ENDOGENOUS MULTIPLE CURRENCIES

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MAY 2002

RWP 02-03

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Antoine Martin is an economist at the Federal Reserve Bank of Kansas City. He would like to thank V.V. Chari especially for his advice. He also thanks Craig Hakkio, Karsten Jeske, Sharon Kozicki, Stacey Schreft, Kei-Mu Yi, seminar participants at the 2002 Missouri Economic Conference, UQAM, and the University of Western Ontario for useful comments. The views expressed in this paper are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

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Abstract

I study a model of multiple currencies in which sellers can choose the currency they will accept. I provide conditions that are necessary and sufficient to avoid indeterminacy of the exchange rate. Under these assumptions, all stable equilibria have the property that all sellers in the same country accept the same currency. Thus stable equilibria are either single currency or national currencies equilibria. I also show that currency substitution occurs as an endogenous response to high growth in the stock of a currency.

JEL classification: F31, F41

Keywords: Multiple Currencies, Currency Substitution

1 Introduction

Starting with the work of Stockman (1980), Helpman (1981), and Lucas (1982), model economies with multiple currencies often impose cash-in-advance constraints. More precisely, it is usually taken as given that purchases of goods produced in a foreign country must be made with the currency accepted in that country, and the currency that is accepted in a particular country is itself taken as given. In this paper, I provide conditions under which these features can arise endogenously and show that currency substitution occurs when the stock of a currency grows at a sufficiently high rate.

Because I do not impose the currency that sellers accept, I have to face the problem of indeterminacy of the exchange rate, first noted by Kareken and Wallace (1981). I provide necessary and sufficient conditions that eliminate the indeterminacy. I impose a cost of accepting more than one type of currency, as well as a foreign exchange transactions cost. The exchange rate indeterminacy disappears if the cost of accepting more than one currency is high enough so that no household ever chooses to pay it.

Under these assumptions, I show that stable equilibria have the property that all sellers in the same country will accept the same currency. This currency might be the same as the currency accepted by sellers in the other country (single currency equilibrium) or it might be different (national currencies equilibrium).

The intuition for these results is that the assumed costs create a network externality: Sellers choose to accept the currency that generates the least transaction costs and the size of these costs depends on the fraction of sellers that accept each of the currencies. In a national currencies equilibrium, sellers find it cheaper to accept the same currency as the other sellers in their country, rather that accepting the currency of the foreign country, because they spend more on the goods sold by the former than the latter.

I also show that, if the stock of a currency grows too fast, a national currencies equilibrium might fail to exist. Under such an equilibrium, an increase in the rate of growth of currency i means that households in country i receive more of that currency as it is newly issued. In other words, the amount of currency i they need to obtain by selling goods diminishes. If it

becomes small enough, it will be cheaper for these households to accept the foreign currency. The amount of money growth that will induce them to switch is lower if the fraction of income they spend on foreign goods is high. This is because they need a lot of foreign currency to make their purchases. An interpretation of this result is that, as trade increases between countries, monetary authorities have less flexibility in choosing high money growth rates. Another way to think about this result is that countries that have close economic ties are more likely to choose a single currency.

There are few papers that consider the endogenous choice of currency. One example is Cooper and Kempf (2000 a and b), but they assume that all sellers in the same country must accept the same currency. Matsui (1998) also develops a model where the choice of currency is endogenous but he imposes that taxes that must be paid in local currency.

Many authors have investigated cash-in-advance models of multiple currencies. For example, Minford (1995) studies a cash-in-advance economy based on Lucas (1980), Boyer and Kingston (1987) study a credit-good, cash-good economy based on Lucas and Stokey (1987). King, Wallace and Weber (1992) have a model with two currencies where some types of agents can accept only one type of currency while others may choose which one they accept. Fisher (1999) builds a multiple-currency model that is based on the cash-and-credit framework developed by Schreft (1992). In all of these cases, cash-in-advance constraints are assumed and thus the existence of a national currencies equilibrium is assumed rather that derived.

More recently, many authors have studied multiple currencies in the context of search models of money. See for example Camera, Craig and Waller (2001), Kocherlakota and Krueger (1999), Matsuyama, Kiyotaki and Matsui (1993), Soller Curtis and Waller (2000), Ravikumar and Wallace (2001), Trejos and Wright (1996 and 2001), Zhou (1997). There has been a lot of work on currency substitution, a survey of which can be found in Giovannini and Turtelboom (1994). Recent work includes Chang (1994), Siebert and Liu (1998), Sturzenegger (1997), Tandon and Wang (1999), Uribe (1997).

In section 2, I study a simple model with only one country and two currencies. In section 3, I look at a two countries model and show that national currencies equilibria exist. In section 4, I prove that fast money growth can lead to currency substitution. Section 5 concludes.

2 A simple model with two currencies

First I consider a simple model and provide necessary and sufficient conditions that eliminate the indeterminacy of the exchange rate. At each date t, $t \geq 0$, a continuum of identical households resides at each location on a circle with circumference of one. Each household in location z, $z \in [0,1]$, is endowed in every period with $\omega > 0$ units of a non-storable, location-specific good. At date $t = 0$, the households hold equal shares of the units of fiat currency outstanding. There are two fiat currencies and the stock of currency i at date t is denoted by $M_{it}, i = 1, 2$.

It is assumed that an household's preferences at each date can be represented by a period utility function $U(W(c_{tz}))$. The argument c_{tz} is a vector with typical element being consumption of good $z, z \in [0,1]$, at date t. Assume that U is twice differentiable, strictly increasing and strictly concave. For tractability, I assume that consumption goods available at the same date are perfect complements. Thus, $W(c_{tz}) = \inf_z\{c_{tz}\}$. Without loss of generality, the utility-maximizing bundle is taken to be $c_{tz} = c_t$ for all z. Thus, the period utility function may be written as $U(c_t)$, and the representative household's objective is to maximize

$$\max \quad \sum_{t=0}^{\infty} \beta^t U\left(c_t\right), \quad 0 < \beta < 1.$$

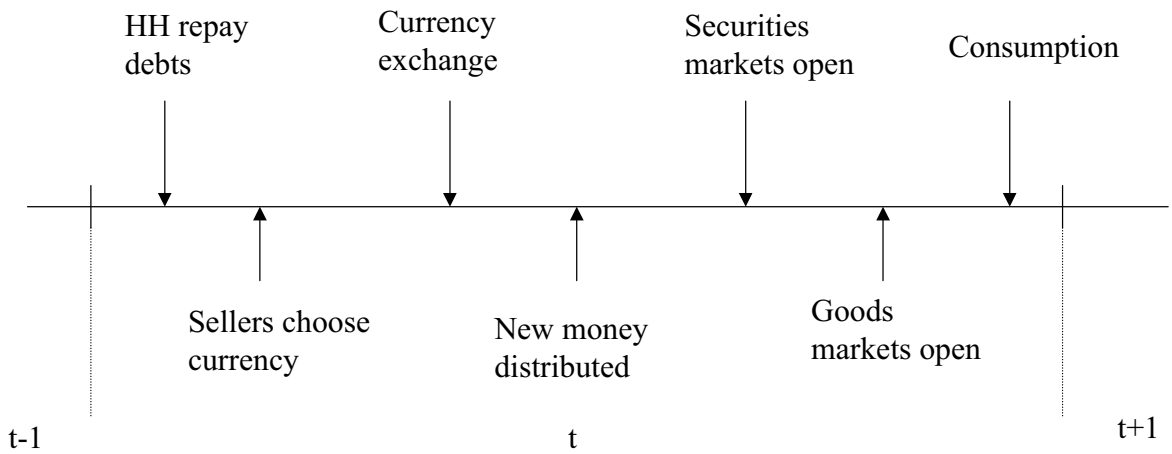
The money supply for each currency evolves according to $M_{it} = \pi_i M_{it-1}$, with $\pi_i \geq 1, i = 1, 2$. Money is distributed equally to each household by means of helicopter drops. In this paper I do not want to assume any preferences for the authority that manages the money supply. I will just consider different levels of π_i and their implications.

Note that there is a perfectly competitive goods market and securities market at each point of the circle. It is convenient to think of each household as consisting of a seller and a shopper. The seller stays home and sells the consumption good, while the shopper travels completely around the circle, buying goods at each point. I impose a cash-in-advance constraint so the the shopper must use cash for all purchases. However, I let the sellers choose which currency they will accept[1].

[1]IThe model can easily be generalized to allow for credit as in Schreft (1992), so that the use of cash is endogenously determined.

The timing, in each period, is as follows: First, households repay outstanding debts. Next sellers decide which currency they will accept and then currency exchange takes place. After that, new money is distributed to all households. Then, the securities markets open and households borrow and lend cash (i.e., exchange currency for claims to currency in the securities market at the next date). After the securities markets close, the goods markets open. Before the period ends, consumption takes place. It is important to note that the securities markets close before the goods markets open, and remain closed until the next period.

Time line



I focus on equilibria in which each good sells for the same price in each currency. I denote by p_t the price of a unit of good in currency 1 at date t and by q_t the price of a unit of good in currency 2 at date t. Let r_{it} denote the net nominal one-period interest rate on securities denoted in currency i at date t.

2.1 The households' problem

Each household solves a two-stage competitive lifetime choice problem. First, at each date, the seller chooses which currency to accept and the shopper chooses an amount of each good to consume. Next, the household chooses sequences for consumption, c_t, holdings of each type of currency at the close of the securities market, m_{it}, holdings of each type of currency at the close of the goods markets, $m_{it}^{'}$, and net lending of one-period securities, denominated in each currency, b_{it}, treating prices, p_t, q_t and r_{it}, parametrically.

So far, I have assumed that it is costless for a seller to accept more than one currency. If all sellers accept both currencies, the households' problem can be written as:

$$\max \quad \sum_{t=0}^{\infty} \beta^t U(c_t), \quad 0 < \beta < 1,$$

subject to

$$\begin{aligned}
b_{1t} + m_{1t} + e_t [b_{2t} + m_{2t}] &\leq m'_{1t-1} + (1 + r_{1t-1}) b_{1t-1} + e_t \left[m'_{2t-1} + (1 + r_{2t-1}) b_{2t-1}\right] \\
&\quad + (\pi_1 - 1) M_{1t-1} + (\pi_2 - 1) e_t M_{2t-1}, \\
m'_{1t} + e_t m'_{2t} &= \left[\bar{\theta} p_t + (1 - \bar{\theta}) e_t q_t\right] \omega + m_{1t} - \theta p_t c_t + e_t [m_{2t} - (1 - \theta) q_t c_t], \\
\theta p_t c_t + (1 - \theta) e_t q_t c_t &\leq m_{1t} + e_t m_{2t}, \\
c_t &\geq 0, \quad \lim_{t \to \infty} \beta^t b_{it} = 0, \quad \lim_{t \to \infty} \beta^t m_{it} = 0, i = 1, 2,
\end{aligned}$$

where θ denotes the fraction of transactions for which the household used currency 1 (currency 2 being used for all other transactions) and $\bar{\theta}$ denotes the fraction of transactions for which this household accepted currency 1.

All constraints are written in terms of currency 1. The first equation states that wealth at the end of the securities trading session, holdings of cash and securities, denominated in each currency, cannot exceed beginning of period wealth. Beginning of period wealth is composed of cash held over from last period, securities bought last period, and injections of new currency. The second equation is an identity that defines end-of-period cash holdings. The third equation is the cash-in-advance constraint. The last three equations are a non-negativity constraint and transversality conditions. The cash-in-advance constraint and the

end-of-period-cash equation can be written as they are because it is costless for sellers to accept more that one currency and for households to change their currency-portfolio allocation. Hence households only care about the total value of their cash holdings, rather than the holdings in each currency

2.2 Equilibrium

A steady state equilibrium is a set of constants $\left(c, m_i^{'}, m_i, b_i, r_i\right), i = 1, 2$, and a value of p_t, q_t and e_t for each $t \geq 0$ for which, given $\omega, M_{10}, M_{20}, \pi_1$ and π_2,

(i) households solve the problem described above;

(ii) The money supply for each currency evolves according to: $M_{it} = \pi_i M_{it-1}, i = 1, 2$;

(iii) each securities markets clear: $b_{it} = 0$ for each location z;

(iv) the money market clears: $M_{it} = m_{it}$;

(v) the foreign exchange market clears: $\theta\left(m_{1t-1}^{1\prime} - m_{1t}^{1}\right) = (1-\theta)\, e_t\left(m_{1t}^{2} - m_{1t-1}^{2\prime}\right)$, and $\theta e_t\left(m_{2t-1}^{1\prime} - m_{2t}^{1}\right) = (1-\theta)\left(m_{2t}^{2} - m_{2t-1}^{2\prime}\right)$, where m_{it}^{j} denotes the holding of currency i by a household whose seller accepts currency j at the close of date t securities market and $m_{it}^{j'}$ denotes the holding of currency i by a household whose seller accepts currency j at the close of date t goods market, $i, j = 1, 2$;

(vi) all goods markets clear: $c_t = \omega$.

An equilibrium with valued fiat money exists since this is an endowment economy with a representative household. In equilibrium, the nominal exchange rate e_t satisfies $p_t = e_t q_t$.

For the rest of the paper I assume that all sellers at one location accept the same currency. Thus, θ denotes the fraction of locations that accepts currency 1. While this saves on notation, I will show that it is without loss of generality. An alternative, and notationally more demanding, specification would be to have a local monopoly at each point of the circle and assume that agents have Dixit-Stiglitz preferences. The result in this paper would hold for this alternative model.

2.3 Indeterminacy of the exchange rate

Proposition 2.1. *If sellers accept both currencies the equilibrium exchange rate is indeterminate.*

The proof of this classic result is omitted. It was first obtained by Kareken and Wallace (1981).

In order to make the sellers decision about which currency to accept interesting, I not impose conditions that eliminate this indeterminacy. I will show that they are necessary and sufficient.

Assumption 1. *A household must pay a fixed cost κ if its seller accepts two currencies.*

Assumption 2. *Foreign exchange transactions are costly: if $m'_{1t-1} \neq m_{1t}$, then the household incurs a real cost equal to $\varepsilon \frac{1}{p_t} |m'_{1t-1} - m_{1t}|$, in terms of good.*

Assumption 3. $\kappa > \frac{N}{2}\varepsilon \left[1 + \frac{N}{2}\varepsilon\right]^{-1} \omega$*, where N, here normalized to 1, is the mass of households in the economy.*

The cost κ must be paid every period. One can think of it as the administrative cost of having to deal with a more complicated accounting system. The decision of which currency to accept is taken every period, and the cost κ is paid in every period the seller decides to accept two currencies. The cost ε can be thought of as a cost of operating the foreign exchange market that must be paid with every transaction. Assumption 3 guarantees that it is always cheaper to pay the cost of foreign exchange than to pay κ and accept both currencies. To obtain the inequality in assumption 3, notice that a household will have to pay $\varepsilon(1-\theta)c_t$ in foreign exchange costs if its seller accepts currency 1 and $\varepsilon\theta c_t$ if it accepts currency 2. It always chooses the cheapest option. Hence, the most the household might have to pay corresponds to the case when these costs are exactly equal. This occurs when $\theta = .5$ and the cost is $\varepsilon c_t \frac{N}{2}$. In equilibrium, c_t is equal to the endowment, ω, minus the amount of resources used to pay for foreign exchange costs. Thus, $c_t = \omega - \varepsilon c_t \frac{N}{2}$. This implies that $c_t = \left[1 + \frac{N}{2}\varepsilon\right]^{-1} \omega$. Provided that assumption 3 holds, both costs can be arbitrarily small, as long as they are strictly positive.

Proposition 2.2. *Under assumptions 1 - 3, there are only three equilibria with valued fiat currency, with either $\theta = 0$, $\theta = 0.5$, or $\theta = 1$.*
If any one of these assumptions does not hold, there are a continuum of equilibria, and the equilibrium exchange rate is indeterminate.

Proof. For a given value of θ, this is just a cash-in-advance economy for which an equilibrium exists. I now show which values of θ are consistent with sellers minimizing the foreign exchange costs of their household.

Given assumptions 1 and 2, assumption 3 guarantees that sellers will accept only one currency because it is always cheaper to pay the foreign exchange cost than to accept two currencies. I now show that θ taking values of $0, 0.5$, or 1 is a necessary condition for an equilibrium. Since foreign exchange transactions are costly, households will choose which currency to accept in order to minimize that cost. Consider a household faced with $\theta \in [0, 0.5)$. It will choose to accept currency 2 because it makes more purchases with currency 2 than with currency 1 and thus it will need to exchange a smaller amount of currency. Since this is true of every household, θ must be equal to zero. Conversely, a household faced with $\theta \in (0.5, 1]$ will choose to accept currency 1, and since this is true for every household, θ must be equal to 1. Finally, a household faced with $\theta = 0.5$ is exactly indifferent between choosing currency 1 or 2 because it makes exactly the same amount of purchases with each currency.

Since equilibria exist only for $\theta = 0$, $\theta = 0.5$, or $\theta = 1$, assumptions 1-3 are sufficient to prevent exchange rate indeterminacy. Now I need to show that assumptions 1-3 are also necessary. If assumption 1 is violated, all sellers accept two currencies and no foreign exchange transaction takes place. Thus, this is similar to the case considered by Kareken and Wallace (1981) and by proposition 2.1, the equilibrium exchange rate is indeterminate. If assumption 2 is violated, the equilibrium exchange rate is uniquely determined by the value of θ. For each θ, this is similar to the "portfolio autarky" case in Kareken and Wallace (1981), as p_t and q_t are determined by $p_t \theta \omega = M_{1t}$ and $q_t (1 - \theta) \omega = M_{2t}$. Also, in equilibrium, $p_t = e_t q_t$. However, the value of θ is not determined in equilibrium.

Now assume that assumptions 1 and 2 hold, but not assumption 3. There are equilibria

such that some sellers accept both currencies. Let η denote the mass of sellers accepting currency 2. Since θ denotes the mass of sellers accepting currency 1, the remainder $(1-\theta-\eta)$ accepts both currencies. In order to minimize their foreign exchange transactions costs, households accepting currency 1 will pay the sellers who accept both currencies with currency 1, and households accepting currency 2 will pay the sellers who accept both currencies with currency 2. In equilibrium, each type of household must have the same costs which implies that $\varepsilon\theta\omega = \kappa = \varepsilon\eta\omega$, and $\theta = \eta$. King, Wallace and Weber (1992) have studied such economies and proved that the equilibrium exchange rate is indeterminate (a consequence of their proposition 1). $\square$

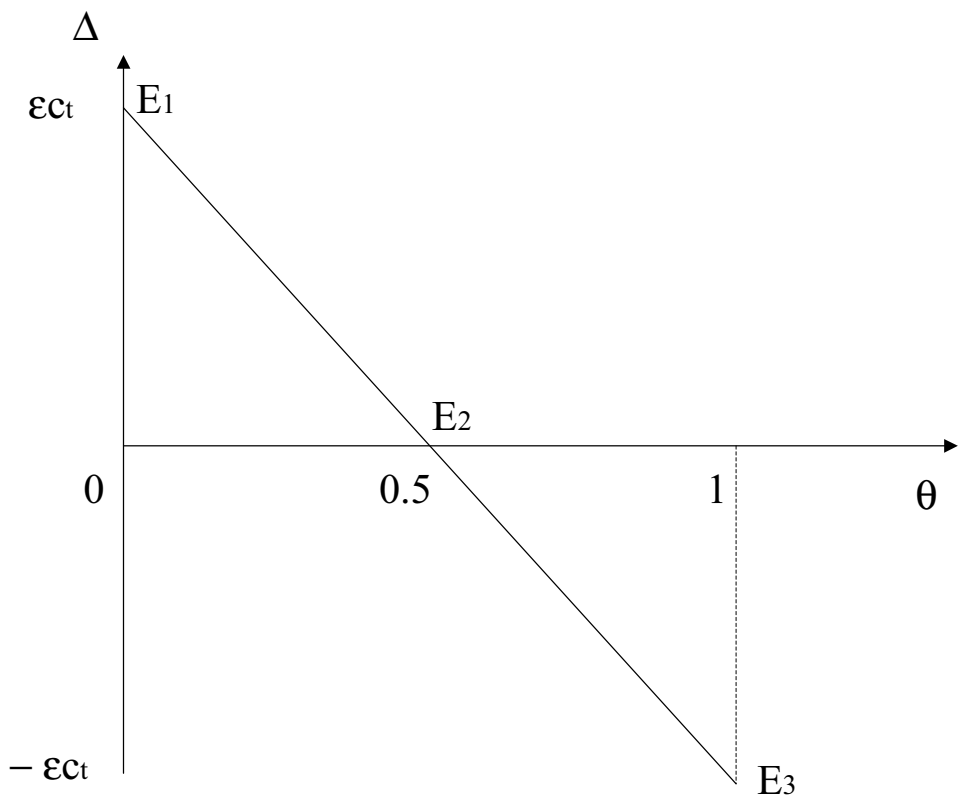
The two currencies will be valued only if exactly half of the sellers accept currency 1 and half the sellers accept currency 2. Otherwise, only one currency will be valued and all sellers will accept that currency. Note that, everything else being equal, welfare is lower in the mixed currency equilibrium because, each period, goods are being wasted, paying for the transaction cost.

In figure 1, the three equilibria of proposition 2.2 are labelled E_1, E_2, and E_3, respectively. This figure graphs $\Delta = \varepsilon(1-2\theta)c_t$, the difference between the cost of accepting currency 1 and the cost of accepting currency 2, as a function of θ. Δ is positive if $\theta \in [0, 0.5)$, because in this case, most sellers accept currency 2, and foreign exchange costs are high for households whose sellers accept currency 1. Conversely, if $\theta \in (0.5, 1]$, then $\Delta < 0$, because most sellers accept currency 1. If $\theta = 0.5$, then $\Delta = 0$ and sellers are indifferent between the two currencies.

Suppose that I had not assumed that all sellers at the same location accept the same currency. Then there would be many equilibria such that exactly half the sellers accept currency 1 and half accept currency 2, with each type arbitrarily distributed around the circle. An example of such an equilibrium has half the sellers in each location accepting currency 1. Another has half the locations with a quarter of the sellers accepting currency 1, while the other locations have three quarters of the sellers accepting currency 1. All these equilibria support the same real allocation, so assuming that all sellers at the same location

accept the same currency is without loss of generality.

Figure 1



As I have done above, I can associate an equilibrium with a value of θ. Abusing terminology, I can say, for example, that $\theta = 1$ is an equilibrium, while $\theta = 0.75$ is not. Let $\hat{\theta}$ denote the common belief of households about the realized value of θ. The equilibria I consider are rational expectations equilibria; they have the property that, $\hat{\theta} = \theta$.

I want to introduce a notion of stability for the equilibria I consider. An equilibrium is unstable if it is not robust to arbitrarily small changes in households beliefs.

Definition 2.3. *An equilibrium θ is unstable if, $\forall \epsilon > 0, |\hat{\theta} - \theta| > \epsilon \Rightarrow \theta$ is not an equilibrium.*

Proposition 2.4. *Equilibria with $\theta = 1$ and $\theta = 0$ are stable, while equilibrium $\theta = .5$ is unstable.*

Proof. As I pointed out in the proof of proposition 2.2, if $\hat{\theta} \in [0, 0.5)$, then $\theta = 0$ is the equilibrium. If $\hat{\theta} \in (0.5, 1]$, then $\theta = 1$ is the equilibrium. This shows that both $\theta = 0$ and

$\theta = 1$ are stable. It also implies that $\theta = 0.5$ is unstable since the only value of $\hat{\theta}$ for which it is an equilibrium is $\hat{\theta} = 0.5$ $\square$

In this section, I showed that one can eliminate exchange rate indeterminacy if foreign exchange transactions are costly (no matter how small the cost) and if accepting more than one currency is sufficiently costly. In this simple model there is no obvious way of thinking about different countries. All households have the same preferences, and consume the same quantities of the same goods. It might not be so surprising then that in the stable equilibria only one currency is valued.

In the next section, I allow consumers to have different preferences. To make this clear, I assume that there are two circles, representing two countries. All households living in the same country have the same preferences, but they can have different preferences than the households in the other country. I show that this is enough to have two valued currencies.

3 Two countries and two currencies

I consider an economy similar to the one of the previous section, but with two countries (represented by two different circles). I prove the existence of a "national currencies" equilibrium, and derives some results about currency substitution.

Each circle is as described above. I will use x_{tz} (y_{tz}) to denote the goods available on the first (second) circle in period t and location z. The endowment of these goods are denoted by ω_x and ω_y, respectively. I assume that there are no markets where households can trade claims on their endowment good.

Preferences of the households in each country can be represented by a utility function, $U(x_t, y_t; \gamma_1)$ in country 1 and $V(x_t, y_t; \gamma_2)$ in country 2, with $\gamma_1, \gamma_2 \in [0, 1]$. I assume that these functions have the following properties:

$$U_y(x_t, y_t; \gamma_1 = 0) = (1 - \gamma)U_y(x_t, y_t; \gamma_1 = \gamma),$$

$$V_x(x_t, y_t; \gamma_2 = 0) = (1 - \gamma)V_x(x_t, y_t; \gamma_2 = \gamma).$$

I want to think of the γ's as capturing some notion of home country bias in consumption. A positive γ_i reduces the marginal utility households in country i get from consuming foreign goods. The larger the γ's, the bigger the bias. In this paper I restrict myself to non negative γ's, but one could, in principle, consider values of γ_1 and γ_2 that are negative. In this case, there would be a foreign country bias. If $\gamma_1 = \gamma_2 = 0$, there is no bias. Another way of thinking of the γ's is as a proxy for how open the countries are to trade. More foreign goods are consumed if the γ's are low.

One can now think of a household as consisting of three members, one seller and two shoppers. The seller stays home and sells the goods the household is endowed with. One shopper goes around the home country circle and makes purchases using cash. The other shopper goes around the foreign country circle.

I assume that new amounts of currency 1 are distributed equally to all households in country 1 while currency 2 is distributed in country 2. I also assume that there is no market where households can trade claims to newly issued currency. Thus, even in the case where all households have the same preferences (if $\gamma_1 = \gamma_2 = 0$ and $U = V$), households in country 1 and 2 are different because they are endowed with different goods and are issued different currencies.

The law of motion for the money supply and the market clearing conditions for the money market and the foreign exchange market are the same as in section 2. Throughout this section I will maintain the assumption that foreign exchange transactions are costly. I assume that both sides to a foreign exchange transaction pay the cost, and that they pay this cost in their endowment goods. The costs need not be the same for households in country 1 and country 2. Let ε_1 denote the cost paid by households in country 1 in terms of good x, and ε_2 denote the cost paid by households in country 2 in terms of good y. Let p_{jt} and q_{jt} denote the price of good j in terms of currency 1 and 2, respectively, $j = x, y$. Let θ_i denote the proportion of sellers accepting currency 1 in country i, $i = 1, 2$. Also, as indicated above, m_{it}^{j} denotes the holdings of currency i by a household whose seller accepts currency j at the close of the date t securities market and $m_{it}^{j'}$ denotes the holding of currency i by a household whose seller accepts currency j at the close of the date t goods market, $i, j = 1, 2$. I use a superscript $*$

to denote variables pertaining to country 2. The condition for the goods markets to clear are

$$\omega_x = x_t + x_t^* + \varepsilon_1 \frac{1}{p_{xt}} \left[\theta_1 \left|m_{1t-1}^{1\prime} - m_{1t}^1\right| + (1-\theta_1)\left|m_{1t-1}^{2\prime} - m_{1t}^2\right|\right],$$
$$\omega_y^* = y_t + y_t^* + \varepsilon_2 \frac{1}{p_{yt}} \left[\theta_2 \left|m_{1t}^{1*\prime} - m_{1t-1}^{1*}\right| + (1-\theta_2)\left|m_{1t}^{2*\prime} - m_{1t-1}^{2*}\right|\right].$$

I can now write the problem of a household. There are 4 different types of households since, in each country, a household has the choice of accepting either currency 1 or currency 2. The problem below is for a household in country 1 that accepts currency 1.

$$\max \quad \sum_{t=0}^{\infty} \beta^t U(x_t, y_t; \gamma_1), \quad 0 < \beta < 1,$$

subject to

$$\begin{aligned}
b_{1t} + m_{1t} + e_t\left[b_{2t} + m_{2t}\right] &\leq m_{1t-1}' + (1+r_{1t-1})\, b_{1t-1} + e_t\left[m_{2t-1}' + (1+r_{2t-1})\, b_{2t-1}\right] \\
&\quad + (\pi_1 - 1)\, M_{1t-1}, \\
m_{1t}' &= p_{xt}\left[\omega_x - \varepsilon_1 \frac{1}{p_{xt}} |m_{1t-1}^{'} - m_t|\right] + m_{1t} - \theta_1 p_{xt} x_t - \theta_2 p_{yt} y_t, \\
e_t m_{2t}' &= e_t m_{2t} - e_t\,(1-\theta_1)\, q_{xt} x_t - e_t\,(1-\theta_2)\, q_{yt} y_t, \\
\theta_1 p_{xt} x_t + \theta_2 p_{yt} y_t &\leq m_{1t}, \\
e_t\,(1-\theta_1)\, q_{xt} x_t + e_t\,(1-\theta_2)\, q_{yt} y_t &\leq e_t m_{2t}, \\
c_t &\geq 0, \quad \lim_{t\to\infty} \beta^t b_{it} = 0, \quad \lim_{t\to\infty} \beta^t m_{it} = 0, i = 1, 2.
\end{aligned}$$

These constraint have the same interpretation as the constraints in section 2. There are now two equations defining end-of-period cash holdings, and two cash-in-advance constraints, one for each currency. The problems of the other households are analogous.

From the first order conditions with respect to b_{1t} and b_{2t}, one gets

$$\frac{1+r_{1t}}{1+r_{2t}} = \frac{e_{t+1}}{e_t},$$

so uncovered interest parity holds because the frictions introduced in the model do not affect

securities. Also, the ratio of marginal utilities, for each type of household, is

$$\frac{U_{xt}(x_t, y_t; \gamma_1)}{(1-\gamma_1)U_{yt}(x_t, y_t; \gamma_1)} = \frac{\theta_1 p_{xt}\left[1 - \frac{\varepsilon_1}{1+\varepsilon_1}\frac{1}{1+r_{1t}}\right] + (1-\theta_1)e_t q_{xt}}{\theta_2 p_{yt}\left[1 - \frac{\varepsilon_1}{1+\varepsilon_1}\frac{1}{1+r_{1t}}\right] + (1-\theta_2)e_t q_{yt}},$$

$$\frac{V_{xt}(x_t^*, y_t^*; \gamma_2)}{(1-\gamma_2)V_{yt}(x_t^*, y_t^*; \gamma_2)} = \frac{\theta_1 p_{xt}\left[1 - \frac{\varepsilon_2}{1+\varepsilon_2}\frac{1}{1+r_{2t}}\right] + (1-\theta_1)e_t q_{xt}}{\theta_2 p_{yt}\left[1 - \frac{\varepsilon_2}{1+\varepsilon_2}\frac{1}{1+r_{2t}}\right] + (1-\theta_2 e_t)q_{yt}},$$

if they accept currency 1, and

$$\frac{U_{xt}(x_t, y_t; \gamma_1)}{(1-\gamma_1)U_{yt}(x_t, y_t; \gamma_1)} = \frac{\theta_1 p_{xt} + (1-\theta_1)e_t q_{xt}\left[1 - \frac{\varepsilon_1}{1+\varepsilon_1}\frac{1}{1+r_{1t}}\right]}{\theta_2 p_{yt} + (1-\theta_2)e_t q_{yt}\left[1 - \frac{\varepsilon_1}{1+\varepsilon_1}\frac{1}{1+r_{1t}}\right]},$$

$$\frac{V_{xt}(x_t^*, y_t^*; \gamma_2)}{(1-\gamma_2)V_{yt}(x_t^*, y_t^*; \gamma_2)} = \frac{\theta_1 p_{xt} + (1-\theta_1)e_t q_{xt}\left[1 - \frac{\varepsilon_2}{1+\varepsilon_2}\frac{1}{1+r_{2t}}\right]}{\theta_2 p_{yt} + (1-\theta_2)e_t q_{yt}\left[1 - \frac{\varepsilon_2}{1+\varepsilon_2}\frac{1}{1+r_{2t}}\right]},$$

if they accept currency 2.

As will become clear below, the nominal amount spent by households on goods x and y plays a central role in the analysis. Consequently, assuming, as I will in the remainder of the section, that households have Cobb-Douglas preferences greatly simplifies the exposition. In that case, the relative amount spent can be read directly from the above first order conditions. Also, as I show below, the logic of all the proofs applies to more general specifications.

Preferences of households living in country 1 will be represented by

$$ln(x_t) + (1-\gamma_1)ln(y_t),$$

while those living in country 2 have utility functions

$$(1-\gamma_2)ln(x_t^*) + ln(y_t^*),$$

where $\gamma_1, \gamma_2 \in [0, 1]$.

In this two country economy, assumption 3 is still sufficient, but might no longer be necessary. A household in country 1 will have to pay $\varepsilon_1 \frac{1}{p_{xt}} e_t \left[(1-\theta_1)q_{xt}x_t + (1-\theta_2)q_{yt}y_t\right]$ in foreign exchange cost if it accepts currency 1, and $\varepsilon_1 \frac{1}{p_{xt}} \left[\theta_1 p_{xt}x_t + \theta_2 p_{yt}y_t\right]$ if it accepts currency 2. Since the households' utility function is Cobb-Douglas, I can write

$$(1-\gamma_1)e_t q_{xt}x_t = (1-\gamma_1)p_{xt}x_t \approx p_{yt}y_t = e_t q_{yt}y_t.$$

This is an approximation, because of the distortion that ε_1 introduces. Because ε_1 and ε_2 are very small, I can ignore them for these calculations.

Consequently, the two costs above can be written as $\varepsilon_1 \left[(1-\theta_1) + (1-\gamma_1)(1-\theta_2)\right] x_t$ and $\varepsilon_1 \left[\theta_1 + (1-\gamma_1)\theta_2\right] x_t$, respectively. The household will choose the cheapest of the two options, so the foreign exchange cost will be highest when the two costs are exactly equal. This will be the case if $(1-2\theta_1) + (1-\gamma_1)(1-2\theta_2) = 0$. It is easy to verify that for any combination of θ_1 and θ_2 that satisfy this relationship, the cost of foreign exchange will be $\varepsilon_1 \left[\frac{2-\gamma_1}{2}\right] x_t$. Following the same steps as in section 2, the maximum transaction cost for households in country 1 can be found to be equal to $\left[1 + \left[\frac{2-\gamma_1}{2}\right] \varepsilon_1\right]^{-1} \omega_x$. Similar calculations imply that the maximum transaction cost for households in country 2 will be $\left[1 + \left[\frac{2-\gamma_2}{2}\right] \varepsilon_2\right]^{-1} \omega_y$.

The cost of accepting two currencies need not be equal in both countries. Let κ_1 and κ_2 denote this cost in country 1 and 2, respectively. Now I can rewrite assumption 3 for this economy.

Assumption 3'. *$\kappa_1 > \varepsilon_1 \left[1 + \left[\frac{2-\gamma_1}{2}\right] \varepsilon_1\right]^{-1} \omega_x$, and $\kappa_2 > \varepsilon_2 \left[1 + \left[\frac{2-\gamma_2}{2}\right] \varepsilon_2\right]^{-1} \omega_y$.*

Proposition 3.1. *Assumptions 1, 2 and 3' are necessary and sufficient conditions to eliminate the exchange rate indeterminacy.*

The proof of this proposition is omitted, as it is essentially identical to part of the proof of proposition 2.2.

Now that there are two countries, I also need to change the definition of equilibrium stability. As above, let $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the common belief of households about the realized value of θ_1 and θ_2, respectively.

Definition 3.2. *An equilibrium $\{\theta_1, \theta_2\}$ is unstable if, $\forall \varepsilon > 0, \min\{|\hat{\theta}_1 - \theta_1|, |\hat{\theta}_2 - \theta_2|\} > \varepsilon \Rightarrow \{\theta_1, \theta_2\}$ is not an equilibrium.*

Now I can determine which values of θ_1 and θ_2 constitute an equilibrium. I consider here the case where the stock of both currencies is constant, and leave the study of the effect of growth in the stock of currencies for the next subsection.

Proposition 3.3. *If $\pi_1 = \pi_2 = 1$, there are nine equilibria with valued currency:*

1) $\theta_1 = \theta_2 = 0$,

2) $\theta_1 = 0$ *and* $\theta_2 \approx \frac{2-\gamma_2}{2}$,

3) $\theta_1 = 0$ *and* $\theta_2 = 1$,

4) $\theta_1 \approx \frac{2-\gamma_1}{2}$ *and* $\theta_2 = 0$,

5) $\theta_1 = \frac{1}{2}$ *and* $\theta_2 = \frac{1}{2}$,

6) $\theta_1 \approx \frac{\gamma_1}{2}$ *and* $\theta_2 = 1$,

7) $\theta_1 = 1$ *and* $\theta_2 = 0$,

8) $\theta_1 = 1$ *and* $\theta_2 \approx \frac{\gamma_2}{2}$,

9) $\theta_1 = \theta_2 = 1$.

If $\gamma_1, \gamma_2 > 0$ *equilibria 1, 3, 7 and 9 are stable.*

If either $\gamma_1 = 0$ *or* $\gamma_2 = 0$ *only equilibria 1 and 9 are stable.*

Proof. Here again, for given values of θ_1 and θ_2, there exists an equilibrium for this standard cash-in-advance economy. I look for the values of θ_1 and θ_2 that are consistent with sellers minimizing the foreign exchange costs of their household.

There are no transactions cost with a single currency, so $\theta_1 = \theta_2 = 0$, and $\theta_1 = \theta_2 = 1$ are equilibria.

Since $\theta_1, \theta_2 \leq 0$, households spend at least as much on the home country good as they will on the foreign country good. This means that it is an equilibrium for all domestic sellers to accept the same currency, even if all foreign seller accept another currency. Thus candidate equilibria 3 and 7 are in fact equilibria.

If half of the households in both countries accept currency 1, and the other half accepts currency 2, then all households are indifferent between the two currencies, and candidate equilibrium 5 is an equilibrium.

Suppose $\theta_1 = 0$ (all households in country 1 accept currency 2), then the foreign exchange cost of accepting currency 1 in country 2 is

$$(1 - \theta_2) q_{yt} y_t^* + q_{xt} x_t^*,$$

while the foreign exchange cost of accepting currency 2 in that country is

$$\theta_2 p_{yt} y_t^*.$$

Households in country 2 are indifferent between the two currencies if these two quantities are equal. Given the Cobb-Douglas utility functions,

$$(1-\gamma_1)e_t q_{xt} x_t = (1-\gamma_1)p_{xt} x_t \approx p_{yt} y_t = e_t q_{yt} y_t,$$

$$e_t q_{xt} x_t^* = p_{xt} x_t^* \approx (1-\gamma_2)p_{yt} y_t^* = (1-\gamma_2)e_t q_{yt} y_t^*.$$

The approximations come from the fact that although ε_1 and ε_2 are very small, they are strictly positive. It is easy to show that the two costs are equal if

$$\theta_2 = \frac{2-\gamma_2}{2}.$$

Thus candidate equilibrium 2 is an equilibrium. Following the same steps, one can easily show that candidate equilibria 4, 6, and 8 are equilibria as well.

Now I show which equilibria are stable. To see that equilibria 1 and 9 are stable, simply consider $\{\hat{\theta}_1, \hat{\theta}_2\} = \{.1, .1\}$ and $\{\hat{\theta}_1, \hat{\theta}_2\} = \{.9, .9\}$. In the former case, all sellers strictly prefer to accept currency 2; in the latter they strictly prefer to accept currency 1.

For equilibrium 3, if $\gamma_1, \gamma_2 > 0$, then $\{\hat{\theta}_1, \hat{\theta}_2\} = \{\frac{\gamma_1}{4}, 1-\frac{\gamma_2}{4}\}$ proves that it is stable. Indeed, since $\frac{\gamma_1}{4} < \frac{\gamma_1}{2}$, households in country 1 strictly prefer currency 2, while since $1-\frac{\gamma_2}{4} > \frac{1-\gamma_2}{2}$, households in country 2 strictly prefer currency 1. If either $\gamma_1 = 0$, or $\gamma_2 = 0$ the equilibrium would no longer be stable. If $\gamma_1 = 0$, then $\hat{\theta}_1 \in (0, 1]$ implies that households in country 1 prefer currency 2, and if $\gamma_2 = 0$, then $\hat{\theta}_2 \in [0, 1)$ implies that households in country 2 prefer currency 1. Similar steps prove that equilibrium 7 is stable if $\gamma_1, \gamma_2 > 0$, and unstable in either $\gamma_1 = 0$, or $\gamma_2 = 0$.

Finally, I can show that all other equilibria are unstable. This is clear from the construction of these equilibria, since they depend on households being exactly indifferent between the two currencies in at least one of the countries. Any deviation from the equilibrium values of θ_1 and θ_2 breaks this indifference. □

Equilibria 1 and 9 are *single currency* equilibria. Equilibrium 7 is a *national currencies* equilibrium, as households in country 1 accept currency 1 and households in country 2 accept currency 2. Equilibrium 3, is a *reversed national currencies* equilibrium, as households in country 1 accept currency 2 (but receive newly issued currency 1), and vice-versa. There are 5 equilibria with *mixed* currencies. In equilibria 2 and 8, all seller in country 1 accept the same currency. In country 2, if just the right number of sellers accept each of the currencies, then sellers in that country are indifferent between accepting currency 1 or 2. Equilibria 4 and 6 are similar, with sellers in country 2 each holding the same currency and sellers in country 1 being indifferent between the two currencies. Finally, in equilibrium 5, half the sellers in each country accept currency 1, and all sellers are indifferent between currency 1 and 2.

The intuition is that sellers try to minimize the cost of foreign exchange for their household. With positive home country biases, households spend more on the home good than on the foreign good, so they will tend to accept the same currency as the other households in the same country. A network externality is created by the costs I have assumed. From the perspective of a given household, these costs create a wedge between the prices paid in each currency. The size of this wedge turns out to be a function of how many sellers accept one currency or the other.

Some of these equilibria can be Pareto ranked because they imply different amounts of goods wasted in transaction costs. Single currency equilibria imply no transaction cost and thus provide the highest welfare, while equilibrium 5 gives rise to the highest transaction costs.

All the equilibria are represented in figure 2 and are denoted by E_1 to E_9 to correspond with proposition 3.3. The lines labelled C_1 and C_2 are given by,

$$(\gamma_1 - 1)\,\theta_2 = \frac{\gamma_1 - 2}{2} + \theta_1$$

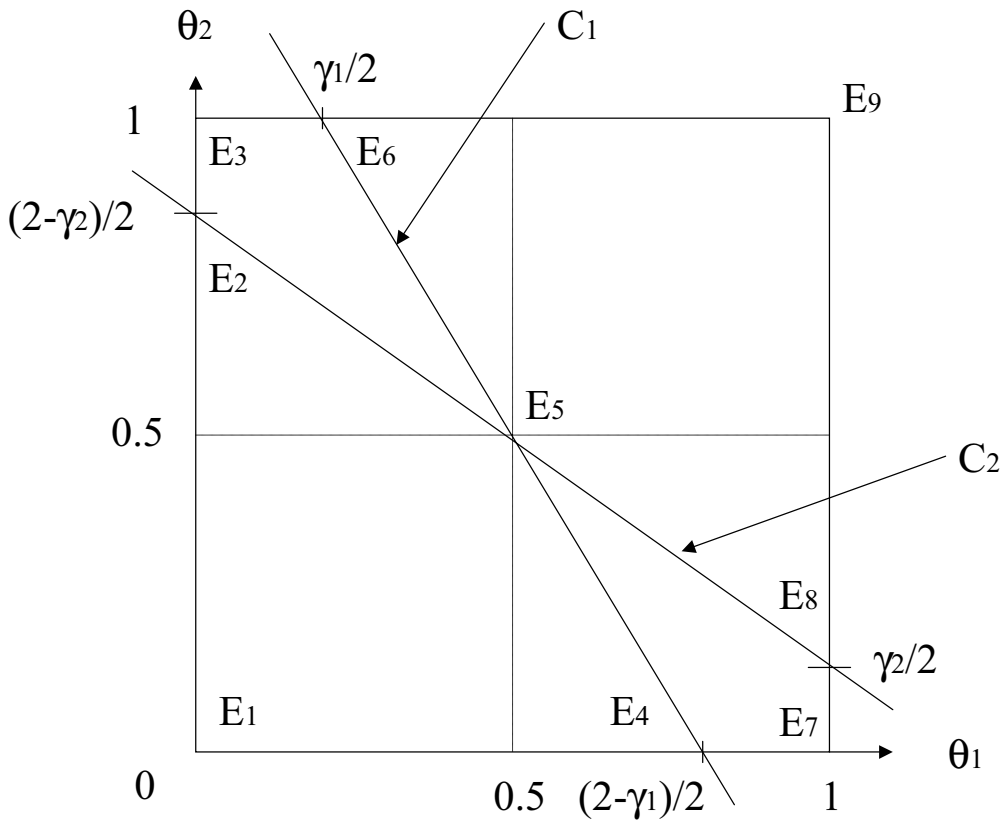
and

$$\theta_2 = \frac{2 - \theta_2}{2} + (\gamma_2 - 1)\,\theta_1,$$

respectively.

They represent pairs θ_1, θ_2 for which households in country 1 and 2, respectively, are indifferent between accepting currency 1 and 2. I call line C_i country i's indifference line, $i = 1, 2$.

Figure 2



The slope of these lines are determined by γ_1 and γ_2, respectively. Consider C_1. If $\gamma_1 = 0$, households in country 1 spend the same value on good x as they do on good y. Thus, if θ_1 increases, θ_2 has to decrease by the same amount for these households to be indifferent between the two currencies. Thus the slope of C_1 will be equal to -1. Suppose now that $\gamma_1 = 1$. Then households in country 1 don't buy good y, and thus θ_2 has no influence on their preference for one or the other currency. In that case, C_1 will be vertical. In general, C_1 gets steeper as γ_1 increases. A similar argument show that C_2 has slope -1 if $\gamma_2 = 0$, gets flatter as γ_2 increases and become horizontal when $\gamma_2 = 1$. Points lower and to the left of C_i are such that households in country i strictly prefer currency 2, while points higher and to the

right of C_i are such that households in country i strictly prefer currency 1, $i = 1, 2$.

Having a home country bias is important for the stability of the national currencies equilibrium. If, for example, $\gamma_1 = 0$, then equilibria 4 and 7 are identical, and so are equilibria 3 and 6. These equilibria are not stable. In that case, the only stable equilibria have a single valued currency. A possible alternative interpretation of the γ terms is as a proxy for openness to trade. Under this interpretation, the results imply that as economies become more open to trade, they are more likely to adopt a single currency. Thus the model provides an explanation of the adoption of the euro.

If should be clear from figure 2 that a result equivalent to proposition 3.3 would hold for more general preferences. Although they would be more difficult to characterize, curves similar to C_1 and C_2 will exists for general utility functions, and their slopes will depend on the γ's. Depending on where these indifference lines lay, not all nine equilibria will exist. However, it is possible to show that if γ_1 and γ_2 are close to 1, they will.

Proposition 3.4. *Assume that the utility functions U and V are twice differentiable, strictly increasing and strictly concave.*
If γ_1 and γ_2 are close to 1 all nine equilibria described in proposition 3.3 *exist.*

Proof. If $\gamma_1 = \gamma_2 = 1$, both countries are in autarky and, from proposition 2.2, there are three equilibria for each country. Combining these yields the nine equilibria of proposition 3.3. By continuity, these equilibria exist if γ_1 and γ_2 are close to 1. □

In this theory, the choice of currency depends on the relative amounts spent on different types of goods. This can help us think of what happens in the case of two countries of different sizes. It is expected that, on average, the fraction of a household's spending on goods from a large country will be big, compared to the fraction spent on goods from a small country. In order for a national currencies equilibrium to exist, the home country bias in the small country will have to be large. If it is too small, only single currencies equilibria will exist. Starting from a national currencies equilibrium, as the small country becomes more open to trade, the sellers in that country might decide to adopt the currency of the big country.

4 Money growth and currency substitution

In this section, I look at the effect of money growth on the equilibria described above. I will pay particular attention to currency substitution. By currency substitution, I mean the switch from a national currencies to a single currency equilibrium[2].

Recall that currency i is issued by country i, $i = 1, 2$, and that there is no market on which households can trade claims on newly issued currency. The effect of growth in the stock of a currency is to decrease the incentive to hold that currency for households that live in the country where the currency is issued. Note that there is no real transfer associated with currency growth because prices move to offset any nominal changes. What is happening, instead, is that households get their cash from a different source. If there is no money growth, households get all their cash from sales, whereas with money growth, part of their cash is newly issued currency. This, in turns, changes the amount households must spend on foreign exchange cost in order to obtain their preferred portfolio. For example, consider a national currencies equilibrium. An increase in π_1 increases the share of currency 1 that households in country 1 receive from the monetary authority, and reduces the share they obtain through sales. As they rely less on their sales to obtain currency 1, the incentive for sellers to accept currency 1 diminishes making currency 2 becomes more attractive. Households in country 2 are completely unaffected by this change.

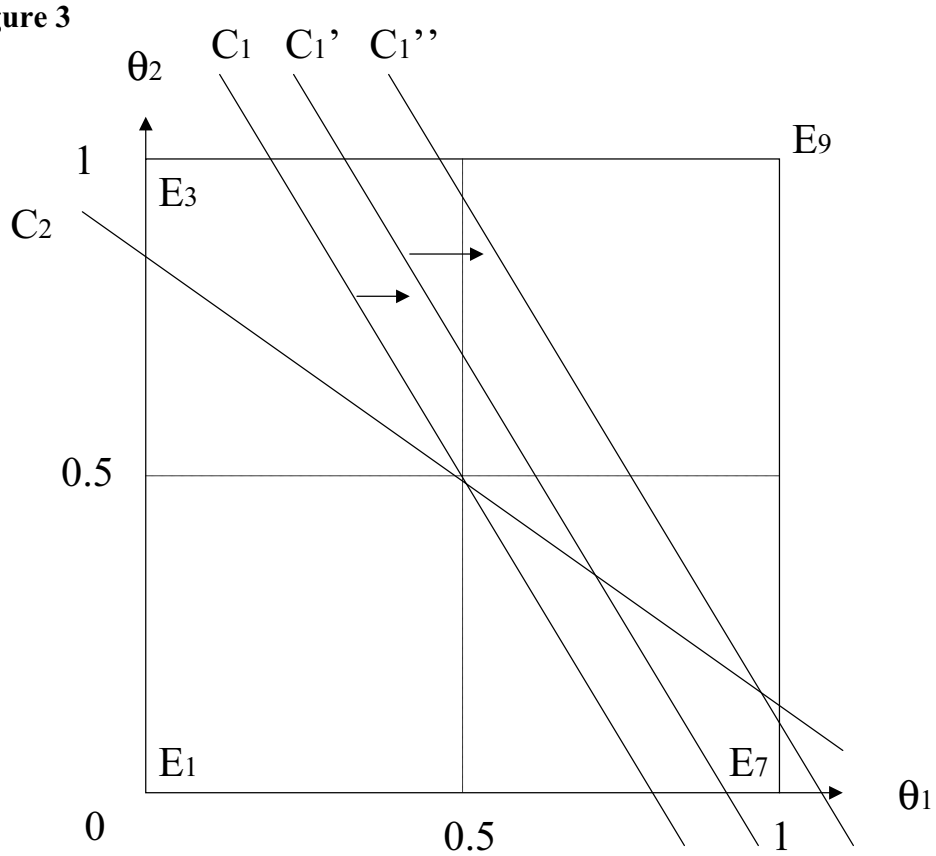
Graphically, this means that an increase in π_1 shifts the line C_1 to the right, as shown in figure 3, while an increase in π_2 shifts the line C_2 to the left. To get some intuition, consider equilibrium of type 4. In such an equilibrium, households in country 1 are indifferent between accepting currency 1 and currency 2. If π_1 increases, households in country 1 are no longer indifferent between the two currencies, because they now receive an extra amount of currency 1. Hence they all prefer currency 2. Only if θ_1 increases, will households again be indifferent between the currencies.

As π_1 increases, the line C_1 moves to the right, say to C_1'. If it increases further, there

[2]There are other definitions of currency substitution in the literature. Giovannini and Turtelboom (1994) provide a good review.

will come a point where equilibria 4 and 7 coincide. For any increase in π_1 above that level, equilibrium 7, the national currencies equilibrium, will fail to exist, as is the case for C_1''. If π_2 increases too much, the line C_2 will move to the left. For high enough π_2, equilibrium 7 will fail to exist. The following proposition gives the precise bounds that trigger currency substitution.

Figure 3



Proposition 4.1. *If either* $\pi_1 - 1 > \frac{\gamma_1}{2-\gamma_1}$, *or* $\pi_2 - 1 > \frac{\gamma_2}{2-\gamma_2}$, *then the national currencies equilibrium fails to exists.*

Proof. Assume $\theta_1 = 1$ and $\theta_2 = 0$. In country 1, the foreign exchange cost of accepting currency 1 is $e_t q_{yt} y_t$ and the cost of accepting currency 2 is $p_{xt} x_t - (\pi_1 - 1) M_{1t}$. From proposition 3.3, we know that if $\pi_1 = 1$, then $p_{xt} x_t - (\pi_1 - 1) M_{1t} > e_t q_{yt} y_t$. However, there is a value of π_1 high enough that this inequality will be reversed. In that case, households in country 1 will prefer currency 2, and the assumed equilibrium will fail to exist. Again,

using the fact that preferences are Cobb-Douglas, it is easily seen that this will happen if $(\pi_1 - 1)M_{1t} > \gamma_1 p_{xt} x_t$. Similar steps will establish that if $(\pi_2 - 1)M_{2t} > \gamma_2 p_{yt} y_t^*$, households in country 2 will prefer currency 1. To complete the proof, note that $\omega_x \approx \frac{M_{1t}}{p_{xt}}$ and $\omega_y \approx \frac{M_{2t}}{q_{yt}}$. Also, $x_t \approx \frac{1}{2-\gamma_1}\omega_x$ and $y_t^* \approx \frac{1}{2-\gamma_2}\omega_y$. All approximations are due to the fact that ε_1 and ε_2 are very small but strictly positive. □

It is interesting to note the role played by the home country bias. If it is high (γ_i high) then it takes faster growth of the currency to get currency substitution to occur. As mentioned above, one can also think of γ_i as a proxy for how open country i is to trade. In this case, a high γ_i means that country i is not very open to trade, as only a small fraction of its consumption comes from the foreign country. In this case, proposition 4.1 indicates that countries that are less open to trade have more room to increase their money supply without triggering currency substitution.

Proposition 4.1 can also be generalized.

Proposition 4.2. *Assume that the utility functions U and V are twice differentiable, strictly increasing and strictly concave.*
There exists $\bar{\pi}_1$ and $\bar{\pi}_2$ such that if either $\pi_1 > \bar{\pi}_1$ or $\pi_2 > \bar{\pi}_2$ or both, then the national currencies equilibrium fails to exists.

Proof. Consider a household in country 1. As π_1 increases, the cost to this households of accepting currency 1 is unchanged. However, the cost of accepting currency 2 is reduced as it now receives some currency 1 through the monetary authority. This means that C_1 has moved to the right. As π_1 increases, the newly issued currency 1 represents a higher fraction of currency 1 available. This fraction tends to 1 as π_1 tends to infinity. Thus there is a π_1 high enough so that households in country 1 will prefer to accept currency 1. Similarly for country 2. □

It is interesting to note that the reverse national currencies equilibrium is stable, and does not disappear with high money growth.

5 Conclusion

This paper studies a model in which sellers can choose the currency they accept. I show that once some assumptions are made to eliminate indeterminacy of equilibrium, the only stable equilibria have familiar features: all sellers in the same country will accept the same currency. They might accept the same currency as sellers in the foreign country, in which case it is a *single currency* equilibrium, or they might accept a different currency, in which case it is a *national currencies* equilibrium.

The model predicts that a national currencies equilibrium might fail to exist if the stock of a currency grows too fast. This can be interpreted as currency substitution. I also show that currency substitution is more likely to occur if home country biases are not too large or, under another interpretation, if countries are more open to trade. Thus, as economic ties between countries grow, we should expect monetary authorities to have less flexibility in choosing high rates of growth for their currencies. For the same reasons, we should expect more countries to choose a single currency, as in Europe.

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