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Abstract

At the zero lower bound, the central bank's inability to offset shocks endogenously generates volatility. In this setting, an increase in uncertainty about future shocks causes significant contractions in the economy and may lead to non-existence of an equilibrium. The form of the monetary policy rule is crucial for avoiding catastrophic outcomes. State-contingent optimal monetary and fiscal policies can attenuate this endogenous volatility by stabilizing the distribution of future outcomes. Fluctuations in uncertainty and the zero lower bound help our model match the unconditional and stochastic volatility in the recent macroeconomic data.

JEL Classification: E32, E52

Keywords: Endogenous Volatility, Zero Lower Bound, Optimal Stabilization Policy

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1 Introduction

Models with nominal rigidities allow aggregate demand to determine output. In response to declines in aggregate demand, monetary policy plays a key role in stabilizing real activity and inflation. Even in the face of significant exogenous shocks, an unconstrained central bank can stabilize the economy using its nominal policy rate. Households internalize this ability of the monetary authority to influence real activity and inflation in all states of the world. In this setting, uncertainty about future exogenous shocks is irrelevant because monetary policy can effectively offset all possible shocks.

At the zero lower bound, however, monetary policy cannot offset further negative shocks but will offset sufficiently large positive shocks. This asymmetry reduces the mean of expected future outcomes and increases their variance. Thus, the zero lower bound generates endogenous volatility. This endogenous volatility leads households to increase their desired savings. With flexible prices, higher desired savings by households would simply lower the real interest rate but leave equilibrium output unchanged. With nominal rigidities, precautionary saving by households reduces aggregate demand further, and keeps the economy at the zero lower bound for a longer period of time. Under standard assumptions about monetary policy, this destabilizing feedback mechanism leads to significant contractions in the economy. In fact, this feedback mechanism may be so powerful that an equilibrium fails to exist. We show that the form of the monetary policy reaction function is crucial for avoiding this catastrophic outcome.

Through this destabilizing feedback mechanism, the distribution of possible future shocks becomes crucially important. Expectations of more volatile shocks further increase the expected variance of future consumption and strengthen the destabilizing feedback loop. Through the endogenous volatility generated by the zero lower bound, small amounts of uncertainty about future exogenous shocks are transformed into large declines in aggregate demand.

This paper illuminates these interactions between the zero lower bound and the uncertainty about future exogenous shocks. We begin by examining the positive economics of changes in expected volatility when the economy is trapped at the zero lower bound. We then examine the normative issues of choosing optimal monetary and fiscal policy to stabilize an economy subject to stochastic volatility and the zero lower bound constraint.

Many policymakers and economists have cited increased uncertainty about the future as a key driver in generating the Great Recession and the subsequent slow recovery. Empirical work by Stock and Watson (2012) and Leduc and Liu (2014) and speeches by policymakers such as Kocherlakota (2010) point to higher uncertainty as the reason for a sizable fraction of the decline in real activity during the Great Recession and the slow subsequent recovery. Basu and Bundick (2012) present a simple model where higher uncertainty about future shocks can cause contractions in all major macroeconomic aggregates. But these papers raise a puzzle: why does uncertainty sometimes have small and sometimes large macroeconomic effects? For example, Bloom (2009) documents a variety of events that generate significant uncertainty about the future. Prior to the Great Recession, however, these events did not seem to spill over dramatically to the broader economy, especially in the post-1984 period.

Our model resolves this puzzle. We show that the level of background uncertainty about the future – just the expectation of future shocks – can assume much greater importance depending on the economic environment. We identify the constraint imposed by the zero lower bound as the key culprit that can transform this normal background noise into a significant downturn. The problematic element is the famous Taylor (1993) rule. This rule interacts with uncertainty and the zero lower bound constraint to create what we term the "contractionary bias." This bias emerges when the zero lower bound prevents the monetary authority from attaining its inflation target on average. We show that higher ex ante uncertainty at the zero lower bound increases this bias and raises the expected average real interest rate. Higher expected real rates reduce output and inflation, making the zero lower bound constraint bind more strongly and creating a destabilizing feedback loop. Our paper thus explains why the effects of uncertainty can be time-varying, and why the existence of uncertainty at the zero lower bound can be catastrophic.

To derive a full set of policy implications, we show that it is crucial to use global solution methods that allow for ex ante uncertainty about future events. The existing literature often fails to uncover the contractionary bias or conflates two conceptually distinct channels: (1) the contractionary bias and (2) the effects of uncertainty *per se* at a given real interest rate. To disentangle these two effects, we need to shift away from simple Taylor rules to rules that allow the central bank to achieve its inflation target on average despite the zero lower bound constraint. These history-dependent rules prevent the average expected real rate from rising simply because the zero lower bound binds in more states of nature. We show that the negative effects of uncertainty *per se* can be substantial when the economy is at the zero lower bound. However, it is the interaction between ex ante uncertainty about future shocks, the zero lower bound, and the Taylor rule that can be devastating. The implication is that monetary policy must follow a rule that may emulate the Taylor rule during normal times, but stabilizes the real interest rate when the zero lower bound constraint binds.

Optimal monetary policy can attenuate the endogenous volatility generated by the zero lower bound. The central bank achieves this outcome via two channels: (1) lowering the expected path of real interest rates and (2) stabilizing the conditional distribution of household consumption. If a contractionary shock is realized, the central bank lowers real rates by committing to a lower path of future nominal interest rates. Households fully internalize this commitment by the central bank to respond to the economy if bad shocks are realized. This state-contingent policy response stabilizes the household's expected distribution of consumption. However, the optimal policy requires maintaining a zero policy rate for an extended period of time. To stabilize the short-run distribution of outcomes at the zero lower bound, the central bank tolerates a higher and more volatile medium-run distribution of inflation. State-contingent government spending, if available, can help stabilize the economy further.

To analyze the quantitative impact of ex ante uncertainty at the zero lower bound, we calibrate and solve a representative-agent, dynamic stochastic general equilibrium model using a global solution method. The model economy is continually hit by first- and second-moment shocks to aggregate demand. We denote a second-moment shock an "uncertainty" shock since it makes forecasting future exogenous shocks more difficult. Qualitatively, this modeling choice allows us to show how expectations about future shocks can assume much greater importance at the zero lower bound. In addition, we show that the interactions between the zero lower bound and these uncertainty shocks are quantitatively important for matching features of recent macroeconomic aggregates. In particular, these two nonlinearities help the model match the unconditional and stochastic volatility of the output gap, inflation, and the nominal interest rate. The model also can generate significant periods of time at the zero lower bound, which is consistent with the recent US experience. The zero lower bound episodes are also characterized by a highly uncertain future liftoff date, which is in line with a recent survey of Federal Open Market Committee (FOMC) participants. Without uncertainty shocks and the zero lower bound, the model struggles to jointly match these features of the recent macroeconomic data.

As an extension, we show that the endogenous volatility generated by the zero lower bound may provide an explanation for the "Forward Guidance Puzzle." Using a similar model with nominal rigidities, Del Negro, Giannoni and Patterson (2013) argues that the model is too responsive to exogenous changes in future interest rates. However, they reach this conclusion using a linearized model where households do not take into account the uncertainty about future consumption. Our paper argues that this omitted variable may be crucially important when the economy is stuck at the zero lower bound. We show that the endogenous volatility generated by the zero lower bound heavily attenuates the response of the economy to exogenous changes in interest rates.

2 Intuition

This section formalizes the intuition from the Introduction using a few key equations from a simple general-equilibrium model. For Section 2 only, we use Taylor series approximations of these equations to show how the zero lower bound endogenously generates volatility. These approximations provide analytical tractability which is unavailable when examining the equations in their original nonlinear form. In Section 4, we show that the intuition from these approximations is consistent with the computational results using the full nonlinear model.

2.1 Household Consumption Under Uncertainty

The household consumption Euler equation highlights how the zero lower bound endogenously generates volatility. Under constant relative risk aversion utility from consumption, the following equation links household consumption C_t to the gross real interest rate R_t^R:

$$1 = \mathrm{E}_t \left\{ \beta R_t^R \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\}, \tag{1}$$

where β is the household discount factor and σ parameterizes intertemporal substitution and risk aversion. Using a third-order Taylor series approximation around the steady state, Appendix A shows that Equation (1) can be written as follows:

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}\left(r_t^r - r^r \right) - \frac{1}{2}\sigma \operatorname{Var}_t c_{t+1} + \frac{1}{6}\sigma^2 \operatorname{Skew}_t c_{t+1}, \tag{2}$$

where lowercase variables denote the log of the respective variable, r^r is the steady state net real interest rate, and $\operatorname{Var}_t c_{t+1}$ and $\operatorname{Skew}_t c_{t+1}$ denotes the conditional variance and skewness of future consumption. For any given real interest rate, households consume less if they expect a more volatile and negatively-skewed distribution of future consumption.

After defining a flexible-price version of Equation (2), Appendix A shows how to derive the following approximate higher-order version of a standard New-Keynesian IS Curve:

$$x_t = \mathrm{E}_t\, x_{t+1} - \frac{1}{\sigma}\left(r_t^r - r_t^n \right) - \frac{1}{2}\sigma \operatorname{Var}_t x_{t+1} + \frac{1}{6}\sigma^2 \operatorname{Skew}_t x_{t+1} \tag{3}$$

where x_t denotes the gap between equilibrium and flexible-price output and r_t^n is the natural real interest rate that would prevail in the flexible-price economy. Iterating Equation (3) forward and taking expectations at time t implies the following solution for current output gap:

$$x_t = -\sum_{i=0}^{\infty} \left(\mathrm{E}_t \, r^n_{t+i} - \mathrm{E}_t \, r^r_{t+i} \right) - \frac{1}{2}\, \sigma \sum_{j=0}^{\infty} \mathrm{Var}_t \, x_{t+1+j} + \frac{1}{6}\, \sigma^2 \sum_{k=0}^{\infty} \mathrm{Skew}_t \, x_{t+1+k} \tag{4}$$

The impact of these higher-order terms on the macroeconomy depends the ability of monetary policy to stabilize the economy. Without a zero lower bound on the nominal interest rate, the monetary authority can always fully offset the effects of uncertainty by setting its policy rate to close the gap between the real and natural real interest rates. In this scenario, the conditional variance and skewness of the output gap are zero since the monetary authority can stabilize the economy in all future states of the world. However, suppose the natural real rate becomes negative and the zero lower bound prevents the central bank from fully stabilizing the economy. Households internalize this reduced ability to offset future contractionary shocks throughout the zero lower bound episode. This asymmetric ability to stabilize the economy endogenously generates a more volatile and negatively-skewed distribution of future output gaps. Increased uncertainty about future shocks increases this asymmetry and leads to a significantly negative output gap today due to precautionary saving by households.

In response to the endogenous volatility at the zero lower bound, the optimal monetary policy can help stabilize the economy. Even though they are constrained today, the monetary authority can offset the higher expected volatility by committing to a lower path of future nominal rates. Lower nominal rates, for any given level of expected inflation, lower real interest rates and help stabilize the output gap. In addition, the monetary authority can promise to further lower nominal rates if bad shocks are realized. Households fully internalize this commitment by the central bank to respond to the economy in bad states of the world. This state-contingent policy response helps stabilize the expected distribution of outcomes. Optimal fiscal policy can also help stabilize the economy by committing to increase government spending if adverse shocks are realized. This additional policy tool helps further stabilize the possible future outcomes for the output gap.

2.2 From Intuition to Model Simulations

The intuition of this section argues that the zero lower bound can transform symmetric background noise about the future into a significant economic downturn. In the following section, we calibrate and solve a nonlinear model and show that the simulated zero lower bound scenarios are consistent with the intuition developed in this section. In addition, we solve for the optimal responses of monetary and fiscal policy under commitment.

3 Model

This section outlines the baseline dynamic stochastic general equilibrium model that we use in our analysis. The baseline model shares many features with the models of Ireland (2003) and Ireland (2011). The model features optimizing households and firms and a central bank that systematically adjusts the nominal interest rate to offset adverse shocks in the economy. We allow for sticky prices using the quadratic-adjustment costs specification of Rotemberg (1982). The baseline model considers fluctuations in the discount factor of households, which we interpret as demand shocks, since they are non-technological in nature.

3.1 Households

In the model, the representative household maximizes lifetime expected utility over streams of consumption C_t and leisure $1 - N_t$. The household receives labor income W_t for each unit of labor N_t supplied in the representative intermediate goods-producing firm. The household also owns the intermediate goods firm, receives lump-sum dividends D_t, and has access to zero net supply nominal bonds B_t and real bonds B_t^R. A nominal bond pays the gross one-period nominal interest rate R_t while a real bond pays the gross one-period real interest rate R_t^R. The household divides its income from labor and its financial assets between consumption C_t and the amount of the bonds B_{t+1} and B_{t+1}^R to carry into next period. The discount factor of the household β is subject to shocks via the stochastic process a_t. An increase in a_t induces households to consume more and work less.

The representative household maximizes lifetime utility by choosing $C_{t+s}, N_{t+s}, B_{t+s+1}$, and B_{t+s+1}^R, for all $s = 0, 1, 2, \ldots$ by solving the following problem:

$$\max \mathrm{E}_t \sum_{s=0}^{\infty} a_{t+s}\beta^s \frac{\left(C_{t+s}^{\eta}(1-N_{t+s})^{1-\eta}\right)^{1-\sigma}}{1-\sigma}$$

subject to the intertemporal household budget constraint each period,

$$C_t + \frac{1}{R_t}\frac{B_{t+1}}{P_t} + \frac{1}{R_t^R}B_{t+1}^R \leq \frac{W_t}{P_t}N_t + \frac{B_t}{P_t} + \frac{D_t}{P_t} + B_t^R.$$

Using a Lagrangian approach, household optimization implies the following first-order conditions:

$$\eta a_t C_t^{\eta(1-\sigma)-1}\left(1-N_t\right)^{(1-\eta)(1-\sigma)} = \lambda_t \tag{5}$$

$$\left(1-\eta\right) a_t C_t^{\eta(1-\sigma)}\left(1-N_t\right)^{(1-\eta)(1-\sigma)-1} = \lambda_t \frac{W_t}{P_t} \tag{6}$$

$$1 = \mathrm{E}_t\left\{\left(\beta\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{R_t P_t}{P_{t+1}}\right)\right\} \tag{7}$$

$$1 = \mathrm{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) R_t^R \right\} \tag{8}$$

where λ_t denotes the Lagrange multiplier on the household budget constraint. Equations (5) - (6) represent the household intratemporal optimality conditions with respect to consumption and leisure, and Equations (7) - (8) represent the Euler equations for the one-period nominal and real bonds.

3.2 Intermediate Goods Producers

Each intermediate goods-producing firm i rents labor $N_t(i)$ from the representative household in order to produce intermediate good $Y_t(i)$. Intermediate goods are produced in a monopolistically competitive market where producers face a quadratic cost of changing their nominal price $P_t(i)$ each period. Firm i chooses $N_t(i)$, and $P_t(i)$ to maximize the discounted present-value of cash flows $D_t(i)/P_t(i)$ given aggregate demand, Y_t, and the price P_t of finished goods. The intermediate goods firms all have access to the same constant returns-to-scale Cobb-Douglas production function. We introduce a production subsidy $\Psi = \theta/(\theta - 1)$ to ensure that the steady state of the model is efficient, where θ is the elasticity of substitution across intermediate goods.

Each intermediate goods-producing firm maximizes discount cash flows using the household stochastic discount factor:

$$\max \mathrm{E}_t \sum_{s=0}^{\infty} \left(\beta^s \frac{\lambda_{t+s}}{\lambda_t} \right) \left[\frac{D_{t+s}(i)}{P_{t+s}} \right]$$

subject to the production function:

$$\left[\frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \leq N_t(i),$$

where

$$\frac{D_t(i)}{P_t} = \Psi \left[\frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - \frac{\phi_P}{2} \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right]^2 C_t$$

The first-order conditions for the firm i are as follows:

$$\frac{W_t}{P_t} N_t(i) = \Xi_t N_t(i) \tag{9}$$

$$\begin{aligned} \phi_P \left[\frac{P_t(i)}{\Pi P_{t-1}(i)} - 1 \right] \left[\frac{P_t C_t}{\Pi Y_t P_{t-1}(i)} \right] &= \Psi(1-\theta) \left[\frac{P_t(i)}{P_t} \right]^{-\theta} + \theta \Xi_t \left[\frac{P_t(i)}{P_t} \right]^{-\theta-1} \\ &+ \phi_P \mathrm{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t} \right) \frac{C_{t+1}}{Y_{t+1}} \frac{Y_{t+1}}{Y_t} \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i)}{\Pi P_t(i)} \frac{P_t}{P_t(i)} \right] \right\}, \end{aligned} \tag{10}$$

where Ξ_t is the multiplier on the production function, which denotes the real marginal cost of producing an additional unit of intermediate good i.

3.3 Final Goods Producers

The representative final goods producer uses $Y_t(i)$ units of each intermediate good produced by the intermediate goods-producing firm $i \in [0, 1]$. The intermediate output is transformed into final output Y_t using the following constant returns to scale technology:

$$\left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \geq Y_t$$

Each intermediate good $Y_t(i)$ sells at nominal price $P_t(i)$ and each final good sells at nominal price P_t. The finished goods producer chooses Y_t and $Y_t(i)$ for all $i \in [0, 1]$ to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to the constant returns to scale production function. Finished goods-producer optimization results in the following first-order condition:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t$$

The market for final goods is perfectly competitive, and thus the final goods-producing firm earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the firm objective function, the aggregate price index P_t can be written as follows:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}$$

3.4 Monetary Policy

We assume a cashless economy where the monetary authority sets the one-period net nominal interest rate $r_t = \log(R_t)$. Due to the zero lower bound on nominal interest rates, the central bank cannot lower its nominal policy rate below zero. In the following results, we show that the form of the monetary policy reaction function is crucial in determining how uncertainty affects the macroeconomy. In our baseline model, we follow the previous literature and assume that the monetary authority sets its policy rate according to the following Taylor (1993)-type policy rule subject to the zero lower bound:

$$r_t^d = r + \phi_\pi \Big(\pi_t - \pi\Big) + \phi_x x_t \tag{11}$$

$$r_t = \max\Big(0, r_t^d\Big) \tag{12}$$

where r_t^d is the desired policy rate of the monetary authority, r_t is the actual policy rate subject to the zero lower bound, π_t is the log of the gross inflation rate, and x_t is the gap between current output and output in the equivalent flexible-price economy.

3.5 Shock Processes

Shocks to the discount rate of households are the exogenous stochastic processes in the baseline model. Large negative innovations to the level this process imply a large decline in aggregate demand, which forces the economy to encounter the zero lower bound. The stochastic processes for these fluctuations are as follows:

$$a_t = (1-\rho_a)\,a + \rho_a a_{t-1} + \sigma^a_{t-1}\varepsilon^a_t \tag{13}$$

$$\sigma^a_t = (1-\rho_{\sigma^a})\,\sigma^a + \rho_{\sigma^a}\sigma^a_{t-1} + \sigma^{\sigma^a}\varepsilon^{\sigma^a}_t \tag{14}$$

ε^a_t is a first moment shock that captures innovations to the level of the household discount factors. We refer to $\varepsilon^{\sigma^a}_t$ and as a second moment or "uncertainty" shock since it captures innovations to the volatility of the exogenous processes of the model. An increase in the volatility of the shock process increases the uncertainty about its future time path. Both stochastic shocks are independent, standard normal random variables.[1]

3.6 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price $P_t(i) = P_t$ and employ the same amount of labor $N_t(i) = N_t$. Thus, all firms have the same cash flows and we define gross inflation as $\Pi_t = P_t/P_{t-1}$ and the markup over marginal cost as $\mu_t = 1/\Xi_t$. Therefore, we can model our intermediate-goods firms with a single representative intermediate goods-producing firm. To be consistent with national income accounting, we define a data-consistent measure of output $Y^d_t = C_t$. This assumption treats the quadratic adjustment costs as intermediate inputs. Fluctuations in household discount factors do not affect the equivalent flexible-price version of the baseline model. Therefore, we define the output gap as data-consistent output in deviation from its deterministic steady state $x_t = \ln(Y^d_t/Y^d)$. In addition, the gross natural real interest rate that would prevail in the equivalent flexible-price economy can be defined as $R^n_t = \beta^{-1}a_t(\mathrm{E}_t a_{t+1})^{-1}$. Thus, shocks to the household discount factor act as fluctuations in the natural real rate for the economy.

3.7 Solution Method

We solve the model using the policy function iteration method of Coleman (1990) and Davig (2004). This global approximation method allows us to model the occasionally-binding zero

[1]We specify the stochastic processes in levels, rather than in logs, to prevent the volatility σ^a from impacting average value of a_t through a Jensen's inequality effect. In the model solution, a_t always remains significantly greater than zero. To ensure that the volatility stays positive, we impose a lower bound $\underline{\sigma}^a = 0.0005$ on the volatility in both the model solution and simulations.

lower bound constraint in an environment where ex ante uncertainty matters for macroeconomic outcomes. Our results show that global methods are crucial in deriving the full set of policy implications from the interactions of uncertainty and the zero lower bound. This method discretizes the state variables on a grid and solves for the policy functions which satisfy all the model equations at each point in the state space. Appendix B contains the details of the policy function iteration algorithm.

3.8 Calibration

Table 1 lists the calibrated parameters of the model. We calibrate the model at quarterly frequency. Since the model shares features with the models of Ireland (2003) and Ireland (2011), we calibrate many of our parameters to match his estimates. To assist in numerically solving the model, we introduce a multiplicative constant into the production function to normalize output Y to equal one at the deterministic steady state. We choose steady-state hours worked N and the model-implied value for η such that the model has a Frisch labor supply elasticity of two. Household risk aversion over the consumption-leisure basket σ is 2. The value for σ implies an intertemporal elasticity of substitution of 0.5, which is consistent with the empirical estimates of Basu and Kimball (2002).

The crucial parameters in our calibration are the parameters that control to stochastic processes for the demand shocks. In conjunction with the monetary policy reaction function, these parameters control the amount of uncertainty about the future endogenous variables faced by the economy. For our initial baseline model, we set the unconditional volatility for $\sigma^a = 0.01$ and the uncertainty shock volatility $\sigma^{\sigma^a} = 0.005$. Thus, a one-standard deviation uncertainty shock increases the volatility of the shocks hitting the economy by 50 percent. However, even after a multiple standard deviation shock uncertainty shock, the volatility of the demand shocks in this baseline economy is significantly smaller than the unconditional maximum likelihood estimate of Ireland (2011). We discuss the rationale for this calibration in detail in Section 4.7.

4 Uncertainty Shocks and the Zero Lower Bound

4.1 Increased Uncertainty Without Change in Realized Volatility

We begin by analyzing the effects of an increase in uncertainty about the future shocks hitting the economy. For these initial impulse responses, we simulate a one standard deviation uncertainty shock, but assume that the economy is hit by no further shocks. This assumption isolates the effects of higher uncertainty about the future without any change in actual realized shock volatility. Figure 1 plots these traditional impulse responses both at the steady state and the zero lower bound. Holding the level of the discount factor shock constant at steady state, a 50 percent increase in the expected volatility of the demand shock causes a one basis point decline in the output gap and a three basis point fall in inflation. Despite the increase in expected shock volatility, households understand that the central bank can effectively stabilize the economy if bad shocks are realized. The ability of the central bank to lower its nominal policy rate limits the spillovers to the macroeconomy.

To compute the traditional impulse response at the zero lower bound, we generate two time paths for the economy. In the first time path, we simulate a large negative first moment demand shock, which causes the zero lower bound to bind for about eight quarters. In the second time path, we simulate the same large negative first moment demand shock, but also simulate a one standard deviation uncertainty shock. We compute the percent difference between the two time paths as the traditional impulse response to the uncertainty shock at the zero lower bound.

The inability of the monetary authority to offset the uncertainty shock magnifies the declines in output and inflation by over an order of magnitude. When the monetary authority is constrained by the zero lower bound, a one standard deviation uncertainty shock causes nearly a one-half percent decline in both the output gap and inflation. Even without any change in actual realized volatility, higher uncertainty about the future can be highly destabilizing at the zero lower bound. The results show that any amount of uncertainty about future shocks can assume much greater importance depending on the current economic conditions.

4.2 Expected Distributions of Future Outcomes

Figure 1 shows that the zero lower bound greatly amplifies the negative effects of the uncertainty shock. However, these traditional impulse responses mask the underlying reasons why uncertainty shocks cause larger contractions at the zero lower bound. Therefore, we now use simulations to show the ex ante distributions of future outcomes that households expect when making their decisions. These results show the exogenous shock volatility transmits to the endogenous volatility of output and inflation. We show that the spillovers to the macroeconomy of higher uncertainty crucially depend on the monetary policy reaction function. Unlike the previous experiments, these alternative impulse responses contain the effects of both higher uncertainty about the future and higher realized shock volatility.

To compute the expected distributions of possible outcomes, we follow the "generalized impulse response" method of Koop, Pesaran and Potter (1996). In addition to simulating the uncertainty shock, we now draw shocks randomly for the life of the impulse response using Equations (13) and (14). We repeat this procedure 50,000 times for both the responses around the steady state and those around the zero lower bound. Figure 2 plots the mean and 80% prediction intervals for the simulations both at and away from the zero lower bound. These intervals show the ex ante distribution of future outcomes that households expect when making their decisions. These alternative responses are also consistent with the rational expectations assumption in the model, since the distribution of actual shocks hitting the economy matches the distribution expected by households.[2]

Away from the zero lower bound, the central bank's Taylor (1993)-type policy rule greatly curtails the spillovers to the macroeconomy. Despite the increase in the exogenous shock volatility, the economy experiences very little increase in the endogenous volatility of output and inflation. By responding to movements in inflation and the output gap, the central bank offsets adverse shocks using its nominal policy rate. Since the central bank remains unconstrained, their ability to offset shocks is symmetric and thus the conditional mean and skewness remain largely unchanged. Away from the zero lower bound, the uncertainty shock simply adds noise to the expectations of future output and inflation without causing a significant economic contraction.

For the same time path of exogenous shock volatility, the zero lower bound endogenously generates large increases in the volatility of output and inflation. Under its simple policy rule, the central bank cannot lower its nominal policy rate to offset contractionary shocks. Since the monetary authority can no longer play its usual stabilizing role, adverse exogenous shocks imply much higher realized volatility in output and inflation. However, the Taylor (1993)-type policy rule implies that the monetary authority will offset expansionary shocks with higher nominal rates. This asymmetric response to shocks endogenously shifts the distribution of outcomes faced by households at the zero lower bound. Since large declines in output and inflation are more likely than the offsetting positive outcomes, the zero lower bound also endogenously causes declines in the conditional mean and implies negative skewness in future outcomes. Households internalize these possible future outcomes, which induces significant precautionary saving. This decline in aggregate demand leads to sizable contractions in output and inflation at the zero lower bound at impact. In addition, the duration of the zero lower bound episode is highly

[2] Under a first-order linearized solution, the mean of the generalized impulse response in Figure 2 would equal the traditional impulse response in Figure 1. However, the nonlinear zero lower bound constraint amplifies contractionary shocks, which induces significant asymmetry when additional shocks hit the economy.

uncertain and may persist even four years after the initial shock. Although the uncertainty shock is relatively short-lived with a half-life of about 4 quarters, the endogenous volatility generated by the zero lower persists for a significant period. At the zero lower bound, higher uncertainty about the future can cause a significant contraction in economic activity.

4.3 Inspecting the Mechanisms

Our previous results show that the endogenous volatility generated by the zero lower bound amplifies and propagates contractionary shocks. We now further inspect the transmission mechanisms of higher uncertainty to the macroeconomy. Under a standard Taylor (1993)-type policy rule, we show that the effects of ex ante uncertainty can be decomposed into two distinct mechanisms: (1) precautionary saving and working by households and (2) a bias in the monetary policy rule which causes higher nominal interest rates on average. We show that the form of the monetary policy reaction function is crucial in determining how these two mechanisms affect the macroeconomy.

4.4 Precautionary Labor Supply & Labor Demand

This section shows how precautionary saving by households lowers output and inflation in the macroeconomy. A more volatile and negatively-skewed expected distribution of consumption induces precautionary saving by the representative household through Equation (2). Since consumption and leisure are both normal goods, lower consumption also induces "precautionary labor supply," or a desire to supply more labor for an given level of the real wage. Figure 3 illustrates this effect graphically in real wage and hours worked space. Denoting the forward-looking marginal utility of wealth by λ_t, an increase in uncertainty raises λ_t, shifting the household labor supply curve outward through a negative wealth effect. This shift in labor supply lowers the real wage, and hence the marginal cost of production. If prices adjust slowly to changing marginal costs, however, firms' markups over marginal cost rise when the household increases its desired labor supply. At a given level of the real wage, an increase in markups decreases the demand for labor from firms.

The equilibrium increase in markups depends crucially on the behavior of the monetary authority. Even in a model without a zero lower bound constraint, Basu and Bundick (2012) shows that labor demand may decrease so much that equilibrium hours worked actually fall after an increase in uncertainty about the future. Since labor is the only input into production in the simple model of this paper, a decline in hours worked implies that output must fall. The zero lower bound further exacerbates this effect since the central bank is unable to offset the increase in markups reducing its policy rate. Thus, the endogenous volatility generated by the zero lower bound leads to further precautionary saving, which results in still higher markups and lower output.

4.5 Contractionary Bias in the Nominal Interest Rate

In addition to the precautionary working mechanism, the interaction between ex ante uncertainty and the zero lower bound can produce an additional source of fluctuations. This additional amplification mechanism, which we define as the contractionary bias in the nominal interest rate distribution, can dramatically affect the economy when uncertainty increases at the zero lower bound. The contractionary bias emerges when the zero lower bound prevents the monetary authority from attaining its inflation goal on average.

For this discussion, assume monetary policy sets its desired policy rate using the following simple rule:

$$r_t^d = r + \phi_\pi \Big(\pi_t - \pi \Big) \tag{15}$$

$$r_t = \max \Big(0, r_t^d \Big) \tag{16}$$

For a given monetary policy rule, the volatility of the exogenous shocks determines the volatility of inflation. Through the monetary policy rule in Equation (15), the volatility of inflation dictates the volatility of the desired nominal policy rate. However, since the zero lower bound left-truncates the *actual* policy rate distribution, more volatile desired policy rates lead to higher average actual policy rates. Figure 4 illustrates this effect by plotting hypothetical distributions of the desired and actual nominal interest rate distributions under low and high levels of exogenous shock volatility. The plot shows that the average actual policy rate is an increasing function of the volatility of the exogenous shocks when monetary policy follows a simple Taylor (1993)-type rule.[3]

Changes in this contractionary bias caused by higher uncertainty have dramatic general-equilibrium effects on the economy. Figure 5 plots the average Fisher relation $r = \pi + r^r$ and the average policy rule under both high and low levels of exogenous shock volatility. The upper-right intersection of the monetary policy rule and the Fisher relation dictates the normal general-equilibrium average levels of inflation and the nominal interest rate. An increase in

[3] Reifschneider and Williams (2000) first discuss this phenomenon and Mendes (2011) analytically proves this result using a simple New Keynesian model. Nakov (2008) and Nakata and Schmidt (2014) also describe this deflationary bias when monetary policy does not attain its inflation target on average when monetary policy is conducted optimally under discretion.

shock volatility shifts the policy rule inward and increases the average nominal interest rate for a given level of inflation. Higher volatility thus raises the average real interest rate, which reduces average output and inflation in the economy. Even if households are risk-neutral, so the precautionary effects discussed in the previous sub-section cease to apply, the contractionary bias implies lower output and inflation through the interaction between higher volatility, the monetary policy rule, and the zero lower bound.

4.6 Quantifying the Mechanisms

We now quantify the effects of the precautionary labor supply and contractionary bias channels. Following the insights in Reifschneider and Williams (2000), slight alterations to our baseline policy rule in Equation (11) can eliminate the contractionary bias mechanism. For example, adding a small weight on the price level automatically removes the contractionary bias. We now assume that the monetary authority conducts policy using the following simple rule:

$$r_t^d = r + \phi_\pi \Big(\pi_t - \pi\Big) + \phi_x x_t + \phi_{pl} \Big(p_t - p^*\Big) \tag{17}$$

$$r_t = \max\Big(0, r_t^d\Big) \tag{18}$$

where p_t is the log of the price level and p^* is the central bank's price level target. This additional term ensures a stable long-run price level by offsetting any deflation with equivalent inflation in the future, thus ensuring that the central bank achieves its inflation target on average. As discussed earlier, this result contrasts with a simple Taylor (1993)-type rule where the zero lower bound causes the average rate of inflation to be below target. This history-dependent rule prevents expectations of nominal rates from rising simply because the zero lower bound binds in a few more states of the world after an increase in exogenous volatility. We set the central bank's response to the price level $\phi_{pl} = 0.1$.

Both the precautionary labor supply and contractionary bias mechanisms are quantitatively significant. Figure 6 replicates the previous traditional impulse responses at the zero lower bound from Section 4.1 both with and without the response to the nominal price level. This exercise allows us to differentiate the effects of precautionary labor supply from those resulting from the contractionary bias channel. Of the 0.45 percent decline in output, about one-third of the decline is attributed to the precautionary saving channel and roughly two-thirds is due to the contractionary bias mechanism. Our results show that the exact form of the monetary policy rule, and how it affects the ex ante average nominal rate, is crucial for determining the general-equilibrium effects of uncertainty at the zero lower bound.

4.7 Should We Remove the Contractionary Bias?

We now show that the destabilizing effects of the contractionary bias may be so powerful that an equilibrium actually fails to exist. When the monetary authority responds only to inflation and output, Figure 5 shows that an increase in exogenous shock volatility shifts the policy rule to the left and increases the average nominal interest rate. For sufficiently high levels of volatility, however, the policy rule shifts far enough such that it no longer intersects the Fisher relation. In this situation, a rational-expectations equilibrium fails to exist because the contractionary bias is too large.

We find that this non-existence result under a Taylor (1993)-type policy rule is not a theoretical curiosity; non-existence occurs if we set the volatility of the exogenous shocks large enough such that the model can match the data. Recall that for our initial model, we set the unconditional volatility for $\sigma^a = 0.01$ and the uncertainty shock volatility $\sigma^{\sigma^a} = 0.005$. If we increase the volatility of the shocks much higher than this level, our numerical solution procedure fails, which is consistent with the non-existence of an equilibrium. However, if we include a small weight on the price level in the monetary policy rule, we are able to solve the model for any level of exogenous shock volatility. Maintaining this lower volatility calibration allows us to solve the model under both policy rules, and decompose the relative contributions of the precautionary working and contractionary bias channels.

Existence of a rational-expectations equilibrium is a desirable property for economists, but it need not hold in the world. Suppose that the world is exactly as described by the model with the simple Taylor rule of Equation (11). What would happen if the exogenous shock volatility increases past the level that causes equilibrium non-existence? We can only analyze this case heuristically. However, intuition suggests that after the increase in expected volatility, households would realize that the ex ante real rate is higher since the zero lower bound binds in a greater number of states. Thus, they would reduce consumption. But the reduction in consumption would lower inflation and thus the average nominal interest rate, making the zero lower bound bind in even more states. Therefore, households would further reduce consumption. This process would continue without converging, until production in the economy had been driven to a vanishingly low level. Thus, fluctuations in uncertainty can create an economic disaster at the zero lower bound, unless the monetary authority switches to a better policy rule than the simple Taylor (1993) rule.[4]

[4]While this economic mechanism is simple to explain, it is difficult to uncover its quantitative implications. To examine the effect of the contractionary bias, the model must incorporate ex ante uncertainty and be solved using a global solution method. Thus, our simple model is an ideal vehicle for exploring these potentially

How should we proceed having identified this channel by which uncertainty at the zero lower bound can have near-infinite economic consequences? We choose a very conservative path in the remainder of the paper, by focusing on monetary policy specifications that remove the contractionary bias channel. We implement this modeling choice for two reasons. First, as we show in the next section, our simple model requires considerably larger exogenous shock volatility than we have used so far if we want to match the unconditional and conditional volatility in key macroeconomic aggregates. However, a rational-expectations equilibrium fails to exist for that calibration if we use a standard Taylor (1993)-type policy rule. Therefore, we use the policy rule in Equation (17) with its response to the price level as our baseline policy rule throughout the rest of the paper. Second and more importantly, the contractionary bias channel is a consequence of examining changes in uncertainty under a particular simple monetary policy rule. For reasons we discuss next, that particular rule probably does not represent the actual conduct of Federal Reserve policy at the zero lower bound. To understand the correct quantitative effects of uncertainty shocks, we need to use a more realistic specification of monetary policy.

We think that Taylor (1993)-type policy rules that only respond to inflation and output are not good descriptions of recent Federal Reserve policy for two reasons. First, these rules have a highly counterfactual property: They imply that the central bank stops responding to the economy once it hits the zero lower bound. Even if the economy is continually hit by bad shocks at the zero lower bound, the central bank will not respond to the economy until conditions improve. This assumption is inconsistent with many actions by policymakers, which have relied on "unconventional" policy tools such as forward guidance about the future conduct of policy and quantitative easing to help stabilize the economy at the zero lower bound.[5] By including a history-dependent state variable like the price level in its policy rule, agents in the economy understand that the central bank will respond to economic outcomes by adjusting the future path of policy. Second, models with simple Taylor rules imply that inflation rates should fall significantly when the economy hits the zero lower bound, but US inflation rates have been surprisingly stable.

We view the incorporation of the price level response as a minimum deviation from standard assumptions that allow us to remove the contractionary bias and allow the central bank to con-

extreme consequences of relatively small uncertainty shocks.

[5]Bundick (2014) discusses this issue in detail and shows that the counterfactual non-response property implicit in simple Taylor-type rules can drive many of the striking results of Eggertsson (2010), Eggertsson (2012) and Christiano, Eichenbaum and Rebelo (2011), including their estimates of large fiscal multipliers and the contractionary effects of increases in full-employment output.

tinue to respond to the economy at the zero lower bound. A potential criticism of our extended monetary policy specification is that the Federal Reserve has adopted a numerical target for inflation, not the nominal price level. Thus, one could argue that our new baseline policy rule may also fail to be a good description of recent monetary policy behavior. While the Federal Reserve has not explicitly adopted a price-level target, we believe that many equilibrium features of this history-dependent rule are consistent with recent central bank behavior. As mentioned previously, the stability of recent inflation provides some evidence that the Federal Reserve has reduced the contractionary bias enough prevent disequilibrium in the actual economy. In addition, in the following section, we show that the moments implied by this simple model under this rule are consistent with both the unconditional and stochastic volatility of key macroeconomic aggregates.

Since we are removing an amplification mechanism, our results throughout the rest of the paper will represent a lower bound on the effects of changes in uncertainty at the zero lower bound. This fact is particularly important to bear in mind when comparing our quantitative estimates of the effects of uncertainty shocks to the analysis of other real shocks at the zero lower bound.

5 Empirical and Model-Implied Moments

We now return to one of the key questions laid out in the Introduction: Are uncertainty shocks important drivers for real activity and inflation? The answer to this question, however, crucially depends on our assumed calibration for the exogenous shock processes. Therefore, we want to ensure that our calibration is reasonable. Given that uncertainty shocks and the zero lower bound generate stochastic volatility in the output gap and inflation, a key litmus test for our model will be its ability to match the time-varying volatility in the data of these key macro aggregates. In this section, we discuss our calibration in detail and argue that the combination of uncertainty shocks and the endogenous volatility generated by the zero lower bound help the model explain key features of the recent data.

To evaluate the model calibration, we compare its simulated moments with their data counterparts along three dimensions. First, we assess the model's ability to match the unconditional volatility in the data as measured by the sample standard deviation. Second, we evaluate the amount of stochastic volatility in key macro aggregates in both the data and in the model. Finally, we examine the model's ability to generate zero lower bound episodes of similar frequency to the most recent macroeconomic data. We use data on the output gap, inflation, and the nominal federal funds rate from 1984-2013. We measure potential output using the Congressional Budget Office estimate and compute the output gap as the percent deviation between actual and potential output. We use the annualized quarterly percent change in the GDP deflator as our measure of inflation.

We estimate stochastic volatility using a simple model-free and non-parametric method based on rolling sample standard deviations. Given a series of simulated or actual data, we estimate a rolling 5-year standard deviation. This procedure provides a time-series of realized volatility estimates for the given data series. Then, we compute the standard deviation of this time-series of estimates. This simple measure provides an estimate of the stochastic volatility in the data series. If the actual data were homoskedastic, the estimates of the 5-year rolling standard deviations should show little volatility and the resulting statistic would be near zero.

To compare the distance between the model-implied moments and their empirical counterparts, we generate small sample bootstrapped confidence intervals from the model. Our empirical moments come from a 30-year sample of quarterly data. We want to determine the likelihood that the moments from this given 30-year sample of data could be generated by our baseline model. To compute the confidence interval for each moment, we simulate the model economy for 30 years after an initial burn in sample of 500 periods.[6] Then, we compute and save all the desired model-implied moments using this small sample of simulated data. We repeat this exercise 1000 times, which provides us with a series of small sample estimates for each moment of interest. In our results, we report the mean and the 90% confidence interval of the estimates for each moment. If the empirical moment falls outside of this model-implied confidence interval, it is highly statistically unlikely that the model is able to generate moments consistent with the data.

We calibrate the exogenous shock volatilities σ^a and σ^{σ^a} such that the model-implied moments are as close as possible to their empirical counterparts. Table 2 shows the empirical and model-implied moments as well as their small sample 90% bootstrapped confidence intervals. Under the monetary policy rule in Equation (17), we find that setting $\sigma^a = 0.02$ and $\sigma^{\sigma^a} = 0.01$ allows the model-implied moments to be consistent with the unconditional volatility in the recent data.[7] We are able to closely match the unconditional volatility of inflation. In addition, the standard deviations for the output gap and nominal interest rate in the data lie well within the confidence intervals generated by the model.

[6] Simulating and dropping this initial sample removes any influence of initial conditions.

[7] All other parameters are calibrated to the values listed in Table 1.

Our calibrated model can closely match the stochastic volatility in the data. Since 1984, the output gap, inflation, and the nominal interest rate in the data all display significant amounts of stochastic volatility. The 5-year rolling standard deviations for the output gap and the short-term policy rate typically fluctuate by over 75 basis points, while the estimates for inflation varies by roughly 50 basis points. The model-implied stochastic volatility for all three variables are close to their counterparts in the data.

Finally, the calibrated simple model spends an amount of time at the zero lower bound similar to the recent experience of the United States. Over the 1984-2013 sample period, the United States economy five years at the zero lower bound.[8] From a 30-year simulation, our economy averages just over three years at the zero lower bound. However, the confidence interval shows an incredible amount of uncertainty about the amount of time the economy is stuck at the zero lower bound. In some simulations, the monetary authority is hardly constrained by the zero lower bound for a given 30-year period. In other simulations, however, the economy can spend over seven years at the zero lower bound. Thus, the actual data falls well within this wide confidence interval generated by the model.

This wide confidence interval for the time at the zero lower bound is consistent with the recent experience of the United States. Since hitting the zero lower bound, policymakers have expressed a great deal of uncertainty when the economy will liftoff from the zero lower bound. Figure 7 plots expected liftoff date for each FOMC participant from the January 2012 Survey of Economic Projections. The Survey shows an almost uniform distribution of possible liftoff dates from less than one year to almost four years in the future. This empirical evidence is consistent with the wide confidence interval in Table 2 and the conditional distributions for the nominal rate in Figures 2 and 8. While this dispersion in policymakers' views does not perfectly align with the representative policymaker structure in the model, this cross-sectional evidence suggests that accurately predicting the amount of time at the zero lower bound may be extraordinarily difficult. Even in the full-information rational expectations setting of our model, the endogenous volatility generated by the zero lower bound makes forecasting equilibrium outcomes very difficult.

We now want to assess the relative contributions of the uncertainty shocks and the zero lower bound in generating the model's fluctuations in the key macro variables. However, since our model contains two highly nonlinear elements (uncertainty shocks and the zero lower bound),

[8]We define being at the zero lower bound as an annualized nominal policy rate less than 25 basis points.

we cannot do a simple variance decomposition as in a linear model. Therefore, we answer this question by solving two alternative versions of our baseline model. In the first alternative, we turn off the stochastic volatility in the exogenous shocks ($\sigma^{\sigma^a} = 0$) leaving all other parameters unchanged. In the second variant, we solve a version of our baseline model with stochastic shock volatility, but do not impose the zero lower bound constraint in Equation (18). These two alternative models allow us to see the relative contribution of the exogenous uncertainty shocks and the zero lower bound in generating the moments of the model. The fourth and fifth columns of Table 2 show the moments implied under these two alternative models.

Time-variation in the exogenous shock volatility is responsible for roughly 40% of the unconditional volatility of output and inflation and over 60% of their time-varying volatility. To compute the relative importance of the uncertainty shocks, we compare the change in each moment relative to the baseline model. For example, the unconditional volatility of the output gap falls by (0.93 - 1.70) $\div$ 1.7 = 45% when uncertainty shocks are absent in the model. Without the exogenous uncertainty shocks, both the unconditional and stochastic volatility in the endogenous macro variables falls dramatically. In almost all of the moments, the empirical moment falls outside of the confidence interval for the model without uncertainty shocks.

In addition, uncertainty shocks are responsible for the model's ability to generate significant periods at the zero lower bound. The model with constant shock volatility struggles to generate significant periods of time at the zero lower bound. Without uncertainty shocks, the model may not even encounter the zero lower bound in a given 30-year period. Even if the economy realizes a bad sequence of shocks such that monetary policy becomes constrained, the amount of time spent at the zero lower bound is significantly less than the actual data and far less uncertain.

The zero lower bound contributes about 20% to the unconditional volatility of output and inflation and at least 30% to their time-varying volatility. The simple model struggles to generate significant fluctuations in the output gap and inflation if monetary policy always remains unconstrained. Without the zero lower bound, the unconditional and stochastic volatility in the key macro aggregates greatly declines and the moments in the data again fall outside the model-implied confidence intervals.[9] Taken together, our results indicate that both time-

[9]Our estimate of the zero lower bound's contribution to the unconditional volatility of output is in line with the work of Ireland (2011) and Gust, López-Salido and Smith (2013). Using likelihood-based methods, these papers to show that output would have been about 20% higher if monetary policy was not constrained during the Great Recession.

varying exogenous shock volatility and the endogenous volatility generated by zero lower bound are important in matching features of the recent macroeconomic data.

6 Optimal Response to Endogenous Volatility

Our previous findings argue that the endogenous volatility generated by the zero lower bound can be an important driver of inflation and real activity. In this section, we examine how an optimal policymaker under commitment responds when the economy faces significant uncertainty about the future. In absence of the zero lower bound, optimal monetary policy can always fully stabilize the economy for any given level of exogenous shock volatility. Therefore, we focus our analysis on the macroeconomic outcomes when the economy hits the zero lower bound. Despite being constrained by the zero lower bound, we show that properly designed forward guidance can limit the spillovers to the macroeconomy.

We begin by examining optimal monetary policy and then examine the added benefits of optimal government spending financed via lump-sum taxation. Appendix C outlines the optimal policy problem and its associated solution. Figure 8 plots the expected distribution of outcomes at the zero lower bound under each alternative policy as well as the simple policy rule given by Equation (17).

6.1 Optimal Monetary Policy Under Commitment

Under optimal monetary policy, the central bank can attenuate the endogenous volatility generated by the zero lower bound. The central bank achieves this outcome via two channels: (1) lowering the expected path of real interest rates and (2) stabilizing the conditional distribution of household consumption. If a bad shock is realized, the central bank commits to lowering the expected path of future nominal interest rates. Lower nominal rates, for any given level of expected inflation, lowers the path of real interest rates through the Fisher relation. As detailed in Equation (4), a lower path of real rates raises consumption (and the output gap) even if the monetary authority remains constrained.

In addition to affecting the expected path of real rates, the central bank uses expectations about future policy to influence the conditional distribution of household consumption. If the economy is hit by contractionary shocks at the zero lower bound, the central bank will continue to respond to the economy using forward guidance about the future path of policy. Households fully internalize this commitment to help stabilize the economy if bad shocks are realized.

Thus, this state contingent policy response stabilizes the household's expected distribution of consumption. The middle row of Figure 8 shows that the lower real rates and the more stabile distribution of future consumption significantly limits the spillovers from the uncertainty shock to the macroeconomy. As opposed to falling by roughly half of a percent under the simple policy rule, the mean response of the output gap falls by less than 10 basis points at impact when policy is conducted optimally.

However, the central bank cannot fully eliminate the endogenous volatility generated by the zero lower bound. The distribution of possible outcomes for both the output gap and inflation still show significant fluctuations after the uncertainty shock. In an effort to stabilize the economy, the central bank maintains a zero nominal policy for an extended period of time. Even under optimal policy, the economy may experience fluctuations and remain stuck at the zero lower bound for four years after the initial shock. However, the central bank is able to limit some of the spillovers to the macroeconomy. The distribution of the output gap remains much more symmetric and less diffuse under optimal policy when compared with the simple policy rule. Under the simple policy rule, the possible outcomes for inflation are volatile, negatively-skewed, and its distribution is highly correlated with the exogenous shock volatility. Under optimal policy, however, the outcomes for inflation are very concentrated around zero for the first year after the shock but then expand modestly with a slightly higher mean and positive skewness for the next few years.

These distribution of outcomes for inflation provides the key insight into the trade-offs faced by the central bank. At impact, the central bank understands that the economy faces the possibility of very bad outcomes due to the uncertainty shock and its endogenous amplification through the zero lower bound. However, the central bank also understands that the higher uncertainty is not permanent as shocks should eventually return to their steady state values. Therefore, the policymaker faces a trade-off between the medium-run distribution of inflation versus the short-run distributions of output and inflation. The central bank optimally accepts a higher and more volatile distribution of future inflation in the future to help offset the downside risks generated by the zero lower bound. The central bank commits to keeping interest rates at zero throughout the life of the uncertainty shock even if the economy experiences significantly expansionary shocks. By keeping rates low, even in the face of expansionary shocks, the central bank tolerates the higher inflation in the medium-run in order to establish a floor under inflation expectations.

6.2 Optimal Fiscal and Monetary Policy

Even under optimal monetary policy, higher uncertainty at the zero lower bound can cause significant fluctuations for the economy. Fiscal policy may play a significant role in stabilizing the economy in this scenario. We assume that the policymaker has access to government spending financed via lump-sum taxation. Figure 7 shows the distribution of possible outcomes under jointly optimal fiscal and monetary policy under commitment.

Access to state-contingent government spending helps stabilize the real economy during period of heightened uncertainty and allows for more effective stabilization of the future inflation. At the onset of the uncertainty shock, optimal government spending increases modestly on average. However, the distribution of possible outcomes for government spending is volatile and slightly positively-skewed after the uncertainty shock. Thus, when the nominal interest rate is constrained, the optimal policymaker uses government spending in a state-contingent manner to help offset shocks that hit the economy. As a result, the distribution of outcomes for the output gap become more stabilized when the policymaker has access to state-contingent government spending. In addition, after the uncertainty shock subsides, the optimal policy slightly lowers government spending on average but continues to use it to offset shocks. Access to this additional policy instrument allows the policymaker to assume less inflation risk and provides more effective stabilization of the medium-run outcomes for the macroeconomy.

7 Discussion and Connections with Literature

Rigorous modeling of the zero lower bound using a global solution method is difficult, especially when the model contains many state variables. This difficultly has lead some researchers, such as Fernández-Villaverde et al. (2013) and Erceg and Lindé (2014), to rely on shooting-type algorithms rather than the global solution methods we employ in this paper. These methods provide quantitative predictions regarding the effect of the zero lower bound but don't suffer from the curse of dimensionality that plagues global methods. At their core, however, both of these works also use a similar structure for nominal rigidities and assume monetary policy follows a standard Taylor (1993)-type rule. Thus, our discussion regarding the contractionary bias and equilibrium existence fully applies to those papers. However, these shooting-type solution methods are unable to investigate these effects and thus cannot derive a full set of policy implications. We view our work, however, as complementary in the following sense: Using our results, researchers wishing to solve models with many state variables can use these shooting-type algorithms but assume monetary policy rules that remove the contractionary bias. This assumption maintains tractability which removing the potential for equilibrium non-existence in the "true" model. As we discuss earlier, we argue the equilibrium properties of these rules are more consistent with recent central bank behavior.

Our discussion of the contractionary bias helps explain the solution difficulty and mechanisms at work in some recent papers in the literature. Recent work by Fernández-Villaverde et al. (2012), Richter and Throckmorton (2014), Nakata (2012), and Johannsen (2013) all solve a similar nonlinear New-Keynesian model with global methods but assume that monetary policy follows a standard Taylor (1993) rule. Fernández-Villaverde et al. (2012) and Richter and Throckmorton (2014) both report difficultly solving their model if they increase the volatility of the exogenous shocks. Our results show that this solution difficulty is likely caused by the contractionary bias being too large to be consistent with a rational-expectations equilibrium. Nakata (2012) and Johannsen (2013) show that higher demand and fiscal uncertainty at the zero lower bound greatly depresses the economy.[10] However, neither of these papers make any adjustments for the contractionary bias. Therefore, their results contain the effects of both the contractionary bias and precautionary working mechanisms.[11]

This study is closest to recent work by Plante, Richter and Throckmorton (2014). Using a similar model but with constant shock volatility, they show that the zero lower bound endogenously generates increased forecast errors about future economic outcomes. Our work differs from theirs in three key respects. First, we characterize the two key mechanisms through which uncertainty about future shocks interacts with the zero lower bound. In addition, Table 2 shows that the simple model struggles to match key features of the data without time-varying exogenous shock volatility. Finally, we examine the optimal response of monetary and fiscal policy to the endogenous volatility generated by the zero lower bound. Nakata (2013) also studies optimal government spending and monetary policy at the zero lower bound. He compares a deterministic economy ($\sigma^a = 0$) to an economy with uncertainty ($\sigma^a > 0$) and shows that optimal government spending under discretion increases when the economy faces uncertainty about the future.

We view our work as highly complementary to other recent work on the Great Recession and

[10]Johannsen (2013) uses a standard New-Keynesian model, but also incorporates physical capital. In Appendix D.2, we show that our results are very similar if we include capital in our model.

[11]This paper is also related to work by Adam and Billi (2006) and Nakov (2008), which use a linearized New-Keynesian model, but solve the model using a global solution method. Thus, the models in these papers are able to capture changes in the conditional mean caused by the presence of uncertainty at the zero lower bound. However, households in their models do not take into account the higher-order moments of consumption, which we show are quantitatively important.

business-cycle models. For example, Christiano, Eichenbaum and Trabandt (2014) argues that the interaction between financial frictions and the zero lower bound is crucial for understanding the economics during the recent recession and recovery. However, they reach this conclusion using a model solution method which relies on certainty equivalence. By contrast, our work allows for uncertainty about future shocks to interact with the zero lower bound to produce significant contractions in economic activity and a highly uncertainty liftoff date from the zero lower bound.

7.1 Optimal Tax Policy & Fiscal Policy Implementation

Optimal interest rate and government spending policy can be effective in limiting the endogenous volatility generated by the zero lower bound. Recent work by Correia et al. (2013) suggests that optimal tax policy can highly effective and more efficient at overcoming the zero lower bound constraint. If we incorporate time-varying consumption and labor taxes in our framework, Equations (6) and (7) would be modified as follows:

$$\frac{1-\eta}{\eta}\frac{1+\tau_t^c}{1-\tau_t^n}\frac{C_t}{1-N_t}=\frac{W_t}{P_t} \tag{19}$$

$$1=\mathrm{E}_t\left\{\beta\frac{a_{t+1}}{a_t}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{C_{t+1}^{\eta}\left(1-N_{t+1}\right)^{1-\eta}}{C_{t+}^{\eta}\left(1-N_t\right)^{1-\eta}}\right)^{1-\sigma}\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{R_t}{\Pi_{t+1}}\right)\right\} \tag{20}$$

Equation (20) shows that a policymaker can offset a decline in aggregate demand (fall in a_t) by either lowering its nominal policy rate R_t or by lowering current consumption taxes τ_t^c. If the policymaker alters its consumption tax policy, they must also raise labor taxes τ_t^n to prevent distortions in the household's intratemporal first-order condition. Correia et al. (2013) shows that optimal tax policy can fully circumvent the zero lower bound constraint if the policy has enough flexibility in its tax instruments and the policymaker can effectively respond to business-cycle fluctuations. Due to the political economy issues surrounding consumptions taxes, however, Christiano, Eichenbaum and Rebelo (2011) express doubts about the ability of fiscal policy to implement the time-varying taxes needed to implement these proposals. Indeed, the uncertainty and delay surrounding Japan's recent consumption tax increase highlights these implementation issues.

Our results suggest that higher uncertainty further exacerbates the implementation of time-varying optimal tax policy. Equation (20) shows that more volatile demand shocks will require a concurrent increase in the volatility of the tax rates. More volatile taxes increase the likelihood that the negative taxes will be required to implement the optimal policy. In addition, our results under optimal government spending show that implementation in an environment with uncertainty shocks may require highly state-contingent policy. In the online Appendix, Table D.1 compares the unconditional and stochastic volatility of government spending under optimal fiscal and monetary policy. The small-sample confidence intervals provide some insight into the types of outcomes required to implement optimal policy. When the economy doesn't hit the zero lower bound too often, the left boundary of the confidence interval shows that government spending is nearly constant and shows no stochastic volatility. However, when the economy spends a large fraction at the time at the zero lower bound, the upper tail of the interval shows that government spending is highly volatile on average and shows significant stochastic volatility. Thus, the presence of uncertainty shocks requires the optimal policymaker to be highly state-contingent in responding to the economy.

7.2 Comparison with News Shocks

Recent work by Jaimovich and Rebelo (2009) and others argues that news about the future can be a key driver of business cycles. News shocks, like our uncertainty shock, change expectations about future shocks for the economy without a change in current fundamentals. In this section, we show that an uncertainty shock at the zero lower bound acts like a negative news shock about the future mean of fundamentals coupled with an endogenous increase in the variance and decline in the skewness of expected future outcomes. [12]

To formally compare uncertainty and news shocks, we solve a version of our model using the following stochastic process for demand shocks:

$$a_{t+1} = (1 - \rho_a)\, a + \rho_a a_t + \sigma^c \varepsilon^a_{t+1} + \sigma^n \varepsilon^a_t \tag{21}$$

This alternative process for aggregate demand features contemporaneous and 1-period ahead news shocks but constant volatility. To compare with our uncertain shock results, we simulate the same decline in aggregate demand in both the uncertainty and news shock economies. Then, we choose the news shock such that both economies have the same expected mean output gap. Figure 9 shows the expected distribution of outcomes for both the news and uncertainty shock economies.

The endogenous volatility created by the zero lower bound implies significantly higher fluctuations for the uncertainty shock economy. Returning to the intuition from Equation (2), a more

[12]Recent work by Ilut and Schneider (2014) argues that exogenous shocks to the "worst-case" shock distribution are a key driver of business cycles. In loose terms, the zero lower bound endogenously generates time-variation in the worst-case outcome by shifting the entire distribution of future consumption.

volatile and negatively-skewed expected output gap produces larger output losses today even if the conditional means are equivalent. In addition, the 1-year ahead output gap shows that the zero lower bound greatly propagates the negative effects of the uncertainty shock. Thus, an uncertainty shock at the zero lower bound produces larger initial losses and its negative effects are significantly more propagated than a similarly-calibrated news shocks.

8 A Solution to the Forward Guidance Puzzle

Our results suggest that the endogenous volatility generated by the zero lower bound has important implications for household decisions. Thus, our results may provide an explanation for "Forward Guidance Puzzle" discussed in Del Negro, Giannoni and Patterson (2013). Using a similar model with nominal rigidities, they argue that the economy is too responsive to exogenous changes in interest rates. However, they reach this conclusion using a first-order linearized model. Under this assumption, households do not take into account the uncertainty about future consumption. We argue that this omitted variable may be crucially important when the economy is stuck at the zero lower bound. Households care about both the current real rate and the distribution of expected future consumption. Thus, current consumption responds less to an exogenous decline in interest rates if expected future consumption also becomes more volatile and negatively-skewed.

To illustrate this idea, we make two changes to our baseline model. First, we replace our rule in Equation (17) with the following specification for monetary policy:

$$r_t^d = \Big(1 - \phi_r\Big) r + \phi_r r_{t-1}^d + \phi_\pi \Big(\pi_t - \pi\Big) + \phi_x x_t + \sigma^r \varepsilon_t^r \tag{22}$$

$$r_t = \max\Big(0, r_t^d\Big) \tag{23}$$

Away from the zero lower bound, this policy rule acts like a Taylor (1993)-type policy rule with interest-rate smoothing and ε_t^r is an exogenous monetary policy shock. When the economy encounters the zero lower bound, however, this history-dependent rule lowers the future path of policy to help offset the previous higher-than-desired nominal rates caused by the nominal constraint. Households fully internalize this future conduct of policy. When desired rates are less than zero, an exogenous shock to the desired rate acts like an exogenous extension of the zero lower bound episode. In addition to changing our monetary policy rule, we replace our uncertainty shock process with two time-invariant calibrations. We denote the first calibration as the perfect foresight calibration with $\sigma^a = \sigma^r = 0$ and the second calibration as our under uncertainty calibration $\sigma^a = 0.03, \sigma^r = 0.0025$. These alterations help make our framework similar to the experiments in the previous literature.

We begin by simulating a large decline in aggregate demand such that both the perfect foresight and uncertainty economies hit the zero lower bound. After the initial shock, we assume that both economies experience no further shocks. We pick the initial demand shock for each economy such that they both experience the same length zero bound episode in equilibrium. We refer to this simulation as the baseline scenario for each economy. We now examine an exogenous extension of the zero lower bound episode. Using the same initial shocks, we simulate a new time path for each economy but also simulate a large negative shock to the desired rate in period nine such that both economies remain at the zero lower bound for an additional few quarters. As with the initial demand shock, we pick the monetary policy shock such that the equilibrium path of the nominal interest rate is the same in both economies.

The endogenous volatility generated by the zero lower bound greatly diminishes the economy's response. At the announcement of interest rate change, Figure 9 shows that the perfect foresight economy surges with a four percent increase in the output gap and a two percent increase in inflation. For the same path of nominal rates, however, the presence of uncertainty attenuates the response of the economy by roughly half. Equation (4) helps provide the intuition for these results. The exogenous extension lowers the path of real interest rates. Under perfect foresight, the household's consumption decision only depends on the path of real interest rates. Under uncertainty, however, the extension also increases the endogenous volatility through the asymmetric ability to offset shocks. This precautionary saving curtails the household's response to the change in interest rates and affects the transmission of policy shock to the macroeconomy.[13]

9 Conclusions

This paper highlights the interactions between the zero lower bound and expectations about future shocks. Our results suggest that even small probabilities of bad outcomes can be destabilizing. To stabilize the economy, the central bank must effectively communicate to households that it will respond using its available policy tools if bad shocks are realized. We also show that the form of the monetary policy rule is crucially important even when the economy is at the

[13] McKay, Nakamura and Steinsson (2014) shows a similar result when households face idiosyncratic, rather than aggregate, uncertainty about future consumption. Vavra (2012) and Aastveit, Natvik and Sola (2013) also find that the responses to exogenous policy shocks may change depending on the state of the economy. However, both of these papers are silent on the effects of the zero lower bound, which currently remains a real constraint on many central banks.

zero lower bound. Rules with good descriptive and normative properties in the past may cause catastrophic outcomes and not be representative of actual policy at the zero lower bound.

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Table 1: Calibration of Baseline Model Parameters

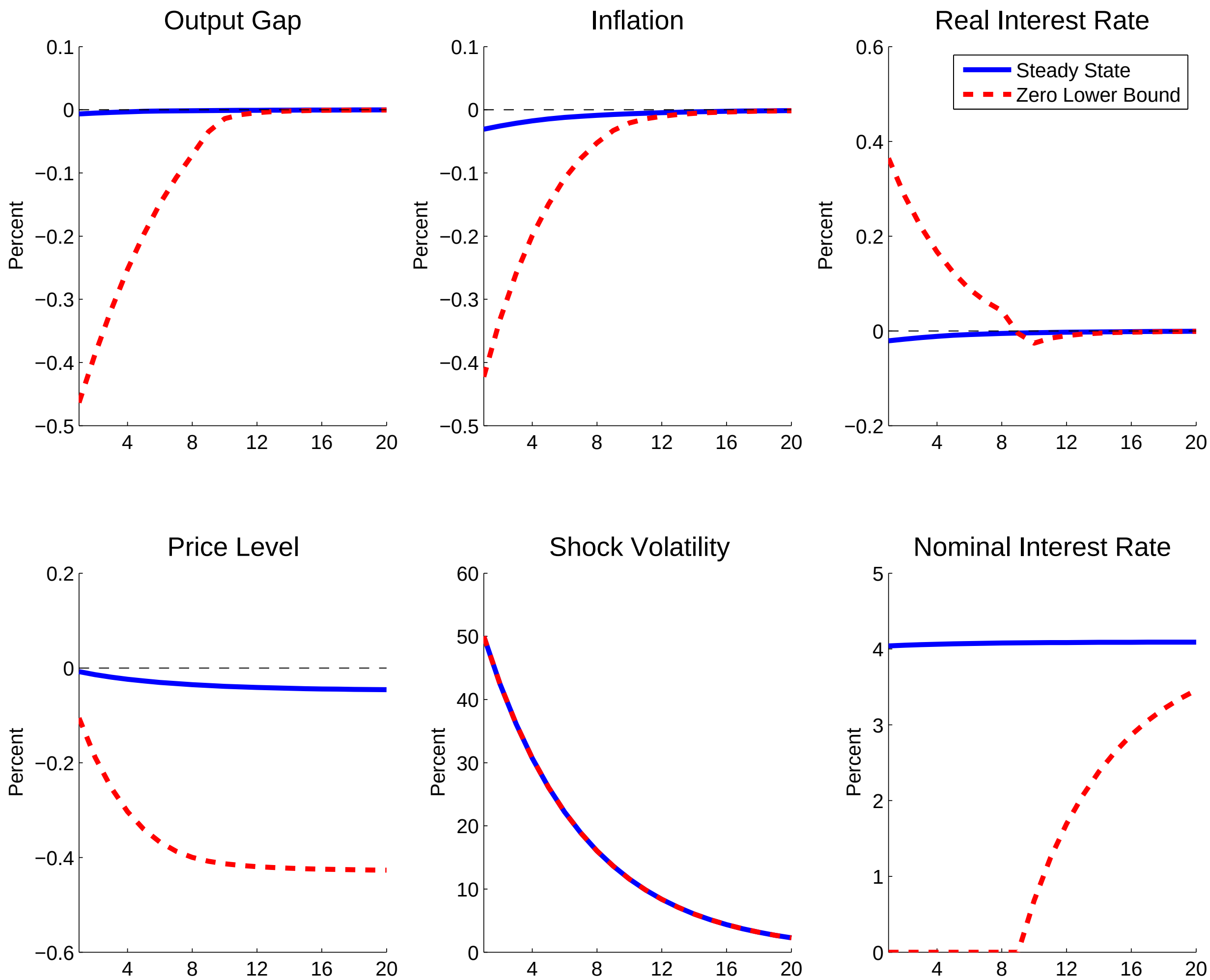
Parameter	Description	Calibrated Value
β	Household Discount Factor	0.99
ϕ_P	Adjustment Cost to Changing Prices	160.0
Π	Steady State Inflation Rate	1.000
σ	Parameter Affecting Household Risk Aversion	2.0
η	Consumption Share in Period Utility Function	0.24
θ	Elasticity of Substitution Intermediate Goods	6.0
ϕ_π	Central Bank Response to Inflation	1.5
ϕ_x	Central Bank Response to Output Gap	0.25
ρ_a	Preference Shock Persistence	0.85
ρ_{σ^a}	Uncertainty Shock Persistence	0.85
σ^a	Steady State Shock Volatility	0.01
σ^{σ^a}	Uncertainty Shock Volatility	0.005

Table 2: Empirical and Model-Implied Moments

Moment	Data 1984 - 2013	Baseline Model	No Stochastic Shock Volatility	No Zero Lower Bound
Unconditional Volatility				
x	2.52	1.70	0.93	1.24
		(0.89, 3.00)	(0.72, 1.21)	(0.80, 1.79)
π	0.98	1.03	0.62	0.83
		(0.62, 1.58)	(0.49, 0.77)	(0.55, 1.17)
r	2.91	2.42	1.89	
		(1.70, 3.28)	(1.53, 2.29)	
Stochastic Volatility				
x	0.77	0.73	0.23	0.41
		(0.28, 1.57)	(0.12, 0.38)	(0.22, 0.70)
π	0.49	0.40	0.14	0.28
		(0.19, 0.77)	(0.09, 0.22)	(0.15, 0.48)
r	0.74	0.72	0.40	
		(0.41, 1.16)	(0.25, 0.58)	
Quarters at Zero Lower Bound	20	13	5	
		(2, 29)	(0, 13)	

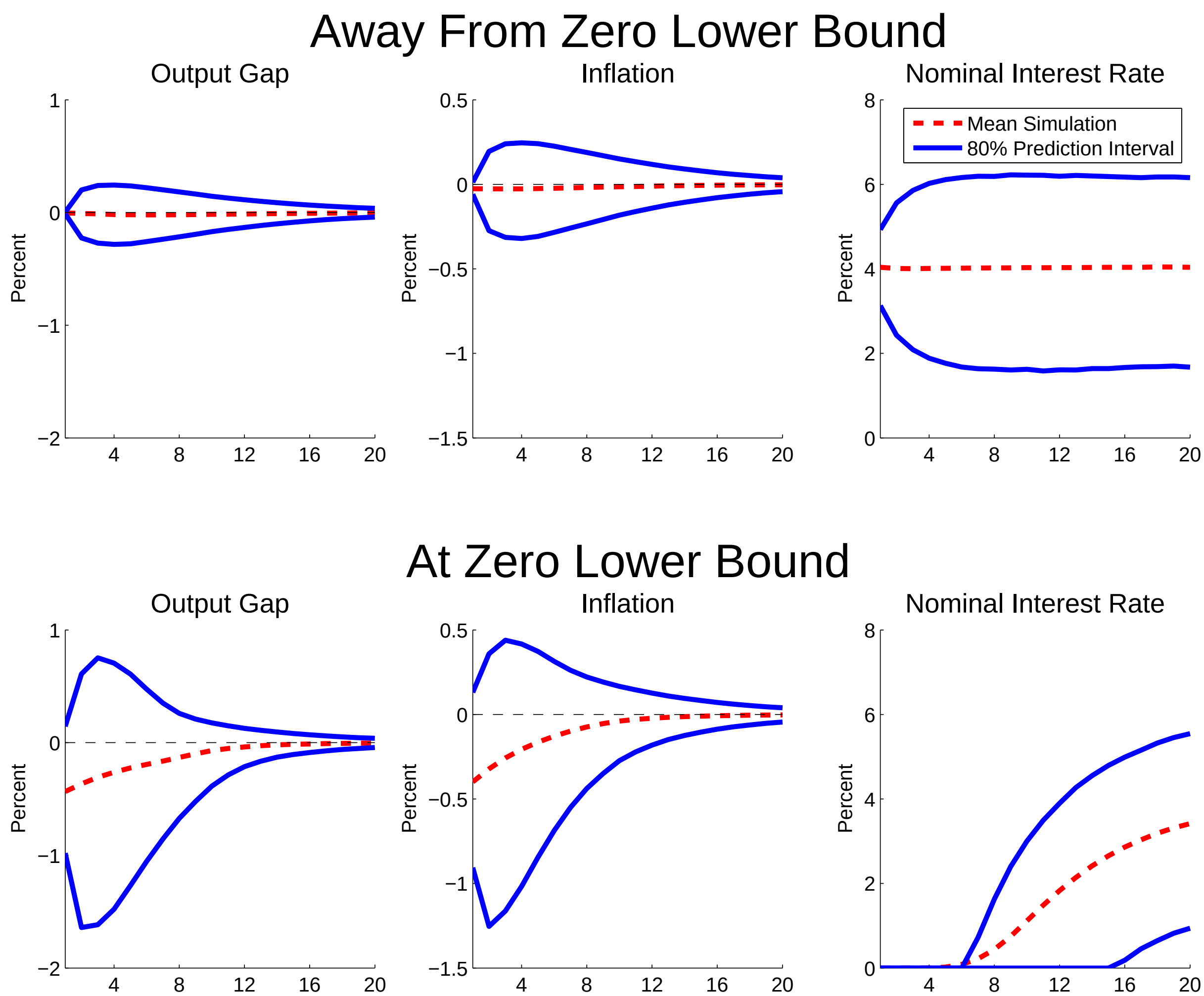
Note: Unconditional volatility is measured with the sample standard deviation. Stochastic volatility is measured by the standard deviation of the time-series estimate for the 5-year rolling standard deviation. The 90% small sample bootstrapped confidence intervals are given in parenthesis.

Figure 1: Impulse Responses to Demand Uncertainty Shock



Note: The output gap, price level, and shock volatility responses are plotted as percent deviations. The real interest rate and inflation are plotted in annualized percent deviations. The nominal interest rate is plotted in annualized percent.

Figure 2: Expected Distributions of Possible Outcomes After Uncertainty Shock



Note: The output gap response is plotted in percent deviations. The inflation response is plotted in annualized percent deviations and the nominal interest rate is plotted in annualized percent.

Figure 3: Transmission of Precautionary Labor Supply to Macroeconomy

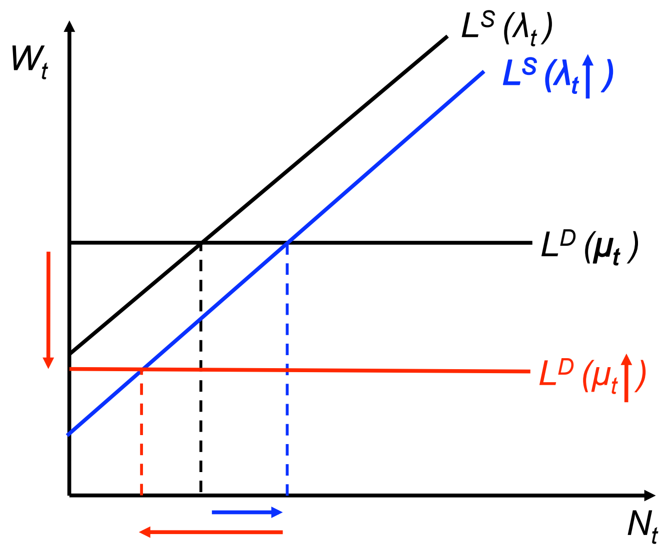


Figure 4: Nominal Interest-Rate Distributions

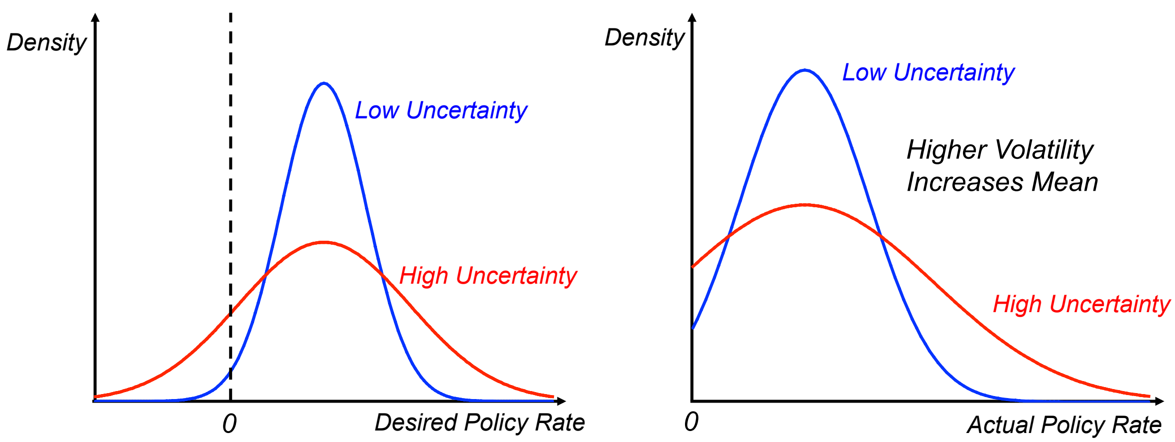


Figure 5: General-Equilibrium Effects of the Contractionary Bias

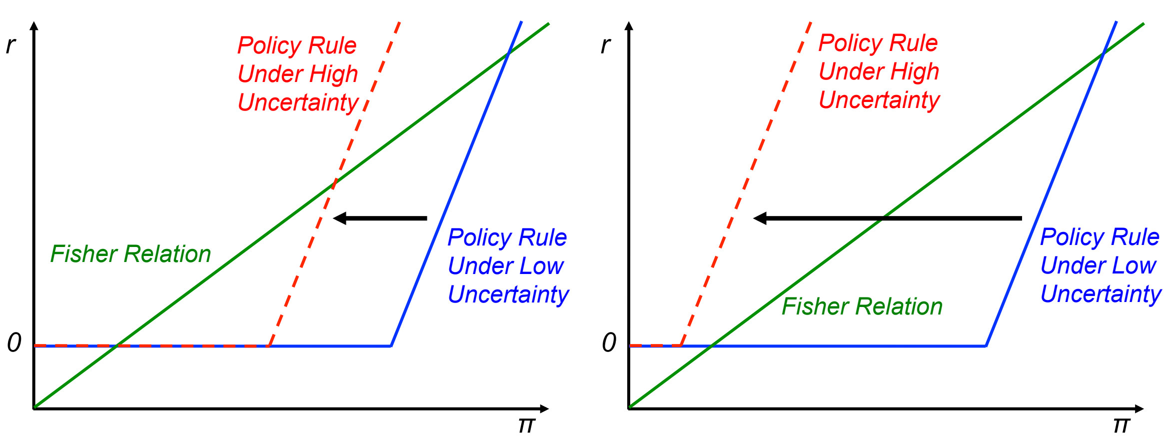


Figure 6: Quantifying the Precautionary Saving and Contractionary Bias Channels

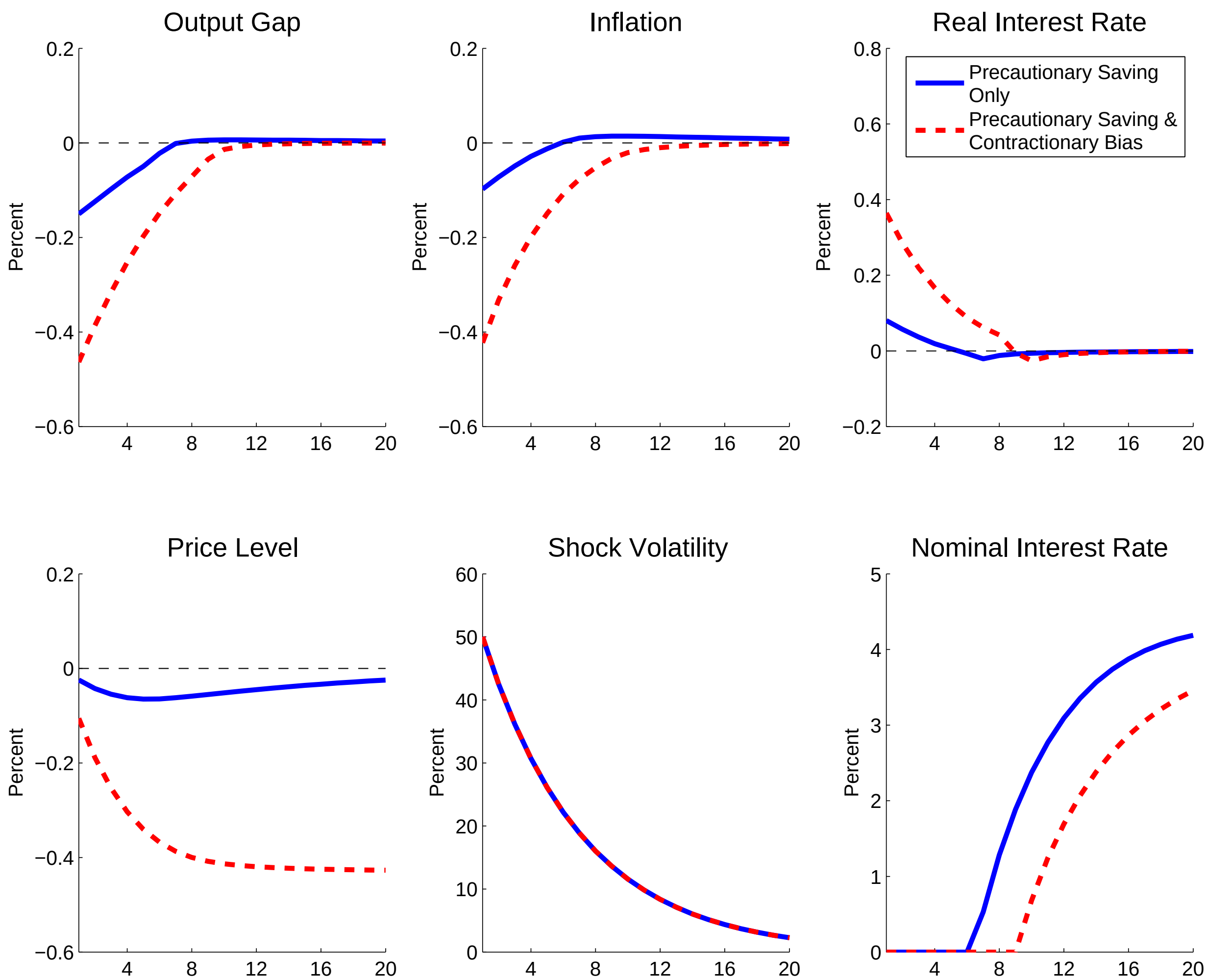


Figure 7: Policymakers' Expectations of Zero Lower Bound Liftoff in January 2012

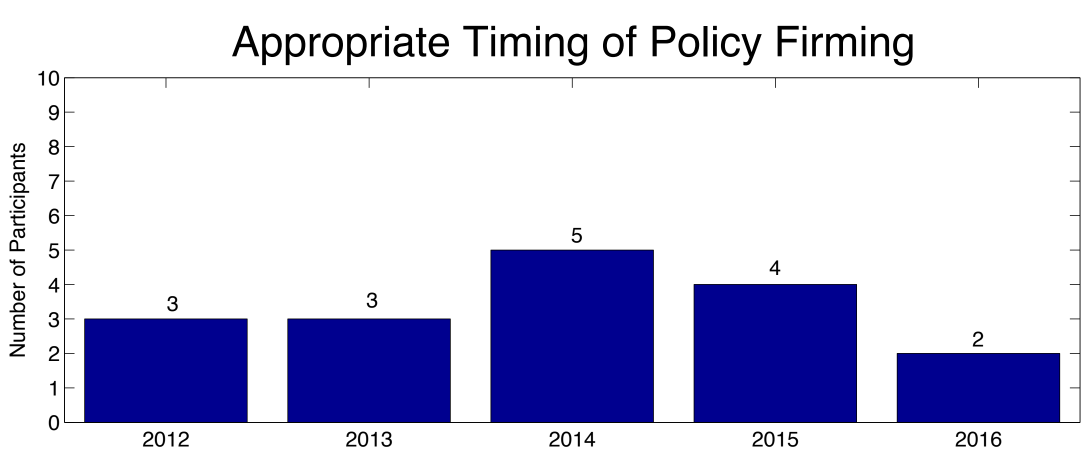
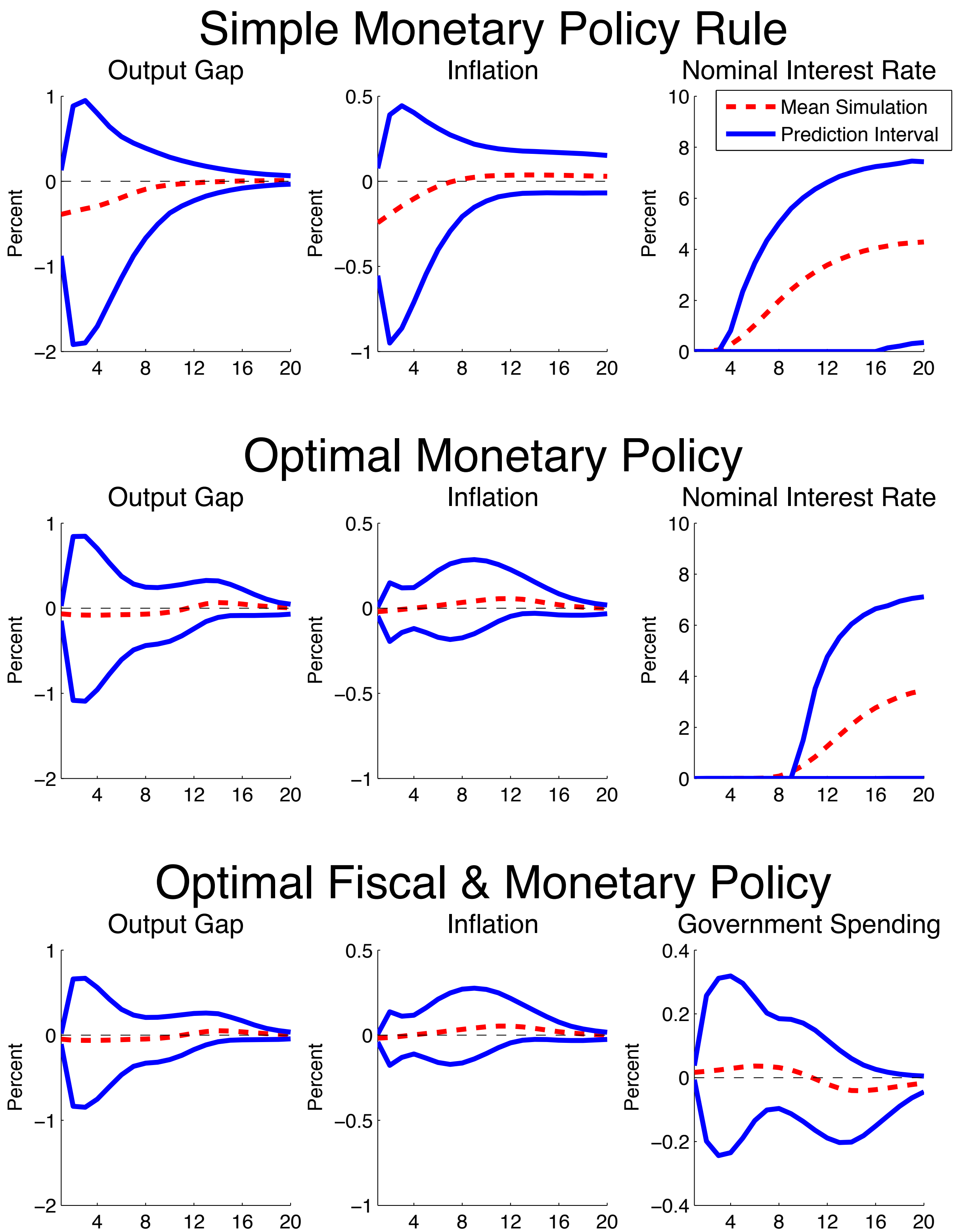


Figure 8: Expected Distributions of Possible Outcomes Under Alternative Policies



Note: The output gap and government spending responses are plotted in percent deviations. The inflation response is plotted in annualized percent deviations and the nominal interest rate is plotted in annualized percent.

Figure 9: Expected Output Gap Distribution at Zero Lower Bound

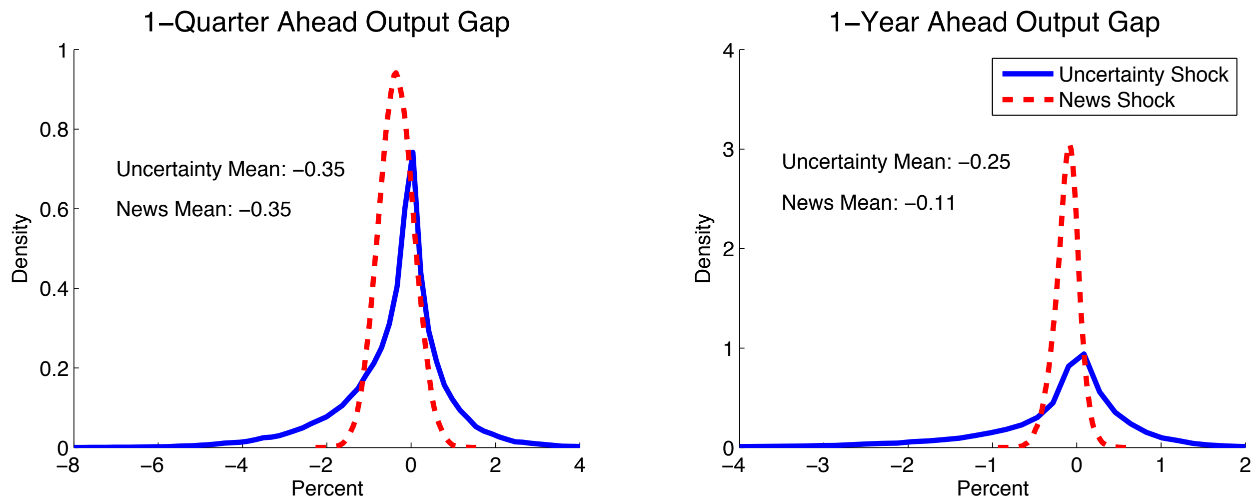
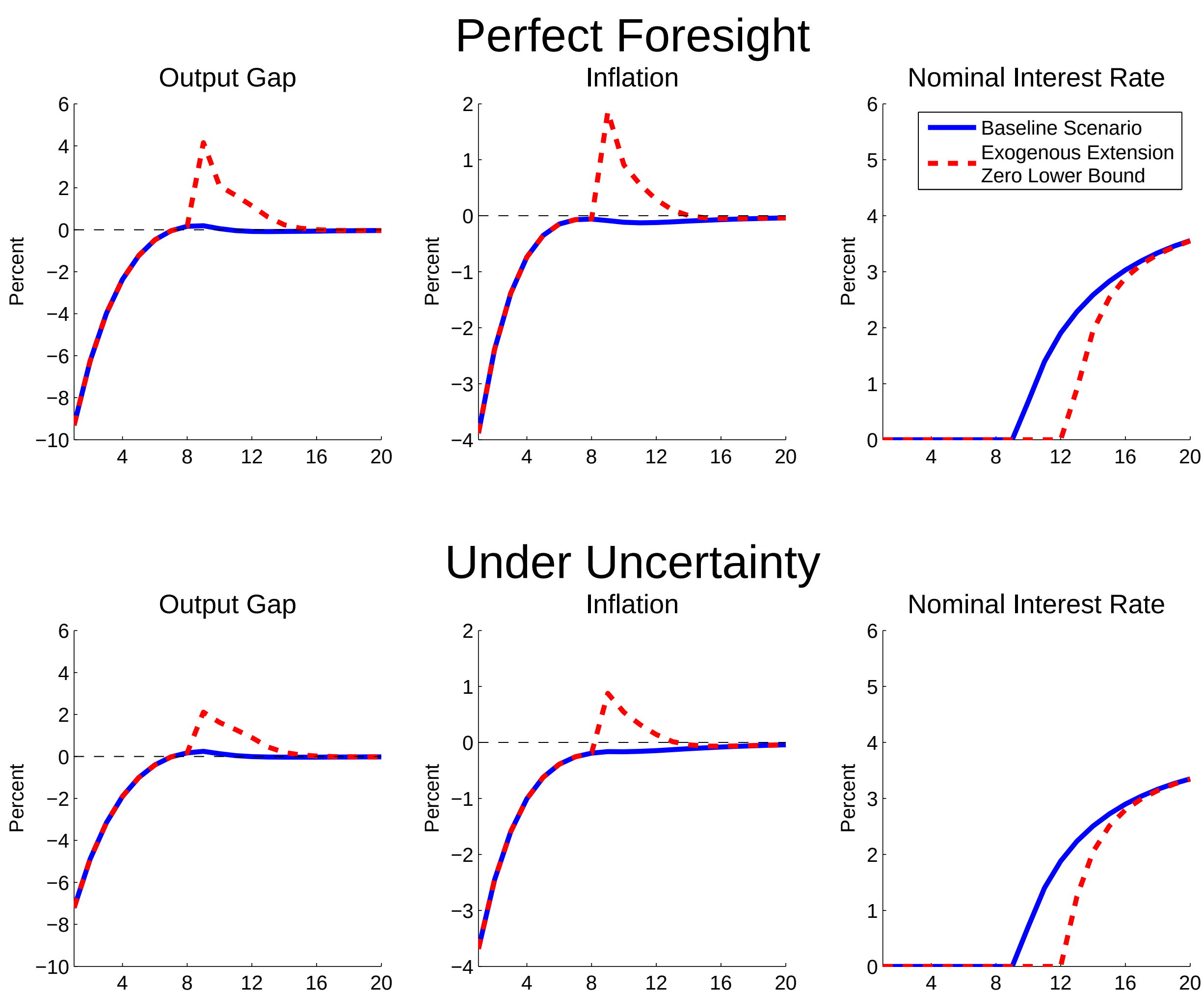


Figure 10: Uncertainty and the Forward Guidance Puzzle



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Technical Appendix

A Derivation of Equations From Intuition Section

This section provides a detailed derivation of the equations from Section 2 of the main text. Using the consumption Euler equation in Equation (1), complete the following steps to derive Equation (2):

1. Multiply and divide the right side of the Euler equation by the steady state values of the real interest rate R^R and consumption C raised to the power $-\sigma$. Apply the natural logarithm and exponential functions inside the conditional expectations. Denote $\hat{X}_t = \log(X_t/X)$ to write the variables in log-deviations from steady state.

$$1 = \mathrm{E}_t \left\{ \beta R_t^R \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right\} = \mathrm{E}_t \left\{ \left(\frac{R_t^R}{R^R} \right) \left(\frac{C_t}{C} \right)^{\sigma} \left(\frac{C_{t+1}}{C} \right)^{-\sigma} \right\}$$

$$1 = \mathrm{E}_t \left\{ \exp\left(\log\left(\frac{R_t^R}{R^R} \right) - \sigma \log\left(\frac{C_{t+1}}{C} \right) + \sigma \log\left(\frac{C_t}{C} \right) \right) \right\}$$

$$1 = \mathrm{E}_t \left\{ \exp\left(\hat{R}_t^R + \sigma \hat{C}_t - \sigma \hat{C}_{t+1} \right) \right\}$$

2. Reorganize, divide by the time t variables, and take the logarithm of both sides.

$$1 = \mathrm{E}_t \left\{ \exp\left(\hat{R}_t^R + \sigma \hat{C}_t \right) \exp\left(-\sigma \hat{C}_{t+1} \right) \right\}$$

$$\left(\exp\left(\hat{R}_t^R + \sigma \hat{C}_t \right) \right)^{-1} = \mathrm{E}_t \left\{ \exp\left(-\sigma \hat{C}_{t+1} \right) \right\}$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log\left(\mathrm{E}_t \left\{ \exp\left(-\sigma \hat{C}_{t+1} \right) \right\} \right)$$

3. Replace $\exp\left(-\sigma \hat{C}_{t+1} \right)$ with its Taylor series expansion around $\hat{C}_{t+1} = 0$ and take conditional expectations at time t.

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log\left(\mathrm{E}_t \left\{ 1 - \sigma \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \hat{C}_{t+1}^2 - \sigma^3 \hat{C}_{t+1}^3 + \dots \right\} \right)$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = \log\left(1 - \sigma \mathrm{E}_t \hat{C}_{t+1} + \frac{1}{2} \sigma^2 \mathrm{E}_t \hat{C}_{t+1}^2 - \sigma^3 \mathrm{E}_t \hat{C}_{t+1}^3 + \dots \right)$$

4. Define $Z = \sigma \mathrm{E}_t \hat{C}_{t+1} - \frac{1}{2}\sigma^2 \mathrm{E}_t \hat{C}_{t+1}^2 + \sigma^3 \mathrm{E}_t \hat{C}_{t+1}^3 - O\left(C_{t+1}^4\right)$ and use the Taylor series expansion of $\log(1-Z) = -Z - (1/2)Z^2 - (1/3)Z^3 - O\left(Z^4\right)$ to expand the previous equation. To compute a third-order approximation, drop all terms that are fourth-order or above. Reorganize the remaining terms to form the conditional variance and conditional skewness:

$$\mathrm{Var}_t \hat{C}_{t+1} = \mathrm{E}_t \hat{C}_{t+1}^2 - \left(\mathrm{E}_t \hat{C}_{t+1}\right)^2$$

$$\mathrm{Skew}_t \hat{C}_{t+1} = \mathrm{E}_t \hat{C}_{t+1}^3 - 3\mathrm{E}_t \hat{C}_{t+1}^2 \mathrm{E}_t \hat{C}_{t+1} + \left(\mathrm{E}_t \hat{C}_{t+1}\right)^3.$$

$$-\hat{R}_t^R - \sigma \hat{C}_t = -\sigma \mathrm{E}_t \hat{C}_{t+1} + \frac{1}{2}\sigma^2 \mathrm{Var}_t \hat{C}_{t+1} - \frac{1}{6}\sigma^3 \mathrm{Skew}_t \hat{C}_{t+1}$$

$$\hat{C}_t = \mathrm{E}_t \hat{C}_{t+1} - \frac{1}{\sigma}\hat{R}_t^R - \frac{1}{2}\sigma \mathrm{Var}_t \hat{C}_{t+1} + \frac{1}{6}\sigma^2 \mathrm{Skew}_t \hat{C}_{t+1}$$

5. Denote variables in logs using lowercase letters and normalize steady state consumption C to equal one to derive Equation (2):

$$c_t = \mathrm{E}_t\, c_{t+1} - \frac{1}{\sigma}\left(r_t^r - r^r\right) - \frac{1}{2}\,\sigma\,\mathrm{Var}_t\, c_{t+1} + \frac{1}{6}\,\sigma^2\,\mathrm{Skew}_t\, c_{t+1}$$

6. Define a flexible-price version of the previous equation. Subtract the flexible-price version from the actual approximated Euler equation in Step 5. Note in a model with only discount factor shocks, flexible-price consumption is constant since prices always fully adjust. Define the data-consistent output gap x_t as the deviation of consumption from steady state:

$$x_t = \mathrm{E}_t\, x_{t+1} - \frac{1}{\sigma}\left(r_t^r - r_t^n\right) - \frac{1}{2}\,\sigma\,\mathrm{Var}_t\, x_{t+1} - \frac{1}{6}\,\sigma^2\,\mathrm{Skew}_t\, x_{t+1}$$

B Numerical Solution Method

To analyze the impact of uncertainty at the zero lower bound, we solve the model using the policy function iteration method of Coleman (1990). This global approximation method allows us to model the occasionally-binding zero lower bound constraint. This section provides the details of the algorithm when monetary policy follows the simple policy rule in Equation (17) and (18) in the main text. The algorithm is implemented using the following steps:

1. Discretize the state variables of the model: $\{a_t \times \sigma_t^a \times P_{t-1}\}$

2. Conjecture initial guesses for the policy functions of the model $N_t = N(a_t, \sigma_t^a, P_{t-1})$, $\Pi_t = \Pi(a_t, \sigma_t^a, P_{t-1})$, $R_t = R(a_t, \sigma_t^a, P_{t-1})$, and $R_t^R = R^R(a_t, \sigma_t^a, P_{t-1})$.

3. For each point in the discretized state space, substitute the current policy functions into the equilibrium conditions of the model. Use interpolation and numerical integration over the exogenous state variables $\left\{a_{t+1} \times \sigma_{t+1}^a\right\}$ to compute expectations for each Euler equation. This operation generates a nonlinear system of equations. The solution to this system of equations provides an updated value for the policy functions at that point in the state space. The solution method enforces the zero lower bound for each point in the state space and in expectation.

4. Repeat Step (3) for each point in the state space until the policy functions converge and cease to be updated.

We implement the policy function iteration method in `FORTRAN` using the nonlinear equation solver `DNEQNF` from the `IMSL` numerical library. When monetary policy follows an alternative specification, the state variables and Euler equations are adjusted appropriately.

C Optimal Policy

C.1 Optimal Monetary Policy Under Commitment

The optimal monetary policy maker under commitment aims to maximize the representative household's utility subject to the constraints of the economy. Some of the constraints include expectations of future variables. Following Khan, King and Wolman (2003), we introduce lagged Lagrange multipliers to make the solutions time-invariant. The augmented Lagrangian for the optimal policy problem under commitment can be written as follows:

$$
\begin{aligned}
L \;=\; & \min_{\{\omega_{t+s}\}_{s=0}^{\infty}} \max_{\{d_{t+s}\}_{s=0}^{\infty}} \mathrm{E}_t \Bigg\{ \sum_{s=0}^{\infty} \beta^s \left(\frac{a_{t+s}}{a_t}\right) \frac{\left(C_{t+s}^{\eta}\left(1-N_{t+s}\right)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} \\
& +\omega_{1t+s}\left(Y_{t+s} - C_{t+s} - \frac{\phi_P}{2}\left(\frac{\Pi_{t+s}}{\Pi}-1\right)^2 C_{t+s}\right) \\
& +\omega_{2t+s}\left(N_{t+s} - \Phi - Y_{t+s}\right) \\
& +\omega_{3t+s}\left(W_{t+s}^R - \frac{1-\eta}{\eta} C_{t+s}\left(1-N_{t+s}\right)^{-1}\right) \\
& +\omega_{4t+s}\left((\theta-1) - \theta W_t^R + \phi_P\left(\frac{\Pi_{t+s}}{\Pi}-1\right)\left(\frac{\Pi_{t+s}}{\Pi}\right)\right)\left(a_{t+s} C^{\eta(1-\sigma)-1}\left(1-N_{t+s}\right)^{(1-\eta)(1-\sigma)} Y_{t+s}\right) \\
& -\omega_{4t+s-1}\left(\phi_P\left(\frac{\Pi_{t+s}}{\Pi}-1\right)\left(\frac{\Pi_{t+s}}{\Pi}\right)\right)\left(a_{t+s} C^{\eta(1-\sigma)-1}\left(1-N_{t+s}\right)^{(1-\eta)(1-\sigma)} Y_{t+s}\right) \\
& +\omega_{5t+s}\left(a_{t+s} C^{\eta(1-\sigma)-1}\left(1-N_{t+s}\right)^{(1-\eta)(1-\sigma)} R_{t+s}^{-1}\right) \\
& -\omega_{5t+s-1}\left(a_{t+s} C^{\eta(1-\sigma)-1}\left(1-N_{t+s}\right)^{(1-\eta)(1-\sigma)} \Pi_{t+s}^{-1}\right) \\
& +\omega_{6t+s}\left(R_{t+s}-1\right)\Bigg\},
\end{aligned}
$$

where $d_t = \left\{Y_t, C_t, N_t, W_t^R, \Pi_t, R_t\right\}$ is the set of decision variables and $\omega_t = \{\omega_{1t}, \omega_{2t}, \omega_{3t}, \omega_{4t}, \omega_{5t}, \omega_{6t}\}$ is the vector of Lagrange multipliers. The final constraint imposes the zero lower bound constraint since the gross nominal policy rate R_t must be greater than or equal to one. After solving for the first-order conditions, the optimal policy problem is solved using the algorithm outlined in Appendix B. To determine the equilibrium real interest rate R_t^R, we also include the Euler equation for a zero net supply real bond as well. The algorithm solves for the policy functions for $N_t = N(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$, $\Pi_t = \Pi(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$,

$R_t = R(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$, $R_t^R = R^R(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$, $\omega_{4t} = \omega_4(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$, and $\omega_{5t} = \omega_5(a_t, \sigma_t^a, \omega_{4t-1}, \omega_{5t-1})$ on a discretized state space for $\{a_t \times \sigma_t^a \times \omega_{4t-1} \times \omega_{5t-1}\}$.

C.2 Optimal Fiscal and Monetary Policy Under Commitment

To solve for jointly optimal fiscal and monetary policy under commitment, we make two changes to the previous Lagrangian from Section C.1. We include utility from government spending in the period utility function of the representative household:

$$\frac{\left(C_{t+s}^{\eta}\left(1-N_{t+s}\right)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} + \psi \frac{G_t^{1-\sigma}}{1-\sigma},$$

where we choose ψ to pin down steady state G/Y to be 20 percent. In addition, the national income accounting identity in the first constraint now includes government spending:

$$Y_{t+s} = C_{t+s} + G_{t+s} + \frac{\phi_P}{2}\left(\frac{\Pi_{t+s}}{\Pi} - 1\right)^2 C_{t+s},$$

where the set of decision variables is $d_t = \left\{Y_t, C_t, N_t, W_t^R, \Pi_t, R_t, G_t\right\}$.

D Additional Results & Extensions

D.1 Model-Implied Moments Under Alternative Policies

Table D.1 computes the moments from our empirical calibration exercise under both our baseline policy rule as well as optimal monetary and fiscal policy. These results echo the findings of the conditional impulse responses in Figure 8 of the main text. Optimal monetary policy can greatly attenuate, but not eliminate, fluctuations in output and inflation. To implement these improved outcomes for the economy, the central bank may have to maintain a zero nominal policy rate for an extended period. In a 30-year simulation, the economy may spend as much as 10 years at the zero lower bound. Access to state-contingent government spending helps further attenuate the endogenous volatility in the output gap.

D.2 Incorporating Physical Capital

Our previous results show that precautionary saving by households leads to significant contractions in real activity and prices at the zero lower bound. However, a potential criticism of our results is that households do not have the ability to save in equilibrium. In our baseline model, households hold zero net supply real and nominal bonds. Higher uncertainty about future consumption increases the desired saving by households. To clear the bond markets, the rates of return on these zero net supply assets must fall. We now address this concern by adding physical capital to the model. We provide a detailed description of the key model equations below and Figure D.1 plots the effect of a one standard deviation uncertainty shock both at and away from the zero lower bound. These results show a very similar decline in total output to the model without physical capital. Even when we incorporate an elastic asset for saving, the endogenous volatility generated by the zero lower bound can generate a large decline in output and all its components.

For the model with capital, we add or modify the following equations from our baseline model.

Intermediate-goods producing firm production function:

$$\left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t \leq K_t(i)^{\alpha} \left[Z_t N_t(i)\right]^{1-\alpha}$$

Capital accumulation equation subject to adjustment costs:

$$K_{t+1}(i) = \left(1 - \delta - \frac{\phi_K}{2}\left(\frac{I_t(i)}{K_t(i)} - \delta\right)^2\right) K_t(i) + I_t(i)$$

Intermediate-goods producing firm first-order conditions for labor and capital demand:

$$\frac{W_t}{P_t} N_t(i) = (1-\alpha)\Xi_t K_t(i)^\alpha \left[Z_t N_t(i)\right]^{1-\alpha} \tag{1}$$

$$\frac{R_t^K}{P_t} K_t(i) = \alpha \Xi_t K_t(i)^\alpha \left[Z_t N_t(i)\right]^{1-\alpha} \tag{2}$$

Household first-order conditions for capital and investment:

$$q_t = \mathrm{E}_t \left\{ \left(\beta \frac{\lambda_{t+1}}{\lambda_t}\right) \left(R_{t+1}^K + q_{t+1} \left(1 - \delta - \frac{\phi_K}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 + \phi_K \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right)\right\} \tag{3}$$

$$\frac{1}{q_t} = 1 - \phi_K \left(\frac{I_t}{K_t} - \delta\right) \tag{4}$$

where $K_t(i)$ denotes physical capital, and q_t is the price of a marginal unit of installed capital. R_t^K/P_t is the marginal revenue product of capital, which is paid to the owners of the capital stock. Our adjustment cost specification is similar to the specification used by Jermann (1998) and Ireland (2003), and allows Tobin's q to vary over time. We calibrate $\phi_K = 20$, which implies an elasticity of I_t/K_t to q_t of 2. Figure D.1 shows the effects of an increased in expected shock volatility without any change in realized shock volatility both at and away from the zero lower bound. In producing Figure D.1, we follow the same procedure outlined in Section 4.1 of the main text.

References

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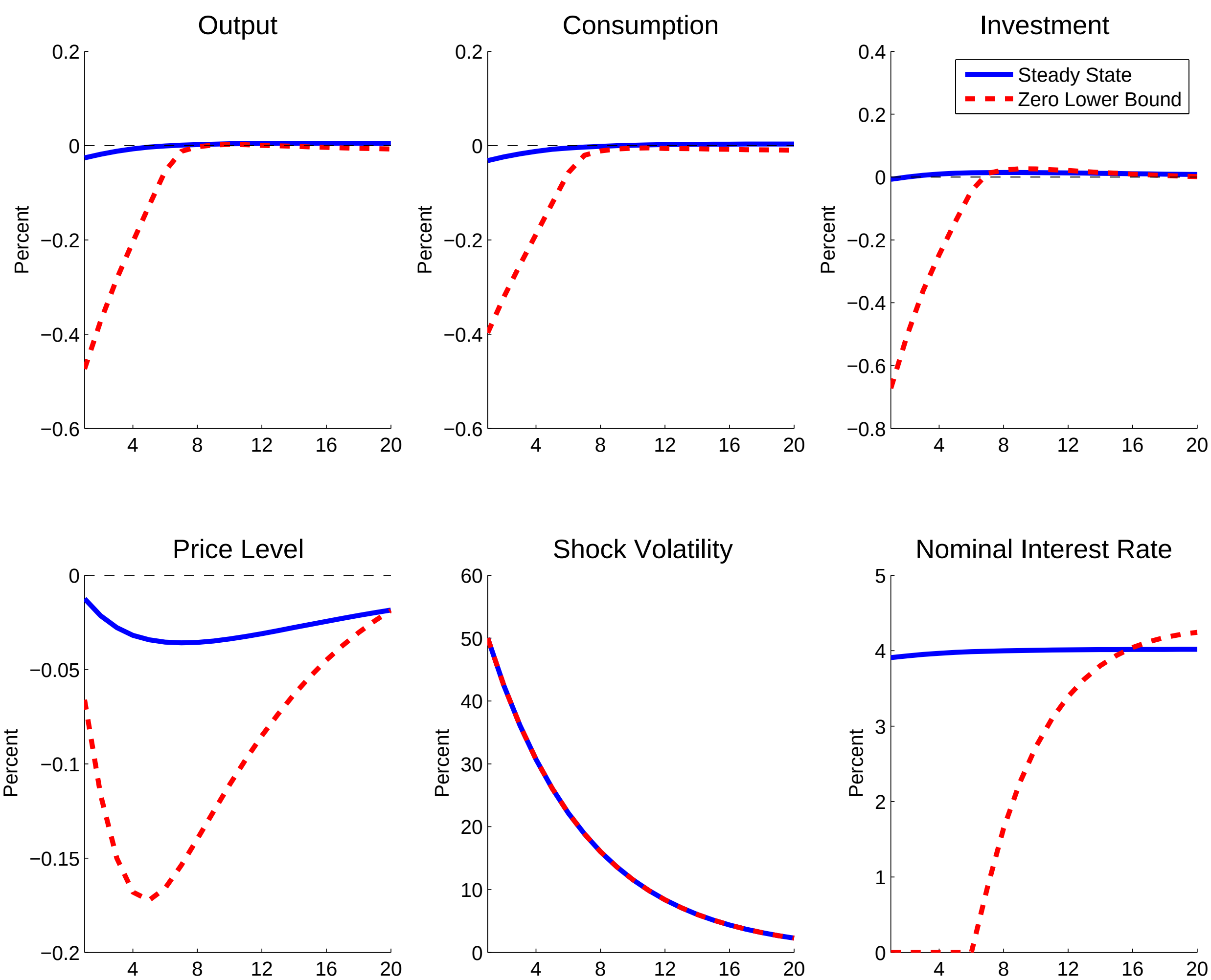
Table D.1: Model-Implied Moments Under Alternative Policies

Moment	Baseline Simple Policy Rule	Optimal Monetary Policy	Optimal Monetary & Fiscal Policy
Unconditional Volatility			
x	1.70	0.51	0.40
	(0.89, 3.00)	(0.03, 1.39)	(0.03, 1.08)
π	1.03	0.14	0.13
	(0.62, 1.58)	(0.01, 0.42)	(0.01, 0.40)
r	2.42	2.65	2.63
	(1.70, 3.28)	(1.87, 3.60)	(1.87, 3.59)
g			0.17
			(0.01, 0.47)
Stochastic Volatility			
x	0.73	0.39	0.30
	(0.28, 1.57)	(0.03, 1.00)	(0.02, 0.78)
π	0.40	0.10	0.09
	(0.19, 0.77)	(0.01, 0.29)	(0.01, 0.27)
r	0.72	0.78	0.78
	(0.41, 1.16)	(0.46, 1.24)	(0.45, 1.24)
g			0.12
			(0.01, 0.35)
Quarters at Zero Lower Bound	13	19	18
	(2, 29)	(2, 44)	(2, 43)

Note: Unconditional volatility is measured with the sample standard deviation. Stochastic volatility is measured by the standard deviation of the time-series estimate for the 5-year rolling standard deviation. The economy is considered at the zero lower bound if the policy rate falls below 25 basis points. The 90% small sample bootstrapped confidence intervals are given in parenthesis.

Figure D.1: Impulse Responses to Uncertainty Shock in Model With Capital



Note: The output, consumption, investment, price level, and shock volatility responses are plotted as percent deviations. The nominal interest rate is plotted in annualized percent.