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Rational Inattention and Dynamics of Consumption and Wealth in General Equilibrium*

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Abstract

This paper derives the general equilibrium effects of rational inattention (or RI; Sims 2003, 2010) in a model of incomplete income insurance (Huggett 1993, Wang 2003). We first show that, under the assumption of CARA utility with Gaussian shocks, the permanent income hypothesis (PIH) arises in steady state equilibrium due to a balancing of precautionary savings and impatience. We explore how the introduction of RI can help the model fit the joint equilibrium dynamics of consumption, income, and wealth. We then contrast RI with habit formation and show that the two models make very different general equilibrium predictions, and that RI is closer to the data. We finally show that the equilibrium welfare costs of incomplete information due to RI are relatively low within the CARA-Gaussian setting.

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1. Introduction

In intertemporal consumption-savings problems, prudent households save today for three reasons: (i) they anticipate future declines in income (saving for a rainy day), (ii) they face uninsurable risks (precautionary savings), and (iii) they are patient relative to the interest rate. In models where only motive (i) is operative, one obtains the "permanent income hypothesis" (PIH) of Friedman (1957), in which consumption is solely determined by permanent income (the annuity value of total wealth) and follows a random walk (see Hall 1978); this statement holds, for example, if households have quadratic utility and have access to a single risk-free bond with a constant return.[1]

In aggregate data, the PIH makes predictions that are inconsistent with the data. Two implications in particular are discussed in Campbell and Deaton (1989), the "excess sensitivity" and "excess smoothness" puzzles. Excess sensitivity arises if current consumption responds to predictable movements in income (or equivalently responds "too much" to transitory changes in income); under the PIH these changes are part of permanent income and therefore should already have had their effect on consumption. Excess smoothness occurs if consumption responds less than one for one to permanent changes in income (equivalent to changes in permanent income). Under the assumption of a single risk-free asset, the two puzzles are manifestations of the same underlying economic forces, as noted in Campbell and Deaton (1989), and their joint test rejects the absence of both.[2]

Regarding motives (ii) and (iii), uninsurable income risk seems to be pervasive in microeconomic data, and general equilibrium models with uninsurable risk often imply impatience of households (that is, they face "low interest rates").[3] There is a straightforward link between uninsurable risk and excess sensitivity – if agents engage in precautionary savings and their risk aversion is affected by wealth, then an increase in income today will lead to a larger increase in current consumption than justified by the increase in permanent income; excess smoothness then arises via the intertemporal solvency condition, since future consumption cannot rise by the required

[1] If utility is quadratic, the random walk nature of consumption is only approximately true, but the PIH still holds.

[2] Attanasio and Weber (1993) show that aggregation errors can generate excess sensitivity, in particular when households of different ages are lumped together. Similarly, Attanasio and Pavoni (2011) show that apparent violations of the intertemporal budget constraint made feasible via insurance markets can deliver excess smoothness in aggregate data. It is substantially more difficult to reject the PIH on individual (or cohort) data, but limitations in quality and coverage of consumption data are particularly serious at low levels of aggregation.

[3] In partial equilibrium, low interest rates are needed to ensure a stationary distribution of wealth (see Carroll 2011). However, Aiyagari (1994) and Huggett (1993) show they arise naturally in general equilibrium, whether the supply of assets is elastic or inelastic in the aggregate.

amount. Ludvigson and Michaelides (2001) study a buffer stock model based on Bewley (1983) and find that quantitatively the model fails to reproduce the excess sensitivity observed in the individual-level data. We are therefore led to look elsewhere for explanations.

Luo (2008) and Luo and Young (2010) introduce the rational inattention hypothesis proposed by Sims (2003) into the basic partial equilibrium PIH environment; RI implies that agents process signals slowly and therefore appear to respond sluggishly to innovations in permanent income. This sluggish response appears to deliver changes in consumption in response to anticipated income changes, and as a result also delivers smaller responses to permanent income changes; that is, the model delivers both excess sensitivity and excess smoothness.[4] Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013) find pervasive evidence consistent with Sims (2003)'s rational inattention theory using the U.S. and European surveys of professional forecasters and other agents, respectively. Crucial for our purposes, the RI model delivers only one new free parameter, which maps directly into the speed of learning (one can think of this parameter as a filter gain).

However, those papers are explicitly partial equilibrium, taking as given a constant exogenous risk-free rate. Wang (2003), using a simple model with constant absolute risk aversion (CARA) utility and risk free assets in zero net supply based on Cabellero (1990), shows that the PIH reemerges in general equilibrium – when decision rules are linear, the equilibrium interest rate exactly balances the forces of precautionary saving and dissaving due to impatience, even in the presence of uninsurable risk, and leads to consumption following a random walk. Due to the linearity of consumption as a function of individual permanent income, Wang (2003) is able to analytically characterize the forces that operate in general equilibrium and show they cancel out, under some mild assumptions about the labor income process.

Our main goal in this paper is to explore the general equilibrium implications of rational inattention in a model with precautionary savings. We first ask the same question from Wang (2003) – namely, whether the PIH reemerges in general equilibrium – in the presence of rational inattention. We study economies with CARA preferences, as they simultaneously generate precautionary savings and linear consumption rules, and characterize the forces that act on the general equilibrium interest rate.[5] We find that the PIH does describe equilibrium consumption behavior in general

[4]The tests of the PIH, while robust to the possibility that agents have more information than the econometrician (see Campbell and Deaton 1989), they may not be robust to the RI implication that the econometrician may actually have more information than the agents (at least at the time of decisions). The reason is that agents with more information will still set consumption changes to be orthogonal to everything the econometrician observes; it is clear that the converse would not necessarily hold.

[5]Constant-relative-risk-aversion (CRRA) utility functions are more common in macroeconomics, mainly due to

equilibrium, with the appropriate substitution of actual permanent income by perceived permanent income. Thus, the delicate cancelling of precautionary and impatience forces found by Wang (2003) carries over unmodified to models with incomplete information about the state, despite the additional precautionary savings that RI generates.[6] This result is robust to the precise way RI is modeled – that is, whether we think of agents as having fixed or flexible speeds of learning.[7]

We then proceed to use the RI model to interpret the dispersion of consumption and wealth (relative to income) that we observe in the data. We construct a panel from the PSID, using the approach from Guvenen and Smith (2014) to impute total consumption from the limited information in the PSID via a demand system estimated on the CEX (see also Blundell, Pistaferri, and Preston 2008). We also use the PSID to construct measures of wealth, and then we compare our measures to those predicted by the model. We find that RI substantially improves the predictions of the model for these relative dispersions. The FI-RE model, for a given income process estimated from micro data, implies that these dispersions are much lower than in the data. With RI, as households become less attentive, relative dispersions in consumption and wealth both rise. Interestingly, we find that the same value of the RI parameter roughly matches both moments, providing an overidentifying test of the model. General equilibrium effects turn out to be less significant when consumers are less information-constrained, acting to slightly reduce the dispersions in the model relative to a partial equilibrium exercise with a fixed interest rate. Furthermore, we find that the welfare losses due to limited attention in general equilibrium are well above that in partial equilibrium because of the general equilibrium channel, but they are still relatively small, about 0.04% of average annual income per month for highly information-constrained consumers.

We next compare our RI model to an alternative method for introducing sluggish movements in consumption, namely habit formation. We model habit formation with one free parameter, which governs the transmission of current consumption into future habit. As the habit parameter increases, consumption changes become sluggish as households try to smooth changes in consump-

balanced-growth requirements. CRRA utility would greatly complicate our analysis because the intertemporal consumption model with CRRA utility and stochastic labor income has no explicit solution and leads to non-linear consumption rules. Introducing RI would then be substantially more difficult and involve approximations of unknown quality.

[6] Luo and Young (2014) document an observational equivalence between rational inattention and signal extraction in linear-quadratic-Gaussian models. Given that we find our CARA model approximately delivers Gaussian uncertainty about the state, we believe that those results would carry over here, meaning our results apply to a very broad class of models.

[7] There is one potential difference between the fixed and flexible learning models – the uniqueness of equilibrium. While we can show that the fixed learning model has a unique equilibrium, the presence of additional forces in the flexible model left us unable to show uniqueness. Nevertheless, we could not find any examples where multiplicity arose.

tion rather than levels. Thus, there is a sense in which the two model frameworks look similar; in fact, Luo (2008) shows that the two models are observationally equivalent at the aggregate level (in terms of consumption growth dynamics), but not at the individual level where RI delivers more consumption volatility. We show that this similarity does not extend to the cross-sectional dispersion moments we examine here, even though the noise shocks aggregate out; unlike RI, habit formation moves both relative dispersions away from their empirical counterparts.[8] In addition, we find that stronger habit formation leads to higher, not lower, interest rates.

This paper is organized as follows. Section 2 constructs a precautionary saving model with a continuum of inattentive consumers who have the CARA utility and face uninsurable labor income. Section 3 solves optimal consumption-saving rules under rational inattention and characterizes the unique general equilibrium of this economy. Section 4 examines how RI affects the interest rate and the joint dynamics of consumption, income, and wealth quantitatively. Section 5 compares the rational inattention model to the habit formation model. In the appendices we provide the key proofs and derivations, and discuss an extension of the basic model to durable goods.

2. A Caballero-Huggett-Wang Economy with Rational Inattention

2.1. A Full-information Rational Expectations Model with Precautionary Savings

Following Caballero (1990) and Wang (2003), we formulate a full-information rational expectations (FI-RE) model with precautionary savings as follows:

$$V\left(a_0, y_0\right) = \max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \left\{ E_0 \left[\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho} \right)^t u(c_t) \right] \right\}, \tag{1}$$

subject to the flow budget constraint

$$a_{t+1} = (1+r)\, a_t + y_t - c_t, \tag{2}$$

where $u(c) = -\exp\left(-\alpha c\right)/\alpha$ is a constant-absolute-risk-aversion utility with $\alpha > 0$, $\rho > 0$ is the agent's subjective discount rate, r is a constant rate of interest, and labor income, y_t, follows a

[8] Relative wealth dispersion is not strictly monotone in the habit parameter, but the change occurs at very high habit levels that cannot be reconciled with observed individual consumption volatility unless substantial measurement error is assumed.

stationary AR(1) process with Gaussian innovations

$$y_t = \phi_0 + \phi_1 y_{t-1} + w_t,\ t \geq 1,\ |\phi_1| < 1, \tag{3}$$

where $w_t \sim N\left(0, \sigma^2\right)$, $\phi_0 = (1-\phi_1)\,\overline{y}$, $\overline{y}$ is the mean of y_t, and the initial levels of labor income y_0 and asset a_0 are given.[9] Solving (1) subject to (2) and (3) yields the following optimal consumption plan:

$$c_t = r\left\{a_t + h_t + \frac{1}{\alpha r^2}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right]\right\}. \tag{4}$$

In this expression,

$$h_t \equiv \frac{1}{1+r}E_t\left[\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j y_{t+j}\right], \tag{5}$$

is human wealth (defined as the discounted expected present value of current and future labor income); evaluating the sum yields

$$h_t = \phi\left(y_t + \frac{\phi_0}{r}\right),$$

where $\phi = 1/\left(1+r-\phi_1\right)$.[10] This consumption function is the same as that obtained in Wang (2003). In the last two terms in (4), $\ln\left(\frac{1+\rho}{1+r}\right)/r\alpha$ measures the relative importance of impatience and the interest rate in determining current consumption, and $\ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)/\left(r\alpha\right)$ measures the amount of precautionary savings determined by the interaction of risk aversion and income uncertainty.

In order to facilitate the introduction of rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model to a univariate model with iid innovations to total wealth. Letting total wealth, $s_t = a_t + h_t$, be defined as a new state variable, we can reformulate the PIH model as

$$v\left(s_t\right) = \max_{c_t}\left\{u\left(c_t\right) + \frac{1}{1+\rho}E_t\left[v\left(s_{t+1}\right)\right]\right\},$$

subject to

$$s_{t+1} = (1+r)\,s_t - c_t + \zeta_{t+1}, \tag{6}$$

[9] Assuming that the individual income shock includes two components, one permanent and the other transitory, does not change the main results in this paper. Here we follow Wang (2003) and adopt specification (3), in order to simplify the algebra. A detailed derivation of the model with the two-income shock specification is available from the corresponding author by request. For the empirical studies on the income specification, see Attanasio and Pavoni (2011).

[10] See Appendix 7.1 for the derivation.

where the time $(t+1)$ innovation to total wealth can be written as

$$\zeta_{t+1} \equiv \frac{1}{1+r} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{j-(t+1)} \left(E_{t+1} - E_t\right) \left[y_j\right], \tag{7}$$

which can be reduced to $\zeta_{t+1} = \phi w_{t+1}$ when we use the income specification, (3), where $v(s_t)$ is the consumer's value function under FI-RE.[11]

2.2. Incorporating Rational Inattention

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above permanent income model with the CARA-Gaussian specification. Under RI, consumers have only finite Shannon channel capacity available to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow. With finite capacity $\kappa \in (0, \infty)$, a random variable $\{s_t\}$ following a continuous distribution cannot be observed without error and thus the information set at time $t+1$, denoted $\mathcal{I}_{t+1}$, is generated by the entire history of noisy signals $\left\{s_j^*\right\}_{j=0}^{t+1}$. Following the literature, we first assume the noisy signal takes the additive form

$$s_{t+1}^* = s_{t+1} + e_{t+1},$$

where e_{t+1} is the endogenous noise caused by finite capacity.[12] We further assume that e_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shocks hitting the economy. The reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint. Agents with finite capacity will choose a new signal $s_{t+1}^* \in \mathcal{I}_{t+1} = \left\{s_1^*, s_2^*, \cdots, s_{t+1}^*\right\}$ that reduces the uncertainty about the variable s_{t+1} as much as possible. Formally, this idea can be described by the information constraint

$$\mathcal{H}\left(s_{t+1} | \mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1} | \mathcal{I}_{t+1}\right) \leq \kappa, \tag{8}$$

where κ is the investor's information channel capacity, $\mathcal{H}\left(s_{t+1} | \mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1} | \mathcal{I}_{t+1}\right)$ is the entropy after observing the new

[11] See Appendix 7.1 for the derivation.

[12] For other types of imperfect information about state variables, see Pischke (1995) and Wang (2004). Pischke (1995) assumes that consumers ignore the aggregate income component, and Wang (2004) assume that consumers cannot distinguish two individual components in the income process.

signal. κ imposes an upper bound on the amount of information flow – that is, the change in the entropy – that can be transmitted in any given period. Finally, following the literature, we suppose that the prior distribution of s_{t+1} is Gaussian.

In a linear-quadratic-Gaussian (LQG) setting, as has been shown in Sims (2003) and Shafieep-oorfard and Raginsky (2013), ex post Gaussian distributions, $s_t|\mathcal{I}_t \sim N\left(E\left[s_t|\mathcal{I}_t\right], \Sigma_t\right)$, where $\Sigma_t = E_t\left[\left(s_t - \widehat{s}_t\right)^2\right]$, are optimal. Here we first assume ex post Gaussian distributions of the true state and Gaussian noise but adopt negative exponential (CARA) preferences. Because both the optimality of ex post Gaussianity and the standard Kalman filter are based on the linear-quadratic-Gaussian (LQG) specification, the applications of these results in the RI models with CARA preferences are only approximately valid.[13] In the next subsection, after solving for the consumption function and the value function under RI, we verify that the loss function due to RI is approximately quadratic and consequently the optimality of the ex post Gaussianity of the state approximately holds in the CARA model.

Since both ex ante and ex post distributions of the state are Gaussian, (8) reduces to

$$\ln\left(|\Psi_t|\right) - \ln\left(|\Sigma_{t+1}|\right) \leq 2\kappa, \tag{9}$$

where $\Sigma_{t+1} = \text{var}_{t+1}\left(s_{t+1}\right)$ and $\Psi_t = \text{var}_t\left(s_{t+1}\right) = (1+r)^2\,\Sigma_t + \text{var}_t\left(\zeta_{t+1}\right)$ are the posterior and prior variances of the state variable, s_{t+1}, respectively. In our univariate model, (9) fully determines the value of the steady state conditional variance Σ:

$$\Sigma = \frac{\text{var}_t\left(\zeta_{t+1}\right)}{\exp\left(2\kappa\right) - (1+r)^2}, \tag{10}$$

which means that Σ is entirely determined by the variance of the exogenous shock $\left(\text{var}_t\left(\zeta_{t+1}\right)\right)$ and finite capacity (κ). To guarantee that the state is stabilizable and the unconditional variance converges, we need to make the following assumption on the value of channel capacity:

Assumption 1

$$\kappa > \ln\left(1+r\right). \tag{11}$$

It is worth noting that this restriction is very weak if r is small; in general equilibrium r will be smaller than ρ, so for short time periods this condition is not restrictive at all.

[13] See Peng (2004), Mondria (2010), and Van Nieuwerburgh and Veldkamp (2010) for applications of CARA preferences in RI models.

Following the steps outlined in Luo and Young (2014), we can compute the Kalman gain in the steady state θ as

$$\theta = 1 - 1/\exp(2\kappa); \tag{12}$$

θ measures the fraction of uncertainty removed by a new signal in each period, and is the only new parameter introduced by the rational inattention framework.[14]

The evolution of the estimated state $\widehat{s}_t$ is governed by the Kalman filtering equation

$$\widehat{s}_{t+1} = (1-\theta)\left((1+r)\widehat{s}_t - c_t\right) + \theta s^*_{t+1}, \tag{13}$$

where $\widehat{s}_t = E_t[s_t]$ is the conditional mean of the state, s_t. Combining (6) with (13) yields

$$\widehat{s}_{t+1} = (1+r)\widehat{s}_t - c_t + \widehat{\zeta}_{t+1}, \tag{14}$$

where

$$\widehat{\zeta}_{t+1} = \theta(1+r)(s_t - \widehat{s}_t) + \theta(\zeta_{t+1} + e_{t+1}) \tag{15}$$

is the innovation to $\widehat{s}_{t+1}$ and is independent of all the other terms on the RHS of (14). $\widehat{\zeta}_{t+1}$ is an MA(∞) process with $E_t\left[\widehat{\zeta}_{t+1}\right] = 0$ and

$$\text{var}\left(\widehat{\zeta}_{t+1}\right) = \Gamma(\theta, r)\,\omega^2_\zeta,$$

where $\omega^2_\zeta = \text{var}_t(\zeta_{t+1})$,

$$\Gamma(\theta, r) = \frac{\theta}{1-(1-\theta)(1+r)^2} > 1 \tag{16}$$

for $\theta < 1$, and

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t}{1-(1-\theta)(1+r)\cdot L} - \frac{\theta e_t}{1-(1-\theta)(1+r)\cdot L} \tag{17}$$

is the estimation error with $E_t[s_t - \widehat{s}_t] = 0$ and $\text{var}(s_t - \widehat{s}_t) = \frac{1-\theta}{1-(1-\theta)(1+r)^2}\omega^2_\zeta$. (Here L denotes the lag operator.) To guarantee that the sum of these infinite series converges, here we need to impose the restriction that $\kappa > 0.5\ln(1+r)$. Note that this condition always holds when we have (11). From (16), it is clear that $\frac{\partial\Gamma}{\partial r} > 0$ and $\frac{\partial\Gamma}{\partial\theta} < 0$.

[14]In Section 3.3 we model RI as confronting the agent with a fixed marginal cost of acquiring channel capacity. Luo and Young (2014) show the two are observationally equivalent with respect to consumption-income dynamics, but there will be an additional force operating on the equilibrium interest rate in the elastic case.

3. General Equilibrium under RI

3.1. Optimal Consumption and Savings Functions

Following the standard procedure in the literature, the consumption function and the value function under RI can be obtained by solving the following stochastic Bellman equation:

$$\widehat{v}\left(\widehat{s}_t\right)=\max_{c_t}\left\{-\frac{1}{\alpha}\exp\left(-\alpha c_t\right)+\frac{1}{1+\rho}E_t\left[\widehat{v}\left(\widehat{s}_{t+1}\right)\right]\right\}, \tag{18}$$

subject to (14)-(17). The following proposition summarizes the main results from the above precautionary-savings model with RI.

Proposition 1. *For a given Kalman gain, θ, the value function is*

$$\widehat{v}\left(\widehat{s}_t\right)=-\frac{1}{r\alpha}\exp\left(-r\alpha\left\{\widehat{s}_t-\frac{1}{r\alpha}\ln\left(1+r\right)+\frac{1}{r^2\alpha}\left[\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}\right), \tag{19}$$

the consumption function is

$$c_t^*=r\left\{\widehat{s}_t+\frac{1}{\alpha r^2}\left[\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}, \tag{20}$$

and the saving function is

$$d_t^*=\left(1-\phi_1\right)\phi\left(y_t-\overline{y}\right)+r\left(s_t-\widehat{s}_t\right)+\frac{1}{r\alpha}\left[\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)-\Psi\left(r\right)\right], \tag{21}$$

where $s_t-\widehat{s}_t$ is an $MA(\infty)$ estimation error process given in (17) and $\Psi\left(r\right)=\ln\left(\frac{1+\rho}{1+r}\right)$.

Proof. See Appendix 7.1 for the derivations. See Appendix 7.2 for the proof that the loss function due to RI is approximately quadratic and the optimality of the ex post Gaussianity of the state approximately holds in the CARA model. ■

Comparing (4) with (20), it is clear that the two consumption functions are identical except that we replace s_t with $\widehat{s}_t$ and ζ_{t+1} ($\equiv \phi w_{t+1}$) with $\widehat{\zeta}_{t+1}$, respectively. First, given that

$$\ln\left(E_t\left[\exp\left(-r\alpha\zeta_{t+1}\right)\right]\right) = \frac{1}{2}\left(r\alpha\right)^2\omega_\zeta^2,$$
$$\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right) = \frac{1}{2}\Gamma\left(\theta,r\right)\left(r\alpha\right)^2\omega_\zeta^2,$$

we can define the precautionary saving premium due to limited attention as

$$P_{ri} \equiv \frac{1}{\alpha r}\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right) - \exp\left(-r\alpha\zeta_{t+1}\right)\right]\right) = \frac{1}{2}\left(\Gamma\left(\theta,r\right)-1\right)r\alpha\omega_\zeta^2, \tag{22}$$

which is clearly decreasing with the degree of attention θ, and is increasing with the coefficient of absolute risk aversion (α) and the persistence and volatility of the income shock (ϕ_1 and σ) for any given θ. Thus, the incomplete information that RI forces upon the households leads to an increase in saving.

To further explore the precautionary savings premium in (22), we isolate the effects of RI on individual consumption and saving by rewriting (20) as

$$c_t^* = r\widehat{s}_t + \frac{1}{r\alpha}\left\{\Psi\left(r\right) - \left[\begin{array}{c}\ln\left(E_t\left[\exp\left(-r\alpha\theta\left(1+r\right)\left(s_t-\widehat{s}_t\right)\right)\right]\right) + \frac{1}{2}\left(r\alpha\theta\omega_\zeta\right)^2 + \\ \frac{1}{2}\left(1-\theta\right)\Gamma\left(\theta,r\right)\left(r\alpha\omega_\zeta\right)^2\end{array}\right]\right\}, \tag{23}$$

where $\Psi\left(r\right) = \ln\left(\frac{1+\rho}{1+r}\right)$ measures the relative importance of impatience to the interest rate in determining optimal consumption (it is greater than 0 if $\rho > r$),

$$\frac{1}{\alpha r}\ln\left(E_t\left[\exp\left(-\alpha r\theta\left(1+r\right)\left(s_t-\widehat{s}_t\right)\right)\right]\right) = r\alpha\theta\left(1-\theta\right)\Gamma\left(\theta,r\right)\left(1+r\right)^2\omega_\zeta^2$$

is the precautionary savings premium due to the time t estimation error, $\left(r\alpha\theta\omega_\zeta\right)^2/2$ is the precautionary savings premium driven by the exogenous fundamental income shocks $\{w_t\}$, and $\left(1-\theta\right)\Gamma\left(\theta\right)\left(r\alpha\omega_\zeta\right)^2/2$ captures the precautionary savings premium driven by the endogenous noise shocks, $\{e_t\}$.[15] Note that when θ converges to 1, the consumption function with RI, (20), reduces to that of the Wang (2003) model, (4). From (20), for finite capacity ($\kappa < \infty$ or $\theta \in (0,1)$), the precautionary saving premium due to fundamental shocks is lower than that in the full-information case, i.e., $\left(r\alpha\theta\omega_\zeta\right)^2/2 < \left(r\alpha\omega_\zeta\right)^2/2$ because of the incomplete adjustment of consumption to the fundamental shock, while we have two new positive terms that increase the total

[15] This result is derived by using Equation (17) and the iid property of the processes $\left\{\widehat{\zeta}_t\right\}$, $\{\zeta_t\}$, and $\{e_t\}$.

savings more than the absolute value of the reduced savings: (i) the premium due to the estimation error and (ii) the premium due to the RI-induced endogenous noise.

Given the time t available information and the fact that $E_t\left[s_t-\widehat{s}_t\right]=0$, the conditional mean of (21) can be written as

$$\widetilde{d}_t=f_t+\left(\frac{1}{2}r\alpha\Gamma\left(\theta,r\right)\omega_\zeta^2-\frac{1}{r\alpha}\Psi\left(r\right)\right), \tag{24}$$

where the first term $f_t=\left(1-\phi_1\right)\phi\left(\widehat{y}_t-\overline{y}\right)$ captures the consumer's demand for savings "for a rainy day", and the second term, $\frac{1}{2}r\alpha\Gamma\left(\theta,r\right)\omega_\zeta^2$, is the certainty equivalent of the innovation to the perceived state, $\widehat{s}_t$.

3.2. Existence and Uniqueness of General Equilibrium

As in Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous agents, of total mass normalized to one, with each agent solving the optimal consumption and savings problem with RI specified in (18). Similar to Huggett (1993), we also make the following assumption:

Assumption 2 *The risk-free asset in our model is a pure-consumption loan and is in zero net supply. The initial cross-sectional distribution of permanent income is a stationary distribution* $\Phi\left(\cdot\right)$.

By the law of large numbers in Sun (2006), provided that the spaces of agents and the probability space are constructed appropriately, aggregate permanent income and the cross-sectional distribution of permanent income $\Phi\left(\cdot\right)$ are constant over time.

Proposition 2. *The total savings demand "for a rainy day" in the precautionary savings model with RI equals zero for any positive interest rate. That is,* $F_t\left(r\right)=\int_{y_t}f_t\left(r\right)d\Phi\left(y_t\right)=0$, *for* $r>0$.

Proof. The proof uses the LLN and is the same as that in Wang (2003). ∎

Proposition 2 states that the total savings "for a rainy day" is zero, at any positive interest rate. Therefore, from (20), for $r>0$, the expression for total savings under RI in the economy at time t is

$$D\left(\theta,r\right)\equiv\frac{1}{r\alpha}\left(\Pi\left(\theta,r\right)-\Psi\left(r\right)\right)=\frac{1}{r\alpha}\left[\frac{1}{2}\left(r\alpha\right)^2\Gamma\left(\theta,r\right)\omega_\zeta^2-\Psi\left(r\right)\right], \tag{25}$$

where $\Pi\left(\theta,r\right)=\frac{1}{2}\left(r\alpha\right)^2\Gamma\left(\theta,r\right)\omega_\zeta^2$ measures the amount of precautionary savings, and $\Psi\left(r\right)$ captures the dissaving effects of impatience. Given (25), an equilibrium under RI is defined by an interest rate r^* satisfying

$$D(\theta, r^*) = 0. \tag{26}$$

The following proposition shows the existence of the equilibrium and the PIH holds in the RI general equilibrium.

Proposition 3. *There exists a unique equilibrium with an interest rate $r^* \in (0, \rho)$ in the precautionary-savings model with RI. In equilibrium, each agent's consumption is described by the PIH, in that*

$$c_t^* = r^* \widehat{s}_t, \tag{27}$$

where $\widehat{s}_t = E[s_t | \mathcal{I}_t]$ is the perceived value of permanent income. The evolution equations of wealth and consumption are

$$\Delta c_{t+1}^* = r^* \widehat{\zeta}_{t+1}, \tag{28}$$

$$\Delta a_{t+1}^* = \frac{1 - \phi_1}{1 + r^* - \phi_1} (y_t - \overline{y}) + r^* (s_t - \widehat{s}_t), \tag{29}$$

respectively, where $\widehat{\zeta}_{t+1}$ is specified in (15) with $E_t\left[\widehat{\zeta}_{t+1}\right] = 0$, $\operatorname{var}\left(\widehat{\zeta}_{t+1}\right) = \Gamma(\theta, r^*)\, \omega_\zeta^2$, and $\Gamma(\theta, r^*) = \frac{\theta}{1-(1-\theta)(1+r^*)^2}$. In the general equilibrium, the value function under RI can be written as

$$\widehat{v}(\widehat{s}_t) = -\frac{1 + r}{r\alpha} \exp(-r\alpha \widehat{s}_t). \tag{30}$$

Proof. If $r > \rho$, the two terms, $\Pi(\theta, r)$ and $-\Psi(r)$, in the expression for total savings $D(\theta, r^*)$, are positive, which contradicts the equilibrium condition, $D(\theta, r^*) = 0$. Since $\Pi(\theta, r) - \Psi(r) < 0$ (> 0) when $r = 0$ ($r = \rho$), the continuity of the expression for total savings implies that there exists at least one interest rate $r^* \in (0, \rho)$ such that $D(\theta, r^*) = 0$. From (20), we can obtain the individual's optimal consumption rule under RI in general equilibrium as $c_t^* = r^* \widehat{s}_t$. Substituting (14) and (27), we can obtain (28). (27) into (2) yields (29). The proof of uniqueness is longer and relegated to Appendix 7.3. ■

The intuition behind Proposition 3 is similar to that in Wang (2003). With an individual's constant total precautionary savings demand $\Pi(\theta, r)$, for any $r > 0$, the equilibrium interest rate r^* must be such that each individual's dissavings demand due to impatience is exactly balanced by their total precautionary-savings demand, $\Pi(\theta, r^*) = \Psi(r^*)$. Figure 1 plots the equilibrium interest rate as a function of θ, given the parameters $\alpha = 2$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.[16]

Given (20) and (26), it is clear that even though the individual increases their total precautionary savings in response to information frictions for a given r, the level of aggregate savings equals zero. That is, RI does not affect the aggregate wealth in the economy, because the equilibrium interest rate is pushed down to counteract this precautionary savings increase.[17] With lower Shannon channel capacity, the equilibrium interest rate is lower.

From the equilibrium condition,

$$\frac{1}{2}\left(r^*\alpha\right)^2 \Gamma\left(\theta, r^*\right)\omega_\zeta^2 - \ln\left(\frac{1+\rho}{1+r^*}\right) = 0, \tag{31}$$

it is straightforward to show that

$$\frac{dr^*}{d\theta} = \frac{r^*\left(2+r^*\right)\left(r^*\alpha\right)^2\omega_\zeta^2}{2\left[1-\left(1-\theta\right)\left(1+r^*\right)^2\right]^2}\left\{\begin{array}{c}\frac{1}{1+r^*} \\ +r^*\alpha^2\omega_\zeta^2\Gamma\left(\theta,r^*\right)\left[\frac{1-\phi_1}{1+r^*-\phi_1}+\frac{r^*(1-\theta)(1+r^*)}{1-(1-\theta)(1+r^*)^2}\right]\end{array}\right\}^{-1} > 0, \tag{32}$$

where $1-\left(1-\theta\right)\left(1+r^*\right)^2 > 0$ and $\ln\left(\frac{1+\rho}{1+r^*}\right) > 0$. It is clear from this expression that r^* is decreasing in the degree of inattention $1-\theta$. The first row of Table 2 reports the general equilibrium interest rates for different values of θ.[18] We can see from the table that r^* decreases as the degree of inattention increases. For example, if θ is reduced from 1 to 0.1, r^* is reduced from 3.27 percent to 2.87 percent. In addition, it is clear that

$$\frac{dr^*}{d\alpha} < 0.$$

That is, the equilibrium interest rate decreases with the degree of risk aversion. From (28) and (29), we can conclude that although both the CARA model and the LQ model lead to the PIH in general equilibrium, risk aversion plays a role in affecting the dynamics of consumption and wealth in the CARA model via the equilibrium interest rate channel.

One might ask what a reasonable value of θ is, and if there is any way to calibrate it outside

[16] In Section 4.1, we will provide more details about how to estimate the income process using the U.S. data. The main result here is robust to the choices of these parameter values.

[17] If we introduced an asset with elastic supply, such as the capital stock in Aiyagari (1994), the same effects would be present but the stock of capital would rise (and the change in the interest rate would be smaller as a result).

[18] Here we also set $\alpha = 2$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

a model. Unfortunately, there is no direct survey evidence on the value of channel capacity of ordinary households in the economics literature, and thus it is not straightforward to answer these questions; estimates of learning capacity exist, but they are not directly useful since we are interested in the capacity that will be devoted to economic activity (specifically, consumption and saving). In lieu of such evidence, we simply note that 0.1 is the value needed to match portfolio holdings in Luo (2010) and is therefore not obviously unreasonable (a caveat can be found in Luo and Young 2016, where a significantly larger number is obtained using recursive utility). Coibion and Gorodnichenko (2015) have the most "model independent" measure of θ, and they find $\theta = 0.5$ provides a good fit for a variety of forecast and survey data, and a variety of other papers obtain a number of different values depending on what facts they bring to bear. We will show below that $\theta = 0.1$ allows us to match some cross-sectional dispersion facts, but are cognizant that this parameter's value is quite uncertain.

3.3. Elastic Attention

Instead of using fixed channel capacity to model finite information-processing ability, one could assume that the marginal cost of information-processing (i.e., the shadow price of information-processing capacity) is fixed. That is, the Lagrange multiplier on (9) is constant. In the univariate case, the objective of the agent with finite capacity in the filtering problem is to minimize the discounted expected mean square error (MSE), $E_t\left[\sum_{t=0}^{\infty}\beta^t\left(s_t-s_t^*\right)^2\right]$, subject to the information-processing constraint, or

$$\min_{\{\Sigma_t\}}\left\{\sum_{t=0}^{\infty}\beta^t\left[\Sigma_t+\lambda\ln\left(\frac{(1+r)^2\Sigma_{t-1}+\omega_\zeta^2}{\Sigma_t}\right)\right]\right\},$$

where Σ_t is the conditional variance at t, λ is the Lagrange multiplier corresponding to (9).[19] Solving this problem yields the optimal steady state conditional variance:

$$\Sigma=\frac{(1+r)^2(1-\beta)\widetilde{\lambda}-1+\sqrt{\left[(1+r)^2(1-\beta)\widetilde{\lambda}-1\right]^2+4\widetilde{\lambda}(1+r)^2}}{2(1+r)^2}\omega_\zeta^2, \tag{33}$$

where $\widetilde{\lambda}=\lambda/\omega_\zeta^2$ is the normalized shadow price of information-processing capacity. It is straightforward to show that as λ goes to 0, $\Sigma = 0$; and as λ goes to ∞, $\Sigma = \infty$. Note that $\frac{\partial\ln\Sigma}{\partial\ln\omega_\zeta^2} < 1$ if we

[19] As in the fixed-capacity case, although we adopt the CARA-Gaussian setting, the loss function due to imperfect-state-observation could be approximately quadratic. See Appendix 7.2 for the proof.

adopt the assumption that λ is fixed, while $\frac{\partial \ln \Sigma}{\partial \ln \omega_{\zeta}^{2}} = 1$ in the fixed κ case. Comparing (33) with (10), it is clear that the two RI modeling strategies are observationally equivalent in the sense that they lead to the same conditional variance if the following equality holds:

$$\kappa\left(r,\widetilde{\lambda}\right) = \ln\left(1+r\right) + \frac{1}{2}\ln\left(1+\frac{2}{\left(1+r\right)^{2}\left(1-\beta\right)\widetilde{\lambda}-1+\sqrt{\left[\left(1+r\right)^{2}\left(1-\beta\right)\widetilde{\lambda}-1\right]^{2}+4\widetilde{\lambda}\left(1+r\right)^{2}}}\right). \tag{34}$$

In this case, the Kalman gain is

$$\theta\left(r,\widetilde{\lambda}\right) = 1-\frac{1}{1+r}\left\{1+\frac{2}{\left(1+r\right)^{2}\left(1-\beta\right)\widetilde{\lambda}-1+\sqrt{\left[\left(1+r\right)^{2}\left(1-\beta\right)\widetilde{\lambda}-1\right]^{2}+4\widetilde{\lambda}\left(1+r\right)^{2}}}\right\}^{-1}. \tag{35}$$

It is obvious that κ converges to its lower limit $\underline{\kappa} = \ln\left(1+r\right) \approx r$ as λ goes to ∞; and it converges to ∞ as λ goes to 0. In other words, using this RI modeling strategy, the consumer is allowed to adjust the optimal level of capacity in such a way that the marginal cost of information-processing for the problem at hand remains constant.

Given this relationship between λ and θ (or κ), in the following analysis we just use the value of θ to measure the degree of attention. It is worth noting that although the above two RI modeling strategies, inelastic and elastic capacity, are observationally equivalent in the "static" sense, they have distinct implications for the model's propagation mechanism if the economy is experiencing regime switching (see Luo and Young 2014). The key difference between the elastic capacity case and the fixed capacity case in this paper is that κ and θ now depend on both the equilibrium interest rate and labor income uncertainty for a given λ. The equilibrium interest rate is now determined implicitly by the following function:

$$D\left(\theta\left(r^{*},\widetilde{\lambda}\right),r^{*}\right) \equiv \frac{1}{r^{*}\alpha}\left[\frac{1}{2}\left(r^{*}\alpha\right)^{2}\Gamma\left(\theta\left(r^{*},\widetilde{\lambda}\right),r^{*}\right)\omega_{\zeta}^{2} - \ln\left(\frac{1+\rho}{1+r^{*}}\right)\right] = 0. \tag{36}$$

Figure 2 illustrates how r^{*} varies with labor income uncertainty, σ, for fixed information-processing cost, λ. It clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with labor income uncertainty. We can see from Table 4 that when the economy becomes more volatile (i.e., larger σ), the Kalman gain (θ) increases while the equilibrium interest rate (r^{*}) decreases. This result is different from that obtained in the fixed capacity case in which θ and r^* move in the same direction. (See Table 2.) The main reason for this result is that income uncertainty affects the equilibrium interest rate via two channels: (i) the direct channel which leads to higher aggregate savings (i.e., the ω_ζ^2 term in (36)) and (ii) the indirect channel which leads to lower aggregate savings (i.e., the $\theta\left(r^*, \widetilde{\lambda}\right)$ term in (36)), and the direct channel dominates.[20] For example, when σ increases from 0.15 to 0.25, the equilibrium interest rate reduces from 2.80 percent to 2.56 percent. (See the third row of Table 4.) Furthermore, elastic attention can have significant effects on the relative volatility of consumption to income. For example, when σ increases from 0.15 to 0.25, this relative volatility declines by 35 percent from 0.46 to 0.30. In contrast, in the FI case, this relative volatility only decreases by 10 percent from 0.29 to $0.2\dot{6}$. (See the fourth and ninth rows of Table 4 for the details.)

4. Empirical and Quantitative Results

In this section we assess our GE-RI model's implications for the dynamics of consumption, income and wealth. To construct empirical counterparts that are comparable with the theoretical moments derived in the model, we construct a panel with individual consumption, income and wealth based on the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX), using the imputation approach from Blundell, Pistaferri, and Preston (2008) and Guvenen and Smith (2014). Then, using the estimated income process, we show the GE-RI model significantly fits the data better than the full information model regarding the consumption and wealth dynamics. Our closed-form solutions explicitly show different channels through which the RI drives the results.

4.1. Empirical Evidence

In order to measure the relative consumption dispersion in the data, $\frac{\text{sd}(\Delta c)}{\text{sd}(\Delta y)}$, we construct a panel data set which contains both consumption and income at the household level. The PSID does not include enough consumption expenditure data to create full picture of household nondurable consumption. Such detailed expenditures are found, though, in the CEX from the Bureau of Labor Statistics. But households in this study are only interviewed for four consecutive quarters and thus do not form a panel. To create a panel of consumption to match the PSID income measures, we use an estimated demand function for imputing nondurable consumption created by Guvenen and Smith (2014). Using an IV regression, they estimate a demand function for nondurable consump-

[20] We have been unable to prove that r^* is unique under elastic capacity, precisely because of the indirect channel. We can show that a necessary condition for multiple equilibria requires the response of θ to changes in r to be very large, and we did not find any examples where this condition was satisfied. See the appendix for details.

tion that fits the detailed data in the CEX. The demand function uses demographic information and food consumption which can be found in both the CEX and PSID. Thus, we use this demand function of food consumption and demographic information (including age, family size, inflation measures, race, and education) to estimate nondurable consumption for PSID households, creating a consumption panel.

Following Blundell, Pistaferri, and Preston (2008), we define the household income as total household income (including wage, financial, and transfer income of head, wife, and all others in household) minus financial income (defined as the sum of annual dividend income, interest income, rental income, trust fund income, and income from royalties for the head of the household only) minus the tax liability of non-financial income. This tax liability is defined as the total federal tax liability multiplied by the non-financial share of total income. Tax liabilities after 1992 are not reported in the PSID and so we estimate them using the TAXSIM program from the National Bureau of Economic Research. Our final household income measure can be expressed as:

$$\text{income measure} = (\text{total HH income} - \text{financial income}) - \text{taxes} \times \frac{\text{total HH income} - \text{financial income}}{\text{total HH income}}.$$

Our household sample selection closely follows that of Blundell, Pistaferri, and Preston (2008) as well.[21] We exclude households in the PSID poverty and Latino subsamples. We exclude households in years of family composition change, change in marital status, or female headship, as well as in years where the head or wife is under 30 or over 65. Households in years with missing education, region, income, and imputed consumption responses are also excluded. We also exclude households in years where they report a negative income or a food consumption level in the top or bottom 5 percent of all reported values in that year. Income and consumption values are then deflated by the CPI to constant $1982-1984$ dollars. Our final panel contains $7,111$ unique households with $58,034$ yearly income responses and $48,990$ imputed nondurable consumption values.[22]

With this constructed panel of household income and consumption, we next drop households in years where year-over-year food consumption changes are more than 20 percent or less than -20

[21] They create a new panel series of consumption that combines information from PSID and CEX, focusing on the period when some of the largest changes in income inequality occurred. For other explanations for observed consumption and income inequality, see Krueger and Perri (2006) and Attanasio and Pavoni (2011).

[22] There are more household incomes than imputed consumption values because food consumption - the main input variable in Guvenen and Smith's nondurable demand function - is not reported in the PSID for the years 1987 and 1988. Dividing the total income responses by unique households yields an average of 7-8 years of responses per household. These years are not necessarily consecutive as our sample selection procedure allows households to be excluded in certain years but return to the sample if they later meet the criteria once again.

percent. To exclude extreme outliers, we then follow Floden and Lindé (2001) and normalize both income and consumption measures as ratios of the mean of each year, and exclude household in bottom and top 1 percent of the distribution of those ratios. Figure 3 shows the relative dispersion of consumption, defined as the ratio of the standard deviation of the consumption change to the standard deviation of the income change between 1980 and 2000. The basic pattern confirms but extends the findings in Blundell, Pistaferri, and Preston (2008) – relative consumption dispersion declines in the 1980s, but this decline stops around 1990.

In order to calculate the relative volatility of wealth to income ratio, $\frac{\text{sd}(\Delta a)}{\text{sd}(\Delta y)}$, we use wealth information included in the PSID data. Notice that the PSID only reports household wealth variables every five years starting in 1983, and then every other year starting in 1998. To be consistent with the model, we construct household wealth in the following way. We use measure of wealth defined as the sum of the net value of liquid assets (checking, savings, money market, etc.), vehicles, home equity, and other assets such as bonds, insurance policies, and trusts. All reported values are again deflated by the CPI to constant $1982-1984$ dollars. We normalize each reported wealth and income value to the mean of the year reported, and then exclude outliers of this distribution at the top and bottom 1 percent. We then take the standard deviations of the change in normalized value from the previous report for both wealth and income to calculate our ratio. Our final panel for wealth and income has $23,630$ observations across 6232 households. This panel is somewhat smaller than our panel of consumption and income due to the limited number of years that wealth measures are reported. Figure 4 reports the results, which shows the relative volatility of wealth to income has been relatively stable in the sample period.

When estimating the income process, we focus on the sample period to the years $1980-1996$, due to the PSID survey changing to a biennial schedule after 1996. To further restrict the sample to exclude households with dramatic year-over-year income changes, we eliminate household incomes with year-over-year level changes in the top and bottom 5 percent of the distribution in each year. Then, again following Floden and Lindé (2001), we normalize household income measures as ratios of the mean for that year and exclude all household values in years in which the income is in the top and bottom 1 percent of the normalized income measure for the year. To eliminate possible heteroskedasticity in the income measures, we follow Floden and Lindé (2001) and regress each on a series of demographic variables in a fixed-effect panel regression to remove variation caused by differences in age and education. We next subtract these fitted values from each measure to create a panel of income residuals. We then use this panel to estimate the household income process as specified by equation (3) by running panel regressions on lagged income. As the last row of Table 1 reports, the estimated values of ϕ_0, ϕ_1, and σ are 0.0005, 0.919, and 0.175, respectively.

4.2. Empirical Implications for the Dynamics of Consumption, Wealth, and Income

Luo (2008) examines how RI affects consumption volatility in a partial equilibrium version of the PIH model presented above. In general equilibrium, since RI affects the equilibrium interest rate, it will have an additional effect on consumption dynamics. Using (29) and (28), we can obtain the key stochastic properties of the joint dynamics of individual consumption, income, and saving. The following proposition summarizes the implications of RI for the relative volatility of consumption to income as well as the relative volatility of financial wealth to income.

Proposition 4. *Under RI, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*}{1+r^*-\phi_1}\sqrt{\frac{\left(1+\phi_1\right)\Gamma\left(\theta,r^*\right)}{2}}, \tag{37}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{1}{\sqrt{2}\left(1+r^*-\phi_1\right)}\sqrt{1-\phi_1+\frac{r^{*2}\left(1-\theta\right)\left(1+\phi_1\right)}{1-\left(1-\theta\right)\left(1+r^*\right)^2}+\frac{2r^*\left(1-\theta\right)\left(1-\phi_1^2\right)}{1-\phi_1(1-\theta)\left(1+r^*\right)}}. \tag{38}$$

Proof. See Appendix 7.1. ■

Expression (37) shows that RI has two opposing effects on consumption volatility. The first effect is direct through its presence in the expression of $\Gamma\left(\theta,r^*\right)$, whereas the second effect is through the general equilibrium interest rate $\left(r^*\right)$ and is thus indirect. Using the expression of $\Gamma\left(\theta,r^*\right)$, it is straightforward to show that the direct effect of RI is to increase consumption volatility. The intuition is very simple: the presence of the RI-induced noise dominates the slow adjustment of consumption in determining consumption volatility at the individual level. In contrast, the indirect effect of RI will reduce consumption volatility because it reduces the general equilibrium interest rate and $\partial\Gamma\left(\theta,r^*\right)/\partial r^* > 0$. Following the literature of precautionary savings and the estimated income process in the preceding subsection, we set $\rho = 0.04$, $\alpha = 2$, $\sigma = 0.175$, and $\phi_1 = 0.919$. The second to fourth rows of Table 2 reports how the interest rate and the relative volatility of consumption and wealth to income vary with θ in general equilibrium. It is clear from the second row of Table 2 that RI can significantly affect the equilibrium interest rate. For example, when θ decreases from 1 to 0.10, r^* decreases from 3.27 percent to 2.87 percent, which is very close to 2.97 percent, the average annual equilibrium real interest rate from 1980 to 1996 estimated in Laubach and Williams (2015) using $1961-2014$ U.S. quarterly data. (Note that when $\theta=0.12$, the equilibrium interest rate obtained in our model is exactly the same as its empirical counterpart.) Here we focus on the $1980-1996$ period because we use it to estimate the income process and the relative volatility of consumption to income.

From the third row of Table 2, the relative volatility of consumption growth to income growth increases with the degree of inattention. For example, when θ decreases from 1 to 0.1, μ_{cy} increases from 0.284 to 0.375, which is the same as the empirical counterpart. It is clear from these results that the direct effect of inattention via the $\Gamma(\theta, r^*)$ term in (37) dominates its indirect general equilibrium effect via r^*. We can get the same conclusion by shutting down the general equilibrium (GE) channel, see the corresponding partial equilibrium (PE) results reported in the same table. Comparing the GE and PE results in Table 2, we can see the values of μ_{cy} are slightly lower in the GE case if the interest rate is fixed as θ decreases. In other words, the general equilibrium effect of RI tends to reduce the volatility of individual consumption in this case.[23]

Another important implication of RI in general equilibrium is that RI leads to more skewed wealth inequality measured by μ_{ay}, the relative volatility of financial wealth to labor income. From the fourth row of Table 2, we can see that when θ is reduced from from 1 to 0.1, μ_{ay} increases from 1.775 to 2.63, which is much closer to the empirical counterpart. (For example, μ_{ay} is 3.11 in 1993 and is 2.59 in 1998.) From (29), it is clear that the main driving force behind this result is the presence of the estimation error, $s_t-\widehat{s}_t$, because $\partial \operatorname{var}(s_t-\widehat{s}_t)/\partial\theta<0$. Note that although $\partial r^*/\partial\theta>0$, the estimation error channel dominates the general equilibrium channel and raises the wealth inequality. Therefore, RI can increase wealth inequality, which makes the model fit the data better.[24]

To briefly summarize the key discussions above, Table 3 compares the performances of the FI-RE model, the general equilibrium rational inattention model (RI-GE), and the partial equilibrium rational inattention model (RI-PE) with the data. Overall, it shows under the estimated income

[23] We cannot examine the stochastic properties of aggregate consumption dynamics because all idiosyncratic shocks (income shocks and RI-induced noise shocks) cancel out after aggregating aross consumers.

[24] The literature has typically found that simple models based on standard CRRA preferences and on uninsurable shocks to labor income cannot account for the observed U.S. wealth distribution. For example, Aiyagari (1994) finds considerably less wealth concentration in a model with only idiosyncratic labor earnings uncertainty. Given the CARA-Gaussian setting, the model here is not suitable to address the issue like why the top 1 percent or 5 percent richest families hold a large fraction of financial wealth in the U.S. economy.

process and at a single value of rational inattention parameter (θ), the GE-RI model can do a significantly better job than the FI-RE model in generating a lower interest rate, a higher consumption volatility, and a higher wealth volatility, bring all of them much closer to the data. In terms of welfare loss, as the last row in Table 3 shows and will be discussed in detail in the next subsection, the partial equilibrium model significantly underestimates the welfare loss, though the welfare loss is generally small.

Table 4 reports how elastic Kalman gain, the general equilibrium interest rate, and the relative volatility of consumption and wealth to income vary with different values of income uncertainty measured by σ (and σ_y). We have reached three key findings. First, it is clear from the second row of Table 4 that the Kalman gain increases with income volatility. For example, if $\lambda = 400$ (the value calibrated to the data as explained in the next paragraph), θ is doubled when σ increases from 0.15 to 0.35. This means agents optimally allocate more attention to the state variable when income uncertainty increases. Second, RI has significant effects on the equilibrium interest rate. For example, r^* declines from 2. 80 percent to 2.23 percent when σ increases from 0.15 to 0.35. It is worth noting that in the elastic capacity case an increase in income volatility affects the equilibrium interest rate via two channels: (i) the direct channel (the ω_ζ^2 term in (36)) and (ii) the indirect channel (the elastic capacity θ term in (36)). The second panel of Table 4 reports the results when we shut down the indirect channel and assume that $\theta = 1$. Comparing the third and sixth rows of Table 4, we can see that the indirect channel is more important when σ is relatively low. For example, given that $\sigma = 0.15$, r^* decreases from 3.39 percent to 2. 80 percent when we switch from the FI economy to the RI economy, whereas r^* only decreases from 2.43 percent to 2.23 percent when $\sigma = 0.35$. Third, the relative volatility of consumption growth to income growth decreases with the value of σ in general equilibrium. That is, consumption becomes smoother when income becomes more volatile. For example, in the equilibrium RI economy, μ_{cy} decreases from 0.46 to 0.25 when σ increases from 0.15 to 0.35.[25]

The last finding highlighted above might provide a potential explanation for the empirical evidence documented in Blundell, Pistaferri, and Preston (2008) that income and consumption inequality diverged over the sampling period they study.[26] To explore this issue in our model, we do the following exercise. First, we divide the full sample into two sub-samples ($1980 - 1986$

[25]It is not surprising that μ_{cy} is greater in the equilibrium RI economy than in the equilibrium FI economy because the value of θ is less than 1 in the RI case. This result is the same as that we obtained in the fixed capacity case and reported in Table 2.

[26]Other mechanisms have been proposed for this decline; see Krueger and Perri (2006) and Athreya, Tam, and Young (2009) for examples.

and 1987 − 1996) and apply the same estimation procedure to re-estimate σ and ϕ_1 (see the first and second rows of Table 1 for the estimation results). Household income is more volatile in late sub-periods than earlier ones. Specifically, the standard deviation of y is 0.386 in the sub-sample (1980 − 1986), while it is 0.427 in the sub-sample (1987 − 1996). The average values of μ_{cy} are 0.46 and 0.30 in the first and second sub-samples, respectively. In the elastic capacity case, using the estimated income processes in the first sub-sample, we first use $\mu_{cy} = 0.46$ to calibrate $\lambda = 400$; the corresponding value of θ is 0.08 in the first sub-sample. Using this calibrated value of λ, we find that μ_{cy} is reduced to 0.36 in the second sub-sample, which is much closer to the empirical counterpart than the value obtained in the fixed capacity case (0.41). Note that here we assume that the marginal information-processing cost is invariant across sub-samples.

4.3. Welfare Losses due to RI in Equilibrium

We now turn to the welfare cost of RI – how much utility does a consumer lose if the actual consumption path he chooses under RI deviates from the first-best FI-RE path in which $\theta = 1$? To answer this question, we follow Barro (2007) and Luo and Young (2010) by computing the marginal welfare cost due to RI. The following proposition summarizes the main result.

Proposition 5. *Given the initial value of the state, $\widehat{s}_0$, the marginal welfare cost* (mwc) *due to RI is given by*

$$\mathrm{mwc}(\theta) \equiv \frac{(\partial v(\widehat{s}_0)/\partial\theta)\,\theta}{(\partial v(\widehat{s}_0)/\partial\widehat{s}_0)\,\widehat{s}_0} = \frac{\theta}{r^{*2}\alpha}\left[r^*\alpha + \frac{1}{(1+r^*)\,\widehat{s}_0}\right]\frac{dr^*}{d\theta}, \tag{39}$$

where $dr^*/d\theta$ is given in (32) and $\widehat{v}(\widehat{s}_0) = -\exp(-r^*\alpha\widehat{s}_0 + \ln(1+r^*))/(r^*\alpha)$. The monthly dollar loss due to deviating from the FI-RE path $(\theta = 1)$ can be written as

$$\$\,\mathrm{loss}(\theta < 1) \equiv \frac{r^*}{12}\mathrm{mwc}(1)(1-\theta)\,\widehat{s}_0. \tag{40}$$

Proof. See Appendix 7.4. Since we are interested in the deviation from the FI-RE path, $\theta = 1$ is considered as the starting point. When the household deviates from 1 to θ, the percentage change in just $(\theta - 1)$. $\widehat{s}_0$ is initial total wealth. Finally, we need to convert the change in the $\widehat{s}_0$ term to monthly rates by multiplying by $r^*/12$. ■

Expression (39) gives the proportionate reduction in the initial level of the perceived state $(\widehat{s}_0)$ that compensates, at the margin, for a percentage decrease in θ (i.e., stronger degree of RI) — in the sense of preserving the same effect on welfare for a given $\widehat{s}_0$. To do quantitative welfare analysis we need to know the value of $\widehat{s}_0$. First, we set $\widehat{y}_0 \equiv E[y_t] = 1$, $\phi_1 = 0.919$, and the ratio of the initial level of financial wealth $(\widehat{a}_0)$ to mean income $(\widehat{y}_0)$ equal to 5.[27] Second, given that $\widehat{s}_0 = \widehat{a}_0 + \widehat{y}_0 / (1 + r^* - \phi_1) + \overline{y}/r^*$, we can calculate the values of the monthly dollar loss (\$ loss) for different values of θ and the corresponding values of the general equilibrium interest rate. The fifth row of Table 2 reports the welfare losses (measured by the dollar) for different degrees of inattention. For example, when $\theta = 0.1$, the welfare loss due to RI in general equilibrium is about \$8.82 per month, or about 0.04% of mean income per month, which is relatively small.[28] This welfare loss decreases to \$4.42 when $\theta = 0.6$, or 0.02% of monthly income. This result is similar to the findings by Pischke (1995), Luo (2008), Luo and Young (2010), and Luo, Nie, and Young (2015), and is robust to changes in the income process and the degrees of patience and risk aversion.[29] Thus, it seems reasonable for agents to devote low channel capacity to observing and processing information because the welfare improvement from increasing capacity would be trivial.

Another implication of the welfare losses due to RI reported in Table 2 is that there is a general equilibrium effect of RI on the welfare loss. For example, when r is set to be 3.27 percent (the general equilibrium interest rate obtained under FI) in the partial equilibrium (PE) case, the monthly dollar loss due to RI is significantly less than that obtained in the GE case.[30] For example, when $\theta = 0.4$, the welfare loss in the GE case is \$6.58, while it is close to 0 in the PE case. The main intuition behind this result is that the general equilibrium channel governed by $dr^*/d\theta$ is shut down in the PE model.

5. Comparison with Habit Formation

An alternative structure that delivers slow consumption dynamics is the habit formation (HF) model of Constantinides (1990). With HF preferences, households try to smooth consumption growth (roughly speaking), rather than the level of consumption; the result is that consumption tends to respond slowly to changes in permanent income. Luo (2008) shows that RI and HF deliver identical aggregate consumption growth movements, but at the individual level RI models deliver

[27] This number varies largely for different individuals, from 2 to 20. 5 is the average wealth/income ratio in the Survey of Consumer Finances 2001. We find that changing the value of this ratio only has minor effects on the welfare calculation.

[28] In our estimation, we normalize the household income to the 1982 unit. The average value of real disposable personal income per capita is \$24,146 from 1980 to 1996.

[29] Pischke (1995) found that in most cases the utility losses due to no information about aggregate income shocks are less than \$1 per quarter in the LQ permanent income model.

[30] See Appendix 7.4 for the derivations of the welfare loss due to RI in partial equilibrium.

more consumption volatility due to the noise shocks (which are canceled out by aggregation).[31]

In this section, we compare the different implications of HF and RI in the general equilibrium framework. Following Alessie and Lusardi (1997), we introduce HF into the FI-RE model specified in Section 2.1 by assuming that the utility function takes the following form:[32]

$$u\left(c_t, c_{t-1}\right) = -\frac{1}{\alpha}\exp\left(-\alpha\left(c_t - \gamma c_{t-1}\right)\right)$$

where $\gamma > 0$ is the habit parameter. Using the same solution method used in Section 2.1, we can solve for the consumption function under HF:

$$c_t = \frac{r\left(1+r-\gamma\right)}{1+r}s_t + \frac{\gamma}{1+r}c_{t-1} + \frac{1}{r\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-\frac{\alpha r\left(1+r-\gamma\right)}{1+r}\zeta_{t+1}\right)\right]\right)\right). \tag{41}$$

(See Appendix 7.5 for the derivation.) The corresponding saving function can thus be written as

$$d_t = \left(1-\phi_1\right)\phi\left(y_t - \overline{y}\right) + \frac{r^2\gamma}{1+r}\frac{\zeta_t}{1-\gamma\cdot L} - \frac{1}{\alpha\left(1-\gamma\right)}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-\frac{r\alpha\left(1+r-\gamma\right)}{1+r}\zeta_{t+1}\right)\right]\right)\right).$$

Following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists a unique equilibrium interest rate r^* such that

$$\ln\left(\frac{1+\rho}{1+r^*}\right) - \frac{1}{2}\left(r^*\alpha\right)^2\widetilde{\Gamma}\left(\gamma, r^*\right)\omega_\zeta^2 = 0,$$

where $\widetilde{\Gamma}\left(\gamma, r^*\right) = \left(\frac{1+r^*-\gamma}{1+r^*}\right)^2 < 1$.[33] In general equilibrium, the consumption and saving functions can be written as

$$c_t^* = \frac{r^*\left(1+r^*-\gamma\right)}{1+r^*}s_t + \frac{\gamma}{\left(1+r^*\right)}c_{t-1}^*,$$
$$d_t^* = \left(1-\phi_1\right)\phi\left(y_t - \overline{y}\right) + \frac{r^2\gamma}{1+r}\frac{\zeta_t}{1-\gamma\cdot L},$$

[31]Otrok (2001) notes that HF models deliver an aversion to high frequency movements in consumption, while RI noise shocks would introduce precisely those kinds of movements. Thus, the spectrum of consumption growth would look very different across the two models. We leave to future work an exploration of which framework can better fit that spectrum.

[32]For simplicity, here we assume that consumers form habits over a composite consumption good. See Ravn, Schmitt-Grohe, and Uribe (2006) for discussions on the deep-habit model in which consumers form habits over individual varieties of goods.

[33]See Appendix 7.5 for a formal proof of uniqueness.

for $t \geq 0$ and given c_{-1}, a_0, y_0, and s_0, and

$$\frac{dr^*}{d\gamma} > 0. \tag{42}$$

That is, the stronger the habit persistence, the higher the equilibrium interest rate.[34] In summary, we can conclude that although both RI and HF lead to slow adjustments in consumption, they have opposite effects on the equilibrium interest rate.[35] RI reduces the equilibrium interest rate, while HF increases it. The second row of Table 5 reports the general equilibrium interest rates for different values of γ.[36] We can see from the table that r^* increases as the degree of habit formation increases. For example, if γ is raised from 0 to 0.9, r^* increases from 3.27 percent to 3.79 percent.

Using the above equilibrium consumption and saving functions, we can obtain the key stochastic properties of the joint dynamics of individual consumption, income, and saving under habit formation. The following proposition summarizes the implications of habit formation for the relative volatility of consumption to income as well as the relative volatility of financial wealth to income:

Proposition 6. *Under habit formation, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*\left(1+r^*-\gamma\right)}{\left(1+r^*\right)\left(1+r^*-\phi_1\right)}\sqrt{\frac{\left[1+\left(\frac{(r^*)^2}{1+r^*}\right)^2\frac{\gamma^2}{1-\gamma^2}\right]\left(1+\phi_1\right)}{2\left[1-\left(\frac{\gamma}{1+r^*}\right)^2\right]}}, \tag{43}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{1}{\sqrt{2}\left(1+r^*-\phi_1\right)}\sqrt{\left(1+\phi_1\right)\left[\frac{1-\phi_1}{1+\phi_1}+\left(\frac{r^{*2}\gamma}{1+r^*}\right)^2\frac{1}{1-\gamma^2}+\frac{2\left(1-\phi_1\right)\left(r^*\right)^2\gamma}{\left(1+r^*\right)\left(1-\gamma\phi_1\right)}\right]}. \tag{44}$$

Proof. See Appendix 7.5 for the derivation. ■

Expressions (43) and (44) show that habit formation affects the relative volatility of consumption and wealth to income via two channels. The first channel is direct through the presence of γ,

[34]In a partial equilibrium model, Alessie and Lusardi (1997) show that the stronger the habit, the smaller the effect of income uncertainty on the precautionary saving term. Seckin (2000) shows that in general one cannot determine the connection between habit and the precautionary savings premium in models with more than two periods.

[35]It is worth noting that the mechanisms of RI and HF that generate slow adjustment are distinct. Under RI, slow adjustment is forced upon the agent due to finite information processing capacity (learning is slow). In contrast, slow adjustment is optimal under HF because consumers are assumed to prefer to smooth consumption growth.

[36]Here we also set $\gamma = 2$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

whereas the second channel is indirect and operates through the equilibrium interest rate r^*. Given the complexity of these expressions, we cannot obtain explicit results about how habit formation affects the relative volatility of consumption and wealth to income. We therefore use the same parameter values as used in the preceding subsection to do a quantitative analysis. Table 5 reports the general equilibrium and partial equilibrium interest rates for different values of γ. It is clear from the table that the relative volatility of consumption to income is decreasing with the degree of habit formation. For example, when γ is increased from 0 to 0.9, μ_{cy} is reduced from 0.284 to 0.079 in general equilibrium; as with RI, the general equilibrium effects are small.

From the fourth row of the table, we can see that the effect of habit formation on the relative volatility of wealth to income is not monotonic because of the general equilibrium channel. If the habit formation preference is not extremely strong ($\gamma < 0.9$), μ_{ay} decreases with the value of γ. For example, if γ is increased from 0 to 0.8, μ_{ay} falls from 1.775 to 1.713 in general equilibrium. Once we shut down the general equilibrium channel, we can see from the sixth row of the table that μ_{ay} slightly increases with the value of γ. That is, the indirect channel may dominate the direct channel for some plausible values of γ. However, values of γ that high cannot be reconciled with some of the large changes in consumption observed at the individual level; in effect, one is forced to suppose that these movements are almost entirely measurement error.[37]

6. Concluding Remarks

In this paper we have studied how rational inattention affects the interest rate and the joint dynamics of consumption and income in a Huggett-type general equilibrium model with the CARA-Gaussian specification. We highlight two main results here in the conclusion. First, RI helps the basic model deliver a good fit to the dispersions of consumption and wealth relative to income; more inattention leads to more dispersion, pushing the models closer to the data, and we find that there is a common value of the inattention parameter that delivers a good fit for both. The effects on the equilibrium interest rate are modest, and work to slightly offset the added dispersion. Second, we compare RI to habit formation, and show that adding habit pushes the model in the wrong direction, as dispersions decrease for plausible values and interest rates rise rather than fall. Thus, if one is interested in models that deliver slow consumption growth dynamics, RI seems to be on

[37]Technically, this statement holds only if the utility function does not permit "effective consumption" $c_t - \gamma c_{t-1}$ to be negative, as would the case with CRRA preferences. CARA preferences are defined for negative values. Nevertheless, we find the general principle likely to be true. Multiplicative habits of the form c_t / c_{t-1}^{γ} permit much larger movements in individual consumption; see Gayle and Khorunzina (2016) for estimates using the PSID that control for measurement error.

better empirical footing than habit formation.

7. Appendix

7.1. Deriving the Consumption, Saving, and Value Functions in the Cabellero-Huggett Model with RI

Given the consumption function (4), the original budget constraint (2) can be rewritten as

$$a_{t+1}+\phi y_{t+1}+\frac{\phi\phi_0}{r}=(1+r)\,a_t+y_t-c_t+\phi\left(\phi_0+\phi_1 y_t+w_{t+1}\right)+\frac{\phi\phi_0}{r}$$
$$=(1+r)\left(a_t+\phi y_t+\frac{\phi\phi_0}{r}\right)-c_t+\zeta_{t+1},$$

where the $(t+1)$-innovation $\zeta_{t+1}=\phi w_{t+1}$ is Gaussian innovation process with mean zero and variance $\phi^2\sigma^2$. Denote $s_t=a_t+\phi y_t+\phi\phi_0/r$, the new budget constraint and the consumption function can be rewritten as

$$s_{t+1}=(1+r)\,s_t-c_t+\zeta_{t+1},$$
$$c_t=r\left\{s_t+\frac{1}{r^2\alpha}\left[\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right]\right\},$$

respectively. Under RI, the first-order condition with respective to c and the Envelope theorem are

$$u'(c_t)=\frac{1}{1+\rho}E_t\left[\widehat{v}'\left(\widehat{s}_{t+1}\right)\right],$$
$$\widehat{v}'\left(\widehat{s}_t\right)=\frac{1+r}{1+\rho}E_t\left[\widehat{v}'\left(\widehat{s}_{t+1}\right)\right],$$

which imply that

$$u'(c_t)=\frac{1}{1+r}\widehat{v}'\left(\widehat{s}_t\right). \tag{45}$$

Conjecture that the value function takes the form

$$\widehat{v}\left(\widehat{s}_t\right)=-\frac{1}{r\alpha}\exp\left(-r\alpha\left(\widehat{s}_t+b\right)\right). \tag{46}$$

Combining the exponential utility, (45) and (46), the candidate optimal consumption is given by

$$c_t^*=r\left[\widehat{s}_t+b+\frac{1}{r\alpha}\ln\left(1+r\right)\right]. \tag{47}$$

Plugging (47) into the utility function gives

$$u\left(c_t^*\right) = -\frac{1}{\alpha}\exp\left(-\alpha c_t^*\right) = -\frac{1}{\alpha}\exp\left(-\alpha r\left[\widehat{s}_t + b + \frac{1}{r\alpha}\ln\left(1+r\right)\right]\right) = \frac{1}{1+r}\widehat{v}\left(\widehat{s}_t\right). \tag{48}$$

Substituting (48) into the Bellman Equation (46) leads to

$$\widehat{v}\left(\widehat{s}_t\right) = \frac{1+r}{1+\rho}E_t\left[\widehat{v}\left(\widehat{s}_{t+1}\right)\right]. \tag{49}$$

Using (46) and (13), (49) implies that

$$c_t^* = r\left\{\widehat{s}_t + \frac{1}{r^2\alpha}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}.$$

Matching coefficients in (47) and (20) gives

$$b = -\frac{1}{r\alpha}\ln\left(1+r\right) + \frac{1}{r^2\alpha}\left\{\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right\}.$$

To verify the optimality of the value function (19) and the consumption function, we need to show that: (a) the consumption function (20) satisfies the following "transversality condition":

$$\lim_{t\to\infty}\left\{E\left[\left(1+\rho\right)^{-t}\widehat{v}\left(\widehat{s}_t\right)\right]\right\} = 0; \tag{50}$$

and (b) $\widehat{v}\left(\widehat{s}_0\right) \geq E\left[\sum_{t=0}^{\infty}\left(1+\rho\right)^{-t}u\left(c_t\right)\right]$ for all feasible consumption plans $\left\{c_t\right\}_{t=0}^{\infty}$. Substituting the consumption rule (20) into (14) gives us

$$\begin{aligned}
\widehat{s}_{t+1} &= \left(1+r\right)\widehat{s}_t - c_t^* + \widehat{\zeta}_{t+1} \\
&= \left(1+r\right)\widehat{s}_t - r\left\{\widehat{s}_t + \frac{1}{\alpha r^2}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\} + \widehat{\zeta}_{t+1} \\
&= \widehat{s}_t + \chi + \widehat{\zeta}_{t+1} \\
&= \widehat{s}_0 + \sum_{i=1}^{t+1}\widehat{\zeta}_i + \left(t+1\right)\chi,
\end{aligned}$$

where $\chi \equiv -\frac{1}{\alpha r}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}r^2\alpha^2\Gamma\left(\theta,r\right)\omega_{\zeta}^2\right]$. Plugging the above expression in the left-hand side of the TVC (20) yields

$$\lim_{t\to\infty}\left\{E\left(1+\rho\right)^{-t}\left[-\frac{1}{r\alpha}\exp\left(-r\alpha\left\{\widehat{s}_t-\frac{1}{r\alpha}\ln\left(1+r\right)+\frac{1}{r^2\alpha}\left[\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}\right)\right]\right\}$$
$$=-\frac{1+r}{r\alpha}e^{\alpha\chi}\lim_{t\to\infty}\left\{\left(1+\rho\right)^{-t}E\left[\exp\left(-r\alpha\widehat{s}_t\right)\right]\right\}$$
$$=-\frac{1+r}{r\alpha}e^{\alpha\chi}\lim_{t\to\infty}\left\{\left(1+\rho\right)^{-t}E\left\{\exp\left[-r\alpha\left(\widehat{s}_0+\sum_{i=1}^{t}\widehat{\zeta}_i+t\chi\right)\right]\right\}\right\}$$
$$=-\frac{1+r}{r\alpha}e^{\alpha\chi}\lim_{t\to\infty}\left\{\left(1+\rho\right)^{-t}\left(\frac{1+\rho}{1+r}\right)^{t}\right\}$$
$$=0.$$

Hence, the consumption function satisfies the transversality condition. Next, let $\{c_t\}_{t=0}^{\infty}$ be any feasible consumption process, and $\{\widehat{s}_t\}_{t=0}^{\infty}$ be the associated process of perceived permanent income. The Bellman equation under RI at time t implies

$$\widehat{v}\left(\widehat{s}_t\right)\geq u\left(c_t\right)+\frac{1}{1+\rho}E_t\left[\widehat{v}\left(\widehat{s}_{t+1}\right)\right].$$

Multiplying through by $(1+\rho)^{-t}$, taking expectations on both sides, using the law of iterated expectations, and rearranging gives

$$E\left[\left(1+\rho\right)^{-t}\widehat{v}\left(\widehat{s}_t\right)\right]-E\left[\left(1+\rho\right)^{-t-1}\widehat{v}\left(\widehat{s}_t\right)\right]\geq E\left[\left(1+\rho\right)^{-t}u\left(c_t\right)\right].$$

Adding the above equation for each t from $t=0$ to $t=T$, for any $T\geq 1$, leaves

$$\widehat{v}\left(\widehat{s}_0\right)-E\left[\left(1+\rho\right)^{-T-1}\widehat{v}\left(\widehat{s}_{T+1}\right)\right]\geq E\left[\sum_{t=0}^{T}\left(1+\rho\right)^{-t}u\left(c_t\right)\right]. \tag{51}$$

Similar to Wang (2003), we restrict attention to $\{c_t\}_{t=0}^{\infty}$ in the family of all feasible plans $\Pi\left(\widehat{s}_0\right)$ satisfying

$$E\left[\sum_{t=0}^{\infty}\left(1+\rho\right)^{-t}u\left(c_t\right)\right]\geq E\left[\sum_{t=0}^{\infty}\left(1+\rho\right)^{-t}u\left(y_t\right)\right]>-\infty, \tag{52}$$

where the second strict inequality comes from the stationarity of the income process $\{y_t\}_{t=0}^{\infty}$ and the nonpositivity and concavity of the utility function. Hence, $E\left[\sum_{t=0}^{\infty}\left(1+\rho\right)^{-t}\left|u\left(c_t\right)\right|\right]<\infty$. With the above condition, the Dominated Convergence Theorem implies that, for any feasible consumption process $\{c_t\}_{t=0}^{\infty}$ satisfying (52),

$$\lim_{T\to\infty}\left\{E\left[\sum_{t=0}^{T}(1+\rho)^{-t}u(c_t)\right]\right\}=E\left[\sum_{t=0}^{\infty}(1+\rho)^{-t}u(c_t)\right]. \tag{53}$$

Taking limits on both sides of (51), using (50) and (53) , we have

$$\widehat{v}(\widehat{s}_0)\geq E\left[\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^t}u(c_t)\right],$$

for all feasible consumption paths $\{c_t\}_{t=0}^{\infty}$. Furthermore, if any one of the feasible paths is replaced by the optimal consumption path $\{c_t^*\}_{t=0}^{\infty}$ described by (20), the above inequality will hold with equality, i.e., $\widehat{v}(\widehat{s}_0)=E\left[\sum_{t=0}^{\infty}(1+\rho)^{-t}u(c_t^*)\right]$.

Using (4), (14) and (20), we can derive the savings function:

$$\begin{aligned}
d_t^* &= ra_t+y_t-c_t^* \\
&= ra_t+y_t-c_t+(c_t-c_t^*) \\
&= ra_t+y_t-r\left[a_t+\phi y_t+\frac{\phi\phi_0}{r}+\frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right]+ \\
&\left\{\begin{array}{c} r\left[a_t+\phi y_t+\phi\frac{\phi_0}{r}+\frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right] \\ -r\left[\widehat{s}_t+\frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right)\right] \end{array}\right\} \\
&= (1-\phi_1)\phi(y_t-\overline{y})+r(s_t-\widehat{s}_t)+\frac{1}{r\alpha}\left[\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)-\ln\left(\frac{1+\rho}{1+r}\right)\right].
\end{aligned}$$

To derive the relative volatility of financial wealth (a_t) and labor income (y_t) in general equilibrium, we first rewrite the above saving equation as follows:

$$\Delta a_{t+1}^*=d_t^*=(1-\phi_1)\phi(y_t-\overline{y})+r^*(s_t-\widehat{s}_t).$$

Taking unconditional variance on both sides yields:

$$\begin{aligned}
\operatorname{var}\left(d_t^*\right) &= \operatorname{var}\left(\left(1-\phi_1\right)\phi\left(y_t-\overline{y}\right)+r^*\left(s_t-\widehat{s}_t\right)\right) \\
&= \operatorname{var}\left(\left(1-\phi_1\right)\phi\left(y_t-\overline{y}\right)\right)+\operatorname{var}\left(r^*\left(s_t-\widehat{s}_t\right)\right)+2\operatorname{cov}\left(\left(1-\phi_1\right)\phi\left(y_t-\overline{y}\right),r^*\left(s_t-\widehat{s}_t\right)\right) \\
&= \left(\frac{1-\phi_1}{1+r^*-\phi_1}\right)^2\operatorname{var}\left(y_t-\overline{y}\right)+r^{*2}\operatorname{var}\left(s_t-\widehat{s}_t\right)+2\frac{r^*\left(1-\phi_1\right)}{1+r^*-\phi_1}\operatorname{cov}\left(y_t-\overline{y},s_t-\widehat{s}_t\right) \\
&= \left[\frac{1-\phi_1}{1+\phi_1}+\frac{\left(1-\theta\right)r^{*2}}{1-\left(1-\theta\right)\left(1+r^*\right)^2}+\frac{2r^*\left(1-\phi_1\right)\left(1-\theta\right)}{1-\phi_1(1-\theta)\left(1+r^*\right)}\right]\frac{\omega^2}{\left(1+r^*-\phi_1\right)^2},
\end{aligned}$$

which is just (38) in the main text, where we use the expression for $s_t-\widehat{s}_t$ specified in (17). Furthermore, using (16) and (28), it is straightforward to show that the relative volatility of consumption growth to income growth is (37) in the main text.

7.2. Optimality of Ex Post Gaussianity under RI

Following Sims (2003), we first define the expected loss function due to limited information-processing capacity as

$$L_t = E_t\left[v_0\left(s_t\right)-v\left(x_t\right)\right], \tag{54}$$

where s_t is the unobservable state variable, x_t is the best estimate of the true state, $v\left(x_t\right) = -\exp\left(-r\alpha b-r\alpha x_t\right)/\left(r\alpha\right)$ is the value function under RI and $v_0\left(s_t\right)=-\exp\left(-r\alpha b-r\alpha s_t\right)/\left(r\alpha\right)$ is the corresponding value function when $\kappa=\infty$. It is straightforward to show that:

$$\begin{aligned}
&\min E_t\left[v_0\left(s_t\right)-v\left(x_t\right)\right] \\
&= \min E_t\left[-\frac{1}{r\alpha}\exp\left(-r\alpha b_0-r\alpha s_t\right)+\frac{1}{r\alpha}\exp\left(-r\alpha b-r\alpha x_t\right)\right] \\
&\simeq \min E_t\left[\begin{array}{c} -\frac{1}{r\alpha}\exp\left(-r\alpha b_0-r\alpha x_t\right)+\exp\left(-r\alpha b_0-r\alpha x_t\right)\left(s_t-x_t\right) \\ -\frac{r\alpha}{2}\exp\left(-r\alpha b_0-r\alpha x_t\right)\left(s_t-x_t\right)^2+\frac{1}{r\alpha}\exp\left(-r\alpha b_0-r\alpha x_t\right) \end{array}\right] \\
&\Longleftrightarrow \min\left\{-\frac{r\alpha}{2}\exp\left(-r\alpha b_0-r\alpha x_t\right)E_t\left[\left(s_t-x_t\right)^2\right]\right\} \\
&\Longleftrightarrow -\frac{r\alpha}{2}\exp\left(-r\alpha b_0-r\alpha x_t\right)\min E_t\left[\left(s_t-x_t\right)^2\right],
\end{aligned} \tag{55}$$

where we use the fact that $x_t = E_t\left[s_t\right]$. The (approximate) loss function under CARA derived above is essentially the same as that obtained in the LQG RI model proposed in Sims (2003). Since the only difference in these two settings is just in the constant coefficient in the loss function, the CARA specification does not affect the optimality of ex post Gaussianity in Sims' LQG setting after we approximate the value functions we obtained in the CARA-Gaussian setting.

Furthermore, we use the following procedure to justify the quadratic approximation, (55):

1. We first conjecture that the quadratic approximation is an accurate approximation for the original exponential loss function, and the higher-order moments in the expansion of the CARA loss function are trivial.

2. Given that the loss function is quadratic, we can obtain the optimality of the ex post Gaussian variables and Gaussian noise and noisy signal (i.e., both x and s are Gaussian).

3. For Gaussian variables, s_t and x_t, the higher-order moments (the third and fourth moments) of the loss function can be written as

$$\frac{(r\alpha)^2}{6}\exp\left(-r\alpha b_0 - r\alpha x_t\right)E_t\left[\left(s_t - x_t\right)^3\right] - \frac{(r\alpha)^3}{24}\exp\left(-r\alpha b_0 - r\alpha x_t\right)E_t\left[\left(s_t - x_t\right)^4\right]$$
$$= -\frac{(r\alpha)^3}{24}\exp\left(-r\alpha b_0 - r\alpha x_t\right)\Sigma^2 \times 3,$$

where we use the facts that for Gaussian variables, we have

$$E_t\left[\left(s_t - x_t\right)^j\right] = \begin{cases} \sigma^j\left(j-1\right)!!, \text{ when } j \text{ is even,} \\ 0, \text{ when } j \text{ is odd.} \end{cases},$$

where $x_t = E_t\left[s_t\right]$ and σ is the standard deviation of x.

4. It is straightforward to calculate that the ratio of the sum of the third and fourth moments to the second moment is:

$$ratio = \frac{-\frac{(r\alpha)^3}{24}\exp\left(-r\alpha b_0 - r\alpha x_t\right)\Sigma^2 \times 3}{-\frac{r\alpha}{2}\exp\left(-r\alpha b_0 - r\alpha x_t\right)\Sigma} = \frac{(r\alpha)^2\Sigma}{4} = 0.0287^2 \times 15.77 = 0.013,$$

where we use the fact that $\Sigma = 15.77$ when $\theta = 10\%$. We can then verify that our guess that the quadratic approximation is accurate is correct.

7.3. Proof of Uniqueness of General Equilibrium in the Benchmark Model

To prove uniqueness, consider the derivative of the precautionary savings aggregate with respect to r: we have

$$\frac{d}{dr}\left(\frac{(r\alpha)^2}{2}\frac{\theta}{1-(1-\theta)(1+r)^2}\left(\frac{\sigma}{1+r-\phi_1}\right)^2\right)=-\frac{\alpha^2 r\theta\sigma^2\left[r(1-\theta)(1-\phi_1)-r^2(1+r)(1-\theta)-\theta(1-\phi_1)\right]}{(1+r-\phi_1)^3\left[\theta-2r(1-\theta)-r^2(1-\theta)\right]^2}.$$

The sign of this derivative equals the sign of

$$F(r)=(1-\theta)r^3+(1-\theta)r^2-(1-\theta)(1-\phi_1)r+\theta(1-\phi_1).$$

At $r=0$ we have $F(0)=\theta(1-\phi_1)>0$ and

$$F'(0)=-(1-\theta)(1-\phi_1)<0.$$

As r converges to ∞, the dominant terms are positive so $F(\infty)>0$ and $F'(\infty)>0$. The question is then (i) does F change sign on the interval $[0,\rho]$ and (ii) how many times? F can only change sign if $F'(r)=0$. Therefore, we look at

$$\begin{aligned}F'(r)&=3(1-\theta)r^2+2(1-\theta)r-(1-\theta)(1-\phi_1)\\&=(1-\theta)\left[3r^2+2r-(1-\phi_1)\right].\end{aligned}$$

Note that $F'(r)=0$ at

$$\widehat{r}=\frac{1}{3}\sqrt{4-3\phi_1};$$

the other critical point is negative and therefore we ignore it. We know that $r^*<\rho$ and $\phi_1\in[0,1]$; therefore, a sufficient condition for uniqueness on $[0,\rho]$ is $\rho<\frac{1}{3}$, since $\widehat{r}\in\left[\frac{1}{3},\frac{2}{3}\right]$.

If $\rho>\frac{1}{3}$, the value of the function at $\widehat{r}$ is key:

$$F(\widehat{r})=(1-\theta)\left(\frac{1}{3}\sqrt{4-3\phi_1}\right)^3+(1-\theta)\left(\frac{1}{3}\sqrt{4-3\phi_1}\right)^2-(1-\theta)(1-\phi_1)\left(\frac{1}{3}\sqrt{4-3\phi}\right)+\theta(1-\phi_1).$$

For this expression to be negative so that the derivative of the RHS changes sign, we need

$$\frac{1}{3}(4-3\phi_1)\left(1+\left(\frac{1}{3}\sqrt{4-3\phi_1}\right)\right)-(1-\phi_1)\sqrt{4-3\phi_1}+\frac{\theta}{1-\theta}(1-\phi_1)<0.$$

or after simplification

$$\frac{1}{3}\sqrt{4-3\phi_1}\left(2\phi_1+\sqrt{4-3\phi_1}-\frac{5}{3}\right)+\frac{\theta}{1-\theta}\left(1-\phi_1\right)<0.$$

The only way this term can be negative is if

$$\frac{5}{3}>2\phi_1+\sqrt{4-3\phi_1}.$$

Minimizing the RHS on the interval $[0,1]$ yields

$$\begin{aligned} 2-\frac{3}{2}\frac{1}{\sqrt{4-3\phi_1}}+z_1-z_2=0, \\ z_1\phi_1=0, \\ z_2\left(1-\phi_1\right)=0, \end{aligned}$$

where z_i are the Lagrange multipliers on the constraints $\phi_1 \geq 0$ and $\phi_1 \leq 1$. If $z_1 = z_2 = 0$ then $\phi_1 = \frac{55}{48} > 1$, so one constraint must bind. It cannot be $\phi_1 = 1$, as then $z_2 > 0$, so the RHS is minimized at $\phi_1 = 0$. At $\phi_1 = 0$ we have $\frac{5}{3} > 2$, which is not true, so the condition is never satisfied. Therefore, the equilibrium is unique for any value of ρ for every $\phi_1 \in [0,1]$ and $\theta \in [0,1]$, subject to the restriction that $1-(1-\theta)(1+r)^2 > 0$. Given that we know $\rho > r$, we need only impose the condition $1-(1-\theta)(1+\rho)^2 > 0$.

For the elastic RI case, the derivatives are too complicated to sign. However, we can obtain a necessary condition that the equilibrium is not unique:

$$\frac{\partial\theta}{\partial r}>\frac{2\theta\left(1-\theta\right)\left(1+r\right)}{r\left(2+r\right)}>0.$$

This condition is derived by noting that the only new term affects the derivative of the precautionary savings aggregate through θ, and noting that if this term is going to have a downward sloping segment it must come through this effect. We can then take the derivatives and isolate this piece, and rearrange it to obtain that it can be negative only if the above inequality is satisfied. For small r this condition seems very strict, and we could not find any cases where it held.

7.4. Computing the Welfare Loss due to RI

Given that the value function under RI in general equilibrium is

$$\widehat{v}(\widehat{s}_0) = -\frac{1}{r^*\alpha}\exp\left(-r^*\alpha\widehat{s}_0 + \ln\left(1+r^*\right)\right), \tag{56}$$

we can compute the following partial derivatives:

$$\frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\theta} = \frac{\exp\left(-r^*\alpha\widehat{s}_0\right)}{r^{*2}\alpha}\left[1 + r^*\alpha\widehat{s}_0\left(1+r^*\right)\right]\frac{dr^*}{d\theta} \text{ and } \frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\widehat{s}_0} = \left(1+r^*\right)\exp\left(-r^*\alpha\widehat{s}_0\right).$$

The marginal welfare cost due to RI can thus be written as:

$$mwc \equiv \frac{\left(\partial v\left(\widehat{s}_0\right)/\partial\theta\right)\theta}{\left(\partial v\left(\widehat{s}_0\right)/\partial\widehat{s}_0\right)\widehat{s}_0} = \frac{\theta}{r^{*2}\alpha}\left[r^*\alpha + \frac{1}{\left(1+r^*\right)\widehat{s}_0}\right]\frac{dr^*}{d\theta},$$

where we use the facts that in general equilibrium (i.e., $\ln\left(\frac{1+\rho}{1+r}\right) = \ln\left(E_t\left[\exp\left(-r^*\alpha\widehat{\zeta}_{t+1}\right)\right]\right)$), and $dr^*/d\theta$ is given in (32).

In the partial equilibrium setting in which $r^* = r$ is fixed,

$$\begin{aligned}\widehat{v}\left(\widehat{s}_t\right) &= -\frac{1}{r\alpha}\exp\left(-r\alpha\left\{\widehat{s}_t - \frac{1}{r\alpha}\ln\left(1+r\right) + \frac{1}{r^2\alpha}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}\right) \\ &= -\frac{1+r}{r\alpha}\exp\left(-r\alpha\widehat{s}_t\right)\exp\left(-\frac{1}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{2}\frac{\theta}{1-\left(1-\theta\right)\left(1+r\right)^2}r\alpha^2\omega_\zeta^2\right).\end{aligned}$$

Note that in partial equilibrium, $\ln\left(\frac{1+\rho}{1+r}\right)$ and $\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)$ do not cancel out. The marginal welfare cost due to RI in partial equilibrium can thus be written as:

$$mwc \equiv \frac{\left(\partial v\left(\widehat{s}_0\right)/\partial\theta\right)\theta}{\left(\partial v\left(\widehat{s}_0\right)/\partial\widehat{s}_0\right)\widehat{s}_0} = \frac{\theta}{\widehat{s}_0}\left(\frac{1}{2}\alpha\omega_\zeta^2\right)\frac{\left(1+r\right)^2-1}{\left[1-\left(1-\theta\right)\left(1+r\right)^2\right]^2},$$

where we use the facts that

$$\begin{aligned}\frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\widehat{s}_0} &= \left(1+r\right)\exp\left(-r\alpha\widehat{s}_0\right)\exp\left(-\frac{1}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{2}\frac{\theta}{1-\left(1-\theta\right)\left(1+r\right)^2}r\alpha^2\omega_\zeta^2\right), \\ \frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\theta} &= \left(1+r\right)\exp\left(-r\alpha\widehat{s}_t\right)\exp\left(-\frac{1}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{2}\frac{\theta}{1-\left(1-\theta\right)\left(1+r\right)^2}r\alpha^2\omega_\zeta^2\right)\frac{\alpha\omega_\zeta^2\left[\left(1+r\right)^2-1\right]}{2\left[1-\left(1-\theta\right)\left(1+r\right)^2\right]^2}.\end{aligned}$$

The month dollar loss can be thus written as:

$$\begin{aligned}\$\,\text{loss}\,(\theta < 1) &\equiv \frac{1}{12} r\text{mwc}\,(1)\,(1-\theta)\,\widehat{s}_0 \\ &= \frac{1}{12} r \left(\frac{1}{2}\alpha\omega_\zeta^2\right)\left((1+r)^2 - 1\right)(1-\theta)\,.\end{aligned}$$

7.5. Solving the Habit Formation Model

As in Alessie and Lusardi (1997), we model habit formation in the Caballero model by assuming that the period utility is defined on $c_t - \gamma c_{t-1}$. The optimization problem for this habit formation model can be specified as follows:

$$\max_{\{c_t\}} U = E_0 \left\{ \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[-\frac{1}{\alpha}\exp\left(-\alpha\left(c_t - \gamma c_{t-1}\right)\right)\right]\right\}, \tag{57}$$

subject to the budget constraint

$$s_{t+1} = (1+r)\,s_t - c_t + \zeta_{t+1}, \tag{58}$$

where $s_t \equiv a_t + \phi\left(y_t + \frac{\phi_0}{r}\right)$ and $\zeta_{t+1} \equiv \phi w_{t+1} = w_{t+1}/\,(1+r-\phi_1)$, and $\gamma > 0$. The Bellman equation for this problem can be written as:

$$v\left(s_t, c_{t-1}\right) = \max_{c_t}\left\{-\frac{1}{\alpha}\exp\left(-\alpha\left(c_t - \gamma c_{t-1}\right)\right) + \frac{1}{1+\rho} E_t\left[v\left(s_{t+1}, c_t\right)\right]\right\}, \tag{59}$$

where $v\,(s,c)$ is the value function.

To solve this Bellman equation, we first conjecture that: $v\left(s_t, c_{t-1}\right) = -\exp\left(-a_0\left(s_t + a_1 c_{t-1} + a_2\right)\right)/a_0$, where a_0, a_1, and a_2 are undetermined coefficients. The FOC w.r.t. c_t is

$$\exp\left(-\alpha\left(c_t - \gamma c_{t-1}\right)\right) = \frac{1-a_1}{1+\rho} E_t\left[\exp\left(-a_0\left(s_{t+1} + a_1 c_t + a_2\right)\right)\right]. \tag{60}$$

The Envelope theorem are:

$$\exp\left(-a_0\left(s_t + a_1 c_{t-1} + a_2\right)\right) = \frac{1+r}{1+\rho} E_t\left[\exp\left(-a_0\left(s_{t+1} + a_1 c_t + a_2\right)\right)\right], \tag{61}$$

$$-\frac{a_1}{\gamma}\exp\left(-a_0\left(s_t + a_1 c_{t-1} + a_2\right)\right) = \exp\left(-\alpha\left(c_t - \gamma c_{t-1}\right)\right). \tag{62}$$

Combining (60) and (61) yields

$$c_t = \frac{a_0}{\alpha} s_t + \left(\frac{a_0 a_1}{\alpha} + \gamma\right) c_{t-1} - \frac{1}{\alpha}\left(\ln\left(\frac{1-a_1}{1+r}\right) - a_0 a_2\right). \tag{63}$$

Combining (60) and (62) yields

$$c_t = \frac{1}{a_0(1-a_1)}\left[a_0 r s_t - a_0 a_1 c_{t-1} + \ln\left(-\frac{a_1(1+\rho)}{\gamma(1-a_1)}\right) - \ln\left(E_t\left(\exp\left(-a_0\zeta_{t+1}\right)\right)\right)\right]. \tag{64}$$

Comparing the two consumption functions, (63) and (64), we can determine the coefficients in the conjectured value function:

$$\begin{aligned}
a_0 &= \frac{\alpha r(1+r-\gamma)}{1+r}, \\
a_1 &= -\frac{\gamma}{1+r-\gamma}, \\
a_2 &= \frac{1+r}{r(1+r-\gamma)}\left\{\frac{1}{\alpha}\ln\left(\frac{1}{1+r-\gamma}\right) + \frac{1}{\alpha r}\left[\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-\frac{\alpha r(1+r-\gamma)}{1+r}\zeta_{t+1}\right)\right]\right)\right]\right\}.
\end{aligned}$$

Substituting these coefficients into (63) yields the consumption function in the main text.

Substituting the state transition equation into the consumption function yields:

$$c_t - r s_t = \gamma(c_{t-1} - r s_{t-1}) - \frac{r\gamma}{r+1}\zeta_t + \frac{1}{r\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-\frac{\alpha r(1+r-\gamma)}{1+r}\zeta_{t+1}\right)\right]\right)\right). \tag{65}$$

Combining (65) with $d_t \equiv r a_t + y_t - c_t$, we can rewrite the individual saving function as follows:

$$\begin{aligned}
d_t &= r a_t + y_t - c_t \\
&= (1-\phi_1)\phi(y_t - \overline{y}) + \frac{r^2\gamma}{r+1}\frac{\zeta_t}{1-\gamma\cdot L} - \frac{1}{\alpha(1-\gamma)}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-\frac{r\alpha(1+r-\gamma)}{1+r}\zeta_{t+1}\right)\right]\right)\right),
\end{aligned}$$

where we use the facts that $\phi_0 = (1-\phi_1)\overline{y}$ and $\phi = 1/(1+r-\phi_1)$.

To prove uniqueness of the equilibrium, note that the derivative of the precautionary savings term is

$$\frac{d}{dr}\left(\frac{(r\alpha)^2}{2}\left(\frac{1+r-\gamma}{1+r}\right)^2\left(\frac{\sigma}{1+r-\phi_1}\right)^2\right) = \frac{\alpha^2 r\sigma^2}{(1+r)^3}\frac{1+r-\gamma}{(1+r-\phi_1)^3}\left[(1+2r-\gamma)(1-\phi_1) + (1+\gamma-\phi_1)r^2\right].$$

Every term is positive if $\gamma < 1$ and $\phi_1 < 1$, so this term has a strictly positive derivative and therefore the equilibrium is unique.

To derive the relative volatility of financial wealth (a_t) and labor income (y_t) in general equilibrium, we first rewrite the above saving equation as follows:

$$\begin{aligned}\Delta a^*_{t+1} = d^*_t &= (1-\phi_1)\,\phi\,(y_t - \bar{y}) + \frac{r^{*2}\gamma}{1+r^*}\frac{\zeta_t}{1-\gamma\cdot L} \\ &= (1-\phi_1)\,\phi\left(\frac{\phi_0}{1-\phi_1} - \bar{y}\right) + \sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^{*2}\gamma}{1+r^*}\phi\gamma^j\right] w_{t-j}.\end{aligned}$$

where we use the facts that $y_t = \phi_0 + \phi_1 y_{t-1} + w_t = \frac{\phi_0}{1-\phi_1} + \sum_{j=0}^{\infty}\phi_1^j w_{t-j}$ and $\zeta_t = \phi w_t$. Taking unconditional variance on both sides yields:

$$\frac{\text{sd}\left(\Delta a^*_{t+1}\right)}{\text{sd}\left(\Delta y_{t+1}\right)} = \frac{1}{\sqrt{2}}\sqrt{(1+\phi_1)\sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^2}{1+r}\phi\gamma^{j+1}\right]^2},$$

where we use the facts that $\frac{\text{sd}(\Delta y_{t+1})}{\text{sd}(w_{t+1})} = \sqrt{\frac{2}{1+\phi_1}}$.

In the next step, we will derive the relative volatility of consumption growth to income growth. We first rewrite the difference of total wealth as

$$\begin{aligned}\Delta s^*_{t+1} &= r^* s^*_t - c^*_t + \zeta_{t+1} \\ &= r^*\left[a^*_t + \phi\left(y_t + \frac{\phi_0}{r^*}\right)\right] - c^*_t + \zeta_{t+1} \\ &= d^*_t + (r^*\phi - 1)\,y_t + r^*\phi\frac{\phi_0}{r^*} + \zeta_{t+1}, \\ &= (1-\phi_1)\,\phi\left(\frac{\phi_0}{1-\phi_1} - \bar{y}\right) + \sum_{j=0}^{\infty}\left[(1-\phi_1)\,\phi\phi_1^j + \frac{r^{*2}}{1+r^*}\phi\gamma^{j+1}\right] w_{t-j} \\ &\quad + (r^*\phi - 1)\left(\frac{\phi_0}{1-\phi_1} + \sum_{j=0}^{\infty}\phi_1^j w_{t-j}\right) + \phi\phi_0 + \phi w_{t+1} \\ &= \frac{\phi_0\,(r^*\phi - 1)}{1-\phi_1} + \phi\phi_0 + \frac{r^{*2}\phi}{1+r^*}\sum_{j=0}^{\infty}\gamma^{j+1} w_{t-j} + \phi w_{t+1},\end{aligned}$$

where we use the fact that $\left[(1-\phi_1)\,\phi + (r^*\phi - 1)\right]\phi_1^j = 0$ and $\phi_0 = (1-\phi_1)\,\bar{y}$. Hence, using (65), we derive the expression for the difference of the equilibrium consumption process, Δc^*_{t+1}:

$$\begin{aligned}\Delta c^*_{t+1} &\equiv c^*_{t+1} - c^*_t \\ &= \frac{\gamma}{(1+r^*)}\Delta c^*_t + \frac{r^*\left(1+r^*-\gamma\right)}{1+r^*}\Delta s^*_{t+1} \\ &= \frac{\gamma}{(1+r^*)}\Delta c^*_t + \frac{r^*\left(1+r^*-\gamma\right)}{1+r^*}\left[\frac{\phi_0\left(r^*\phi-1\right)}{1-\phi_1} + \phi\phi_0 + \frac{r^{*2}\phi}{1+r^*}\sum_{j=0}^{\infty}\gamma^{j+1}w_{t-j} + \phi w_{t+1}\right]. \quad (66)\end{aligned}$$

Taking unconditional variance on both sides yields:

$$\operatorname{var}\left(\Delta c^*_{t+1}\right) = \left(\frac{\gamma}{1+r^*}\right)^2 \operatorname{var}\left(\Delta c^*_t\right) + \left(\frac{r^*\left(1+r^*-\gamma\right)\phi}{1+r^*}\right)^2\left[1+\left(\frac{r^{*2}\gamma}{1+r^*}\right)^2\sum_{j=0}^{\infty}\left(\gamma^2\right)^j\right]\operatorname{var}\left(w_{t+1}\right),$$

which reduces to (43) in the main text.

7.6. An Extension to Incorporate Durable Consumption

Following Bernanke (1985), we consider an FI-RE version of the PIH model which includes both durable and nondurable consumption. The optimizing decisions of a representative consumer in the RE-PIH model with durables goods can be formulated as

$$\max_{\{c_t,k_{t+1}\}_{t=0}^{\infty}} \left\{E_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t,k_t\right)\right]\right\}, \quad (67)$$

subject to the budget constraint

$$a_{t+1} = (1+r)\,a_t + y_t - c_t - e_t, \quad (68)$$

and the accumulation equation for durables

$$k_{t+1} = (1-\delta)\,k_t + i_t, \quad (69)$$

where $u\left(c_t,k_t\right) = -\exp\left(-\alpha_c c_t\right)/\alpha_c - \varrho\exp\left(-\alpha_k k_t\right)/\alpha_k$ is the utility function, c_t is consumption of nondurables, k_t is the stock of durables goods, i_t is the purchase of durable goods, $\beta = 1/\left(1+\rho\right)$ is the discount factor, $R = 1+r$ is the constant gross interest rate, δ is the depreciation rate of durable goods, $\alpha_c > 0$, $\alpha_k > 0$, and $\varrho > 0$.

To incorporate RI, following the same procedure used in the our benchmark model and Luo, Nie, and Young (2014), we define a new state variable (s) as:

$$s_t \equiv a_t + \frac{1-\delta}{1+r} k_t + \frac{1}{1+r-\phi_1}\left(\frac{\phi_0}{r} + y_t\right), \tag{70}$$

which is governed by the following evolution equation:

$$s_{t+1} = (1+r)\, s_t - c_t - \frac{r+\delta}{(1+r)} k_{t+1} + \zeta_{t+1},$$

where $\zeta_{t+1} = \phi w_{t+1} = \frac{1}{1+r-\phi_1} w_{t+1}$ is the innovation to s_{t+1}.

Following Luo, Nie, and Young (2014), we formulate the optimization problem for the typical household under RI:

$$v\left(\widehat{s}_t\right) = \max_{\{c_t, k_{t+1}\}} \left\{E_t\left[u\left(c_t, k_t\right) + \beta v\left(\widehat{s}_{t+1}\right)\right]\right\}$$

subject to

$$\widehat{s}_{t+1} = (1+r)\, \widehat{s}_t - c_t - \frac{r+\delta}{1+r} k_{t+1} + \widehat{\zeta}_{t+1},$$

where $\widehat{\zeta}_{t+1}$ is defined in (15) and $\widehat{s}_0$ is given, The following proposition summarizes the results from the above dynamic program:

Proposition 7. *Under RI, the functions of nondurable consumption and the stock of durable accumulation are:*

$$c_t = H_c \widehat{s}_t + \Omega_c + \widehat{\Pi}_c, \tag{71}$$

$$k_{t+1} = \Omega + \frac{\alpha_c}{\alpha_k} c_t, \tag{72}$$

respectively, where $H_c = r\left(1 + \frac{r+\delta}{1+r}\frac{\alpha_c}{\alpha_k}\right)^{-1}$, $\Omega = -\frac{1}{\alpha_k}\ln\left(\frac{r+\delta}{\varrho}\right)$, $\Omega_c = -\left(1 - \frac{1-\delta}{1+r}\right)\left(1 + \frac{r+\delta}{1+r}\frac{\alpha_c}{\alpha_k}\right)^{-1}\Omega$, $\widehat{\Pi}_c = -\widehat{\Pi}/r$, *and*

$$\widehat{\Pi} \equiv \frac{1}{\alpha_c}\ln\left(\beta\left(1+r\right)\right) + \frac{\alpha_c}{2}\left(\frac{r}{1+r-\phi_1}\right)^2\left(1 + \frac{r+\delta}{1+r}\frac{\alpha_c}{\alpha_k}\right)^{-2}\Gamma\left(r, \theta\right)\phi^2\sigma^2. \tag{73}$$

Proof. See Online Appendix. ∎

Given the original budget constraint and the decision rules, the expression for individual saving, $d_t \equiv ra_t + y_t - c_t - [k_{t+1} - (1-\delta) k_t]$, can be written as:

$$d_t = \frac{r}{1+r-\phi_1}\left[1-\left(1+\frac{\alpha_c}{\alpha_k}\right)\left(1+\frac{r+\delta}{1+r}\frac{\alpha_c}{\alpha_k}\right)^{-1}\right]\widehat{\zeta}_t + r\left(s_t-\widehat{s}_t\right) + \left[\frac{1+r}{H_c}-\left(1+\frac{\alpha_c}{\alpha_k}\right)\right]\widehat{\Pi} + \frac{r}{1+r-\phi_1}\left(y_t-\overline{y}\right), \tag{74}$$

where $s_t - \widehat{s}_t$ is defined in (17), respectively.

After aggregating across all consumers using the same law of large number we applied in the benchmark model, all the idiosyncratic shocks (including the fundamental income shocks and endogenous shocks due to RI) are canceled out and we obtain the following expression for aggregate saving:

$$D(\theta, r) \equiv \left[\frac{1+r}{H_c}-\left(1+\frac{\alpha_c}{\alpha_k}\right)\right]\widehat{\Pi}, \tag{75}$$

where $\widehat{\Pi}$ is defined in (73). The following proposition proves the existence of a general equilibrium and shows the PIH holds in such an equilibrium.

Proposition 8. *There exists at least one equilibrium with an interest rate $r^* \in (0, \rho)$ in the RI precautionary-savings model with durables. In any such equilibrium, aggregate saving is zero:*

$$\frac{1}{2}\left(\alpha_c r^*\right)^2\left(1+\frac{\delta+r^*}{1+r^*}\frac{\alpha_c}{\alpha_k}\right)^{-2}\Gamma\left(r^*,\theta\right)\omega_\zeta^2 - \ln\left(\frac{1+\rho}{1+r^*}\right) = 0.$$

The PIH holds since consumption follows

$$c_t = H_c\widehat{s}_t + \Omega_c.$$

Proof. The proof is the same as that in the benchmark model. Here we need to assume that $\frac{1+r^*}{H_c} - \left(1+\frac{\alpha_c}{\alpha_k}\right) \neq 0$. Since this extension adds another force to the interest rate determination, we cannot prove uniqueness. ■

We now examine how rational inattention affects the equilibrium interest rate in the CARA-Gaussian setting with durable consumption. Luo, Nie and Young (2014) show that if $\frac{\alpha_c}{\alpha_k} = \frac{r+\delta}{\varrho}$ holds, then the model here is observationally equivalent to the LQ Gaussian model with durables. Using the estimated parameters in Bernanke (1985) ($\delta = 0.08$, and $\varrho = 0.0286$) and $\alpha_c = 2$ used in Caballero (1991) and Wang (2000), we calibrate the CARA parameter on durable consumption: $\alpha_k = 1.63$. Given the following parameter values: $\alpha_c = 2$, $\alpha_k = 1.63$, $\sigma = 0.175$, $\phi_1 = 0.92$, $\rho = 0.04$,

$\delta = 0.08$, and $\varrho = 0.0286$, Figure **??** illustrates how r^* varies with the value of θ. The figure also clearly shows that the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with the degree of inattention, which is consistent with the conclusion obtained in our benchmark model without durable goods.

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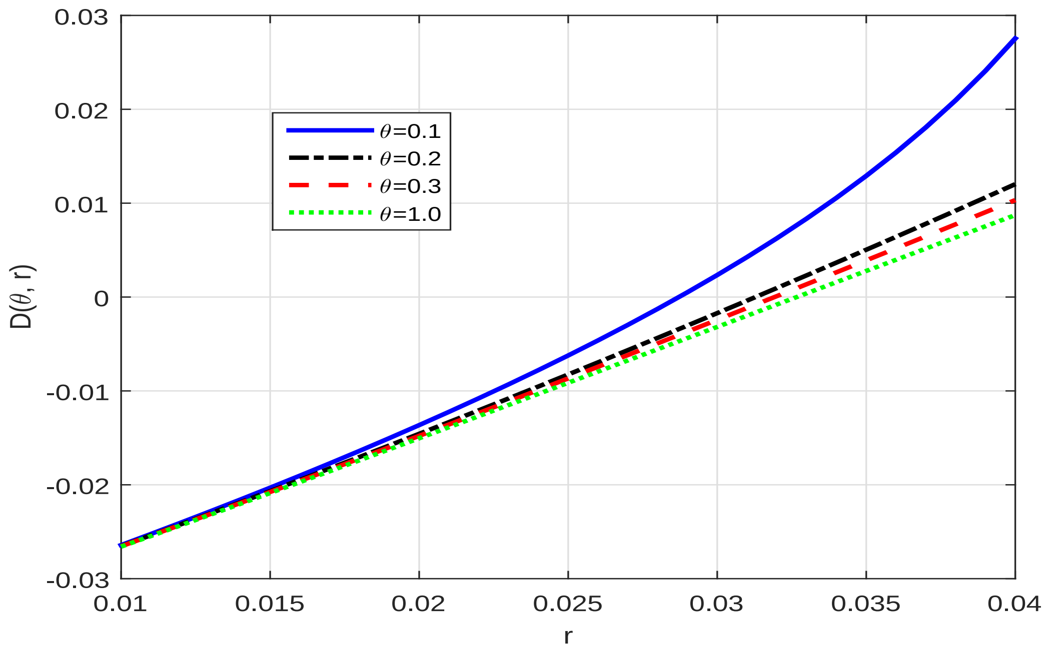

Figure 1. *Effects of RI on Aggregate Saving*

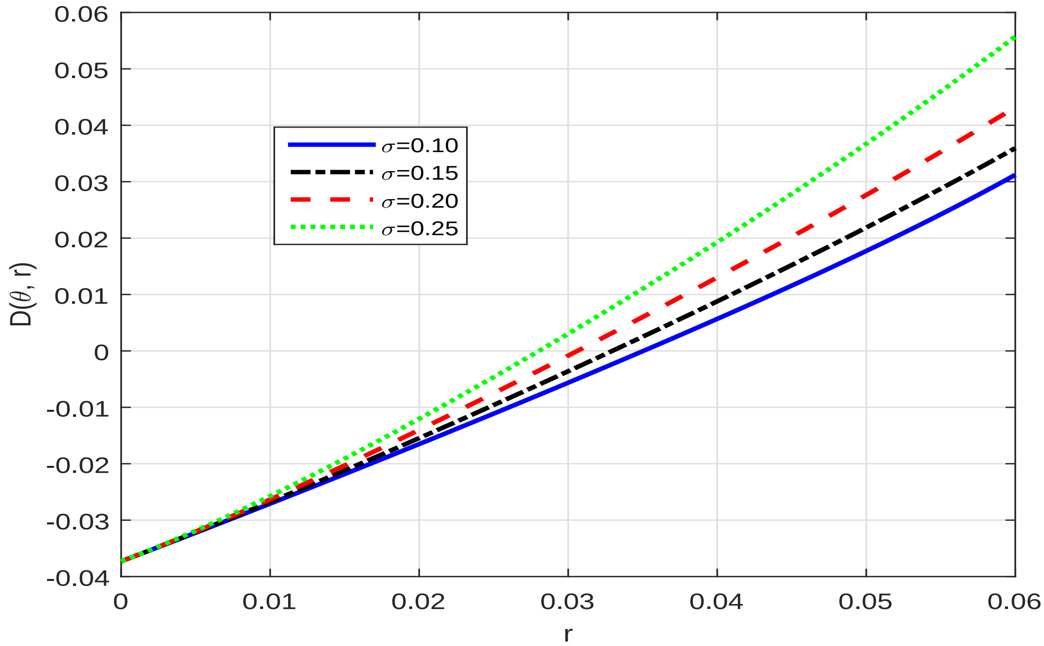

Figure 2. *Effects of Income Volatility on the Interest Rate in GE (Elastic κ)*

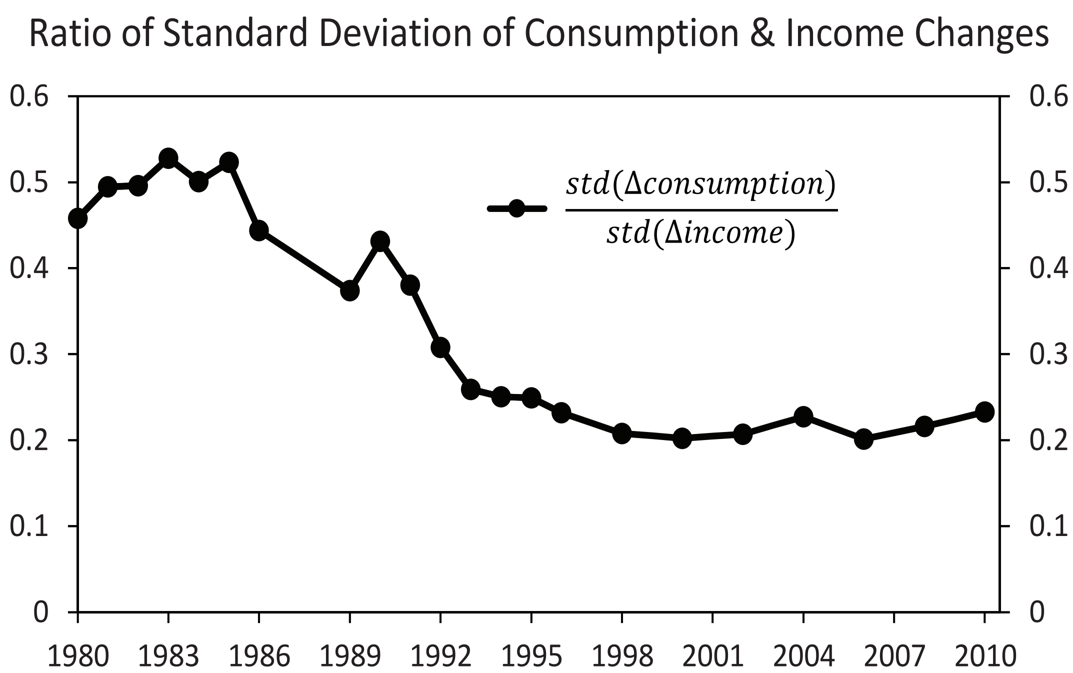

Figure 3. *Relative Consumption Dispersion*

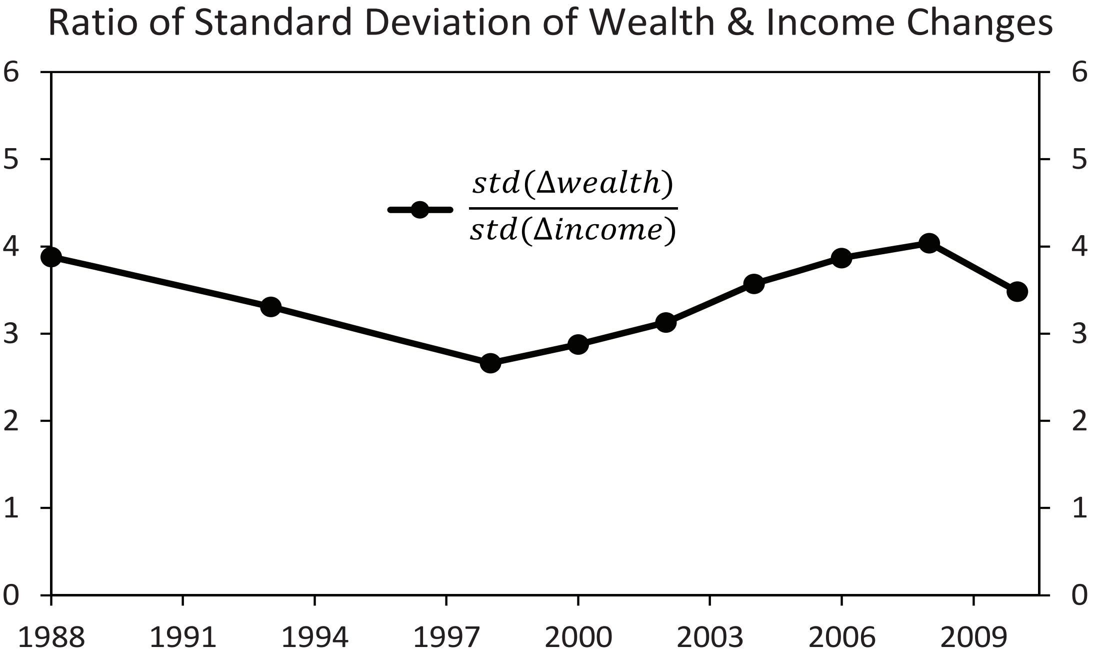

Figure 4. *Relative Dispersion of Wealth to Income*

Table 1. *Estimation of the Income Process*

	std (ϵ_{it})	ϕ_{it}	std (y_{it})
Period 1 (1980 − 1986)	0.154	0.917	0.386
Period 2 (1987 − 1996)	0.175	0.912	0.427
Full Period (1980 − 1996)	0.175	0.919	0.444

Table 2. *Implications of RI for Interest rates, the Relative Volatility of Consumption and Wealth to Income, and Welfare*

	θ	10%	20%	40%	60%	100%
GE	r^*	2.87%	3.13%	3.22%	3.25%	3.27%
	μ_{cy}	0.375	0.319	0.296	0.289	0.284
	μ_{ay}	2.630	2.219	1.964	1.863	1.775
	\$loss	8.82	8.52	6.58	4.42	0
PE	μ_{cy}	0.448	0.332	0.299	0.291	0.284
($r = 3.27\%$)	μ_{ay}	2.722	2.216	1.959	1.861	1.775
	\$loss	3.92×10^{-4}	3.49×10^{-4}	2.62×10^{-4}	1.74×10^{-4}	0

Table 3. *Model Comparison*

	Data	FI-RE	RI-GE ($\theta = 0.1$)	RI-PE ($\theta = 0.1$)
r^*(%)	2.97	3.27	2.87	n.a.
μ_{cy}	0.38	0.28	0.38	0.45
μ_{ay}	3.28	1.78	2.63	2.72
\$loss	n.a.	0	8.82	3.92×10^{-4}

Table 4. *Implications of RI (Elastic κ)*

	σ (σ_y)	0.15 (0.38)	0.25 (0.64)	0.35 (0.89)
GE-RI ($\lambda = 400$)	θ	8%	12%	16%
	r^*	2.80%	2.56%	2.23%
	μ_{cy}	0.46	0.30	0.25
	μ_{ay}	2.94	2.48	2.34
GE-FI ($\theta = 1$)	r^*	3.39%	2.89%	2.43%
	μ_{cy}	0.29	0.26	0.23
	μ_{ay}	1.76	1.84	1.92

Table 5. *Implications of Habit Formation*

	γ	0	0.4	0.6	0.8	0.9
GE	r^*	3.27%	3.57%	3.69%	3.77%	3.79%
	μ_{cy}	0.284	0.201	0.160	0.113	0.08
	μ_{ay}	1.775	1.731	1.717	1.713	1.720
PE	μ_{cy}	0.284	0.189	0.146	0.101	0.07
($r = 3.27\%$)	μ_{ay}	1.775	1.777	1.780	1.786	1.794