HOW DOES LABOR MOBILITY AFFECT INCOME CONVERGENCE?

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Abstract

Labor mobility is introduced into the neoclassical growth model. For a small open economy with capital intensity below its steady-state level, outmigration directly contributes to faster income convergence but also creates a disincentive for gross capital investment. At low relative income levels, the latter disincentive effect tends to dominate so that labor mobility can actually slow the speed of income convergence.

JEL classification: O410, F430, J610

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1 Introduction

How does labor mobility affect income convergence?

Intuitively, individuals’ moving in search of higher wages might be expected to increase the speed at which such wages are equilibrated across regions. But empirical research has produced mixed results on the link between labor mobility and the speed of income convergence. Barro and Sala-i-Martin (1991, 1992, 1995) estimate the speed of convergence for U.S. states to be nearly identical to that for nation-states. Their explanation is that the presumed higher labor mobility across U.S. states is nevertheless too low to affect the speed of convergence. But many have suggested that the Barro and Sala-i-Martin cross-sectional empirical methodology is misspecified (e.g., Islam, 1995; Caselli, Esquivel, and Lefort, 1996; Quah, 1996). Using high-frequency dynamic panel-data techniques, Evans and Karras (1996) and Evans (1997) indeed find substantially higher speeds of income convergence for U.S. states than for countries.

Little theoretical work to date has explored the effect of labor mobility on income convergence. The main exception is Braun (1993), discussed extensively in Barro and Sala-i-Martin (1995), which shows that the asymptotic speed of convergence is directly proportional to the degree of labor mobility. But in dynamic systems with multiple state variables (i.e., non-jumping variables), asymptotic results often poorly characterize dynamics even relatively close to the steady state (Evans, 1997; Eichner and Turnovsky, 1999; Rappaport, 2000).

The present paper points out that the intuition on labor mobility’s positive contribution to income convergence misses an offsetting negative contribution: the exit of labor from poorer economies lowers the return to capital there and thus slows gross capital formation. Numerical results from a neoclassical growth model modified to include labor mobility show this disincentive effect can be quite large. Especially at income levels far below their steady state, the disincentive effect of outmigration on investment may dominate its direct effect, so that labor mobility can actually slow wage growth.
2 Neoclassical Growth with Mobile Labor

The model developed herein is a slight variation on the standard neoclassical growth framework (Ramsey, 1928; Cass, 1965; Koopmans, 1965). The world is assumed to be composed of two open economies, one large and one small. Besides size, the economies are otherwise identical, except that the large one is assumed to be at its long-run steady state whereas the small one will start with capital intensity below the shared steady-state level. As the small economy transitions to its steady state, labor is assumed to migrate from the small to the large economy at a rate proportional to the net present value difference in labor income. Numerical results show the effect of varying the degree of this proportionality.

The remainder of this section describes firm behavior, labor migration, the large-economy steady state, the system of equations governing the small economy’s dynamics, and the small-economy steady state.

2.1 Firms

Within each of the two economies, perfectly-competitive firms employ a constant-returns-to-scale (CRS) production function that combines capital and labor, $K_i$ and $L_i$, to manufacture a numeraire good, $Y$. As CRS implies an indeterminate firm size, I write instead the aggregate economy production and capital evolution functions,

\[ Y_i(t) = AK_i(t)^\alpha L_i(t)^{1-\alpha} \]

\[ \dot{K}_i(t) = I_i(t) - \delta K_i(t) \]

\[ i = \{\text{large, small}\} \]

Here, $A$ measures aggregate total factor productivity, $\alpha$ measures the share of income accruing to the owners of capital, and $\delta$ measures the rate of capital depreciation. All three parameters are assumed to be the same between the two economies. For simplicity, (1) abstracts from any sort of technological progress.
Firms choose their level of employment and gross investment to maximize the net present value of future cash flows,

\[ V_i(t) = \int_t^\infty \left( Y_i(s) - w_i(s) L_i(s) - I_i(s) - \left( \frac{I_i(s)}{K_i(s)} \right)^2 \right) e^{-r(s-t)} ds \]  

Along the lines of Abel (1982) and Hayashi (1982), (3) posits a total adjustment cost that increases quadratically in the rate of gross investment. The real interest rate, \( r \), is assumed to remain at its steady-state value. Setting up and solving the Hamiltonian associated with firms’ maximization problem gives standard results. In particular, the first-order condition with respect to \( I_i(t) \) implies that firms’ rate of gross investment is a linearly increasing function of the shadow value of capital, \( q_i(t) \), (i.e., the capital co-state variable).

\[ \frac{I_i(t)}{K_i(t)} = \frac{q_i(t) - 1}{b} \]  

Let capital per worker be given by \( k_i(t) = \frac{K_i(t)}{L_i(t)} \). The first-order condition associated with \( L_i(t) \) can be inverted to give the wage at which firms are willing to fully employ all workers.

\[ w_i(t) = (1 - \alpha) A k_i(t)^\alpha \]  

Substituting the investment function, (4), into the capital evolution of equation, (2), and into the first-order condition with respect to the shadow value of capital gives

\[ \dot{k}_i(t) = \left( \frac{q_i(t) - 1}{b} - \delta - \frac{\dot{L}_i(t)}{L_i(t)} \right) k_i(t) \]  

\[ \dot{q}_i(t) = (r + \delta) q_i(t) - \alpha A k_i(t)^{-(1-\alpha)} - \frac{(q_i - 1)^2}{2b} \]  

### 2.2 Labor Mobility

Labor is assumed to enter or exit the small economy at a rate directly proportional to the log difference between the small and large economies of the net present value of wage income.

\[ \frac{\dot{L}_{\text{small}}(t)}{L_{\text{small}}(t)} = \mu (\log(h_{\text{small}}(t)) - \log(h_{\text{large}}(t))) \]
\[ h_i(t) \equiv \int_t^{\infty} w_i(s) e^{-r(s-t)} ds \]

Here, \( \mu \) is a constant measuring the degree of labor mobility. The higher \( \mu \) is, the more rapidly labor responds to differences in labor wealth.

The labor mobility function, (8), is similar to that assumed by Braun (1993) and simplifies the framework of Rappaport (2000, 2004), in which individuals compare their utility from living in each of two localities and a friction proportional to net migration slows labor movement. Such a friction might arise, for instance, from the slow adjustment of local housing stock. While admittedly arbitrary, the present setup nicely isolates the effect of labor mobility on income convergence.

### 2.3 Large-Economy Steady State

Because the large economy remains at its steady state, individuals’ consumption choices do not need to be modeled explicitly. Rather, assumed full employment implies that the only feedback from individual preferences to firm behavior is mediated by the steady-state real interest rate, \( r \). In a more fully-developed neoclassical framework, the steady-state real interest rate would equal the rate at which individuals discount future utility flows.

The size distinction between the two economies implies that migration out of or into the small economy does not affect the population of the large one. For simplicity, natural population growth is assumed to be zero. Hence the large economy’s population remains constant.

Setting each of (6) and (7) equal to zero and substituting \( \dot{L}_{\text{large}}(t) / L_{\text{large}}(t) = 0 \) gives standard solutions for steady-state large-economy capital intensity and the steady-state large-economy shadow value of installed capital. As all parameters are identical between the two economies and small-economy population growth of zero will also turn out to hold at the small-economy steady state, I drop subscripts for steady-state values.

\[ q^* = 1 + \delta b \] (9)
Identical steady-state labor wealth in both economies is given by

\[ h^* = \frac{w^*}{r} \]  

(11)

where \( w^* \) is given by substituting \( k^* \) into (5).

### 2.4 Dynamic System and Small Economy Steady State

A self-referential dynamic system can now be written which describes the evolution of the four variables, \( L_{small}(t), k_{small}(t), q_{small}(t), \) and \( h_{small}(t) \). The first and third associated equations are given by (8) and (7) above. The second associated equation is given by substituting (8) into (6). And the equation of motion for labor wealth is derived by simple differentiation of its definition above.

\[ \dot{h}_{small}(t) = rh_{small}(t) - w_{small}(t) \]  

(12)

The small-economy steady state is defined by the constancy of each of these four system variables. So by definition, small-economy population growth is zero, which in turn implies that the small-economy steady-state capital intensity and shadow value of capital are indeed given by (9) and (10) above.

Finally, small-economy steady-state population, \( L_{small}^* \), is indeterminate. As soon as small-economy capital intensity attains its steady-state value, so does small-economy labor wealth in turn implying that \( \dot{L}_{small}(t) = 0 \). But in the present CRS environment, this can occur at any small economy population. The history dependence of small-economy steady-state population will be made more intuitive in the discussion of the numerical results below. Such history dependence is consistent with the empirical finding in Blanchard and Katz (1992) that labor-demand shocks permanently affect employment levels of U.S. states.
3 Factor Mobility and Income Convergence

Numerical solutions to the present growth system richly characterize the time paths of population and wages as the small economy transitions from an initial wage level below its steady state. While labor mobility directly contributes to faster income convergence, the exit of labor also creates a disincentive for gross capital investment.\(^1\) Especially at low initial wage levels, this disincentive effect can outweigh the direct contribution, so that labor mobility actually slows income convergence.

3.1 Calibration

The numerical exercise that follows illustrates the large disincentive effect of labor mobility on gross capital formation and also the misleading nature of asymptotic summary measures of dynamic systems. Its goal is not to pin down an exact quantitative contribution of labor mobility to the speed of income convergence.

System parameters are set at benchmark values. In particular, the capital income share is set to one third \((\alpha = \frac{1}{3})\), the rate of capital depreciation is set to 6 percent \((\delta = 0.06)\), and the real interest rate is set to 3 percent \((r = 0.03)\). All qualitative results are extremely robust to varying these.

Qualitative results will prove somewhat more sensitive to the assumed capital installation friction, \(b\) in (3). This friction serves the key purpose of slowing capital mobility so that small-economy wages do not instantaneously converge to their steady state (Barro, Mankiw, and Sala-i-Martin, 1995). I parameterize this capital mobility by the steady-state shadow value of capital, \(q^*\), which for a given rate of capital depreciation maps one-to-one with \(b\) by (9). As a baseline value, I set \(q^* = 1.48\). This is high relative to time series aggregate average values of installed capital reported in Summers (1981) and Blanchard, Rhee, and Summers (1993), but consistent with

\(^1\)More precise language would refer to “per capita output convergence” or “wage convergence” rather than “income convergence”. In a permanent-income setting in which individuals cannot fully insure against local shocks, steady-state capital income will differ between the small and large economies.
average firm values reported in Barnett and Sakellaris (1998). The interaction of capital and labor mobility on the speed of income convergence is explored in detail below.

The appropriate degree of labor mobility will obviously vary depending on context. For instance, labor mobility within a nation-state tends to be much higher than labor mobility between nation-states. But even restricted to a locality within the United States, empirical estimates leave wide latitude in parameterizing labor mobility. Looking at the relationship between net migration and initial wage levels for U.S. states for each decade from the 1900s through the 1980s, Barro and Sala-i-Martin (1991) suggest that the net migration response to current income differences is below $\mu = \frac{1}{25}$. Regressing net migration on a constructed measure of expected income differences, Greenwood et al. (1991) find a response equivalent to $\mu = \frac{1}{5}$. Using a different methodology to control for future income differences, Gallin (2004) presents results that suggest that U.S. labor mobility may be in the range $1.5 \leq \mu \leq 2$. These estimates are biased towards finding low labor mobility, since they assume that no part of observed wage differences compensate for varying local quality of life.

2 Two caveats apply to calibrating the steady-state shadow value of capital. First is that observed average values of capital equal the shadow value of capital only when production and adjustment costs are homogeneous of degree one (Hayashi, 1981). Second is that the steady-state value, $q^* = 1.48$, is based on the absence of taxes. In reality, the shadow value of capital encompasses any marginal taxes firms face from investing. For a given capital intensity, taxes on corporate profits and on capital gains tend to lower the shadow value of capital; investment tax credits, accelerated capital depreciation schedules, and taxes on dividends tend to increase the shadow value of capital (Summers, 1981; Abel, 1982). Adjusting for such tax considerations tends to increase observed average values of capital (Summers, 1981; Barnett ans Sakellaris, 1998). In other words, adjustment costs are probably higher than suggested by non-tax-adjusted average values of capital.

3 Gallin (2004) estimates that a 1 percent wage difference that lasts for one year only induces a net migration rate of 0.09 percent. Assume a 3 percent real interest rate and a 30 year time horizon, a one-period 1 percent wage difference implies a 0.049 percent labor wealth difference. So Gallin’s result implies that such a 0.049 percent labor wealth difference suffices to induce a 0.09 percent migration rate, in turn implying $\mu = 1.85$.

4 In sharp contrast, the compensating differential literature (e.g., Rosen, 1979; Roback, 1982; Gyourko and Tracy, 1989, 1991) makes the identifying assumption that all of observed wage differences
For illustrative purposes, transition dynamics immediately below are shown when labor is completely immobile, \( \mu = 0 \), as well as for “intermediate” labor mobility, \( \mu = \frac{1}{4} \), and “high” labor mobility, \( \mu = 4 \). The subsequent subsection on robustness more systematically discusses the effect of varying labor mobility from zero to well above \( \mu = 4 \). To give more intuition on the degree of labor mobility, it is helpful to think about the implied labor wealth difference needed to induce a 1 percent rate of net migration. With the intermediate labor mobility calibration, a 4 percent real wealth difference will do so. In other words, when labor wealth in the large economy is 1.04 times that in the small economy, migration from the latter to the former will be at a 1 percent annual rate. With the high labor mobility calibration, a 0.25 percent real wealth difference suffices to induce a 1 percent rate of net migration. And, of course, with zero labor mobility, even an infinite labor wealth difference cannot induce any migration.

### 3.2 Transition Dynamics

Figure 1 shows dynamics for each of the three enumerated degrees of labor mobility as the small economy transitions to its steady state from an initial wage that is 60 percent of its steady-state (and the large-economy) level.

Figure 1 Panel A shows the alternative time paths of small-economy population. With zero labor mobility (\( \mu = 0 \)), population remains constant during the transition. But with intermediate and high labor mobility (\( \mu = \frac{1}{4} \) and \( \mu = 4 \)), the small economy experiences rapid outmigration. With intermediate labor mobility, people initially depart the small economy at a 2.3 percent annual rate, and population eventually falls to 79 percent of its initial level by the time wages attain their steady-state (and large-economy) level (Table 1 Columns 2 and 5). With high labor mobility, initial outmigration occurs at a 22.5 percent annual rate, and population eventually falls to 31 percent of its initial level.

While the outmigration of labor directly contributes to increasing small-economy wages, it also creates a disincentive for gross investment. In other words, the outflow compensate for unobserved quality-of-life differences.
of labor drives down the marginal product of capital in turn driving down the shadow value of installed capital, on which investment depends. The result, shown in Panel B, is that investment is higher the lower is labor mobility. With zero labor mobility, the initial investment rate, \( \frac{I}{K} \), is 26.8 percent (Table 1 Column 3). With intermediate and high labor mobility, initial investment falls respectively to 24.8 percent and 15.3 percent.\(^5\)

From a theoretical perspective, at very high levels of labor mobility the positive direct contribution to faster wage growth must outweigh the indirect negative contribution. After all, as labor mobility becomes infinite, population can immediately jump down to the level at which wages in the small economy are equal to those in the large one.

What the numerical results show is that at low and intermediate levels of labor mobility, the direct positive and indirect negative contributions from labor mobility approximately cancel each other out. With zero labor mobility, the initial rate of wage growth is 6.9 percent; with intermediate labor mobility, it is 7.0 percent (Table 1 Column 4). Along the transition, intermediate-mobility wages remain no more than 1 percentage point higher than zero-mobility wages. But further increasing the degree of labor mobility allows the direct effect to dominate. With high labor mobility, the initial rate of wage growth is 10.6 percent and along the transition, high-mobility wages are as much as 9 percentage points higher than zero-mobility wages.

The speed at which wages converge to their steady state is a helpful metric for comparing the combined direct and indirect effects of labor mobility.\(^6\) Essentially, this is just the wage growth rate normalized by the log distance of wages from their

\(^5\)The initial shadow value of capital is 3.14 with zero labor mobility, 2.98 with intermediate labor mobility, and 2.22 with high labor mobility. Because of the linear relationship linking gross investment and the shadow value capital, (4), the transition time paths for the two variables look exactly the same.

\(^6\)In particular, the speed of convergence allows for comparisons near the steady state, where transitional growth approaches zero for all calibrations.
steady state. More formally,

$$\Lambda (\log w(t) - \log w^*) \equiv -\frac{d}{dt} \left( \log w(t) - \log w^* \right)$$

$$= \frac{\dot{w}(t)}{w(t)} \frac{\log w(t) - \log w^*}{\log w^* - \log w(t)}$$

The speed of convergence proves more sensitive to the degree of labor mobility the closer the small economy is to its steady state. Put differently, the indirect disincentive effect of labor mobility on the speed of convergence is greater the further the small economy is from its steady state. Figure 1 Panel D shows the transition speed of convergence for the three labor mobility levels. The initial speed of convergence is 13.5 percent with zero labor mobility versus 13.8 percent with intermediate labor mobility (Table 1 Column 6). So the initial difference in the speed of convergence between the two is just 0.3 percent. But as the transition proceeds, this difference increases to 1.1 percent. Similarly, the difference between the zero-labor-mobility and high-labor-mobility convergence speeds increases from 7.7 percent initially to 12.3 percent asymptotically.

Figure 2 Panel A shows that at distances further from the steady state than the initial 60 percent relative income assumed in Figure 1, the disincentive effect of labor mobility dominates the direct effect so that the speed of convergence actually decreases as labor becomes more mobile. With small-economy wages at 20 percent their steady-state level, convergence speed is 41.7 percent with zero labor mobility falling to 38.8 percent with intermediate labor mobility falling to 29.2 percent with high labor mobility (Table 1 Column 9). Then, as the transition proceeds, this initial negative relationship between labor mobility and income convergence reverses.

This reversal of the relationship between labor mobility and the speed of convergence along the transition path powerfully highlights how asymptotic relationships can poorly characterize system dynamics. In the present case, Figure 2 shows that the asymptotic dynamics for the speed of convergence inaccurately describes the dynamics for the speed of convergence at income levels even moderately below the steady state. More generally, for dynamic systems with multidimensional transition mani-
folds, the speed of convergence can vary widely, even very close to the steady state (Evans, 1997; Eichner and Turnovsky, 1999; Rappaport, 2000).7

3.3 Robustness: The Effect of Factor Mobility on Convergence and Labor Wealth

The negative net effect of labor mobility on the speed of income convergence at low relative incomes is extremely robust to the model’s parameters. The result holds for substantial variations in the capital factor income share, $\alpha$; the rate of capital depreciation, $\delta$; the real interest rate, $r$; and the degree of capital mobility ($q^\ast$). 8

At extremely high levels of labor mobility, however, the positive direct effect overwhelms the negative disincentive effect. Regardless of parameters or relative income, sufficiently large increases in labor mobility always cause the speed of convergence to increase. As labor mobility becomes infinite, the system can essentially jump right to its steady state. With the parameters underpinning the transitions shown in Figure 2 Panel A, increasing labor mobility above $\mu = 4$ causes the speed of convergence at 20 percent relative income to increase.

Similarly, as the small economy transitions to its steady state, the positive direct effect of labor mobility comes to dominate its negative disincentive effect. Intuitively, 7Multidimensional transition manifolds are characteristic of dynamic systems with multiple state (i.e., non-jumping) variables. A necessary condition to reach some steady state from any initial combination of state variables is that the dimensionality of the set of steady states plus the dimensionality of the transition manifold to each of these sum to the number of state variables. The present system has two state variables (capital and labor); it has a one-dimensional manifold to each of a one-dimensional set of steady states. In the working paper version of the present model (Rappaport 2000), with three state variables (capital, labor, and asset wealth) and a two-dimensional manifold to each of a one-dimensional set of steady states, the speed of income convergence can vary dramatically even within 1 percent of the system steady state.

8In contrast, the negative net effect proves less robust to a slight generalization of the present model. Allowing for a capital adjustment cost that increases at substantially greater than the quadratic specification implicit in (3), as in Rappaport (2002), the direct positive contribution of labor mobility to convergence speed dominates its negative disincentive effect at all relative income levels.
as the system approaches its steady state, the disincentive effect goes to zero since the cumulative future exit of labor goes to zero. Hence, the numerically derived asymptotic speed of convergence is strictly increasing in labor mobility (Table 1 Column 7). A corresponding analytical result is derived by Braun (1993).

The negative net effect of labor mobility on the speed of income convergence also diminishes as capital becomes less mobile. The higher the adjustment friction slowing capital investment, the more the direct effect of labor’s outmigration outweighs its disincentive effect. With high capital mobility, \( q^* = 1.12 \), going from zero to high labor mobility causes the speed of convergence at 60 percent relative income to rise by just 3.2 percentage points (Table 2 Panel A, Column 6). With low capital mobility, \( q^* = 2.96 \), doing so causes the speed of convergence at 60 percent relative income to rise by 10.8 percentage points (Table 2 Panel B, Column 6). But even with low capital mobility, at small-economy relative wages well below the steady state, labor mobility’s disincentive effect on investment dominates (Figure 2 Panel B and Table 2 Panel B, Column 9).

Although labor mobility may slow the initial speed of convergence, the numerical results suggest that labor mobility always increases initial labor wealth. In other words, the lower net present value of wages associated with slower convergence during the early part of transitions seems always to be dominated by the higher net present value of wages associated with faster convergence during the later part of transitions. For instance, with the base calibration and small-economy initial relative income of 20 percent, the initial speed of convergence falls as labor mobility increases from \( \mu = 0 \) to \( \mu = 4 \); but over this same range, initial relative wealth relative to its steady-state level increases from 82.9 percent to 86.9 percent (Table 1, Columns 9 and 10). Despite much effort, I have not found a counterexample in which an increase in labor mobility lowers initial labor wealth. While there is no obvious reason why the initial slower wage growth due to labor mobility could not dominate the later faster wage growth due to it, the strong numerical regularity that the faster wage growth dominates suggests a theoretical result.

The size of the increase in labor wealth from increasing labor mobility depends
inversely on the degree of capital mobility. With high capital mobility and initial relative income of 20 percent, initial labor wealth is fairly insensitive to the degree of labor mobility. Going from zero to high labor mobility causes relative wealth to increase by just 0.8 percentage points (Table 2 Panel A, Column 10). But with low capital mobility, the wealth effect from higher labor mobility is more substantial. Going from zero to high labor mobility causes relative wealth at 20 percent relative income to increase by 9.4 percentage points (Table 2 Panel B, Column 10).

The increase in labor wealth from increasing capital mobility proves far larger than that from increasing labor mobility. Paralleling the result immediately above, the size of the wealth effect from capital mobility depends inversely on the degree of labor mobility. But even with high labor mobility, going from low to high capital mobility (from $q^* = 2.96$ to $q^* = 1.12$) causes relative wealth at 20 percent relative income to increase by 5.7 percentage points (Table 2 Panel B versus Table 2 Panel A, bottom highlighted row). With zero labor mobility, going from low to high capital mobility causes relative wealth at 20 percent relative income to increase by 14.3 percentage points (Table 2 Panel B versus Table 2 Panel A, top highlighted row).

4 Conclusions

Introducing labor mobility into the neoclassical growth model shows that outmigration creates a powerful disincentive for gross capital investment. This disincentive effect at least partly offsets the positive direct contribution of labor mobility to faster income convergence. At low relative income levels, the disincentive effect can dominate, so that increasing labor mobility slows the speed of income convergence. Nevertheless, the numerical results suggest that increasing labor mobility always increases labor wealth, though by a relatively small amount when capital installation costs are low.

The small-economy transitions also serve as a powerful example of how asymptotic relationships can poorly characterize system dynamics. Near the steady state, increasing labor mobility always increases the speed of convergence. But away from
the steady state, this relationship is commonly reversed.

From a practical perspective, the results show that even in a completely homogeneous environment, the increase in labor wealth to remaining poor-economy residents from the widespread exit of others may be relatively small. In a more complicated world, remaining residents might actually suffer, for instance if high-human-capital individuals were the first to emigrate. Policies that lower the frictions to capital investment can help avoid such negative outcomes.

Bibliography


### Table 1: Summary Numerical Results

Numerical results for a small economy transitioning from an initial wage 60% of the large-economy level (columns 2 through 8) and from an initial wage 20% of the large-economy level (columns 9 and 10). Except for labor mobility enumerated in Column 1, all parameters are the same as in Figure 1. Highlighted rows correspond to labor mobility levels in Figures 1 and 2.

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<th>Initial Relative Wealth</th>
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Table 2: Summary Numerical Results for Alternative Levels of Capital Mobility

Numerical results for a small economy transitioning from an initial wage 60% of the large-economy level (columns 2 through 8) and from an initial wage 20% of the large-economy level (columns 9 and 10). Except for capital and labor mobility, all parameters are the same as in Figures 1 and 2. Highlighted rows correspond to labor mobility levels in Figures 1 and 2. Capital mobility in Panel B is the same as that in Figure 2 Panel B.

A. High Capital Mobility (q* = 1.12)

<table>
<thead>
<tr>
<th>Labor Mobility (μ)</th>
<th>Initial Growth Rates</th>
<th>Steady-State Pop</th>
<th>Convergence Speed t=0</th>
<th>Convergence Speed t=∞</th>
<th>Initial Relative Wealth</th>
<th>Convergence Speed t=0</th>
<th>Convergence Speed t=∞</th>
<th>Initial Relative Wealth</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>0.460 0.133</td>
<td>1</td>
<td>0.261 0.166</td>
<td>0.943</td>
<td>0.797</td>
<td>0.900</td>
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</tr>
<tr>
<td>1/32</td>
<td>-0.002 0.458 0.133</td>
<td>0.990</td>
<td>0.261 0.166</td>
<td>0.943</td>
<td>0.794</td>
<td>0.900</td>
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<tr>
<td>1/16</td>
<td>-0.004 0.457 0.133</td>
<td>0.980</td>
<td>0.261 0.167</td>
<td>0.943</td>
<td>0.791</td>
<td>0.900</td>
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<tr>
<td>⅛</td>
<td>-0.007 0.453 0.133</td>
<td>0.960</td>
<td>0.261 0.169</td>
<td>0.944</td>
<td>0.786</td>
<td>0.900</td>
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</tr>
<tr>
<td>¼</td>
<td>-0.014 0.446 0.133</td>
<td>0.923</td>
<td>0.261 0.172</td>
<td>0.944</td>
<td>0.777</td>
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<tr>
<td>½</td>
<td>-0.027 0.434 0.134</td>
<td>0.859</td>
<td>0.262 0.179</td>
<td>0.945</td>
<td>0.759</td>
<td>0.901</td>
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<tr>
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<td>0.265 0.191</td>
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<td>0.901</td>
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<td>0.620</td>
<td>0.272 0.214</td>
<td>0.950</td>
<td>0.674</td>
<td>0.903</td>
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<tr>
<td>4</td>
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<td>0.478</td>
<td>0.293 0.254</td>
<td>0.955</td>
<td>0.608</td>
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<tr>
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<td>0.962</td>
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<tr>
<td>16</td>
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<td>0.295</td>
<td>0.420 0.424</td>
<td>0.970</td>
<td>0.540</td>
<td>0.928</td>
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<tr>
<td>32</td>
<td>-0.726 0.176 0.281</td>
<td>0.257</td>
<td>0.554 0.579</td>
<td>0.977</td>
<td>0.594</td>
<td>0.943</td>
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</tr>
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</table>

B. Low Capital Mobility (q* = 2.96)

<table>
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<tr>
<th>Labor Mobility (μ)</th>
<th>Initial Growth Rates</th>
<th>Steady-State Pop</th>
<th>Convergence Speed t=0</th>
<th>Convergence Speed t=∞</th>
<th>Initial Relative Wealth</th>
<th>Convergence Speed t=0</th>
<th>Convergence Speed t=∞</th>
<th>Initial Relative Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>1</td>
<td>0.082 0.052</td>
<td>0.863</td>
<td>0.256</td>
<td>0.757</td>
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<td></td>
</tr>
<tr>
<td>1/32</td>
<td>-0.004 0.183 0.042</td>
<td>0.925</td>
<td>0.083 0.054</td>
<td>0.865</td>
<td>0.251</td>
<td>0.758</td>
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</tr>
<tr>
<td>1/16</td>
<td>-0.008 0.179 0.043</td>
<td>0.862</td>
<td>0.084 0.056</td>
<td>0.867</td>
<td>0.246</td>
<td>0.758</td>
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</tr>
<tr>
<td>⅛</td>
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<td>0.761</td>
<td>0.085 0.061</td>
<td>0.870</td>
<td>0.237</td>
<td>0.760</td>
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</tr>
<tr>
<td>¼</td>
<td>-0.031 0.163 0.045</td>
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<td>0.089 0.068</td>
<td>0.877</td>
<td>0.223</td>
<td>0.764</td>
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</tr>
<tr>
<td>½</td>
<td>-0.056 0.149 0.048</td>
<td>0.485</td>
<td>0.097 0.082</td>
<td>0.888</td>
<td>0.205</td>
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<td>0.113 0.105</td>
<td>0.904</td>
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<td>0.790</td>
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<tr>
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<td>0.142 0.141</td>
<td>0.922</td>
<td>0.187</td>
<td>0.817</td>
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<tr>
<td>4</td>
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<td>0.258</td>
<td>0.190 0.196</td>
<td>0.940</td>
<td>0.207</td>
<td>0.851</td>
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</tr>
<tr>
<td>8</td>
<td>-0.362 0.090 0.131</td>
<td>0.237</td>
<td>0.261 0.276</td>
<td>0.955</td>
<td>0.255</td>
<td>0.884</td>
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<tr>
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<tr>
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<td>0.221</td>
<td>0.515 0.555</td>
<td>0.976</td>
<td>0.457</td>
<td>0.937</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Transition Dynamics with Labor Mobility

Parameters:

- Capital Share $\alpha = 1/3$
- Rate of Capital Depreciation $\delta = 0.06$
- Real Interest Rate $r = 0.03$
- Steady-State Shadow Value of Capital $q^* = 1.48$

Figure shows dynamics as the small economy transitions from an initial wage 60% of the large-economy level.
Figure 2: Factor Mobility and Convergence Speed

Figure shows dynamics as the small economy transitions from an initial wage 20% of the large-economy level. Except for the "low" capital mobility as indicated in Panel B, all parameters are the same as in Figure 1.

A. Base Capital Mobility ($q^* = 1.48$)

B. Low Capital Mobility ($q^* = 2.96$)