect on equilibrium portfolio allocations and welfare of increases in μ, given values of $(g-\tau)/(\omega-\tau)$ that are feasible for the parameterized economy.

The numerical analysis was conducted for a wide range of the parameter space. All parameterizations with $R > 1$ yielded the same qualitative results, and $R > 1$ is assumed in the analysis. Thus, only the results for two parameterizations are presented here, one with $\rho < 1$ and one with $\rho > 1$. When $\rho < 1$, the substitution effect of a change in the nominal interest rate on the reserve-deposit ratio dominates the income effect, which is generally thought to be the case. However, many estimates of the coefficient of relative risk aversion put it between one and two. In each example shown, the distribution of x is assumed to be Prob($x = x_l$) = Prob($x = x_h$) = 1/2.

Figure 3 depicts the seigniorage Laffer curves net of the government's interest obligations for select values of μ for $\rho < 1$. When $\mu = 0$, 100 percent of seigniorage revenue is available to finance the deficit since there are no bonds on which to pay interest. Thus, the Laffer curve has its traditional shape, reflecting the dominance of the substitution effect of a change in the nominal interest rate on reserve demand. That is, for a given value of μ, as the inflation rate rises, the nominal interest rate rises as well. Because the substitution effect dominates, this reduces the reserve-deposit ratio. The rising inflation rate and falling reserve-deposit ratio give the curve its typical hump shape.

As μ increases, the real interest rate rises to restore equilibrium in the bond market. Together, the increase in the stock of bonds and the real interest rate drive up the government's interest obligations. With more seigniorage revenue going to finance the government's interest expenses, less revenue is available to finance a deficit. Thus, the maximum budget deficit consistent with equilibrium shrinks, shifting the Laffer curve down. As μ increases, the government's interest expense, which is increasing at a decreasing rate, becomes the more dominant term in the government's budget constraint. Its role in the budget constraint ultimately distorts the shape of the curve. This is apparent in Figure 3.

Figure 4 shows the effect of changes in μ on welfare for various values of $(g-\tau)/(\omega-\tau)$ and the same parameter values used to construct Figure 3. For a given value of $(g-\tau)/(\omega-\tau)$, welfare is decreasing in μ on the good side of the Laffer curve for the same reasons described in the local analysis: the benefits that bonds provide in terms of insurance against the risky return on storage are offset by the costs in reduced insurance against relocation due to the higher inflation rate. Similarly, for a given μ, the bigger the primary budget deficit, the more revenue the government must raise via seigniorage. As a result, welfare declines as the primary budget deficit increases.

Figures 5 and 6 show the seigniorage Laffer curves, net of interest obligations, and welfare for the case of $\rho > 1$. The income effect of a change in the nominal interest is dominant when $\rho > 1$. This means that along a Laffer curve in Figure 5, as the inflation rate rises, the nominal interest rate rises also and drives up the reserve-deposit ratio. As a result, when $\mu = 0$, so there are no interest obligations to finance, the dominant income effect causes the Laffer curve to slope up everywhere. But as before, as μ increases, the government must use an increasing share of its seigniorage revenue to make the promised interest payments on its bonds. The Laffer curve again shifts down and its shape becomes distorted. In this case, however, it takes on a more typicial shape as μ increases because the government's interest obligations, which increase with the inflation rate, drag the right end of the curve down more than the left. This allows the possibility of the economy being on the bad side of the Laffer curve when $\mu > 0$, a possibility that does not exist when $\mu = 0$.

Figure 6 shows welfare as a function of μ for the parameter values used to construct Figure 5. Welfare is again decreasing in μ, assuming the economy is operating on the good side of the Laffer curve. This is due solely to the fact that when $R > 1$, regardless of the value of ρ, the Laffer curve shifts down as μ increases.

The results apparent in Figures 3 through 6 do not hold if the economy is on the bad side of the Laffer curve or if $R < 1$. On the bad side of the Laffer curve the results are reversed, as discussed in section 6.1. As μ increases, the demand for money rises, the inflation tax falls, and welfare increases. And if $R < 1$, then welfare also increases, exactly as in section 6.1, because the government earns interest on its bonds. These interest earnings allow the government to finance its expenditures with a lower inflation rate, so the Laffer curve shifts up and bank deposits offer better insurance against the risk of relocation.

7. Steady States with $I > 1$ and Tax Financing

This section considers the social value of government debt when the government must finance its obligations by varying τ, which in this context is a nondistortionary tax. Under this financing scenario, the rate of money creation is held fixed as μ is varied. Only the local analysis with $\mu = 0$, as in Section 6.1, is presented here. The results from the global analysis (with $\mu > 0$) are the same as those from the local analysis, as is the intuition for them, so they are not presented.

To examine the social value of risk-free government debt when interest obligations on the debt are tax financed, τ is allowed to change along with μ while the rate of money creation, and hence the steady-state inflation rate, remains constant. Under this scenario, the envelope theorem implies that

$$W'(\mu)=-(\omega-\tau)^{-\rho}\left(\frac{\partial\tau}{\partial\mu}\right)\left\{\pi\left[\left(\frac{\gamma^m}{\pi}\right)\left(\frac{p_t}{p_{t+1}}\right)\right]^{1-\rho}+(1-\pi)\int_{\underline{x}}^{\overline{x}}\left[\frac{R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)}{1-\pi}\right]^{1-\rho}f(x)\,dx\right\}$$
$$+\left(\frac{\partial R}{\partial\mu}\right)(\omega-\tau)^{1-\rho}(1-\pi)^{\rho}\gamma^b\int_{\underline{x}}^{\overline{x}}\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-\rho}f(x)\,dx.$$

Evaluating this derivative at $\mu=0=\gamma^b$ gives

$$W'(0)=-(\omega-\tau)^{-\rho}\left(\frac{\partial\tau}{\partial\mu}\right)\bigg|_{\mu=0}\left\{\pi\left[\left(\frac{\gamma^m}{\pi}\right)\left(\frac{p_t}{p_{t+1}}\right)\right]^{1-\rho}+(1-\pi)\int_{\underline{x}}^{\overline{x}}\left[\frac{x\left(1-\gamma^m\right)}{1-\pi}\right]^{1-\rho}f(x)\,dx\right\}.$$

The sign of $W'(0)$ is the opposite of the sign of $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0}$. Thus, it remains to determine the sign of $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0}$.

Differentiating the government budget constraint (4.3) with respect to μ yields

$$(R-1)\frac{\partial\gamma^b}{\partial\mu}+\gamma^b\frac{\partial R}{\partial\mu}=\left(1-\frac{p_t}{p_{t+1}}\right)\frac{\partial\gamma^m}{\partial\mu}+\left[\frac{\omega-g}{(\omega-\tau)^2}\right]\frac{\partial\tau}{\partial\mu}.$$

Evaluating this expression at $\mu=0=\gamma^b$ and using $\left(\partial\gamma^b/\partial\mu\right)\big|_{\mu=0}=\gamma^m$ gives

$$(R-1)\gamma^m=\left(1-\frac{p_t}{p_{t+1}}\right)\frac{\partial\gamma^m}{\partial\mu}\bigg|_{\mu=0}+\left[\frac{\omega-g}{(\omega-\tau)^2}\right]\frac{\partial\tau}{\partial\mu}\bigg|_{\mu=0}. \tag{7.1}$$

Differentiating (3.4) with respect to μ yields

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\gamma^m\right)^{-(1+\rho)}\frac{\partial\gamma^m}{\partial\mu}$$
$$=(1-\pi)^{\rho}\int_{\underline{x}}^{\overline{x}}\left\{x\left[R\gamma^b+x\left(1-\gamma^m-\gamma^b\right)\right]^{-(1+\rho)}\left[(R-x)\frac{\partial\gamma^b}{\partial\mu}+\gamma^b\frac{\partial R}{\partial\mu}-x\frac{\partial\gamma^m}{\partial\mu}\right]f(x)\,dx\right\}.$$

Evaluating this at $\mu=0=\gamma^b$ and using $\left(\partial\gamma^b/\partial\mu\right)\big|_{\mu=0}=\gamma^m$ yields

$$\begin{aligned}\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}&\left(\gamma^m\right)^{-(1+\rho)}\frac{\partial\gamma^m}{\partial\mu}\bigg|_{\mu=0}\\&=(1-\pi)^{\rho}\gamma^m\left(1-\gamma^m\right)^{-(1+\rho)}\int_{\underline{x}}^{\overline{x}}(R-x)x^{-\rho}f(x)\,dx\\&-(1-\pi)^{\rho}\left(1-\gamma^m\right)^{-(1+\rho)}\left(\frac{\partial\gamma^m}{\partial\mu}\bigg|_{\mu=0}\right)\int_{\underline{x}}^{\overline{x}}x^{1-\rho}f(x)\,dx.\end{aligned} \tag{7.2}$$

Finally, when $\mu=0=\gamma^b$, equation (4.6) implies that

$$\int_{\underline{x}}^{\overline{x}}(R-x)x^{-\rho}f(x)\,dx=0.$$

Thus, (7.2) reduces to

$$\pi^{\rho}\left(\frac{p_t}{p_{t+1}}\right)^{1-\rho}\left(\frac{1-\gamma^m}{\gamma^m}\right)^{(1+\rho)}\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}=-\left(1-\pi\right)^{\rho}\left(\left.\frac{\partial\gamma^m}{\partial\mu}\right|_{\mu=0}\right)\int_{\underline{x}}^{\bar{x}}x^{1-\rho}f\left(x\right)dx.$$

Clearly, this condition implies that $\left(\partial\gamma^m/\partial\mu\right)\big|_{\mu=0}=0$. Therefore, from (7.1),

$$\left[\frac{\omega-g}{\left(\omega-\tau\right)^2}\right]\left.\frac{\partial\tau}{\partial\mu}\right|_{\mu=0}=\left(R-1\right)\gamma^m.$$

Proposition 3 follows directly:

Proposition 3. Given $R>1$, $\left(\partial\tau/\partial\mu\right)\big|_{\mu=0}>0$ and thus $W'\left(0\right)<0$.

In short, if $R>1$, and if interest payments on the government debt can be financed with nondistorting taxation, then steady-state welfare is not increased locally by raising government debt levels above zero. Any benefit bonds offer in terms of from better insurance against portfolio risk is outweighed by the cost of the taxation needed to finance the government's interest payments on the bonds.

If $R<1$, this result is reversed. But here, deeper issues arise. If $R<1$, and if τ can be varied, the best policy may be to keep $p_t/p_{t+1}\geq 1$, which may drive agents away from investing in risky storage altogether. Hence, this possibility is not examined further.

8. Conclusion

This paper considers whether an economy is better off with a stock of risk-free government bonds solely because the bonds allow for greater portfolio diversification and thus better insurance against risk. It finds that it is rational for banks to hold interest-bearing risk-free government bonds if they are available. But the benefits from doing so are generally outweighed by the increased cost to society arising from the higher tax rates needed to finance the government's interest payments on those bonds. Only under a number of very specific conditions—including that the economy be operating on the bad side of the Laffer curve—does the introduction of interest-bearing, risk-free bonds raise welfare. It is never optimal, however, to introduce risk-free bonds that do not bear interest (that is, the Friedman rule is not optimal).

Since markets are complete here under appropriate assumptions, these results raise the question of whether some form of market incompleteness is necessary for the availability of government bonds to raise welfare. Certainly, most of the literature on the social value of government debt has assumed incomplete markets in the form of borrowing constraints. That literature easily finds a role for government bonds in relaxing those constraints.

There is another possible role for risk-free government debt, a role not considered here. Some financial market analysts have argued that risk-free government bonds are socially beneficial because their rate of return is a risk-free rate that serves as a benchmark in pricing more risky assets. Others, however, have argued that even if there were such a benefit to having risk-free government, the private sector could easily provide an asset with that attribute. This is certainly a theoretical possibility. Whether it is also a practical possibility is unclear. If it is, then there is no necessary role for the government in issuing bonds even if there is a role for risk-free debt.

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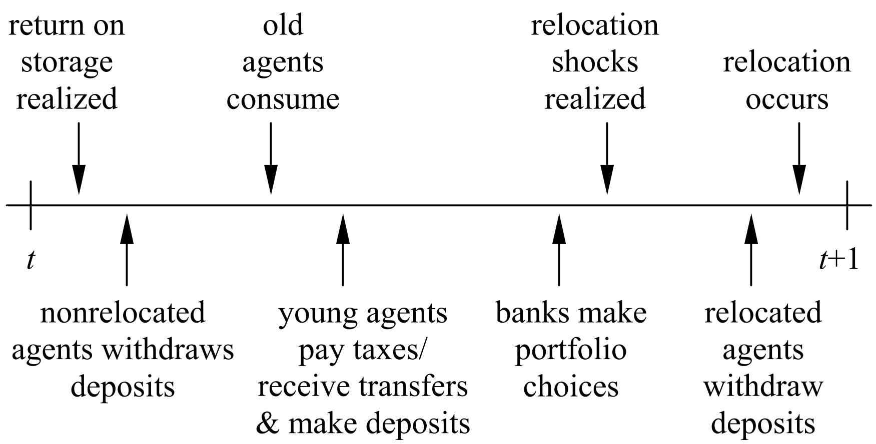

Figure 1—The Timing of Events

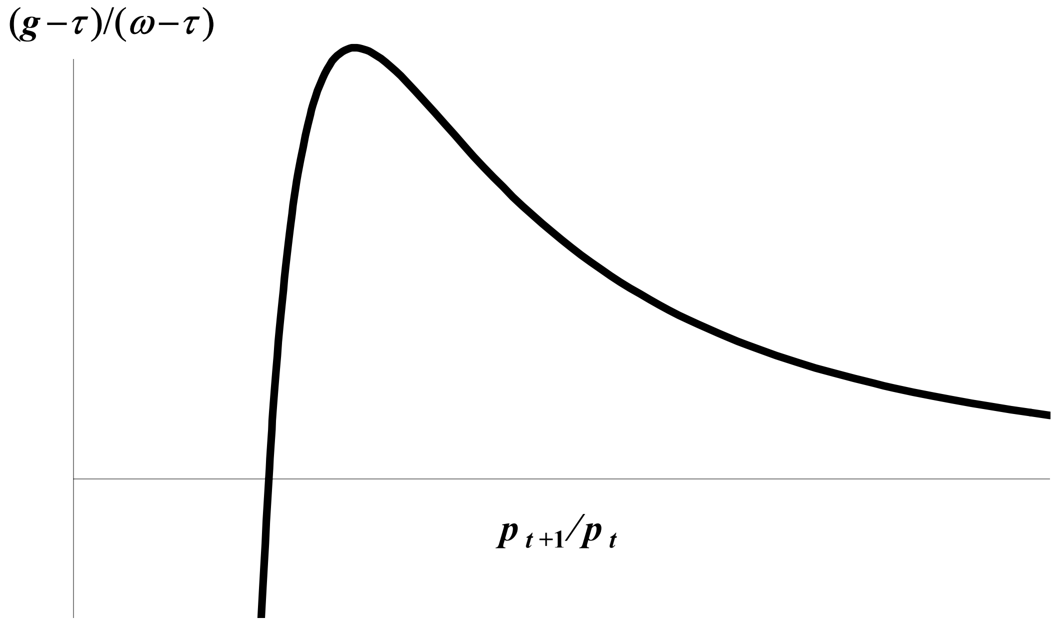

Figure 2—The Seigniorage Laffer Curve

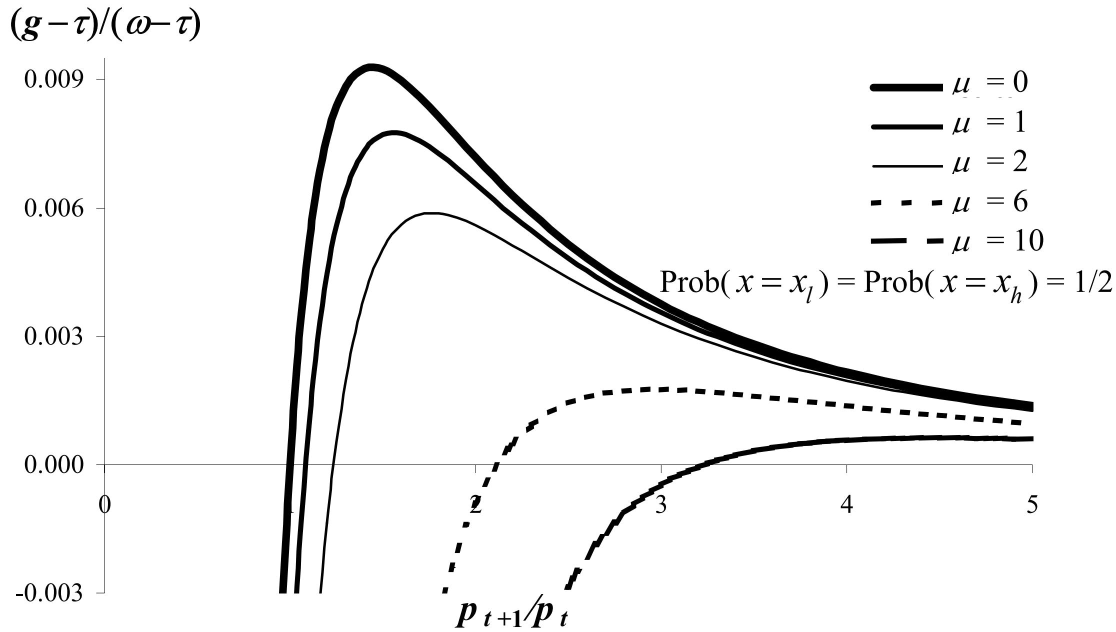

Figure 3—Seigniorage Laffer Curves Net of Interest Obligations for ρ = 0.3, π = 0.1, x_l = 0.1, x_h = 1.56, *q* = 0.2

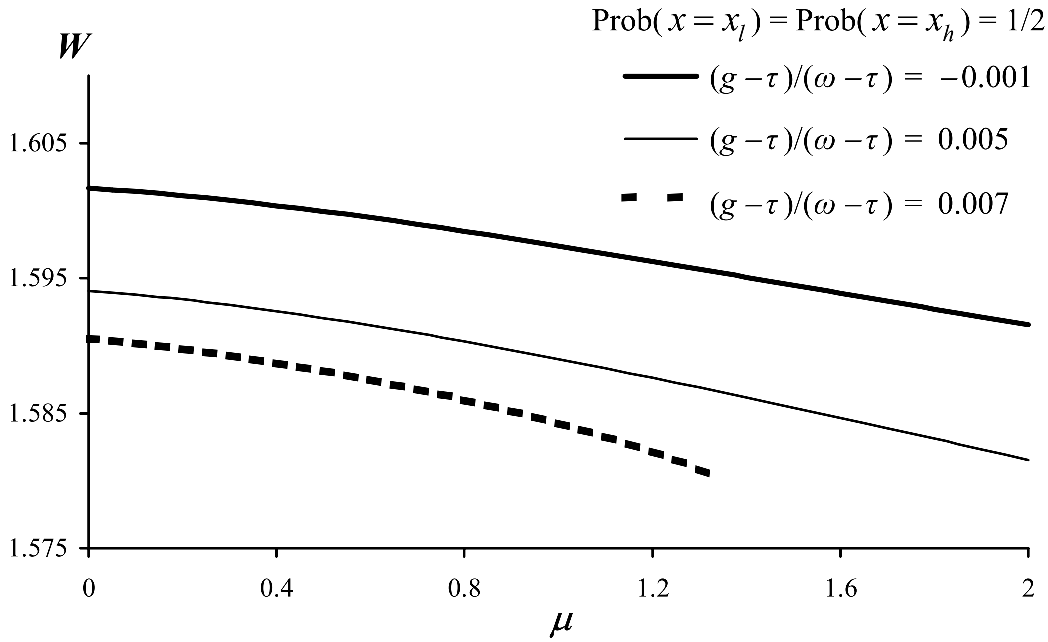

Figure 4—Welfare for ρ = 0.3, π = 0.1, x_l = 0.1, x_h = 1.56, q = 0.2

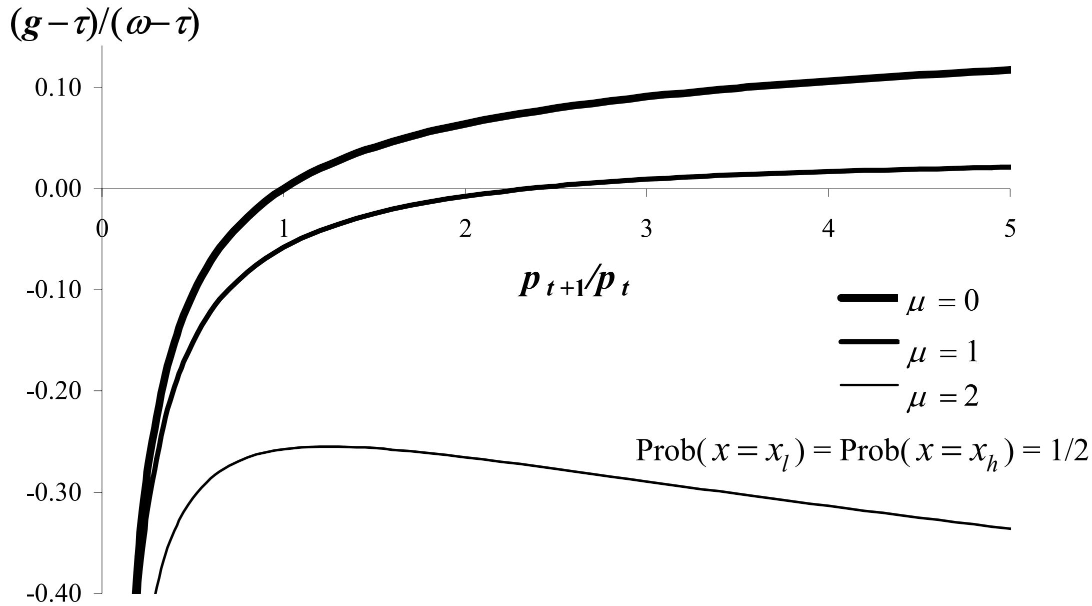

Figure 5—Seigniorage Laffer Curves Net of Interest Obligations
for ρ = 1.2, π = 0.1, x_l = 0.4, x_h = 5.0, q = 0.2

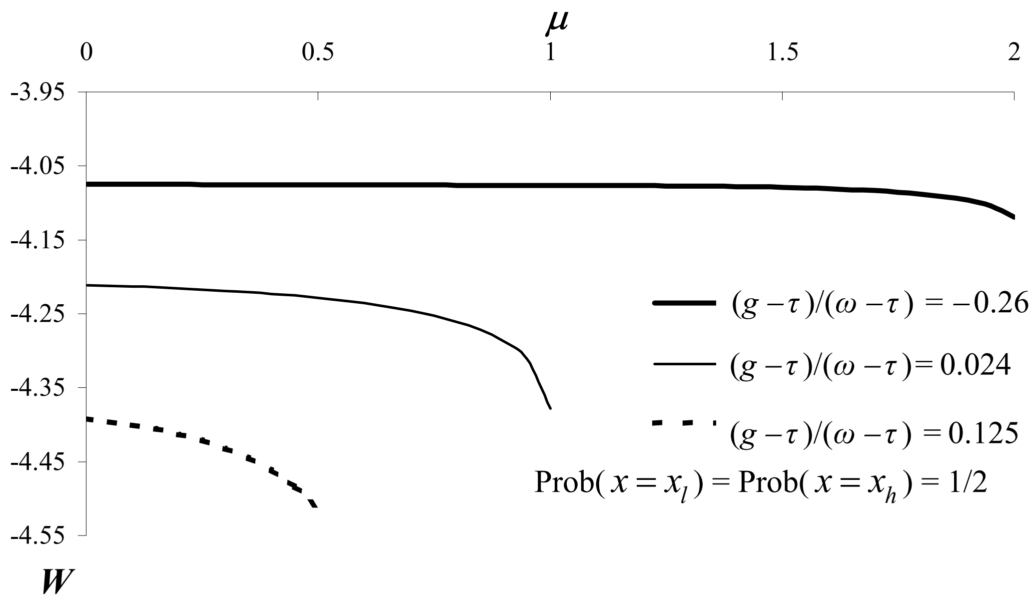

Figure 6—Welfare for ρ = 1.2, π = 0.1, x_l = 0.4, x_h = 5.0, *q* = 0.2