NEXUS, THROWBACKS, AND THE WEIGHTING GAME

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Abstract

This paper modifies a model proposed by Anand and Sansing (2000) to explain why states have chosen different formulas for corporate income apportionment. I demonstrate that nexus assumptions and allocation rules can have significant effects on the outcomes of the model, and are important considerations in analyzing the impetus for and effects of apportionment competition.

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1. Introduction

There is an old saying which suggests that for any problem one may encounter or attempt to solve, the “devil is in the details.” That is, while we may be inclined to focus our policy efforts broadly on the issues, often it is how we approach the smaller details and technical nuances that determines our probability of success. Perhaps nowhere is this more the case than in state corporate income tax apportionment policies.

In a 2000 article, Bharat Anand and Richard Sansing set out to explain why states have chosen different formulas for corporate income apportionment. Their primary explanation is that the choice of formula reflects the desire of states to tax immobile capital. Specifically, they assert that “[natural resource importing states] will tend to tax firms that do not produce – but simply sell – in [the] state” (193), while “natural resource exporting states will tend to tax firms selling out of the state” (194). They also find that aggregate welfare is maximized when states choose a uniform formula, regardless of what that formula is, and that this coordinated solution is not sustainable in equilibrium.

Anand and Sansing’s results are derived from a two-jurisdiction (“states” \(x\) and \(y\)) equilibrium model of location choice which consists of three types of firms. One firm produces and sells exclusively in state \(x\), another produces and sells in state \(y\), and a third multijurisdictional firm produces in \(x\) and sells in \(y\). Further, the statutory corporate income tax rate \((\tau)\) in each state is the same. Clearly the two entirely local firms face a uniform tax rate that is invariant to the apportionment formula, as noted by Anand and Sansing. What is less clear, however, is the tax rate faced by the multijurisdictional firm. Anand and Sansing assume that “a multijurisdictional firm has nexus in both states, so its tax liability depends on the apportionment formula used by state \(x\) and state \(y\)” (186).

In this paper, I present a modification and extension of Anand and Sansing’s model where the multijurisdictional firm does not have taxable nexus in state \(y\) and argue that this is a
more likely state of affairs. Results from this modified model differ from Anand and Sansing’s results in important ways. First, under plausible circumstances, states acting in their own interests may in fact reach a Nash equilibrium that maximizes aggregate welfare (the coordinated solution). Additionally, results from the modified model demonstrate that (1) uniform apportionment formulas are not generally necessary to maximize aggregate welfare, and (2) rather than increasing their sales factors to 100 percent, under my nexus assumptions, “importing states” are expected to reduce their sales factor weights to zero. Results from the modified model are consistent with Anand and Sansing’s results in that “exporting states” generally have an incentive to lower the sales factor weight, and for own-demand sufficiently low (relative to importing states), will reduce their sales factor weights to zero.

The modified model demonstrates that “throw-back” provisions also are an important consideration in analyzing the impetus for and effects of apportionment competition. Throw-backs allocate sales that are not taxable in any other state, perhaps due to protections provided by federal law, as discussed below, to the state from which the product was shipped. If a throw-back provision is in place in either state (or both), aggregate welfare is independent of apportionment formulae, and therefore any equilibrium is consistent with aggregate welfare maximization. If only one state has a throw-back provision, welfare in that state is independent of its apportionment formula, but state welfare in the no-throw-back state will be strictly increasing in its apportionment formula. Finally, with throwbacks in both states, the model reduces to the case of an origin-based production tax in each state, and apportionment formulae become irrelevant to production decisions, revenue collections, and welfare.

The paper proceeds as follows. In section 2 I discuss Public Law 86-272 [P. L. No. 86-272 (1959)], which plays a large role in governing nexus for state corporate income tax purposes and serves as the basis for my argument that Anand and Sansing’s multijurisdictional firm is not likely to have nexus in the “importing” state. I then present the modified model and discuss the results in section 3, followed by concluding remarks in section 4.
2. **Nexus Rules for State Corporate Income Taxation**

While nexus rules are for the most part left up to individual states, federal law [P. L. No. 86-272 (1959)] has established that a state may not tax the income of a corporation whose only business activity within the state consists of soliciting orders for sales of tangible personal property that are filled and shipped from outside the state.\(^1\) That is, *the existence of sales in a state is insufficient activity to create nexus in that state for income tax purposes.* Activities that are deemed “ancillary” to the solicitation of orders are also insufficient to create nexus.\(^2\) Given P.L. 86-272, Anand and Sansing’s multijurisdictional firm would not appear to have taxable nexus in state \(y\).\(^3\)

One might reasonably argue that the propositions of Anand and Sansing are consistent with a multijurisdictional firm that has a very small amount of productive activity and all of its sales in state \(y\). However, I would argue that in many cases, the firm would then have a strong incentive to shift all of its productive activity to \(x\) so as to avoid the establishment of taxable nexus in \(y\). The exception is the case in which state \(x\) has a throw-back provision in its tax code and a higher (effective) tax rate than \(y\), which I describe below. Nevertheless, many states do not have throwback provisions (some of which only recently abandoned them), which renders this scenario less likely, and certainly not the rule.\(^4\) For all of these reasons, I believe that the

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\(^1\) The impetus for this law was a Supreme Court decision that deemed an Iowa company to have nexus in Minnesota because it maintained a sales office there, although orders were approved and filled in Iowa [*Northwestern States Portland Cement Co. v. Minnesota* (358 U.S. 450 (1959))].

\(^2\) The Multistate Tax Commission, in their “Statement of Information Concerning Practices of Multistate Tax Commission and Signatory States Under Public Law 86-272,” has delineated those activities that are not considered the solicitation of orders or ancillary thereto and thus likely to cause a corporation to lose its protection under P.L. 86-272.

\(^3\) It is, however, possible for the multistate firm to have nexus in \(y\) without payroll or property factors in that state. For example, the South Carolina Supreme Court ruled that the exploitation of intangible property within the state by Geoffrey, Inc. was sufficient to create income tax nexus, although the company had no representatives or tangible property in South Carolina [*Geoffrey, Inc. v. South Carolina* (313 S.C. 15, 437 S.E.2d 13 (1993))]. Moreover, activities of a firm’s in-state “representatives,” if not protected under P.L. 86-272, may be sufficient to create nexus even if they are not employees of the firm [*Tyler Pipe v. Washington* (483 U.S. 232 (1987))].

\(^4\) Currently 21 of the 46 states that impose a corporate income tax do not have a throw-back provision in their tax code (Commerce Clearing House, CCH Internet Research Network, http://business.cch.com). States that have most recently abandoned throwback provisions (Smith, 2000) include Michigan (effective 1997), Nebraska (effective 1997), and Arizona (effective 1998). This provision is discussed in some detail below.
analysis is more realistic when the multijurisdictional firm is assumed not to have taxable nexus in $y$.

Before turning to the specific implications of P.L. 86-272 for Anand and Sansing’s model and findings, it is helpful to first discuss the implications of this legislation for apportionment policy more generally.

The implications of P.L. 86-272 for state corporate income apportionment policy are both broad and significant. The most obvious of these are on the revenue side. As noted by Anand and Sansing, whether a firm’s overall tax liability increases or decreases with a heavier sales factor weight depends on whether it is production- or sales-intensive in the state. If it is production-intensive, its tax liability declines, and the acting state loses revenue. Of course, there will be sales-intensive firms in the state as well, and one might expect that the effects would more or less cancel out, rendering an overall impact that is close to revenue-neutral, especially in a static sense.\(^5\) The problem is that some of the sales-intensive firms in the state are likely to be protected under P.L. 86-272. To the extent that this is the case, the state will suffer tax revenue losses from production-intensive firms without offsetting gains from sales-intensive firms. This result is best demonstrated by considering the extreme case where firms are either entirely production-oriented (no sales in the state) or entirely market-oriented (no production), in which case a single-factor sales formula would drive corporate tax revenues to zero if market firms do not have nexus in the state.

On the surface, the protection offered to corporations under P.L. No. 86-272 would seem to matter little for the economic development consequences of formula manipulation, but in reality it can matter a great deal. The clincher is the existence of throw-back provisions. To the extent that a firm markets its products to states in which it is protected under P.L. No. 86-272 (or

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\(^5\) Static revenue neutrality will not, however, generally be the case. Ignoring the revenue implications of P.L. 86-272, discussed below, the static revenue effect would be positive for sales-intensive states and negative for production-intensive states, which Anand and Sansing deemed “importing” and “exporting” states, respectively. This change in revenues is an immediate effect that does not arise from changes in firm behavior. In that sense it is static. A dynamic revenue effect arises when firms alter their sales and production decisions in response to heavier sales factor weights.
to the federal government), throw-back provisions in the state of production transform the sales tax component of the formula-apportioned corporate income tax into a production tax, as generally occurs when sales are sitused at origin.\(^6\) By increasing the sales factor weight, the state of production runs the risk of discouraging increased capital and labor utilization there.

With greater sales factor weights in the production state, the firm now has the unusual incentive to locate payroll and property in the market state so as to establish taxable nexus there. To illustrate, consider a simple example where a firm locates all of its productive activity in state A and splits its sales between states A and B. Suppose further that A and B initially have uniform tax rates and equally weighted three-factor apportionment formulas, and that state A has a throw-back provision in its corporate tax code. The firm does not have nexus in state B, and hence 100 percent of its income is taxable in state A. Now suppose that state A imposes a single factor sales formula and state B retains its equally weighted formula. By establishing a modicum of property and payroll in state B, the firm’s state-B sales, which are 50 percent of its total, will be taxed at a much lower rate, thereby reducing the firm’s overall tax liability. Of course, the development consequences may be minimal, as the firm only needs to establish a minimum physical presence in state B, and state-A retains a tax advantage for production under its single-factor sales formula scheme.\(^7\) The revenue consequences, however, are certain to be severe. The 50 percent of the firm’s sales that were once allocated to state A are now allocated to state B.

Recognizing these disincentives, some states recently have abandoned throw-back provisions in the tax code,\(^8\) which allows a portion of firm sales to go untaxed anywhere in the country. Continuing with the example immediately above, suppose that state A drops its throw-back provision. From a development perspective, the state benefits not only from eliminating the production tax effect on the firm’s sales in state B, but also from directly making state B less

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\(^{6}\) See Edmiston (2002) for a discussion of situsing rules and their relevance for apportionment policy.

\(^{7}\) If investment is lumpy, however, the development consequences could be significant.

\(^{8}\) See note 3.
attractive. Without a throw-back provision in state A, the firm’s income tax liability is reduced significantly because only 50 percent of its income is apportioned for tax purposes (recall there is a single-factor sales formula in A). Were the firm to locate productive activity sufficient to create a physical presence in state B, it would then suffer a 33 percent increase in its tax liability.\(^9\) The disincentive for establishing a physical presence in state B is exacerbated if B imposes a single factor sales formula, in which case the firm would see 100 percent of its income apportioned, and an associated 100 percent increase in its tax liability.


In an effort to further explore the implications of P.L. 86-272 for state apportionment policy, I modify Anand and Sansing’s model to account for the existence of this legislation. In particular, I derive results from Anand and Sansing’s model under the assumption that the multijurisdictional firm does not have nexus in state \(y\).

3.1 The Anand and Sansing Model

The model is a two-jurisdiction (“states” \(x\) and \(y\)) model with a single tradable good, where production can occur in either state. One unit of capital (the sole depreciable input) is required to produce one unit of output, and the price of capital is given by

\[ C(q_i) = \alpha q_i \text{[AS.1]}, \]

where \(q_i\) is aggregate production in state \(i\) (Anand and Sansing’s equation [1], denoted [AS.1]) and \(\alpha\) is a constant. The cost of capital therefore is \(rq_yC(q_i)\), where \(r\) is the return to equity capital, and economic profits for any firm \(i\) in the absence of taxes are given by

\[ \pi_i = p_iq_i + p_jq_{ij} - rq_yC(q_i), \]

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\(^9\) It is assumed here that the amount of activity shifted to state B is sufficiently small to leave the property and payroll factors approximately zero in that state. Thus \(\phi_y \approx (1/3)(0.50) + (1/3)(0) + (1/3)(0) \approx 0.167\), which implies that 2/3 rather than 1/2 of the firm’s income will be apportioned for tax purposes. Recalling that states A and B are assumed to have uniform statutory tax rates, this represents a 33.3 percent increase in the firm’s overall tax liability.
where production in state $i$ that is sold in state $j$ is denoted $q_{ij}$. Demand ($\delta$) is perfectly inelastic, but differs across states, and it is assumed, without loss of generality, that demand in state $y$ exceeds demand in state $x$: $\delta_y > \delta_x$. Under Anand and Sansing’s assumption that firms do not produce and/or sell in multiple states, and ruling out mergers, four types of firms are possible: (1) $xx$ firms which produce in $x$ and sell in $x$, (2) $xy$ firms which produce in $x$ and sell in $y$, (3) $yx$ firms which produce in $y$ and sell in $x$, and (4) $yy$ firms which produce in $y$ and sell in $y$.

**Taxes.** Formula-apportioned corporate income taxes are levied at a rate $\tau$ on normal profits. The apportionment of income to any state $i$ is based on a weight of $\omega_x$ applied to productive factors (capital employed in $i$) and a weight of $1 - \omega_x$ applied to sales.

**Equilibrium.** Anand and Sansing demonstrate that under the condition of their model (multijurisdictional firms have nexus in both states), only three types of firms exist: $xx$, $yy$, and $xy$ because there is no incentive for a firm to produce in $y$ and sell in $x$. They are thus left with a competitive equilibrium characterized by

\[
\begin{align*}
[3] & \quad p_x(1 - \tau) = rC(q_x) = r\alpha(q_{xx} + q_{xy}) \quad \text{[AS.2]} \\
[4] & \quad p_y(1 - \tau) = rC(q_y) = r\alpha q_{yy} \quad \text{[AS.3]} \\
[5] & \quad p_y(1 - \omega_x - \tau(1 - \omega_y)) = rC(q_y) = r\alpha(q_{xx} + q_{xy}) \quad \text{[AS.4]} \\
[6] & \quad q_{xy} + q_{yy} = \delta_y \quad \text{[AS.5]} \\
\end{align*}
\]

and

\[
\begin{align*}
[7] & \quad q_{xx} = \delta_x \quad \text{[AS.6]},
\end{align*}
\]

the solution to which is given by

\[
\begin{align*}
[8] & \quad p_x = \frac{\alpha(\delta_x + q_{xy})r}{1 - \tau} \quad \text{[AS.7]} \\
[9] & \quad p_y = \frac{\alpha(\delta_y + q_{xy})r}{1 - \tau} \quad \text{[AS.8]}
\end{align*}
\]
3.2 The Anand & Sansing Scenario Under P.L. 86-272

First consider the implications for the model equations ([3] – [7]) and the competitive equilibrium (equations [8] – [10]). Of course, the existence of P.L. 86-272 has no bearing on equations [3] and [4] because it is irrelevant to entirely local firms. The multijurisdictional firm, however, receives protection under P.L. 86-272 because it does not have taxable nexus in state $y$. This implies that equation [5] becomes:

$$p_x (1 - \tau \omega_x) = rC(q_x) = \omega r (q_{xx} + q_{yy})$$

The equilibrium prices (equations [8] and [9]) are derived from [3] and [4] and hence remain unchanged. The equilibrium output for the multijurisdictional firm [10] under my assumption is given by

$$q_{xy} = \frac{(\delta_x + \delta_y)(1 - \tau) + \tau \delta_y (\omega_x - \omega_y)}{(2 - 2\tau + \tau(\omega_y - \omega_x))}$$

Anand and Sansing’s no-entry condition (Remark 1) must then be reevaluated, which is satisfied when (under my assumptions):

$$\pi_{yx} = p_y (1 - \tau \omega_y) - r \alpha q_{xy} \leq 0$$

Substituting for $p_x$ and $q_{xy}$ (where $q_{yy} = \delta_y - q_{xy}$) yields:

$$\left[\frac{\tau r (\tau \omega_x \omega_y - \omega_y + 2 - \tau - \omega_x)}{(1 - \tau)(2 - \tau \omega_x)}\right] (\delta_x + \delta_y) \leq 0$$

which is satisfied only in the case where $\omega_x = \omega_y = 1$. Thus, in general, the $yx$ firm will have an incentive to produce and the no-entry condition is violated. How, then, might the equilibrium be characterized under P.L. 86-272? Intuition suggests that, given an upward-sloping supply curve and $\delta_y > \delta_x$, equilibrium would be characterized by two multijurisdictional firms, $xy$ and $yx$, and a single local firm $yy$. 

Consider first firms producing for state $x$ consumption. Local firms face a tax rate of $\tau$, while firms located in $y$ and selling in $x$ face a tax rate $\tau \omega_y \leq \tau$. In a competitive environment, the firms may co-exist only when $\omega_y = 1$. In this case, the distribution of production across local and multijurisdictional firms is irrelevant because firms are indifferent to producing locally or out-of-state for the in-state market.\(^\text{10}\) In the case where $\omega_y < 1$, the local firm $(xx)$ will be driven out and demand in $x$ will be met by production of the multijurisdictional firm $yx$ exclusively ($q_{yx} = \delta_x$). The same is not necessarily the case for state $y$, however. Given the upward-sloping supply curve and the case where $\omega_x < 1$, we should expect that the $xy$ firm would produce $\delta_y$, and the production of any remaining demand ($\delta_y - \delta_x$) would be shared between the $xy$ firm and the $yy$ firm. Specifically, production beyond $\delta_x$ will be undertaken by the $xy$ firm until the additional capital costs outweigh the tax advantages, at which point a share of any remaining production will be undertaken by the $yy$ firm. To illustrate more formally, I characterize an equilibrium in light of P.L. 86-272 where three firms exist: two multistate firms $xy$ and $yx$, and a single local firm $yy$.\(^\text{11}\)

3.3 Equilibrium Analysis Under P.L. 86-272

Given two multistate firms $xy$ and $yx$, and a single local firm $yy$, the model equations are given by:

\[ p_x(1 - \tau \omega_y) - r\alpha(q_{yx} + q_{yy}) = 0 \]  
\[ p_y(1 - \tau \omega_x) - r\alpha(q_{xx} + q_{xy}) = 0 \]

\(^\text{10}\) The only restriction is that total production must be balanced across the two states in equilibrium. Given the upward sloping supply function, a production decision by one of the four firms will pre-determine the production levels of the other firms. Suppose, for example, that $\delta_y \geq (1/2)\delta_x$ and the local firm in $x$ decides to produce at a level $q_{xx} = \delta_x / 4$. The remainder of the demand in $x$ will be met by the $yx$ firm, which implies that $q_{yx} = (3/4)\delta_x$. It follows that the $xy$ firm will produce $q_{yx} = q_{xx} + (\delta_y - \delta_x)/2 = \delta_x / 4 + \delta_y / 2$ and the local firm $yy$ will produce $q_{yy} = \delta_y / 2 - \delta_x / 4$. The restriction $\delta_y \geq (1/2)\delta_x$ is specific to this example; else the local firm in $x$ will produce more than one-quarter of the state $x$ demand.

\(^\text{11}\) An equilibrium with positive production by four firms $(xx, xy, yx, yy)$ exists only when $\omega_x = \omega_y = 1$. Otherwise, production of the local firm in the low-demand state will always be driven to zero. For the sake of brevity, a formal argument is not made here; however, one can simply look at the analysis below in the case where $\delta_x > \delta_y$ and see that $xx$ will survive and $yy$ will exit.
Substituting [18] – [20] into [15] – [17] leaves three equations in three unknowns \((p_x, p_y, \text{ and } q_{xy})\), the solution to which is given by:

\[
\begin{align*}
\begin{split}
p_x = \frac{\alpha r(\delta_x + \delta_y)}{2 - \tau - \tau \omega_x}
\end{split} \\
p_y = \frac{\alpha r(\delta_x + \delta_y)(1 - \tau)}{(2 - \tau - \tau \omega_x)(1 - \tau \omega_y)} = p_y \left( \frac{1 - \tau}{1 - \tau \omega_y} \right)
\end{align*}
\]

\[
q_{xy} = \left[ \frac{1 - \tau \omega_x}{2 - \tau - \tau \omega_x} \right] (\delta_x + \delta_y)
\]

This equilibrium is properly characterized if a local state-\(x\) firm has no incentive to produce, which requires that

\[
p_x (1 - \tau) - r \alpha q_{xy} \leq 0,
\]

which, substituting [22] and [23], gives:

\[
r \alpha \tau (\delta_x + \delta_y) \left[ \frac{2 - \tau - \omega_x - \omega_y + \tau \omega_x \omega_y}{(2 - \tau - \tau \omega_x)(1 - \tau \omega_y)} \right] \leq 0
\]

If \(\omega_x = \omega_y = 1\), [15] holds with equality, but is strictly negative for all other cases, which means the \(xx\) firm will not enter.

**Proposition 1'**: The coordinated solution is given by \(\omega^*_x = 1\) and is invariant to \(\omega_y\).

In the modified equilibrium, the aggregate social welfare function \((W_T)\) is given by:\(^{12}\)

\(^{12}\) Equation [26] corresponds to Anand and Sansing’s equation 17 [AS.17].
where \((A_i - p_i)\delta_i\) is consumer surplus in state \(i\) \((A_i\) is the reservation price for buyers in \(i\)), producer surplus in state \(i\) is

\[
W^p_i = \int_0^{q_i} (C(q_i) - C(u))du = \frac{r \alpha q_i^2}{2},
\]

and combined tax revenues in states \(x\) and \(y\) are given by

\[
\tau \left[ p_x q_{xy} + p_y \omega_x q_{xy} + p_y \omega_y \delta_x \right]
\]

Incorporating [21] – [23] and differentiating with respect to \(\omega_x\) yields the first-order condition for \(\omega_x\):

\[
\frac{\partial W^T}{\partial \omega_x} = \frac{\tau^2 r \alpha (\delta_x + \delta_y)^2 (1 - \tau - \omega_x + \tau \omega_y)}{(2 - \tau \omega_x - \tau)^3} = 0,
\]

the solution to which is \(\omega_x^* = 1\). Given that \(\frac{\partial W^T}{\partial \omega_x}\) is strictly increasing over the range \(\omega_x \in [0,1]\), I know this solution is a maximum. Because \(\frac{\partial W^T}{\partial \omega_y} = 0\) always, aggregate welfare is invariant to changes in \(\omega_y\). The idea that aggregate welfare is invariant to \(\omega_y\) is puzzling on the surface; however, if aggregate welfare is broken down into its component parts, it can be seen that

\[
\frac{\partial W^x}{\partial \omega_y} = \frac{-\tau (1 - \tau) r \alpha \delta_x (\delta_x + \delta_y)}{(2 - \tau - \tau \omega_y)(1 - \tau \omega_y)} = -\frac{\partial W^y}{\partial \omega_y}
\]

That is, any increase in \(\omega_y\) decreases consumer surplus in state \(x\) through increases in consumer prices in that state. However, this increase in \(\omega_y\) increases tax collections in state \(y\) by the same magnitude, representing a simple transfer from state \(x\) residents to state \(y\) residents and leaving aggregate welfare unchanged.
**Proposition 2':** Producer surplus in state \( x \) unambiguously increases with a decrease in the production factor \( \omega_x \), but producer surplus in state \( y \) is invariant to its production factor \( \omega_y \).

The term \( \omega_y \) does not appear in the formulas for producer surplus in either state, and hence producer surplus is neither state will change with \( \omega_y \). As \( \omega_x \) is altered the change in producer surplus in state \( x \) \( (W_P^x) \) is given by:

\[
\frac{\partial W_P^x}{\partial \omega_x} = -\alpha r \tau (\delta_x + \delta_y) \frac{(1-\tau)(1-\tau \omega_x)}{(2-\tau-\tau \omega_x)^3},
\]

which is strictly negative for \( 0 \leq \omega_x \leq 1 \) and \( \tau < 1 \).

**Proposition 3':** Welfare in state \( y \) is strictly increasing in \( \omega_y \), but the relationship between state-\( x \) welfare and \( \omega_x \) is ambiguous with respect to sign.

The change in state-\( y \) welfare (for any \( \omega_y \)) arising from a change in its apportionment formula is given by:

\[
\frac{\partial W^y}{\partial \omega_y} = \frac{\tau(1-\tau)r\alpha \delta_x (\delta_x + \delta_y)}{(2-\tau-\tau \omega_x)(1-\tau \omega_y)^2},
\]

which is strictly positive for \( \tau < 1 \), and thus the welfare-maximizing apportionment formula in state \( y \) is given by \( \omega_y^* = 1 \). As demonstrated in [30], changes in the state-\( y \) apportionment formula affects only tax collections, and does so positively. By increasing the weight on productive factors in their apportionment formula, state-\( y \) collects additional revenues while shifting the burden of those revenues to consumers in state \( x \).

The change in state-\( x \) welfare arising from a change in its apportionment formula is ambiguous in sign (see appendix). However, by [32], \( \omega_y^* = 1 \). Substituting into \( \frac{\partial W^y}{\partial \omega_x} = 0 \) (see appendix) and solving for \( \omega_x \) yields the welfare-maximizing value of \( \omega_x \) for state \( x \):
Hence,

**Proposition 4**: the Nash equilibrium is given by:

\[
\omega^*_x = \min \left( \max \left( 0, \frac{-(1-\tau)\delta_x - \delta_y}{\tau \left( 2-\tau \right) \delta_y + (1-\tau)\delta_x} \right), 1 \right)
\]

\[
\omega^*_y = \min \left( 1, \frac{-(1-\tau)\delta_x - \delta_y}{\tau \left( 2-\tau \right) \delta_y + (1-\tau)\delta_x} \right)
\]

For \( \delta_y > \delta_x \) and \( 1/(3-2\tau) < \omega^*_x \leq 1 \), \( \omega^*_x \) is strictly increasing in the tax rate \( \tau \). Further, for \( \delta_y > \delta_x [((1+\tau)/(1-\tau))] \), \( \omega^*_x = 1 \). Currently the maximum statutory corporate tax rate is approximately 10 percent across states.\(^{13}\) At this rate, given \( \delta_y > \delta_x \), the state \( x \) weight on productive factors will range between 35.7 percent and 100 percent. Moreover, the Nash equilibrium is \( \omega^*_x = \omega^*_y = 1 \) if state \( x \) demand is less than 82 percent of state \( y \) demand, which is consistent with the coordinated solution. In this case equilibrium is consistent with production by all four firm types, and firms are indifferent to location.

### 3.4 The Case of Throw-Back Provisions

The modified model in Section 3.3 assumes that neither state has a throw-back provision in its tax code. Given the implications of throwback provisions (or lack thereof) for economic development and corporate tax collections, I next consider the Anand & Sansing model when (1) both states have throw-back provisions, and (2) only one state has a throw-back provision.

#### 3.4.1 Both States Have Throwback Provisions

If both states have throwback provisions in their tax code, apportionment factor weights will be entirely irrelevant, and all firms will face a uniform tax rate of \( \tau \). There are no tax advantages for producing and/or selling in one state *versus* another, and equilibrium could be characterized by any combination of 2, 3, or 4 firms, so long as production is equally divided.

\(^{13}\) Although Iowa has a maximum tax rate of 12 percent and North Dakota has a maximum rate of 10.5 percent, federal taxes are deductible (50 percent and 100 percent, respectively) in both states. All other states have maximum rates below 10 percent (Federation of Tax Administrators, http://www.fta.org/).
between the two states, thereby minimizing production cost. Specifically, equilibrium will be characterized by

\[ q_{xx} + q_{xy} = q_{yx} + q_{yy} = \frac{\delta_x + \delta_y}{2} \]

\[ p_x = \frac{\alpha r (\delta_x + \delta_y)}{2(1 - \tau)} = p_y \]

3.4.2 One State Has a Throwback Provision

The more interesting case is when only one state has a throwback provision. I show here the case where only state \( x \) has a throwback provision, but note that because each result is independent of the relative magnitudes of \( \delta_x \) and \( \delta_y \), identical results hold if state \( y \) has a throwback provision and state \( x \) does not. One simply has to swap \( x \) and \( y \) in the results.

In the case of a throwback provision in state \( x \) only, equilibrium is characterized by the following:

\[ p_x (1 - \tau \omega_y) - r \alpha (q_{yx} + q_{yy}) = 0 \]

\[ p_y (1 - \tau) - r \alpha (q_{xx} + q_{xy}) = 0 \]

\[ p_y (1 - \tau) - r \alpha (q_{xx} + q_{yy}) = 0 \]

\[ q_{xx} = 0 \]

\[ q_{yx} = \delta_x \]

\[ q_{yy} = \delta_y - q_{xy} \]

Substituting [40] – [42] into [37] – [39] leaves three equations in three unknowns (\( p_x \), \( p_y \), and \( q_{xy} \)), the solution to which is given by:

\[ p_y = \frac{\alpha r (\delta_x + \delta_y)}{2(1 - \tau)} \]

\[ p_x = \frac{\alpha r (\delta_x + \delta_y)}{2(1 - \tau \omega_y)} = p_y \left( \frac{1 - \tau}{1 - \tau \omega_y} \right) \]
\[ q_{xy} = \frac{1}{2} (\delta_x + \delta_y) \]

The no entry condition is the same as that in [24]. Making the appropriate substitutions from [44] and [45], gives:

\[ -r \alpha \tau (\delta_x + \delta_y) \left[ \frac{(1 - \omega_y)}{2(1 - \tau \omega_y)} \right] \leq 0 \]

which is satisfied for \( \tau \leq 1, \omega_y < 1 \).

The coordinated solution in this case is invariant to \( \omega_y \) (Proposition 1). Any increase in \( \omega_y \) reduces consumer surplus in state \( x \) \([ (A^x - p_x)\delta_x ]\), and therefore welfare in \( x \), by an amount \( p_x \delta_x \), and increases corporate tax collections in state \( y \), and therefore welfare in state \( y \), by the same amount.

Unless \( \omega_y = 1 \), the entire amount of production for state \( x \) consumption will be produced in state \( y \) by its multistate firm; thus, a change in \( \omega_y \) (as long as \( \omega_y < 1 \)) will not lead to changes in state \( y \) production levels, and hence, will not lead to changes in producer surplus in \( y \). Thus, producer surplus in both states are independent of apportionment formulas (Proposition 2).

Given that \( p_y \) is not a function of \( \omega_y \), consumer surplus is invariant to \( \omega_y \). Having noted that tax collections are increasing in \( \omega_y \), it must be the case that \( \partial W^y / \partial \omega_y > 0 \) (Proposition 3), and the Nash equilibrium is given by \( \omega_y^* = 1 \) and any value of \( \omega_x \) (Proposition 4). Of course, the no-entry condition ([34]) will hold with equality in this equilibrium, which means, in the end, that all firms will be indifferent to location and the existence of all four types of firms is consistent with equilibrium.

4. Conclusion

The results from the modified model suggest that P.L. 86-272 is an important consideration in analyzing the impetus for and effects of apportionment competition. First, Anand and Sansing demonstrate that importing states will tend to lower their weight on
productive factors (to zero actually) in an effort to tax firms that “do not produce – but simply sell – in that state” (193). I would argue that in many (if not most) cases, importing states cannot tax those firms. Because only the entirely local firm has taxable nexus in the state, Anand and Sansing’s state $y$ should be indifferent to its formula, which I show to be the case. Secondly, Anand and Sansing demonstrate that (for $\omega_y$, “low enough”) state $x$ has an incentive to increase $\omega_x$ away from the coordinated solution. I show in the modified model (with no throwbacks) that while the state $x$ incentive is ambiguous, if $\delta_y$ is sufficiently greater than $\delta_x$ (at least 22 percent larger with statutory tax rates at 10 percent or lower), $\omega^*_x = 1$. While this solution for state $x$ agrees with the Anand and Sansing result, it is in fact consistent with the coordinated solution in the modified case. Finally, given the irrelevance of $1^* = \omega_y$ in aggregate welfare, results from the modified model suggest that when some sales go untaxed, uniform apportionment formulas are not required for a welfare maximum.

The modified model demonstrates that throw-back provisions also are an important consideration in analyzing the impetus for and effects of apportionment competition. If a throw-back provision is in place in either state (or both), aggregate welfare is independent of apportionment formulae, and therefore any equilibrium is consistent with aggregate welfare maximization. If only one state has a throw-back provision, welfare in that state is independent of its apportionment formula, but state welfare in the no-throw-back state will be strictly increasing in its apportionment formula. Finally, with throwbacks in both states, the model reduces to the case of an origin-based production tax in each state, and apportionment formulae have no meaning or importance whatsoever. Table 1 provides summary results from the Anand and Sansing model and each extension.

Of course, it should be noted that both Anand and Sansing’s results and mine are restricted to cases where multijurisdictional firms produce in one state and sell in another. Formula-apportioned corporate income taxes are extremely difficult to model in an analytical setting because of their complicated structure, and additional complexities in the model would require a numerical approach (see Edmiston, 2002). Moreover, it should be kept in mind that
apportionment decisions are made in a political environment. A reasonable argument could be made that many apportionment policies are formulated by simply bowing to political pressure. That is, in many states single corporations or industries have provided much of the impetus for changing the apportionment formula by threatening to reduce or eliminate their presence in the state. Examples include Raytheon in Massachusetts, Eastman Chemical in Tennessee, and Intel in New Mexico. Further, Pomp (1999) suggests that “it is widely accepted that Maine moved to a double-weighting at the request of the paper industry, Georgia adopted it for its automobile industry, Florida shifted in response to its military contractors, and West Virginia was responding to the request of its mining industry” (942). Anand and Sansing’s empirical results may be picking up this very result, as immobile firms cannot offer a credible threat to leave, nor can they be encouraged to enter. This phenomenon may also explain the use of either optional formulas or industry-specific formulas in some natural resource exporting states, including Kansas, Mississippi, Louisiana, Colorado, and New Mexico.
### Table 1
Summary Results
Anand & Sansing (2000) and Extensions

<table>
<thead>
<tr>
<th>Throw-back Provisions</th>
<th>Coordinated Solution ((W^*_{\text{max}}))</th>
<th>(\partial W^*/\partial \omega_x)</th>
<th>(\partial W^*/\partial \omega_y)</th>
<th>(\partial W^*/\partial \omega_x)</th>
<th>(\partial W^*/\partial \omega_y)</th>
<th>Nash Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>N/A</td>
<td>(\omega_x = \omega_y = 1)</td>
<td>(+)</td>
<td>(+)</td>
<td>(+)</td>
<td>(-)</td>
<td>(\omega_y = 0);(\omega_x \in (0,1])</td>
</tr>
<tr>
<td></td>
<td>Multistate firms have nexus in the market state (Anand &amp; Sansing)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No throw-backs</td>
<td>(\omega_x = 1)</td>
<td>(+)</td>
<td>(= 0)</td>
<td>(ambiguous)</td>
<td>(+)</td>
<td>(\omega_y = 1);(\omega_x \in (0,1])</td>
</tr>
<tr>
<td>Throw-back in x</td>
<td>invariant to (\omega_x) and (\omega_y)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(+)</td>
<td>(\omega_y = 1)</td>
</tr>
<tr>
<td>Throw-back in y</td>
<td>invariant to (\omega_x) and (\omega_y)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(+)</td>
<td>(= 0)</td>
<td>(\omega_x = 1)</td>
</tr>
<tr>
<td>Throw-backs in x and y</td>
<td>invariant to (\omega_x) and (\omega_y)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>N/A</td>
</tr>
</tbody>
</table>
APPENDIX


Given $\omega_x^* = 1$, the first-order condition for maximizing state $x$ welfare with respect to the weight on productive factors in its apportionment formula is given by:

$$\frac{\partial W^x}{\partial \omega_x} \bigg|_{\omega_x = 1} = -r\alpha \tau (1-\tau)(\delta_x + \delta_y)\delta_x - \frac{r\alpha \tau (1-\tau \omega_x)^2(\delta_x + \delta_y)^2}{(2-\tau - \tau \omega_x)^2(1-\tau \omega_x)^3} + \frac{2r\alpha \tau^2 \omega_x(1-\tau \omega_x)(\delta_x + \delta_y)^2}{(2-\tau - \tau \omega_x)^3}$$

$$= \frac{r\alpha \tau^2 \omega_x(\delta_x + \delta_y)^2}{(2-\tau - \tau \omega_x)^2} = 0$$

Solving for $\omega_x \in [0,1]$ yields:

$$\omega_x^* = \min \left\{ \frac{-[(1-\tau)\delta_x - \delta_y]}{\tau[(2-\tau)\delta_y + (1-\tau)\delta_x]} , 1 \right\}$$

Of course, $\omega_x^* = 1$ for $\frac{-[(1-\tau)\delta_x - \delta_y]}{\tau[(2-\tau)\delta_y + (1-\tau)\delta_x]} > 1$, which is true for $\delta_y > \delta_x[(1+\tau)/(1-\tau)]$. 
REFERENCES


