Search with Wage Posting under Sticky Prices

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Abstract

This paper examines the implications of interacting pricing frictions, labor market frictions, and consumption risk by comparing variants of a New Keynesian model. The model variants make alternative assumptions about whether hiring and pricing decisions occur within the same firm or across different firms, and whether workers pool income. Nonetheless, each model implies the same contract is offered to workers, making model comparisons transparent. The economy’s response to changes in unemployment benefits or persistently below-target inflation depends on whether hiring and pricing decisions are integrated. Meanwhile, the dynamics following technology or monetary shocks are shaped both by firm- and worker-level assumptions.

Keywords: Search, Matching, Inflation, Sticky Prices, Heterogeneity, Incomplete markets

JEL: E10, E30, E50, J60.

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1 Introduction

Following the seminal work of Merz (1995) and Andolfatto (1996), a large literature has emerged examining the quantitative importance of incorporating labor market frictions into dynamic stochastic general equilibrium (DSGE) models. More recently, this literature has extended into New Keynesian (NK) models in order to study monetary policy in the presence of nominal rigidities and labor market frictions (Chéron and Langot, 2000; Walsh, 2005; Trigari, 2006; Blanchard and Galí, 2010, for example). The implications of this literature for macroeconomic questions depend crucially on the degree to which these frictions interact, and the extent to which workers are able to insure against the risk of unemployment. Thus, two separate and distinct types of assumptions play critical roles in the implications of NK models with these frictions.

First, on the production side, firms set prices with nominal rigidities and hire workers in a frictional labor market; these two frictions can either be separated into distinct types of firms or integrated within the same firm. The former assumption, following Walsh (2005) or Trigari (2006), has wholesale firms that hire labor in a frictional labor market and produce a competitively priced good, and monopolistically competitive retail firms that purchase the aforementioned good to produce a differentiated good while facing pricing frictions. This modeling assumption thus splits the key frictions of hiring and pricing into separate entities. The latter assumption, following Kuester (2010) or Thomas (2011), has a single firm that hires in a frictional labor market, sets prices, and produces a differentiated product. How these frictions interact matters for how the production side of the economy responds to changes in policy or the response to shocks.

Second, on the household side, workers can either be combined into a large household that pools consumption or left as individuals to self-insure against earnings and unemployment risk in a frictional labor market. The former assumption, is common in NK models (Walsh, 2005; Trigari, 2006; Kuester, 2010; Thomas, 2011), and potentially affects individual incentives to work relative to the case without risk sharing, which can lead to differences in aggregates. The latter assumption, found in the growing literature merging heterogeneous agent models with New Keynesian constructs (Gornemann et al., 2016; Ravn and Sterk, 2016; Kaplan et al., 2018), often entails computational complexity that limits other features of the model that may be of interest. Additionally, the differing risk sharing assumptions necessarily lead to different marginal utilities of consumption, and this will have consequences for dynamic choices like pricing and hiring through the stochastic discount factor used by firms to evaluate future profits. Thus, assumptions on the household side
have production side implications and vice versa.

Motivated by these alternative modeling assumptions, this paper examines the quantitative implications of interacting price rigidities, labor market frictions, and consumption risk in NK models. It considers two alternative environments on the production side of the economy and two on the household side. On the production side, in one setup the same firm makes pricing and hiring (PH) decisions subject to frictions at both margins. In the second, the hiring and pricing decisions are separated into wholesale and retail firms (WR), respectively. On the household side, in one setup independent individuals (II) make their own choices while facing idiosyncratic employment and wage risk, and in the second all agents live in a representative household (RH) that pools all risk. In all cases we assume firms make take-it-or-leave-it offers to workers with Greenwood et al. (1988) preferences. The combination of this contracting environment and preferences with no wealth effect on labor supply allows the consideration of heterogeneity without having to keep track of the complicated joint distribution of wealth, employment, wages, and prices. A key message from the analysis is that alternative assumptions about integrating pricing and hiring decisions and the degree of risk sharing matters greatly even in our very stylized framework.

For example, how the economy responds to changes in the level of unemployment benefits, or the costs of persistently low inflation depends greatly on the environment firms operate in and less on the specifics of the household side. When firms hire in a frictional labor market and also face sticky prices (PH), increases in unemployment benefits lead to higher average wages, lower vacancy posting, and as a result higher unemployment and lower output. In contrast, separating pricing and hiring decisions (WR) leads to virtually no change in response to changes in unemployment benefits. Similarly, if inflation is persistently below the explicit inflation target, firms making pricing and hiring decisions (PH) respond with a larger change in labor demand than when the decisions are separated (WR), leading to more substantial losses from low inflation.

In contrast, both the assumptions on the firm and household side matter in subtle ways for understanding the propagation of technology and monetary policy shocks. In the face of technology shocks, the responses of vacancies and unemployment are larger with a representative household (RH) than with independent individuals (II). This result holds regardless of the assumptions on the firm side. Given monetary shocks, the responses of vacancies and unemployment are particularly dampened in the independent individual case when pricing and hiring decisions are separated (II-WR). Changing the assumptions on either the household or firm side leads to greater responses of vacancies and unemployment.
The critical feature generating these results lies in the trade-offs faced by firms who control prices and hiring decisions and how income pooling shapes the stochastic discount factor used by firms. In models that integrate pricing and hiring decisions, firms internalize how changing their prices affects demand for their goods and hence how much labor they require. As a result, comparatively less adjustment occurs through prices and more occurs through labor demand. Models that allow for income pooling amplify shocks through the stochastic discount factor as in this case it depends on consumption, individual hours worked, and aggregate employment. In contrast, models without income pooling have less propagation as in this case the stochastic discount factor does not depend on aggregate employment.

The remainder of the paper is as follows. Section 2 provides a brief overview of the literature using the different environments discussed in this paper. Section 3 describes the model environments, focusing on the contrasting assumptions for households and firms. Section 4 parameterizes each model. Section 5 shows how the implications for unemployment benefits or below-target inflation greatly depend on the household- and firm-side assumptions. Section 6 highlights the differences in response to shocks, and Section 7 concludes.

2 Related Literature

The production-side environments studied in this paper draw mainly on the literature for New Keynesian models with frictional labor markets. The wholesaler-retailer environment is well-known, having been studied by, for example, Walsh (2005), Trigari (2006), Sveen and Weinke (2008), Blanchard and Galí (2010), and Christiano et al. (2016), among many others. On the other hand, Kuester (2010), Barnichon (2010), Thomas (2011), and Lago Alves (2018) study environments where firms face trade-offs from both pricing and hiring frictions.\footnote{Evidence presented by Klenow and Malin (2010) suggest strong linkages between pricing and hiring decisions.}\footnote{An additional possibility is, in the presence of multi-worker firms, firms would use over-hiring to strategically reduce wages (see Dossche et al., 2019).} This paper facilitates comparisons between the alternative production frameworks by assuming firms post take-it-or-leave-it contracts in the vein of Postel-Vinay and Robin (2002). This contract ensures that all workers are indifferent between employment and unemployment and as a result the optimal contract is independent of aggregate labor market tightness. Hence, comparing allocations across models is transparent as they do not reflect macro-level differences in aggregate labor market tightness, but rather how the micro-level production structure relates pricing to hiring.\footnote{Evidence presented by Klenow and Malin (2010) suggest strong linkages between pricing and hiring decisions.}
tioned literature only considers representative household (RH) environments for the worker-side. Hence, our paper contributes to this literature by examining how the integration of pricing and hiring within the same firm interacts with the degree of idiosyncratic risk faced by the worker.\(^3\)

Indeed, by considering environments with independent individuals (II) versus a representative household (RH), this paper also relates to the rapidly growing literature on heterogeneous agent New Keynesian (HANK) models such as Gornemann et al. (2016), Ravn and Sterk (2016), and Kaplan et al. (2018), among others. The independent individual environment is similar in spirit to the HANK literature but admittedly simpler in order to accommodate a more complex production sector. The combination of Greenwood et al. (1988) preferences that eliminate the wealth effect on labor supply and take-it-or-leave-it offers eliminates the need to keep track of the distribution of wealth, as workers who are indifferent across employment states do not save in equilibrium. A direct consequence is that solving the household side of the economy does not require approximation techniques like those of Krusell and Smith (1998). In this sense, our simplified framework is more closely related to the spender-saver or two-agent New Keynesian (TANK) models following the seminal work of Campbell and Mankiw (1989) and more recent contributions like Galí et al. (2007), Bilbiie (2008), Debortoli and Galí (2018), and Bilbiie (2020). In those models a fixed fraction of the population is hand-to-mouth while the rest have access to capital markets and hence their consumption follows a standard Euler equation. In the II models presented in this paper, all agents in equilibrium behave as if they are hand-to-mouth in spite of the fact that they face idiosyncratic employment risk. Importantly, particularly in the II-PH model, while the distribution of wealth is degenerate, the distributions of consumption, hours worked, and earnings are not, and hence this model still generates heterogeneity. Critically, because the frequency of price adjustment is Calvo-style, tracking the distribution of prices, and hence hours or consumption, is tractable. Overall, the simplifying assumptions in the II model allow for a transparent investigation of the implications of idiosyncratic employment risk when hiring and pricing decisions are integrated versus separated, which is not the focus of the aforementioned literature.

Lastly, while useful for studying cyclical fluctuations as in the present case, GHH preferences

\(^3\)While making workers indifferent between employment states makes the analysis of employment volatility potentially vacuous, aggregating consumption and income and making decisions at the household level, as in the standard representative household framework, does not guarantee that all workers individually are at least as well off as the unemployed. Indeed, at a point in time, some workers may be working more than they would like given the consumption they receive. By contrast, the II allocation with take-it-or-leave-it offers naturally mimics that achieved by a household that runs employment lotteries (Hansen, 1985; Rogerson, 1988; Rogerson and Wright, 1988), since differences in consumption are offset by differences in hours worked in order to make all individuals indifferent between employed and unemployed states.
are inconsistent with balanced growth because they have no wealth effect on labor supply. The analysis in this paper therefore can be viewed as modeling detrended, stationary fluctuations. For long-run analysis, one would have to appeal to preferences like those in Boppart and Krusell (2020), which are consistent with balanced growth and where GHH preferences are a limiting case.

3 The Models

This section presents the models, which are variants of a conventional New Keynesian monetary framework augmented with frictional employment. On the household side, two alternatives are considered: one with independent individuals (II) subject to idiosyncratic employment risk, and the more conventional representative household (RH) which pools all idiosyncratic risk. Similarly, on the production side there are two separate environments: one where the same firm makes pricing and hiring (PH) decisions subject to frictions at both margins, and the typical wholesaler-retailer (WR) construct that splits these frictions into two distinct entities.4

3.1 Two Versions of the Household

This section outlines two versions of the household, with independent individuals or a representative household, to highlight the implications of income pooling for the macroeconomy. In each case the household supplies labor, consumes, and saves using a single risk-free asset. The independent individual (II) version follows the Bewley (1986), Huggett (1993), Aiyagari (1994) tradition and assumes individuals operate in isolation and face idiosyncratic earnings risk stemming from employment and hours volatility. As seen below, however, under suitable assumptions about initial asset holdings, the combination of GHH preferences (Greenwood et al., 1988) and take-it-or-leave-it offers make the savings decision in the II version trivial and thus tracking the distribution of wealth unnecessary. Hence, the II model becomes highly tractable along the lines of TANK economy in Debortoli and Galí (2018) or the PRANK economy in Acharaya and Dogra (2020). The representative household (RH) version follows Merz (1995) and Andolfatto (1996), among others, and assumes these decisions are taken within a large household that pools idiosyncratic risk.

4A full set of derivations is in the Online-Only Appendix.
3.1.1 Independent Individuals (II)

In this setup, there is a unit mass of individuals, indexed by \( i \in [0, 1] \), which have GHH preferences over consumption \( c_{i,t} \) and hours worked \( h_{i,t} \) of the form

\[
U(c_{i,t}, h_{i,t}) = \left( c_{i,t} - \varphi h_{i,t}^{1+1/\psi} \right)^{1-\gamma} - 1, \tag{1}
\]

where \( \gamma \) is the constant of relative risk aversion, \( \varphi \) is the disutility of labor, and \( \psi \) is the Frisch elasticity of labor supply. Individuals discount future utility by a factor \( \beta \).

As noted before, the use of GHH preferences, which eliminate any wealth effect on labor supply, provides multiple benefits in the current framework. First, they greatly simplify the contractual environment, as the presence of wealth effects on labor supply would imply that firms would vary their wage offering depending upon the wealth of the worker. Wealth effects on labor supply would also counterfactually imply asset-rich individuals preferring unemployment over employment.\(^5\) Second, along with the assumption on the contracting environment and given symmetric initial conditions on bond-holdings, these preferences imply that in equilibrium no savings occurs, which makes the problem of tracking the wealth distribution trivial.\(^6\) Alternatively, one would have to resort to the perfect consumption insurance assumption as in the RH model described next, or approximation techniques (Gornemann et al., 2016).

Individuals purchase consumption goods at price \( P_t \) and buy nominal bonds \( B_t \) which have gross return \( R_t \) in period \( t + 1 \). They also own shares in a mutual fund that owns all other firms in the economy; the mutual fund pays real dividends \( D_t \).\(^7\) Finally, they pay real lump sum taxes equal to \( T_t \).

The employment status \( n_{i,t} \) of each individual varies between being unemployed \( (n_{i,t} = u) \) and employed \( (n_{i,t} = e) \). In each period a fraction \( n_t \) of individuals are employed, and \( u_t = 1 - n_t \) are unemployed.\(^8\)

Unemployed individuals work zero hours \( (h_{i,t}^u = 0) \), collect real unemployment benefits from

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\(^5\) In contrast, Mustre-del-Río (2015) finds that for prime age males employment is roughly flat with household wealth.

\(^6\) This assumption relates to Chatterjee (1994), who notes that in a standard neoclassical growth model, an equal initial distribution of wealth will persist over time.

\(^7\) Given symmetric initial conditions, in equilibrium all individuals own equal shares in the mutual fund and no trading occurs, so this result is imposed from the outset for simplicity. The Online-Only Appendix shows derivations including a market for mutual fund shares.

\(^8\) For simplicity, the model abstracts from the participation decision and assumes all non-employed individuals actively search for employment.
the government equalling $b$, and search for employment the subsequent period, which occurs in equilibrium with probability $s_t$. If $\mathbb{E}_t$ denotes the expectations operator conditional on time $t$ information, an unemployed worker’s problem is therefore

$$W^u_{i,t} = \max_{c^u_{i,t}, B^u_{i,t}} \left\{ \left( \frac{c^u_{i,t}}{1 - \gamma} \right)^{1 - \gamma} - 1 \right\} + \beta \mathbb{E}_t \left[ s_t W^e_{i,t+1} + (1 - s_t) W^u_{i,t+1} \right]$$  \hspace{1cm} (2)

subject to

$$c^u_{i,t} + B^u_{i,t} P_t + T_t = b + \frac{R_{t-1} B_{i,t-1}}{P_t} + D_t. \hspace{1cm} (3)$$

Employed workers, on the other hand, work positive hours $h_{i,t}$ and are paid a real compensation level $\omega_{i,t}$. Their existing job ends with exogenous probability $\delta$, in which case they enter unemployment the following period, and with probability $(1 - \delta)$ they remain employed. An employed worker’s problem is therefore

$$W^e_{i,t} = \max_{c^e_{i,t}, B^e_{i,t}} \left\{ \left( \frac{c^e_{i,t} - \varphi h_{i,t}^{1+1/\psi}}{1 - \gamma} \right)^{1 - \gamma} - 1 \right\} + \beta \mathbb{E}_t \left[ (1 - \delta) W^e_{i,t+1} + \delta W^u_{i,t+1} \right]$$  \hspace{1cm} (4)

subject to

$$c^e_{i,t} + \frac{B^e_{i,t}}{P_t} + T_t = \omega_{i,t} + \frac{R_{t-1} B_{i,t-1}}{P_t} + D_t. \hspace{1cm} (5)$$

Note that employed workers do not choose $h_{i,t}$, as hours are determined within the contracting environment with the firm.

Standard optimality conditions yield Euler equations for the unemployed and employed

$$\lambda^u_{i,t} = \beta \mathbb{E}_t \left[ \frac{s_t \lambda^e_{i,t+1} + (1 - s_t) \lambda^u_{i,t+1}}{\Pi_{t+1}} \right] R_t,$$

and

$$\lambda^e_{i,t} = \beta \mathbb{E}_t \left[ \frac{(1 - \delta) \lambda^e_{i,t+1} + \delta \lambda^u_{i,t+1}}{\Pi_{t+1}} \right] R_t. \hspace{1cm} (6)$$

respectively, where $\Pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate. The marginal utilities of consumption for the unemployed and employed are respectively given by

$$\lambda^u_{i,t} = (c^u_{i,t})^{-\gamma}, \hspace{0.5cm} \text{and} \hspace{0.5cm} \lambda^e_{i,t} = \left( c^e_{i,t} - \varphi h_{i,t}^{1+1/\psi} \right)^{-\gamma}. \hspace{1cm} (7)$$

Workers receive take-it-or-leave-it offers in the vein of Postel-Vinay and Robin (2002) and Cooper et al. (2007) from firms in a frictional environment described below. Given symmetric initial conditions on bond-holdings, the optimal contract equalizes the value of employment $W^e_{i,t}$
and unemployment $W_{i,t}^u$, which implies

$$
\omega_{i,t} = b + \varphi h_{i,t}^{1+1/\psi}.
$$

As a result of the assumed preferences and contract, in equilibrium, the marginal utilities of consumption are symmetric across employment states: $\lambda_t = \lambda_{i,t}^u = \lambda_{i,t}^e$.

### 3.1.2 Representative Household (RH)

The second setup for the household is one where a large or representative household (RH) also has GHH preferences, but makes all consumption and labor decisions at the aggregate level. Keeping all other aspects unchanged, this model still implies the same contract as the II model.

In this version of the problem, the household enters the period with $n_t$ of its members employed, and has a value function given by

$$
V_t(n_t) = \left( C_t - \varphi \int_0^{n_t} h_{i,t}^{1+1/\psi} \omega_{i,t} di \right)^{1-\gamma} \frac{1}{1-\gamma} - 1 + \beta \mathbb{E}_t V_{t+1}(n_{t+1}).
$$

Since the household pools all income to choose aggregate consumption and bond holdings, it faces the budget constraint

$$
C_t + \frac{B_t}{P_t} + T_t = b (1 - n_t) + \int_0^{n_t} \omega_{i,t} di + \frac{R_{t-1} B_{t-1}}{P_t} + D_t.
$$

Standard optimality conditions show that the marginal utility of consumption is now

$$
\lambda_t = \left( C_t - \varphi \int_0^{n_t} h_{i,t}^{1+1/\psi} \omega_{i,t} di \right)^{-\gamma},
$$

while the optimal choice of bonds—which end up being in zero net supply—is given by

$$
\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\Pi_{t+1}} = 1.
$$

In this setup, the marginal utility of consumption differs due to the fact that the household is aggregating income to make consumption and savings decisions. As a result, the stochastic discount factor used by firms and the relationship between expected inflation and the nominal interest rate depend on the household’s decisions.
All employed workers in the household receive wage posting offers from their firms in the period. The benefit to the household of having the \( n_t \)-th worker employed is given by the envelope condition of (9) with respect to \( n_t \):

\[
V_n(n_t) = -\varphi h_n^{1+1/\psi} \lambda_t + (\omega_{n,t} - \beta) \lambda_t + [(1 - \delta) - s_t] \beta \mathbb{E}_t V_{n,t+1}(n_{t+1}).
\]  

(13)

The first term is the loss in utility to the household due to one of its workers providing additional hours, the second term is the gain in utility for receiving compensation rather than unemployment benefits, and the third term is the expected future benefit of having an employed worker rather than an unemployed one.

When receiving wage offers, the household treats all workers as symmetric. Equivalently, the take-it-or-leave-it assumption implies that firms will give wage offers that make the above envelope condition equal to zero for all \( i \) in \([0, n_t]\). In other words, for any employment level, the contract makes the household indifferent at the margin between having one more worker employed or not. As a result, the optimal contract for each worker is identical to equation (8) in the II problem.

3.1.3 Discussion

Despite key differences in the risk sharing environments, both the II and RH setups have identical forms of the optimal contract. In particular, when firms make take-it-or-leave-it offers to workers, the contract makes compensation vary with hours worked but not aggregate labor market tightness. Thus, cyclical variation in compensation is solely due to changes in labor demand through hours worked \( h_{i,t} \), which as seen next will depend on whether the hiring firm also sets prices or not.

However, one important difference between the the II and RH models pertains to marginal utility, which as shown in the quantitative sections is key for firm decisions through the stochastic discount factor. As shown in equation (11), income pooling in the RH model makes marginal utility depend on consumption, individual hours worked, and the total number of employed workers. In contrast, because there is no large household construct in the II model, marginal utility only depends on consumption and individual hours worked, as shown in equation (7). Thus, even if consumption and hours worked were equal across frameworks, marginal utility, and hence the stochastic discount factor, would still differ across models.
3.2 Two Versions of Intermediate Good Production

Next, two versions of the production sector—one with firms that both hire and set prices, and one with a wholesaler-retailer structure—are presented to show the quantitative relevance of integrating price-setting and hiring within one firm. In both versions, hiring firms make take-it-or-leave-it offers to attract workers in a frictional labor market. Additionally, both versions have a final goods producer that combines differentiated products into a bundle, which is then sold to consumers. In the pricing and hiring (PH) environment, firms hire workers in a frictional labor market to produce a good which is sold subject to sticky prices to the final good producer. The integration of these two decisions subject to frictions along both margins follows the work of Kuester (2010), Barnichon (2010), Thomas (2011), and Lago Alves (2018), among others. In the wholesaler-retailer environment (WR), by contrast, wholesalers hire workers in a frictional labor market to produce and sell a competitively priced wholesale good to retailers, who then sell their repackaged good subject to sticky prices to final goods producers. This separation of the hiring and pricing decisions is similar to Walsh (2005), Trigari (2006), and Sveen and Weinke (2008), among others.

3.2.1 Pricing and Hiring (PH) Firms

This first structure integrates the pricing and hiring decisions within the same firm. Specifically, in this pricing and hiring (PH) firm environment, firms hire in a frictional labor market, and when matched with a worker, sell their output subject to sticky prices. As a result, the firm internalizes the effects of each friction on both decisions. For example, the firm takes into account the stickiness of prices when making hiring and contracting decisions. Similarly, conditional on being matched, the firm takes into account the marginal cost of an additional hour of work and the probability of job separation when resetting prices. Thus, integrating both the hiring and pricing decisions within a single firm reveals how frictions on one margin affect decisions on the other.

Demand arises from final good producers, which operate competitively, purchasing $Y_{j,t}$ from $j \in [0, n_t]$ operating intermediate goods firms and combining them into final output $Y_t$ using a technology with constant elasticity of substitution $\epsilon$:

$$
Y_t = n_t \left( \frac{1}{n_t} \int_0^{n_t} Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.
$$

(14)

Standard cost minimization implies that the demand for each intermediate good $Y_{j,t}^{d}$ depends on
its relative price according to
\[ Y_{j,t}^d = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} Y_t n_t. \] (15)

The aggregate price level is related to individual prices by
\[ P_t^{1-\epsilon} = \frac{1}{n_t} \int_0^{n_t} P_{j,t}^{1-\epsilon} dj. \] (16)

Intermediate goods firms are indexed by \( j \), and produce using a linear technology
\[ Y_{j,t}^s = Z_t h_{j,t}, \] (17)

where \( h_{j,t} \) is hours at firm \( j \) and productivity \( Z_t \) follows
\[ \log Z_t = \rho z \log Z_t - 1 + \sigma z \varepsilon z,t. \] (18)

Firms sell their output at price \( P_{j,t} \) and are subject to a Calvo friction when setting prices. Firms employ a single worker; conditional on being matched with a worker the firm negotiates a contract \( \Upsilon_{j,t} = (\omega_{j,t}, h_{j,t}) \) that determines a compensation level \( \omega_{j,t} \) and an hours requirement \( h_{j,t} \). Firms face a two-stage problem: in the first stage they set prices and in the second stage they contract with labor and produce.

In the second stage, given a price \( P_{j,t} \), firms make a take-it-or-leave-it offer to their worker. They choose a contract \( \Upsilon_{j,t} \) to maximize current period profits subject to their demand (15), the constraint that they must meet demand at the posted price \( Y_{j,t}^s \geq Y_{j,t}^d \), and the worker’s participation constraint. As noted, regardless of whether the household sector is populated by independent individuals or a representative household, the optimal contract follows equation (8).

Given the optimal contract, in the first stage a matched firm can re-optimize its price subject to a Calvo friction. The value of an operating firm with price \( P_{j,t} \) is given by
\[ J_t(P_{j,t}) = \left( \frac{P_{j,t}}{P_t} \right) Y_{j,t}^d - \omega_{j,t} + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1}(P_{j,t}) + (1 - \zeta) J_{t+1}(P^*_t) \right], \] (19)

where \( \beta \frac{\lambda_{t+1}}{\lambda_t} \) denotes the stochastic discount factor, \( \zeta \) the probability of not re-optimizing prices, and \( P^*_t \) denotes the optimal price set by a firm that can re-optimize at time \( t \). Since the optimal compensation scheme depends on hours, and firms must meet demand at the posted price, the
value is given by

\[ J_t (P_{j,t}) = \left( \frac{P_{j,t}}{P_t} \right)^{1-\epsilon} \frac{Y_t}{n_t} - b - \varphi \left( \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t} \right)^{1+1/\psi} + \beta (1 - \delta) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1} (P_{j,t}) + (1 - \zeta) J_{t+1} (P_{t+1}^*) \right]. \] (20)

This expression makes explicit the fact that prices, by pinning down demand, consequently pin down hours, and hence total compensation, through the relationship

\[ h_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{Z_t n_t}. \] (21)

A firm that can re-optimize prices, hence, takes this dependence of hours and compensation on the relative price, with the optimal reset price $P_t^*$ satisfying

\[ \mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \zeta (1 - \delta))^k \frac{\lambda_{t+k}}{\lambda_t} \left\{ (1 - \epsilon) \left( \frac{P_t^*}{P_{t+k}} \right)^{1-\epsilon} \frac{Y_{t+k}}{n_{t+k}} + \epsilon \left( 1 + \frac{1}{\psi} \right) \varphi (h_{t+k}^*)^{1+1/\psi} \right\} \right] = 0, \] (22)

where $h_{t+k}^* = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{Y_{t+k}}{Z_{t+k} n_{t+k}}$, represents the optimal hours choice at time $t+k$ given a reset price $P_t^*$. This optimal reset equation is similar to that found in typical Calvo price-setting environments. However, in the current environment the firm’s marginal cost is the marginal compensation paid to the worker it is matched with, which in turn depends on the evolution of the firm’s relative price. Thus, while a high relative price in the future lowers demand, the firm can reduce costs by decreasing hours worked, and hence compensation, from the matched worker.

Firms post vacancies at cost $\kappa$, which are filled with probability $q_t$ and become productive the following period. At the beginning of $t+1$ price adjustment occurs, then contracting and production. New entrants inherit a price level in period $t$ equal to the aggregate price level ($P_{j,t} = P_t$), and receive a Calvo shock before production in $t + 1$.9 Because of free entry, firms post vacancies until the vacancy posting cost equals the expected return, which implies

\[ \kappa = q_t \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[ \zeta J_{t+1} (P_t) + (1 - \zeta) J_{t+1} (P_{t+1}^*) \right]. \] (23)

9 An alternative assumption where entrants always optimally set prices would generate slightly different dynamics between employment and inflation, and in fact might lead to larger differences between the models considered. However, that assumption would also lead to a different evolution of the price level than in standard New Keynesian models and those with labor search (for example, Kuester, 2010).
This expression highlights how expectations of future price-setting behavior affect the incentive to enter the market and hence vacancy creation.

3.2.2 Wholesaler-Retailer (WR)

The second setup is a wholesaler-retailer environment, which separates the pricing and hiring decisions across two types of firms. In this version, wholesalers hire labor in a frictional labor market using take-it-or-leave-it offers and produce a competitively priced good. Monopolistically competitive retail firms, who face Calvo price frictions, purchase the wholesale good and convert it into a differentiated good for sale to the final good producer. Thus, unlike in the PH environment, in this framework hiring firms do not internalize the infrequent adjustment of prices when making hiring decisions. Similarly, price-setting firms do not internalize the time cost of searching for labor and the marginal cost of production they face is notably different.

As in the PH model, demand stems from competitive final good producers, purchasing $Y_{j,t}$ from $j \in [0,1]$ retailers and combining into final output $Y_t$ using a CES technology. Since there are a constant unitary mass of retailers, the equations characterizing behavior are the same as with PH, (14), (15), and (16), except with $n_t = 1$.

Focusing on the wholesaler problem, they operate a linear technology as in equation (17) to produce a wholesale good $Y^w_t$, taking the price of the wholesale good $P^w_t$ as given. Again, regardless of the household sector being independent individuals or a representative household, wholesalers face a contracting environment that produces an optimal contract of the form of equation (8). A key difference of this model relative to the PH version is that, since the wholesaler is a perfectly competitive firm with symmetry, all employed workers have identical contracts $\Upsilon_t = (\omega_t, h_t)$. Given this contract, wholesalers choose hours to maximize

$$J_t = \max_{h_t} \frac{P^w_t}{P_t} Z_t h_t - b - \varphi h_t^{1+1/\psi} + \beta(1 - \delta)\lambda_t \lambda_{t+1} J_{t+1}.$$  

(24)

The first-order condition with respect to hours implies that hours depend on the relative price of wholesale goods and the level of technology by

$$h_t = \left( \frac{P^w_t}{P_t} \frac{Z_t}{\varphi (1 + 1/\psi)} \right)^{\psi}.$$  

(25)

\textsuperscript{10}Given symmetry of the wholesale firms in this environment, the subscript $j$ is omitted for simplicity.
Comparing this expression to equation (21) under the PH model reveals important differences across the two models. In the PH model, hiring firms are price setters and demand for their differentiated good is decreasing in their relative price, so are hours worked. In contrast, in the WR model, hiring firms are price takers and because supply for their good is increasing in the relative price of wholesale goods, so are hours worked. In partial equilibrium, technology shocks also have differential effects on hours worked across models. In the PH model, because output is pinned down given relative prices, any increase in productivity is labor saving and hence hours worked fall. In contrast, in the WR model an increase in productivity increases hours since each existing employment match is more valuable given prices.

Next, under the WR model the free-entry condition takes the usual form

$$\kappa = q_t \beta \mathbb{E}_t \left( \frac{\lambda_{t+1}}{\lambda_t} \right) J_{t+1}. \quad (26)$$

Comparing this equation to (23), the free entry condition under the PH model, highlights that frictions related to price adjustment do not directly affect vacancy creation in the WR model, while they have a direct impact in the PH model.

To close the WR model, retailers face a standard problem summarized by the optimal reset price condition

$$\mathbb{E}_t \left[ \sum_{k=0}^{\infty} (\beta \zeta)^k \left( \frac{\lambda_{t+k}}{\lambda_t} \right) \left( 1 - \epsilon \right) \left( \frac{P^*_t}{P_{t+k}} \right) + \epsilon \left( 1 + \frac{1}{\psi} \right) \varphi^{h_{t+k}^{1/\psi}} \frac{h_{t+k}^{1/\psi}}{Z_{t+k}} \mathbb{E}_t \left[ \left( \frac{P^*_t}{P_{t+k}} \right)^{-\epsilon} \right] \right] = 0, \quad (27)$$

where the expression uses the fact that marginal costs, dependent on $P^w_t$, can be written as a function of hours using equation (25).

### 3.2.3 Discussion

There are two important differences between the PH and WR models, which can be seen by comparing their optimal reset price equations. First, as seen in equation (22) of the PH model, because intermediate firms face pricing and labor market frictions, they discount future revenues both by the expected duration of the current price, which depends on $\zeta$, and the expected duration of the current match, which depends on $\delta$. In contrast, as seen in equation (27) of the WR model, retail firms do not care about match duration when setting their prices. Second, marginal costs are notably different across the two models. In the WR model, marginal costs depend on the
relative price of wholesale goods. This price in turn is related to the marginal disutility of work at \( h_t \), which is common to all workers and determined in general equilibrium and thus not directly dependent on the firm’s chosen relative price. In contrast, in the PH model marginal costs depend on the marginal disutility of hours worked for the matched worker \( h_{jt} \), which—rather than being dependent solely on general equilibrium outcomes—depend critically on the firm’s relative price. As a consequence, in the PH model, price-setting firms directly consider the impact that pricing decisions have on future hours demanded and hence marginal costs, while no such trade-off exists for price-setting firms in the WR model.

3.3 Closing the Models

Each model is closed with identical assumptions on the matching process for labor markets, monetary and fiscal policy, and the market clearing condition.

3.3.1 Vacancy Posting and the Labor Market

Matches \( m_t \) depend upon the number of unemployed workers \( u_t \) and vacancies \( v_t \) according to

\[
    m_t = \sigma_m u_t^\alpha v_t^{1-\alpha},
\]

where \( \sigma_m \) governs the efficiency of the matching function, and \( \alpha \) is the elasticity of matches with respect to the number of unemployed workers. The job filling rate is \( q_t = m_t/v_t \), while the job finding rate is \( s_t = m_t/u_t \). New matches take one period to form, and existing matches are destroyed at an exogenous rate \( \delta \). Consequently, employment evolves according to

\[
    n_t = (1 - \delta) n_{t-1} + m_{t-1}.
\]

3.3.2 Policy and Market Clearing

Monetary policy follows a Taylor Rule, setting the nominal rate \( R_t \) according to

\[
    \frac{R_t}{R_{ss}} = \left( \frac{R_{t-1}}{R_{ss}} \right)^{\rho_r} \left( \frac{\Pi_t}{\Pi_{ss}} \right)^{(1-\rho_r)\gamma_\pi} \exp \left( \sigma_r \varepsilon_{r,t} \right),
\]

where \( \Pi_{ss} \) indicates the inflation target, \( R_{ss} \) the nominal rate target, \( \rho_r \) the degree of interest rate persistence, \( \gamma_\pi \) the response to inflation, and \( \varepsilon_{r,t} \) denotes a monetary policy shock.
Fiscal policy adjusts lump sum taxes to balance the budget, and since its only payments are unemployment benefits $b$, then $u_t b = T_t$. Market clearing requires that aggregate output equals aggregate consumption, so $Y_t = C_t$.\footnote{A full definition of the equilibrium is provided in the Online-Only Appendix.}

## 4 Calibration and Moment Matching

Having laid out the models in Section 3, this section calibrates them. The four model variants considered (II-PH, II-WR, RH-PH, and RH-WR), have many common elements but a few slight differences to consider when choosing parameters. The calibration strategy thus takes three steps. In the first step, a number of parameters that are common across models are chosen based on typical values in the literature. In the second step, some key labor market parameters are chosen, possibly differently across models, to match certain steady state targets. Finally, the third step is to calibrate the level of unemployment benefits $b$ differently across models to match wage and hours data in the US economy.

Table 1 lists the first set of parameters, which are fixed at standard values and common across all models. Assuming the model period is a quarter, the discount factor $\beta$ is set to imply a steady state real interest rate of 2%. The coefficient of risk aversion $\gamma$ is set to 2 as is standard in the literature, while the Frisch elasticity of labor supply on the intensive margin $\psi$ is set to 0.5 as suggested by Chetty et al. (2011). The probability of not re-optimizing prices $\zeta$ is set to match a median price duration of six months as reported in Bils and Klenow (2004). Following Gertler et al. (2008), the elasticity of substitution across goods is $\epsilon = 10$, which implies a steady state markup of 11%. Consistent with empirical estimates in Shimer (2005) and den Haan et al. (2000), the quarterly separation rate is 10 percent. The elasticity of the matching function with respect to unemployment $\alpha$ is set to 0.5, which is the midpoint of values typically cited in the literature.

A full definition of the equilibrium is provided in the Online-Only Appendix.
### Panel A: Calibrations and Targets

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
<th>II-PH</th>
<th>II-WR</th>
<th>RH-PH</th>
<th>RH-WR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>Disutility of labor</td>
<td>$h_{j,ss} = 1/3$</td>
<td>2.70</td>
<td>2.70</td>
<td>2.07</td>
<td>2.70</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Matching efficiency</td>
<td>$u_{ss} = 0.11$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$b/\omega_{ss}$</td>
<td>SS Replace. Ratio</td>
<td>$\sigma(w)/\sigma(Y) = 0.53$</td>
<td>0.45</td>
<td>0.80</td>
<td>0.62</td>
<td>0.01</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Vacancy posting</td>
<td>$q_{ss} = 0.70$</td>
<td>1.01</td>
<td>1.28</td>
<td>0.46</td>
<td>1.33</td>
</tr>
</tbody>
</table>

### Panel B: Performance of Models

<table>
<thead>
<tr>
<th>Moment</th>
<th>Description</th>
<th>US Data</th>
<th>II-PH</th>
<th>II-WR</th>
<th>RH-PH</th>
<th>RH-WR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(w)/\sigma(Y)$</td>
<td>Rel Vol of Avg Wage</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>$\sigma(H)/\sigma(Y)$</td>
<td>Rel Vol of Agg Hours</td>
<td>0.79</td>
<td>0.79</td>
<td>0.45</td>
<td>2.86</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Lastly, the parameters governing shocks and monetary policy are also set to standard values.

Panel A of Table 2 lists the parameters calibrated to match steady state values, and how these differ across models. The disutility of hours worked $\varphi$ is such that steady state hours worked per employed person equals $1/3$. Given that preferences have identical forms across models, this target implies $\varphi = 2.7$ for each model. Following Blanchard and Diamond (1990), the targeted steady state unemployment rate is 11 percent, which includes both individuals who are categorized as unemployed and those out of the labor force who want a job. Following den Haan et al. (2000), the steady state worker finding rate is 70 percent. These assumptions directly pin down the matching efficiency parameter $\sigma_m$. Again, since the matching function and evolution of employment are identical across models, this produces $\sigma_m = 0.75$. Given the calibration and targets, the vacancy posting cost $\kappa$ is implied from the steady state free-entry condition in each model, and these differ across models due to differences in unemployment benefits $b$.

The calibration of the unemployment benefits $b$ requires a relative level, in this case the steady state replacement ratio, which is defined as the ratio of unemployment benefits divided by the average compensation per worker in steady state. There are a wide range of values in the literature, as Shimer (2005) considers a value of 0.4 while Hagedorn and Manovskii (2008) consider a value close to one. Rather than choose a value within this range, the replacement ratio is chosen so that the volatility of hourly wages, relative to output, matches US data. As can be seen from Panel A of Table 2 the implied replacement ratios vary significantly across the models, as despite the identical optimal contract, the demand for hours and general equilibrium effects also differ across models. Panel B of Table 2 highlights the tensions faced by some of the models in matching both
the volatility of average hourly wages and aggregate hours.\footnote{The relevant data used are for the non-farm sector, and consist of real output, aggregate hours, and real compensation per hour, all since 1951Q1-2016Q3 and are HP filtered using a smoothing parameter of 1600.} While the goal of this paper is not to evaluate the empirical performance of each model, these comparisons help highlight the mechanisms at play across frameworks.

The models all hit the relative volatility of the average wage by construction. But the untargeted moment of the relative volatility of aggregate hours differs substantially across models reflecting the different production (PH or WR) and household (II or RH) structures. The II-PH model essentially replicates the value in the data. Meanwhile, the II-WR and RH-WR models underpredict the data by about half and the RH-PH model has about four times the value in the data.

Focusing first on the importance of integrating pricing and hiring decisions, note that relative to the PH models, the WR models generate lower volatility of aggregate hours given the volatility of average wages. As equation (25) shows, in the WR framework, the hours of each worker are completely symmetric. In contrast, as seen in equation (21), in the PH structure price dispersion induces hours dispersion because pricing and hiring are integrated within the same firm. Thus, this additional link in the PH models, is responsible for the greater volatility in aggregate hours.

Turning next to the importance of income pooling, note that the RH-PH model generates slightly too much volatility in aggregate hours compared to the II-PH model. In the RH-PH model, income pooling changes the behavior of the stochastic discount factor used by firms in their pricing and entry decisions and this amplifies the response of aggregate hours in response to shocks relative to what occurs in the II-PH model. Recall, in the RH model marginal utility, equation (11), depends on consumption, individual hours worked, and the total number of employed workers in the household. In contrast, in the II model by virtue of the fact that individuals operate in isolation, marginal utility, equation (7), does not depend on employment. Hence, changes in aggregate employment get amplified in the RH-PH model, but not in the II-PH model.

5 Comparative Statics: The Effects of Policy

To begin illustrating the quantitative implications of the different environments, this section turns to the analysis of two different types of policy changes using comparative statics at the steady state of each model. First, motivated by the recent increase in the generosity of unemployment benefits as a response to the COVID-19 pandemic, this section examines the aggregate impact of changes in the level of unemployment benefits. Second, motivated by the fact that the US has had inflation below
the Federal Reserve’s 2 percent target for a majority of the expansion following the Great Recession, it considers how changes in the steady state level of inflation affect economic outcomes. The results suggest that firm-level assumptions matter greatly for these policy questions. Models that integrate hiring and pricing decisions imply nondegenerate responses to changes in unemployment benefits and significant output losses and higher unemployment in response to persistently below target inflation. In contrast, models that separate these decisions lead to essentially no quantitative responses to these policy changes.

5.1 Unemployment Benefits

In part motivated by the increase in the level of unemployment benefits as part of the Coronavirus Aid, Relief, and Economic Security (CARES) act, this section considers whether the different models imply different macroeconomic outcomes in response to this policy change. Figure 1 shows the effects of increasing or decreasing the level of unemployment benefits $b$ by up to 5 percent across models.\footnote{By comparison, the additional $600 per week provided by the Federal Pandemic Unemployment Compensation (FPUC) component of the CARES act lead to roughly a 155 percent increase in average benefits.} An immediate take-away from Figure 1 is that the policy implications from the different modeling frameworks vary significantly.

First, PH models exhibit significant responses to changes in the level of unemployment benefits. In these models, increases in $b$ raise the average wage, lower vacancies, raise unemployment, and lower output. These results are in a similar vein to the empirical findings of Hagedorn et al. (2013) regarding the general equilibrium effects of unemployment benefit extensions during the Great Recession. They conclude that extending the duration of unemployment benefits raised equilibrium wages, which subsequently lead to a contraction in vacancy creation and a rise in unemployment. In the present context, when the level of benefits is increased, existing firms with pricing power pass the increase in labor costs on to demand by raising prices. As demand shrinks, this alters the incentives of new firms to post vacancies, which leads to a large drop in unemployment and output.

In contrast, the WR structure that splits pricing and search frictions into separate firms leads to negligible effects when benefits change. While it is still true that changes in benefits translate into changes in wages since the optimal compensation contract is the same, the pass-through of this change in labor costs is fairly limited. Recall that in this framework hiring firms do not have pricing power. As a result, aggregate quantities like output, vacancies and unemployment barely change.
Notes: Shows the effects of a ±5% change in unemployment benefits $b$ for models with combinations of independent individuals (II) or a representative household (RH), with pricing and hiring firms (PH) or wholesaler-retailers (WR).

5.2 Persistently Below-Target Inflation

Next, given the behavior of inflation over the past two decades, this section considers the implications of inflation persistently below the Federal Reserve’s 2 percent target. In a model with indexation of prices, having an inflation target that differs from the level of indexation generates a distortion in relative prices (for example, Ascari and Ropele, 2009). However, instead of considering the effects of raising the inflation target when prices are not indexed, persistently low inflation can be interpreted in the model as an inflation target below 2 percent, while prices continue to be indexed at 2 percent. This assumption generates a gap where re-optimizing firms are increasing their prices at a lower rate than non-optimizing firms.

Figure 2 therefore shows the effects of a lower inflation target, keeping fixed the fact that firms index their prices at the baseline calibration of 2 percent annually. As a result, firms that do not re-optimize their prices index them by relatively too much, leading to prices that tend to be too
high, which subsequently stifles demand. The analysis in this section complements Lago Alves (2018), which notes that positive trend inflation can help reconcile the Shimer (2005) puzzle.

Overall, the effects of inflation misses mimic the effects of unemployment benefit changes in that the PH models have significant responses to lower steady state inflation, while the WR models do not. The PH models once again show much stronger effects in response to the policy change. Lower demand induced by relatively too-high pricing generates less entry by firms and as as result vacancies decline, leading to higher unemployment. This higher unemployment feeds back to lower demand, leading to a further contraction in demand. In total, the decreases in demand lead to much lower output levels. The differences between the representative household (RH) and independent individuals (II) are mostly magnitudes, except for the behavior of average wages. In both models average compensation and average hours fall to partly offset the fall in demand. However, in the II-PH model the decline in hours relative to compensation is smaller, and so the average hourly

\[14\]
rate—the ratio of compensation to hours—falls as well. In contrast, in the RH-PH model hours fall by more relative to compensation, and so the average hourly rate rises.

Interestingly, the II-PH model broadly captures the concerns of the expansion in the 2010s. As the economy expanded at a modest pace, inflation remained below the 2 percent target, leading to concerns about lower output and higher labor market slack, along with stagnant wages. The II-PH model accounts for these observations precisely because price-setting firms internalized lower demand by reducing hiring activity, which further lowers demand and keeps inflation muted.

In contrast to the PH models, the separation of labor market and pricing frictions in the WR models leads to negligible changes in the economy. Indeed, pricing misses have little effect on the labor market, which subsequently leads to little changes in demand, which reaffirms the non-importance of relatively too-high pricing when inflation is below target. Therefore, as steady state inflation declines, there is little movement in aggregates.

6 Impulse Responses

This section complements the comparative static analysis from the previous section and examines the dynamics associated with technology and monetary policy shocks. Unlike the results from the previous section, the results shown below suggest both firm- and household-level assumptions shape the propagation of shocks through the economy. In particular, the results highlight how the household-level assumptions, through the stochastic discount factor, change the dynamics of inflation and unemployment.

6.1 Technology Shock

Figure 3 shows the behavior of the models to a one standard deviation positive innovation in total factor productivity. Because the analyzed models are parsimonious and lack certain real rigidities, most of the change in aggregate output occurs in the first period.\textsuperscript{15} However, each model differs in how the instantaneous increase in demand is accommodated.

Focusing first on the II-PH model, employment is pre-determined in the first period since matches take a period to become productive. Existing firms that can re-optimize their price face a trade-off between changing prices and changing hours and hence compensation of their matched worker. With higher aggregate demand due to the shock, a firm could increase prices, reduce hours

\textsuperscript{15}In a framework similar to the RH-WR model, Trigari (2006) finds that consumption habits help undo some of the immediate response.
and compensation, and therefore sell fewer goods at a higher markup. Alternatively, they could decrease prices, increase hours and compensation, and sell more goods at a lower markup. On net, the incentives for the latter dominate, so inflation barely moves while individual hours and wages rise. Consumption and aggregate output rise sharply from higher compensation, leading to a decline in marginal utility. However, given GHH preferences, the rise in hours mutes the decline in marginal utility, which is a key distinguishing feature between II and RH models.

Indeed, turning to the RH-PH model reveals how the pooling of consumption changes some of the previously documented dynamics. With income pooling, hours and wages barely move in response to the productivity shock. Thus, marginal utility falls by much more in this model compared to the II-PH model. The combination of a muted response in hours of existing firms and lower marginal utility, which makes future profits move valuable, results in an explosion of firm
entry that persistently lowers the unemployment rate. The explosion of entry and the rise in the value of future profits induces forward-looking price-setters to reduce their prices and so inflation falls sharply on impact and remains persistently below its steady state value. Importantly, the main reason why price-setters react to firm entry is because of the integration of pricing and hiring decisions within the same firm.

To see the importance of the pricing and hiring link, note the downward response of inflation in the RH-WR model is much more muted compared to the RH-PH model. Since this is also an RH model, marginal utility falls drastically as consumption rises and hours barely move. The fall in marginal utility and the relative invariance of hours still causes an explosion in wholesaler firm entry and a fall in the unemployment rate as in the RH-PH model. However, due to the WR block of this model, retailers do not internalize the explosion in wholesaler firm entry and so inflation moves by much less in this model.

Finally, the responses of the II-WR model reiterate many of the previously highlighted mechanisms. Because of the WR block, retailers face no trade-off between prices and hours and so inflation rises sharply on impact. This sharp rise in inflation mutes demand and so hours worked rise by much less compared to the II-PH model. On net, output still rises by more than hours worked in this model and so marginal utility falls a bit more in comparison to the II-PH model. Critically, though, the sharp rise in inflation and the initially higher path of the nominal rate erode the positive effect that lower marginal utility has on future profits, through the discount factor. As a result, vacancies rise by less in this model compared to the II-PH model, where inflation barely moves on impact.

Across models, the effect of the positive productivity shock on output, hours, and wages is relatively short-lived, since higher productivity induces firm entry. This additional entry lowers unemployment in subsequent periods, but also impacts price-setting and hours choices. In particular, with greater competition, the incentive to lower prices and sell more goods grows. As a result, non-price setting firms end up with high relative prices, which causes the hours of their workers to fall, leading to a dampening effect on compensation and hence aggregate demand.

To summarize, after a technology shock, the responses of both macroeconomic and labor market variables vary across models. Like in Thomas (2011), a comparison of PH and WR models show the importance of integrating pricing and hiring decisions within the same firm, as the former produces

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16Note that unemployment falls in response to a positive technology shock because there is no participation margin. This is in contrast to models like Gali (2011) where unemployment actually rises in response to a positive technology shock as more workers enter the labor force.
more muted responses of inflation to a productivity shock compared to the latter. Meanwhile, comparing the II and RH models highlights that the single representative household construct, by pooling income between the employed and unemployed, mutes the effects of productivity shocks on output, average hours worked, and wages while in turn generating larger fluctuations in vacancies and unemployment. The response of the latter variables reflects the fact that marginal utility moves more in the RH model and this distorts forward-looking decisions like vacancy posting through the stochastic discount factor.

The behavior of the stochastic discount factor, in particular, makes the differences in dynamics between the II and RH models presented here different than typical HANK and RANK model comparisons. For example, in Gornemann et al. (2016) the stochastic discount factor used by firms is derived from the marginal utility of a mutual fund investor, which in turn distributes profits to households in the form of dividends. In Kaplan et al. (2018), firms discount profits using the rate of return on illiquid assets, which is determined in equilibrium to clear the physical capital market. In contrast, because of the simplifying assumptions imposed here, the stochastic discount factor is always a function of individual or household marginal utility, highlighting how the differing risk sharing assumptions distort firm-level decisions and have important aggregate consequences.

6.2 Monetary Policy Shock

Figure 4 shows the behavior of the models to a one standard deviation contractionary innovation to monetary policy. While the II-WR model is the biggest outlier, the figure reiterates many of the key mechanisms at play across each model.

In the II-WR model, since retail firms do not internalize the impact of pricing decisions on the labor market, the decline in inflation is much larger compared any other model. This substantial decline in prices mutes some of the impact of tighter monetary policy on output, so hours worked fall less drastically. In addition, the sharp drop in inflation nearly undoes the effects of the policy shock on the nominal rate, with the rate increasing only slightly in equilibrium. Ultimately, the sharper drop in inflation induces a smaller decline in output and hours, which leads to a smaller increase in marginal utility. Since marginal utility rises by less, future profits are not discounted much more. As a result, forward-looking wholesalers reduce vacancy creation by less and unemployment rises less dramatically compared to all other models.

In the II-PH model, output, hours worked, and unemployment all fall by much more compared to its WR counterpart. Critically, because PH firms internalize how pricing decisions affect the
labor market the decline in inflation in this model is much more muted. As a result, demand and hours work must fall by much more, and ultimately unemployment rises by much more as well.

Turning finally to the RH-PH and RH-WR models, the household’s ability to pool income mutes many of the effects seen in the II models. Here the pooling of income means that aggregate demand declines less following the shock, which therefore tempers the prices versus hours adjustment faced by firms. As a result, inflation, average hours worked, and wages fall by less than in the II models. Inflation in the RH-WR model, in particular, barely falls compared to the sharp drop in the II-WR model. While in both cases retail firms can slash prices, income pooling in the RH-WR model ameliorates the decline in demand and as a result reduces the need to cut prices.\textsuperscript{17} Overall, as the

\textsuperscript{17}A similar mechanism explains the difference in inflation between the RH-PH and II-PH models, but there the possibility of adjusting hours and prices makes any drop in inflation smaller and hence the differences across models only modest.
declines in inflation are very slight in the RH models, most of the effects of the monetary policy shock are passed directly to an increase in the nominal rate. This increase, coupled with the interest rate inertia in the policy rule, leads to a more gradual increase in output in subsequent periods, and distorts the forward-looking behavior of firms through the stochastic discount factor. Thus, the ensuing decline in vacancies and rise in unemployment is then slightly more pronounced in the RH models than the II versions. This again is particularly true when comparing the RH-WR and II-WR models, which exhibit the biggest discrepancy in inflationary dynamics and in concert the largest difference in unemployment dynamics.

As with a productivity shock, the effects of a monetary policy shock are short-lived. Lower demand, paired with a higher marginal utility of consumption, causes firms to contract the number of vacancies posted, and hence unemployment increases in the period following the shock. The decline in the number of operating firms, however, dampens the need for further price reductions, which helps hours, wages, and therefore output to quickly return to their steady state values.

In sum, these results highlight that both macroeconomic and labor market variables vary across models in how they respond to monetary shocks. In the II-WR model, in particular, the lack of income pooling combined with the disassociation of pricing and hiring delivers a very large response in inflation. This strong price response absorbs much of the shock and thus less adjustment occurs through real variables like unemployment and vacancies. Changing either the household or production side affects these conclusions. With income pooling, as in the RH-WR model, the fall in demand is muted and as a consequence so is the fall in inflation. Thus, more adjustment occurs through unemployment and vacancies. Similarly, when pricing and hiring are integrated, as in the II-PH model, less adjustment occurs through prices and hence inflation falls by less. Again, more adjustment occurs through unemployment and vacancies.

The general smoothness of inflation echoes back to Trigari (2009) who shows that the NK model augmented with search frictions produces much more muted responses to inflation compared to the standard version with a neoclassical labor market. The results for the II-WR model reveal that the lack of risk sharing stemming from the II assumption can lead to greater inflation volatility, highlighting the importance of the RH assumption in delivering smooth inflation within the class of WR models with search frictions.

That integrating pricing and hiring frictions also generates smoothness in inflation and greater volatility in unemployment and vacancies is reminiscent of the results reported in Kuester (2010) and Thomas (2011). The results highlighted here suggest this result is robust to the RH assumption.
Indeed, the paths of inflation, unemployment, and the nominal rate are very similar across the II and RH environments when the firm-side is of the PH form.

7 Conclusion

This paper examines the quantitative implications of two common assumptions in the New Keynesian literature with frictional labor markets: separating pricing and hiring decisions across firms in the face of frictions at both margins, and allowing for income pooling by agents who live in a large representative household. A key message from the analysis is that both assumptions matter greatly for the answers to key policy questions and the dynamics of the macroeconomy in response to technology and monetary shocks.

For example, the integration of pricing and hiring decisions within the same firm matters for unemployment insurance policy and the costs of persistently below-target inflation. Models where firms set both prices and hire workers imply significant macroeconomic responses to changes in unemployment benefits or persistently below-target inflation. In contrast, models that separate these decisions imply essential no response in either of these scenarios.

Additionally, all of these assumptions shape the propagation of shocks. In response to technology shocks, the responses of vacancies and unemployment are larger with a representative household, that pools income, than with independent individuals, that cannot. In response to monetary policy shocks, the responses of vacancies and unemployment are muted when individuals cannot pool income and when pricing and hiring decisions are separated; changing either of these assumptions increases the response of vacancies and unemployment.

While the frameworks studied are purposefully parsimonious, a number of extensions left for future research may remain tractable. For example, the II-PH model could be extended to allow for ex-ante fixed heterogeneity in worker types such as education or ability.\(^\text{18}\) Lastly, an interesting extension would be extending the PH environments in a TANK framework.

\(^{18}\)For example, if there are two types of workers, such as low and high ability, there would be two optimal reset prices dependent on the worker type. This combined with the iid Calvo shock assumption would keep the distribution of prices, hours, and earnings tractable. Implicitly, we are assuming firms learn about the worker type after matching with the worker and all separations remain exogenous.
References


