

**INTERNATIONAL TRANSMISSION OF ANTICIPATED INFLATION UNDER  
ALTERNATIVE EXCHANGE-RATE REGIMES**

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**ABSTRACT**

This paper studies the international transmission of anticipated inflation. A two-country, two-good, two-currency, cash-in-advance model is used to examine analytically and numerically the consequences of changes in a country's inflation rate. Domestic monetary policy influences real activity at home through an inflation-tax channel. These real effects are transmitted to the foreign country via fluctuations in the real exchange rate. Under a flexible nominal exchange rate, inflation is a beggar-thy-neighbor policy. Under a fixed nominal exchange rate, each country suffers a welfare loss when one country inflates. The quantitative results are fairly insensitive to variations in the cash-credit mix used to finance investment expenditures.

## **INTERNATIONAL TRANSMISSION OF ANTICIPATED INFLATION UNDER ALTERNATIVE EXCHANGE-RATE REGIMES\***

As the world's economies become more interconnected, a thorough understanding of how foreign policies affect domestic activity becomes crucial. Whether and how macroeconomic policy actions can be transmitted across countries and the extent to which a flexible exchange rate may insulate an economy from foreign disturbances are important issues for today's monetary policymakers. This paper addresses the macroeconomic connections between countries by highlighting the role of anticipated inflation in the international transmission of monetary-policy fluctuations.

A two-country, two-good, two-factor, two-currency model is developed to study the long-run consequences of monetary policy in an open economy. In the model, money is nonneutral because capital accumulation and goods purchases (but not labor purchases) must be financed with money holdings. Domestically, an increase in expected inflation raises the rate at which capital and labor incomes are taxed, altering the steady-state capital and labor stocks. The effects of the inflation tax are transmitted to the domestic country's trading partner. The international inflation-tax transmission channel depends on the substitutability of foreign goods for domestic goods and on the nominal exchange-rate regime. The long-run implications of changes in a country's inflation-tax rate are examined analytically and numerically to provide both qualitative and quantitative predictions from the model.

Studies on the transmission of monetary-policy fluctuations across countries are part of a broad literature concerning monetary nonneutralities. Some authors working in this area view the real effects of money as arising from nominal rigidities. Svensson and van Wijnbergen (1989),

Obstfeld and Rogoff (1994), and Stockman and Ohanian (1995) emphasize the role of price rigidity in the international transmission mechanism. Fender and Yip (1994) incorporate wage rigidity in their static, two-country model of monetary transmission.

Other authors contend that monetary nonneutralities arise because of differences in the timing of information and transactions. Using closed-economy models, Lucas (1990) and Fuerst (1992) study the link between money and real activity when incomplete information about monetary injections produces liquidity effects. Schlagenhaut and Wrase (1995) and Ho (1993) emphasize the role of liquidity effects in the determination of exchange rates and the international transmission of disturbances between countries.

Another group of authors argues that money is nonneutral because inflation acts as a differential tax on market goods versus nonmarket goods. Most studies on the inflation tax use a closed-economy setting.<sup>1</sup> Notable exceptions include Stockman (1985), Roldos (1992), and Kimbrough (1992). Stockman (1985) develops a two-good, three-factor, small-open-economy model where the monetary-growth rate determines the sectoral allocation of resources and the pattern of trade. Roldos (1992) modifies Stockman's (1985) model by allowing for factor specificity in production. The small-open-economy assumption invoked in the Stockman and Roldos studies permits only a limited discussion of the international transmission of inflation taxes.<sup>2</sup>

Kimbrough (1992) bridges the gap between the inflation-tax literature and research on the international transmission of monetary policy. He develops a Ricardian continuum-of-goods model with cash-in-advance (CIA) constraints and variable labor supply to study the effects of inflation in a two-country world. In Kimbrough's model, where goods must be purchased with currency, an increase in the inflation rate in one country influences both countries' relative wage rates and

employment levels. Thus, inflation alters the relative price of home and foreign goods, in turn altering patterns of specialization and trade. Kimbrough, however, addresses neither capital accumulation nor exchange-rate regimes, factors others identify as playing an important role in the international transmission of monetary policy.<sup>3</sup>

In this paper, we generalize several aspects of the existing literature on the effects of an inflation tax in an open economy. The paper abandons the small-country assumption made by Stockman (1985) and Roldos (1992), and instead considers a two-country framework. It addresses qualitatively and quantitatively the importance of capital accumulation, endogenous labor supply, and alternative exchange-rate regimes in the domestic and international monetary-transmission mechanisms. It also examines the implications of varying the cash-credit mix used to finance investment expenditures. Finally, the paper analyzes the welfare cost of inflation taxes across countries and exchange-rate policies.

The paper is organized as follows. Section I develops the model, while section II characterizes the steady-state equilibrium. Section III discusses the effects of the inflation tax on the labor-supply decision. Section IV describes the correlation between inflation and capital accumulation and the international-transmission mechanism. Section V examines numerically the domestic and international consequences of changes in a country's inflation-tax rate. Section VI verifies the robustness of the results to the specification of the CIA constraints. Section VII briefly concludes.

## I. THE MODEL

Consider a two-country world economy. The home country produces and exports X, while the foreign country produces and exports Y. Production of X and Y requires labor and capital. Factors are fully employed in each country and are immobile internationally. Households in the two countries purchase both X and Y. Each country has its own central bank that issues a national currency. All transactions require cash and are conducted in the seller's currency. Prices adjust to clear all markets.

Assume that domestic and foreign representative agents have identical utility functions and that home and foreign goods are produced with the same technologies. Let us first consider the home country's problem.<sup>4</sup> The home-country's representative household maximizes the discounted sum of utility over the infinite horizon,

$$(1) \quad U = \sum_{t=0}^{\infty} \beta^t U(C_{xt}, C_{yt}, H_{xt}),$$

where  $\beta = 1/(1+D)$  is the discount factor,  $C_{it}$  is consumption of good  $i$  at time  $t$  ( $i = x, y$ ), and  $H_{xt}$  is leisure time taken in period  $t$ .  $U$  is assumed to be homothetic, concave, and twice continuously differentiable.

The supply side of the home economy is described by a representative firm that produces X according to a constant-returns-to-scale production function:

$$(2) \quad X_t = F(L_{xt}, K_{xt}).$$

$L_{xt}$  denotes labor used in producing X. The home-country representative agent is assumed to have one unit of time to allocate between labor and leisure activities. Therefore,  $H_{xt} = 1 - L_{xt}$ . X can be

either consumed or invested in X-sector capital.  $K_{xt}$  denotes capital used in X-production. Capital in place depreciates at rate  $\delta$ .

The representative household enters period  $t$  holding  $M_t$  units of domestic currency that consist of the amount held over from period  $t-1$  ( $M_{t-1}^d$ ) plus any monetary transfers ( $J_t$ ) received from the home government at the beginning of period  $t$ :<sup>5</sup>

$$(3) \quad M_t = M_{t-1}^d + J_t.$$

The home country also has a stock of foreign currency that it uses to purchase good Y:

$$(4) \quad N_t = N_{t-1}^d.$$

There are no financial assets other than home and foreign currencies. Asset markets are assumed to be incomplete in that monetary injections cannot be traded across households in different countries. However, currencies are exchanged to facilitate international trade in goods.<sup>6</sup>

The home household maximizes utility subject to the sequence of budget constraints,

$$(5) \quad M_t + e_t N_t + P_{xt} F_L(L_{xt}, K_{xt}) L_{xt} + P_{xt} F_K(L_{xt}, K_{xt}) K_{xt} = M_t^d + e_t N_t^d + P_{xt} (C_{xt} + K_{x,t+1}^d + (1-\delta)K_{xt}) + e_t P_{yt} C_{yt},$$

where  $P_{xt}$  is the nominal price of good X in units of home currency,  $P_{yt}$  is the nominal price of Y in units of foreign currency,  $K_{x,t+1}^d$  is capital purchased for use in the production of  $X_{t+1}$ , and  $e_t$  is the nominal exchange rate defined as the relative price of foreign currency in terms of domestic currency.

Current-period transactions are also restricted by two CIA constraints:

$$(6) \quad M_t = P_{xt} (C_{xt} + K_{x,t+1}^d + (1-\delta)K_{xt})$$

$$(7) \quad e_t N_t = e_t P_{yt} C_{yt}.$$

Equation (1) is maximized with respect to consumption ( $C_{xt}$ ,  $C_{yt}$ ), leisure ( $H_{xt}$ ), the portfolio of assets ( $M_t^d$  and  $N_t^d$ ), and the investment decision ( $K_{x,t+1}^d$ ) subject to sequences of the budget constraint and CIA constraints. Letting  $\xi_t$ ,  $N_t$ , and  $\lambda_t$  be the LaGrange multipliers on equations (5) through (7), respectively, the first-order conditions for a maximum in the home country are

$$(8) \quad U_1(C_{xt}, C_{yt}, H_t) = (\xi_t N_t) P_{xt}$$

$$(9) \quad U_2(C_{xt}, C_{yt}, H_t) = (\xi_t \lambda_t) e_t P_{yt}$$

$$(10) \quad U_3(C_{xt}, C_{yt}, H_t) = \xi_t P_{xt} F_L(L_{xt}, K_{xt})$$

$$(11) \quad \xi_t (\xi_{t+1} N_{t+1}) = \xi_t$$

$$(12) \quad \frac{e_{t+1}}{e_t} = \frac{\xi_t}{\xi_{t+1} \lambda_{t+1}}$$

$$(13) \quad \xi_{t+1} P_{x,t+1} F_K(K_{t+1}) = \xi_t (\xi_{t+1} N_{t+1}) P_{x,t+1} (1 + \delta)$$

## II. STEADY-STATE EQUILIBRIUM

The world economy is in equilibrium when goods, factor, and money markets clear. Factor-market clearing requires that capital is fully employed and that the quantity of labor supplied equals the quantity of labor demanded in both countries. Goods-market clearing requires balanced trade. The money markets clear when  $M_t^s = M_t^c + M_t^d$  and  $N_t^s = N_t^c + N_t^d$ .

Let the home money stock grow at rate  $B$ , and let the foreign money stock grow at rate  $\mu$ . Since the two countries are identical *ex ante*,  $B = \mu$  in the initial steady state. As consumption is constant in the steady state, equations (8), (9), and (10) imply that there exist unique steady-state values of  $(\xi/N_t)P_{xt}$ ,  $(\xi/2_t)e_t P_{yt}$ ,  $N_t P_{xt}$ , and  $\xi P_{xt}$ . Combining these expressions with (11) yields

$$(14) \quad N_t = (\xi/\$)[(P_{x,t}/P_{xt})\&\$].$$

$N_t$  is positive, unless there is deflation at the rate  $(1-\$)$ .<sup>7</sup> Assume that  $N_t$  is strictly positive, which implies that the home CIA constraint (6) binds. Since  $B = \mu$  in the initial steady state,  $N_t = N_t^* = 2_t = 2_t^* > 0$ , implying that equations (6\*), (7), and (7\*) also hold with equality.<sup>8</sup> Therefore, in the steady state, the velocity of both monies is fixed at unity,  $P_x$  grows at rate  $B$ , and  $P_y$  grows at rate  $\mu$ .

Free trade in goods ensures that

$$(15) \quad \frac{U_2(C_{xt}, C_{yt}, H_t)}{U_1(C_{xt}, C_{yt}, H_t)} = \frac{e_t P_{yt}}{P_{xt}} = \frac{U_2(C_{xt}^c, C_{yt}^c, H_t^c)}{U_1(C_{xt}^c, C_{yt}^c, H_t^c)}.$$

Define  $q_t$  as the relative price of Y in terms of X in the home country. In equilibrium,  $q_t/P_{yt}/P_{xt} = U_2(t)/U_1(t)$ . The law of one price implies that  $P_{yt} = q_t P_{xt}^*$ . Thus,  $q_t = e_t [P_{yt}^c/P_{xt}^c]$  is the real

exchange rate.<sup>9</sup> As the real exchange rate is constant in the steady state, and because  $B=\mu$  in the initial steady state, the nominal exchange rate is also constant in the initial steady state.

### III. TRANSMISSION OF THE FOREIGN INFLATION TAX WITHOUT INVESTMENT

The implications of changes in the foreign money-growth rate are investigated through a series of comparative steady-state exercises.<sup>10</sup> Inflation acts as a tax in the model for two reasons. First, inflation distorts the labor-leisure choice. Second, inflation alters the capital-accumulation decision. Although labor and capital play symmetric roles in production, inflation affects the two factors differently. The consequences of the inflation tax for labor are isolated by concentrating first on inflation's distortion of the labor-leisure choice while abstracting from investment. The capital-accumulation decision is then endogenized, and the general-equilibrium effects of the inflation tax are analyzed. The international transmission of anticipated inflation is considered first with flexible exchange rates and subsequently with fixed exchange rates to highlight the importance of the exchange-rate regime in the transmission mechanism.

Assume that the nominal exchange rate is flexible and that the world economy is initially in a steady-state equilibrium with equal domestic and foreign money-growth rates ( $B=\mu$ ). Both countries produce the same level of output, and consumption of X and Y is split equally between home and foreign residents. Now, let there be a permanent, anticipated increase in the foreign money-growth rate ( $\mu$ ).

With the  $K_x$  stock fixed at  $\bar{K}$  and the  $K_y$  stock fixed at  $\bar{K}^f$ , equations (13) and (13\*) drop out of the model. Substituting equations (8\*) and (14\*) into equation (10\*) produces an expression linking the foreign money-growth rate to foreign employment in the steady state:

$$(16) \quad G_L(\bar{K}^{(t)}, L^{(t)}) = \frac{(1-\mu)U_3(C_x^{(t)}, C_y^{(t)}, H^{(t)})}{\$U_2(C_x^{(t)}, C_y^{(t)}, H^{(t)})}$$

Equation (16) shows an inverse relationship between foreign inflation rates and the quantity of labor supplied to the Y industry; an increase in  $\mu$  requires a reduction in  $L^*$  to ensure that (16) holds.<sup>11</sup> This inverse relationship reflects foreign households' substitution of leisure for goods. In the model, leisure is a "credit" good, while consumption is a cash good. Therefore, inflation acts as a differential tax applied to goods purchased through the market (X and Y) but not to leisure. Since an increase in  $\mu$  raises the relative price of goods in terms of leisure, it causes foreign agents to work fewer hours.

As  $\mu$  and foreign employment covary negatively, an anticipated increase in the foreign inflation-tax rate reduces foreign supply. Consequently, an increase in  $\mu$  reduces consumption of Y around the world, and the higher foreign inflation tax is transmitted to the home country by means of a change in the relative price of the foreign good. Thus, when inflation acts as a tax on market activities (but not on leisure activities), a flexible exchange rate alone cannot insulate an economy from foreign monetary disturbances. This result stands in contrast to the results of most equilibrium open-economy macroeconomic models.<sup>12</sup>

Besides affecting production and consumption of the foreign-produced good, an increase in the foreign money-growth rate may also affect each country's consumption of the home-produced good.<sup>13</sup> Whether transmission to the home country's consumption of X is positive or negative depends on how the decline in the consumption of Y affects home wealth relative to foreign wealth.

To identify the wealth effects, define the intertemporal elasticity of substitution between X and Y as  $\sigma_{xy} = \sigma_{xy} / [d(X/Y)/(X/Y)]/[dq/q]$ .

Assume that  $\sigma_{xy}$  is a positive constant, which implies that equation (1) is a CES function in

consumption of X and Y. The equation for  $\epsilon_{xy}$  shows that the decrease in Y output from the higher foreign inflation tax requires an increase in the real exchange rate.

If  $\epsilon_{xy}$  equals one (the Cobb-Douglas case), the increase in  $\mu$  renders both countries equally less wealthy in the new steady state. The increase in the foreign money-growth rate causes both countries to consume smaller (but equal) amounts of Y but causes no change in either country's consumption of X. In this case, the transmission of the increase in the foreign inflation tax takes the form of a change in the home country's consumption of Y due to an increase in the real exchange rate. If  $\epsilon_{xy}$  exceeds one, the change in the foreign inflation tax is transmitted to the home country by means of the negative shock to Y and consequent substitution of X for Y. In this scenario, the increase in  $\mu$  leads to a terms-of-trade improvement for the foreign country, but this improvement is not large enough to offset the reduction in foreign output. Although both countries are less wealthy in the new steady state, the foreign country is in a worse position than the home country. Accordingly, the home country consumes more X and less Y, while the foreign country consumes less of both goods. Finally, if  $\epsilon_{xy}$  is less than one, an increase in  $\mu$  leads to a large terms-of-trade improvement for the foreign country, which offsets the reduction in foreign output. Thus, the foreign country is wealthier than the home country, and there is negative transmission of the increase in the foreign inflation rate to the home country's consumption of both X and Y.

The effect of an increase in  $\mu$  on the nominal exchange rate between the home and foreign currencies depends on the relative magnitudes of two opposing effects: the relative-price effect and the money-demand effect. An increase in  $\mu$  raises the relative price ( $q$ ) of foreign goods in terms of home goods, which in turn increases  $e$ . By contrast, because an increase in  $\mu$  reduces foreign output and raises home output, the demand for foreign currency falls and the demand for home currency

rises, decreasing  $e$ . Therefore, whether  $e$  rises or falls depends on which effect dominates. For the nominal and real exchange rates to covary positively, as is revealed by empirical studies, the relative-price effect must outweigh the money-demand effect.

Now consider a fixed nominal exchange-rate regime. Such regimes have been shown empirically to play an important role in the international transmission of monetary policies.<sup>14</sup> Following Lucas (1982), the equation describing a fixed nominal exchange rate is

$$(17) \quad \left( \frac{M_t^s + R_t}{N_t^s + S_t} \right) \frac{Y_t^s}{X_t^s} q_t = \frac{P_{xt}}{P_{yt}} q_t$$

where  $R$  represents reserves of the home currency and  $S$  represents reserves of the foreign currency sufficient to maintain the exchange rate at  $\bar{e}$ . Thus,  $M_t^s + R_t$  denotes the amount of home currency and  $N_t^s + S_t$  denotes the amount of foreign currency held by private agents. Assume that the home-country's central bank pegs its currency to the foreign currency and adjusts  $B$  to keep the exchange rate fixed.

An increase in foreign inflation ( $\mu$ ) requires an equal increase in the home inflation rate ( $B$ ) to maintain the exchange-rate peg in the long run. From the home counterpart to (16), the subsequent increase in  $B$  decreases employment in the home country. The smaller labor stock in the home country is associated with a lower level of  $X$  output, which reduces  $X$  consumption in both countries. Therefore, with a fixed exchange-rate regime, an increase in the foreign inflation tax adversely affects both production *and* consumption in the home country.<sup>15</sup>

As there is no change in the real exchange rate (a higher  $\mu$  is associated with a higher  $q$ , while a higher  $B$  is associated with a lower  $q$ ), both countries bear the burden of the foreign inflation tax equally. Output of  $X$  and  $Y$  fall by the same amount, and consumption of the two goods is split equally between the countries. Unlike the flexible exchange-rate case, the monetary transmission mechanism does not depend on the intratemporal elasticity of substitution and does not redistribute wealth between countries in the fixed exchange-rate case. Therefore, the nominal exchange-rate regime is an important determinant of how monetary disturbances are transmitted across countries.

#### IV. TRANSMISSION OF THE FOREIGN INFLATION TAX WITH INVESTMENT

The previous section concentrated on partial equilibrium exercises that abstracted from the investment decision to isolate the effect of the foreign inflation tax on the labor-leisure choice. Now, let us allow agents in each country to accumulate capital and use general equilibrium exercises to investigate the domestic and international consequences of an increase in the foreign country's inflation-tax rate. Optimal investment by foreign agents in  $K^*$  is given by (13\*). Substituting (14\*) into (13\*) and rearranging gives the modified golden rule for capital accumulation in the foreign country:<sup>16</sup>

$$(18) \quad G_K(K^*, L^*) = (D^* \mu^*) [(1 + D^*) (1 + \mu)].$$

Assume that the nominal exchange rate is flexible and that the world economy is initially in a steady-state equilibrium where the two countries have equal inflation-tax rates. Now, let there be an anticipated, permanent increase in  $\mu$ . As revealed by equation (18), an increase in the foreign money-growth rate raises the marginal product of  $K^*$ , implying that higher foreign inflation rates are

associated with lower steady-state Y-capital stocks. As noted by Stockman (1981, 1985) and Roldos (1992), inflation taxes investment by driving a wedge,  $[(1+\mu)(1+D^*)]$ , between the rate at which individuals want to substitute intertemporally and the rate at which the production and transaction technologies allow them to transfer resources intertemporally. Therefore, an increase in  $\mu$  raises the rate at which  $K^*$  is taxed and causes a reduction in the Y-capital stock.

In the partial equilibrium exercises in the preceding section, an increase in the foreign inflation rate unambiguously reduces foreign employment. However, when investment is endogenous, the effect of an increase in inflation on labor supply is ambiguous. To see this, assume that utility is additively separable across commodities, totally differentiate (16) and (18), and combine the two resulting expressions to get an equation linking the change in the foreign labor stock to changes in  $\mu$ :<sup>17</sup>

$$(19) \quad \frac{dL^f}{d\mu} = \frac{1}{(1+\mu)U_{33}} \left[ U_3 - U_2 \frac{K^f}{L^f} (1 + \mu(1 + D^*)) + U_{22} \frac{dC_y^f}{d\mu} \right].$$

An increase in the foreign inflation-tax rate lowers domestic real wages, as  $\mu$  and the capital-to-labor ratio in Y covary negatively. The first term in (19) is negative, reflecting the substitution effect of the reduction in real wages. The second term describes another substitution effect of the inflation tax: the distortion of the labor-leisure choice. This term is also negative as an increase in inflation makes leisure less expensive relative to market activities. The third term is positive and reflects the wealth effect of lower real wages.

If the combined substitution effects and the income effect of a change in  $\mu$  are offsetting, an increase in the foreign inflation-tax rate does not alter the steady-state level of foreign employment.

In this case, the foreign inflation tax only affects the foreign country's capital stock. By contrast, if the two substitution effects outweigh the income effect,  $L^*$  falls unambiguously as the foreign inflation-tax rate rises. The reduction in foreign employment reduces the marginal product of  $K^*$ , which offsets to some extent the direct effect of the inflation tax on the marginal product of the foreign capital stock. If the income effect outweighs the combined substitution effects (the foreign country is on the backward-bending portion of its labor-supply curve),  $L^*$  rises unambiguously with the foreign inflation-tax rate. The increase in foreign employment raises the marginal product of  $K^*$ , which reinforces the direct effect of the increase in the foreign inflation tax on the marginal product of  $K^*$ . In this case, the change in foreign output from an increase in  $\mu$  is ambiguous as  $K^*$  falls and  $L^*$  rises. Furthermore, as the change in production of  $Y$  is ambiguous, the change in consumption of  $Y$  (in both countries) is ambiguous. To simplify discussion, the remainder of this section assumes that the combined substitution effects are larger than the income effect.<sup>18</sup>

By (18), a higher foreign money-growth rate is associated with a lower steady-state  $K^*$ . If the combined substitution effects exceed the income effect, equation (19) implies a smaller foreign labor stock at higher foreign inflation rates. Consequently, foreign output falls as  $\mu$  rises. As there is less  $Y$  available in the world, transmission of foreign inflation to the home country comes through an increase in the real exchange rate. Thus, wealth and consumption of  $Y$  in both countries fall. The nominal exchange rate between home and foreign currencies will increase with  $\mu$  as long as the relative-price effect outweighs the money-demand effect of a higher foreign inflation tax.

As in the analysis with capital held fixed, an increase in the foreign inflation rate alters the pattern of consumption of  $X$  across the two countries.<sup>19</sup> The change in the amount of  $X$  consumed in each country depends on the value of the intratemporal elasticity of substitution. If  $\sigma_{xy}$  equals one,

both countries consume equal but smaller amounts of  $Y$ . In this case, an increase in  $\mu$  does not affect either country's consumption of  $X$ . By contrast, if  $\sigma_{xy}$  exceeds one, the home country consumes more  $X$  and less  $Y$ , while the foreign country consumes less of both goods. If  $\sigma_{xy}$  is less than one, the consumption pattern across countries is reversed.

When the home country desires to maintain a fixed nominal exchange rate, it must raise its money-growth rate to match the increase in  $\mu$ . By the home-country counterpart of equation (18), the increase in  $B$  causes a reduction in the  $X$ -capital stock. If the substitution effects outweigh the income effect of a change in  $B$ , employment in the home country falls. Consequently, the increase in the home money-growth rate causes a reduction in  $X$  production and consumption in both countries. Therefore, as in the model with exogenous capital stocks, an increase in the foreign inflation tax negatively affects both consumption and production in the home country when the exchange rate between the two currencies is fixed.

## V. QUANTITATIVE EVALUATION OF THE MODEL

Quantitative predictions from the theoretical model are obtained with numerical exercises. First, functional forms for technology and preferences are chosen. Second, parameter values are assigned. Third, a benchmark steady state is computed by applying a nonlinear equation solver to the set of first-order conditions and constraints described in Section I.<sup>20</sup> Finally, starting from the benchmark position, the quantitative effects of changes in the foreign country's inflation rate ( $\mu$ ) are evaluated.

The home-country representative agent's instantaneous utility function is assumed to take a constant-relative-risk-aversion (CRRA) form:

$$(1) \quad U(C_{xt}, C_{yt}, H_{xt}) = \frac{1}{R} \left[ C_{xt}^{\alpha(1-\phi)} C_{yt}^{(1-\alpha)(1-\phi)} H_{xt}^{\phi} \right]^R$$

where  $0 < \alpha < 1$  and  $0 < \phi < 1$ . This functional form imposes unitary elasticity of substitution between  $X$  and  $Y$  in consumption ( $\phi$ ) and unitary elasticity of substitution between composite consumption and leisure ( $\alpha$ ), where composite consumption is defined as  $(C_x^{\alpha(1-\phi)} C_y^{(1-\alpha)(1-\phi)})^{1/\phi}$ . The home-country firm's technology is specified as Cobb-Douglas in domestic capital and labor:

$$(2) \quad X_t = K_{xt}^\beta L_{xt}^{1-\beta}$$

where  $0 < \beta < 1$ . The foreign country's utility and production functions are assumed to take the same forms.

Table 1 presents the benchmark steady-state parameter values. The parameterization follows closely that in Schlagenhauf and Wrase (1995).<sup>21</sup> The parameters take the same values for both countries, which is a good approximation to a situation where the U.S. is the home country and an OECD aggregate is the foreign country. As can be seen in Table 1, capital's share of income ( $\beta$ ) is set to 0.35, consistent with post-WWII U.S. data. The rate of depreciation of capital in the U.S. is about 2.5 percent per quarter (10 percent per year). This implies a value of 0.025 for  $\delta$ . The discount factor ( $\beta$ ) is 0.99 in the benchmark. This yields a real rate of return of 1 percent per quarter (or 4 percent per year), which is in accordance with U.S. data this century. Leisure's share in the utility function ( $\phi$ ) is assigned a value of 0.76, implying that households spend about one-third of their productive time on market activities as estimated by Ghez and Becker (1975).<sup>22</sup> The share of domestically produced goods in consumption ( $\alpha$ ) is set to 0.50, indicating that households consume

domestic and foreign goods in equal proportions. Following Schlagenhaut and Wrase (1995), the utility curvature parameter ( $R$ ) is set to -1.0. Finally, the average money-growth rates ( $\mu$  and  $B$ ) are set to 0.018, which is consistent with Schlagenhaut and Wrase's (1995) findings for the U.S. and Europe during the post-WWII period.

Table 2 compiles the long-run effects of 1 percent, 5 percent, and 10 percent increases in the foreign inflation-tax rate when the nominal exchange rate between the home and foreign currencies is flexible. The first six rows of Table 2 list the effects on home-country variables: consumption of  $X$  ( $C_x$ ), consumption of  $Y$  ( $C_y$ ), capital ( $K_x$ ), labor ( $L_x$ ), output ( $X$ ), and welfare ( $W$ ). The next six rows show the effects on foreign-country variables. Finally, the last two rows present the effects on the nominal exchange rate ( $e$ ) and the real exchange rate ( $q$ ). All results are reported in percentage changes from the benchmark steady state.

As can be seen in Table 2, the numerical exercises reinforce the analytical results when  $\tau_{xy} = \tau_h = 1$ . In the long run, an increase in  $\mu$  decreases the quantities of labor and capital supplied to foreign production, acts as a negative shock to foreign output, increases the real exchange rate, decreases the nominal exchange rate, and decreases consumption of  $Y$  in both countries.<sup>23</sup> The results in Table 2 suggest that an increase in the foreign inflation-tax rate has a sizable impact on the foreign economy and on the home economy's consumption of the foreign good. While the entries in the table may seem small, they should be interpreted as percentage changes in U.S. and OECD-aggregate variables, given the parameterization of the model. If we consider an OECD aggregate as the foreign economy, a 1 percent increase in the OECD inflation rate would lead to a 0.44 percent reduction in OECD real GDP, which at the current levels of real GDP would be several billion dollars. Table 2 also reveals that as the differential between the foreign and home inflation-tax rates rises, the stronger

the effects of the foreign inflation tax become. For instance, a 1 percent increase in  $\mu$  reduces foreign output 0.44 percent, while a 10 percent increase in  $\mu$  reduces foreign output 3.67 percent.

To complete the discussion of the consequences of anticipated inflation and to obtain policy prescriptions from the model, it is necessary to assess the effects of inflation taxes on national welfare levels.<sup>24</sup> We measure the foreign country's welfare cost (gain) from an increase in the domestic inflation rate as the percentage change in composite goods consumption ( $\hat{c}^*$ ) required to leave the representative agent as well off as under the benchmark steady state.<sup>25</sup> Define  $\hat{a}^*$  as the level of utility attained by the foreign agent under the benchmark steady state. Define labor supplied to foreign production as  $y^*$  under the higher foreign money growth steady state. The welfare cost (gain) for the foreign agent is  $W^* = \hat{c}^*/\hat{c}^*$ , where  $\hat{c}^*$  solves the equation:  $\hat{a}^* = \hat{c}^* (\hat{c}^*)^{\alpha} (\hat{y}^*)^{1-\alpha}$ . The home country's welfare cost (gain) is computed similarly.

Table 2 shows the quantitative effects of an increase in  $\mu$  on welfare levels in the two countries. Interestingly, Table 2 reveals that an increase in the foreign inflation-tax rate is a beggar-thy-neighbor policy. The foreign country experiences a welfare gain from inflating, while the home country experiences a welfare loss. Thus, it appears that a country can export its inflation tax to its trading partner when the nominal exchange rate is flexible. Although worldwide welfare is reduced, either country finds a unilateral increase in its inflation-tax rate an attractive policy, as it is welfare-enhancing from its perspective.

Table 3 reports the long-run effects of increases in the foreign inflation rate when the nominal exchange rate is fixed. Table 3 reveals that an increase in  $\mu$  reduces production and consumption of both goods in both countries. Given the parameterization of the model, the two countries bear the

burden of the foreign inflation tax equally. Each country suffers a welfare loss, implying that the increase in foreign inflation is not a beggar-thy-neighbor policy in this case. Interestingly, the numerical analysis shows that the home country is relatively better off with a fixed exchange rate, as opposed to a flexible exchange rate, when the foreign country inflates. The difference in the welfare effects under the alternative exchange-rate policies may provide one reason the home country might want to fix the value of its currency vis-à-vis the value of the foreign currency. Of course, the home country's adoption of a fixed exchange rate is not a welfare-enhancing policy from the foreign country's perspective.

## VI. SPECIFICATION OF THE CIA CONSTRAINTS

An increase in the foreign inflation rate affects the foreign economy through two avenues; it distorts the labor-leisure choice *and* the capital-accumulation decision. A reasonable criticism of the model is that cash is required for transactions involving capital goods. Verifying the robustness of the results to this assumption is relatively simple. Assume that capital purchases can be financed with credit and/or cash. Define  $\alpha$  as the fraction of current investment purchases in the home country (foreign country) that must be financed with cash.<sup>26</sup> Allowing for different mixtures of investment financing alters the specification of the domestic-currency CIA constraints and the Euler equations for capital. For the foreign country, equation (6\*) becomes

$$(6^*) \quad N_t^f \leq P_{yt}^f [C_{yt}^f + \alpha (K_{y,t+1}^d + (1-\delta^f)K_{yt}^f)],$$

and equation (13\*) becomes

$$(13^*) \quad \left( P_{y,t}^c G_K^c(t\%1) \xi_{t1} \% (\xi_{t1} \% 2_{t1}^c) P_{y,t1}^c (1\&^*(c)) \right)' (\xi \% 2_t^c) P_{yt}^c.$$

The modified golden rule for capital accumulation in the foreign country is now given by:

$$(18') \quad G_K(K^c, L^c)' (D^c \% ^*(c)) [1\% (c) ((1\% D^c)(1\% \mu) \& 1)].$$

To determine how sensitive the quantitative results are to different mixtures of investment financing, the numerical exercises in Tables 2 and 3 were repeated with several alternative values for  $\alpha$  ( $\alpha = 1, 0.75, 0.50, 0.25,$  and  $0$ ).<sup>27</sup> Table 4 compiles the long-run effects of an increase in  $\mu$  under the different assumptions about  $\alpha$  when the nominal exchange rate is flexible. Table 5 assembles the results when the nominal exchange rate is fixed. It is immediately apparent from Tables 4 and 5 that foreign inflation taxes have their strongest effects on foreign capital and output the more cash is required in transactions involving capital goods. For example, when  $\alpha = 1$ , a 10 percent increase in  $\mu$  reduces the steady-state foreign capital stock by 5.91 percent. In contrast, when  $\alpha = 0.25$ , the same increase in  $\mu$  reduces the steady-state foreign capital stock by only 2.90 percent. This occurs because as  $\alpha$  moves toward zero, one avenue for the foreign inflation tax to affect the foreign economy is being shut down. Notice that even when capital is a credit good ( $\alpha = 0$ ), an acceleration of inflation in the foreign country reduces the amount of foreign capital accumulated. According to equation (18'), when  $\alpha = 0$ , the marginal product of foreign capital is independent of the foreign inflation rate ( $G_K = (D^* + ^*)$ ) in the steady state. An increase in the foreign inflation rate still affects the foreign capital stock indirectly through its distortion of the labor-leisure choice. An increase in  $\mu$  reduces the

quantity of labor supplied to foreign production, which causes the foreign capital stock to fall such that the marginal product of capital remains constant at  $(D^{+*})$ .

Besides the effects on foreign capital, Tables 4 and 5 reveal that increases in the foreign inflation-tax rate have only slightly stronger effects on the other variables in the model as more cash is used to finance investment. For instance, steady-state welfare levels in the two economies remain virtually unchanged as  $\zeta$  is varied from 1 to 0. Overall, the quantitative results are fairly insensitive to the specification of the CIA constraints. If labor is elastically supplied and cash is required to purchase consumption goods, an acceleration of one country's inflation rate affects both economies regardless of the cash-credit mix used to finance investment expenditures.

## VII. CONCLUSIONS

This paper studies the long-run effects of monetary policy on domestic and international real economic activity by emphasizing the role of inflation as a tax. The analysis extends the existing literature on inflation taxes in open economies in several ways. First, it replaces the small-country framework employed in most other studies with a two-country framework. Second, it investigates both analytically and numerically the consequences of widening inflation-tax differentials across countries. Third, it emphasizes the endogenous responses of the factors of production to changes in inflation-tax rates at home and abroad. Fourth, it addresses the role of the nominal exchange-rate regime in the international inflation-tax transmission mechanism. Fifth, it examines qualitatively and quantitatively the implications of varying the cash-credit mix used to finance investment expenditures. Finally, the paper analyzes the welfare costs of inflation taxes across countries and exchange-rate policies.

The main lesson to be taken from the theoretical analysis and the numerical exercises is that inflation taxes have important long-run implications for both the country of origin and its trading partner. Domestically, an increase in expected inflation reduces steady-state investment expenditures, hours worked, and output. The international transmission mechanism depends on the substitutability of home and foreign goods in consumption and the nominal exchange-rate regime. A flexible exchange-rate system will not insulate a country from inflation taxes abroad. When the nominal exchange rate is flexible, foreign inflation taxes alter the home country's consumption of goods but not the home country's production decision. By contrast, when the nominal exchange rate is fixed, foreign inflation reduces the home country's level of output. Hence, in the fixed exchange-rate case, an increase in the foreign money-growth rate acts as a negative shock to the world's supply of goods. If the home and foreign countries are identical, they will bear the burden of the inflation tax equally.

In the theoretical model, leisure is a credit good, while capital and consumption are cash goods. The numerical exercises reveal that the quantitative predictions of the model are fairly insensitive to alternative assumptions about the cash-credit mix used in transactions involving capital goods. Of course, inflation taxes have stronger effects the more cash is required to finance investment expenditures. However, even when capital goods are purchased on credit, an increase in a country's inflation rate still affects real activity both at home and abroad.

Several interesting conclusions can be drawn from the welfare analysis. Under a flexible nominal exchange rate, a unilateral increase in a country's inflation rate is a beggar-thy-neighbor policy. Although worldwide welfare is reduced, inflation is a welfare-enhancing policy from the country of origin's perspective. Under a fixed nominal exchange rate, a country cannot increase its own welfare at the expense of its trading partner. The numerical analysis reveals that the home

country is relatively better off when it ties its currency to the foreign currency when the foreign country inflates. The different welfare effects under the alternative exchange-rate policies suggest that this analysis may be useful in studying such topics as disputes over, and coordination of, monetary policies across countries.

The results in this paper are subject to several caveats that could be addressed by subsequent research in this area. We briefly sketch a few ways that our analysis could be extended. In focusing on the long-run consequences of anticipated increases in inflation, the short-run dynamics are lost. However, an intuitively appealing view of the domestic and international transmission mechanisms is gained. Analyzing the transitional responses of the real variables in the two economies to changes in inflation-tax rates would be a worthwhile and significant extension.

The analysis has abstracted from international financial markets. In the comparative steady-state exercises considered in the paper, there is no explicit role for asset markets as there is no uncertainty and the countries are identical *ex ante*. If we were to introduce uncertainty about changes in inflation-tax rates, this would provide motivation for trades in international financial assets, which may alter the results. Further motivation for asset trade could be provided by assuming that the two countries have different subjective probabilities about inflation-tax rates, different degrees of risk aversion, and/or different production functions.

The introduction of international financial markets would allow agents to transfer resources over time by borrowing and lending. If we were to focus on the transitional dynamics from one steady state to the next, consideration of asset trades would be crucial in the flexible exchange-rate case. Without international markets for borrowing and lending, changes in the foreign inflation rate lead to periods in which consumption of the foreign good is high or low as the economies move to

the new steady state. The ability to trade bonds internationally may permit the foreign country to finance its expenditures with external debt rather than automatically altering its consumption path in the face of higher inflation-tax rates.

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**TABLE 1**  
**BENCHMARK PARAMETERS**

<b>Parameter</b>	<b>Description</b>	<b>Value</b>
" = " *	Capital share	0.35
* = * *	Depreciation rate	0.025
\$ = \$ *	Discount factor	0.99
R = R *	Utility curvature	-1.00
< = < *	Leisure share	0.76
O = O *	Consumption share	0.50
$\mu = B$	Money-growth rate	0.018

**TABLE 2**  
**LONG-RUN EFFECTS OF AN INFLATION TAX IN A TWO-COUNTRY MODEL WITH A FLEXIBLE**  
**NOMINAL EXCHANGE RATE**  
 (percentage changes)

<b>Percent Increase in the Foreign Money-Growth Rate (<math>\mu</math>)</b>	<b>1%</b>	<b>5%</b>	<b>10%</b>
<b>Home Country</b>			
$C_x$	0	0	0
$C_y$	-0.32	-1.50	-2.91
$K_x$	0	0	0
$L_x$	0	0	0
$X$	0	0	0
$W$	-0.04	-0.18	-0.35
<b>Foreign Country</b>			
$C_x^*$	0	0	0
$C_y^*$	-0.32	-1.50	-2.91
$K_y^*$	-0.65	-3.11	-5.91
$L_y^*$	-0.28	-1.26	-2.43
$Y$	-0.44	-1.90	-3.67
$W^*$	0.02	0.08	0.15
<b>Exchange Rates</b>			
$e$	-0.38	-1.86	-3.57
$q$	0.31	1.51	2.96

**TABLE 3**  
**LONG-RUN EFFECTS OF AN INFLATION TAX IN A TWO-COUNTRY MODEL WITH A FIXED**  
**NOMINAL EXCHANGE RATE**  
 (percentage changes)

<b>Percent Increase in the Foreign Money-Growth Rate (<math>\mu</math>)</b>	<b>1%</b>	<b>5%</b>	<b>10%</b>
<b>Home Country</b>			
$C_x$	-0.32	-1.50	-2.91
$C_y$	-0.32	-1.50	-2.91
$K_x$	-0.65	-3.11	-5.91
$L_x$	-0.28	-1.26	-2.43
$X$	-0.40	-1.90	-3.67
$W$	-0.01	-0.10	-0.20
<b>Foreign Country</b>			
$C_x^*$	-0.32	-1.50	-2.91
$C_y^*$	-0.32	-1.50	-2.91
$K_y^*$	-0.65	-3.11	-5.91
$L_y^*$	-0.28	-1.26	-2.43
$Y$	-0.40	-1.90	-3.67
$W^*$	-0.01	-0.10	-0.20
<b>Exchange Rates</b>			
$e$	0	0	0
$q$	0	0	0

**TABLE 4**  
**SENSITIVITY OF THE RESULTS TO THE SPECIFICATION OF THE CIA CONSTRAINT**  
**FLEXIBLE NOMINAL EXCHANGE RATE**  
 (percentage changes)

Percent Increase in $\mu$	$\zeta = \zeta^* = 1$			$\zeta = \zeta^* = 0.75$			$\zeta = \zeta^* = 0.50$			$\zeta = \zeta^* = 0.25$		
	1%	5%	10%	1%	5%	10%	1%	5%	10%	1%	5%	10%
<b>Home Country</b>												
$C_x$	0	0	0	0	0	0	0	0	0	0	0	0
$C_y$	-0.32	-1.50	-2.91	-0.29	-1.36	-2.65	-0.25	-1.21	-2.35	-0.21	-1.07	-2
$K_x$	0	0	0	0	0	0	0	0	0	0	0	0
$L_x$	0	0	0	0	0	0	0	0	0	0	0	0
$X$	0	0	0	0	0	0	0	0	0	0	0	0
$W$	-0.04	-0.18	-0.35	-0.03	-0.17	-0.32	-0.03	-0.15	-0.29	-0.03	-0.13	-0
<b>Foreign Country</b>												
$C_x^*$	0	0	0	0	0	0	0	0	0	0	0	0
$C_y^*$	-0.32	-1.50	-2.91	-0.29	-1.36	-2.65	-0.25	-1.21	-2.35	-0.21	-1.07	-2
$K_y^*$	-0.65	-3.11	-5.91	-0.55	-2.58	-4.91	-0.39	-2.00	-3.92	-0.29	-1.50	-2
$L_y^*$	-0.28	-1.26	-2.43	-0.23	-1.21	-2.29	-0.23	-1.12	-2.14	-0.23	-1.02	-2
$Y$	-0.44	-1.90	-3.67	-0.34	-1.67	-3.22	-0.30	-1.45	-2.77	-0.25	-1.20	-2
$W^*$	0.02	0.08	0.15	0.01	0.08	0.15	0.02	0.08	0.15	0.02	0.08	0
<b>Exchange Rates</b>												
$e$	-0.38	-1.86	-3.57	-0.33	-1.62	-3.11	-0.29	-1.41	-2.72	-0.26	-1.26	-2
$q$	0.31	1.51	2.96	0.28	1.37	2.69	0.25	1.23	2.41	0.22	1.09	2



## ENDNOTES

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1. Cooley and Hansen (1989) develop a closed-economy, real-business-cycle model that incorporates inflation as a tax on market goods and simulate the effects of changes in the money-growth rate. Their simulations reveal that a permanent increase in the annual inflation rate from 0 to 10 percent reduces steady-state output and employment by 2.4 percent. Lucas (1990) uses the evidence on money demand from time-series studies to calculate the implied welfare cost of inflation in the US economy. His estimates suggest that the inflation tax imposes a considerable real cost. Stockman (1981) uses a one-good, closed-economy model to illustrate how changes in the money-growth rate influence the equilibrium capital stock. In his model, higher inflation rates are associated with lower capital stocks because capital accumulation must be financed with cash.

2. The effects of inflation taxes in a two-country model are similar to the effects in a small-open-economy model from the small country's perspective. Obstfeld and Rogoff (1996, p. 660) state that a small-open-economy model does not yield the interesting cross-country transmission effects of a two-country model.

3. See, for example, Stockman (1993) and Bordo and Eichengreen (1993).

4. The foreign representative agent solves an identical problem. Foreign variables and equations are denoted with an asterisk.

5.  $M_t/M_{t-1} + J_t$  is the agent's nominal money holdings after the monetary transfer at the beginning of period  $t$ .  $M_t^d$  is the amount of money held at the end of period  $t$ . In equilibrium,  $M_t^d = M_t$ . Substituting  $M_{t-1}^d = M_{t-1}$  into the definition of  $M_t$  gives (3).

6. Heuristically, a government distributes cash (injects money) only to its own citizens and does

not distribute cash to foreign citizens. At the very beginning of period  $t$ , the home-country representative agent empties his “vending machine,” which contains the proceeds from period  $t-1$  sales of home-produced goods. As the seller’s currency convention holds, these proceeds are in home-currency units. The home agent trades some of his proceeds to obtain the foreign currency necessary to buy foreign goods in period  $t$ . Currency trades take place in an underlying currency-exchange market at the nominal exchange rate,  $e_t$ .

7. This result is analogous to that in Cooley and Hansen (1989).  $N_t$  is nonnegative when the gross money-growth rate  $(1+B)$  is greater than or equal to  $\beta$ . Note that restricting  $N_t$  to be strictly positive is equivalent to assuming that the nominal interest rate in the home country is strictly positive.

8. The pareto optimum for this two-country economy occurs when the home and foreign money-growth rates are such that all of the CIA constraints are slack. This is achieved when  $(1+\mu) = (1+B) = \beta$ , which is the rule for the optimal quantity of money. Intuitively, the pareto optimal allocation in the world economy is the same as the pareto optimal allocation in an identical economy without the two monies.

9. For simplicity, the expression for the real exchange rate assumes that  $\lambda_t = N_t$ . This assumption is not critical for the analysis that follows. When home and foreign money-growth rates are equal, the two countries are identical, and  $\lambda_t = N_t$  by default. If home and foreign money-growth rates are not equal, differences in  $\lambda_t$  and  $N_t$  can arise. If  $\lambda_t > N_t$ , then  $e_t = q_t [P_{xt} / P_{yt}] [(\beta N_t) / (\beta \lambda_t)]$ . As will be shown in the next section, an anticipated increase in the foreign inflation rate raises the real exchange rate regardless of whether  $\lambda_t = N_t$ . As  $\lambda_t$  represents the marginal value of an extra unit of foreign currency, an increase in the foreign money-growth rate lowers  $\lambda_t$  in the steady state. Therefore, in a steady state where the foreign money stock grows at a faster rate than the home money stock, both

the real exchange rate and  $z_t$  act to increase the steady-state level of the nominal exchange rate. Thus, the assumption that  $z_t = N_t$  affects the quantitative, but not the qualitative, predictions of the model.

10. Abel (1986) linearizes the single difference equation for Stockman's (1981) one-sector, closed-economy model to study the speed of adjustment to the new steady state after a change in the inflation rate. Equations (11)-(13) and their foreign counterparts can be simplified into three third-order difference equations for the capital stocks and the exchange rate for which analytical solutions are not available. Thus, it is not possible to conduct Abel's procedure for the two-country, two-good, two-factor, two-currency model considered here. This paper focuses instead on comparative steady-state exercises that are analytically tractable. Analyzing the transition between steady states would be a useful exercise to undertake. However, simulating the dynamics of the model is beyond the scope of the present paper, and this exercise is left for subsequent work.

11. Consumption of X and Y may also have to be altered to ensure that (16) holds after an increase in  $\mu$ . Recognize, however, that modifications to the consumption pattern require a change in labor, which causes a change in production. Thus, the discussion concentrates first on the change in employment required to maintain (16). After the effect on labor is isolated, the impact of an increase in  $\mu$  on consumption of X and Y is discussed.

12. See Stockman (1983, 1993) for a discussion of the conditions necessary to ensure that a flexible exchange-rate system protects a country from foreign monetary disturbances.

13. Define  $\dagger$  ( $\dagger^*$ ) as composite consumption of X and Y in the home (foreign) country. Denote the intratemporal elasticity of substitution between  $\dagger$  and leisure in the home country as  $\sigma_{\dagger, \text{th}}$ . For ease of exposition, we assume that  $\sigma_{\dagger, \text{th}} = \sigma_{\dagger^*, \text{th}}^* = 1$  throughout our analysis. This assumption implies that an increase in  $\mu$  will not affect production of X. To see this, combine equations (9), (10), and

$e_t = q_t [P_{xt}/P_{yt}] [(S_t N_t)/(S_t^2)]$  to get an expression for the marginal rate of substitution between consumption of Y and consumption of leisure in the home country:  $U_2(t)/U_3(t) = [q_t(1+B)]/[F_L(t)]$ . An increase in  $\mu$  increases  $U_2(t)$ . As will be shown, an increase in  $\mu$  also increases the real exchange rate ( $q$ ). As long as  $\tau_{th} = 1$ , the quantity of leisure consumed by the home country need not change to ensure that the expression for the marginal rate of substitution holds with equality. Equivalently, as long as  $\tau_{th} = 1$ , an increase in  $\mu$  does not change the quantity of labor supplied to home production nor the quantity of X produced. If  $\tau_{th} < 1$ , then an increase in  $\mu$  could potentially affect the home country's production of X.

14. Stockman (1983) and Glick, Kretzmer, and Wihlborg (1995) find that the degree of nominal exchange rate flexibility is an important determinant of the cross-country transmission of foreign monetary disturbances.

15. Svensson and van Wijnbergen (1989) stress the importance of the exchange-rate regime in determining whether home output is affected by foreign monetary shocks. They develop a two-country model of monetary transmission where firms behave as monopolistic competitors and output prices are rigid. In the region where the CIA constraints bind and the full-capacity constraints are slack (which is comparable to the situation described here), they also find transmission to home production when the nominal exchange rate is fixed but not when the nominal exchange rate is flexible.

16. Substituting  $P_{y,t+1} = (1+\mu)P_{yt}$  into (14\*) yields  $(S_t N_t)/(S_t^2) = (1+\mu)$ . Using this equation and recognizing that  $S_t P_{yt}$  and  $N_t P_{yt}$  are constant in the steady state, (13\*) can be reduced to (18).

17. To derive equation (19), recognize that constant returns to scale in Y production implies  $(G_{LL} G_{KK} + G_{KL}^2)/G_{KK} = 0$  and  $G_{LK}/G_{KK} = K_y/L_y$ . The assumption of constant returns to scale in

production implies that the elasticity of substitution between labor and capital in Y does not appear in (19).

18. Cooley and Hansen (1989) provide empirical evidence that shows a negative correlation between inflation rates and labor supply in a cross-section of countries.

19. As argued in the exercises with capital held fixed, an increase in  $\mu$  does not affect the quantity of labor supplied to home production in the steady state as long as  $\sigma_h = 1$ . Since the home-country counterpart to (18),  $F_K(K_x, L_x) = (D + \mu)[(1+D)(1+B)]$ , does not depend on the foreign money-growth rate, there is no transmission of the foreign inflation tax to the home country's capital stock in the steady state. Consequently, an increase in  $\mu$  does not alter the home country's production of X.

20. The numerical exercises were conducted with the non-linear equation solver in MATLAB, Version 4.0. The programs are available from the authors by request.

21. This parameterization has been employed extensively in international general-equilibrium models. See, for instance, Backus, Kehoe, and Kydland (1992).

22. To understand the reasoning behind  $\sigma_h \leq 0.76$ , consider the following example. Assume agents sleep 8 hours per day, leaving 16 hours available for nonsleep activities. Ghez and Becker (1975) calculate that agents spend approximately one-third of their nonsleep time working. This implies that agents work on average 5.3 hours per day ( $5.3 = 16/3$ ). The 5.3 hours at work comprise 22 percent of a 24-hour day ( $0.22 = 5.3/24$ ). Normalizing total time to unity yields a leisure share of approximately 0.76, as is standard in the real business cycle literature.

23. The nominal and real exchange rates covary negatively in the numerical exercises due to the parameterization of the utility function and the specification of the CIA constraints. The exchange rates will covary positively only if the relative-price effect outweighs the money-demand effect. This requires that the income elasticity of foreign money demand ( $\epsilon$ ) exceeds the inverse of  $\sigma_{xy}$ . In the

simulations, this condition is not fulfilled because  $\phi_{xy} = 1$ .

24. As noted by Kimbrough (1992), a discussion of the effect of the foreign inflation tax on national welfare levels is reminiscent of discussions of commercial policy in traditional trade models. Inflation acts as a tax on the factors of production that are indirectly traded through free trade in goods. In this sense, the foreign inflation tax is a substitute for a tariff.

25. This welfare-cost measure follows that used by Cooley and Hansen (1989). Cooley and Hansen (1989) specify their benchmark steady state as the Pareto optimal allocation and measure the welfare cost of introducing a distortionary inflation tax. In contrast, we start from a nonoptimal allocation and measure the welfare cost of exacerbating the distortion.

26. Stockman (1981) assumes  $\phi = 1$ , while Lucas (1980) assumes  $\phi = 0$ . Koenig (1987, 1989), Wang and Yip (1992), Palivos, Wang, and Zhang (1993), and Palivos and Yip (1995) allow for  $0 < \phi < 1$  in their closed-economy models.

27. Investment is a cash good ( $\phi = 1$ ) in Tables 2 and 3. The results from Tables 2 and 3 are replicated in Tables 4 and 5 to facilitate comparison with cases when  $\phi < 1$ .