House Prices, Heterogeneous Banks and Unconventional Monetary Policy Options

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Abstract

This paper develops a financial mechanism which integrates housing and the real economy through housing-secured debt. In this environment, movements in home prices are amplified through both borrowers and banks’ balance sheets, leading to a self-reinforcing credit/liquidity crunch. When placed within a traditional business cycle model, this financial structure quantitatively captures empirical relationships the traditional financial accelerator mechanism struggles to explain and the qualitative predictions of the model are consistent with dynamic responses from a VAR. The model provides a framework to examine the ability of QE policies and equity injections into big banks to mitigate a housing bust. Although both are effective, the nuances of the policies are important. A prolonged asset purchase program is preferable to a short-term equity injection; however, the model suggests the equity injections may have been necessary to prevent an economic collapse at the acute stage of the 2008 Financial Crisis.

Keywords: Financial Crises, Financial Frictions, Unconventional Monetary Policy, Housing

JEL Codes: E32, E44, G01, G21

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1 Introduction

The largest economic contractions in the United States over the last 100 years coincided with falling nationwide home prices and financial crises. Although the Great Depression and the Great Recession both featured housing busts, the policy responses that followed were dramatically different. Following the 2008 Financial Crisis, the Federal Reserve and U.S. Government took the exceptional actions of purchasing housing secured debt (Mortgage-Backed Securities) and injecting equity into the largest, most complex U.S. financial firms. Although it is generally believed that these actions had a mitigating effect on the economic downturn, the channels through which these policies are transmitted to the real economy are not as well known. In this paper, I provide a description of one such channel through which swings in home prices can lead to real economic cycles. I then exploit this channel to spell out how unconventional policies such as large scale purchases of secured debt and equity injections into complex banks can soften the economic blow of a housing bust.

Key to unraveling the transmission mechanism of unconventional monetary policy is specifying how housing and the real economy are connected. To do so, I follow the approach of Iacoviello (2005) and Liu, Wang, and Zha (2013) and employ the financial sector as the linchpin connecting housing to production. Although both of these studies focus on the transmission of home prices through the financial sector, financial firms in these constructs are not explicitly modeled. The role of highly complex intermediaries in fueling and amplifying the 2003-2013 housing cycle is part of the narrative of the 2008 Financial Crisis, but has rarely been exploited in equilibrium models to conduct policy experiments.¹ I find that specifying banks as having one of two types: (1) complex or (2) simple helps to understand how movements in home prices can have powerful effects on financing premiums and hence production. This modeling choice leads to the policy implication that the effectiveness of unconventional policy interventions depends on the degree of heterogeneity within the financial sector. In other words, the avenue through which housing swings are amplified within the financial sector is the same path that transmits unconventional policy.

To further support the finding that home price swings are amplified by the redistribution of assets between complex and simple banks, I compare the model’s amplification sequence against estimated vector autoregressions. The resulting impulse response functions from the data provide qualitative support for the model’s predictions and show the all important degree of heterogeneity between financial firms in the simulated model is quantitatively comparable to that found in the data. Moreover, I show the financial integration of the housing and production sector produces theoretical correlations that the traditional financial accelerator mechanism in Bernanke, Gertler, and Gilchrist (1999) struggles to explain, including the correlation of finance premiums with home prices, investment and output.

Finally, I use the equilibrium model to conduct a variety of policy experiments by varying asset purchase programs based on: whether they are announced or unannounced ahead of

¹A recent exception to this can be found in the interpretation of “experts” in Brunnermeier and Sannikov’s (2014) model.
time, whether the pace of asset purchases ends abruptly or is tapered off slowly, whether the proceeds of the asset purchases are re-invested by the central bank and how fast the central bank unwinds its balance sheet following a large scale asset purchase program. The largest policy effect on output, home prices and finance premiums (and the largest remittances to taxpayers) comes through an asset purchase program which slowly tapers purchases, re-invests the proceeds from the purchases and unwinds the balance sheet slowly. Comparing this QE strategy to a short-term injection of equity into complex banks, I find that both are effective at mitigating the economic contraction. However, the long-term nature of the QE policies offers an accelerated economic recovery while the short-term nature of equity injections are most effective at softening the downturn at the acute stage of the crisis.

The remainder of the paper is as follows. First, I present some empirical facts regarding the behavior of secured lending during the Great Recession and use these facts to motivate the financial structure employed in the equilibrium model. After presenting the model and studying the dynamics to various shocks, I turn to the data to examine the validity of the model’s predictions. Here I compare the model’s implied correlations with those found in the data and estimate a series of vector autoregressions to further examine the empirical consistency of the equilibrium model. After ensuring the model produces features consistent with the data, I conclude with a host of unconventional policy experiments which shed light on how unconventional policies should be conducted to ensure maximum effectiveness.

2 Secured Lending and Liquidity During the Great Recession

In this section, I seek to highlight the behavior of secured lending preceding and following the 2008 Financial crisis. I show two key facts: (1) The amount of secured debt outstanding as a share of all credit is correlated with home prices and (2) the concentration of secured lending between complex and simple banks depends on home prices. These relationships motivate the debt contract I specify in the general equilibrium model below. In what follows I focus on relative effects by studying the share of secured debt to unsecured debt and the share of assets held between complex and simple banks. Studying level effects around the 2008 Financial Crisis can be misleading because many quantities and prices were rising in the years up to the crisis and falling thereafter. By studying the relative effects, I am able to focus on movements above and beyond those of a base quantity.

2.1 Secured Debt Issuance is Correlated with Real Home Prices

I provide two examples of the first point. I plot the outstanding amount of asset-backed commercial paper (ABCP), relative to all commercial paper, against real home prices in Figure 1a. This market has received a great deal of attention in the wake of the financial crisis due to the fact that outstanding ABCP quickly outpaced unsecured commercial paper while home prices were rising, but the market for commercial paper essentially evaporated when home prices began their descent. In fact, the sample correlation between the share of ABCP outstanding relative to all commercial paper and real home prices is 73%. More
generally focusing on secured debt markets, I plot the amount of secured debt outstanding relative to the liabilities of non-financial firms in Figure 1b. This shows the expansion and contraction in secured lending around the 2008 Financial Crisis extends to secured lending more generally. In fact, the correlation of real home prices and this share of secured lending is 95%.

Figure 1: Real home prices (normalized to pre-recession peak) and the relative quantities of secured debt.

This reduced form analysis suggests that although credit contracted during the Great Recession, there was also a compositional change in the type of debt being issued. When home prices were rising secured debt issuance was growing and when home prices fell so too did the amounts of secured debt. I implement this feature in the equilibrium model below by specifying a debt-contracting problem that makes borrowers prefer to issue debt secured by a fixed quantity of collateral. However, the amount of secured debt they can issue depends on the market value of this collateral. To link secured debt to home prices, I assume this collateral is in the form of housing. Therefore, home prices and secured debt issuance are mechanically linked in the model.

Furthermore, the debt-contract in the model implies that as borrowers make the relative shift towards unsecured debt, finance premiums rise and, in turn, default rates increase. Although the details of this mechanism will be more fully fleshed out in the remainder of the paper, this creates an adverse feedback which amplifies the severity of downturns within the current period and propagates their effects into the future – as illustrated in Figure 2.
2.2 Secured Lending Concentration in Complex Banks is Correlated with Real Home Prices

In addition to the compositional effects on the demand side of credit, in Figure 3 I show there was also a compositional change amongst suppliers of credit around the 2008 financial crisis. To study the differential behavior across banks, I partition the set of commercial banks by their complexity. Inherently, I assume the most complex banks are the largest commercial banks. Two factors motivate me to make this classification. First, large commercial banks have trading desks within their firms which actively take short and long positions on various contracts at the will of the market. These broker-dealers act as market makers for securities and derivatives that are inherently complicated. In fact, the Office of the Comptroller of the currency, which is tasked with regulating derivatives markets acknowledges the absolute and comparative advantage in derivatives and securities markets these firms have over their smaller counterparts:

...because the highly specialized business of structuring, trading, and managing the full array of risks in a portfolio of derivatives transactions requires sophisticated tools and expertise, derivatives activity is appropriately concentrated in those few institutions that have made the resource commitment to be able to operate the business in a safe and sound manner. Typically, only the largest institutions have the resources, both in personnel and technology, to support the requisite risk management infrastructure. (OCC, 1998-2012)

The second reason for this partition of banks is motivated from a policy analysis standpoint. The largest banks were the focus of the initial round of equity injections in known as TARP. Of the initial $115 Billion dollars in equity injections paid out on October 28, 2008, these banks received $100 Billion. Therefore, understanding the behavior of these
large financial firms is central to better understanding the effectiveness of the TARP equity injections.

Figure 3a plots the share of credit exposure held by: J.P. Morgan Chase, Bank of America, HSBC, Citi Bank and Wells Fargo relative to all commercial banks. This measure of credit exposure, all other things equal, would increase in times of financial turmoil since losses from default would rise. However, this was not the case for the largest financial firms. In fact, these institutions were actually decreasing their credit risk exposure when home prices began falling. I interpret this as evidence that these large and complex banks, who serve as market makers for many financial products, are exiting the market during times of financial turmoil. Adrian and Shin (2013) study the behavior of large commercial and investment banks over the last 15 years. They reach a similar conclusion, “...intermediaries are shredding risk and withdrawing credit precisely when the financial system is under the most stress, thereby serving to amplify the downturn”. Figure 3a shows this statement applies more specifically to the housing market, as the correlation between the share of credit exposure held by these large dealer banks and home prices is 89%.

The behavior of the distribution of credit exposure and home prices suggests a linkage between market based funding and home prices. One connection is the collateralized nature of derivatives contracts. The International Capital Market Association notes that, prior to the financial crisis, interest-rate contracts were often collateralized by mortgage backed securities (ICMA, 2013). However, direct data on the types of collateral posted for the contracts is not readily available. Therefore, I turn to a more direct measure of lending secured by real
estate. In Figure 3b, I show a similar redistribution occurred using data from the Federal Reserve’s Y9-C series, loans secured by real estate. This data series includes residential loans secured by real estate such as home-equity lines of credit and retail mortgages, in addition to commercial mortgages and mortgage-backed securities holdings. Therefore, it is a noisier measure of credit extended to firms and includes loans that larger complex banks would not necessarily be more efficient in handling. However, the same general pattern holds as the share of loans secured by real estate held by the largest banks and real home prices is correlated at 65%.

To implement this redistribution across banks into the general equilibrium model, I follow an approach similar in spirit to the heterogeneous investor model of Adrian and Shin (2010). In particular, I specify the complex banks in the model as having an efficiency advantage in terms of liquidating collateral upon default relative to simple banks. This advantage makes the complex banks naturally the dominant players in the secured lending market. However, these banks face an moral hazard problem which limits their leverage in these markets. When home prices fall, this endogenously worsens their moral hazard problem, forcing them to deleverage leaving the simple banks to absorb the collateralized lending.

![Diagram](image)

Figure 4: When complex banks reduce their secured lending activity, a positive feedback loop of fire sales and large complex banks deleveraging is initiated.

Although the reduced form analysis carried out in this section can say little about the implications of this redistribution, the equilibrium model shows the bank heterogeneity coupled with the redistribution generates a fire sale effect. As the simple banks absorb the shortfall in secured lending created by the deleveraging of the complex banks, the value of the borrowers collateral decreases as the simple banks pass on the higher marginal cost of expected liquidations. This drop in collateral values leads to higher finance premiums and default rates which serves to further tighten the moral hazard constraint of the complex banks. This decrease in the leverage of complex banks creates an illiquidity effect in the secured lending market and therefore serves to amplify the fall in home prices and rise in finance premiums, ultimately leading to less production. However, the role of unconventional monetary policy becomes clear. If the central bank steps in to purchase the shortfall in the demand for secured debt created by the complex banks deleveraging, then the illiquidity effect can be reversed. Moreover, injecting equity into the complex banks prevents them
from withdrawing credit, creating a host of unconventional policy options for the central bank in response to the financial amplification. The next section spells out the equilibrium model in detail.

3 A DSGE Model with Integrated Housing and Financial Markets

The model features a standard infinitely-lived household and perfectly competitive housing producing firms. However, the goods producing firms are noticeably different from standard RBC models. In particular, short-lived entrepreneurs are the only agents that have access to a production technology which requires capital and labor. The timing of the model is such that capital must be purchased in period $t$ to be used in the production process in period $t+1$. Since these agents only live for two periods, and I assume they have no first period endowment, they must borrow (secured and unsecured) funds to purchase capital which is used in an uncertain investment project. Although the household’s are the ultimate supplier of the funds for the entrepreneurs, I assume household’s deposit funds in a monopolistically competitive banking sector which ultimately loans the funds to entrepreneurs for their investment projects. Therefore, banks exist in the model because they possess the monitoring and liquidation technologies necessary to provide loans to entrepreneurs. Although the banks pay identical deposit rates set by the central bank and have identical monitoring technologies, they differ in terms of their liquidation technologies. This bank heterogeneity interacts with the entrepreneur’s debt contract to generate large amplification effects and a transmission mechanism for nontraditional monetary policy. In what follows, all lower case variables are real and all uppercase variables are nominal.

3.1 Household

The household earns wages by renting labor to goods producers $l_t$ and home builders $l_h$. Additionally, they earn non-labor income from banks in the form of dividends $div_t$ and principal plus interest payments $d_{t-1}^d r_{t-1}^d$ on last periods deposits. The monetary authority may transfer any revenue back to the household in lump-sum form via $\tau_t$. This income can be saved in the form of bank deposits $d_t$ or spent on consumption $c_t$ and housing $p_h h_t$. Also, any non-depreciated housing stock, $(1 - \delta_h) h_{t-1}$, can be resold at the nominal market price $p_h$. The resulting budget constraint in any period $t = 0, 1, 2, ...$ is given by,

$$c_t + p_h^t [h_t - (1 - \delta_h) h_{t-1}] + d_t \leq w_t l_t + w^h_t l_h^t + d_{t-1}^r r_{t-1}^d + div_t + \tau_t.$$

Although I follow Iacoviello (2005) and Iacoviello and Neri (2010) by modeling a housing market, I do not include home equity borrowing constraints on the household side. Where they focus on the wealth effects of housing price changes in terms of financing consumption, I focus more so on real-estate as collateral for investment and the interaction of house prices, secured debt and the distribution of credit supplied between banks. That being said, in general equilibrium changes in home prices can impact consumption via traditional income
and substitution effects, but not through a home equity channel. Adding this channel would only strengthen the effects from the secured commercial debt I model in this paper.

The household maximizes their lifetime expected utility subject to the flow budget constraint above. The household’s lifetime expected utility is specified by

\[ U = \sum_{i=0}^{\infty} \beta^i \mathbb{E}_t \left\{ \ln (ct+i) + \eta^h_{t+i} \ln (ht+i) + \eta^l_l \ln (1 - lt+i) + \eta^l_h \ln (1 - lh_{t+i}) \right\} , \]

where \( \eta^h_h \) represents a shift in the elasticity of demand for housing. I specify this as an exogenous process following a first order auto-regressive process, in line with Iacoviello and Neri (2010).

\[ \ln (\eta^h_h) = (1 - \rho_{\eta^h_h}) \bar{\eta} + \rho_{\eta^h_h} \ln (\eta^h_h_{t-1}) + \epsilon^\eta_{t+1} \quad \epsilon^\eta_{t} \sim \mathcal{N}(0, \sigma_{\eta^h_h}) \quad (1) \]

3.2 Goods Production

The goods producing sector is comprised of a continuum of entrepreneurs. Entrepreneurs have limited resources to finance capital required to produce the final good so they must borrow from banks. A financial friction arises whereby entrepreneurs borrow funds from banks this period to purchase capital used in production next period. Their output next period is subject to idiosyncratic productivity disturbances only observable by banks after paying a monitoring cost.

Entrepreneur’s Debt Contract: The Demand Side Financial Friction

A continuum of entrepreneurs \( j \in \mathbb{R}_+ \) supply wholesale goods to retailers using capital and labor. Entrepreneurs only live for 2 periods and only care about their second period utility. In the first period they have no endowment and no technology but they have a unit of labor supply. In the second period of their lives they are endowed with 1 unit of an asset which can be narrowly thought of as land, \( \bar{n} \), which can be transformed into housing only by entrepreneurs and banks. However, capital must be purchased this period to be useful tomorrow. Denote the cost of capital purchased in period \( t \) by entrepreneur \( j \) by \( k^g_j \). To purchase this capital the entrepreneur will receive financing from the banking sector. More specifically, the entrepreneur uses wages earned today \( w^e_t \) and pledges tomorrows endowment \( \bar{n}^j \) as collateral for a secured loan in the amount \( p^s_i \bar{n}^j \) and the remaining portion of the capital purchase is financed with an unsecured loan in the amount \( b^u_t \). More concretely,

\[ k^g_j = p^s_i \bar{n}^j + b^u_t + w^e_t \quad (2) \]

is entrepreneur \( j \)’s balance sheet constraint.

Without default, distinguishing between secured and unsecured loans is trivial. However, in the second period of their life, entrepreneurs are subjected to an idiosyncratic productivity shock \( \omega^j_{t+1} \) which is i.i.d. across entrepreneurs and time. I assume throughout the analysis in
This paper, \( \omega_{t+1} \sim \ln\mathcal{N}\left(\frac{-(\sigma^\omega)^2}{2}, \sigma^\omega_t\right) \) with CDF at time \( t \) denoted by \( F_t(\omega_{t+1}) \). The choice of parameters implies \( \mathbb{E}\{\omega_{t+1}\} = 1 \) so that in the aggregate this idiosyncratic shock has no direct impact on production, but the existence of uncertainty at the firm level impacts aggregate output through financial imperfections (BGG). To capture exogenous increases in the cross-sectional dispersion of idiosyncratic productivity shocks I allow \( \sigma^\omega_t \) to vary over time. I posit the simple auto-regressive process,

\[
\ln(\sigma^\omega_t) = (1 - \rho_{\sigma^\omega}) \sigma^\omega_{t-1} + \rho_{\sigma^\omega} \ln(\sigma^\omega_{t-1}) + \varepsilon_t^{\sigma^\omega} \quad \varepsilon_t^{\sigma^\omega} \sim \mathcal{N}(0, \sigma_{\sigma^\omega})
\]

for this demand-side risk shock. Christiano, Motto, and Rostagno (2013) show that such shocks have played a significant role in shaping the U.S. business cycle. Moreover, these shocks prove useful in the empirical analysis of the paper as they provide a structural interpretation for exogenous increase in the external finance premium.

Since projects are financed before the idiosyncratic productivity shock can be observed by either the entrepreneur or the bank, entrepreneurs who receive a low productivity value will default upon their loan. Denote the real gross interest rate on unsecured loans by \( r_{b,j}^t \) and denote the gross real return on capital common to all entrepreneurs by \( r_{k}^{t+1} \). Then for any entrepreneur \( j \), we can define the cut-off value of \( \bar{\omega}_j \) by the equation

\[
\bar{\omega}_j r_{k}^{t+1} = b_j r_{b,j}^t.
\]

This equation defines the minimum level of productivity needed to pay back the unsecured loan. Implicit is the assumption that unsecured debt is senior to secured debt, allowing the debt contract to maintain the form and properties inherent in the debt contract within Bernanke et al. (1999). For entrepreneur \( j \), the loan will be repaid if \( \omega_j \geq \bar{\omega}_j \) and will otherwise be defaulted upon. However, the bank can not observe the level of productivity without paying an auditing cost in proportion \( \mu^a \in (0,1) \) to the entrepreneur’s revenue.\(^2\) Banks who do not pay for auditing never find out if the entrepreneur actually received a low productivity draw or if they simply chose to renege on their loan. Given this arrangement, the optimal debt-contract (without stochastic monitoring) dictates that banks will audit only defaulting entrepreneurs and only entrepreneurs who receive a bad-draw will default on their loans.

To make matters more explicit I define the expected revenue to the bank for loaning \( b_j^t \) to entrepreneur \( j \) in (5). This expected revenue is comprised of 2 terms, the first of which is non-defaulting loan revenue and the second is revenue net of auditing costs on non-performing loans.

\[
\text{Payment to Bank} = \left(1 - F_t(\bar{\omega}_j)ight) b_j^t r_{b,j}^t + \left(1 - \mu^a\right) \int_0^{\bar{\omega}_j} \omega_j dF_t(\omega_j) r_{k}^{t+1} k_j^t
\]

\(^2\)This follows from Townsend (1979), but has been popularized in this context by Carlstrom and Fuerst (1997) and Bernanke et al. (1999).
For the bank to be willing to make this loan, this expected pay-off must be at least equal to bank’s cost of making the loan. In Bernanke et al. (1999) the cost of making the loan is simply the cost of obtaining the funds via deposits - \( r^c_t = r^d_t \). However since the banking sector in this model has market power this is no longer the case. In the appendix, I show how the rate charged to entrepreneurs \( r^c_t \) is related to the bank’s marginal cost and mark-up there over using the approach laid out in Hafstead and Smith (2012). Then the individual rationality constraint of the bank can therefore be expressed as,

\[
b_t^r t^e_t = (1 - F_t(\omega_{t+1}^j)) b_t^r t^b_t + (1 - \mu^a) \int_0^{\omega_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) r^k_{t+1} k^j_t. \tag{6}
\]

We can simplify this expression (and the resulting entrepreneur’s optimization problem) by defining the following terms. First let \( G_t(\omega_{t+1}^j) \) be defined as the expected productivity value for defaulting entrepreneurs.

\[
G_t(\omega_{t+1}^j) = \int_0^{\omega_{t+1}^j} \omega_{t+1}^j dF_t(\omega_{t+1}^j) \tag{7}
\]

Also let \( \Gamma_t(\omega_{t+1}^j) \) be defined as the expected share of entrepreneurial profits going to the bank gross of auditing costs.

\[
\Gamma_t(\omega_{t+1}^j) = (1 - F_t(\omega_{t+1}^j)) \omega_{t+1}^j + G_t(\omega_{t+1}^j) \tag{8}
\]

Now I can combine (6) with (4), (7) and (8) to rewrite the bank’s individual rationality as,

\[
b_t^r t^e_t = (\Gamma_t(\omega_{t+1}^j) - \mu^a G_t(\omega_{t+1}^j)) r^k_{t+1} k^j_t. \tag{9}
\]

We can now formally state the problem faced by entrepreneur \( j \). To keep the debt-contract tractable, I assume the entrepreneur is risk-neutral with regards to aggregate consumption. In particular, I assume they seek to maximize their income and then allocate that income between consumption and housing services. Entrepreneur \( j \) therefore seeks to maximize total income\(^3\) subject to the bank’s individual rationality constraint.

\[
\max_{k^j_t, \omega_{t+1}^j} \quad (1 - \Gamma_t(\omega_{t+1}^j)) r^k_{t+1} k^j_t
\]

subject to

\[
[k^j_t - p_t\bar{n}^j - w^c_t] r^c_t = (\Gamma_t(\omega_{t+1}^j) - \mu^a G_t(\omega_{t+1}^j)) r^k_{t+1} k^j_t
\]

The solution to this optimization problem pins down the cut-off value \( \omega_{t+1}^j \) and the entrepreneur’s demand for capital \( k^j_t \).\(^4\) The problem is identical in nature to the problem entrepreneurs face in Bernanke et al. (1999) who show the optimal debt contract has the property that the default rate and external finance premium move inversely with net-worth. In this model, the net-worth component is replaced with the value of collateral, implying that \( \partial \omega_{t+1}^j / \partial p_t^c < 0 \) - finance premiums and default rates will move in the opposite direction of collateral prices.

\(^3\)Notice the entrepreneur’s income can be re-written as \( \int_{\omega_{t+1}^j}^{\infty} \omega_{t+1}^j dF(\omega_{t+1}^j) r^k_{t+1} k^j_t - (1 - F_t(\omega_{t+1}^j)) r^c_t b_t^r = (1 - \Gamma_t(\omega_{t+1}^j)) r^k_{t+1} k^j_t \) where I use (4) and (8).

\(^4\)The first order conditions for this problem are in the appendix.
Aggregate Goods Production

The previous section describes the firm-level behavior in the goods producing sector, specifically it describes the debt-contract problem faced by each producer. In this section, I describe the industry wide behavior. In what follows, I assume the existence of a retail sector which is monopolistically and purchases inputs from entrepreneurs. The existence of these retailers implies the entrepreneurs and laborers receive a gross markdown on their capital returns and wages. However, I do not assume retailers face any type of pricing friction as to keep the focus on the financial friction. Hence, modeling this sector simply allows me to align my calibration of entrepreneur leverage and auditing cost closely with Bernanke et al. (1999) and Christiano et al. (2013) who also model a retailer with market power.

Each entrepreneur (in the second period of their life) has access to the production technology,

\[ y_j^t = \omega_j^t z_t (k_{j,t-1})^{\alpha_g} (l_{g,j}^t)^{1-\alpha_g}, \]

which can be aggregated over due to constant returns to scale. The aggregate goods production technology in any given period \( t \) is specified as:

\[ y_t = z_t (k_{t-1})^{\alpha_g} (l_{g,t}^t)^{1-\alpha_g}, \tag{10} \]

where \( z_t \) is an exogenous technology process which affects all entrepreneurs equally. I assume this technology follows a first-order autoregressive process.

\[ \ln (z_t) = (1 - \rho) z + \rho \ln (z_{t-1}) + \varepsilon_t^z \]

\[ \varepsilon_t^z \sim \mathcal{N}(0, \sigma_z) \tag{11} \]

The gross real nominal return on holding a unit of capital from period \( t-1 \) to period \( t \) is defined by

\[ r_t^k = \alpha_g \frac{y_t}{k_{t-1}} \frac{1}{\mu^r} + (1 - \delta)^k, \tag{12} \]

where I utilize the aggregate marginal product of capital from the Cobb-Douglas specification above - \( mp_k = \alpha_g \frac{y_t}{k_{t-1}} \) and \( \mu^r \) is the retailers gross markdown on the entrepreneur’s goods.

The labor aggregate in the production function is a composite of labor supplied by the household, \( l_t \), and labor supplied by this period’s young entrepreneurs, \( l_{e,t}^t \),

\[ l_t^g = (l_{e,t}^t)^{\alpha_e} (l_t)^{1-\alpha_e}. \tag{13} \]

This implies the wage paid to the household’s labor and the wage paid to entrepreneurial labor are given by,

\[ w_t = (1 - \alpha_g)(1 - \alpha_e) \frac{y_t}{l_t} \frac{1}{\mu^r} \tag{14} \]

\[ w_{e,t} = (1 - \alpha_g)\alpha_e \frac{y_t}{l_t} \frac{1}{\mu^r} \tag{15} \]

I calibrate \( \alpha_e = 0.01 \) so that in equilibrium the household receives the majority of wages and variations in collateral values are the primary sources of movement in entrepreneur’s balance.
The aggregate income of entrepreneurs in period $t$ is $(1 - \Gamma_{t-1}(\hat{\omega}_t)) r^k_t k_{t-1}$. I assume entrepreneurs, like the household, receive utility from consuming both the consumption good and housing services. Unlike households, entrepreneurs have an endowment of non-tradeable housing - $\bar{n}^j$. Recall however, this endowment was leveraged last period to secure a loan in the amount $p^n_{t-1} \bar{n}^j$. Hence, entrepreneurs who are able, choose to payback the secured loan with interest $p^n_{t-1} \bar{n}^j r^d_{t-1}$ and then convert $\bar{n}^j$ into housing services one for one. If they don’t payback the secured loan then they default on this contract and the bank takes possession of the collateral $\bar{n}^j$.

I assume in the aggregate, all the entrepreneurs who did not default on their unsecured loan, payback their secured loan and use the rest of their income on the consumption good. Those who defaulted on the unsecured loan have lost all income and hence do not consume anything. More specifically, the aggregate real consumption of entrepreneurs is given by

$$c^e_t = (1 - \Gamma_{t-1}(\hat{\omega}_t)) r^k_t k_{t-1} - (1 - F_{t-1}(\hat{\omega}_t)) p^n_{t-1} \bar{n}^j r^d_{t-1}.$$  \hfill (16)

“What micro-level preferences would give rise to this aggregate consumption behavior?” is an interesting question. In the appendix I describe one possible micro-structure that would lead to this aggregate consumption behavior. An appealing aspect of this description is the existence of a single default rate in the economy.\footnote{That is to say, the default rate on unsecured loans is the same as the default rate on secured loans. I choose this as a starting point although this assumption can be relaxed.}

### 3.3 New Housing Production

I assume new housing is produced in a purely competitive market and free from financial frictions. In particular, housing producers combine labor $l^h_t$ with housing specific technology, $z^h$ in the production technology,

$$h^\text{new}_t = z^h (l^h_t)^{1-\alpha_h}.$$  \hfill (17)

I model housing specific technology independent of technology in the goods producing sector since much of the economic growth over the last two decades has been IT-driven and housing production is a non IT-intensive industry. Moreover, this specification allows for goods technology process, $z_t$, to play a significant role in determining output without implying a counterfactual negative correlation between home prices and GDP (see for example Davis and Heathcote (2005)). The resulting demand for labor from the housing sector takes the form

$$w^h_t = (1 - \alpha_h) p^h_t h^\text{new}_t h^h_t.$$  \hfill (18)

### 3.4 Banking Sector: The Supply Side Financial Friction

There are a unit measure of banks in the model each belonging to one of two types. I distinguish bank types by a superscript ‘c’ for complex banks and a superscript ‘s’ for...
simple banks (who make up $\nu$ share of the population). Complex banks represent the large commercial banks in the data. These banks are more productive with repossessed collateral pledged by entrepreneurs to secure loans and hence place a relatively higher value on the collateral relative to their simple counterparts. However, this efficiency creates a moral hazard problem for borrowers due to the possibility of complex banks prematurely repossessing collateral and absconding with the profits. If this occurs, the entrepreneur’s are dependent on the government to fine the complex banks, seizing a portion of their collateral. To this extent, bank capital mitigates the moral hazard concerns and allows the complex banks to hold more collateralized loans. An amplification effect emerges from the endogenous tightening and loosening of this moral hazard constraint which forces complex banks to adjust their holding of collateralized assets in response to movements in home prices.

For ease of exposition I describe factors common to both types of banks before describing each type’s optimizing behavior. In particular, all banks have some degree of market power, face a balance sheet constraint and remit a fraction of profits each period back to the household in the form of dividends. In what follows I generically refer to bank $i$ to reference one of the infinitely many identical banks within either type.

Each bank possesses a degree of market power which is captured by assuming a Dixit-Stiglitz type aggregator function. As Hafstead and Smith (2012) point out, this has the simplifying feature that all banks serve all entrepreneurs and therefore face the same ex-ante and ex-post default rates. More specifically, aggregate loans are a CES index

$$b_t = \left( \int_0^1 b_t(i) \frac{\theta_b-1}{\theta_b} \right)^{1/\theta_b-1}$$

(19)

where $\theta_b$ is the elasticity of substitution between different bank loans and is calibrated to match aggregate lending rates. The corresponding price index which is dual to this quantity index is given by

$$r_t^b = \left( \int_0^1 r_t^b(i)^{1-\theta_b} \right)^{1/(1-\theta_b)}.$$  

(20)

This specification of the aggregate indexes implies each bank $i$ of type $\zeta \in \{c, s\}$ faces the downward sloping demand for loans,

$$b_t^\zeta(i) = \left( \frac{r_t^{b,\zeta}(i)}{r_t^b} \right)^{-\theta_b} b_t.$$  

(21)

Each bank $i$ must not only satisfy their demand for loans, but they must also abide to the balance sheet constraint,

$$b_t^\zeta(i) + p_t^\zeta n_t^\zeta(i) = d_t^\zeta(i) + \frac{bh_{t-1}^\zeta(i)}{\pi_t}$$

(22)

which simply states that assets (bank loans) must equal liabilities (bank deposits) plus bank capital, respectively.
To summarize this, banks of type $ζ$ pay out a time varying fraction $γ_t^ζ$ of period $t$ profits as dividends, invest the remaining fraction in bank capital and lose a fraction of bank capital to depreciation. Following Gerali, Neri, Sessa, and Signoretti (2010), I assume bank investment decisions are made independently from bank profit maximization, however, I assume this fraction is time varying. In particular, I assume that $γ_t^ζ = \frac{\gamma_t^ζ}{π_t}$ so that each period a constant amount of new equity is injected into the banks from shareholders. This implies realistically that banks respond to falling profits by paying out a smaller share of profits in dividends. Before proceeding to a specific description of each type of bank’s problem, it useful to summarize these transfers by defining real investment in the banking sector:

$$\nu_t^ζ = ν\bar{γ}^ζ + (1 − ν)\bar{γ}^c.$$  (24)

**Complex Bank**

The complex bank enters each period $t$ with inflows consisting of maturing unsecured loans $r_{t-1}^b(i)b_{t-1}^c(i)$ and maturing secured loans $p_{t-1}^n n_{t-1} d_{t-1}^d$, of which, $(1 − F_{t-1}(\bar{ω}_t))$ will be repaid in full. Denote the real income of all borrowers who are unable to repay last periods loan by $φ_{t-1}(\bar{ω}_t)$. The complex bank will receive the fraction $b_{t-1}^c(i)$ of $φ_{t-1}(\bar{ω}_t)$ net of auditing costs $μ^a$ for the unsecured loan defaults. Additionally, the complex bank repossesses $F_{t-1}(\bar{ω}_t)n_{t-1}^c(i)$, which is the collateral posted on the secured loans who defaulted. This repossessed collateral is transformed in to housing using the technology common to all banks, $h_t^c(i) = z^*n_{t-1}^c(i)$. Finally, the complex bank also has incoming deposits totaling $d_{t-1}^r(i)$. At the same time, the productive bank has outflows of newly originated unsecured and secured loans totaling $b_t(i) + p_t^n n_t^c(i)$ and maturing deposits from period $t − 1$ totaling $d_{t-1}^r(i)r_{t-1}^d$. This is stated more concisely below in (27) which defines the complex bank’s period $t$ nominal profits.

$$\Pi_t^c(i) = (1 − F_{t-1}(\bar{ω}_t)) r_{t-1}^{b,c} b_{t-1}^{c-1}(i) + (1 − μ^a) \frac{b_{t-1}^c(i)}{b_{t-1}} φ_{t-1}(\bar{ω}_t)$$
$$+ (1 − F_{t-1}(\bar{ω}_t)) p_{t-1}^n n_{t-1}^c(i) r_{t-1}^d + F_{t-1}(\bar{ω}_t)p_t^h z^*n_{t-1}^c(i)$$
$$− b_t^c(i) − p_t^n n_t^c(i) − d_{t-1}^r(i) r_{t-1}^d + d_t^r(i).$$  (25)

The productive bank’s ability to liquidate repossessed collateral, $n_t^c$, at zero marginal cost raises a moral hazard problem. In particular, if the productive bank were to prematurely claim default on all the secured loans originated in period $t$ and repossess the collateral the following period, they would earn a gross real return totaling $n_t^cE_t \{ z^* p_{t+1}^h − (1 − F_t(\bar{ω}_{t+1}) r_t^d p_t^h) \}$.  

I assume that raising bank capital is expensive to the extent that adjusting the amount of capital raised is infinitely costly for banks. Banks pay the profits remaining after replenishing their capital stock out as dividends. Therefore, bank capital evolves according to the following law of motion.

$$bk_t^ζ(i) = γ_t^ζ Π_t^ζ(i) + (1 − δ^{bk}) \frac{bk_{t-1}^ζ(i)}{π_t}$$  (23)

To summarize this, banks of type $ζ$ pay out a time varying fraction $γ_t^ζ$ of period $t$ profits as dividends, invest the remaining fraction in bank capital and lose a fraction of bank capital to depreciation. Following Gerali, Neri, Sessa, and Signoretti (2010), I assume bank investment decisions are made independently from bank profit maximization, however, I assume this fraction is time varying. In particular, I assume that $γ_t^ζ = \frac{\gamma_t^ζ}{π_t}$ so that each period a constant amount of new equity is injected into the banks from shareholders. This implies realistically that banks respond to falling profits by paying out a smaller share of profits in dividends.
The first term represents income from the selling the unlawfully repossessed collateral and the second term subtracts the foregone income that would have been received from entrepreneurs paying back their loans.

If the productive bank chooses to abscond with the assets, I assume as in Kiyotaki and Moore (1997) that entrepreneurs themselves have imperfect enforcement over the contract and instead rely on the monetary authority to fine the complex banks, transferring a portion of their capital to entrepreneurs. I make the non-trivial assumption the amount of capital the monetary authority takes from the bank is proportional to the value of collateral. The idea being that if the value of the stolen property is low, the fines are smaller and if the value of the stolen property is higher the fines are larger. This conveniently captures the observed behavior of regulators when dealing with the illegal activities of complex banks. When asset prices are low and the economy is contracting, regulators can not credibly commit to severely fine, and hence de-capitalize large banks, instead they actually inject capital into these firms at times of distress. Regulators wait until the economy and asset prices have recovered to level large fines on large complex banks for their misbehavior. Thus, the incentive for complex banks to prematurely liquidate their borrower’s collateral is eliminated when the fines exceeds the gross return on premature liquidations. If this is not the case, then complex banks have the incentive to prematurely liquidate the borrower’s collateral and therefore borrowers will only take secured loans from complex banks up to the point where the incentive to illegally liquidate their collateral is eliminated.

\[ \frac{p_t^c b_t^c(i)}{G} \geq n_t^c \mathbb{E}_t \left\{ z r_t^h p_{t+1}^h - (1 - F_t(\omega_{t+1}) r_t^d n_t^d) \right\} \]  

(26)

When (26) holds with equality, the complex bank will be limited in how many secured loans it can make. Moreover, this constraint will endogenously loosen and tighten in response to various macroeconomic shocks which affect home prices or default rates. Let \( \Lambda_t \) denote the household’s stochastic discount factor used for valuing future real payments. The problem faced by the productive bank is then defined below:

\[
\max_{\{r_t^c, b_t^c, n_t^c, d_t^c\}} \sum_{j=0}^{\infty} \mathbb{E}_t \{ \Lambda_{t+j} \Pi_t^c(i) \} \quad \text{subject to} \quad (21), (22), (26).
\]

Due to the complications that arise from solving a model with an occasionally binding constraint, I calibrate the model so that the complex bank’s moral hazard constraint binds in the non-stochastic steady-state.

Simple Bank

The simple bank is identical to the productive bank with one noticeable exception - they incur an increasing marginal cost when repossessing collateral. To make matters more concrete, when a secured loan defaults the unproductive bank repossesses collateral $F_{t-1}(\bar{\omega}_t)n_{t-1}^s(i)$. Unlike their complex counterparts, the simple bank liquidates this collateral while bearing an increasing marginal cost. On defaulted secured loans, the simple bank transforms repossessed collateral into housing yielding revenue $p_n^s t(i)$ at a resource cost of $\mu^{l,s} (n_{t-1}^s(i))^{\chi^{l,s}}$. This captures the heterogeneity between commercial banks (empirically found by (Wheelock and Wilson, 2012; Bos and Kolari, 2013)) with regards to their ability to evaluate and trade complex assets. Most notably, as simple banks increase their holding of secured loans, the value of the collateral will fall due to the increasing marginal cost. Hence, the market liquidity of collateral depends on the distribution of these assets (i.e. it depends on who is holding the assets). A point first made by Kiyotaki and Moore (1997) and applied to here to housing secured credit.

With this exception, the simple bank’s profit function is very similar to the complex bank’s:

$$\Pi^s_t(i) = (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^b s(i) b_{t-1}^s(i) + (1 - \mu^{b,s}) b_{t-1}^s(i) \phi_{t-1}(\bar{\omega}_t)$$

$$+ (1 - F_{t-1}(\bar{\omega}_t)) p_n^s t(i) r_{t-1}^d$$

$$+ F_{t-1}(\bar{\omega}_t) \left[ p^h_z n_{t-1}^s(i) - \mu^{l,s} (n_{t-1}^s(i))^{\chi^{l,s}} \right]$$

$$- b^s_t(i) - \chi^{b,s} b^s_t(i) - p_n^s t(i) - d_{t-1}^s(i) r_{t-1}^d + d^s_t(i)$$

Notice the lack of productivity spills over to unsecured loans. The parameter $\chi^{b,s}$ is calibrated to match the average share of resources allocated to financial intermediation. The increasing resource cost of repossessing collateral implies the simple bank is not subject to a moral hazard constraint. In particular, if any single simple bank $i$ attempted to issue a large amount of secured loans at a given market price $p_n^s$ and prematurely liquidate all of the collateral, their cost of liquidating the assets would quickly exceed what they paid for them. Therefore the existence of these increasing marginal cost eliminates any incentive to prematurely liquidate collateral.

Let $\Lambda_t$ denote the household’s stochastic discount factor used for valuing future real payments. The problem faced by the simple bank is then defined below:

$$\max_{\{r_{t+j}^b i(i), b_{t+j}^b(i), n_{t+j}^s(i), d_{t+j}^s(i)\}} \sum_{j=0}^{\infty} \mathbb{E}_t \left\{ \Lambda_{t+j} \Pi^s_{t+j}(i) \right\}$$

subject to \[(21)\] and \[(22)\].

3.5 Central Bank

The central bank is charged with setting a monetary policy rule. Although the economy features no nominal rigidities, inflation does not completely factor out of the model. In
particular, As for the monetary policy instrument I assume the central bank follows the simple interest rate rule whereby the gross nominal rate \( R_t^d = r_t^d \mathbb{E}_t \{ \pi_{t+1} \} \) on one-period deposits adjusts to itself lagged, the inflation rate, and the growth rate of real GDP:

\[
\left( \frac{R_t^d}{R_{t-1}^d} \right) = \left( \frac{R_{t-1}^d}{R_{t-1}^d} \right)^{\rho_r} \left( \frac{\pi_t}{\pi} \right)^{\psi_{\pi}} \left( \frac{gdp_t}{gdp_{t-1}} \right)^{\psi_{gdp}}.
\]  

(28)

3.6 Market Clearing

Sections 3.1 - 3.5 describe the optimal behavior of all agents in the economy. A symmetric competitive equilibrium is defined as a sequence of quantities, prices and Lagrange multipliers (shadow prices) which satisfy all optimality conditions, policy rules and market clearing conditions. In particular, the demand for housing must equate the supply of housing on the market which consists of newly built homes, repossessed collateral being liquidated on the housing market and non-depreciated housing from last period. Put more simply,

\[
h_t = h^\text{new}_t + F_{t-1}(\bar{\omega}_t)z^r (\nu n^s_{t-1} + (1 - \nu)n^c_{t-1}) + (1 - \delta^h)h_{t-1}.
\]  

(29)

The above expression can be further simplified by noting that the market for secured lending clears when

\[
\bar{n} = \nu n^s_t + (1 - \nu)n^c_t,
\]

(30)

where the left hand side is the aggregate endowment of entrepreneurs. By the ex-ante symmetry among entrepreneurs this is required to equal \( \bar{n} = \bar{n}_j \) for all entrepreneurs \( j \). Similarly, this ex-ante symmetry also implies the demand for capital by entrepreneurs is identical, or \( k_t = k^i_t \) for all entrepreneurs \( j \). I assume that capital can be transformed one for one from the final good and depreciates at rate \( \delta^k \). Therefore, capital evolves according to,

\[
k_t = i_t + (1 - \delta^k)k_{t-1}.
\]  

(31)

Since I do not include adjustment costs, the price of capital equals the price of the final good at all times. Adjustment costs in the production of capital could easily be added, as in Bernanke et al. (1999). However, in this model, they are not needed to generate an amplification effect. Instead, the asset price spirals occur from the redistribution of assets between banks. With this description of the model, the goods market clearing condition is satisfied whenever,

\[
y_t = c_t + c^s_t + i_t + i^b_t + \mu^s \phi_{t-1}(\bar{\omega}_t) + \nu \mu^s \left( n^s_{t-1}(i) \right) \chi^s + \nu \chi^b h^s_t ,
\]

(32)

which stipulates that the consumption good must be either consumed by the household or the entrepreneur, invested in bank capital, or used to audit or repossess the collateral of defaulting entrepreneurs. It is useful for the purpose of calibration and model inference to define real GDP in this multi-sector model:

\[
gdp_t = c_t + i_t + p^h_t h^\text{new}_t.
\]  

(33)
4 Model Calibration, Moments and Dynamics

This section examines the empirical properties of the equilibrium model by assigning parameter values so that the model mimics the behavior of the U.S. economy and financial sector. Although many of the model’s parameters can be assigned values to match the non-stochastic steady state to averages in the data, parameters governing the shock processes only affect higher order moments of the model. To calibrate these parameters I perform a moments matching exercise. The results of this exercise suggest the model can explain the second moments of finance premiums, home prices output and investment. This is noteworthy to the extent that the Bernanke et al. (1999) model struggles to match these higher-order moments, regardless of the calibrated strength of financial accelerator mechanism. In particular, strengthening the financial accelerator mechanism of Bernanke et al. (1999) aids the model’s supply shocks to match the data, but at the detriment of the empirical plausibility of the model’s demand shocks. This tension is highlighted by impulse response analysis.

4.1 Calibration

The model is calibrated to match characteristics of the U.S. economy from 1998 to 2011 and each time period is interpreted as one quarter. In order to numerically solve the model, there are 23 non-shock parameters and 15 shock parameters which must first be assigned values. Beginning with the household’s parameters I calibrate $\beta = .9875$ as to match up the steady state deposit rate in the model with the average rate on 3-Month U.S. Treasury Bills. I set the utility on non-housing leisure and housing leisure, $\eta^h = 4.6034$ and $\eta^l = 1.43$ which matches the share of labor supplied in housing equal to 5%, the U.S. average using data from the BLS and the total share of time spent working equal to $\frac{1}{3}$. Finally, the last of the preference parameters $\eta^t = .2219$ calibrates the steady state real price of housing so that consumption’s share of gdp, $c_{gdp} = .80$, which is the average ratio of personal consumption expenditures to personal consumption expenditures and private investment for the U.S. Similarly, setting $\delta^h = .0189$ implies the share of housing wealth to annual gdp, $\frac{p^h}{4 \times p_{gdp}} = 1.4$.

On the production side, I set the share of income going to labor in the goods producing sector, $1 - \alpha_g = .7$ and the same share in the housing sector $1 - \alpha_h = .8$ following Iacovello and Neri (2010). In order to imply a 20% markup of retail goods over the entrepreneur’s goods, I set $\mu^r = 1.2$, which is consistent with a CES over inputs of 6. I normalize the collateral endowment of entrepreneurs, $\bar{n} = 1$. As for the financial accelerator parameters, I collectively set $\mu^a = .135$ and $\sigma^\omega = .18$. The auditing cost parameter falls between the value from Christiano et al. (2013) and Bernanke et al. (1999) and the steady state value of the variance of the idiosyncratic productivity shock implies an annual steady state default rate of $F(\bar{w}) = .01$ which is the average default rate on C & I loans secured by real-estate using date obtained from the St. Louis Fed’s FRED database.

With regards to the banking sector, I normalize each bank’s share of the population to be equal by setting $\nu = \frac{1}{2}$. The depreciation rate on bank capital is set at $\delta^b = .04$ which
implies the leverage ratio of large banks is $14.7$ The value for $\nu \chi^{b,s} = 0.004954$ is obtained from Hafstead and Smith (2012) who create a time series of banking productivity in loan intermediation. I set the steady-state rate of return on entrepreneurial loans equal to the average prime loan rate, $r^e = 1.017$. This pins down the elasticity of substitution between bank loans, $\theta_b = 263.75.8$ There is little agreement over the real return on capital, I set it equal to equal to $10\%$ per annum which also matches the annualized return on small-cap stocks, representing firms who are likely to be financially constrained, using data from Morningstar.

I set $\bar{\gamma}^c = 0.0063$ which ensures the share of housing-secured assets held by the productive banks, $\frac{n^c}{n^c + n^s} = 0.92$, the average share of total credit exposure concentrated in large banks. Similarly, setting $\bar{\gamma}^s = 0.0261$ ensures the leverage ratio of unproductive banks is $0.3$, the average value in the data according to the Office of the Comptroller of the Currency’s (OCC) Quarterly Derivatives Report. Additionally, I set $\chi^{l,s} = 1.07$, implying strictly convex cost of repossessing/liquidating collateral for the unproductive bank. This together with setting $\mu^{l,s} = 0.0871$ calibrates the steady state price of the housing-secured assets so that $\frac{\nu n^p + \mu b t}{p^* n^s} = 2.5$, or the average ratio of C&I loans plus total credit exposure to total credit exposure. Finally, normalizing $z^r = 1$ and setting $z^h = 1.14$ implies the real-estate owned share, or REO share, $\frac{F(\omega)z^h n^c + F(\omega)z^h n^s}{F(\omega)z^r n^c + F(\omega)z^r n^s} = 0.1775$ which is the value in the data according to RealtyTrac. As for the policy parameters, steady state gross inflation is set equal to unity and $\rho_r = 0.85, \psi_\pi = 1.5, \psi_{gdp} = 0.125$.

The remaining shock parameters can not be pinned down by matching steady state values and therefore they are calibrated using a moments matching exercise. In particular, I choose $\rho_z = 0.9325, \sigma_z = 0.0260, \rho_{q^h} = 0.9899, \sigma_{q^h} = 0.0307, \rho_{\sigma^w} = 0.9015$ and $\sigma_{\sigma^w} = 0.0491$ to match the model’s covariance matrix and first-order autocorrelation of: the external finance premium (proxied by the spread between BAA corporate bond-rate and 10-year treasuries), real GDP$^9$, real private investment and real home prices. This exercise not only pins down values for the model’s driving shocks, but since the estimation strategy is over identified, it allows for an empirical examination of the model’s performance relative to the standard financial accelerator model.

### 4.2 Cyclical Properties and the Financial Accelerator

In this section I analyze the behavior of the following four variables in both the model and the data:

- **External Finance Premium** $efp_t = \mu^a \phi_{t-1} \frac{(\omega_t)}{b_{t-1}}$
- **Real GDP** $gdp_t$

---

7For a a point of reference, Gerali et al. (2010) set the depreciation rate on bank capital to 0.08.
8Hafstead and Smith (2012) find a similarly high value for $\theta_b = 260$.
9The standard deviation of housing supply shocks, when included, were consistently pushed to 0 in the moments matching exercise. Real GDP is measured as the model equivalent. Hence, I sum personal consumption expenditures and private investment and deflate the resulting series by the civilian population over the age of 16 and the personal consumption expenditures excluding food and energy price deflator.
- Real house prices = $p_t^h$
- Private Investment = $i_t + p_t^h h_{t}^{new}$.

Real GDP, real house prices and private investment are defined in the previous section detailing the equilibrium model. However, the external finance premium is not a standard feature of business cycle models and therefore deserves an explanation of how it emerges from the equilibrium model and how it is measured in the data.

The external finance premium is the interest rate premium charged to borrowers to compensate lenders for the agency cost associated with, in this model, the borrower having more information about the outcome of the investment project than the lender. In other words, agency cost manifests itself in this model through the cost of verifying the entrepreneur’s idiosyncratic productivity value. To see where the external finance premium appears in the model, consider the first order condition for either bank type, complex or simple, when choosing what loan rate to charge per dollar borrowed $b_{t-1}$ in period $t-1$:

$$\bar{r}_{t-1}^e = \left( \frac{\theta_b - 1}{\theta_b} \right) (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^b + \frac{(1 - \mu^a) \phi_{t-1}(\bar{\omega}_t)}{b_{t-1}}. \tag{34}$$

where $\bar{r}^e$ denotes the marginal cost of the the bank obtaining funds and making the loan, $r^b$ is the interest rate set by the bank on the loan and $\Phi_{t-1}(\bar{\omega}_t)$ are the real revenue recovered from defaulting entrepreneurs. A fraction, $\mu^a$, of this recovered revenue is spent on auditing cost which are thrown away (i.e. not transferred to anyone else in the economy) – these are the agency costs. Therefore, it is clear what the first order condition for a bank when choosing what loan rate to charge will look like without any agency costs:

$$\bar{r}_{t-1}^e = \left( \frac{\theta_b - 1}{\theta_b} \right) (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^b + \frac{\phi_{t-1}(\bar{\omega}_t)}{b_{t-1}}. \tag{35}$$

Therefore, the interest rate premium charged to borrowers to compensate lenders for the agency cost associated with auditing defaulting borrowers is:

$$efp_t = \mu^a \frac{\phi_{t-1}(\bar{\omega}_t)}{b_{t-1}}, \tag{36}$$

which is the external finance premium within the model. In the data I proxy this unobservable variable using the spread between BAA corporate debt and ten-year treasuries. This method follows the approach of Christiano et al. (2013) and Carlstrom, Fuerst, Ortiz, and Paustian (2012). The former citation verifies their results are consistent with the constructed finance premium measure of Gilchrist and Zakrajsek (2012) and the BAA-10 year treasury spread. The other variables are readily observable and their construction is defined in Section 6.2.

The model fits the data reasonably well with nearly all the moments in the confidence interval. Comparing this model to the baseline Bernanke et al. (1999) model augmented with a housing sector, it becomes clear why the traditional financial accelerator mechanism must
Table 1: Cyclical Properties of the Model

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>Bernanke et al. (1999)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma(\text{EFP}) )</td>
<td>1.16</td>
<td>1.08**</td>
<td>0.55</td>
</tr>
<tr>
<td>( \sigma(\text{GDP}) )</td>
<td>0.08</td>
<td>0.14</td>
<td>0.10**</td>
</tr>
<tr>
<td>( \sigma(\text{p}^h) )</td>
<td>0.18</td>
<td>0.15**</td>
<td>0.11</td>
</tr>
<tr>
<td>( \sigma(\text{Investment}) )</td>
<td>0.24</td>
<td>0.43</td>
<td>0.26**</td>
</tr>
<tr>
<td>( \rho(\text{EFP, GDP}) )</td>
<td>-0.48</td>
<td>-0.62**</td>
<td>0.26</td>
</tr>
<tr>
<td>( \rho(\text{EFP, p}^h) )</td>
<td>-0.72</td>
<td>-0.57**</td>
<td>0.28</td>
</tr>
<tr>
<td>( \rho(\text{EFP, Investment}) )</td>
<td>-0.55</td>
<td>-0.78*</td>
<td>0.07</td>
</tr>
<tr>
<td>( \rho(\text{GDP, p}^h) )</td>
<td>0.73</td>
<td>0.89*</td>
<td>0.86</td>
</tr>
<tr>
<td>( \rho(\text{GDP, Investment}) )</td>
<td>0.98</td>
<td>0.89</td>
<td>0.82</td>
</tr>
<tr>
<td>( \rho(\text{p}^h, \text{Investment}) )</td>
<td>0.75</td>
<td>0.71**</td>
<td>0.65</td>
</tr>
<tr>
<td>( \rho(\text{EFP}) )</td>
<td>0.93</td>
<td>0.92**</td>
<td>0.85</td>
</tr>
<tr>
<td>( \rho(\text{GDP}) )</td>
<td>0.93</td>
<td>0.95**</td>
<td>0.93**</td>
</tr>
<tr>
<td>( \rho(\text{p}^h) )</td>
<td>0.97</td>
<td>0.99*</td>
<td>0.98**</td>
</tr>
<tr>
<td>( \rho(\text{Investment}) )</td>
<td>0.91</td>
<td>0.91**</td>
<td>0.81</td>
</tr>
</tbody>
</table>

The data correlations and confidence intervals are computed using a non-informative prior on an estimated VAR(2).

** implies the moment is within the 68% posterior coverage area.

* implies the moment is within the 90% posterior coverage area.

be adjusted to analyze the financial crisis. The Bernanke et al. (1999) financial contract assumes the borrower's wealth is liquid, therefore (real-estate) secured debt is absent in the model. The assumption in this model of secured lending, with a fixed amount of collateral and heterogeneous lenders, generates a market illiquidity effect during economic downturns, as originally highlighted in Kiyotaki and Moore (1997), which implies finance premiums move opposite of home prices, investment and output on average. Instead, economic contractions are amplified Bernanke et al. (1999) with convex investment adjustment cost which generates asset price spirals. This generates a technological illiquidity effect, since scaling down the capital stock is costly – resulting in lower capital prices. However, by making the feedback effect exclusive to the goods market, there is no explicit interaction with asset prices outside of capital prices. For this reason, a change in technology or preferences over housing fails to generate the observed empirical interaction between home prices, finance premiums, investment and output.

4.3 Model Dynamics: Impulse Response Analysis

Bernanke et al.'s (1999) counterfactual correlation between output and financing premiums stems largely from the documented puzzle that their debt-contract generates an fall in
the finance premium following a negative technology shock (Shen, 2011). This is so because entrepreneurs in their model want to deleverage due to the lower real returns on investment, making default less likely. The model presented in this paper is able to generate the opposite relationship – the one which is observed in the data. The reason is that a decrease in technology in this model lowers the real return on investment as well, but it also lowers home prices (due to wealth effects). The effect on collateral values of lower home prices is sufficiently strong to decrease the value of the entrepreneur’s collateral enough to overcome their desire to deleverage.\footnote{Although, there is some over shooting of the finance premium due to the entrepreneur’s desire to disinvest, this effect diminishes with more persistent technology shocks.} The counterfactual relationship between finance premiums and output and investment can be overcome in the Bernanke et al. (1999) model with very large adjustment costs so that the decreased capital expenditures results in disproportionately lower capital prices, causing entrepreneur’s net worth to significantly deteriorate. Simulations with $\chi_{ac} = 10$ for example are able to generate correlations of $\rho(EFP, GDP) = -30\%$ and $\rho(EFP, Investment) = -63\%$, but the relationship between home prices and finance premiums is still positive.

Figure 5: Impulse response functions from the Bernanke et al. (1999) financial accelerator model and the equilibrium model to an exogenous decrease in the level of technology.
The difficulty Bernanke et al. (1999) has in capturing the relationship between home prices and finance premiums can be understood by the fact that housing demand shocks are the primary driver of home prices and these disturbances also generate counterfactual relationships between home prices and finance premiums. In particular, an exogenous decrease in home prices lowers the wage rate in the housing sector, leading to an increase in the labor supplied to the goods producing sector. This results in a higher real return on capital for a given level of investment, causing finance premiums to fall as default becomes less likely. Higher investment adjustment costs will have the opposite effect here as large values of $\chi_{ac}$ only serve to worsen the problem by leading to higher capital values and even lower finance premiums. Therefore, the Bernanke et al. (1999) debt-contract will struggle to simultaneously capture the relationship between house prices, finance premiums and investment and output regardless of the mix of supply and demand shocks.

On the other hand, the debt contract specified in this paper generates the observed relationship between finance premiums and home prices following a housing demand shock. This is clear from the relationship between home prices and borrower’s collateral values. When home prices fall, so too do collateral values, forcing borrowers to issue relatively
more unsecured debt to achieve the same level of capital expenditures. Therefore, finance premiums rise to compensate lenders for the now great possibility that a borrower will receive a productivity value which will result in default and costly monitoring.

5 Amplification and Propagation: Fire sales and Foreclosures

Having established the ability of the equilibrium model to capture the joint behavior of finance premiums, home prices and real activity, I now investigate the amplification and propagation effects generated in response to exogenous shocks. In this section, I show the model generates a fire sale effect whereby the entrepreneur attempts to pledge their entire endowment of housing to the banking sector each period for a secured loan, but the size of loan received depends critically on the secured lending capacity of the complex banks. When an aggregate downturn occurs complex banks, which are most efficient at collateralized lending, withdraw from the market serving to amplify the downturn as less efficient banks absorb this credit demand. The heterogeneity is key here as unlike the complex banks, the simple banks can only liquidate collateral at an increasing marginal cost, therefore, the more secured lending they absorb the more the value of collateral falls. This section highlights how this effect amplifies downturns by generating illiquidity in secured lending markets.

I highlight this amplification effect in figure 7 which illustrates the effect of the binding capital constraint on secured lending. The diagram shows the effect of a drop in home prices on the value of collateral. The equation determining the value of collateral in the model is given by the simple banks first order condition for $n^s_t$,

$$p^n_t = \frac{1}{r^n_t} \mathbb{E}_t \left\{ z^r p^n_{t+1} - \chi^{l,s} \mu^{l,s} n^s_t (\chi^{l,s} - 1) \right\}, \quad (37)$$

which for $\chi^{l,s} > 1$ looks like a typical demand curve. If expected home prices fall, this will shift down the demand curve for collateral. Without any redistribution effect, collateral values fall from $p^n_t$ to $p^n_s'$ - this is the dynamic captured in the figure on the left.

To understand the amplification effect stemming from the redistribution of secured lending notice two things. (i) First, due to the positive marginal cost of liquidating collateral, collateral values fall by more (in percentage terms) than expected home prices. That is, $\mathcal{E}_{p^n_t, \mathbb{E}_t \{p^n_{t+1}\}} > 1$.

$$\mathcal{E}_{p^n_t, \mathbb{E}_t \{p^n_{t+1}\}} = \frac{\partial p^n_t}{\partial \mathbb{E}_t \{p^n_{t+1}\}} \mathbb{E}_t \{p^n_{t+1}\} = \frac{z^r \mathbb{E}_t \{p^n_{t+1}\}}{z^r \mathbb{E}_t \{p^n_{t+1}\} - \chi^{l,s} \mu^{l,s} n^s_t (\chi^{l,s} - 1)} > 1 \quad (38)$$

(ii) Second, the debt contract described in section 3.2 shows that as the value of the en-
Figure 7: The graph on the left illustrates the impact on collateral values when home prices drop, without any redistribution effect. The graph on the right highlights the additional fall in collateral values (the Fire Sale effect) that results when the complex bank must reduce its share of secured lending due to the endogenously tightening moral hazard constraint.

entrepreneur’s pledgeable assets falls, the probability of default increases.\(^\text{11}\)

\[
\frac{\partial F_t(\bar{\omega}_{t+1}^j)}{\partial p^n_t} = \frac{\partial F_t(\bar{\omega}_{t+1}^j)}{\partial \bar{\omega}_{t+1}^j} \frac{\partial \bar{\omega}_{t+1}^j}{\partial p^n_t} < 0 \tag{39}
\]

These two effects, (i) and (ii) (Eqs. 38 and 39), both act to tighten the binding moral hazard constraint for the complex bank (Eq. (26)) and hence the complex bank share falls. This is illustrated in the graph on the right of figure 7. In particular, a drop in home prices induces not only a direct fall in collateral values from \(p^n_1\) to \(p^n'_1\) but a further drop to \(p^n_2\) due to an endogenous reduction in \(n^c_t\) (and the downward sloping demand for \(n^s_t\) due to the strictly convex costs). This is the beginning a multiplier effect of sorts. As \(p^n_t\) falls by more than \(E_t\{p^h_{t+1}\}\), the moral hazard constraint tightens further inducing further reductions in \(n^c_t\). All the while, as these forces act to push down the value of collateral, and equivalently the amount of secured lending, borrower’s face an increasing external finance premium. This is the amplification effect highlighted by the difference between the two sets of IRFs in Figure 8.

For each structural shock, I present the model’s response when the complex bank’s moral hazard constraint binds (the solid-red lines) and when this constraint is relaxed (the dashed-blue lines). Notice that when the constraint is relaxed, the complex banks intermediate all

\(^{11}\)The first partial derivative is positive due to the monotonicity of CDFs and the second partial derivative is negative due to the structure of the optimal debt contract.
of the secured lending since they are significantly more productive. Hence, for this model, the Complex Bank Share variable is constant.

Figure 8: Impulse response functions from the equilibrium model. The solid red lines denote the dynamic responses when the complex bank’s moral hazard constraint binds and the dashed blue lines are the dynamics when this constraint is relaxed.

Figure 8 displays the equilibrium model’s response of the endogenous variables to a detrimental risk, technology and housing demand shock. The dynamics are noticeably different when the moral hazard constraint binds compared to the more efficient allocation whereby complex banks hold all of the housing-secured assets. In particular, the response of finance premiums, real GDP and home prices are amplified. Changes in the risk-characteristics of borrowers or the household’s preferences towards housing generate 30%-50% greater movement of real GDP when assets are redistributed between banks. Even technology shocks
lower GDP by 10% more at peak when complex banks are capital constrained. The ampli-
plified movement in GDP is driven by (a) more resources spent on auditing bankrupt en-
trepreneurs which lowers consumption, (b) less investment due to higher finance premiums
and (c) lower home prices due to more foreclosures (the liquidation of borrowers collateral on
the housing market). The amplification of home prices also results in amplified movement
in collateral values. However, there is a secondary amplification effect on collateral values
through the aforementioned fire sale effect which causes collateral values to fall by more than
they otherwise would in the presence of no bank heterogeneity (see Figure 7).

One common theme through all the impulse response functions is an amplification effect
stemming from the redistribution of secured lending. Although, I have highlighted the
static amplification, there is also a dynamic feature at work which makes the moral-hazard
constrained model more persistent. Since repossessed collateral ultimately is liquidated on
the housing market, this increase in the supply of homes lowers home prices into the future
to the extent that housing does not depreciate immediately. These effects re-enforce one
another over time, propagating downturns to become more protracted than they otherwise
would be in a model without this financial friction. Ultimately though, these dynamic
amplification effects are powered by restarting, period by period, the engine that drives the
static multiplier. The joint feedback effects of foreclosures and fire sales are schematically
presented in Figure 9.

![Diagram](image)

Figure 9: The outer loop is the feedback effect from using housing as collateral for secured lending. This loop illustrates that when home prices fall, defaults rise causing foreclosures and a further reduction in home prices. The inner loop is the fire sale effect whereby the bank heterogeneity creates a secondary drop in collateral values, above and beyond that driven by falling home prices, which acts to intensify the feedback loop.
6 VAR Tests of the Model’s Amplification Mechanism

In this section, I estimate a series of vector autoregressions (VARs) to examine the empirical plausibility of the equilibrium model’s predictions. The variables included in the VARs are the same variables examined in the model’s impulse response functions: the external finance premium, real GDP, real home prices and the share of total credit exposure concentrated in complex banks. All variables are at a quarterly frequency from 1998:Q2 to 2011:Q4. As specified in log-levels, the Akaike information criterion selects 2 lags for the VAR model. In what follows I first lay-out the model’s testable predictions, then I go on to describe the data and the structural identification before presenting the impulse response functions for the various models.

6.1 The Model’s Empirical Implications

The DSGE model developed in the previous section posits an amplification effect stemming from the redistribution of secured lending between large-complex and small-simple banks. In particular, the amplification mechanism posits that an initial economic downturn, in which home prices fall, causes secured lending of complex banks to fall which in turn causes finance premiums to rise.

1. A decrease in real home prices decreases the concentration of secured loans in complex banks.
2. A decrease in the concentration of secured loans in complex banks causes finance premiums to rise.
3. A rise in finance premiums lowers home prices and output.

This mechanism is self re-enforcing. At each completion of the cycle output falls; therefore, as the cycle feeds back on itself output falls further and further which deepens the recession. To test this qualitative aspect of the model, I use the approach employed in Ludvigson (1998). The econometric strategy aims to sequentially identify shocks which trace out the steps of the financial accelerator mechanism. As robustness checks, I also test the accelerator effect with a Factor Structural VAR (FSVAR) and with an alternative measure of the share of secured lending of complex banks. The results provide further empirical support for the integrated housing and financial structure I put forth in the equilibrium model.

6.2 Data Description

One of the model’s key variables, the external finance premium is unobservable. However, following the recent strategy of Christiano et al. (2013) and Carlstrom et al. (2012), I use the spread between BAA corporate bonds and 10-year Treasuries to proxy this unobservable

\footnote{The time series is limited by the availability of net credit exposure data from the OCC’s Quarterly Derivatives Report (start date) and the end date coincides with the onset of changes in the Call Reports Y-9-C that would make aggregation difficult – such as the addition of credit unions to the reports per the Dodd-Frank Act. For the removal of trends, I use data going back to 1987:Q1.}
variable. As for real GDP, I use the model equivalent definition. I sum personal consumption expenditures and private investment (the sum of residential and non-residential investment in the model) and divide the resulting series by the personal consumption expenditures excluding food and energy price deflater. I measure real home prices using the Case-Shiller National Composite Home Price Index divided by the personal consumption expenditures excluding food and energy price deflater. Both real GDP and real home prices reveal evidence of a unit root at the 10% confidence level using an ADF test. I therefore, remove any deterministic and/or stochastic trend by taking the difference between the log of the original series and the HP-filtered trend.

Finally, I follow Breuer (2000) and construct the complex bank share variable using the Office of the Comptroller of the Currency’s (OCC) Quarterly Derivatives Report which tracks the derivative activity of the 25 largest U.S. commercial banks (in terms of notional derivatives held) and the commercial banking sector as a whole. Specifically, I sum the total credit exposure of the largest commercial banks and divide this by the total credit exposure of all commercial banks. The banks I deem as ‘complex’ are the five financial firms which consistently hold the largest amount of notional derivatives. These five banks include: JP Morgan Chase, Citibank, Bank of America, HSBC, and Wells Fargo.\(^{13}\) In the baseline model, the variable Complex Bank Share refers to the share of total credit risk held by these five banks relative to all U.S. commercial banks. There are two motivations for using this data series to measure the complex bank share in the model.

The first is due to the fact that secured debt in the model looks like a repurchase agreement. In particular, it is a sequence of transactions whereby the lender purchases the asset from the buyer with the agreement the buyer will purchase back the asset at a higher price in the future. This price differential is effectively the interest rate on the contract. Unfortunately, firm level data on repurchase agreements is limited (starting on Y-9C forms in 2002), therefore I turn to a similar product. Interest-rate swaps and repurchase agreements are similar products with the exception of whether the collateral actually trades hands. Therefore, I use total credit exposure to proxy the interest rate contract activity of financial firms. In particular, credit exposure proxies the credit actually extended on interest rate contracts since it takes into account bilateral netting which would not represent a flow of credit. The credit exposure measure includes all derivatives contracts (including FX and Equity), but these contracts are dominated by interest rate contracts which were often secured by MBSs.\(^{14}\)

The second reason stems from the documented heterogeneity across banks in derivatives markets. In particular, the amplification effect from the redistribution of secured lending between large complex banks and small simple banks is due to technological/efficiency dif-

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\(^{13}\)One challenge to tracking these firms over time is dealing with mergers, acquisitions and the financial crisis. I handle these issues by adding the off-balance sheet asset’s of acquired banks to the acquiring bank’s assets to create (as much as is possible) a consistent time series. See Table (2) for more details on how this group evolves over time.

ferences between these types of firms. Therefore, the model predicts the accelerator effect should be present in asset classes where complex banks have an productivity advantage over other banks. This is precisely the case in the derivatives market as cited in the beginning of this paper, “Typically, only the largest institutions have the resources, both in personnel and technology, to support the requisite risk management infrastructure [to operate in the derivatives markets].” (OCC, 1998-2012). Moreover, recent estimates of scale efficiency in the financial sector, including off-balance sheet activities, have found that large banks have a technological advantage over smaller banks (Wheelock and Wilson, 2012; Bos and Kolari, 2013). As a robustness check, I also compute the large bank share using data obtained directly from the FR Y-9C Consolidated Financial Statements for Bank Holding Companies collected quarterly by the Federal Reserve.\footnote{I am indebted to Bob DeYoung for recommending this data source.}

6.3 Baseline Test for the Accelerator Mechanism

In what follows, I explain how I apply Ludvigson’s (1998) procedure to test the financial amplification mechanism in this paper. The first step in testing the amplification mechanism using this approach is to identify the impact a contractionary shock which decreases home prices (i.e. the shock to home prices) has on the share of credit exposure held by large banks variable. If this share falls with home prices, I then proceed to step two which estimates the response of the external finance premium to an exogenous decrease in the large bank share when home prices are removed from the VAR. The idea is to examine how a change in the share of secured loans held by complex banks impacts the external finance premium – independent of the endogenous response of home prices to this change in the composition of lending. Finally, if the finance premium increases when the complex bank share decreases, I estimate the response of real GDP and real home prices to an exogenous increase in the external finance premium, excluding the large bank share.\footnote{The results are robust to alternatively removing both home prices and the large bank share from the VAR and estimating the impact an increase in the external finance premium has on real GDP.} Again, the idea is to trace out the steps in the amplification mechanism while controlling for the endogenous reaction of the share of credit exposure held by complex banks.

To carry out these tests, I estimate a sequence of vector auto-regressions. The first VAR includes all four variables in the following order: the external finance premium, real GDP, real home prices, and the complex bank share. The ordering is relevant for the identification of shocks using a Cholesky decomposition. Placing the external finance premium first is motivated from the equilibrium model, in which the external finance premium in period $t$ depends largely on period $t-1$ fundamentals and therefore does not react in the equilibrium model on impact to changes in home prices. The most delicate issue of this ordering lies in the decision of whether to place real GDP before real home prices or vis-a-versa. The equilibrium model implies both have a non-zero response to housing demand and technology shocks. I follow Iacoviello (2005) and order GDP before real home prices; however, all of the results for this recursive model are robust to reversing the ordering of real GDP and real home prices. As a robustness check, I also show results for a factor structural VAR which
treats real GDP and real home prices symmetrically. The conclusions are the same from both models.

Since the focus is on the response of the complex bank share, I order this variable last to leave its response unconstrained. I use this ordering throughout the VARs presented in the section, with the appropriate variable removed for each step of the accelerator mechanism. In particular, the second step analyzes the impulse response functions of the three variable VAR: the external finance premium, real GDP and the complex bank share. Finally, for the third step of the accelerator mechanism, I estimate the three variable VAR: the external finance premium, real GDP and real home prices.17

6.4 Baseline Impulse Response Functions

Impulse response functions trace out the path of the variables in periods \( t = 0, 1, 2, \ldots \) in response to a one time structural disturbance in period \( t = 0 \). Confidence bands are computed using Monte Carlo integration techniques assuming a normal likelihood and uninformative prior.

In order to test the model’s amplification prediction the first step is to examine if a drop in home prices tightens the complex banks’ moral hazard constraint forcing them to shed credit exposure relative to other banks. Interestingly, the identified shock decreases real GDP and increases the external finance premium. The equilibrium model predicts that regardless of the structural shock which would set off such a sequence of events (housing demand, technology or risk shock), the share of secured lending concentrated in complex banks will fall. Figure 10 confirms this finding, as the drop in home prices sets off a chain of events which raises finance premiums and lowers real output, and importantly lowers the share of credit exposure in complex banks.

The second step in the accelerator mechanism, and in fact the model’s key prediction, is that changes in the distribution of secured lending between large complex banks small simple banks alters the finance terms offered to borrowers. For example, if the assets shift to less productive banks, which are the simple banks, finance premiums will rise as the liquid value of the borrowers collateral falls when it is held by these unproductive intermediaries. The data confirms this prediction in Figure (11). An exogenous decrease in the share of assets held by large banks increases the external finance premium, indicative of worsening financing conditions and consistent with the accelerator mechanism in the equilibrium model.

Since the VAR has confirmed the first two steps of the accelerator mechanism, I proceed to the third step to examine how rising finance premiums affect the macroeconomy. In particular, the model predicts that rising financing costs will lead to an increase in defaults and ultimately lower home price due to the increased supply of homes on the market. Moreover,

\footnote{As a robustness check of this approach, I also estimated a sequence of bi-variate VARs and found the results to be robust. In particular, the bivariate VAR of: real home prices and the complex bank share, the bivariate VAR: the external finance premium and the large bank share and the bivariate VAR: the external finance premium and real GDP all displayed statistically significant responses consistent with the accelerator mechanism put forth in the paper.}
the existence of higher financing costs leads to less investment. Both of these movements exact a negative effect on real GDP. This is confirmed in Figure (12) As home prices fall so too does real GDP due to both the direct effect home prices have on GDP and through the indirect effect on residential investment and ultimately consumption. Therefore the impulse response functions show that when home prices fall, credit exposure shifts from large complex banks which magnifies the movement of finance premiums and, in turn, amplifies the movement of output and home prices, starting the cycle over. In summary, the results above provide evidence which supports the hypothesis that the redistribution of secured lending activity between banks magnifies the movement of finance premiums, house prices and output across the business cycle, supporting the model’s predictions.

6.5 Robustness Check: A Factor Structural VAR

The model’s implications described above calls for the identification of 3 distinct structural shocks using four variables. Since the number of desired structural shocks is less than the number of variables for which there are model implications, I employ a factor structural
vector auto-regression following Gorodnichenko (2005). This robustness check is appropriately fitting here for a couple of reasons.

First, the four variables in the VAR behave qualitatively similar to technology and housing demand shocks in the equilibrium model as shown in Figure 8. For this reason, imposing timing restrictions at any horizon to distinguish these shocks proves difficult. Moreover, there is no need to disentangle these shocks to test the model’s predictions that an aggregate contraction which decreases real home prices. Therefore, I choose to simply recognize macroeconomic disturbances as a single factor, which allows me to test the model’s prediction along this dimension without imposing arbitrary timing restrictions on the behavior of output and real home prices. Blanchard and Quah (1989) make a similar argument that shocks can be aggregated when they elicit qualitatively similar dynamics.\(^{18}\) Second, the

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\(^{18}\)Their argument is a bit more formal. To summarize, they show that so long as the dynamic responses of the variables in the VAR to the aggregated shocks differ up to a scalar lag distribution (The responses need not be identical nor proportional) then the shocks can be aggregated.
external finance premium in the model is unobservable. Hence, by using a factor-structural VAR, I can explicitly include measurement error terms to ensure this proxy for the external finance premium does not contaminate the structural shocks of study. In addition to the macroeconomic factor discussed above, I identify a shock which exogenously increases the external finance premium and an shock which exogenously decreases the complex bank share.

To identify these three shocks, I impose a recursive scheme that ensures global identification is achieved\(^\text{19}\) and allows for home prices and real GDP to behave symmetrically to all the shocks. In the model, the ex-post observable external finance premium in period \(t\) is determined by fundamentals in period \(t - 1\). For this reason, I order the spread, which proxies the external finance premium first in the VAR to match this feature of the model. Next I order real GDP and then home prices. The ordering between these two variables is

\(^{19}\)Since the \(3 \times 3\) sub-matrix of \(A\) excluding the last row is lower triangular, we can ensure global identification (Anderson, 2003; Anderson and Rubin, 1956).
innocuous since they are treated symmetrically in the identification scheme. Finally, I order the share of total credit exposure held by large complex banks. This recursive ordering is consistent with the equilibrium model, as discussed above.

Figure 13: FSVAR Model: Median Impulse Response to various contractionary shocks with 68% and 90% confidence bounds.

To summarize the identification scheme, let $e_t$ denote the $4 \times 1$ vector of reduced form VAR residuals. Let $\epsilon_t$ denote the $3 \times 1$ vector of structural shocks and let $v_t$ denote the $4 \times 1$ vector of measurement errors which ensures the rank between the reduced form shocks and the identify structure match. The matrix $A$, is a $4 \times 3$ loading matrix which relates the structural factors to the reduced form residuals. Summarizing this,

$$e_t = A\epsilon_t + v_t$$  \hspace{1cm} (40)
where

\[
A = \begin{bmatrix}
  a_{1,1} & 0 & 0 \\
  a_{2,1} & a_{2,2} & 0 \\
  a_{3,1} & a_{3,2} & 0 \\
  a_{4,1} & a_{4,2} & a_{4,3}
\end{bmatrix}
\] (41)

I assume the measurement error terms, \(v_t\) are independent of the structural factors \(\epsilon_t\) at all leads and lags and I also assume the covariance matrix of the measurement errors, \(\Psi\), is diagonal. Equation (40) is estimated using maximum likelihood techniques.\(^{20}\)

The results from the estimated FSVAR show that both, the VAR approach and the FS-VAR approach lead to similar conclusions in this case. In particular, the aggregate downturn which simultaneously decreases real home prices and real GDP, also increases the external finance premium and lowers the complex bank’s share of credit exposure. The implication of the redistribution in credit supply is the key model prediction. The results in Step 2 show that, as the model predicts, decreasing the share of credit exposure in complex banks increases financing costs. Not surprisingly, higher financing costs lead to lower output and home prices in Step 3.

### 6.6 Robustness Check: Alternative Measures of the Complex Bank Share

As a second robustness check, I develop an alternative measure of the complex bank share using data from the Federal Reserve’s Y-9C report. Ideally, I would like to obtain a time series of the amount of debt securities issued by bank holding companies which are secured by mortgage backed securities, as this most closely aligns with the interpretation of these variables in the model when I consider unconventional monetary policy in Section 7. Unfortunately, this data has only been collected as of 2008 in the call reports. To get a rough measure of the credit banks have extended which is secured, either directly or indirectly, by housing I use the reported level of loans secured by real-estate. Impulse response function estimates which refer to ‘Alternative Complex Bank Share’ refers to the share of loans secured by real estate held by the five banks listed above relative to all bank holding companies which file the Y-9C report.

The impulse responses with this alternative measure of the complex bank share generate the same sign as those of the baseline estimate; however, there is noticeably less statistical significance overall. The first step reveals that a drop in real home prices decaes the share of housing-secured lending intermediated by complex banks, although the effect is delayed. With sufficient evidence that changes in real home prices alter the distribution of secured

\(^{20}\)In particular, assuming \(\epsilon_t\) are i.i.d. \(Normal\), the log-likelihood equation is given by,

\[
log(\mathcal{L}) = -\frac{4T \ln(\pi)}{2} - \frac{T}{2} \ln(|AA' + \Psi|) - \frac{1}{2} \sum_{t=1}^{T} e_t'(AA' + \Psi)^{-1} e_t.
\]
lending between banks, I proceed to Step 2. A drop in the share of secured lending by complex banks leads to higher finance premiums. The response of finance premiums loses some statistical significance in this version. However, the median response is still positive as is the majority of the posterior density for the first 6 quarters. The third step is identical to Step 3 in the baseline model, but the results are sported again for completeness.

Interpreting these results leads me to conclude that there is broad evidence for the notion that aggregate downturns which elicit drops in real home prices are amplified through the financial sector. However, the exact magnitudes depend on the types of assets considered. More complex assets such as derivatives have stronger redistribution effects and lead to larger
amplifications relative to more general asset classes such as loans secured by real estate. This is consistent with the notion that the degree of amplification from the redistribution of assets between agents depends on their relative productivity differences, as emphasized in Section 5 and originally in Kiyotaki and Moore (1997).

7 Unconventional Monetary Policy

The analysis of the equilibrium model’s amplification mechanism and the results from the VAR tests of this mechanism effect suggest that bank heterogeneity creates an amplification effect generated by the redistribution of assets. This would suggest that unconventional monetary policies may be effective by altering the distribution of secured lending and therefore collateral values. Essentially suggesting the central bank can overcome the friction generated by (a) the superior nature of secured lending to overcome the auditing friction and (b) the inability of large-complex financial firms to intermediate the market’s demand for secured debt by acting as a third lender (complex and simple banks being the other two) and making loans secured by houses (i.e. purchase mortgage backed securities). However, this conclusion also suggests the effectiveness of unconventional monetary policy will depend on the degree of bank heterogeneity. Therefore, before examining alternative unconventional monetary policies, I examine whether the amount of heterogeneity within the model is consistent with that observed in the data.

To this end, I estimate a bi-variate VAR including: the external finance premium and the share of credit exposure (or loans secured by real estate) held by large complex banks. The degree to which collateral values fall, and thus finance premiums rise, when the complex bank share changes is determined by the convexity of the simple bank’s liquidation cost function \( \chi_{l,s} \). Since this is simply calibrated, drawing policy conclusions without empirically confirming the degree of bank heterogeneity may be misleading. Therefore, I then simulate the equilibrium model for 500 periods and take the last 55 observations. I estimate the same bi-varaite VAR using this model’s simulated data and plot the median impulse response functions from the simulated data on top of the confidence bands of computed using the actual data. The results in Figure 15 confirm the degree of heterogeneity within the model is consistent with what is found in the data. This provides some comfort the magnitudes of the unconventional monetary policy interventions studied in this section are empirically reasonable.

I now turn the focus to analyzing the unprecedented actions taken by policy makers in the wake of the 2008 Financial Crisis through the lens of this empirically verified model. Since the model features large financial firms (‘Too Big to Fail’ banks) and housing secured debt (Mortgage Backed Securities), it can be used to analyze the relative effectiveness of:

(i) Central Bank purchases of mortgage backed securities such as QE1 and QE3,

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21 As Reis (2009) clearly explains, for unconventional monetary policy to be effective the central bank has to be endowed with a special ability. In this model, I assume the central bank is as productive as the complex banks at liquidating collateral, however, they face no enforcement problems that would limit their participation in the secured lending market.
When undertaking a large scale asset purchase program, there are several dimensions of policy to consider, the least of which is managing expectations and providing guidance about the asset purchases. Therefore, in Figure 16, I consider three different approaches to carrying a quantitative easing program. All of these policies vary with regards to their reinvestment policy and the pace at which the central bank purchases assets.

**Abrupt End:** The central bank announces in period 9 that it will purchase assets at a constant rate each period. In period 19 the central bank announces that all asset purchases will be ceased in period 21.
Figure 16: Quantitative Easing: The above figure provides the model’s dynamics under various quantitative easing policies. The vertical dashed line in period 9 denotes when the asset purchases are announced while the vertical dashed lines denote when the end/tapering of the asset purchases are announced.

**Slow End:** The central bank announces in period 9 that it will purchase assets at a constant rate each period. In period 19 the central bank announces that the pace at which it purchases assets will geometrically decline to zero.

**Reinvest + Slow End:** The central bank announces in period 9 that it will purchase assets at a constant rate. In period 19 the central bank announces that the pace at which it purchases assets will geometrically decline to zero. Agents know that in every period, a fraction of the proceeds will be reinvested in more assets.

Analyzing these three alternative LSAP policies in Figure 16 reveals the nuances of the asset purchase programs have non-trivial effects on how they impact output, home prices and finance premiums. In particular, abruptly ending the asset purchases leads output back to its pre-policy path the period after the purchases end. Home prices and collateral values return
shortly thereafter while finance premiums return sharply and overshoot their no policy path. On the other hand, slowly ending the asset purchase program has the benefit of lengthening the asset purchases and therefore agents expect collateral values to remain supported into the future. This propagation of slowly winding-down the purchases is intensified when the central bank reinvests the proceeds from the asset purchases in more assets. In this case, the policy has its largest effects as the reinvestment policy compounds the size of the program.

**Announcement Effects**

The effects of reinvesting proceeds appear both at announcement and after the asset purchase program begins in Figure 16. The significant announcement effect of quantitative easing been the focus of most empirical studies on large-scale asset purchases (Gagnon, Raskin, Remache, and Sack, 2011; Bauer and Rudebusch, 2014). Krishnamurthy and Vissing-Jorgensen (2011) find that such announcements induce a fall in the BAA-10 year treasury spread, although it is quantitatively small. The simulations from the model are consistent with the notion that purchasing housing-secured debt lowers risk premia on announcement; however, larger effects appear when the purchases actually begin. Therefore, although the model validates an announcement effect of QE policies, there is an even more significant purchase effect that most event studies don’t consider. However, the absolute size of both these effects depends critically on how the asset purchases are carried out and communicated.

**Taper Tantrum**

In June 2013, then Federal Reserve Chairman Bernanke announced that economic conditions had improved to the point that asset purchases may cease later in 2013. In reaction to this news, the economy stalled a bit as exchange rates increased along with long term interest rates (Neely, 2014). A similar reaction is incited within the equilibrium model. The second set of vertical-dashed lines in Figure 16 denote when the tapering of asset purchases are announced. It is clear from the path of the variables the economy has an adverse reaction upon the announcement of tapering. The effect is most pronounced, once again, when the proceeds are reinvested. Intuitively this happens because the policy change is compounded into the future since a reduction in the flow of purchases ultimately leads to a reduction in the size of the reinvestments.

**Unwinding the Balance Sheet**

Debt in the equilibrium model is one-period, however, each period a fraction of projects defaults leaving $F_{t-1}(\hat{\omega}_t)n_{t-1}^{cb}$ assets for the central bank to liquidate on the housing market. In the simulations to this point I have assumed the central bank liquidates this collateral each period (jut as the private banks do) so that stock of assets on its balance sheet are equal to its new purchases, $p_t n_{t}^{ch}$. I relax that assumption in what follows and assume the central bank has a storage technology that allows it to delay liquidation and grow its balance sheet over time. In Figure 17, I consider the effects of only liquidating a fraction of the foreclosed assets each period. The macroeconomic effects of the asset purchases are larger when the central bank slowly unwinds its balance sheet. Meanwhile, the total tax payer remittances from this policy are essentially unchanged, but slightly larger under the Slow Sell Off policy.
Equity Injections vs. Quantitative Easing

In addition to purchasing assets that central banks don’t typically purchase, such as mortgage backed securities, policy makers also collectively decided to inject equity into the largest financial firms. To the extent that these banks were capital constrained, such policies are generally believed to have a stimulatory effect by increasing credit provided by these firms. In this equilibrium model, it is the case that large complex banks are limited in how much secured lending they can intermediate with a fixed amount of capital. Assuming this constraint binds, then injecting equity is dollar for dollar equivalent to the central bank providing the secured lending that large banks can’t. However, one crucial difference is the expectations of these alternative policies. In particular, the QE programs are prolonged while
the equity injections are assumed to be short lived. Consistent with the timing of the TARP legislation, the equity injections are calibrated to be repaid within 7 quarters of the policy. Thus, the short lived nature of the capitalization of the banks, limits the effectiveness of these policies in shortening the duration of a protracted downturn. This is evident in Figure 18.

One could easily argue that larger and more prolonged equity injections could be just as effective as a prolonged large-scale asset purchase program. However, the political ramifications of equity injections makes such adjustments unrealistic. In totality, QE policies are
preferable to equity injections as they provide sufficient stimulus to offset the crisis without carrying political costs. Although not modeled here, equity injections carry a stigma of

(i) Directly benefiting Wall Street as opposed to Main Street and

(ii) The government taking an ownership stake in a private firm.

**Were equity injections necessary?**

The above conclusion that a persistent QE policy outperforms a short-lived equity injection into big banks raises a natural question: Were the equity injections necessary? In other words, the results above suggest the economy would be in roughly the same condition with or without the TARP injections. Recall however the circumstances under which TARP was passed. The economy was quickly deteriorating and urgent action was required to prevent a total collapse of the financial system. Chairman Ben Bernanke’s reply to a weekend extension for congress to debate the equity injections was famously recounted as, “If we don’t do this [pass TARP] tomorrow, we won’t have an economy on Monday.” This paper’s inclusion of heterogeneous banks allows me to analyze the stabilizing role of capitalizing big banks.

In particular, the strength of the asset-price spiral from the redistribution of assets is calibrated by the slope of the unproductive bank’s demand for secured assets, \( \chi^{l,s} \). The baseline calibration is set to \( \chi^{l,s} = 1.07 \). As this value increases the impact of asset redistribution is strengthened. In fact, for slightly larger values of \( \chi^{l,s} \) the model becomes indeterminate so that no unique rational expectations equilibrium exists. The amplification effect becomes so strong that without counter-cyclical policy there are multiple equilibria.

![Figure 19: If the fire sale effect is strong enough, for example if the simple banks demand curve is \( d_s' \), the economy becomes indeterminate. Determinacy can be restored with countercyclical equity injections.](image)

Interestingly, this indeterminacy can be remedied with counter-cyclical equity injections.

---

[^22]: See for example, Sorkin (2010)
In particular,

\[ bk_{t}^{ch} = \theta_{bk} \left( \log(p^{n}_{t}) - \log(\bar{p}^{n}) \right) \]

with \( \theta_{bk} < 0 \) is sufficient to restore determinacy. This systematic recapitalization of complex banks eliminates the multiple equilibria induced by a high-degree of bank heterogeneity. In this sense, the role served by the equity injections in October of 2008 to the largest U.S. banks is clearly understated in the preceding simulations. Although counter cyclical asset purchases have a similar stabilizing effect, it seems likely that in reality capital injections can be implemented more rapidly as opposed to asset purchases which require choosing which assets to buy. Thus, equity injections are justified as a triage treatment at the acute stage of the 2008 financial crisis while large-scale asset purchase programs can be understood as a prolonged therapy for the economy which shortens the time needed to fully recover.

8 Conclusion

Understanding the sources, consequences and policy response to the 2008 Financial Crisis is of critical importance to policy makers. If for no other reason, because financial crises seem to reoccur with devastating economic consequences. Although there were many dimensions to the most recent U.S. business cycle, the housing boom and bust and the role played by highly leveraged-complex financial institutions in the cycle are defining features of the financial crisis. Largely because of these defining roles, housing-secured debt and big banks have also been at center stage of the policy response to the ensuing Great Recession. The Federal Reserve has deviated from its traditional policy of manipulating short-term interest rates to stabilize the economy to purchasing large quantities of mortgage backed securities and the joint policy coordinated by the Federal Reserve, the Treasury Department and the FDIC to inject equity into large commercial banks is equally exceptional. Understanding how these policies transmit to an economy crippled in the wake of a housing bubble is largely the focus of this paper.

Before analyzing the policy response to a deep economic contraction, I develop an equilibrium model which bridges the housing sector to the production side of the economy via the financial sector. The modeling choice is empirically motivated by the behavior of secured lending and home prices over the decade startling the 2008 Financial Crisis. The data provides evidence that not only do secured lending and home prices co-move, but so too does the concentration of secured asset in complex commercial banks. To capture these facts in the equilibrium model, I specify a debt contract where, unlike in Bernanke et al. (1999), the entrepreneur’s net worth is in the form of a housing endowment.

The value of the entrepreneur’s endowment is composed of two parts, it’s fundamental price if liquidated on the housing market and the cost of liquidation – which varies depending on the liquidation technology of the lender. Aligning the model with the data, I specify a capital constraint on the complex banks in the model which endogenously makes their capacity to make secured loans co-move with home prices. When home prices fall, a feedback loop is initiated in which borrowers collateral falls in value because (a) home prices fall and (b) complex banks reduce the secured lending. The latter effect, leads to amplified fall in
collateral values associated with market illiquidity and fire sales as in \( ? \). Lower collateral values lead to worse financing terms and more defaults – which leads to more collateral liquidated on the housing market and further reductions in home prices.

When this financial mechanism is placed within an otherwise standard real business cycle model, the resulting model implied second moments are largely consistent with the joint behavior of external finance premiums, real GDP, real home prices and investment. Meanwhile, I show the standard Bernanke et al. (1999) financial accelerator naively coupled with a housing market will generally struggle to simultaneously capture the joint behavior of the external finance premium, real GDP and real home prices – for a range of values of the financial-accelerator-governing parameters. Then using vector autoregressions, I test the amplification mechanism put forth in this paper using impulse response functions in the data. The resulting impulse response functions provide further evidence the interaction between house prices, complex banks and financing cost has consequences for the real economy. However, these empirical tests also suggest the degree of the amplification effect depends on the degree heterogeneity between the banks.

After testing the model against the data, I turn the focus to the ultimate question of how large scale purchases of housing-backed debt and equity injections into large-complex banks mitigates a housing generated liquidity-credit crunch. Through the lens of the equilibrium model, the effectiveness of these policies is limited degree of heterogeneity in the financial sector. If all banks are identical, then altering the distribution of assets between banks can have no real effects. Unconventional monetary policy in this case could only be effective if the central bank offered entrepreneurs collateral values above market prices, similar to the conclusion of Williamson (2012). However, when the amount of liquidity the financial sector can create is dependent on the distribution of assets between banks, unconventional monetary can have powerful effects by simply valuing borrowers collateral at the market price. This of course, assumes the central bank faces no moral hazard problem like the complex banks and are more efficient than the simple banks. In this case, a large-scale asset purchase program which reinvests the proceeds, tapers slowly and slowly unwinds a large central bank balance sheet can have substantial stimulatory effects.

Injecting equity into the complex financial firms has a similar effect, but political constraints outside the scope of the model make prolonged equity injections difficult to propose. When thinking about the role played by equity injections in the 2008 TARP legislation, the model can justify it as a rapid response to quickly deteriorating economic conditions. In fact, for some calibrations, systematically re-capitalizing complex financial firms can imply large welfare gains by eliminating the multiplicity of equilibria. However, moral hazard issues that arise from such policies may be a cause for concern as such considerations are abstracted from. Along this dimension, and many others, considerable work remains to understand appropriate policy response to a housing bust and ensuing recession.

Despite the recent attention economists have devoted to understanding macro-financial linkages, including this paper, there are many unanswered questions. One essential question is when should a central bank intervene into financial markets. Most models used to evaluate
unconventional monetary policy feature a steady-state which is itself inefficient. Hence, without a severe downturn, intervention would be justified on the basis of efficiency. However, few would argue that constant central bank intervention into private financial markets is optimal due to the cost of managing a large portfolio of complex assets. Better understanding the scenarios in which intervention is deemed necessary from a welfare standpoint is needed, but only after we better understand the costs of such interventions.

Additionally, in this paper, complex banks interact with regulators in a simplistic manner. In reality regulating a complex bank is a dynamic problem in which regulators balance financial efficiency and innovation against systemic risk. Understanding these trade-offs is important for the development of macroprudential policy. Moreover, when banks misbehave, the process of fining these institutions typically involve negotiations between the banks and the Justice Department. Modeling these negotiations in a richer Nash Bargaining framework would yield considerably more insights into the emergence of the moral hazard problem I specify in this paper. Finally, the paper focus primarily on “mopping-up” after a crash, but gives little attention to preventive policies. Central banks are eager to develop tools, outside of interest rates, to limit financial risk – but more guidance is needed what these tools should be and how to use them.

Beyond macroprudential considerations, the debt-contract structure I propose in this paper has implications for business cycle analysis as well. In particular, Christiano et al. (2013) argue the Bernanke et al. (1999) financial accelerator mechanism with risk-shocks can explain the majority of business cycle movements. However, this conclusion results from discipling the estimation with credit and the external finance premium, among other variables, as observable variables. The empirical facts set forth in this paper suggest the amount secured-credit relative to all credit is an important variable for understanding the behavior of the external finance premium. Also including home prices in the estimation strategy, as in Iacoviello and Neri (2010), may lead to very different conclusions about the sources of business cycles. In particular, I show in this paper the debt contract is capable of capturing the observed relationship between real GDP and finance premiums following technology and preference shocks, in addition to risk shocks. Better understanding the importance of risk shocks relative to traditional drivers of the business cycle would be a worthwhile endeavor within this framework.
References


Gorodnichenko, Y. (2005): “Reduced-Rank Identification of Structural Shocks in VARs,” Macroeconomics 0512011, EconWPA.


A VAR Data and Complete Results

Table 2: Bank Mergers and Acquisitions

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<th>Banks Excluded from the set of Commercial Banks</th>
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<td>Goldman Sachs</td>
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1. Bank of America NT and SA merges with Nationsbank NA
2. HSBC enters the U.S. commercial bank market by acquiring Republic NB of New York
4. Chase Manhattan Bank and Morgan Guaranty TR CO of NY merge to form JP Morgan Chase Bank
5. Wachovia acquires First Union National Bank
6. JP Morgan Chase acquires Bank One National ASSN
7. Goldman Sachs becomes a commercial bank
8. JP Morgan Chase acquired Bear Stearns due to extreme financial distress. Beginning in 2008 Q1, JP Morgan Chase experienced increase exposure through repo transactions with Bear Stearns and the ultimate acquisition of Bear Stearns and all exposures. This is very difficult to account for since Q1 to SEC filings report National values, not total credit exposure. Hence, from 2008 Q1-2008 Q4 I use a linear interpolation to assess JP Morgan Chase’s total credit exposure.
B Baseline Equilibrium Model

In this section I provide the full set of equations which defines the dynamic equilibrium model.

Household - 5

\[
\frac{w^h_t}{c_t} = \frac{\eta^h}{1 - \frac{\mu^h}{\mu^t}} \tag{B.1}
\]

\[
\frac{w_t}{c_t} = \frac{\eta^l}{1 - l_t} \tag{B.2}
\]

\[
\frac{\eta^h}{h_t} = \frac{\tilde{p}_t^h}{c_t} - \beta \mathbb{E}_t \left\{ \frac{(1 - \delta^h)p_{t+1}^h}{c_{t+1}} \right\} \tag{B.3}
\]

\[
\frac{1}{r^d_t} = \mathbb{E}_t \{ \Lambda_{t+1} \} \tag{B.4}
\]

\[
\Lambda_t = \beta \frac{c_t-1}{c_t} \tag{B.5}
\]

Aggregate Goods Production - 7

\[
y_t = z_t (k_{t-1})^{\alpha_g} (l^g_t)^{1-\alpha_g} \tag{B.6}
\]

\[
l^g_t = (l_t)^{(1-\alpha_e)} (l^e_t)^{\alpha_e} \tag{B.7}
\]

\[
w_t = (1 - \alpha_g)(1 - \alpha_e) \frac{y_t}{l_t} \frac{1}{\mu^c} \tag{B.8}
\]

\[
w^c_t = (1 - \alpha_g)\alpha_e \frac{y_t}{l_t} \frac{1}{\mu^c} \tag{B.9}
\]

\[
r^{k}_t = \alpha_g \frac{y_t}{k_t} \frac{1}{\mu^c} + (1 - \delta^k) \tag{B.10}
\]

\[
c^e_t = (1 - \alpha^n_t) (1 - \Gamma_{t-1}(\bar{\omega}_t)) r_k^c k_{t-1} \tag{B.11}
\]

\[
\alpha^n_t = \frac{(1 - F_{t-1}(\bar{\omega}_t)) p^n_{t-1} r^d_{t-1} \bar{n}}{(1 - \Gamma_{t-1}(\bar{\omega}_t)) r_k^c k_{t-1}} \tag{B.12}
\]

Capital Producers - 1

\[
i_t = k_t - (1 - \delta^k)k_{t-1} \tag{B.13}
\]
Goods Production: Firm-level Debt Contract - 9

In this section I provide the equations which determine the debt contract. Moreover, I provide a description of the individual entrepreneur’s problem which leads to the aggregate entrepreneur’s consumption rule defined in equation (16). In particular, suppose entrepreneur \(j\) has preferences over consumption and housing given by a Cobb-Douglas utility function,

\[
U_t(j) = c_t(j)^{1-\alpha^n_t} \cdot n_t(j)^{\alpha^n_t}
\]

with

\[
\alpha^n_t = \frac{(1 - F_{t-1}(\bar{\omega}_t)) \cdot p^n_{t-1} \cdot r^d_{t-1} \cdot \bar{n}}{(1 - \Gamma_{t-1}(\bar{\omega}_t)) \cdot r^j_t \cdot k_{t-1}}.
\]

Since entrepreneur \(j\) takes the aggregate default rate \(F_{t-1}(\bar{\omega}_t)\) and the aggregate choice of capital, \(k_t\) as given, this Walrasian demand bundles given these preferences has the well-known property that the expenditure shares on consumption and housing will equal their weights in the Cobb-Douglas utility function:

\[
c_t(j) = (1 - \alpha^n_t) \cdot (1 - \Gamma_{t-1}(\bar{\omega}_t)) \cdot r^j_t \cdot k_{t-1}.
\]

Aggregating over this equation implies

\[
c_t = \int_0^\infty (1 - \alpha^n_t) \cdot (1 - \Gamma_{t-1}(\bar{\omega}_t)) \cdot r^j_t \cdot k_{t-1} \cdot dj
\]

\[
= (1 - \alpha^n_t) \cdot (1 - \Gamma_{t-1}(\bar{\omega}_t)) \cdot r^j_t \cdot k_{t-1}
\]

\[
= (1 - \Gamma_{t-1}(\bar{\omega}_t)) \cdot r^j_t \cdot k_{t-1} - (1 - F_{t-1}(\bar{\omega}_t)) \cdot p^n_{t-1} \cdot r^d_{t-1} \cdot \bar{n}.
\]

The second equality follows from the ex-ante homogeneity among entrepreneurs implying they all will choose the same default cut-off, \(\bar{\omega}_t\), and the same level of capital expenditures, \(k^j_t\).

\[
\mathbb{E}_t \left\{ \Gamma'_t(\bar{\omega}_{t+1}) \cdot r^k_{t+1} \cdot k_t \right\} = \lambda_t \mathbb{E}_t \left\{ (\Gamma'_t(\bar{\omega}_{t+1}) - \mu^a G'_t(\bar{\omega}_{t+1})) \cdot r^k_{t+1} \cdot k_t \right\}
\]

\[
\mathbb{E}_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \cdot r^e_{t+1} \right\} = \lambda_t \mathbb{E}_t \left\{ r^e_{t+1} - \Gamma_t(\bar{\omega}_{t+1}) \cdot r^k_{t+1} \right\}
\]

\[
+ \lambda_t \mathbb{E}_t \left\{ \mu^a G_t(\bar{\omega}_{t+1}) \cdot r^k_{t+1} \right\}
\]

\[
r^e_t (k_t - p^n_t \cdot \bar{n} - w^e_t) = \mathbb{E}_t \left\{ (\Gamma_t(\bar{\omega}_{t+1}) - \mu^a G_t(\bar{\omega}_{t+1})) \cdot r^e_{t+1} \cdot p_t \cdot k_t \right\}
\]

\[
b_t = k_t - p^n_t \cdot \bar{n} - w^e_t
\]

\[
z_{t+1} = \frac{ln(\bar{\omega}_{t+1}) + .5 \cdot (\sigma^\omega_t)^2}{\sigma^\omega_t}
\]

\[
G_t(\bar{\omega}_{t+1}) = \Phi^N(z_{t+1} - \sigma^\omega_t)
\]

\[
\Gamma_t(\bar{\omega}_{t+1}) = \Phi^N(z_{t+1} - \sigma^\omega_t) + \bar{\omega}_{t+1} \cdot (1 - \Phi^N(z_{t+1}))
\]
\[ G'(\bar{\omega}_{t+1}) = \left( \frac{1}{\sigma^2} \right) e^{-\frac{z_t^2}{2}} \]
\[ \Gamma'(\bar{\omega}_{t+1}) = 1 - \Phi^{N}(z_{t+1}) \]  

**Housing Production:** 
\[ h_{t}^{\text{new}} = z_t^h(l_t^{h})^{(1-\alpha_h)} \]  
\[ w_t^h = p_t^h(1-\alpha_h)\frac{h_{t}^{\text{new}}}{l_t} \]  

**Complex Bank:** 
\[ bk_{t}^c = n_t \mathbb{E}_t \left\{ z_r p_{t+1}^h (1 - \tilde{F}_t(\bar{\omega}_{t+1})) p_{t+1}^n r_{t+1}^d \right\} \]  
\[ \mathbb{E}_t \{ \Lambda_{t+1} r_{t+1}^{e,c} \} = \mathbb{E}_t \{ \Lambda_{t+1} r_{t}^{e,d} \} \]  
\[ \mathbb{E}_t \{ \Lambda_{t+1} r_{t}^{e,c} \} = \left( \frac{\theta_b - 1}{\theta_b} \right) \mathbb{E}_t \{ \Lambda_{t+1} (1 - \tilde{F}_t(\bar{\omega}_{t+1})) \} r_{t}^{b,c} \]  
\[ b_{t}^{c} = \left( \frac{r_{t}^{b,c}}{r_{t}^{b}} \right)^{-\theta_b} b_{t} \]  
\[ b_{t}^{c} = d_{t}^{c} + \frac{bk_{t-1}^{c}}{\pi_t} - p_{t}^{n} n_{t}^{c} \]  
\[ \Pi_{t}^{c} = (1 - F_{t-1}(\bar{\omega}_{t})) r_{t-1}^{b,c} b_{t-1}^{c} \]  
\[ + (1 - \mu^b) \frac{b_{t-1}^{c}}{b_{t-1}} \phi_t(\bar{\omega}_{t+1}) \]  
\[ + (1 - F_{t-1}(\bar{\omega}_{t})) r_{t-1}^{d} p_{t-1}^n n_{t-1}^{c} \]  
\[ + F_{t-1}(\bar{\omega}_{t}) p_{t}^{h} z_r n_{t-1}^{c} - p_{t}^{n} n_{t}^{c} \]  
\[ - r_{t-1}^{d} a_{t-1}^{c} + d_{t}^{c} - b_{t}^{c} \]  
\[ i_{t}^{bk,c} = \delta^{c} \]  
\[ bk_{t}^c = i_{t}^{bk,c} + (1 - \delta^{bk}) \frac{bk_{t-1}^s}{\pi_t} \]
Before stating the aggregate banking equilibrium conditions, I first derive the rate the bank charges to entrepreneurs per dollar loaned. First, denote the ex-post rate received by lenders per dollar of unsecured lending. This is equal to the loan rate paid by successful entrepreneurs plus the residual revenue of defaulting entrepreneurs net of auditing costs:

\[
\bar{r}_t = (1 - F_t(\bar{\omega}_{t+1})) r_{t+1} + (1 - \mu^a) \phi_t(\bar{\omega}_{t+1}) \bar{b}_t .
\]  

Letting \( \bar{b}_t \) denote the aggregate marginal cost of the banking sector of lenders per dollar of unsecured lending. Then, from each bank type’s \( \zeta \in s, c \) first order condition, the ex-post rate received by lenders per dollar of unsecured lending as a function of their marginal cost is equal to:

\[
\bar{r}_t = \left( \frac{\theta_b}{\theta_b - 1} \right) \bar{r}_t + \left( 1 - \frac{1}{\theta_b - 1} \right) (1 - \mu^a) \phi_t(\bar{\omega}_{t+1}) \frac{\bar{b}_t}{b_t} .
\]
This expression confirms that as \( \theta_b \to \infty \) and the banking sector becomes perfectly competitive, then the rate received by lender’s per dollar of unsecured lending reduces to their marginal cost of funds. In this case, the lender’s individual rationality constraint reduces to that of Bernanke et al. (1999), as \( r_t^e = \bar{r}_t = r_t^d \) when lenders face no marginal cost other than obtaining the funds from depositors.

\[
\bar{r}_t^e = \left[ \nu \left( \tilde{r}_t^{e,U} \right)^{1-\theta_b} + (1-\nu) \left( \tilde{r}_t^{e,P} \right)^{1-\theta_b} \right]^{\frac{1}{1-\theta_b}} \tag{B.41}
\]

\[
r_t^e = \left( \frac{\theta_b}{\theta_b - 1} \right) \bar{r}_t^e - \left( \frac{1}{\theta_b - 1} \right) \left( 1 - \mu_z^a \right) \phi_t(\bar{\omega}_{t+1}) \tag{B.42}
\]

\[
r_t^b = \left[ \nu \left( r_t^{b,s} \right)^{1-\theta_b} + (1-\nu) \left( r_t^{b,p} \right)^{1-\theta_b} \right]^{\frac{1}{1-\theta_b}} \tag{B.43}
\]

\[
d_t = \nu d_{t+1}^u + (1-\nu) d_t^p \tag{B.44}
\]

\[
i_t^{bk} = \nu i_t^{bk,s} + (1-\nu) i_t^{bk,c} \tag{B.45}
\]

\[
\phi_{t-1}(\bar{\omega}_t) = G_{t-1}(\bar{\omega}_t)r_t^{s,k}p_{t-1}k_{t-1} \tag{B.46}
\]

**Market Clearing - 4**

\[
h_t = h_t^{new} + F_{t-1}(\bar{\omega}_t)z_t^\tau \bar{n} + (1-\delta^h)h_{t-1} \tag{B.47}
\]

\[
\bar{n} = \nu n_t^c + (1-\nu)n_t^s \tag{B.48}
\]

\[
y_t = c_t + c_t^e + i_t + i_t^{bk} + \mu_z^a \phi_{t-1}(\bar{\omega}_t) + \nu F_{t-1}(\bar{\omega}_t)\mu_z^{l,s} \left( N_t^{u,s} \right) \chi^{l,s} + \nu \chi^{b,s}h_t^g \tag{B.49}
\]

\[
gdp_t = c_t + i_t + p_t^bh_t^{new} \tag{B.50}
\]

**Monetary Policy - 2**

\[
\left( \frac{R_t^d}{R_t^d} \right) = \left( \frac{R_{t-1}^d}{R_t^d} \right)^{\rho_r} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\psi_r} \left( \frac{gdp_t}{gdp_{t-1}} \right)^{\psi_{gdp}} \tag{B.51}
\]

\[
R_t^d = r_t^d \mathbb{E}_t \{ \pi_{t+1} \} \tag{B.52}
\]

**Exogenous Shocks - 3**

\[
\ln (\eta_t^h) = (1-\rho_{\eta^h}) \bar{\eta}^h + \rho_{\eta^h} \ln (\eta_{t-1}^h) + \varepsilon_t^h \tag{B.53}
\]

\[
\ln (z_t) = (1-\rho_z) \bar{z} + \rho_z \ln (z_{t-1}) + \varepsilon_t^z \tag{B.54}
\]

\[
\ln (\sigma_t^\omega) = (1-\rho_{\sigma^\omega}) \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln (\sigma_{t-1}^\omega) + \varepsilon_t^\omega \tag{B.55}
\]
C Equilibrium Model of Unconventional Monetary Policy

In this section I provide the full set of equations which defines the dynamic equilibrium model used for the unconventional monetary policy simulations in Section 7.

Household - 6

\[
\frac{w^h_t}{c_t} = \frac{\eta^h}{1 - l^h_{t\text{new}}} \quad (C.1)
\]

\[
\frac{w_t}{c_t} = \frac{\eta^l}{1 - l_t} \quad (C.2)
\]

\[
\frac{v^h_t}{b_t} = \frac{P^h_t}{c_t} - \beta E_t \left\{ (1 - \delta^h)P^h_{t+1} \right\} \quad (C.3)
\]

\[
\frac{1}{r^d_t} = E_t \{ \Lambda_{t+1} \} \quad (C.4)
\]

\[
m_t = \eta_m c_t \left( \frac{R^d_t - 1}{R^d_t} \right) \quad (C.5)
\]

\[
\Lambda_t = \beta \frac{c_{t-1}}{c_t} \quad (C.6)
\]

Aggregate Goods Production - 7

\[
y_t = z_t (k_{t-1})^{\alpha_g} (l^l_t)^{1-\alpha_g} \quad (C.7)
\]

\[
l^l_t = (l_t)^{(1-\alpha_e)} (l^m_t)^{\alpha_e} \quad (C.8)
\]

\[
w_t = (1 - \alpha_g)(1 - \alpha_e) \frac{y_t}{l_t} \frac{1}{\mu^r} \quad (C.9)
\]

\[
w^e_t = (1 - \alpha_g)\alpha_e \frac{y_t}{l^e_t} \frac{1}{\mu^r} \quad (C.10)
\]

\[
r^k_t = \alpha_g \frac{y_t}{k_t} \frac{1}{\mu^r} + (1 - \delta^k) \quad (C.11)
\]

\[
c^e_t = (1 - \alpha^n_t)(1 - \Gamma_{t-1}(\bar{\omega}_t)) r^k_t k_{t-1} \quad (C.12)
\]

\[
\alpha^n_t = \frac{(1 - F_{t-1}(\bar{\omega}_t)) p^n_{t-1} r^d_{t-1} \bar{n}}{(1 - \Gamma_{t-1}(\bar{\omega}_t)) r^k_t k_{t-1}} \quad (C.13)
\]

Capital Producers - 1

\[
i_t = k_t - (1 - \delta^k)k_{t-1} \quad (C.14)
\]
Goods Production: Firm-level Debt Contract - 9

\begin{align*}
\mathbb{E}_t \{ \Gamma_t'(\bar{\omega}_{t+1}) r_{t+1}^k k_t \} &= \lambda_t^e \mathbb{E}_t \{ (\Gamma_t'(\bar{\omega}_{t+1}) - \mu^a G_t'(\bar{\omega}_{t+1})) r_{t+1}^k k_t \} \quad (C.15) \\
\mathbb{E}_t \{[1 - \Gamma_t'(\bar{\omega}_{t+1})] r_{t+1}^k \} &= \lambda_t^e \mathbb{E}_t \{ r_{t+1}^k - \Gamma_t'(\bar{\omega}_{t+1}) r_{t+1}^k \} \\
+ \lambda_t^e \mathbb{E}_t \{ \mu^a G_t'(\bar{\omega}_{t+1}) r_{t+1}^k \} \quad (C.16) \\
r_{t}^k (k_t - p_t^n \bar{n} - w_{t}^c) &= \mathbb{E}_t \{ (\Gamma_t'(\bar{\omega}_{t+1}) - \mu^a G_t'(\bar{\omega}_{t+1})) r_{t+1}^k k_t \} \\
b_t &= k_t - p_t^n \bar{n} - w_{t}^c \quad (C.17) \\
z_{t+1} &= \frac{\ln(\bar{\omega}_{t+1}) + .5 (\sigma_{t}^o)^2}{\sigma_{t}^o} \quad (C.19) \\
G_t'(\bar{\omega}_{t+1}) &= \Phi^N(z_{t+1} - \sigma_{t}^o) \quad (C.20) \\
\Gamma_t'(\bar{\omega}_{t+1}) &= \Phi^N(z_{t+1} - \sigma_{t}^o) + \bar{\omega}_{t+1} (1 - \Phi^N(z_{t+1})) \quad (C.21) \\
G_t'(\bar{\omega}_{t+1}) &= \left( \frac{1}{\sigma_{t}^o \sqrt{2\pi}} \right) e^{-\frac{z_{t+1}^2}{2}} \quad (C.22) \\
\Gamma_t'(\bar{\omega}_{t+1}) &= 1 - \Phi^N(z_{t+1}) \quad (C.23)
\end{align*}

Housing Production: - 2

\begin{align*}
h_{t}^{new} &= z_{t}^{h}(l_{t}^{h})^{(1-\alpha_h)} \quad (C.24) \\
w_{t}^{h} &= p_{t}^{h} (1 - \alpha_h) \frac{h_{t}^{new}}{l_{t}^{h}} \quad (C.25)
\end{align*}

Complex Bank - 8

\begin{align*}
bk_{t}^{c} &= n_{t}^{c} \mathbb{E}_t \left\{ \frac{z_{t}^{*} p_{t+1}^{h} - (1 - F_t(\bar{\omega}_{t+1})) p_{t+1}^{h,k} d_t}{p_t^{n}} \right\} \quad (C.26) \\
\mathbb{E}_t \{ \Lambda_{t+1} r_{t}^{c,e} \} &= \mathbb{E}_t \{ \Lambda_{t+1} r_{t}^{d} \} \quad (C.27) \\
\mathbb{E}_t \{ \Lambda_{t+1} r_{t}^{c,e} \} &= \left( \frac{\theta_b - 1}{\theta_b} \right) \mathbb{E}_t \{ \Lambda_{t+1} (1 - F_t(\bar{\omega}_{t+1})) \} \quad (C.28) \\
+ \mathbb{E}_t \left\{ \Lambda_{t+1} \frac{(1 - \mu^a) \phi_t(\bar{\omega}_{t+1})}{b_t} \right\} \\
b_{t}^{c} &= \left( \frac{r_{t}^{c,e}}{r_{t}^{b,c}} \right)^{-\theta_b} b_t \quad (C.29) \\
b_{t}^{c} &= d_t^{c} + \frac{bk_{t-1}^{c} - p_t^{n} n_{t}^{c}}{\pi_t} \quad (C.30)
\end{align*}

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\[
\Pi_t^c = (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^{b,c} b_{t-1}^c \\
+ (1 - \mu^a) \frac{b_{t-1}^c}{b_{t-1}} \phi_t(\bar{\omega}_{t+1}) \\
+ (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^d p_{t-1}^n n_{t-1}^c \\
+ F_{t-1}(\bar{\omega}_t) p_t^b z^n_t n_{t-1}^c - p_t^n n_{t}^c \\
- r_{t-1}^d d_{t-1}^c + d_t^c - b_t^c \\
\]
\[
i_t^{bk,c} = z_t^c \\
b_t^{bk,c} = t_t^{bk,c} + (1 - \delta^{bk}) \frac{bk_{t-1}^s}{\pi_t} \\
\]

Simple Bank - 8
Aggregate Bank - 6

\[ r_t^c = \left[ \nu \left( \bar{r}_t^{c,U} \right)^{1-\theta_b} + (1 - \nu) \left( \bar{r}_t^{c,P} \right)^{1-\theta_b} \right]^{1/1-v_b} \] \hspace{1cm} (C.42)

\[ r_t^c = \left( \frac{\theta_b}{\theta_b - 1} \right) \bar{r}_t^c - \left( \frac{1}{\theta_b - 1} \right) (1 - \mu^c) \phi_t(\bar{\omega}_{t+1}) \] \hspace{1cm} (C.43)

\[ r_t^b = \left[ \nu \left( r_t^{b,s} \right)^{1-\theta_b} + (1 - \nu) \left( r_t^{b,p} \right)^{1-\theta_b} \right]^{1/1-v_b} \] \hspace{1cm} (C.44)

\[ d_t = \nu d_t^b + (1 - \nu) d_t^r \] \hspace{1cm} (C.45)

\[ i_t^{bk} = \nu i_t^{bk,s} + (1 - \nu) i_t^{bk,c} \] \hspace{1cm} (C.46)

\[ \phi_{t-1}(\bar{\omega}_t) = G_{t-1}(\bar{\omega}_t) r_t^k p_{t-1} k_{t-1} \] \hspace{1cm} (C.47)

Market Clearing - 4

\[ h_t = h_t^{new} + F_{t-1}(\bar{\omega}_t) z^r (\nu n_t^c + (1 - \nu) n_t^s + n_t^{cb,sell}) + (1 - \delta^h) h_{t-1} \] \hspace{1cm} (C.48)

\[ \bar{n} = \nu n_t^c + (1 - \nu) n_t^s + n_t^{cb} \] \hspace{1cm} (C.49)

\[ y_t = c_t + c_t^e + i_t + i_t^{bk} + \mu^a \phi_{t-1}(\bar{\omega}_t) + \nu F_{t-1}(\bar{\omega}_t) \mu^{l,s} (n_t^{l,s})^{\chi^{l,s}} + \nu \chi^{b,s} b_t^s \] \hspace{1cm} (C.50)

\[ gdp_t = c_t + i_t + p_t^h h_t^{new} \] \hspace{1cm} (C.51)

Monetary Policy - 12

\[
\left( \frac{R_t}{\tilde{R}_t} \right) = \left( \frac{R_{t-1}^d}{\tilde{R}_{t-1}^d} \right)^{\rho^c} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\psi^s} \left( \frac{gdp_t}{gdp_{t-1}} \right)^{\psi_{gdp}}
\]

\[ = n_t^{cb,\text{buy}} + n_t^{cb,\text{taper}} \] \hspace{1cm} (C.52)

\[ n_t^{cb,\text{buy}} = n_{t-1}^{cb,\text{buy}} + \varepsilon_{t-2}^{n,cb,\text{buy}} \] \hspace{1cm} (C.53)

\[ n_t^{cb,\text{taper}} = \rho^\text{taper} n_{t-1}^{cb,\text{taper}} + \varepsilon_{t-2}^{n,cb,\text{taper}} \] \hspace{1cm} (C.54)

\[ n_t^{cb,\text{stock}} = F_{t-1}(\bar{\omega}_t) n_{t-1}^{cb} + \varepsilon_{t-1}^{n,cb,\text{sell}} + n_{t-1}^{cb,\text{stock}} + n_{t-1}^{cb,\text{sell}} \] \hspace{1cm} (C.55)

\[ n_t^{cb,\text{sell}} = \rho^\text{selling} n_{t-1}^{cb,\text{stock}} + \varepsilon_{t-1}^{n,cb,\text{sell}} \] \hspace{1cm} (C.56)

\[ b_t^{\text{inj}} = \varepsilon_t^{bk,\text{inj}} \] \hspace{1cm} (C.57)

\[ p_t n_t^{cb} = m_t + \frac{nu_{t-1}^{cb}}{\bar{\pi}_t} - \delta^{cb} - b^{\text{inj}} \] \hspace{1cm} (C.58)
\[ \Pi_t^{cb} = m_t - d_t^{cb} - \frac{m_{t-1}}{\pi_t} + d_{t-1}^{cb} r_{t-1}^{d} - p^n n_t^{cb} \]
\[ + (1 - F_{t-1}(\bar{\omega}_t)) r_{t-1}^{d} p_{t-1}^n n_t^{cb} + p_t^h z_t^{cb} \]
\[ + \frac{\delta^{inj} b_k^{cb}_{t-1}}{\pi_t} - b_k^{inj} \]
\[ nw_t^{cb} = \Pi_t^{cb} + (1 - \tau^{cb}) \left( n w_{t-1}^{cb} \right) \]
\[ \tau_t = \tau^{cb} \frac{n w_{t-1}^{cb}}{\pi_t} - \Pi_t^{cb} \]  \hspace{1cm} (C.60, C.61, C.62)

**Exogenous Shocks - 3**

\[ \ln(\eta_t^h) = (1 - \rho_\eta^h) \bar{\eta}^h + \rho_\eta^h \ln(\eta_{t-1}^h) + \varepsilon_t^\eta^h \]  \hspace{1cm} (C.63)
\[ \ln(z_t) = (1 - \rho_z) \bar{z} + \rho_z \ln(z_{t-1}) + \varepsilon_t^z \]  \hspace{1cm} (C.64)
\[ \ln(\sigma_t^\omega) = (1 - \rho_{\sigma^\omega}) \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln(\sigma_{t-1}^\omega) + \varepsilon_t^{\sigma^\omega} \]  \hspace{1cm} (C.65)
D Bernanke, Gertler and Gilchrist (1999) Equilibrium Model

In this section I provide the full set of equations which defines the dynamic equilibrium model of Bernanke et al. (1999) augmented with a simple housing sector.

Household - 5

\[
\frac{w^h_t}{c_t} = \frac{\eta^h}{1 - l_t^{new}} \tag{D.1}
\]

\[
\frac{w_t}{c_t} = \frac{\eta^l}{1 - l_t} \tag{D.2}
\]

\[
\frac{\eta^h_t}{h_t} = \frac{p^h_t}{c_t} - \beta E_t \left\{ \frac{(1 - \delta^h)p^h_{t+1}}{c_{t+1}} \right\} \tag{D.3}
\]

\[
\frac{1}{\rho_t^d} = E_t \{ \Lambda_{t+1} \} \tag{D.4}
\]

\[
\Lambda_t = \beta \frac{c_{t-1}}{c_t} \tag{D.5}
\]

Aggregate Goods Production - 8

\[
y_t = z_t \left( k_{t-1} \right)^{\alpha_g} \left( l^g_t \right)^{1 - \alpha_g} \tag{D.6}
\]

\[
l^g_t = \left( l_t \right)^{1 - \alpha_e} \left( l^c_t \right)^{\alpha_E} \tag{D.7}
\]

\[
w_t = (1 - \alpha_g)(1 - \alpha_e) \frac{y_t}{l^c_t} \frac{1}{\mu^r} \tag{D.8}
\]

\[
w^e_t = (1 - \alpha_g) \frac{y_t}{l^c_t} \frac{1}{\mu^r} \tag{D.9}
\]

\[
r_t^k = \frac{\alpha_g \frac{w_t}{k_t} \frac{1}{\mu^r} + (1 - \delta^k)q_t}{q_{t-1}} \tag{D.10}
\]

\[
c^e_t = (1 - \gamma^e) v_t \tag{D.11}
\]

\[
v_t = (1 - \Gamma_t)(\bar{\omega}_t) r_t^k q_{t-1} k_{t-1} \tag{D.12}
\]

\[
nw_t = \gamma^e v_t + w^e_t \tag{D.13}
\]

Capital Producers - 3

\[
i_t = k_t - (1 - \delta^k) k_{t-1} \tag{D.14}
\]
\[ q_t = 1 + \chi_{ac} \left( \frac{i_t}{k_{t-1}} - \delta^k \right) \quad \text{(D.15)} \]

\[ ac_t^k = \frac{\chi_{ac}}{2} \left( \frac{i_t}{k_{t-1}} - \delta^k \right)^2 \quad \text{(D.16)} \]

**Goods Production: Firm-level Debt Contract - 9**

\[ E_t \left\{ (\Gamma_t'(\bar{\omega}_{t+1}) - \mu^a G_t'(\bar{\omega}_{t+1})) r^k_{t+1} q_t k_t \right\} = \lambda e E_t \left\{ \mu^a G_t'(\bar{\omega}_{t+1}) r^k_{t+1} q_t k_t \right\} \quad \text{(D.17)} \]

\[ E_t \left\{ (1 - \Gamma_t(\bar{\omega}_{t+1})) r^k_{t+1} \right\} = \lambda e E_t \left\{ \mu^a G_t(\bar{\omega}_{t+1}) r^k_{t+1} q_t k_t \right\} + \lambda e E_t \left\{ \mu^a G_t'(\bar{\omega}_{t+1}) r^k_{t+1} q_t k_t \right\} \quad \text{(D.18)} \]

\[ r^d_t (q_t k_t - n w_t) = E_t \left\{ (\Gamma_t'(\bar{\omega}_{t+1}) - \mu^a G_t'(\bar{\omega}_{t+1})) r^k_{t+1} q_t k_t \right\} \quad \text{(D.19)} \]

\[ b_t = q_t k_t - n w_t \quad \text{(D.20)} \]

\[ \phi_{t-1}(\bar{\omega}_t) = G_{t-1}(\bar{\omega}_t) r^k_{t-1} k_{t-1} \quad \text{(D.21)} \]

\[ z_{t+1} = \frac{\ln(\bar{\omega}_{t+1}) + 0.5 (\sigma^w)^2}{\sigma^w} \quad \text{(D.22)} \]

\[ G_t'(\bar{\omega}_{t+1}) = \Phi_N(z_{t+1} - \sigma^w_t) \quad \text{(D.23)} \]

\[ \Gamma_t'(\bar{\omega}_{t+1}) = \Phi_N(z_{t+1} - \sigma^w_t) + \bar{\omega}_{t+1} (1 - \Phi_N(z_{t+1})) \quad \text{(D.24)} \]

\[ G_t'(\bar{\omega}_{t+1}) = \frac{1}{\sigma^w_t \sqrt{2\pi}} e^{-\frac{z_{t+1}^2}{2}} \quad \text{(D.25)} \]

\[ \Gamma_t'(\bar{\omega}_{t+1}) = 1 - \Phi_N(z_{t+1}) \quad \text{(D.26)} \]

**Housing Production: - 2**

\[ h^e_t = z_t^h (l^h_t)^{1-\alpha_h} \quad \text{(D.27)} \]

\[ w^h_t = p^h_t (1 - \alpha_h) \frac{h^e_t}{l^h_t} \quad \text{(D.28)} \]

**Market Clearing - 3**

\[ h_t = h^e_t + (1 - \delta^h) h_{t-1} \quad \text{(D.29)} \]

\[ y_t = c_t + c^e_t + i_t + a c^k_t + \mu^a \phi_{t-1}(\bar{\omega}_t) \quad \text{(D.30)} \]

\[ gdp_t = c_t + i_t + p^h_t h^e_t \quad \text{(D.31)} \]
Monetary Policy - 2

$$\left( \frac{R_t^d}{R^d} \right) = \left( \frac{R_{t-1}^d}{R^d} \right)^{\rho_r} \left( \frac{\pi_t}{\pi} \right)^{\psi_r} \left( \frac{gdpt}{gdpt_{-1}} \right)^{\psi_{gdp}}$$ \quad (D.32)

$$R_t^d = r_t^d \mathbb{E}_t \{ \pi_{t+1} \}$$ \quad (D.33)

Exogenous Shocks - 3

$$\ln (\eta^h_t) = \left( 1 - \rho_{\eta^h} \right) \eta^h + \rho_{\eta^h} \ln (\eta^h_{t-1}) + \varepsilon^h_t$$ \quad (D.34)

$$\ln (z_t) = \left( 1 - \rho_z \right) z + \rho_z \ln (z_{t-1}) + \varepsilon^z_t$$ \quad (D.35)

$$\ln (\sigma^\omega_t) = \left( 1 - \rho_{\sigma^\omega} \right) \sigma^\omega + \rho_{\sigma^\omega} \ln (\sigma^\omega_{t-1}) + \varepsilon^\sigma_t$$ \quad (D.36)
## E Calibration Results

Table 3: Baseline Model Calibration Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td>( \beta ) 0.9875</td>
</tr>
<tr>
<td>Disutility of Goods Labor</td>
<td>( \eta^l ) 1.43</td>
</tr>
<tr>
<td>Disutility of Housing Labor</td>
<td>( \eta^h ) 4.60</td>
</tr>
<tr>
<td>Housing Demand Shock SS</td>
<td>( \eta^{hs} ) 0.22</td>
</tr>
<tr>
<td>Housing Depreciation Rate</td>
<td>( \delta^h ) 0.0189</td>
</tr>
<tr>
<td>Labor’s Share of Goods Production</td>
<td>( 1 - \alpha_g ) 0.70</td>
</tr>
<tr>
<td>Labor’s Share of Housing Production</td>
<td>( 1 - \alpha_g ) 0.80</td>
</tr>
<tr>
<td>Retailer’s Gross Markup</td>
<td>( \mu^r ) 1.2</td>
</tr>
<tr>
<td>Entrepreneur’s Housing Endowment</td>
<td>( \bar{n} ) 1</td>
</tr>
<tr>
<td>Monitoring/Auditing Cost</td>
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<tr>
<td>SS Variance Entrepreneur’s Dispersion</td>
<td>( \bar{\sigma}^\omega ) 0.18</td>
</tr>
<tr>
<td>Share of Simple Banks</td>
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</tr>
<tr>
<td>Bank Capital Depreciation Rate</td>
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<tr>
<td>Marginal Labor Used to Produce a Loan</td>
<td>( \chi^{b,s} ) 0.0009</td>
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<tr>
<td>Bank Loan Elasticity of Substitution</td>
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<tr>
<td>Complex Bank Capital Investment</td>
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<tr>
<td>Simple Bank Capital Investment</td>
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<td>Curvature of Simple Bank Liquidation Cost</td>
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<td>Level of Simple Bank Liquidation Cost</td>
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<td>Level of Bank Liquidation Technology</td>
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<tr>
<td>Level of New Housing Production Technology</td>
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<tr>
<td>Monetary Policy Smoothing Parameter</td>
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<tr>
<td>Monetary Policy Inflation Reaction</td>
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<tr>
<td>Monetary Policy Output Reaction</td>
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<tr>
<td>Technology Shock</td>
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<tr>
<td>Technology Shock</td>
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<td>Housing Demand Shock</td>
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<td>Housing Demand Shock</td>
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<td>Risk Shock</td>
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<tr>
<td>Risk Shock</td>
<td>( \sigma_{\sigma^\omega} ) 0.0491</td>
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Table 4: Unconventional Policy Calibration Results

<table>
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<th>QE - Abrupt End</th>
<th>Parameter</th>
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<tbody>
<tr>
<td>Fraction of CB Net Worth Remitted</td>
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<td>Reinvestment Policy</td>
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<tr>
<td>Start of Purchases</td>
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<tr>
<td>End of Purchases</td>
<td>$\varepsilon^{n,cb,\text{buy}}_{19,t-2}$</td>
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<thead>
<tr>
<th>QE - Slow End</th>
<th>Parameter</th>
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<tbody>
<tr>
<td>Fraction of CB Net Worth Remitted</td>
<td>$\tau^{cb}$</td>
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<tr>
<td>Pace of Balance Sheet Unwinding</td>
<td>$\rho^{selling}$</td>
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<tr>
<td>Reinvestment Policy</td>
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<tr>
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<tr>
<td>End of Purchases</td>
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<tr>
<td>End of Purchases</td>
<td>$\varepsilon^{n,cb,\text{taper}}_{19,t-2}$</td>
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<th>QE - Slow End + Reinvest</th>
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<td>Pace of Balance Sheet Unwinding</td>
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<tr>
<td>Reinvestment Policy</td>
<td>$\rho^{reinvest}$</td>
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<td>$\varepsilon^{n,cb,\text{taper}}_{19,t-2}$</td>
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<thead>
<tr>
<th>QE - Slow End + Reinvest + Slow Sell Off</th>
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<td>Reinvestment Policy</td>
<td>$\rho^{reinvest}$</td>
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<tr>
<td>Start of Purchases</td>
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<tr>
<th>Equity Injections</th>
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<tr>
<td>Fraction of CB Net Worth Remitted</td>
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<td>Pace of Equity Payback</td>
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<td>Initial of Injection</td>
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<tr>
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<tr>
<td>Discount Rate</td>
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<tr>
<td>Disutility of Goods Labor</td>
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<tr>
<td>Disutility of Housing Labor</td>
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<tr>
<td>Housing Demand Shock SS</td>
<td>$\eta^h$ 0.17</td>
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<tr>
<td>Housing Depreciation Rate</td>
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<td>Labor’s Share of Goods Production</td>
<td>$1 - \alpha_g$ 0.70</td>
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<tr>
<td>Labor’s Share of Housing Production</td>
<td>$1 - \alpha_g$ 0.80</td>
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<tr>
<td>Retailer’s Gross Markup</td>
<td>$\mu^r$ 1.2</td>
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<tr>
<td>Entrepreneur’s Survival Probability</td>
<td>$\gamma^e$ 0.9545</td>
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<tr>
<td>Monitoring/Auditing Cost</td>
<td>$\mu^a$ 0.208</td>
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<tr>
<td>SS Variance Entrepreneur’s Dispersion</td>
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<tr>
<td>Capital Adjustment Cost Elasticity</td>
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<tr>
<td>Monetary Policy Smoothing Parameter</td>
<td>$\rho_r$ 0.85</td>
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<td>Monetary Policy Inflation Reaction</td>
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<tr>
<td>Monetary Policy Output Reaction</td>
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<td>Technology Shock</td>
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<tr>
<td>Risk Shock</td>
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