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Determinacy under Inflation Targeting Interest Rate Policy in a Sticky Price Model with Investment (and Labor Bargaining)

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Determinacy under Inflation Targeting Interest Rate Policy in a Sticky Price Model with Investment (and Labor Bargaining)[*]

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Abstract

In a sticky price model with investment spending, recent research shows that inflation-forecast targeting interest rate policy makes determinacy of equilibrium essentially impossible. We examine a necessary and sufficient condition for determinacy under interest rate policy that responds to a weighted average of an inflation forecast and current inflation. This condition demonstrates that the average-inflation targeting policy ensures determinacy as long as both the response to average inflation and the relative weight of current inflation are large enough. We also find that interest rate policy which responds solely to past inflation guarantees determinacy when its response satisfies the Taylor principle and is not large. These results still hold even when wages and hours worked are determined by Nash bargaining.

Keywords: inflation targeting interest rate policy, investment, indeterminacy of equilibrium, cost channel of monetary policy, labor bargaining.

JEL codes: E22, E24, E52

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INTRODUCTION

Recent research has shown that investment activity induces a critical implication for inflation targeting interest rate policy in sticky price models. Carlstrom and Fuerst (2005) derive a necessary condition for (local) determinacy of equilibrium in a Calvo (1983)-style sticky price model and find that inflation-forecast targeting interest rate policy makes determinacy essentially impossible.[1] Even when the policy response to an inflation forecast is active (i.e. satisfies the Taylor principle), indeterminacy is induced due to a cost channel of monetary policy that stems from investment spending, as emphasized by Kurozumi and Van Zandweghe (2008). This channel leads a rise in the real interest rate to increase the expected future real rental price of capital via a no-arbitrage condition between bonds and capital and hence to raise the expected future real marginal cost of production, which feeds into inflation expectations in the New Keynesian Phillips curve. Consequently, active policy responses to the inflation forecast, which increase the real interest rate, make inflation expectations self-fulfilling, thereby causing indeterminacy.[2]

This indeterminacy is a critical issue for central banks, since they are concerned about expected future inflation rather than current inflation, as indicated by empirical studies such as Clarida, Galí and Gertler (1998, 2000). We therefore take as given the empirical finding that interest rate policy is based at least to some degree on an inflation forecast. Because Carlstrom and Fuerst (2005) show that the policy response to current inflation possesses desirable properties in terms of determinacy in a sticky price model with investment, we investigate interest rate policy that targets a weighted average of an inflation forecast and current inflation.[3] Specifically, we examine a necessary and sufficient condition for this interest rate policy

[1] Huang and Meng (2007) obtain a similar indeterminacy result using a quadratic price adjustment cost model.

[2] To avoid this indeterminacy, Kurozumi and Van Zandweghe (2008) propose introducing a response to current output or interest rate smoothing into the policy. Huang, Meng, and Xue (2009) numerically investigate the role of the policy response to current output for determinacy in a Calvo-style sticky price model with sticky wages and firm-specific capital.

[3] Determinacy properties of the average-inflation targeting interest rate policy are also analyzed by Zanetti

to ensure determinacy. This condition demonstrates that the average-inflation targeting policy guarantees determinacy as long as both the response to average inflation and the relative weight of current inflation are large enough. The average-inflation targeting policy inherits the determinacy properties of current-inflation targeting interest rate policy, which ensures determinacy when its response is sufficiently large. We also show that past-inflation targeting interest rate policy guarantees determinacy when its response satisfies the Taylor principle and is not large.

These results still hold, even when wages and hours worked are determined by Nash bargaining as Zanetti (2006) suggests, rather than being determined in a competitive labor market as our baseline model assumes. Recent literature has seen a surge of interest in the role of labor markets in sticky prices models. The bulk of this literature has introduced labor market search and matching frictions along the lines of Mortensen and Pissarides (1994).[4] Specifically, firms pay a cost of posting vacancies in order to adjust their employment, and wages and hours worked are determined by bargaining between firms and workers. In sticky price models with investment, however, incorporating such labor market frictions makes determinacy conditions hard to examine analytically, because it adds one more predetermined variable, i.e. employment.[5] Adopting the Nash bargaining over wages and hours worked is thus motivated as a first step toward the analysis of determinacy in sticky price models with investment and labor market search and matching frictions while retaining the analytical characterization of determinacy conditions. Because the Nash bargaining outcome is privately efficient, hours worked satisfy the same condition as in the baseline model while wages play only a distributive role. Then, equilibrium dynamics is independent of an equilibrium condition for wages, as shown later. Therefore, the model with Nash bargaining has exactly the same determinacy properties

(2006). Nessén and Vestin (2005) study inflation and output gap variability under outcome-based average-inflation targeting policies that take the form of targeting rules rather than instrument rules.

[4]See e.g. Krause and Lubik (2007), Trigari (2009), and Walsh (2005).

[5]In Kurozumi and Van Zandweghe (2010), we find that in a sticky price model without investment, labor market search and matching frictions almost always induce indeterminacy of equilibrium under inflation-forecast targeting interest rate policy which satisfies the Taylor principle.

as the baseline model.[6]

The remainder of the paper proceeds as follows. The next section presents the model, Section 2 shows the results, and Section 3 concludes.

1 THE MODEL

The model is a generalization of Carlstrom and Fuerst (2005) in that interest rate policy responds to a weighted average of an inflation forecast and current inflation. In the economy there are a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a monetary authority. This section describes each agent's behavior in turn.

The household is infinitely lived with preferences over consumption c_t, real money balances $m_t = M_t/P_t$, and hours worked L_t, represented by

$$\sum_{t=0}^{\infty} \beta^t U(c_t, m_t, L_t) = \sum_{t=0}^{\infty} \beta^t \left[V(c_t, m_t) - L_t\right],$$

where $\beta \in (0, 1)$ is the subjective discount factor, M_t is nominal money balances held at the end of period t, and P_t is the price of final goods. The period utility function U is separable between hours worked and the other arguments, and the elasticity of labor supply is infinite.

The household enters period t with a capital stock K_{t-1}, nominal money balances M_{t-1}, and nominal one-period bonds B_{t-1}, which pay the gross nominal interest rate R_{t-1}. The household starts period t by trading bonds and renting out capital and labor respectively at the real rental price r_t and at the real wage rate w_t. Subsequently, the household purchases final goods for consumption c_t and investment $[K_t - (1-\delta)K_{t-1}]$, where $\delta \in (0, 1)$ is the depreciation rate of capital. The household receives profits D_t from firms and a lump-sum transfer T_t from the monetary authority. Thus, the household faces its budget constraint

$$M_t + B_t + P_t c_t + P_t[K_t - (1-\delta)K_{t-1}] = M_{t-1} + B_{t-1}R_{t-1} + P_t(w_t L_t + r_t K_{t-1}) + D_t + T_t.$$

The first-order conditions for the household's optimal decisions on labor supply, investment,

[6] This demonstrates the failure of Proposition 1 of Zanetti (2006), as explained later.

consumption, and money holdings are given by

$$\frac{1}{U_c(t)} = w_t, \tag{1}$$

$$U_c(t) = \beta U_c(t+1)\left(r_{t+1} + 1 - \delta\right), \tag{2}$$

$$U_c(t) = \beta U_c(t+1)\frac{R_t}{\pi_{t+1}}, \tag{3}$$

$$\frac{U_m(t)}{U_c(t)} = \frac{R_t - 1}{R_t}, \tag{4}$$

where $U_c(t)$ and $U_m(t)$ denote the marginal utility of consumption and real money balances in period t and $\pi_t = P_t/P_{t-1}$ is the gross inflation rate.

On the production side, there is a continuum of intermediate-good firms $i \in [0,1]$, each of which produces one kind of differentiated good and sells the product to the representative final-good firm under monopolistic competition.

The final-good firm produces output Y_t by choosing a combination of intermediate inputs $\{y_t(i)\}$ so as to maximize profits $P_tY_t - \int_0^1 P_t(i)y_t(i)di$ given intermediate-good prices $\{P_t(i)\}$ and the CES production technology $Y_t = (\int_0^1 y_t(i)^{(\eta-1)/\eta}di)^{\eta/(\eta-1)}$ with the elasticity $\eta > 1$. The first-order conditions for the final-good firm's profit maximization imply that its demand for each intermediate good is $y_t(i) = Y_t(P_t(i)/P_t)^{-\eta}$, while perfect competition in the final-good market implies that its price satisfies

$$P_t = \left(\int_0^1 P_t(i)^{1-\eta}di\right)^{\frac{1}{1-\eta}}. \tag{5}$$

Also, the final-good market clearing condition is

$$Y_t = c_t + K_t - (1-\delta)K_{t-1}. \tag{6}$$

Each intermediate-good firm i produces output $y_t(i)$ by choosing a cost-minimizing combination of capital and labor for the pair of the real rental price and the real wage rate $\{r_t, w_t\}$ and the Cobb-Douglas production technology $y_t(i) = (K_{t-1}(i))^{\alpha}(L_t(i))^{1-\alpha}$ with the cost share of capital $\alpha \in (0,1)$. In the presence of competitive labor and rental capital markets, all intermediate-good firms choose an identical capital-labor ratio and face the same real marginal cost z_t. Therefore, it follows that $r_t = z_t \alpha x_t^{1-\alpha}$ and

$$w_t = \frac{z_t(1-\alpha)}{x_t^{\alpha}}, \tag{7}$$

where $x_t = L_t/K_{t-1}$. Also, aggregating the Cobb-Douglas production technology over intermediate-good firms yields $Y_t d_t = K_{t-1}^{\alpha} L_t^{1-\alpha} = K_{t-1} x_t^{1-\alpha}$, where $d_t = \int_0^1 (P_t(i)/P_t)^{-\eta} di$ measures the intermediate-good price dispersion, and hence the final-good market clearing condition (6) becomes

$$\frac{K_{t-1} x_t^{1-\alpha}}{d_t} = c_t + K_t - (1-\delta) K_{t-1}. \tag{8}$$

Combining the labor supply condition (1) and the real wage rate condition (7) yields

$$U_c(t) = \frac{x_t^{\alpha}}{z_t(1-\alpha)}, \tag{9}$$

and hence the investment and consumption Euler equations (2) and (3) can be rewritten as

$$\frac{x_t^{\alpha}}{z_t} = \beta \frac{x_{t+1}^{\alpha}}{z_{t+1}} \left(z_{t+1} \alpha x_{t+1}^{1-\alpha} + 1 - \delta \right), \tag{10}$$

$$\frac{x_t^{\alpha}}{z_t} = \beta \frac{x_{t+1}^{\alpha}}{z_{t+1}} \frac{R_t}{\pi_{t+1}}. \tag{11}$$

Facing the final-good firm's demand, each intermediate-good firm sets the price of its product as in Calvo (1983). Each period a fraction $1-\nu \in (0,1)$ of intermediate-good firms can reoptimize prices, while the remaining fraction ν charges the previous-period prices adjusted for steady-state gross inflation π. Thus all intermediate-good firms that can reoptimize prices in period t face the same problem

$$\max_{P_t(i)} \sum_{j=0}^{\infty} \nu^j \frac{\beta^j \Lambda_{t+j}}{\Lambda_t} \left[Y_{t+j} \left(\frac{P_t(i)\, \pi^j}{P_{t+j}} \right)^{-\eta} \right] \left(P_t(i)\, \pi^j - P_{t+j} z_{t+j} \right),$$

where $\Lambda_t = U_c(t)/P_t$ is the marginal utility of one dollar. The first-order condition for this problem is given by

$$P_t(i) = \frac{\eta}{\eta-1} \frac{\sum_{j=0}^{\infty} (\nu\beta\pi^{-\eta})^j \Lambda_{t+j} P_{t+j}^{\eta+1} Y_{t+j} z_{t+j}}{\sum_{j=0}^{\infty} (\nu\beta\pi^{1-\eta})^j \Lambda_{t+j} P_{t+j}^{\eta} Y_{t+j}}. \tag{12}$$

The monetary authority conducts Taylor (1993)-style interest rate policy that responds to a weighted average of an inflation forecast and current inflation

$$\log R_t = \log R + \tau \left[\gamma \log \pi_{t+1} + (1-\gamma) \log \pi_t - \log \pi \right], \tag{13}$$

where R denotes the steady-state gross nominal interest rate, $\tau > 0$ is the policy response to the weighted average inflation, and $\gamma \in [0,1]$ is the weight of the inflation forecast relative to current inflation. This policy is a generalization of Carlstrom and Fuerst (2005), who consider the two special cases of $\gamma = 0, 1$. While Carlstrom and Fuerst report that the policy response to current inflation possesses desirable properties in terms of determinacy in a sticky price model with investment, empirical studies such as Clarida, Galí, and Gertler (1998, 2000) suggest that central banks are concerned about expected future inflation rather than current inflation. The average-inflation targeting policy takes this concern into account by placing weight on the inflation forecast and simultaneously inherits the determinacy properties of the current-inflation targeting policy.

The equilibrium conditions are now given by (4), (5), (7)$-$(13), and the law of motion of the intermediate-good price dispersion d_t. Since this dispersion is of second order under the Calvo-style staggered price-setting and its steady-state value is one, log-linearizing the equilibrium conditions and rearranging the resulting equations yields

$$\hat{w}_t = \hat{z}_t - \alpha \hat{x}_t, \tag{14}$$

$$\hat{m}_t = \eta_c \hat{c}_t - \eta_R \hat{R}_t, \tag{15}$$

$$\hat{K}_t = (1 + c/K)\hat{K}_{t-1} + (1-\alpha)(Y/K)\hat{x}_t - (c/K)\hat{c}_t, \tag{16}$$

$$-\sigma^{-1}\hat{c}_t + \chi \hat{m}_t = \alpha \hat{x}_t - \hat{z}_t, \tag{17}$$

$$\alpha \hat{x}_t - \hat{z}_t = [1 - \beta(1-\delta)(1-\alpha)]\hat{x}_{t+1} - \beta(1-\delta)\hat{z}_{t+1}, \tag{18}$$

$$\alpha \hat{x}_t - \hat{z}_t = \hat{R}_t + \alpha \hat{x}_{t+1} - \hat{\pi}_{t+1} - \hat{z}_{t+1}, \tag{19}$$

$$\hat{\pi}_t = \lambda \hat{z}_t + \beta \hat{\pi}_{t+1}, \tag{20}$$

$$\hat{R}_t = \tau \left[\gamma \hat{\pi}_{t+1} + (1-\gamma)\hat{\pi}_t \right], \tag{21}$$

where hatted variables denote log-deviations from steady-state values, $\eta_c, \eta_R > 0$ measure the consumption elasticity and the interest rate semielasticity of money demand, $c/K, Y/K > 0$ are steady-state ratios of consumption and output to capital, $\sigma > 0$ measures the intertemporal elasticity of substitution in consumption, χ represents the degree of non-separability of the period utility function between consumption and real money balances, and $\lambda = (1-\nu)(1-\beta\nu)/\nu > 0$ is the real marginal cost elasticity of inflation.

Combining (15)$-$(17) to substitute out $\hat{c}_t$ and $\hat{m}_{t+1}$, we obtain

$$\begin{aligned}\hat{K}_t &= (1+c/K)\hat{K}_{t-1} + \{(1-\alpha)(Y/K) + [\alpha\sigma/(1-\sigma\eta_c\chi)](c/K)\}\,\hat{x}_t \\ &\quad - [\sigma/(1-\sigma\eta_c\chi)](c/K)\hat{z}_t + [\sigma\eta_R\chi/(1-\sigma\eta_c\chi)](c/K)\hat{R}_t. \qquad (22)\end{aligned}$$

Since the real wage rate condition (14) is static and the rate $\hat{w}_t$ appears only there, equilibrium dynamics is determined by the system of five equations (18)$-$(22) with five variables $\hat{\pi}_t, \hat{x}_t, \hat{z}_t, \hat{K}_{t-1}, \hat{R}_t$.

2 RESULTS

In this section, we present and illustrate a necessary and sufficient condition for determinacy of equilibrium under the average-inflation targeting interest rate policy. We then examine whether past-inflation targeting interest rate policy ensures determinacy. Moreover, we investigate whether introducing Nash bargaining over wages and hours worked into the model alters implications for determinacy.

2.1 NECESSARY AND SUFFICIENT CONDITION FOR DETERMINACY

Analyzing the system of five log-linearized equilibrium conditions (18)$-$(22) with five variables $\hat{\pi}_t, \hat{x}_t, \hat{z}_t, \hat{K}_{t-1}, \hat{R}_t$ leads to the following proposition, which provides a necessary and sufficient condition for determinacy under the average-inflation targeting interest rate policy (21).

Proposition 1 *Let $a_1 = 1-\beta(1-\delta)(1-\alpha)$ and $a_2 = 1-\beta(1-\delta)$. In the model, a necessary* *and sufficient condition for determinacy of equilibrium consists of the following three*[7]

$$1 < \tau, \tag{23}$$

$$\tau(2\gamma - 1) < 1 + \frac{2a_2(1+\beta)}{\lambda(a_1+\alpha)}, \tag{24}$$

$$0 < (1-\gamma)\tau^2 + \frac{(1-\gamma)\{\beta(1-\alpha)(a_2)^2 - \alpha[\lambda a_1 + (1-\beta)a_2]\} - \alpha\beta a_2}{\alpha\lambda[(1-\gamma)\alpha + \gamma a_1]}\tau + \frac{\beta a_2[\alpha\lambda + (1-\beta)a_2]}{\alpha\lambda^2[(1-\gamma)\alpha + \gamma a_1]}$$
$$\text{or} \quad 3\beta a_2 \; < \; |\gamma\lambda a_1 \tau - [\lambda a_1 + (1+\beta)a_2]| \, . \tag{25}$$

Proof. See Appendix A. ■

Condition (23) is the Taylor principle, which suggests that the nominal interest rate should be raised by more than the increase in any weighted average of the inflation forecast and current inflation. Note that the necessary and sufficient condition for determinacy depends on the cost share of capital α, the subjective discount factor β, the capital depreciation rate δ, the probability of not reoptimizing price ν, and the policy parameters τ, γ, but not on the other model parameters, e.g. the intertemporal substitution elasticity σ and the money-related parameters η_c, η_R, χ.

In the two special cases of $\gamma = 0, 1$, Proposition 1 can be reduced to the next two corollaries.

Corollary 1 *In the case of inflation-forecast targeting interest rate policy, i.e.* $\gamma = 1$ *in* (21)*, a necessary and sufficient condition for determinacy of equilibrium is the Taylor principle* (23) *and*

$$\tau \; < \; 1 + \frac{a_2}{\lambda} \min\left\{ \frac{1-\beta}{\alpha}, \; \frac{2(1+\beta)}{a_1+\alpha} \right\}. \tag{26}$$

Corollary 2 *In the case of current-inflation targeting interest rate policy, i.e.* $\gamma = 0$ *in* (21)*, a necessary and sufficient condition for determinacy of equilibrium is the Taylor principle* (23) *and*

$$0 \; < \; \tau^2 + \frac{\beta(1-\alpha)(a_2)^2 - \alpha(\lambda a_1 + a_2)}{\alpha^2\lambda}\tau + \frac{\beta a_2[\alpha\lambda + (1-\beta)a_2]}{(\alpha\lambda)^2}. \tag{27}$$

[7]To be precise, this condition is sufficient for determinacy but only generically necessary. Throughout this paper, consideration of non-generic boundary cases is omitted.

Further, if $\lambda a_1 > (2\beta - 1)a_2$, only the Taylor principle (23) *is the necessary and sufficient condition.*[8]

Unlike these Corollaries, Propositions 1 and 2 of Carlstrom and Fuerst (2005) provide a necessary, but not sufficient, condition for determinacy.

To illustrate our necessary and sufficient condition in Proposition 1, we use the same calibration of model parameters as Carlstrom and Fuerst, except the probability of not reoptimizing price ν: $\alpha = 1/3$, $\beta = 0.99$, and $\delta = 0.02$. Since the actual value of ν is controversial in empirical literature, we examine three values of $\nu = 1/2, 2/3, 3/4$, which imply that firms reoptimize prices, on average, once every two, three, and four quarters, respectively.[9] Under these calibrations we have $\lambda a_1 > (2\beta - 1)a_2$. Figure 1 shows a region of the pair of the policy response to average inflation and the relative weight of the inflation forecast $\{\tau, \gamma\}$ that guarantees determinacy. The average-inflation targeting interest rate policy (21) ensures determinacy as long as τ is sufficiently large and γ is sufficiently small, that is, both the response to average inflation and the relative weight of current inflation are large enough. When γ is close to one, the average-inflation targeting policy (21) induces indeterminacy, even if the Taylor principle (23) is satisfied, due to the cost channel of monetary policy explained before.

2.2 PAST-INFLATION TARGETING INTEREST RATE POLICY

We turn next to the analysis of past-inflation targeting interest rate policy

$$\log R_t = \log R + \tau \left(\log \pi_{t-1} - \log \pi \right). \tag{28}$$

Although there is difficulty in explicitly deriving determinacy conditions for this policy, we have the following useful result.

Proposition 2 *In the model with past-inflation targeting interest rate policy* (28)*, the necessary and sufficient condition for determinacy of equilibrium depends on the cost share of capital*

[8] This part of Corollary 2 follows from Proposition 2 of Carlstrom and Fuerst (2005).

[9] Carlstrom and Fuerst (2005) use the value of $\lambda = 1/3$, which implies that $\nu = 0.57$ if $\beta = 0.99$.

α, the subjective discount factor β, the capital depreciation rate δ, the probability of not reoptimizing price ν, and the response to past inflation τ, but not on the other model parameters.

Proof. See Appendix B. ■

The same calibration of model parameters as used above then shows that the past-inflation targeting interest rate policy (28) guarantees determinacy when its response τ satisfies the Taylor principle (23) and is not large. For instance, the three values of the probability of not reoptimizing price, $\nu = 1/2, 2/3, 3/4$, yield the determinacy regions $1 < \tau < 1.35$, $1 < \tau < 2.02$, and $1 < \tau < 3.02$, respectively. The past-inflation targeting policy (28) induces indeterminacy unless it meets the Taylor principle. Even when the Taylor principle is satisfied, this policy generates no stable equilibrium if its response to past inflation is too large.

2.3 NASH BARGAINING OVER WAGES AND HOURS WORKED

Recent literature has seen much interest in implications of labor market search and matching frictions for sticky price models, as mentioned before. In these models, firms pay a cost of posting vacancies in order to adjust their employment, and wages and hours worked are determined by bargaining between firms and workers. This subsection examines whether introducing Nash bargaining over wages and hours worked into our baseline model alters determinacy conditions, as addressed by Zanetti (2006). Because it is difficult to analytically investigate determinacy in sticky price models with investment and labor market search and matching frictions, our model with the Nash bargaining is motivated as a first step toward such an analytical investigation. To this end, a representative wholesale intermediate-good firm and a continuum of retail intermediate-good firms are introduced.[10] The wholesale firm bargains with the representative household over wages and hours worked and produces output y_t under perfect competition

[10] If the original intermediate-good firms bargain with the household, the model is highly intractable due to the firms' Calvo-style staggered price setting. The distinction between wholesale and retail intermediate-good firms is inconsequential for the robustness exercise, since this distinction can be analogously introduced in the baseline model without affecting the equilibrium conditions.

using the Cobb-Douglas production technology $y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$. The retail firms purchase the wholesale goods at the real price z_t and differentiate them at no cost to set their prices on the Calvo-style staggered basis.

The Nash bargaining sets the pair of the real wage rate and hours worked $\{w_t, L_t\}$ so as to maximize a weighted product of surpluses from production

$$\left(w_t L_t - \frac{L_t}{P_t \Lambda_t}\right)^{\theta} \left(z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - w_t L_t\right)^{1-\theta}, \tag{29}$$

where the first and the second terms represent respectively the household's and the wholesale firm's surpluses and $\theta \in [0, 1]$ is the household's relative bargaining power. Since $P_t \Lambda_t = U_c(t)$ and $x_t = L_t / K_{t-1}$, the first-order conditions for the wage rate and hours worked become

$$w_t = \theta \frac{z_t}{x_t^{\alpha}} + (1-\theta) \frac{1}{U_c(t)}, \tag{30}$$

$$w_t = \theta \frac{z_t}{x_t^{\alpha}} + (1-\theta) \frac{z_t (1-\alpha)}{x_t^{\alpha}}. \tag{31}$$

The condition (30) shows that the real wage $w_t L_t$ is composed, for the fraction θ, by the wholesale firm's real revenues $z_t L_t / x_t^{\alpha} = z_t K_{t-1}^{\alpha} L_t^{1-\alpha}$ and, for the remaining fraction $1 - \theta$, by the household's labor disutility in terms of final goods $L_t / U_c(t) = L_t / (P_t \Lambda_t)$. More importantly, using (31) to substitute out w_t from (30) yields the same labor input condition as (9). This suggests that hours worked satisfy the same equilibrium condition as in the baseline model while the wage plays a distributive role.

The equilibrium conditions in the presence of the Nash bargaining are given by (4), (5), (8)−(13), (31), and the law of motion of the intermediate-good price dispersion d_t. Log-linearizing the equilibrium conditions and rearranging the resulting equations yields the same equations as (14)−(21), and therefore equilibrium dynamics is determined by the system of five equations (18)−(22) with five variables $\hat{\pi}_t, \hat{x}_t, \hat{z}_t, \hat{K}_{t-1}, \hat{R}_t$. Consequently, introducing the Nash bargaining into the baseline model never alters the determinacy implications presented before.

Proposition 3 *In the model with Nash bargaining over the real wage rate and hours worked, a necessary and sufficient condition for determinacy of equilibrium is the same as that given in*

Proposition 1. Also, the determinacy properties of past-inflation targeting interest rate policy (28) *are the same as in the baseline model.*

This result demonstrates the failure of Proposition 1 of Zanetti (2006), which claims that in the presence of the Nash bargaining, the inflation-forecast targeting interest rate policy, the current-inflation targeting interest rate policy, and the past-inflation targeting interest rate policy all induce indeterminacy for any value of the policy response to inflation. That result is due to the fact that the period $t+1$ real wage rate equation is erroneously included in the dynamic system of equations shown in Appendix A of Zanetti (2006).

It is important to stress that if a definitional equation like the wage rate condition (14) is included in a dynamic system then one should be very careful with the Blanchard and Kahn (1980) root-counting approach for equilibrium determinacy, i.e. matching up the number of unstable eigenvalues with the number of non-predetermined variables. While this approach works for most cases, it can sometimes go wrong. Sims (2002) shows that the correct condition for determinacy is that the column space spanned by the lag matrix in a system is contained in the row space spanned by the lead matrix. Lubik and Schorfheide (2003) give a simple example of how this computation can be done by hand.

3 CONCLUDING REMARKS

In a sticky price model with investment, we have examined a necessary and sufficient condition for determinacy of equilibrium under interest rate policy that targets a weighted average of an inflation forecast and current inflation. This condition has shown that such average-inflation targeting policy ensures determinacy as long as both the response to average inflation and the relative weight of current inflation are large enough. We have also shown that past-inflation targeting interest rate policy guarantees determinacy when its response satisfies the Taylor principle and is not large. These results still hold even when wages and hours worked are determined by Nash bargaining rather than being determined in a competitive labor market.

APPENDIX A

This appendix presents the proof of Proposition 1. Let $a_1 = 1 - \beta(1-\delta)(1-\alpha)$ and $a_2 = 1 - \beta(1-\delta)$. Using the average-inflation targeting interest rate policy (21) to substitute out $\hat{R}_t$ from (19) and (22) and letting $\tilde{z}_t = \hat{z}_t - \alpha\hat{x}_t$, the system of five equations (18)–(22) can be reduced to

$$\begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{x}_{t+1} \\ \tilde{z}_{t+1} \\ \hat{K}_t \end{bmatrix} = C \begin{bmatrix} \hat{\pi}_t \\ \hat{x}_t \\ \tilde{z}_t \\ \hat{K}_{t-1} \end{bmatrix} = \begin{bmatrix} 1/\beta & -\alpha\lambda/\beta & -\lambda/\beta & 0 \\ C_{21} & C_{22} & C_{23} & 0 \\ C_{31} & C_{32} & C_{33} & 0 \\ C_{41} & C_{42} & C_{43} & 1 + c/K \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{x}_t \\ \tilde{z}_t \\ \hat{K}_{t-1} \end{bmatrix}, \quad (32)$$

where $C_{21} = (1-a_2)C_{31}/a_2$, $C_{22} = (1-a_2)C_{32}/a_2$, $C_{23} = -1 - \lambda(1-a_2)(\tau\gamma - 1)/(\beta a_2)$, $C_{31} = \tau(1-\gamma) + (\tau\gamma - 1)/\beta$, $C_{32} = -\alpha\lambda(\tau\gamma - 1)/\beta$, and $C_{33} = 1 - \lambda(\tau\gamma - 1)/\beta$.[11]

Then we can show that four eigenvalues of the matrix C are $1 + c/K (> 1)$ and three solutions to the cubic equation

$$\mu^3 + b_2\mu^2 + b_1\mu + b_0 = 0, \quad (33)$$

where $b_2 = -1 - 1/\beta + \lambda a_1(\tau\gamma - 1)/(\beta a_2)$, $b_1 = 1/\beta + \lambda[-\alpha(\tau\gamma - 1) + a_1\tau(1-\gamma)]/(\beta a_2)$, and $b_0 = -\alpha\lambda\tau(1-\gamma)/(\beta a_2)$.

Because the system (32) contains only one predetermined variable, $\hat{K}_{t-1}$, it follows from Proposition 1 of Blanchard and Kahn (1980) that the necessary and sufficient condition for determinacy of equilibrium is that the cubic equation (33) has exactly one solution inside the unit circle and the other two outside the unit circle. By Proposition C.2 of Woodford (2003), this is the case if and only if either of the following two cases is satisfied.

$$\text{Case I}: \quad 1 + b_2 + b_1 + b_0 < 0, \ -1 + b_2 - b_1 + b_0 > 0;$$

$$\text{Case II}: \quad 1 + b_2 + b_1 + b_0 > 0, \ -1 + b_2 - b_1 + b_0 < 0, \ (b_0)^2 - b_0 b_2 + b_1 - 1 > 0 \ \text{or} \ |b_2| > 3.$$

The three conditions in Case II can be reduced to (23)–(25), respectively.

[11] The forms of C_{41}, C_{42}, and C_{43} are omitted, since they are not needed in what follows.

To complete the proof of this proposition, we show that Case I never holds. Assume first that $0 \le \gamma \le 1/2$. Then, the second condition in Case I induces a contradiction

$$0 \ge \tau(2\gamma - 1) > 1 + \frac{2a_2(1+\beta)}{\lambda(\alpha + a_1)} > 1.$$

Assume next that $1/2 < \gamma \le 1$. Combining the two conditions in Case I yields a contradiction

$$1 \ge 2\gamma - 1 > \tau(2\gamma - 1) > 1 + \frac{2a_2(1+\beta)}{\lambda(\alpha + a_1)} > 1.$$

Thus, (23)−(25) are the necessary and sufficient condition for determinacy.

APPENDIX B

This appendix presents the proof of Proposition 2. Let $a_2 = 1 - \beta(1-\delta)$. Using the past-inflation targeting interest rate policy (28) to substitute out $\hat{R}_t$ from (19) and (22) and letting $\tilde{z}_t = \hat{z}_t - \alpha\hat{x}_t$, the system of five equations (18)−(20), (22), and (28) can be rewritten as[12]

$$[\,\hat{\pi}_{t+1}\ \hat{x}_{t+1}\ \tilde{z}_{t+1}\ \hat{K}_t\ \hat{\pi}_t\,]' = D\,[\,\hat{\pi}_t\ \hat{x}_t\ \tilde{z}_t\ \hat{K}_{t-1}\ \hat{\pi}_{t-1}\,]', \tag{34}$$

where

$$D = \begin{bmatrix} 1/\beta & -\alpha\lambda/\beta & -\lambda/\beta & 0 & 0 \\ (a_2-1)/(\beta a_2) & \alpha\lambda(1-a_2)/(\beta a_2) & -1+\lambda(1-a_2)/(\beta a_2) & 0 & \tau(1-a_2)/a_2 \\ -1/\beta & \alpha\lambda/\beta & 1+\lambda/\beta & 0 & \tau \\ 0 & D_{42} & D_{43} & 1+c/K & D_{45} \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Because the system (34) contains two predetermined variables, $\hat{K}_{t-1}$ and $\hat{\pi}_{t-1}$, it follows from Proposition 1 of Blanchard and Kahn (1980) that the necessary and sufficient condition for determinacy of equilibrium is that the matrix D has two eigenvalues inside the unit circle and the other three outside the unit circle. Then we can show that five eigenvalues of the

[12] The forms of D_{42}, D_{43}, and D_{45} are omitted, since they are not needed in what follows.

matrix D are $1 + c/K (> 1)$ and four eigenvalues of the matrix

$$\tilde{D} = \begin{bmatrix} 1/\beta & -\alpha\lambda/\beta & -\lambda/\beta & 0 \\ (a_2 - 1)/(\beta a_2) & \alpha\lambda(1 - a_2)/(\beta a_2) & -1 + \lambda(1 - a_2)/(\beta a_2) & \tau(1 - a_2)/a_2 \\ -1/\beta & \alpha\lambda/\beta & 1 + \lambda/\beta & \tau \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

where $\lambda = (1 - \nu)(1 - \beta\nu)/\nu$ and $a_2 = 1 - \beta(1 - \delta)$. This matrix $\tilde{D}$ depends on the cost share of capital α, the subjective discount factor β, the capital depreciation rate δ, the probability of not reoptimizing price ν, and the policy response to past inflation τ, but not on the other model parameters. Therefore, this property is true for the necessary and sufficient condition for determinacy.

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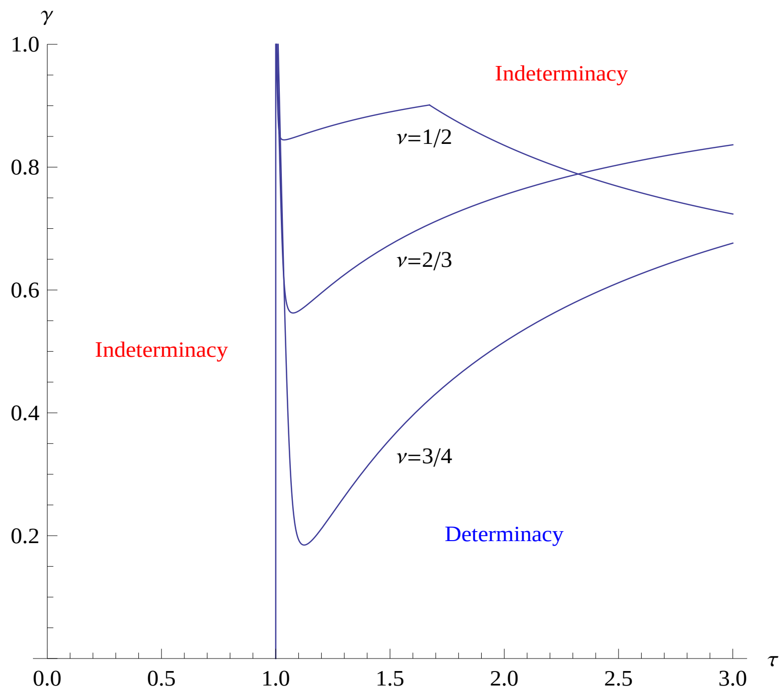

Figure 1: Region of the pair of the policy response to the weighted average inflation and the relative weight of the inflation forecast $\{\tau, \gamma\}$ that guarantees determinacy of equilibrium.