The natural real rate of interest—the level of the real federal funds rate most consistent with the Federal Reserve’s statutory mandates of maximum sustainable employment and stable prices—is a key guidepost for monetary policy decisions. But most approaches used to estimate the natural rate have not kept pace with the Federal Open Market Committee’s (FOMC) rapidly expanding set of monetary policy tools. From 2008 to 2014, the FOMC purchased large amounts of Treasury and agency mortgage-backed securities to put downward pressure on longer-term interest rates and ease overall financial conditions. However, existing measures of the natural real rate, also known as $r^*$, do not explicitly account for the additional accommodation these unconventional policies may provide.

In this article, we provide two estimates of the natural real rate that account for the Fed’s balance sheet and, more generally, the broad state of U.S. financial conditions. Since the goal of the 2008–14 asset purchases was to ease financial market conditions by reducing bond yields, we use bond premiums to gauge the ease or tightness of financial markets. More specifically, we derive our first estimate of $r^*$ from a statistical model that explicitly incorporates term and risk premiums from bond markets. We then produce a second, purely data-driven estimate

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of \( r^* \) by looking for a common component across multiple variables that have been plausibly linked to the natural rate, including bond premiums. While we construct these two estimates of \( r^* \) quite differently, they yield similar results. Both estimates reveal that the natural rate reached historically low values during the 2007–09 financial crisis and recession but rebounded more recently as the economy improved and financial conditions eased.

Our results suggest bond premiums are an important determinant of the natural real rate and lead to highly cyclical estimates. In particular, our estimates from both approaches show that a reduction in bond premiums increases the natural real rate. All else equal, lower bond premiums can provide an additional source of policy accommodation by reducing financing costs for housing, consumer durables, and investment projects. Therefore, if the economy is operating at full employment and inflation rests at the FOMC’s 2 percent longer-run objective, a change in bond premiums may require offsetting changes in the real federal funds rate to keep the economy on an even keel.

Section I motivates the inclusion of bond premiums in models of the natural real rate of interest and reviews the relationships between the Federal Reserve’s balance sheet, bond premiums, and the natural rate. Section II presents a model-based estimate of \( r^* \) that augments the popular Laubach and Williams model with bond premiums. Section III presents a purely data-driven approach for estimating \( r^* \). Section IV highlights how \( r^* \) is related not only to the Federal Reserve’s balance sheet but also to other factors that shape global financial markets.

I. The Balance Sheet, Bond Premiums, and the Natural Real Rate

Assets held by the Federal Reserve increased from around $900 billion in 2007 to nearly $4.5 trillion in 2014. The balance sheet initially expanded during the financial crisis, when the Federal Reserve provided short-term liquidity to banks to fulfill its role as lender of last resort. However, the recession that followed proved too severe for conventional policy tools to address. To provide additional accommodation, the Fed began expanding its balance sheet further by purchasing substantial Treasury and agency mortgage-backed securities, a policy known as large-scale asset purchases or, more commonly, quantitative
easing (QE). The last of three rounds of QE ended in October 2014, but the FOMC maintains a large portfolio of agency debt and Treasury securities by reinvesting proceeds of maturing securities. Consequently, the size and composition of the balance sheet continues to influence financial market conditions.

Some members of the FOMC have explicitly argued that changes to the balance sheet may influence the natural rate through their effects on bond premiums (Fischer). While multiple event studies have confirmed a relationship between the balance sheet and bond premiums, less is known about the empirical relationship between bond premiums and the natural real rate.

The link between the Federal Reserve’s balance sheet and bond premiums

Since the goal of QE was to ease financial market conditions largely by reducing bond yields, most event studies have focused on how two bond premiums, the term premium and the risk premium, respond to announced changes in the asset purchase programs. The term premium measures the extra compensation investors require to hold a long-term government bond instead of buying a sequence of short-term government bonds. The risk premium measures the extra return investors require to hold a bond with some risk of default instead of holding a Treasury security of a similar maturity. The sum of the term and risk premium is therefore equal to the spread between corporate bond rates and the average of the expected future path of short-term interest rates.

Many event studies have concluded that the FOMC was successful in reducing the level of the term premium by expanding its balance sheet. Gagnon and others show that the cumulative effect of FOMC announcements regarding QE1, a round of asset purchases from November 2008 to March 2010, lowered the term premium on Treasury securities by about 50 basis points. Using a similar approach, Abrahams and others find that the cumulative effect of FOMC announcements for all three rounds of asset purchases plus the maturity extension program—which increased the average maturity of the FOMC’s balance sheet without altering its size—decreased the term premium by about 110 basis points.
Evidence on the ability of large-scale asset purchases to reduce risk premiums is more mixed. Gagnon and others find that FOMC announcements during the QE1 program depressed risk premiums on corporate bonds by almost 20 basis points. In contrast, Krishnamurthy and Vissing-Jorgenson show that large-scale asset purchases actually raised risk premiums by lowering Treasury yields more than corporate bond yields. However, both event studies focus on changes in bond yields over a one- to two-day window, which may be too short to capture meaningful movements in risk premiums. A short window may be valid for highly liquid Treasury securities, but risky corporate debt changes hands less frequently. To check this possibility, Edgerton looks at how corporate bond yields reacted over a longer window around QE announcements and finds more meaningful reductions in risk premiums.¹

*The link between bond premiums and the natural real rate*

Monetary policy makers have previously highlighted a relationship between bond premiums and the natural real rate of interest. For example, both former Federal Reserve Chair Bernanke (2006) and former Governor Stein expressed the view that monetary policy makers may need to monitor, and possibly offset, changes in bond premiums when the economy is operating near levels consistent with the Fed’s dual mandate. However, there is little empirical evidence for the relationship between \( r^* \) and bond premiums, particularly for term premiums, which are the primary channel through which asset purchases operate.

Economic theory predicts that an increase in bond premiums lowers the natural real rate. The widely cited Smets and Wouters model of the U.S. economy features adverse “risk shocks” that, like increases in bond premiums, raise the return on bonds relative to the interest rate controlled by the central bank. Smets and Wouters’ model predicts that increases in these bond premiums reduce the natural rate one for one: higher bond premiums in the model cause consumers to save more in the present and delay consumption for the future. Since postponed consumption decreases current demand, policymakers must lower real policy rates to prevent a slowdown in the economy.² Woodford and Curdia’s model of credit frictions similarly shows that policymakers should offset shifts in risk spreads, but not necessarily one for one.³
Monetary policy makers have also advocated for adjusting short-term policy rates to counteract shifts in bond premiums. For example, Bernanke (2006) suggests “to the extent that the decline in forward rates can be traced to a decline in the term premium . . . the effect is financially stimulative and argues for greater monetary policy restraint, all else being equal. . . . thus, when the term premium declines, a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices.” Similarly, a joint paper by Taylor and Federal Reserve Bank of San Francisco President Williams suggests short-term policy rates may need to be lowered to offset increases in risk spreads following the recent financial crisis. And Stein argues from a financial stability perspective that “all else being equal, monetary policy should be less accommodative . . . when estimates of [term and credit] risk premiums in the bond market are abnormally low.”

Despite these views, the empirical relationship between the natural real rate and term premiums is not well understood. While many economists and policymakers believe lower term premiums are stimulatory, Hamilton and Kim find that lower term premiums actually predict slower GDP growth. But Rudebusch, Sack, and Swanson show that regression models such as Hamilton and Kim’s can be sensitive to the empirical specification and the sample period.

Unlike the term premium, empirical evidence widely supports the idea that rising risk premiums dampen future economic activity. In the closest paper to ours, Kiley finds an inverse relationship between risk spreads and his estimate of the natural real rate. However, his model doesn’t include term premiums, which are a primary channel through which asset purchases are thought to operate (Bernanke 2012b). Pescatori and Turunen take a related approach by positing that global savings, economic policy uncertainty, and the equity risk premium all affect the natural real rate. While these factors are likely to capture some elements that drive bond premiums, they may not fully capture how a central bank’s asset purchases alter the relative demand for bonds or their yields.
II. A Model of Bond Premiums and the Natural Real Rate of Interest

The current mix of monetary policy tools employed by the FOMC warrants a fresh look at how bond premiums influence the natural rate. Since the natural real rate of interest is not observable, we use a semistructural model to explore the link between bond premiums and $r^*$. Our approach therefore follows Laubach and Williams, who developed a stylized model of the U.S. economy to estimate low-frequency movements in the natural rate. However, our model accounts for bond premiums. By doing so, our estimates take into account not only medium-term growth prospects and aggregate demand conditions but also current financial market conditions as measured by financing premiums implied by corporate and government bond yields.

An overview of the model

Laubach and Williams’ model identifies the natural real rate of interest using an estimated investment-savings (IS) equation. The IS equation relates the output gap—the percent difference between the level of real GDP and its potential level—to the real interest rate gap—the difference between the real effective federal funds rate and the natural real rate. The IS equation posits a negative relationship between the real interest rate gap and the output gap. More specifically, the IS equation suggests that an increase in the real interest rate above the natural rate leads to a decline in real GDP below its potential level.

The relationship implied by the IS equation can be used to infer the natural real rate. Suppose, for example, that the economy is initially operating at potential with no real interest rate gap. If output falls persistently below its potential level, then the model would infer that the real interest rate has risen above the natural rate, thereby turning the real interest gap positive. Conversely, if output persists above potential, then the model would infer that the real interest rate has fallen below the natural rate, thereby turning the real interest rate gap negative. By setting the real federal funds rate equal to the natural real rate, monetary policy makers can keep the economy from slowing or overheating. In this sense, the natural real rate provides a guidepost for monetary policy.

If the output gap and the real effective federal funds rate were observable, we could directly extract a measure of the natural real rate from
an estimated IS equation. However, the output gap is unobservable.\textsuperscript{5} To infer whether GDP is above or below its potential level, Laubach and Williams use an accelerationist Phillips curve. This version of the Phillips curve relates the change in the inflation rate to the output gap. More specifically, the accelerationist curve implies that rising inflation is due to output exceeding its potential level, while falling inflation is due to output falling below potential.\textsuperscript{6}

In this way, data on output and inflation can be used to measure potential output—which, in turn, can be used to infer the natural real rate of interest.

\textit{Model specification}

The IS equation and Phillips curve can be directly estimated in principle, but certain features of the data can make them challenging to model in practice. Laubach and Williams’ IS equation, for instance, assumes that the output gap depends in part on lags of itself, reflecting that the U.S. economy has momentum when expanding or contracting. In addition, Laubach and Williams’ IS equation assumes that the economy is slow to adjust to the real interest rate gap, reflecting the long-standing notion that monetary policy influences the economy with a lag of one to two quarters. Given these assumptions, the IS equation that enters the model is expressed mathematically as:

\begin{equation}
    y_t = a_1 y_{t-1} + a_2 y_{t-2} + a_r r_t + \epsilon_{t-1},
\end{equation}

where $y_t$ denotes the output gap, $r_t$ denotes the real effective federal funds rate, $r^*$ denotes the natural real rate of interest, and $\epsilon_t$ is a statistical error with a standard deviation of $\sigma_1$ included to capture noise in the data. The terms $a_1$ and $a_2$ measure the persistence of output gap deviations, while $a_r$ measures the sensitivity of the output gap to the real interest rate gap.

As with the IS equation, Laubach and Williams specify several features of the data when modeling the Phillips curve. First, they incorporate eight quarters of lagged inflation, as U.S. inflation can be slow to adjust to policy changes (Christiano, Eichenbaum, and Evans). Second, they incorporate oil and import prices as control variables, as factors outside of the output gap can also have an effect on inflation. Finally,
they incorporate the output gap with a one-quarter lag, consistent with the idea that prices are slow to adjust to slack in the economy. The Phillips curve that enters the model is expressed mathematically as:

\[
\pi_t = \sum_{i=1}^{8} b_i \pi_{t-i} + b_y \tilde{y}_{t-1} + b_{imp} \pi_{t}^{imp} + b_{oil} \pi_{t}^{oil} + \epsilon_t^2,
\]

where \(\pi\) denotes the quarterly inflation rate as measured by the price index for personal consumption expenditures excluding food and energy, \(\tilde{y}\) denotes the output gap, \(\pi^{oil}\) denotes oil import price inflation, \(\pi^{imp}\) denotes inflation in core import prices, and \(\epsilon_t^2\) is a statistical error with a standard deviation of \(\sigma^2\) included to capture noise in the data.

Laubach and Williams impose the restrictions that \(\sum b_i = 1\) and that the coefficients on lags two through four are equal as are the coefficients on lags five through eight during the estimation. The coefficients \(b_{imp}\) and \(b_{oil}\) measure the effect of changes in import and energy prices on core inflation, while \(b_y\) measures the sensitivity of inflation to changes in the output gap.

**Determinants of the natural real rate of interest**

Standard models of economic growth predict that the natural real rate varies positively with the economy’s trend growth rate, denoted here by \(g\), leading Laubach and Williams to specify:

\[
r^*_t = c_g g_t + z_t,
\]

where \(c_g\) measures the sensitivity of the natural rate to trend growth. Laubach and Williams include the \(z\) term to capture other factors that are difficult to quantify but may affect the natural rate through aggregate demand channels, including expectations of fiscal deficits, the health of household and firm balance sheets, and demand emanating from abroad.

We augment Laubach and Williams’ expression for \(r^*_t\) to include the term premium, \(tp\), and the risk premium, \(rp\), from bond markets. Specifically, we specify the natural real rate as:

\[
r^*_t = c_g g_t + z_t + c_{tp} tp_t + c_{rp} rp_t.
\]

Neither the term premium nor the risk premium are perfectly observable, so we employ commonly used estimates instead. For the term
premium, we use the estimate for the 10-year U.S. Treasury security from Adrian, Crump, and Moench (2013a).\(^7\) For the risk premium, we use the difference between Moody’s index of BAA corporate bonds and the 10-year constant-maturity U.S. Treasury security. The coefficients \(c_{tp}\) and \(c_{rp}\) measure the potential influence that term and risk premiums have on \(r^*\). We expect both coefficients to be negative.

Finally, we specify the statistical process for the unobserved variables in the model, which include \(z\), other unobservable demand factors that affect \(r^*\); \(y^*\), the natural log of potential output; and \(g\), the trend growth rate. Following Laubach and Williams, we assume these unobserved variables evolve according to:

\[
\begin{align*}
z_t &= z_{t-1} + \varepsilon_t^3, \\
y^*_t &= y^*_{t-1} + g_{t-1} + \varepsilon_t^4, \\
\end{align*}
\]

and

\[
\begin{align*}
g_t &= g_{t-1} + \varepsilon_t^5.
\end{align*}
\]

In each of these equations, the terms \(\varepsilon_t^3\), \(\varepsilon_t^4\), and \(\varepsilon_t^5\) are unexpected shocks to the unobserved aggregate demand factor, the natural log of potential output, and trend growth with standard deviations equal to \(\sigma^3\), \(\sigma^4\), and \(\sigma^5\), respectively.\(^8\)

**Estimates of the natural real rate of interest**

Our model estimates reveal that a decline in bond premiums increases the natural real rate of interest. Table 1 reports the full set of parameter estimates and standard errors. The first column of results shows estimates from our unrestricted model. As expected, the coefficient on the risk premium, \(c_{rp} = -0.78\), is negative, suggesting a lower risk premium would yield a higher natural real rate. The coefficient on the term premium, \(c_{tp} = -1.54\) is also negative, giving empirical weight to Bernanke’s and Stein’s views that reductions in the term premium raise the natural real rate. The coefficients on the risk and term premiums are both near -1, suggesting bond premiums may have an economically significant effect on \(r^*\). Finally, the coefficient on trend growth, \(c_g = 5.69\), is positive, as expected. The magnitude of the coefficient
## Table 1
Parameter Estimates for Natural Real Rate Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unrestricted model</th>
<th>Preferred model</th>
<th>Model without bond premiums</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>1.65*** (0.16)</td>
<td>1.62*** (0.12)</td>
<td>1.66*** (0.10)</td>
</tr>
<tr>
<td>$a_2$</td>
<td>−0.75*** (0.14)</td>
<td>−0.71*** (0.11)</td>
<td>−0.72*** (0.10)</td>
</tr>
<tr>
<td>$a_3$</td>
<td>−0.06** (0.03)</td>
<td>−0.07*** (0.02)</td>
<td>−0.05*** (0.02)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.56*** (0.06)</td>
<td>0.56*** (0.06)</td>
<td>0.55*** (0.06)</td>
</tr>
<tr>
<td>$b_2$</td>
<td>0.34*** (0.08)</td>
<td>0.34*** (0.08)</td>
<td>0.34*** (0.08)</td>
</tr>
<tr>
<td>$b_3$</td>
<td>0.11** (0.05)</td>
<td>0.11** (0.05)</td>
<td>0.12*** (0.04)</td>
</tr>
<tr>
<td>$b_{	ext{mp}}$</td>
<td>0.03*** (0.01)</td>
<td>0.03*** (0.01)</td>
<td>0.03*** (0.01)</td>
</tr>
<tr>
<td>$b_{	ext{ol}}$</td>
<td>0.003** (0.001)</td>
<td>0.003*** (0.001)</td>
<td>0.003** (0.001)</td>
</tr>
<tr>
<td>$c_g$</td>
<td>5.69 (8.63)</td>
<td>4.00 (4.00)</td>
<td>4.00 (4.00)</td>
</tr>
<tr>
<td>$c_{t}$</td>
<td>−1.55* (0.81)</td>
<td>−1.00 (1.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>$c_{rp}$</td>
<td>−0.78 (1.27)</td>
<td>−1.00 (1.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.25*** (0.10)</td>
<td>0.27*** (0.09)</td>
<td>0.27*** (0.08)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0.80*** (0.04)</td>
<td>0.80*** (0.04)</td>
<td>0.80*** (0.04)</td>
</tr>
<tr>
<td>$\sigma_4$</td>
<td>0.64*** (0.05)</td>
<td>0.63*** (0.05)</td>
<td>0.63*** (0.05)</td>
</tr>
</tbody>
</table>

Log likelihood: −519.61
Null hypothesis for likelihood ratio test: —
P-value for likelihood ratio test: 0.92

* Significant at the 10 percent level
** Significant at the 5 percent level
*** Significant at the 1 percent level

Notes: All models are estimated via maximum likelihood from 1962:Q1 to 2016:Q3. Details regarding the estimation procedure are available in Appendix A. Standard errors are in parentheses.
suggests that annualized trend growth ($c_g/4$) affects the natural real rate (expressed in annualized percentage points) nearly one for one.\textsuperscript{9}

The second column of results in Table 1 shows our preferred estimation results, which restrict the parameters so that $c_g = 4$ and $c_{rp} = c_{tp} = -1$. The likelihood ratio test, which compares how well the unrestricted model fits the data compared with this restricted version of the model, reveals that these restrictions cannot be rejected by the data. The restriction that $c_g = 4$ implies that changes in trend growth have a one-for-one effect on the natural real rate; this restriction emerges from many macroeconomic models and is supported within our framework by the estimates in Laubach and Williams. The restriction that $c_{rp} = c_{tp} = -1$ also emerges from theoretical models of the macroeconomy, such as in the model developed by Smets and Wouters. This restriction implies that changes in bond yields, emanating from both term and risk premiums, have a one-for-one negative effect on $r^*$. The relative fit of this preferred model suggests that bond premiums may have an equal and economically significant influence on the natural real rate when compared with trend growth.

We also find some evidence against a version of the model that removes bond premiums from the estimation of the natural real rate. In particular, the third column of results in Table 1 shows estimates from a model that restricts, $c_{rp} = c_{tp} = 0$, thereby eliminating bond premiums as a determinant of the natural real rate. Chart 1 plots these estimates alongside two other estimates of the natural real rate series: our unrestricted estimate and our preferred estimate. The chart shows very little difference between our unrestricted and preferred estimates. However, the estimate without bond premiums differs significantly from the other two. A more formal comparison using a likelihood ratio test reveals that the model fit deteriorates when risk and term premiums are excluded, suggesting they are important determinants of the natural real rate.\textsuperscript{10} As a result, the remainder of this section focuses on our preferred model estimates.

The cyclicality of the natural real rate

Our preferred estimate of $r^*$ is procyclical, rising in economic expansions and declining in recessions. The cyclicality of the natural real rate reflects that bond premiums are countercyclical. Chart 2 plots the
**Chart 1**

Natural Real Rate Estimates

Note: Gray bars denote NBER-defined recessions.
Sources: Bureau of Economic Analysis (BEA), Bureau of Labor Statistics (BLS), Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, Moody’s, National Bureau of Economic Research (NBER), and authors’ calculations. All data sources accessed through Haver Analytics.

**Chart 2**

The Cyclicality of Bond Premiums

Note: Gray bars denote NBER-defined recessions.
Sources: BEA, BLS, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, Moody’s, NBER, and authors’ calculations. All data sources accessed through Haver Analytics.
unemployment rate alongside the term and risk premiums to illustrate that bond premiums tend to follow the unemployment rate: they rise in and around recessions and decline in expansions. The driving forces behind risk and term premiums can offer some insight into why this pattern emerges.

Risk premiums arise from both the risk of default investors face in corporate debt markets and their tolerance for bearing such risks. Both factors tend to make risk premiums strongly countercyclical. Slowing economic growth (as in recessions) can weigh on firms’ balance sheets, increasing their risk of defaulting on corporate bonds and thereby increasing risk premiums. In addition, Gilchrist and Zakrajšek (2012) show that declining investor sentiment toward risks is associated with slowing economic activity and may perhaps play a causal role in propagating economic downturns.

Similarly, the term premium arises, in part, from the risk that realized short-term interest rates could differ from their expected future values. Prior to the 1990s, the primary risk for bond holders was unexpected inflation, which eats into the purchasing power of a bond’s nominal coupon payments. But today, the primary risk may be uncertainty related to the near-term growth outlook. Adrian, Crump, and Moench (2013b) show the term premium is highly correlated with measures of interest rate uncertainty. As a consequence, term premiums rise in recessionary periods along with other financial market measures of uncertainty such as the Chicago Board of Exchange’s Volatility Index (VIX) and, the counterpart for U.S. government bond markets, the Merrill Lynch Option Volatility Estimate (MOVE) index.11

*Decomposing factors that drive the natural real rate*

Our model posits four variables that affect the natural real rate: the term premium \((tp)\), the risk premium \((rp)\), trend growth \((g)\), and other aggregate demand factors \((z)\). Chart 3 plots the natural real rate over bars showing the contribution of each component to the natural real rate. While term and risk premiums made strong negative contributions to the natural rate during the 2007–09 financial crisis and recession, they have since ebbed from their post-recession highs, leading to a rise in \(r^*\). However, the level of \(r^*\) has been weighed down by trend
growth and other aggregate demand factors despite easing financial market conditions during the economic recovery.

The economy’s rate of trend growth has persistently declined since the end of the 20th century. We estimate that the economy’s trend growth rate has slowed from 3 percent per year in the mid-1990s—a period of rapid technological advancement and adoption—to 2 percent in the mid-2000s and to just 1.7 percent in the mid-2010s. This decline in potential growth is consistent with the observation by Stock and Watson (2016) that demographic forces due to an aging U.S. population, together with the slowing rate of growth in output per worker, are acting as a headwind to economic growth. By our estimates, the reduction in the economy’s long-run growth capacity has reduced the natural real rate by 1.3 percentage points since the mid-1990s.

Our estimate of the economy’s potential growth rate from this top-down approach aligns well with the estimate from the Congressional Budget Office (CBO). The CBO regularly publishes estimates of the U.S. economy’s potential growth rate using a growth accounting perspective. In particular, the CBO attempts to estimate the economy’s productive capacity based on sectoral data and then aggregates this back to a
measure of aggregate output. This bottom-up approach predicts that the growth rate of potential output was about 1.6 percent in the third quarter of 2016, very near our estimate of 1.7 percent for the same period.

In addition to the decline in trend growth, latent aggregate demand factors, captured by the \( z \) term in the natural real rate equation, also appear to be acting as a headwind to \( r^* \) in recent years. Interpreting the factors influencing \( z \) is difficult, because by assumption, this variable captures components of aggregate demand that are unobservable. One often-cited factor restraining the economy during the most recent expansion is the stance of fiscal policy. Although government spending supported GDP growth in the initial years of the recession, it became a drag on growth in subsequent years (Bernanke 2012a; Yellen; Stock and Watson 2016).

Our estimate of \( z \) seems to be capturing the stance of fiscal policy among other possible elements of aggregate demand. Chart 4 plots our time-series of \( z \) against the two-year centered moving average of government spending’s (arithmetic) contribution to GDP growth to capture not only past spending, but also its contributions to aggregate growth over the next year. The two series are tightly correlated over our estimation sample. This suggests that during much of the economic expansion, past and expected future reductions in government spending have contributed to weak aggregate demand and thereby weighed on the natural rate.

**Uncertainty in our estimates of \( r^* \)**

One caveat to our interpretations is that our estimate of the natural real rate is not very precise. Chart 5 shows our point estimate surrounded by 90 percent confidence bands. The average range between the upper and lower confidence band is about 5 percentage points, but in the most recent period, the range exceeds 7 percentage points. In other words, the uncertainty associated with our estimate of the natural rate is high on average, but especially high for the most recent estimate. Another source of uncertainty, not captured in Chart 5, is model specification. For example, when Laubach and Williams change the specification of the latent aggregate demand process, the resulting estimate of the natural rate becomes more cyclical.

These uncertainties are not unique to our estimates. Any model-based approach can produce imprecise estimates that vary substantially with
Chart 4
The Link between Aggregate Demand Factors and Fiscal Policy

Note: Gray bars denote NBER-defined recessions.
Sources: BEA, BLS, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, Moody’s, NBER, and authors’ calculations. All data sources accessed through Haver Analytics.

Chart 5
Uncertainty Surrounding the Natural Real Rate

Note: Gray bars denote NBER-defined recessions.
Sources: BEA, BLS, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, Moody’s, NBER, and authors’ calculations. All data sources accessed through Haver Analytics.
different vintages of data and different model specifications (Laubach and Williams; Clark and Kozicki; Holston, Laubach, and Williams). As Clark and Kozicki point out, these issues make statistical estimates of the real rate less reliable in practical policy applications. To address this shortcoming, we develop an alternative, data-driven estimate of the natural real rate as a cross-check on our model-based analysis.

III. A Data-Driven Approach to Estimating the Natural Real Rate of Interest

To derive an alternative estimate of the natural real rate, we look for a common component across numerous variables that economists and policymakers have associated with the natural real rate. This approach removes the uncertainty surrounding model specification, as it requires us to make minimal assumptions.

We estimate what we call “the natural real rate factor,” denoted by $f$, using a statistical technique called principle component analysis. Principle component analysis enables us to consolidate information across 24 variables plausibly related to the natural real rate, including long-term real interest rates, trend-growth estimates from the CBO, demographic trends, measures of economic policy uncertainty, measures of the U.S. credit and housing cycle, cyclically adjusted price-to-earnings ratios as a measure of investor sentiment, measures of the supply of global savings into U.S. financial markets, a measure of government regulations, and both quantitative and qualitative measures of the ease or tightness of U.S. financial markets (which include the risk and term premium used in our model-based estimate). The natural real rate factor is constructed as a weighted average of the 24 variables. The complete list of variables, along with a description of any transformations, is included in Appendix B.

Since the variables have very different units, means, and standard deviations, each variable is first normalized to have a mean equal to zero and a standard deviation equal to one. As a result, our estimate of the natural real rate factor, $f$, also has a mean of zero and a standard deviation of one. Therefore, a reading of $f = 0$ means the natural real rate equals its historical average, while the historical average of our $r^*$ estimate is about 2 percent.
To illustrate the relationship between the 24 variables and \( f \), Chart 6 reports the factor loadings—that is, the correlation between the normalized variable and \( f \). The chart ranks the variables by the size of the absolute value of the correlation. Blue bars denote a positive factor loading, while green bars denote a negative factor loading. As expected, consistent with the model-based estimate of \( r^* \), the correlation between the term and risk premium and our natural rate factor is negative. Also as expected and consistent with the model-based estimate, growth in real potential GDP is positive and has the second largest correlation with \( f \). The Federal Reserve Bank of Kansas City’s Labor Market Conditions Indicators (LMCI) Activity index, a broad measure of labor market conditions, has the largest correlation with \( f \), presumably reflecting the cyclicality of \( r^* \). Finally, the correlation between \( f \) and three three other measures of financial conditions—the Kansas City Financial Stress Index (a broad measure of financial stress), the share of banks
reporting tighter standards for commercial loans, and the growth rate of nonfinancial credit—indicate tighter credit conditions reduce $f$.

Although our approaches to estimating $r^*$ and $f$ are vastly different, they yield similar interpretations of the natural rate. To compare these two estimates of the natural real rate on the same chart, Chart 7 plots $r^*$ on the left axis and $f$ on the right axis. Our data-driven estimate of the natural real rate closely tracks our model-based estimate with a correlation coefficient of 0.89. And much like our model-based estimate, the data-driven estimate was still running below its historical average as of the second quarter of 2016.

The similar conclusions reached from both a model-based and data-driven approach provide some confidence in our assessment of the natural real rate in recent decades. In particular, both estimates are highly cyclical, rising in expansions and falling in recessions. Both estimates also reached their sample lows during the recent financial crisis but have trended up in recent years. In all, the timing and magnitude of the movements are broadly consistent with our previous analysis linking the ease or tightness of financial conditions to the natural real rate. Since $r^*$ and $f$ are so highly correlated, we focus on $r^*$, which has a meaningful level interpretation for the natural real rate, for the remainder of the article.
IV. Movements in the Natural Real Rate and Events in Financial Markets

Many factors can influence bond premiums—and, in turn, the natural real rate of interest. To illustrate this, Chart 8 highlights how $r^*$ has been influenced by four prominent events that significantly changed the ease or tightness of U.S. financial market conditions: changes in the supply of global savings (often referred to as the “global savings glut”), the global financial crisis, changes in expectations about the size of the Fed’s balance sheet that occurred in spring 2013 (an event now referred to as the “taper tantrum”), and the 2014 oil price collapse.

The global savings glut

From June 2004 to February 2005, the FOMC increased the target federal funds rate by 150 basis points, but the yield on the 10-year Treasury security fell by more than 50 basis points. At the time, then-Chair Greenspan called the diverging paths of long-term and short-term rates a “conundrum.” In 2007, then-Chair Bernanke proposed the global savings glut as an explanation for the puzzling decline in long-term rates. Bernanke hypothesized that a global savings imbalance led to large inflows of foreign savings into U.S. capital markets, driving up the price of both safe and risky assets and thereby lowering their yield.

Consistent with Bernanke’s hypothesis, the term premium fell by more than 2 percentage points from 2004 to 2006, while risk premiums declined by nearly 0.5 percentage point. Together, these declines led to a more than 2.5 percentage point increase in our estimate of $r^*$. Warnock and Warnock use data on foreign official purchases of U.S. securities to show that foreign purchases lowered the yield on the 10-year Treasury security during 2004–06 by more than 80 basis points. Moreover, they find that foreign purchases have larger effects on BAA-rated U.S. corporate bonds than Treasury securities, suggesting foreign inflows also played a role in depressing risk premiums during these years. This evidence, viewed through the lens of our model of the natural real rate, suggests that an influx of foreign funds into U.S. capital markets applied meaningful upward pressure on the natural real rate over this period.
The global financial crisis

Our estimate of the natural real rate fell precipitously over the 2006–08 period. The sharpest decline came in the fourth quarter of 2008, when the global financial crisis intensified. A full discussion of the events that increased turmoil in financial markets over this period is beyond the scope of this article; instead, we focus on some clearly identifiable events in the second half of 2008 that led to a sharp rise in bond premiums.

In September 2008, the Federal Housing Finance Agency placed Fannie Mae and Freddie Mac into conservatorship, Lehman Brothers filed for bankruptcy, and the Federal Reserve extended AIG an $85 billion rescue package. This sequence of events amplified already high risk aversion, sending the risk premium to a post-war high.

The term premium remained elevated throughout September but reached new highs in October due in large part to increased uncertainty over the policy response to the unfolding crisis. As the financial crisis intensified, the Federal Reserve voted to cut its target for the federal funds rate by 50 basis points in a coordinated move with other central banks. The rate reduction, which was announced after an unscheduled conference call, sparked uncertainty over the timing and size of further

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**Chart 8**
The Natural Real Rate and Changes in U.S. Financial Market Conditions

![Chart showing changes in U.S. financial market conditions](chart8.png)

Sources: BEA, BLS, Federal Reserve Bank of New York, Board of Governors of the Federal Reserve System, Moody’s, NBER, and authors’ calculations. All data sources accessed through Haver Analytics.
interest rate cuts. As a result, measures of near-term expected interest rate volatility, such as the MOVE index, rose sharply along with the term premium. Tense political negotiations over the Troubled Asset Relief Program (TARP) added further policy uncertainty. The initial TARP bill failed to pass the House of Representatives, raising concerns in financial markets over how long it would take Congress to agree on a policy response.

The rise in term and risk spreads during the crisis was associated with what was arguably the largest tightening in U.S. financial conditions since the Great Depression. Our estimate of the natural real rate commensurately declined to nearly −4 percent in the fourth quarter of 2008. Although we estimate that the natural real rate became negative in the four previous recessions, the unusually large decline suggests that deeply negative policy rates would have been needed to fully stabilize the economy. Therefore, through the lens of our model, unconventional monetary policy can be viewed as an attempt to reduce the output gap by narrowing the real rate gap when the natural real rate is deeply negative and nominal interest rates have reached their effective lower bound.

The taper tantrum

One of the most vivid illustrations of the link between the FOMC’s balance sheet and the natural rate came in the spring of 2013. Unlike previous QE programs, the Federal Reserve’s third round of asset purchases (referred to as QEIII) was an open ended bond-buying program with no preset size or end date. In May 2013, then-Chair Bernanke suggested during congressional testimony that the current pace of asset purchases might be tapered in the “next few meetings” if the U.S. economy continued to improve. Bernanke reiterated this assessment in a press conference after the June FOMC meeting. Together, these comments pulled forward the expected timing of reductions in the monthly flow of asset purchases and increased uncertainty about when policy accommodation would be reduced.12 As a result, financial markets tightened considerably during this time. From May to June 2013, the term premium jumped nearly 40 basis points and continued to rise through the end of the year, at which time the FOMC began tapering
asset purchases. Our estimate of the natural real rate declined nearly 1 percentage point over the second half of 2013 due to this rise in the term premium.

The 2014 oil price collapse

A 70 percent collapse in oil prices from 2014 to 2016 sent energy firms into financial distress and tightened overall financial conditions. From 2011 to mid-2014, West Texas Intermediate (WTI) oil prices averaged about $100 per barrel. But growing U.S. oil production, together with the November 2014 announcement that the Organization of the Petroleum Exporting Countries (OPEC) was not willing to play the role of swing producer, sent prices tumbling by 60 percent in just one year. After recovering to around $60 per barrel in mid-2015, spot prices for WTI fell again on the heels of an announced agreement with Iran that would enable the country to once again supply global markets with oil. Prices continued to fall through 2015, breaching $30 per barrel in early 2016 amid growing concerns that demand for oil was faltering. At the same time, concerns about China were growing as its economy transitioned to a more consumer-oriented growth model.13

The risk premium rose nearly 120 basis points from the peak in oil prices in the second quarter of 2014 to the trough in the first quarter of 2016. Corporate bond spreads peaked in 2016:Q1 as creditors grew concerned that low oil prices would hamper oil producers’ ability to repay their debt. Comments in the January 2016 Senior Loan Officer Opinion Survey noted oil and gas producers as a particular industry of concern and cited the energy industry as one reason for tightening credit standards on commercial loans. Capital markets tightened similarly over this period. Chart 9 shows that the rise in risk spreads was initially concentrated in the energy sector but spilled over to non-energy firms as well, thereby tightening overall credit conditions. Consequently, our estimate of the natural real rate declined throughout 2015. After cresting at nearly 1.5 percentage points to start the year, by the end of 2015, the natural real rate had fallen to an estimated 0.5 percentage point.14 As oil prices rose through 2016, risk spreads narrowed for both energy and non-energy firms, and $r^*$ recovered.
V. Conclusion

Recent estimates of the natural real rate of interest show a persistent decline from its historical average of about 2 percent. The prospect of a persistently low natural real rate of interest has numerous ramifications for monetary policy makers. For example, as lower rates are required to keep the economy operating at potential, encounters with the effective lower bound may become more frequent and longer lasting. Widely cited estimates from Laubach and Williams’ model, which links the natural real rate to both the economy’s trend growth rate and persistent aggregate demand factors, suggest the natural real rate of interest has been declining for several decades and is currently near zero. However, the model does not explicitly account for the influence of financial market conditions on the natural real rate.

In this article, we augment the Laubach and Williams model with measures of bond premiums to capture the relationship between U.S. capital markets and the natural real rate of interest. We find evidence of a meaningful negative relationship between term and risk premiums and the natural real rate. To the extent that some of the recent movements in bond premiums can be traced to the FOMC’s asset purchases, our findings suggest a link between the FOMC’s balance sheet...
and the level of the natural real rate of interest. However, a multitude of non-monetary factors can also drive bond premiums. Therefore, a broader interpretation of our results is that changes in financial market conditions, emanating from changes in the risk appetite of investors or the supply of global savings, may require a monetary policy response.

In addition to our model-based estimate, we also provide a data-driven estimate of the natural real rate. This alternative approach produces a natural real rate factor, \( f \), that is highly correlated with our model-based estimate of \( r^* \) but requires few modeling assumptions. The strong correlation between \( f \) and \( r^* \)—despite very different estimation techniques—further supports our interpretation of the link between financial market conditions and the natural real rate.

Our resulting estimates of the natural real rate are much more cyclical than most other estimates of \( r^* \). While we estimate that the U.S. economy’s rate of potential growth has been steadily declining for several decades, the time variation in financial market conditions outweighs this long-term decline in trend growth. As a consequence, our natural real rate estimates fell sharply during and after the recent recession, but have also risen steadily in line with the recovery and ongoing economic expansion. Nevertheless, a sustained return of \( r^* \) to its historical average seems unlikely due to the apparent deceleration in trend growth over the past 20 years.
Appendix A
Model Estimation Details

In this appendix, we describe the estimation strategy for our model-based estimates of the natural real rate. Equations (1)–(5) in the text form the basis of a state-space model, with equations (1) and (2) serving as the measurement equations and equations (3), (4), and (5) serving as the transition equations. In principle, this state-space model can be directly estimated via maximum likelihood using the Kalman filter. However, in practice, the estimates of $\sigma_3$ and $\sigma_5$ are typically pushed to zero due to the so-called “pile-up” problem (Stock). Therefore, we follow Laubach and Williams and peg the signal to noise ratios:

$$\lambda_z = \frac{|a_1|}{\sqrt{2}} \frac{\sigma_3}{\sigma_y} = 0.058 \quad \text{and} \quad \lambda_g = \frac{\sigma_5}{\sigma_y} = 0.042$$

These values for $\lambda_z$ and $\lambda_g$ are the estimated values from Laubach and Williams, who use a multistep estimation procedure. First they model potential GDP as a random walk with drift so that trend growth is a constant. With this specification, they find an estimate of $\lambda_g$ by performing a structural break test on the intercept term in a regression of the growth rate of potential GDP on a constant and then use the look-up table (Table 3) in Stock and Watson (1998). They then use this estimate of $\lambda_g$ in a second-stage estimation that assumes $z_t$ is constant to similarly arrive at an estimate of $\lambda_z$. In particular, they perform a structural break test on the intercept term in a regression of the resulting output gap series on two lags of itself and a two-quarter average of the lagged real rate and then use the look-up table (Table 3) in Stock and Watson (1998).

With Laubach and Williams’ estimates of $\lambda_z$ and $\lambda_g$ in hand, we estimate the state-space model via maximum likelihood using the Kalman filter. The estimation is performed in RATS version 9.0. All results reported in the figures are calculated using the smoothed (two-sided) states. The model is estimated from 1962:Q1 through 2016:Q3.

The data used in the estimation are as follows. We measure output as 100 times the natural log of real GDP. Inflation is the annualized quarterly percent change of the core PCE price deflator (prior to 1959, we use the PCE price deflator, since the core PCE price deflator...
is not available). For the control variables in the Phillips curve, we measure import price movements by the difference between the annualized quarterly percent change in the price deflator for non-petroleum imports and inflation, and we measure oil-price movements by the difference between the change in the import price of crude oil and inflation. The import and oil price series are obtained from Laubach and Williams’ regular updates reported on the Federal Reserve Bank of San Francisco website. We calculate the real federal funds rate as the difference between the nominal effective federal funds rate and a statistical forecast of inflation over the next year using an AR(3) model estimated over a 10-year rolling window. Prior to 1965, we use the Federal Reserve Bank of New York’s discount rate series instead of the nominal effective federal funds rate, since the funds rate regularly falls below the discount rate over this period.
Appendix B

Data-Driven Natural Real Rate Estimation Details

In this appendix, we list each variable used in our factor analysis. In addition, we also provide the source for each variable along with any transformations made to the variables. All variables that are available daily or monthly are first averaged to a quarterly frequency before further calculations. The data we use can be generally classified into one of 10 categories:

**Real interest rates**

- Real long-term interest rate: The yield on the 10-year constant-maturity U.S. Treasury security (BOG) minus the median SPF forecast for 10-year-ahead CPI inflation (SPF).
- Real federal funds rate: The nominal effective federal funds rate (BOG) minus the year-over-year percent change in the CPI inflation rate (BLS).
- Long-term inflation expectations: The median forecast for 10-year-ahead CPI inflation (SPF).

**Real trend growth**

- The quarterly year-over-year percent change in potential GDP (CBO).

**Real economic activity**

- The year-over-year percent change in real output per hour in the nonfarm business sector (BLS).
- The year-over-year percent change in aggregate weekly hours of production and nonsupervisory employees (BLS).
- The year-over-year percent change in the civilian labor force (BLS).
- LMCI: Activity (KC Fed)
- LMCI: Momentum (KC Fed)

**Uncertainty**

- Economic Policy Uncertainty Index (Baker, Bloom, and Davis)
- Economic Policy Uncertainty Index: Tax Code Expirations sub index (Baker, Bloom, and Davis)
Demographics

- The year-over-year percent change in the civilian population ages 16–64 (BLS).
- The 16–64 civilian population divided by the civilian population (BLS).
- The year-over-year percent change in the number of households (Commerce Department).

Asset prices

- The year-over-year percent change in the market value of credit outstanding to the nonfinancial sector (BIS) minus the year-over-year percent change in the CPI inflation rate (BLS).
- The year-over-year percent change in the S&P CoreLogic Case-Shiller Home Price Index (S&P) minus the year-over-year percent change in the CPI inflation rate (BLS).

Supply and demand for loans

- Net percentage of domestic respondents tightening standards for C&I loans to small firms (SLOOS).
- Net percentage of domestic respondents reporting stronger demand for consumer loans (SLOOS).

Financial market conditions

- Federal Reserve Bank of Kansas City Financial Stress Index.
- Risk premium: The difference between Moody’s index of BAA corporate bonds (Moody’s) and the 10-year constant-maturity U.S. Treasury security (BOG).
- Term premium: The estimate for the 10-year U.S. Treasury security (Adrian, Crump, and Moench 2013a).

Government regulation

- Percentage of firms reporting government regulation as their single most important problem (NFIB).
Global savings glut

- U.S. current account as a share of GDP (BEA).

All series are obtained from Haver Analytics. The natural rate factor is the first principle component of the standardized version of these 24 variables.
Endnotes

1Gilchrist and Zakrajšek (2013) use a different estimation approach but also find that QE announcements that lowered yields on government bonds led to a reduction in the overall level of credit risk in the economy. In addition, Hamilton and Wu find that shifting the composition of the FOMC’s balance sheet toward longer-maturity Treasury securities could lower term and risk premiums using a term-structure model.

2In a recent paper, Carlstrom, Fuerst, and Paustian more carefully model the theoretical underpinnings of the term premium and find the same normative prescription for monetary policy makers: changes in the term premium should be offset by changes in policy rates.

3Doh presents a careful overview of the arguments for and against central banks counteracting swings in asset prices, including risk spreads.

4Since the slope of the yield curve is composed of the term premium and the difference between expected future short-term rates and current short-term rates, this finding is consistent with the stylized fact that a downward sloping (inverted) yield curve is a harbinger of a recession.

5The real effective federal funds rate is also unobservable, since nominal interest rates have to be adjusted for inflation expectations (which are themselves not readily observable). However, following Laubach and Williams, we proxy inflation expectations using, at each point in time, a statistical forecast of inflation over the next year. Therefore, our estimation treats the real effective federal funds rate as a known quantity.

6The term “accelerationist” is used to describe this relationship, because a positive output gap is associated with rising inflation and hence an accelerating price level.

7We use Adrian, Crump, and Moench’s (2013) estimate of the term premium, rather than Kim and Wright’s estimate, because the model estimation begins in 1962.

8The estimation procedure pegs the values of $\sigma_3$ and $\sigma_5$ following Laubach and Williams. More details are available in Appendix A.

9The standard errors in the equation for the natural real rate should be interpreted with caution. In particular, since $z_t$ is non-stationary (follows a random walk), spurious correlations could be driving the results (Granger and Newbold; Laubach and Williams).

10The p-value for the likelihood ratio test comparing our preferred model with the model without bond premiums suggests that removing bond premiums does not alter the fit by a statistically significant amount. However, if we specify the model with a single parameter governing the effects of bond premiums by restricting $c_p$ to equal $c_{rp}$, a restriction that cannot be rejected by the data, then
we can easily reject the model that excludes bond premiums with a high degree of statistical significance.

11 The VIX measures the implied uncertainty over the stock market during the next 30 days according to options prices.

12 The April 2013 Survey of Primary Dealers suggests financial market participants expected the FOMC to reduce its pace of asset purchases sometime in 2014. However, by July, the expected timing of tapering had been pulled forward to September 2013.

13 Nie (2016) provides a summary of this expected transition and potential outcomes for Chinese growth.

14 While risk premiums continued to rise in the first quarter of 2016, term-premiums fell as well that quarter due to concerns of the growth prospects for major emerging market economies which resulted in demand for safe U.S. Treasuries. These safe-haven flows helped to prevent a further decline in the natural rate at the start of 2016.
References


