Search with Wage Posting under Sticky Prices

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Abstract

This paper studies the interaction between nominal rigidities, labor market frictions, and consumption risk in a model where firms face sticky prices and post wage contracts to attract risk averse workers in a frictional labor market. Comparing a calibrated version of the model with two alternative versions—one that separates search and pricing frictions between two types of firms, and one in which a representative household makes consumption and employment decisions at an aggregate level—highlights the importance of integrating labor market and price-setting frictions with individual consumption risk. Separating search and pricing frictions between wholesale and retail sectors increases movements in inflation while muting those in labor markets and other macroeconomic variables. Meanwhile, using a representative household model significantly diminishes the effects of shocks on output and inflation, but increases the effects on vacancies and unemployment.

Keywords: Search, Matching, Inflation, Sticky Prices, Heterogeneity, Incomplete markets

JEL: E10, E30, E50, J60.

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1 Introduction

After the Great Recession, labor markets recovered at an anemic rate, with weak goods demand ostensibly being a reason for their sluggish recovery. According to this view, producers facing weak demand responded by lowering labor demand rather than lowering prices, in part due to nominal rigidities. These decisions in turn lead to higher unemployment and lower earnings for the employed, further weakening aggregate demand. Overall, the hiring and price-setting decisions of firms, and the consumption decisions of individuals in response to changes in the labor market all interacted to produce tepid growth even years after the recession ended.

This paper considers the macroeconomic implications of unified pricing, labor demand, and imperfect risk sharing in a model with sticky prices, search frictions in the labor market, and risk averse workers. It develops a framework in which firms make pricing decisions and post take-it-or-leave-it contracts to hire risk averse workers. The model developed therefore contrasts sharply with two typical constructs that limit the interaction between these relevant frictions. In particular, a wholesaler-retailer model that considers both labor market frictions and pricing decisions but separates these frictions into two separate entities limits the extent of their interaction. Additionally, a representative household model that considers labor market frictions in the presence of nominal rigidities but assumes workers are members of a household that makes decisions at an aggregate level distorts how aggregate demand responds to movements in the unemployment rate.

A key message from the analysis is that separating pricing and hiring frictions and consumption smoothing motives matters greatly even in a parsimonious framework. The results in this paper indicate that the typical assumptions of a wholesaler-retailer or representative construct—which may be used primarily for modeling convenience—may not be innocuous in a more rigorous empirical analysis and could lead to different conclusions about the relative importance of different shocks or frictions affecting the macroeconomy.

For example, in response to technology or monetary policy shocks, the model developed in this paper generates less movement in inflation and greater movements in labor and other macroeconomic variables compared to a model with the wholesaler-retailer structure. This
result occurs because firms in the baseline model act in the labor market and face nominal rigidities, leading to trade-offs between price and hours adjustment. In contrast, the wholesaler-retailer model separates pricing frictions into retail firms and labor market frictions into wholesale firms, so firms do not internalize how these frictions interact. Thus, retailers adjust their prices in response to shocks to a greater extent, limiting the effect on labor markets and the macroeconomy more generally. Meanwhile, comparing the developed model to a representative household model reveals that the latter generates much smaller responses in output and inflation to shocks, while producing slightly larger fluctuations in vacancies and unemployment after both types of shocks. This is because the representative household’s ability to pool consumption risk makes it more willing to accept changes in employment status of its members.

The assumption that firms post take-it-or-leave-it contracts, which is both analytically convenient and empirically plausible, plays a key role in the results. From an analytical standpoint, this contract ensures that all workers are offered their value of unemployment, which is independent of the firm’s product price, and consequently independent of whether the firm is a price-setter as in the baseline model, or a price-taker as in the wholesaler-retailer model. Hence, comparing allocations across models is transparent as they solely reflect changes in firm hiring behavior. By contrast, if wages were determined by commonly used Nash bargaining, the two models would trivially produce different allocations as hiring firms and workers would be splitting a surplus that varies based on the economy’s structure.1 From an empirical standpoint, survey evidence from Hall and Krueger (2012) suggests take-it-or-leave-it offers are common in the U.S. labor market. For example, they find that only about a third of all workers bargained over pay with their current employer, the remainder presumably considering their job offers to be take-it-or-leave-it. Additionally, among those who did not bargain 40 percent had precise knowledge about pay when they first met with their employer, a sign of wage posting.2 Thus, the assumed wage rule appears to be at least

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1 See for example, Kuester (2010), Barnichon (2010), Thomas (2011), Lago Alves (2012), and Dossche et al. (2014), for papers that combine search and Calvo pricing frictions within the same firm when wages are Nash bargained. Krause and Lubik (2007) allow for surplus splitting and quadratic price adjustment.

2 It is worth highlighting that these are average figures and there is significant heterogeneity in contractual arrangements. For example, bargaining is more common among the college educated and less likely for those without a high school diploma. Recent job losers and blue collar workers are also less likely to bargain over pay.
as plausible as Nash bargaining.

An additional by-product of wage posting is tractability on the individual side. In combination with the assumed preferences that eliminate the wealth effect on labor supply, wage posting eliminates the need to keep track of the distribution of wealth, as workers who are indifferent across employment states do not save in equilibrium. A direct consequence is that solving the household problem does not require approximation techniques like those of Krusell and Smith (1998) that are typically required to study business cycles with individual heterogeneity under incomplete markets.

This paper builds upon the literature analyzing the role of frictional unemployment for inflation dynamics by developing a framework that allows firms to jointly make pricing and hiring decisions to attract risk averse workers. Typically, papers specify labor market arrangements that preclude the direct interaction of frictions stemming from the job search process and frictions associated with infrequent price adjustment. For example, Walsh (2005), Trigari (2006), Sveen and Weinke (2008), and Christiano et al. (2016) assume a wholesaler-retailer structure, where hiring in the frictional labor market is done by wholesale firms, whereas prices are set separately by monopolistic retail firms that face nominal rigidities. By separating these decisions, the wholesaler-retailer structure obscures the impact of their interactions. In the baseline model developed in this paper, firms making the pricing decisions also make hiring and wage posting decisions, leading to a clear link between these decisions. In particular, because search frictions make labor a firm-specific factor in the short-run, pricing decisions at the firm-level critically depend on the curvature of labor disutility from the existing worker.\(^3\) Further, evidence presented by Klenow and Malin (2010) suggest strong linkages between pricing and hiring decisions.

The previously cited literature typically assumes, for tractability, perfect consumption insurance across individuals when modeling labor market dynamics under sticky prices. A key contribution of the current paper is developing a modeling framework where perfect consumption insurance is not necessary. Comparing the baseline model with a single representative household model reveals that the latter implies significantly smaller responses of output, hours, and wages, but larger unemployment responses, to shocks than the baseline

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\(^3\)Kuester (2010) obtains a similar result when wages and prices are infrequently bargained.
model where individuals act independently.

The remainder of the paper is as follows. Section 2 describes the baseline model with single agent households and firms that both hire and set prices. Section 3 outlines two alternative models—one with a wholesaler-retailer structure and one with a representative household assumption—that provide useful comparisons with the baseline model. Section 4 discusses calibration of the three models, and Section 5 highlights their differences in response to shocks. Finally, Section 6 concludes.

2 Baseline Model

This section presents the model, which is a variant of the conventional New Keynesian monetary framework. The key differences are in the labor market and the contractual environment, as well as the lack of a representative household. Hiring firms match with unemployed workers via random search. Rather than assuming firms and workers bargain over wages, hiring firms post take-it-or-leave-it offers that stipulate compensation and hours worked. Crucially, and in contrast to environments that separate these frictions into two different types of firms, the same firms that employ workers and produce are also subject to nominal rigidities in the form of a Calvo price-setting friction. Lastly, individuals operate in isolation, rather than within a large household construct that aggregates individuals’ income and makes all consumption and labor decisions at a household level.

The following subsections describe the model: individuals who supply labor and consume, final goods producers that bundle intermediate goods, intermediate goods firms that set prices and hire workers in a frictional labor market by posting wages, the evolution of the labor market, followed by policy and market clearing conditions. This section concludes with a discussion about the preferences that allow for abandoning the large household or perfect consumption insurance assumption while maintaining tractability.
2.1 Individuals

There is a unit mass of individuals, indexed by $i \in [0, 1]$. They have Greenwood et al. (1988) (GHH henceforth) preferences over consumption $c_{i,t}$ and hours worked $h_{i,t}$ of the form

$$U (c_{i,t}, h_{i,t}) = \frac{(c_{i,t} - \varphi h_{i,t}^{1+1/\psi})^{1-\gamma} - 1}{1 - \gamma},$$

where $\gamma$ is the constant of relative risk aversion, $\varphi$ is the disutility of labor, and $\psi$ is the Frisch elasticity of labor supply. Individuals discount future utility at rate $\beta$.

The use of GHH preferences, which eliminate any wealth effect on the labor supply, provides multiple benefits in the current framework. First, it greatly simplifies the contractual environment, as the presence of wealth effects on labor supply would imply that firms would vary their wage offering depending upon the wealth of the worker. Wealth effects on labor supply would also counterfactually imply asset-rich individuals preferring unemployment over employment.\(^4\) Second, along with the assumption on the contracting environment, GHH preferences eliminate the need for a perfect consumption insurance assumption typical in New Keynesian models with search (for example, Walsh (2005), Sveen and Weinke (2008), Kuester (2010), and Thomas (2011)). In those models, unemployed individuals are better off compared to employed individuals, as both enjoy the same level of consumption while the former also enjoy leisure.\(^5\) In contrast, the current preference and contractual specification implies that unemployed individuals are no better off than employed individuals.

Individuals purchase consumption goods at price $P_t$ and buy nominal bonds $B_t$ which have gross return $R_t$ in period $t + 1$. They also own shares in a mutual fund that owns all other firms in the economy; the mutual fund pays real dividends $D_t$.\(^6\) Finally, they pay real lump sum taxes equal to $T_t$.

The employment status $n_{i,t}$ of each individual varies between being unemployed ($n_{i,t} = u$) and employed ($n_{i,t} = e$). In each period a fraction $n_t$ of individuals are employed, and

\(^4\)In contrast, Mustre-del-Río (2015) finds that for prime age males employment is roughly flat with household wealth.

\(^5\)See Rogerson and Wright (1988) for a related analysis.

\(^6\)Given symmetric initial conditions, in equilibrium all individuals own equal shares in the mutual fund and no trading occurs, so this result is imposed from the outset for notational simplicity. The online-only appendix shows derivations for the full model, including with a market for mutual fund shares.
$u_t = 1 - n_t$ are unemployed.\footnote{For simplicity, the model abstracts from the participation decision and assumes all non-employed individuals actively search for employment.}

Unemployed individuals work zero hours ($h_{i,t}^u = 0$), collect real unemployment benefits from the government equalling $b$, and search for employment the subsequent period, which occurs in equilibrium with probability $s_t$. If $\mathbb{E}_t$ denotes the expectations operator conditional on time $t$ information, an unemployed worker’s problem is therefore

$$W_{i,t}^u = \max_{c_{i,t}^u, B_{i,t}^u} \left\{ \frac{(c_{i,t}^u)^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E}_t \left[ s_t W_{i,t+1}^e + (1 - s_t) W_{i,t+1}^u \right] \right\}$$

subject to

$$c_{i,t}^u + \frac{B_{i,t}^u}{P_t} + T_t = b + \frac{R_{t-1} B_{i,t-1}}{P_t} + D_t. \tag{3}$$

Employed workers, on the other hand, work positive hours $h_{i,t}$ and are paid a real compensation level $\omega_{i,t}$. Their existing job ends with exogenous probability $\delta$, in which case they enter unemployment the following period, and with probability $(1 - \delta)$ they remain employed. An employed worker’s problem is therefore

$$W_{i,t}^e = \max_{c_{i,t}^e, B_{i,t}^e} \left\{ \frac{(c_{i,t}^e - \phi h_{i,t}^{1+1/\psi})^{1-\gamma} - 1}{1 - \gamma} + \beta \mathbb{E}_t \left[ (1 - \delta) W_{i,t+1}^e + \delta W_{i,t+1}^u \right] \right\}$$

subject to

$$c_{i,t}^e + \frac{B_{i,t}^e}{P_t} + T_t = \omega_{i,t} + \frac{R_{t-1} B_{i,t-1}}{P_t} + D_t. \tag{5}$$

Note that employed workers do not choose $h_{i,t}$, as hours are determined by the firm.

Standard optimality conditions yield Euler equations for the unemployed and employed

$$\lambda_{i,t}^u = \beta \mathbb{E}_t \left[ \frac{s_t \lambda_{i,t+1}^e + (1 - s_t) \lambda_{i,t+1}^u}{\Pi_{t+1}} \right] R_t, \text{ and } \lambda_{i,t}^e = \beta \mathbb{E}_t \left[ \frac{(1 - \delta) \lambda_{i,t+1}^e + \delta \lambda_{i,t+1}^u}{\Pi_{t+1}} \right] R_t, \tag{6}$$

respectively, where $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate. The marginal utilities of
consumption for the unemployed and employed are given by

\[ \lambda_{i,t}^u = (c_{i,t}^u)^{-\gamma}, \quad \text{and} \quad \lambda_{i,t}^e = (c_{i,t}^e - \varphi h_{i,t}^{1+1/\psi})^{-\gamma}, \] (7)

respectively. Given symmetric initial conditions on bond-holdings, the optimal contract to be discussed equalizes the value of employment \( W_{e,i,t} \) and unemployment \( W_{u,i,t} \), which implies

\[ c_{i,t}^u = c_{i,t}^e - \varphi h_{i,t}^{1+1/\psi}. \] (8)

As a result of the assumed preferences and contract, in equilibrium, the marginal utilities of consumption are symmetric across employment states:

\[ \lambda_t = \lambda_{i,t}^u = \lambda_{i,t}^e. \] (9)

### 2.2 Final Good Producers

Final good producers operate competitively, purchasing \( Y_{j,t} \) from \( j \in [0, n_t] \) operating intermediate goods firms and combine them into final output \( Y_t \) using a technology with constant elasticity of substitution \( \epsilon \):

\[ Y_t = n_t \left( \frac{1}{n_t} \int_0^{n_t} Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}. \] (10)

Standard cost minimization implies that the demand for each intermediate good \( Y_{j,t}^d \) depends on its relative price according to

\[ Y_{j,t}^d = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{n_t}. \] (11)

The aggregate price level is related to individual prices by

\[ P_t^{1-\epsilon} = \frac{1}{n_t} \int_0^{n_t} P_{j,t}^{1-\epsilon} dj. \] (12)
2.3 Intermediate Goods Producers

Intermediate goods firms are indexed by $j$, and produce using a linear technology

$$Y^*_j = Z_j h_j,$$  \hspace{1cm} (13)

where $h_j$ is hours at firm $j$ and productivity $Z_j$ follows

$$\log Z_j = \rho_z \log Z_{j-1} + \sigma_z \varepsilon_{z,t}.$$  \hspace{1cm} (14)

Firms sell their output at price $P_j$ and are subject to a Calvo friction when setting prices. Firms employ a single worker; conditional on being matched with a worker the firm negotiates a contract $\Upsilon_j = (\omega_j, h_j)$ that determines a compensation level $\omega_j$ and an hours requirement $h_j$. Firms face a two-stage problem: in the first stage they set prices and in the second stage they contract with labor and produce.

In the second stage, given a price $P_{j,t}$, firms make a take-it-or-leave-it offer to their worker. They choose a contract $\Upsilon_{j,t}$ to maximize current period profits subject to their demand (11), the constraint that they must meet demand at the posted price $Y^*_j \geq Y^d$, and their matched worker’s participation constraint $W_i \leq W_e$. Since the firm will always choose to make the participation constraint bind, then for symmetric initial conditions on asset holdings, the value functions for an unemployed individual (2) and an employed one (4) imply the optimal contract satisfies

$$\omega_{j,t} = b + \varphi h_{j,t}^{1+1/\psi}. \hspace{1cm} (15)$$

This equation reveals that the equilibrium compensation contract when firms make take-it-or-leave-it offers to workers makes compensation solely dependent on hours worked and independent of aggregate labor market tightness. Thus, cyclical variation in compensation is solely due to changes in labor demand through hours worked $h_{j,t}$, which will also depend on the firm’s set price.

Given the optimal contract from (15), in the first stage a matched firm can re-optimize its price subject to a Calvo friction. The value of an operating firm with price $P_{j,t}$ is given
by

\[ J_t(P_{j,t}) = \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} - b - \varphi \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \]

\[ + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1}(P_{j,t}) + (1 - \zeta) J_{t+1}(P^*_t) \right]. \tag{16} \]

where \( \beta \frac{\lambda_{t+1}}{\lambda_t} \) denotes the stochastic discount factor, \( \zeta \) the probability of not re-optimizing prices, and \( P^*_t \) denotes the optimal price set by a firm that can re-optimize at time \( t \). Since the optimal compensation scheme depends on hours, and firms must meet demand at the posted price, the value is given by

\[ J_t(P_{j,t}) = \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} - b - \varphi \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \]

\[ + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1}(P_{j,t}) + (1 - \zeta) J_{t+1}(P^*_t) \right]. \tag{17} \]

This expression makes explicit the fact that prices, by pinning down demand, consequently pin down hours, and hence total compensation, through the relationship

\[ h_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t}. \tag{18} \]

A firm that can re-optimize prices, hence, takes this dependence of hours and compensation on the relative price, with the optimal reset price \( P^*_t \) satisfying

\[ \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \zeta (1 - \delta))^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ (1 - \epsilon) \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\epsilon} \frac{Y_{t+k}}{n_{t+k}} + \epsilon \left( 1 + \frac{1}{\psi} \right) \varphi h_{t+k}^{\epsilon+\frac{1}{\psi}} \right\} \right] = 0, \tag{19} \]

where \( h_{t+k}^* = \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon} \frac{Y_{t+k}}{Z_{t+k} n_{t+k}} \), represents the optimal hours choice at time \( t + k \) given a reset price \( P^*_t \). This optimal reset equation is similar to that found in typical Calvo price-setting environments. However, in the current environment the firm’s marginal cost is the marginal compensation paid to the worker it is matched with, which in turn depends on the evolution of the firm’s relative price \(^8\). Thus, while a high relative price in the future lowers demand, the firm can reduce costs by decreasing hours worked, and hence compensation, from the matched worker.

\(^8\)Appendix A provides a full derivation of this expression.
2.4 Vacancy Posting and the Labor Market

Firms post vacancies at cost $\kappa$, which are filled with probability $q_t$ and become productive the following period. At the beginning of $t + 1$ price adjustment occurs, then contracting and production. New entrants inherit a price level in period $t$ equal to the aggregate price level ($P_{j,t} = P_t$), and receive a Calvo shock before production in $t + 1$.

Because of free entry, firms post vacancies until the vacancy posting cost equals the expected return, which implies

$$
\kappa = q_t \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1} (P_t) + (1 - \zeta) J_{t+1} (P_{t+1}^*) \right].
$$

(20)

Matches $m_t$ depend upon the number of unemployed $u_t$ workers and the number of vacancies $v_t$ according to

$$
m_t = \sigma_m u_t^\alpha v_t^{1-\alpha},
$$

(21)

where $\sigma_m$ governs the efficiency of the matching function, and $\alpha$ is the elasticity of matches with respect to the number of unemployed workers. The job filling rate is $q_t = m_t/v_t$, while the job finding rate is $s_t = m_t/u_t$. New matches take one period to form, and existing matches are destroyed at an exogenous rate $\delta$. Consequently, employment evolves according to

$$
n_t = (1 - \delta) n_{t-1} + m_{t-1}.
$$

(22)

Figure 1 summarizes the timing of the model.

2.5 Policy and Market Clearing

Monetary policy follows a Taylor Rule, setting the nominal rate $R_t$ according to

$$
\frac{R_t}{R_{ss}} = \left( \frac{R_{t-1}}{R_{ss}} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi_{ss}} \right)^{(1-\rho_r)\gamma_s} \exp (\sigma_r \varepsilon_{r,t}),
$$

(23)

An alternative assumption where entrants always optimally set prices would generate slightly different dynamics between employment and inflation, and in fact might lead to larger differences between the models considered. However, that assumption would also lead to a different evolution of the price level than in standard New Keynesian models and those with labor search (for example, Kuester (2010)), so symmetry in pricing between existing and new firms is sufficient for the analysis in this paper.
where $\Pi_{ss}$ indicates the inflation target, $R_{ss}$ the nominal rate target, $\rho_r$ the degree of interest rate persistence, $\gamma_{\pi}$ the response to inflation, and $\varepsilon_{r,t}$ denotes a monetary policy shock.

Fiscal policy adjusts lump sum taxes to balance the budget, and since its only payments are unemployment benefits $b$, then

$$u_t b = T_t.$$  \hfill (24)

Market clearing requires that aggregate output equals aggregate consumption

$$Y_t = C_t,$$ \hfill (25)

while aggregate consumption is the consumption of all individuals

$$C_t = C_t^e + C_t^u = \int_0^{n_t} c_{i,t}^e di + \int_{1-u_t}^1 c_{i,t}^u di.$$ \hfill (26)

Finally, the optimal contract and the relationship between relative prices and hours helps in defining particular objects of interest. For example, based on equation (18) individual
hours depend on relative prices, so aggregate hours satisfy

\[ H_t = \int_0^{n_t} h_{j,t} \, dj = \int_0^{n_t} \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_{t,n_t}} \, dj. \]  

(27)

In addition, the average hourly wage is similarly dependent on relative prices through the optimal contract and the expression for individual hours

\[ w_t = \frac{\int_0^{n_t} \omega_{j,t} \, dj}{\int_0^{n_t} h_{j,t} \, dj} = \frac{\int_0^{n_t} \left( b + \varphi \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_{t,n_t}} \right)^{1+1/\psi} \, dj}{H_t}. \]  

(28)

2.6 Discussion

The presentation of the model concludes with a brief emphasis on the importance of GHH preferences to the analysis. The contract posted by firms will always make their matched workers’ participation constraint bind, which equalizes the value of being employed \( W_{e,i,t} \) or unemployed \( W_{u,i,t} \), period-by-period, and for all possible aggregate and individual states. As a result, the optimal contract always equalizes the instantaneous utility across employment states:

\[ U(c_{u,i,t}, 0) = U(c_{e,i,t}, h_{e,i,t}). \]  

(29)

where consumption is chosen optimally.

Without further restrictions on preferences, the model above would still lack tractability. For example, in an identical environment but with preferences of the form considered by King et al. (1988), the optimal contract would still satisfy equation (29), but the contract would lead to consumption and savings motives that differed across individuals, and hence would require keeping track of individuals’ assets.\(^{10}\) So, the second characteristic of preferences for tractability is that, under the optimal contract, the marginal utility of consumption for employed and unemployed are equal for all possible states:

\[ U_c(c_{u,i,t}, 0) = U_c(c_{e,i,t}, h_{e,i,t}), \]  

(30)

\(^{10}\)There has been some progress in considering sticky-price models with more general types of heterogenous consumers, but these advances require significant computational complexity (Gornemann et al. (2012)).
where hours are chosen under the optimal contract. With this additional restriction, the marginal utilities of consumption are always identical, leading all individuals to have equal consumption and savings motives regardless of whether they are employed or not. As a consequence, and paired with a symmetric initial condition on assets, all individuals will continue to have symmetric asset holdings.

This discussion highlights the fact that the exact specification of GHH preferences is not essential, but satisfies the necessary restrictions to keep the model tractable without resorting to a perfect consumption insurance assumption.

3 Two Alternative Models

Having presented the baseline model, this section sketches two alternative frameworks to the baseline model, both of which serve as useful bases of comparison. Importantly, the form of the optimal contract in these two models is identical to that in the baseline model (15), so any differences in outcomes directly reflect the particular deviation of each model from the baseline, rather than indirectly by affecting the wage determination process.

The first alternative separates the pricing and labor market frictions into distinct entities. Specifically, in this wholesaler-retailer (WR) model, perfectly competitive wholesalers enter the labor market, contract using wage posting as in the baseline model, and produce a wholesale good; monopolistically competitive retail firms then purchase this good and sell it to the final goods firm while facing sticky prices. This wholesaler-retailer structure helps isolate the importance of the interaction of pricing and labor market frictions for behavior in the baseline model.

The second alternative returns to a single firm facing both labor market and pricing frictions, but assumes there is partial consumption insurance, with a single representative household (RH) making all the consumption and employment decisions, rather than each individual operating in isolation. This representative household structure helps to show that in the baseline model, the fact that movements in an individual’s consumption—rather than pooled consumption across the household—play a key role in generating fluctuations.
3.1 Wholesaler-Retailer (WR) Model

To isolate the importance of allowing pricing and labor market frictions to interact within the firm, the first alternative model uses a customary wholesaler-retailer structure, but with posted wages. Wholesale producers hire labor in the frictional labor market using wage posting and produce a competitively priced good. Monopolistically competitive retail firms face Calvo price frictions and purchase the wholesale good and convert it into a differentiated good. The remaining aspects are the same as the baseline.

Focusing on the wholesaler problem, they operate a linear technology

\[ Y_t^w = Z_t h_t, \tag{31} \]

taking the price of the wholesale good \( P_t^w \) as given.\(^{11}\) Since the problem of individuals remains unchanged, wholesalers face a similar contracting environment as in the original model, and hence produce an optimal contract identical to the original model

\[ \omega_t = b + \varphi h_t^{1+1/\psi}, \tag{32} \]

where the only difference is that, since the wholesaler is a perfectly competitive firm, all employed workers have identical contracts \( \Upsilon_t = (\omega_t, h_t) \). Given this contract, wholesalers choose hours to maximize

\[ J_t = \max_{h_t} \frac{P_t^w}{P_t} Z_t h_t - b - \varphi h_t^{1+1/\psi} + \beta (1 - \delta) E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \tag{33} \]

The first-order condition with respect to hours implies that hours depend on the relative price of wholesale goods and the level of technology by

\[ h_t = \left( \frac{P_t^w}{P_t} Z_t \right)^{\psi} \frac{\varphi (1 + 1/\psi)}{\varphi (1 + 1/\psi)} \tag{34} \]

Comparing this expression to equation (18) under the baseline model, reveals important

\(^{11}\)Given symmetry of the wholesale firms in this environment, the subscript \( j \) is omitted for notational simplicity.
differences across the two models. In the baseline model, hiring firms are price setters and demand for their differentiated good is decreasing in their relative price, so are hours worked. In contrast, in the WR model, hiring firms are price takers and because supply for their good is increasing in the relative price of wholesale goods, so are hours worked. In partial equilibrium, technology shocks also have differential effects on hours worked across models. In the baseline model, because output is pinned down given relative prices, any increase in productivity is labor saving and hence hours worked fall. In contrast, an increase in productivity in the WR model increases hours since each existing employment match is more valuable given prices.

Next, under the WR model the free-entry condition takes the usual form

\[ \kappa = q_t \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}. \]  

Comparing this equation to (20), the free entry condition under the baseline model, highlights that frictions related to price adjustment do not directly affect vacancy creation in the WR model, while they have a direct impact in the baseline model.

To close the WR model, retailers face a standard problem summarized by the optimal reset price condition

\[
\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \gamma)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ (1 - \epsilon) \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\epsilon} + \epsilon \left( \frac{P^w_{t+k}}{P_{t+k}} \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon} \right) \right\} Y^d_{t+k} \right] = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \gamma)^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ (1 - \epsilon) \left( \frac{P^*_t}{P_{t+k}} \right)^{1-\epsilon} + \epsilon \left( 1 + \frac{1}{\psi t} \right) \frac{h^{1/\psi}_{t+k}}{P_{t+k}} \right\} Y^d_{t+k} \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon} \right] = 0, \tag{36}
\]

where the second equality uses the fact that marginal costs, dependent on \( P^w_t \), can be written as a function of hours using equation (34). There are two important differences between the optimal reset price equations in the WR model (36) and the baseline model (19). First, in the baseline model, because intermediate firms face pricing and labor market frictions, they discount future revenues both by the expected duration of the current price, which depends on \( \gamma \), and the expected duration of the current match, which depends on \( \delta \). In contrast, in the WR model, retail firms do not care about match duration when setting their prices. Second,
marginal costs are notably different across the two models. In the WR model, marginal costs depend on the relative price of wholesale goods. This price in turn is related to the marginal disutility of work at \( h_t \), which is common to all workers and determined in general equilibrium and thus not directly dependent on the firm’s chosen relative price. In contrast, in the baseline model marginal costs depend on the marginal disutility of hours worked for the matched worker \( h_{jt} \), which—rather than being dependent solely on general equilibrium outcomes—depend critically on the firm’s relative price. As a consequence, in the baseline model, price-setting firms directly consider the impact that pricing decisions have on future hours demanded and hence marginal costs, while no such trade-off exists for price-setting firms in the WR model.

### 3.2 Representative Household (RH) Model

In both the baseline and WR models discussed above, individuals operate in isolation without the benefit of consumption insurance, which in turn affects the optimal contract offered by firms and the intertemporal consumption decision. This feature contrasts with typical New Keynesian models with labor market search, such as Sveen and Weinke (2008), Kuester (2010), and Thomas (2011), where a large household provides consumption insurance that equalizes consumption across individuals, independent of employment status. As a result, in those models unemployed individuals within the household are strictly better off than the employed, since they get to enjoy equalized consumption while also enjoying leisure. In contrast, in both the baseline and WR models, individuals are indifferent between states of employment by virtue of the optimal contract.\(^{12}\) Thus, to highlight the importance of having workers make individual consumption and labor supply decisions, this section considers two alternative specifications that assume some form of consumption insurance.

First, Appendix B shows derivations of the optimal contract in a version of the environment in which a large household provides the same level of consumption to individuals regardless of employment status. This setup mimics Sveen and Weinke (2008), Kuester (2010),

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\(^{12}\)The baseline and WR setups would naturally mimic that achieved by a household that ran employment lotteries (Hansen (1985), Rogerson (1988), and Rogerson and Wright (1988)), since differences in consumption offset differences in hours worked in order to make all individuals indifferent between employed and unemployed states.
and Thomas (2011), and implies that the unemployed are better off than the employed. It also implies a contract different from the one that comes out of both the baseline and WR models. This difference is due to the fact that consumption insurance at the household level allows employed workers to increase their outside option during negotiations. Moreover, adding consumption insurance in this way eliminates the tractability of the setup.

Second, a more tractable option is to consider a framework where a representative household has preferences analogous to those in equation (1), and makes all consumption and labor decisions at the household level. Keeping all other aspects unchanged, this alternative model implies the same contract as the baseline and WR models, and therefore, that the employed are no worse off compared to the unemployed.

In this version of the problem, the household enters the period with \( n_t \) of its members employed, and has a value function given by

\[
V_t(n_t) = \left( C_t - \varphi \int_0^{n_t} h_{i,t}^{1+1/\psi} di \right)^{1-\gamma} - \frac{1}{1-\gamma} + \beta \mathbb{E}_t V_{t+1}(n_{t+1}).
\] (37)

Since the household pools all income to choose aggregate consumption and bond holdings, it faces the budget constraint

\[
\frac{C_t}{P_t} + \frac{B_t}{P_t} + T_t = b(1-n_t) + \int_0^{n_t} \omega_{i,t} di + \frac{R_{t-1}B_{t-1}}{P_t} + D_t.
\] (38)

Standard optimality conditions show that the marginal utility of consumption is now

\[
\lambda_t = \left( C_t - \varphi \int_0^{n_t} h_{i,t}^{1+1/\psi} di \right)^{-\gamma},
\] (39)

while the optimal choice of bonds—which end up being in zero net supply—is given by

\[
\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\Pi_{t+1}} = 1.
\] (40)

These aspects of the equilibrium are similar to the baseline model, with the important exception that the marginal utility of consumption is different, and hence so is the stochastic discount factor used by firms, as well as the exact relationship between expected inflation
and the nominal interest rate.

All employed workers in the household receive wage posting offers from their firms in the period. The benefit to the household of having the \( n_t \)th worker employed is given by the envelope condition of (37) with respect to \( n_t \):

\[
V_n (n_t) = -\varphi h_{n,t}^{1+1/\psi} \lambda_t + (\omega_{n,t} - b) \lambda_t + [(1 - \delta) - s_t] \beta \mathbb{E}_t V_{n,t+1} (n_{t+1}) .
\]

The first term is the loss in utility to the household due to one of its workers providing additional hours, the second term is the gain in utility for receiving compensation rather than unemployment benefits, and the third term is the expected future benefit of having an employed worker rather than an unemployed one.

When receiving wage offers, the household treats all workers as symmetric. Equivalently, the take-it-or-leave-it assumption implies that firms will give wage offers that make the above envelope condition equal to zero for all \( n_t \). In other words, for any employment level, the contract makes the household indifferent at the margin between having one more worker employed or not. As a result, the optimal contract for all workers is identical to that in the original problem

\[
\omega_{i,t} = b + \varphi h_{i,t}^{1+1/\psi} .
\]

Since the optimal contract remains unchanged in this framework, so do firms’ problems. The only difference is that, since the household has a given level of consumption and disutility from the work by its members, the marginal utility of consumption, and hence how it values consumption across periods and states, is potentially different from the baseline model. Because the household can pool resources for consumption insurance and manage the disutility of labor for all employed workers, the stochastic discount factor used in firms’ price-setting equation (19) and the free entry condition (20) will alter those firms’ choices and hence aggregate outcomes. Thus, a comparison between this model and the baseline model reveals how the degree of consumption insurance afforded to individuals affects firm-level intertemporal decisions like hiring and pricing.
4 Calibration

Having laid out the baseline model in Section 2 and sketched the two alternative models in Section 3, this section now turns to calibrating the models. The calibration strategy takes three steps. In the first step, a number of parameters that are common across models are chosen based on typical values in the literature. In the second step, some key labor market parameters are chosen, possibly differently across models, to match certain steady state targets. Finally, the third step is to calibrate the level of unemployment benefits \(b\) differently across models to match wage and hours data in the US economy.

Table 1 lists the first set of parameters, which are fixed at standard values. Assuming the model period is a quarter, the discount factor \(\beta\) is set to imply a steady state real interest rate of 2%. The coefficient of risk aversion \(\gamma\) is set to 2 as is standard in the literature, while the Frisch elasticity of labor supply on the intensive margin \(\psi\) is set to 0.5 as suggested by Chetty et al. (2011). The probability of not re-optimizing prices \(\zeta\) is set to match a median price duration of six months as reported in Bils and Klenow (2004). Following Gertler et al. (2008), the elasticity of substitution across goods is \(\epsilon = 10\), which implies a steady state markup of 11%. Consistent with empirical estimates in Shimer (2005) and den Haan et al. (2000) the quarterly separation rate is 10 percent. The elasticity of the matching function with respect to unemployment \(\alpha\) is set to 0.5, which is the midpoint of values typically cited.

Table 1: Standard Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>Discount factor</td>
<td>0.9951</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>Risk aversion</td>
<td>2</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Frisch elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Prob. not re-optimizing prices</td>
<td>0.66</td>
</tr>
<tr>
<td>(\epsilon)</td>
<td>Elasticity of substitution</td>
<td>10</td>
</tr>
<tr>
<td>(\delta)</td>
<td>Separation rate</td>
<td>0.1</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>Matching function elasticity</td>
<td>0.5</td>
</tr>
<tr>
<td>(\Pi_{ss})</td>
<td>Inflation target</td>
<td>1</td>
</tr>
<tr>
<td>(\rho_r)</td>
<td>Policy persistence</td>
<td>0.6</td>
</tr>
<tr>
<td>(\gamma_\pi)</td>
<td>Response to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>(\rho_z)</td>
<td>Technology persistence</td>
<td>0.95</td>
</tr>
<tr>
<td>(\sigma_r)</td>
<td>Std Dev MP shock</td>
<td>0.0025</td>
</tr>
<tr>
<td>(\sigma_z)</td>
<td>Std Dev technology shock</td>
<td>0.007</td>
</tr>
</tbody>
</table>
Table 2: Additional Calibrated Parameters and Targets

Panel A: Calibrated to Steady State Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
<th>Baseline</th>
<th>WR</th>
<th>RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Disutility of labor</td>
<td>$h_{js} = 1/3$</td>
<td>2.7</td>
<td>2.7</td>
<td>2.7</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Matching efficiency</td>
<td>$u_{ss} = 0.11$</td>
<td>0.7526</td>
<td>0.7526</td>
<td>0.7526</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy posting</td>
<td>$q_{ss} = 0.70$</td>
<td>1.0124</td>
<td>1.1752</td>
<td>0.8249</td>
</tr>
</tbody>
</table>

Panel B: Calibrated to Relative Vol of Average Wage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
<th>Baseline</th>
<th>WR</th>
<th>RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Unemployment Benefits</td>
<td>$\frac{\sigma(w)}{\sigma(Y)} = 0.530$</td>
<td>0.0816</td>
<td>0.0239</td>
<td>0.1097</td>
</tr>
<tr>
<td>$b/\omega_{ss}$</td>
<td>SS Replacement Ratio</td>
<td>Implied from $b$</td>
<td>0.4493</td>
<td>0.1926</td>
<td>0.5231</td>
</tr>
</tbody>
</table>

Panel C: Performance of Models

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>US Data</th>
<th>Baseline</th>
<th>WR</th>
<th>RH</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sigma(w)}{\sigma(Y)}$</td>
<td>Relative Vol of Average Wage</td>
<td>0.530</td>
<td>0.530</td>
<td>0.530</td>
<td>0.530</td>
</tr>
<tr>
<td>$\frac{\sigma(H)}{\sigma(Y)}$</td>
<td>Relative Vol of Aggregate Hours</td>
<td>0.790</td>
<td>0.7947</td>
<td>0.4494</td>
<td>0.8598</td>
</tr>
</tbody>
</table>

in the literature. Lastly, the parameters governing shocks and monetary policy are also set to standard values.

Panel A of Table 2 lists the parameters calibrated to match steady state values, and how these differ between the baseline and two alternative models. The disutility of hours worked $\varphi$ is such that steady state hours worked per employed person equals $1/3$. Given that preferences have identical forms across models, this target implies $\varphi = 2.7$ for each model. Following Blanchard and Diamond (1990), the targeted steady state unemployment rate is 11 percent, which includes both individuals who are categorized as unemployed and those out of the labor force who want a job. Following den Haan et al. (2000), the steady state worker finding rate is 70 percent. These assumptions directly pin down the matching efficiency parameter $\sigma_m$. Again, since the matching function and evolution of employment are identical across models, this produces $\sigma_m = 0.7526$. Lastly, given the calibration and targets, the vacancy posting cost $\kappa$ is implied from the steady state free-entry condition in each model, and these differ across models due to differences in unemployment benefits $b$.

Panel B of Table 2 shows the calibration of the level of unemployment benefits $b$. Rather than calibrate this value directly, the calibration focuses on the steady state replacement
ratio, which is defined as the ratio of unemployment benefits divided by the average compensation per worker in steady state, so \( \frac{b}{\omega_{ss}} \). There are a wide range of values in the literature. For example, Shimer (2005) considers a value of 0.4 while Hagedorn and Manovskii (2008) consider a value close to one. In the baseline model, linking the pricing and hiring decisions of monopolistically competitive firms puts a restriction on possible values for the replacement ratio that depends on technology and preferences. The value of the firm (17) in steady state must be positive, so the steady state replacement ratio must satisfy

\[
\frac{b}{\omega_{ss}} < 1 - \frac{(\epsilon - 1)}{\epsilon (1 + 1/\psi)}.
\]  

(43)

Intuitively, if the replacement ratio is too large, then the benefits of being unemployed are large enough to make hiring workers infeasible. The upper bound depends on firms’ pricing power, \( \epsilon \), and how sensitive are hours worked to changes in compensation, \( \psi \). If the elasticity of substitution \( \epsilon \) is low, then firms have more pricing power and expect to make more profits, allowing a larger replacement ratio. Likewise, as the Frisch elasticity \( \psi \) falls, then the amount of compensation needed to induce more hours is lower, also allowing a larger replacement ratio. Given the calibration in Table 1, the replacement ratio in the baseline model must be less than 0.7. A similar restriction exists for the WR model, but the upper bound is slightly lower due to the lack of market power for hiring firms in this framework; the restriction for the RH model is the same as the baseline model.

With these restrictions in mind, the replacement ratios in each model are calibrated to match the volatility of average hourly wages, as shown in Panel C of Table 2. This objective is not arbitrary, but instead chosen to highlight how the baseline model can match moments that related frameworks typically have trouble matching (see Sveen and Weinke (2008)). The implied replacement ratios vary significantly across the models, with a value of 0.4493 in the baseline model, 0.1926 in the WR model, and 0.5231 in the RH model.

Panel C in Table 2 also highlights a second, untargeted moment in the US data, which is the relative volatility of aggregate hours. Despite the fact that this moment is not used in the calibration, the baseline model nearly replicates the value in the data of 0.79, while

\footnote{The relevant data used are for the nonfarm sector, and consist of real output, aggregate hours, and real compensation per hour, all since 1951Q1-2016Q3 and are HP filtered using a smoothing parameter of 1600.}

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the WR model severely underpredicts and the RH model slightly overpredicts the volatility of aggregate hours.

The equations for aggregate hours (27) and the average hourly wage (28) in the baseline model highlight how changes in the unemployment benefits $b$ affect both of these moments. In particular, by acting as a fixed cost in the compensation of a worker, $b$ directly impacts the level of compensation and hence the average wage. On the other hand, $b$ only affects aggregate hours indirectly through both the optimal contract and general equilibrium effects. A higher value of $b$ lowers the volatility of the wage by making a larger portion of the wage a fixed cost regardless of the hours choice.

Relative to the baseline model, the WR model generates far lower volatility of aggregate hours given the volatility of average wages. As equation (34) shows, in this alternative, the hours of each worker are completely symmetric, while under the baseline model, aggregate hours have more volatility because each workers’ hours inherit additional volatility based on the relative price of the firm. In other words, the baseline model is able to generate more volatility in aggregate hours because it has both cross-sectional and time-series variation in individual hours, whereas the WR model only has the latter.

The RH model, on the other hand, does have both cross-sectional and time-series variation in individual hours, but it generates slightly too much volatility in aggregate hours. The reasons for this higher volatility are due to indirect general equilibrium effects, rather than direct ones. As the following section will discuss, the consumption smoothing motive at the household level distorts the stochastic discount factor used by firms in their pricing and entry decisions, which mutes the response of average hours per worker and amplifies the response of employment in response to shocks relative to the baseline model. On net, larger swings in employment under the RH model generate higher volatility in aggregate hours than those in the baseline model.

To better understand how varying the replacement ratio across the three models considered affects the volatility of wages and aggregate hours, Figure 2 shows how these moments vary as a function of the replacement ratio. The top panel shows that increasing the replacement ratio decreases the relative volatility of the average wage in all three models. As discussed, as the replacement ratio increases, the portion of compensation that is fixed,
and unrelated to hours increases, which lowers the volatility of the wage rate in response to shocks.

Meanwhile, the bottom panel shows how aggregate hours responds as the replacement ratio changes. The key implication of these graphs is that the baseline and RH models both perform fairly well in hitting both aggregate hours and wage volatility. Due to the slope of the lines in the RH model, there is a more significant trade-off between matching the volatility of wages and hours. On the other hand, in the baseline model, the volatility of hours is relatively close to the data for a wide range of replacement ratios, suggesting less
of a trade-off. Finally, the volatility of aggregate hours in the WR model is far from the data over a range of values. Indeed, only for a replacement ratio above 0.6 does this model generate empirically plausible movements in aggregate hours, at the expense of implying too little volatility in wages. Conversely, for a replacement ratio near zero the WR model over-predicts the volatility of wages, but under-predicts the volatility of aggregate hours. Missing in these two dimensions often calls for sticky wages to lessen the fluctuations of wages and increase those of hours; by contrast, by integrating pricing and labor market behavior the baseline and RH models appears to mitigate some of these issues.

5 Impulse Responses

This section examines the dynamics associated with different shocks. Again the main comparisons are between the baseline, Wholesaler-Retailer (WR), and Representative Household (RH) models.

5.1 Technology Shock

Figure 3 shows the behavior of the three models to a one standard deviation positive innovation in total factor productivity. Because the analyzed models are parsimonious and lack certain real rigidities, most of the change in aggregate output occurs in the first period. However, each model differs in how the instantaneous increase in demand is accommodated.

Focusing first on the baseline model, employment is pre-determined in the first period since matches take a period to become productive. Existing firms that can re-optimize their price face a trade-off between changing prices and changing hours and hence compensation of their matched worker. With higher aggregate demand due to the shock, a firm could increase prices, reduce hours and compensation, and therefore sell fewer goods at a higher markup. Alternatively, they could decrease prices, increase hours and compensation, and sell more goods at a lower markup. On net, the incentives for the latter dominate, so inflation barely moves while individual hours and wages rise. Consumption and aggregate output rise

\[14\] In a framework similar to the WR model, Trigari (2006) finds that consumption habits help undo some of this result.
sharply thanks to higher compensation.

However, the effect of the positive productivity shock on output, hours, and wages is relatively short-lived, since higher productivity induces firm entry. This additional entry lowers unemployment in subsequent periods, but also impacts price-setting and hours choices.\textsuperscript{15} In particular, with greater competition, the incentive to lower prices and sell more goods grows.

\textsuperscript{15}Note that unemployment falls in response to a positive technology shock because there is no participation margin. This is in contrast to models like Galí (2011) where unemployment actually rises in response to a positive technology shock as more workers enter the labor force.
As a result, non-price setting firms end up with high relative prices, which causes the hours of their workers to fall, leading to a dampening effect on compensation and hence aggregate demand.

Turning next to the WR model, the main difference in this framework is that existing retail and wholesale firms face separate frictions and do not face a trade-off between increasing prices or increasing individual hours worked as in the baseline model. Therefore, inflation rises on impact, which mutes aggregate demand and vacancy posting activity compared to the baseline model. However, the instantaneous overshooting of inflation is paid back in subsequent periods, as the entry of wholesale firms due to higher productivity leads to lower prices of the wholesale good, leading to lower inflation and stronger aggregate demand. As a result, the path of unemployment closely resembles the baseline path one year out.

Lastly, the responses in the RH model have the same channels as in the baseline model, but the magnitudes of the incentives differ due to how the pooling of consumption affects intertemporal decisions. On impact, aggregate consumption rises in the RH model by less than in the baseline model because the representative household pools consumption between the employed and unemployed and hence the marginal utility of consumption is lower. With aggregate demand barely rising and productivity increasing, existing firms respond by reducing hours and wages. However, with expectations of higher productivity and lower wages, vacancies rise more in this model compared to the baseline. These expectations also induce existing firms in the first period to cut prices as future profits are now more valuable and price adjustment is infrequent.

To summarize, after a technology shock, the responses of both macroeconomic and labor market variables vary across models. Critically, a comparison of baseline and WR models show the importance of integrating pricing and hiring frictions within the same firm, as the former produces more muted responses of inflation to a productivity shock compared to the latter. Meanwhile, comparing the baseline model to the RH model highlights that the single representative household construct, by pooling consumption between the employed and unemployed, mutes the effects of productivity shocks on output, average hours worked, and wages while in turn generating larger fluctuations in vacancies and unemployment. As was noted in Section 4, these fluctuations end up producing too much volatility in aggregate
hours given wage volatility.

5.2 Monetary Policy Shock

Figure 4 shows the behavior of the three models to a one standard deviation innovation to monetary policy and reiterates many of the key mechanisms at play across each model.

In the baseline model, the positive shock to the nominal rate, all else equal, lowers individuals’ demand in favor of savings, and hence the level of aggregate demand falls. As
in the case following a productivity shock, existing firms that can re-optimize their prices face a trade-off between adjusting prices or hours. On net, firms use both margins and price adjusters, in particular, lower their prices. The lower demand and now higher relative prices of non-optimizing firms leads to a decline in hours and wages. The feedback from lower compensation causes the ensuing decline in output to be large, while the decline in the inflation rate mitigates some of the upward pressure on the nominal rate from the shock.

Again, as in the case with the productivity shock, the effects of a monetary policy shock end up being short-lived. The lower demand, paired with a higher marginal utility of consumption, causes firms to contract the number of vacancies posted, and hence unemployment increases in the period following the shock. The decline in the number of operating firms, however, dampens the need for further price reductions, which helps hours, wages, and therefore output to quickly return to their steady state values.

The effects of the monetary policy shock in the WR model mimic those seen in the productivity case. Since retail firms do not internalize the impact of pricing frictions on the labor market, the decline in inflation is much larger. This substantial decline in prices mutes some of the impact of tighter monetary policy on output, so hours worked and wages fall less drastically compared to the baseline. In addition, the sharp drop in inflation nearly undoes the effects of the policy shock on the nominal rate, with the rate increasing only slightly in equilibrium. Ultimately, the sharper drop in inflation generates a smaller decline in vacancies and hence the rise in unemployment is less stark.

Turning finally to the RH model, the household’s ability to pool consumption mutes many of the effects seen in the baseline model. Here the pooling of consumption means that aggregate demand declines less following the shock, which therefore tempers the prices versus hours adjustment faced by firms. As a result, inflation, average hours worked, and wages fall by less than in the baseline model. The decline in inflation is very slight, meaning most of the effects of the monetary policy shock are passed directly to an increase in the nominal rate. This increase, coupled with the interest rate inertia in the policy rule, leads to a more gradual increase in output in subsequent periods, and distorts the forward-looking behavior of firms through the stochastic discount factor. The ensuing decline in vacancies and rise in unemployment is then slightly more pronounced in the RH model than the baseline model.
In sum, these results highlight that both macroeconomic and labor market variables vary across models in how they respond to monetary shocks. The WR model, by separating labor and pricing frictions into two separate types of firms, produces more drastic swings in inflation than the baseline model. Meanwhile, the ability of the household to pool consumption, as in the RH model, generates smaller adjustments in the output, hours, wages, and inflation, than when individuals operate independently, as in the baseline model. As a result, the RH model generates slightly more variation in vacancies and unemployment compared to the baseline model.

6 Conclusion

This paper has considered the macroeconomic implications of the interaction between infrequent price adjustment, labor market frictions, and consumption risk. In the New Keynesian model analyzed, risk averse workers randomly search for jobs and monopolistically competitive firms post take-it-or-leave-it wage contracts taking into account infrequent adjustment of their own price. By allowing for wage posting by firms, the model provides a direct link between pricing and hiring behavior at the micro level. Meanwhile, risk aversion and imperfect capital markets highlight the importance of consumption smoothing for aggregate price and labor market dynamics.

A comparison of the baseline model with a model that separates pricing and hiring across wholesale and retail firms reveals key differences. In the face of technology or monetary policy shocks the response of inflation is larger in this alternative model compared to the baseline as retail firms do not internalize how changing their prices affects the labor market and hence aggregate demand. As more adjustment occurs through prices, labor market variables move by less compared to the baseline model.

Meanwhile, a comparison of the baseline model with a model that allows for a single representative household that makes all decisions shows how the pooling of consumption affects aggregate demand and hence labor market. The household’s ability to pool consumption between employment states leads to a more gradual and longer-lived period of adjustment in aggregate demand. Forward-looking firms anticipate this response, and hence vacancies
and unemployment adjust by more.

A key message from the analysis is that separating pricing and hiring frictions and consumption smoothing motives matters greatly even in a parsimonious framework. While a full structural estimation of the model is left for future research, the results in this paper indicate that the typical assumptions of a wholesaler-retailer or representative construct—which may be used primarily for modeling convenience—may not be innocuous in empirical analysis and could lead to different conclusions about the relative importance of different shocks or frictions affecting the macroeconomy.

References


## Appendices

### Appendix A  Optimal Reset Price in Baseline Model

This Appendix provides the derivation of equation (19) in the main text.
In the first stage, firms can re-optimize their prices subject to a Calvo friction. If a firm re-optimizes in period $t$, it solves

$$J^*_t = \max_{P_{j,t}} \left\{ \left( \frac{P_{j,t}}{P_t} \right)^{d_j - \omega_j + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} [\zeta J_{t+1} (P_{j,t}) + (1 - \zeta) J^*_t + 1]} \right\} ,$$

subject to the fact that they must meet demand at the posted price

$$Y^s_{j,t} = Y^d_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{n_t},$$

goods are produced according to the production function

$$Y^*_{j,t} = Z_t H_{j,t},$$

and hours and compensation must satisfy the optimal contract

$$\omega_{j,t} = b + \varphi H_{j,t}^{1+\frac{1}{\psi}}.$$

Plugging in the constraints produces

$$J^*_t = \max_{P_{j,t}} \left\{ \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} - b - \varphi \left( \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \right)^{1+\frac{1}{\psi}} \right\} + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} [\zeta J_{t+1} (P_{j,t}) + (1 - \zeta) J^*_t + 1] \right\}$$

and denoting the optimal reset price by $P^*_t$, the first order condition is

$$(1 - \epsilon) \left( \frac{P^*_t}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} + \epsilon \left( 1 + \frac{1}{\psi} \right) \varphi \left( \left( \frac{P^*_t}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \right)^{1+\frac{1}{\psi}} + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \zeta P^*_t \left( J_{t+1} (P_{j,t}) + (1 - \zeta) J^*_t + 1 \right) = 0.$$

A firm that doesn’t reset its price has value

$$J_1 (P_{j,t}) = \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} - b - \varphi \left( \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \right)^{1+\frac{1}{\psi}} + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} [\zeta J_{t+1} (P_{j,t}) + (1 - \zeta) J^*_t + 1] .$$
and the envelope condition is

\[ J_{P,t}(P_{j,t}) = (1 - \epsilon) \left( \frac{P_{j,t}}{P_{t}} \right)^{-\epsilon} y_{t+1} \frac{y_{t+n_t}}{Z_{t+n_t}} \left( \frac{P_{j,t}}{P_{t}} \right)^{-\epsilon} \left( \frac{y_{t+n_t}}{Z_{t+n_t}} \right)^{1+\frac{1}{\psi}} + \beta (1 - \delta) \mathbb{E}_t \frac{y_{t+n_t}}{Z_{t+n_t}} \zeta J_{P,t+1}(P_{j,t}). \]  

(51)

Combining this envelope condition for a price-setting firm with the first order condition, and writing in infinite summation form yields

\[ \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \zeta (1 - \delta))^k \frac{y_{t+n_t}}{Z_{t+n_t}} \left( 1 - \epsilon \right) \left( \frac{P_{j,t}}{P_{t+k}} \right)^{1-\epsilon} \frac{y_{t+k}}{n_{t+k}} + \epsilon \left( 1 + \frac{1}{\psi} \right) \varphi \left( \left( \frac{P_{j,t}}{P_{t+k}} \right)^{-\epsilon} \frac{y_{t+k}}{Z_{t+k} n_{t+k}} \right)^{1+\frac{1}{\psi}} \right] = 0. \]  

(52)

### Appendix B  Consumption Insurance

This Appendix characterizes a problem similar to those in typical New Keynesian models with labor search, such as Sveen and Weinke (2008), Kuester (2010), and Thomas (2011), where a large household provides consumption insurance and controls individual labor decisions. Because consumption is equalized across employment states, but labor disutility is not, the employed are strictly worse off than the unemployed.

In this consumption insurance setup, the household aggregates individuals’ utility and income to pool consumption. It has a value function given by

\[ V_t(n_t) = \int_{0}^{n_t} \left( C_t - h_{t,1+\psi}^t di \right)^{1-\gamma} - 1 \frac{1}{1-\gamma} di + (1 - n_t) \frac{(C_t)^{1-\gamma} - 1}{1-\gamma} + V_{t+1}(n_{t+1}) \]  

(53)

and faces the budget constraint

\[ C_t + \frac{B_t}{P_t} + T_t = b (1 - n_t) + \int_{0}^{n_t} \omega_{i,t} di + \frac{R_{t-1} B_{t-1}}{P_t} + D_t. \]  

(54)

Standard optimality conditions show the marginal utility of consumption is now

\[ \lambda_t = \int_{0}^{n_t} \left( C_t - h_{t,1+\psi}^t di \right)^{-\gamma} di + (1 - n_t) C_t^{-\gamma}, \]  

(55)

which shows the fact that increases in consumption are distributed across all employed and
unemployed individuals. Similar to the RH model as described in the main text, the benefit to the household of having the $n_t - th$ worker employed is given by the envelope condition of (53) with respect to $n_t$:

$$V_{n_t}(n_t) = \left(\frac{C_t - h_{n,t}^{1+1/\psi} \psi dt}{1 - \gamma}\right)^{1-\gamma} - \left(\frac{C_t}{1 - \gamma} - 1\right) + \lambda_t \left(\omega_{n,t} - b\right) + \left[(1 - \delta) - s_t\right] \beta \mathbb{E}_t V_{n,t+1}(n_{t+1}).$$

(56)

The first term is the gain in utility from having an employed worker, the second term is the loss of utility from no longer having an unemployed individual, the third term is the gain in utility from having compensation rather than benefits, and the fourth term is the expected future benefit of having an employed worker rather than an unemployed one. Comparing this expression to (41) in the main text highlights some important differences. The representative household in the main text is comparing the marginal disutility of additional hours worked versus their marginal benefit in terms of additional earnings. The large household here is comparing the total utility cost—both consumption and hours worked—of one more individual employed versus the marginal benefit in terms of additional earnings. Thus, even if employment and the marginal utility of consumption were the same across both large household arrangements, the optimal choice of hours worked would be different as each household values additional employment differently.

Since the household treats all workers as symmetric, the optimal contract makes the household indifferent between having each worker employed or not, meaning the envelope condition is zero for all workers in every period. As a result, the optimal contract is given by

$$\omega_{n,t} = b + \left(\frac{C_t^{1-\gamma} - \left(C_t - h_{n,t}^{1+1/\psi} \psi dt\right)^{1-\gamma}}{\lambda_t (1 - \gamma)}\right)$$

(57)

which is an obviously different form from the optimal contract in the baseline, WR, and RH models. This expression clearly demonstrates how having consumption insurance in a manner similar to Sveen and Weinke (2008), Kuester (2010), and Thomas (2011), alters the optimal contract given to a worker and makes it now dependent on the level of aggregate consumption. In other words, income from other workers now affects the bargaining power of an individual worker, distorting the optimal contract relative to when the individual operates
without consumption insurance. Importantly, however, while employed workers benefit from consumption insurance, they are still worse off compared to the unemployed.

Further, the optimal contract under this form of consumption insurance means that the relative simplicity in the firms’ problem and aggregation that was present in the other models is lost, making the full equilibrium dynamics intractable under non-separability of consumption and leisure.