Financing Modes and Lender Monitoring

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Financing modes and lender monitoring

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Abstract

Bank and nonbank modes of financing can emerge endogenously in a simple borrower-lender framework without regulatory arbitrage or policy interventions. Lenders gravitate towards insured financing (bank deposits) for low-liquidation value projects but tend to choose uninsured deposits in funding projects of relatively higher quality (higher liquidation value). The co-existence of banks and shadow banks in the absence of regulatory intervention speaks to their importance as alternative modes of financial intermediation. It explores the scope and limits of regulation in determining the size and location of shadow banking as opposed to designing regulation aimed at “choking off shadow bank activities.”

*JEL codes: C78, D82, G11.

Key words: Liquidation value; lender monitoring; deposit insurance. shadow banking.

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1 Introduction

Designing financial regulation in response to financial crises is not without its challenges. Enhanced capital and liquidity regulation within the regulated banking system can mitigate risk-taking by banks (Admati et al., 2013; Carlson et al., 2015). However, increased bank regulation has often been accompanied with regulatory arbitrage and the migration of risks to nonbanks or the unregulated shadow banking system. Unlike regulated banks, nonbanks do not have access to central bank liquidity facilities or issue insured liabilities. The lack of these facilities makes nonbanks ill-equipped to take on additional risk away from the regulated banking system. This raises important questions about how economies should design policy and regulation so that risk-taking is optimally distributed between banks and nonbanks (Rajan, 2006; Allen, Goldstein, and Jagtiani, 2018).

This paper constructs a generalized theoretical framework that shows how bank and shadow bank modes of financing can emerge endogenously in a simple borrower-lender framework without regulatory arbitrage or policy interventions. It is widely believed that shadow banks are a creation of financial regulation and regulatory arbitrage (Plantin, 2015; Hanson et al., 2015). The co-existence of banks and shadow banks in the absence of regulatory intervention or regulatory arbitrage speaks to their importance as alternative modes of financial intermediation. It opens up the possibility that shadow banks can be welfare improving even in the absence of crude regulation. Moreover, it explores the

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3 We use the terms “nonbank” and “shadow bank” interchangeably. This is consistent of the definition of shadow banking used by the Financial Stability Board (FSB) who include all “credit intermediation involving entities and activities outside the regular banking system.”

4 IMF (2014, pp. 68) gives an accounting of run risks when it notes that “these risks are usually greater at shadow banks because they have no formal official sector liquidity backstops and are not subject to bank-like prudential standards and supervision.”

5 The most notable example of this is the creation of money funds in the United States. The origin of money funds is often attributed to the reach for yield as Regulation Q as bank interest rates were capped at the time by Federal Reserve Regulation Q, and Treasury bills were subject to $10,000 minimum investments (Bouveret et al., 2022).
scope and limits of regulation in determining the size and location of shadow banking as opposed to designing regulation aimed at “choking off shadow bank activities” (Ordonez, 2018).

We begin with a simple one-period model of lending, where borrowers (firms or entrepreneurs) need external financing to make productive investments that are heterogeneous in terms of their liquidation (salvage) values. Lenders monitor borrowers which improves their investment outcome because monitoring prevents borrowers from shirking for private benefits (Holmström and Tirole, 1997). However, monitoring is costly and lender’s monitoring effort is unverifiable so that lenders are subject to a moral hazard problem (Martínez-Miera and Repullo, 2017).

On the funding side, lenders’ capital is insufficient for productive investment, so that lenders need to supplement their capital with other liabilities. Lenders finance their liabilities with debt issued to market investors for whom monitoring is prohibitively costly. Herein lies the rationale behind intermediated financing as opposed to direct financing by investors. Total credit supplied to the borrower comprises the lenders’ paid-in capital plus liabilities. When investments fail, the liquidation value obtained by the bank is insufficient to payoff investors and this creates the need for insured financing. Lenders can choose one of two funding modes: insured or uninsured liabilities. Lenders have limited liability and therefore do not compensate holders of uninsured liabilities beyond the liquidation values of investment returns. Lenders with insured financing pay a deposit insurance premia ex ante that finances all expected deposit insurance payouts.

We demonstrate how financing is distributed across banks and nonbanks in this simple setting. In particular, lenders’ optimal choice between insured and uninsured liabilities classify them as banks and nonbanks (shadow banks) respectively. Stated differently, lenders’ optimal choice of funding mode—whether to be a bank or a shadow bank—is determined in our model by borrowers’ investment opportunities and lenders’ paid-in capital. The role of paid-in capital in enhancing monitoring incentives is well documented. Increased capital forces lenders to internalize their cost of default thereby reducing the limited liability problem lenders face from their debt financing (Allen et al., 2011). The role of heterogeneity in liquidation values is less understood. In a comprehensive study of firm financing in the United States, Lian and Ma (2021) find that firm borrowing is
predominantly based on cash flows from firms’ operations (cash flow–based lending), as opposed to asset-based or collateralized lending. Such widespread prevalence of earnings-based constraints is the dominant motivation behind our assumption of heterogeneity in liquidation values.

The model finds that monitoring by lenders is higher under deposit insurance when it is paid for by the lender. Banks monitor with greater intensity so that the increase in expected payoff offsets the ex-ante insurance premium which covers the shortfall between the liquidation value of the investment and the deposit guarantee. Because a higher liquidation value implies a smaller premium, optimal monitoring under deposit insurance decreases with the liquidation value of investment. In contrast, optimal monitoring under uninsured liabilities does not depend on liquidation values because no additional payoffs are needed beyond liquidation values.

For a given level of paid-in capital, lenders tend to choose uninsured deposits to fund entrepreneurs’ investments of better quality (higher liquidation value). In an economy where lenders are matched with borrowers of varying quality, the lender chooses the funding mode that maximizes the surplus generated from the loan contract. For higher quality borrowers this tends to be uninsured deposits because a higher liquidation value reduces the need for a higher monitoring intensity. Conversely, insured deposits tend to be the choice of funding mode for low quality entrepreneurs (low liquidation values) because greater monitoring intensity reduces the need for insurance payouts and thereby lowers the expected premium for banks.

2 Related literature

This paper is related to a large theoretical literature that explores the interactions between bank and shadow bank financing. The difference in insured and uninsured modes is often taken to be a defining feature of the distinction between banks and nonbanks (Hanson, Shleifer, Stein, and Vishny, 2015; LeRoy and Singhania, 2020; Donaldson, Piacentino, and Thakor, 2021; Chrétien and Lyonnet, 2017). Because this is an endogenous choice in our model, we can explain the co-existence of traditional banks with insured deposits and shadow banks with uninsured deposits (LeRoy and Singhania, 2020; Chrétien and
Lyonnet, 2017). In these models, banks and shadow banks coexist either because of a government subsidy (LeRoy and Singhania, 2020) or the the avoidance of regulatory cost (Chrétien and Lyonnet, 2017). In our setting, the coexistence prevails even when deposits are privately insured by lenders. Moreover, in a similar vein to Hanson et al. (2015), our model finds that traditional banks tend to hold relatively more illiquid assets (low liquidation value) whereas shadow banks hold more liquid assets.

This paper is also closely related to the work on monitored finance by Martínez-Miera and Repullo (2017), who find that safer entrepreneurs borrow from non-monitoring shadow banks whereas riskier entrepreneurs borrow from monitoring banks. In our model, monitoring levels are also greater for lenders that choose insured liabilities, and these lenders gravitate towards lending to riskier borrowers (with low liquidation values). However, in contrast to their optimal contract where banks monitor but nonbanks do not, in our model all lenders monitor borrowers, but the equilibrium monitoring intensity varies with the lenders’ choice of funding mode. As a result, the demarcation as to how banks and shadow banks monitor and supply credit to borrowers of varying quality emerges directly from whether their liabilities are insured or uninsured.

Finally, our model can also speak to the vast literature on how capital regulation can increase monitoring intensity and improve outcomes (Allen, Carletti, and Marquez, 2011). By encouraging monitoring, bank capital reduces the premium that needs to be offered to depositors, and thus provides a rationale for holding capital that acts through the bank’s liabilities. In particular, Allen et al. (2011) show that the fairly priced deposit insurance premium plays the same role as the deposit rate in the case of uninsured liabilities in providing liability side discipline. This rationalizes why differences can arise between incentives from insurance that is fairly priced and insurance that is funded by public financing.

3 The model

Consider a single-period economy that comprises three classes of risk-neutral agents—a continuum of penniless borrowers (firms or entrepreneurs), lenders (financial intermediaries), and a continuum of identical investors, each of measure 1. Borrowers have invest-
ment projects that require external finance which can come only from the lender. The lender, in turn, has insufficient capital and has to raise funds from investors. Investors’ outside option is the risk free rate, \( r_f \). The basic structure of the model is built upon Holmström and Tirole (1997).

**Borrowers** Each borrower has a project that is scalable and requires a investment at \( t = 0 \). A dollar invested in the project yields a verifiable stochastic cash flow \( \tilde{y} \) at \( t = 1 \) given by

\[ \tilde{y} = \begin{cases} y & \text{success with probability } p \\ \lambda & \text{failure with probability } 1 - p \end{cases} \]

where \( y > \lambda > 0 \) and \( p \in (0, 1) \) are constant parameters.

Each borrower can choose between two non-verifiable actions – “behave” and “shirk” – that influence the probability of success and may also yield a private benefit. When the borrower behaves, the project succeeds with a greater probability but yields no private benefit. In contrast, shirking implies low probability of success (normalized to 0) but strictly positive private benefit, \( b \). The payoff structure is as follows.

<table>
<thead>
<tr>
<th></th>
<th>Behave</th>
<th>Shirk</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of success</td>
<td>( p \in (0, 1) )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Private benefit when $1 is invested</td>
<td>0</td>
<td>( b &gt; 0 )</td>
<td></td>
</tr>
</tbody>
</table>

Although all borrowers are identical with respect to the per unit cash flow in the event of success, \( y \), they differ in terms of their per unit liquidation value, \( \lambda \). Let \( F(\lambda) \) be the measure of borrowers, or equivalently, projects with liquidation value less than \( \lambda \) with \( f(\lambda) = F'(\lambda) > 0 \) for all \( \lambda \in [\lambda_{\min}, \lambda_{\max}] \) with \( \lambda_{\min} \geq 0 \). We shall refer to a borrower with liquidation value \( \lambda \) as a “type-\( \lambda \) borrower”. The liquidation value, \( \lambda \), can be interpreted as one measure of borrower risk with risk decreasing in \( \lambda \).\(^6\)

\(^6\)The term borrower risk should be interpreted with caution. The variance of per unit cash flow when borrower behaves is given by \( \text{var}(\tilde{y}) = p(1 - p)(y - \lambda)^2 \) which is decreasing in \( \lambda \). Meanwhile, the project yields a constant cash flow, \( \lambda \), when the borrower shirks and that has zero variance.
We assume that the project is economically viable only when the borrower does not shirk.

A1. \( py + (1 - p)\lambda > 1 > \lambda + b. \)

**Lenders.** Borrowers do not have any wealth and must finance their entire investment from a lender. Each lender can fund a limited set of borrowers, taken to be one for simplicity. Each lender has equity capital \( k > 0 \) and grants a loan of \( 1 + k \) to a borrower, of which $1 is the lender’s liability. Lenders differ in terms of their equity capital \( k \in [k_{\text{min}}, k_{\max}] \). The distribution function of lender capital is given by \( G(k) \) with \( g(k) = G'(k) > 0 \) for all \( k \in [k_{\text{min}}, k_{\max}] \). We refer to a lender with capital \( k \) as a “type-\( k \) lender”.

The action chosen by an borrower – behave or shirk – depends on lender monitoring. Following Holmström and Tirole (1997), we assume that lender monitoring effort, \( m \), can induce a borrower to behave with probability \( m \in [0, 1] \). But monitoring is costly with the cost of monitoring given by \( c(m) = \frac{1}{2} m^2 \). Lenders cannot commit ex-ante to a given level of monitoring, which creates a moral hazard problem. Each borrower is protected by limited liability, and as a result, the contract between a type-\( k \) lender and a type-\( \lambda \) borrower is a contingent debt contract wherein the lender receives \( \min\{R, \lambda\}(1 + k) \).

We assume that

A2. \( py + (1 - p)\lambda < r_f + b, \)

where \( r_f \) is the risk-free rate, the opportunity cost of funds.

Note that the competitive rate, \( R_0 \), which solves \( (pR_0 + (1 - p)\lambda)(1 + k) = r_f(1 + k) \) is the lowest feasible loan rate. Therefore it follows that \( p(y - R_0) = py + (1 - p)\lambda - r_f < b \), which is the same as Assumption A2. This means that, in the absence of lender monitoring, all borrowers choose to shirk even at the lowest loan rate, \( R_0 \). In other words, lender monitoring is necessary in mitigating borrower misbehavior. Lenders as financial intermediaries enjoy advantages over market investors whose monitoring effort is assumed to be prohibitively costly. Herein lies the rationale behind financial intermediation in the model.

**Investors.** Lenders intermediate between market investors as the provider of funds and borrowers as the end-user of funds. All investors are ex-ante identical. Each lender
promises rate $r > 0$ to the investor, which it can deliver if the project is a success. We assume that

A3. $\lambda(1 + k) \leq r_f$ for all $k$,

Assumption A3 implies that, if the project fails, each type-$k$ lender loses their income to the investor.

## 4 Lender financing modes and contracts

There are two financing modes (regimes) available to each lender which always borrows $1$ from an investor. In one regime, denoted by $S$ (shadow bank), lenders finance investment through uninsured liabilities. In the other regime, denoted by $T$ (traditional bank), the lender issues insured deposits. A lender can insure its deposits by paying the deposit insurer (e.g. the FDIC) an insurance premium, $\pi > 0$. The financing modes are determined contractually. As a first step, we derive the optimal financial contract under each financing mode separately. Next, we determine the optimal choice of financing mode for a given borrower-lender pair, whose types are denoted by $(\lambda, k)$.

### 4.1 Uninsured liabilities

Under regime $S$, the expected utilities of the type-$\lambda$ borrower and the type-$k$ lender in a borrower-lender pair $(\lambda, k)$ are given by

$$U(R, m) = mp(y - R)(1 + k) + (1 - m)b(1 + k),$$

$$V(R, r, m) = mp(R(1 + k) - r) - \frac{1}{2}m^2 - krf.$$

respectively. We first determine the financing rate, $r$, and lender monitoring, $m$, as functions of the loan rate, $R$.

Following Pennacchi (2006), we determine the optimal financing rate by risk-neutral valuation method. Let $q$ be the “risk-neutral probability” of the no-default state. A type-$k$ lender’s assets in the event of success and failure are given by $R(1 + k)$ and $\lambda(1 + k)$,
respectively. The risk-neutral probability $q$ is then derived as

$$1 + k = \frac{qR(1 + k) + (1 - q)\lambda(1 + k)}{r_f} \iff q = \frac{r_f - \lambda}{R - \lambda}.$$ 

Thus, the competitive uninsured financing rate, $r$ solves

$$1 = \frac{qr + (1 - q)\lambda(1 + k)}{r_f} \iff r(R) = \theta(\lambda, k)R + (1 - \theta(\lambda, k))r_f, \quad (1)$$

where $\theta(\lambda, k) \equiv 1 - \frac{\lambda k}{r_f - \lambda}$. 

Next, the lender’s monitoring incentive constraint is given by

$$m(R) = \arg\max_m \left\{ mp[R(1 + k) - r(R)] - \frac{1}{2} m^2 - kr_f \right\} = p[R(1 + k) - r(R)]. \quad (2)$$

Now, let $U(R) \equiv U(R, m(R))$ and $V(R) \equiv V(R, r(R), m(R))$. The optimal financial contract for the borrower-lender pair, $(\lambda, k)$ solves

$$\phi^S(\lambda, k, v) = \max_R \{U(R) \mid V(R) = v\}, \quad (3)$$

where $v \geq 0$ is the lender’s outside option. The (maximum) value function under uninsured financing, $\phi^S(\lambda, k, v)$ is the Pareto frontier for a given borrower-lender pair $(\lambda, k)$.

**Lemma 1** Optimal lender monitoring under uninsured financing is given by

$$m^S(\lambda, k, v) = \sqrt{2(v + kr_f)} \quad \text{for } v^S \leq v \leq \bar{v}^S,$$

where $v^S > 0$. The optimal monitoring under uninsured financing is invariant to the liquidation value, $\lambda$, and increases with lender capital, $k$ and lender’s outside option, $v$.

Because lender uses $\lambda(1 + k)$ to repay depositors in the default state, the optimal monitoring does not depend on the liquidation value. An increase in lender capital implies greater participation of lender in investment. Hence, lender monitoring increases with $k$. Finally, higher outside option means it is easier to incentivize the lender to increase monitoring, and hence, monitoring is increasing in $v$. 

9
4.2 Insured deposits

Under regime $T$, the expected utilities of the type-$\lambda$ borrower and the type-$k$ lender in a borrower-lender pair $(\lambda, k)$ are given by

$$U(R, m) = mp(y - R)(1 + k) + (1 - m)b(1 + k),$$
$$V(R, r, m) = mp(R(1 + k) - r - \pi) - \frac{1}{2} m^2 - kr_f.$$  

respectively. When deposits are insured, each depositor receives $r = r_f$.  

Next, we determine the insurance premium, $\pi$. Note that bank’s income in the failure state, $\lambda(1 + k)$, is given to the depositors as part of the repayment. Because $r_f > \lambda(1 + k)$, the deposit insurer makes the depositors whole at the disbursement cost of $r_f - \lambda(1 + k)$. On the other hand, the deposit insurer receives $\pi$ in the no-default state. Because bank receives $R(1 + k)$ with probability $mp$ and $\lambda(1 + k)$ with probability $m(1 - p) + (1 - m) = 1 - mp$, the expected margin of the deposit insurer is given by $mp\pi - (1 - mp)(r_f - \lambda(1 + k))$. Assuming that the deposit insurer earns a margin $\alpha \geq 0$, $\pi$ solves

$$mp\pi - (1 - mp)(r_f - \lambda(1 + k)) = \alpha.$$  

(4)

With $\alpha = 0$, we obtain the fairly priced deposit insurance. Now substituting $r = r_f$ and the $\pi$ from (4) into $V(R, r, m)$, we obtain

$$V(R, m) = mp(R - \lambda)(1 + k) - \frac{1}{2} m^2 - kr_f = \mu(\lambda, k),$$

where $\mu(\lambda, k)$ is the insurer’s margin plus the disbursement cost, which is decreasing in both $\lambda$ and $k$.

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7Bank’s limited liability constraints are given by $R(1 + k) - r_f - \pi \geq 0$ and $\lambda(1 + k) - r_f - \pi \geq 0$ in the no-default and default states, respectively. However, the limited liability binds only in the default state, and hence bank’s expected income (gross of monitoring cost and opportunity cost of funds) is given by

$$mp(R(1 + k) - r_f - \pi) + (1 - mp)(\lambda(1 + k) - r_f - \pi) = mp(R(1 + k) - r_f - \pi).$$
Lender’s monitoring incentive constraint is given by

\[ m(R) = \arg\max_m \left\{ m p(R - \lambda)(1 + k) - \frac{1}{2} m^2 - kr_f - \mu(\lambda, k) \right\} = p(R - \lambda)(1 + k). \]  

(5)

With \( U(R) \equiv U(R, m(R)) \) and \( V(R) \equiv V(R, m(R)) \), the optimal financial contract for the borrower-lender pair, \((\lambda, k)\) solves

\[ \phi^T(\lambda, k, v) = \max_R \{ U(R) \mid V(R) = v \}. \]  

(6)

The (maximum value) function under insured deposit financing, \( \phi^T(\lambda, k, v) \) is the Pareto frontier for a given borrower-lender pair \((\lambda, k)\).

**Lemma 2** The optimal bank monitoring under insured financing is given by

\[ m^T(\lambda, k, v) = \sqrt{2(v + kr_f + \mu(\lambda, k))} \text{ for } v^T \leq v \leq \overline{v}^T, \]

where \( v^T > 0 \). Optimal monitoring under insured deposits decreases with the liquidation value, \( \lambda \), and increases with bank capital, \( k \) and bank’s outside option, \( v \).

An important difference between the two financing modes is that, unlike uninsured liabilities, the optimal lender monitoring depends of the liquidation value when deposits are insured because under this funding mode, \( \mu(\lambda, k) \) is paid by the lender. A higher \( \lambda \) implies a lower disbursement cost, and consequently, lower monitoring. From Lemmas 1 and 2, we obtain that

**Proposition 1** Bank monitoring for any given borrower-lender pair \((\lambda, k)\) is higher under insured financing.

Under deposit insurance, apart from the foregone cost of funds, \( kr_f \) and the outside option, \( v \), each type-\( k \) bank incurs an additional cost, \( \mu(\lambda, k) \) which reflects insurer’s margin, \( \alpha \), plus the disbursement cost, \( r_f - \lambda(1 + k) \). This additional cost incentivizes lenders to monitor with greater intensity when financing is insured.
4.3 Optimal choice of financing modes

Any given borrower-lender pair $(\lambda, k)$ will choose uninsured liabilities over insured deposits if and only if $\phi^S(\lambda, k, v) \geq \phi^T(\lambda, k, v)$. Because the choice of financing mode is endogenous, the Pareto frontier of the pair $(\lambda, k)$ is given by

$$u = \phi(\lambda, k, v) \equiv \max\{\phi^S(\lambda, k, v), \phi^T(\lambda, k, v)\}.$$ 

We obtain the following result.

**Proposition 2** Given any borrower-lender pair $(\lambda, k)$, there is a unique $\hat{v}$ such that the pair chooses uninsured liabilities if and only if $v > \hat{v}$.

Figure 1 illustrates the optimal choice of funding modes by a given borrower-lender pair. Lenders’ monitoring incentive problem is more severe when liabilities are not insured. Low values of bank’s outside option (under both funding modes) means lower bargaining power of lenders, and consequently, it is more difficult to incentivize lenders to monitor. As a result, the insured deposit funding mode tends to dominate for low values of $v$. By contrast, when lender’s bargaining power is high (i.e., high $v$), it is easier to incentivize the lender to monitor and the funding mode with uninsured liabilities tends to dominate.

5 Market equilibrium with identical lenders

We model our economy as a matching market wherein one type-$\lambda$ borrower and one type-$k$ lender is matched pairwise (one-to-one matching). Formally, let $\lambda : [k_{\min}, k_{\max}] \rightarrow [\lambda_{\min}, \lambda_{\max}]$ be a one-to-one function that maps each $k$ to one $\lambda$, so that $\lambda = \lambda(k)$. In the matching market, each type-$\lambda$ borrower chooses a type-$k$ lender. Given that the lender consumes $v(k)$, each type-$\lambda$ borrower solves

$$u(\lambda) = \max_k \phi(\lambda, k, v(k)).$$

(7)

Therefore, the outside option of lenders is endogenous in the matching market. In equilibrium, if a lender finances a borrower, then it must be the case that its equilibrium utility is higher than the utility offers made by any other borrower in the market, which is
the outside option of this given lender. Lenders’ “outside option” should be distinguished from their “reservation utility”, which is the utility to any lender if it does not enter into any lending relationships with borrowers (i.e., the lender is “unmatched”). On the other hand, the outside option of a type-\(k\) lender (when matched with a type-\(\lambda\) borrower) is the maximum utility offered by any other borrower, say of type-\(\lambda'\). Let \(v_0 \geq 0\) denote the reservation utility of all lenders in the economy. In the matching stage, each borrower, while solving the maximization problem (7), must take into account the constraint \(v(k) \geq v_0\).

We start with a simple market structure where all lenders are of the same size, that is, \(k = k_0\) for all \(k\). Therefore, the solution to the maximization problem (7) is trivial: (a) a lender can be matched randomly to a borrower; (b) \(v(k) = v_0\) for all \(k\); and (c) \(u(\lambda) = \phi(\lambda, k_0, v_0)\) which is strictly increasing in \(\lambda\).

The equilibrium lender financing modes are determined as follows. Let \(\hat{v}\) is defined as

Figure 1: The Pareto frontier of an arbitrary borrower-lender pair \((\lambda, k)\).
(as illustrated in Figure 1)

\[
\phi^S(\lambda, k_0, \hat{v}) = \phi^T(\lambda, k_0, \hat{v}),
\]

which defines implicitly the function, \( \hat{v}(\lambda) \). Note that all type-\( \lambda \) borrowers on \( \hat{v}(\lambda) \) are indifferent between the two financing modes. We call \( \hat{v}(\lambda) \) the indifference locus, which is the downward-sloping curve in Figure 2. The indifference locus divide the \( \lambda-v \) space into two disjoint regions: above the locus all borrower-lender pair choose uninsured financing, and below the locus all matches chooses insured financing. Because the equilibrium utility of lenders is constant at \( v_0 \), the borrower type that is ‘indifferent’ between the two financing modes is determined by \( \hat{v}(\lambda^*) = v_0 \), which is unique.

![Figure 2: Equilibrium choice of lender financing modes](image)

**Proposition 3** In the equilibrium of the borrower-lender matching market with homogeneous banks, there is a unique borrower type \( \lambda^* \in [\lambda_{\min}, \lambda_{\max}] \) such that all borrower-lender pairs \( (\lambda, k_0) \) choose the insured (uninsured) financing mode according as \( \lambda < (>) \lambda^* \). Therefore, the fraction of borrowers financed by insured deposits is given by \( F(\lambda^*) \).
Note that the indifferent borrower type, $\lambda^*$ is sensitive to changes in bank capital, $k_0$, the risk-free rate, $r_f$ and insurer margin, $\alpha$.

6 Conclusion

This paper develops a generalized framework for analyzing how lender risk-taking varies with the mode of financing. The competitive equilibrium in a model of monitored financing demonstrates how insured banks will monitor more intensely and gravitate towards projects with lower liquidation values (riskier illiquid assets) while uninsured nonbanks or shadow banks will monitor less intensely but lend to borrowers with high liquidation values. From a policy standpoint it follows that, in the absence of regulation on banks or nonbanks, it is optimal for banks to monitor relatively riskier projects intensively. This distribution of risks between banks and nonbanks is constrained efficient in the sense that a social planner subject to the same moral hazard problem as banks and entrepreneurs would not be able to improve outcomes.

Appendix

Proof of Lemma 1

With contingent debt contract, $\min\{R, \lambda\}(1+k)$, under uninsured financing, the expected utilities of any type-$\lambda$ borrower and any type-$k$ lender are respectively given by:

\[
U(R, m) = mp(y - R)(1 + k) + (1 - m)b(1 + k),
\]

\[
V(R, r, m) = mp(R(1 + k) - r) - \frac{1}{2} m^2 - kr_f.
\]

By the risk-neutral valuation method, the optimal deposit rate is given by

\[
r(R) = \theta(\lambda, k)R + (1 - \theta(\lambda, k)) r_f,
\]

where $\theta(\lambda, k) \equiv 1 - \frac{\lambda k}{r_f - \lambda}$.
From the above, it follows that

$$R(1 + k) - r(R) = \frac{kr_f(R - \lambda)}{r_f - \lambda}.$$  \hfill (8)

On the other hand, from the monitoring incentive compatibility constraint, we obtain

$$m(R) = p(R(1 + k) - r(R)) = \frac{pkr_f(R - \lambda)}{r_f - \lambda}.$$  \hfill (9)

$$\iff R(m) = \lambda + \frac{m(r_f - \lambda)}{pkr_f}.$$  \hfill (10)

Substituting $R \equiv R(m)$ and $r \equiv r(R(m))$ into the expressions of expected utility, the maximization problem of the borrower reduces to

$$\max_m U(m) \equiv \left\{ b + m(p(y - \lambda) - b) - \left( \frac{r_f - \lambda}{kr_f} \right) m^2 \right\} (1 + k),$$  \hfill (11)

subject to $V(m) \equiv \frac{1}{2} m^2 - kr_f = v.$  \hfill (12)

Solving for $m$ from (12), we get the expression of $m^S(\lambda, k, v)$. Let $\phi^S(\lambda, k, v)$ be the value function of the above maximization problem, which is given by $U(m^S(\lambda, k, v))$. First, note that $V'(m) = m \geq 0$. Therefore, $V(m)$ reaches its maximum at $m = 1$. Thus,

$$v \leq V(1) = \frac{1}{2} - kr_f \equiv \tau^S$$

Next, the Lagrangean of the above maximization problem is given by:

$$\mathcal{L}(m, \zeta^S) = U(m) + \zeta^S[V(m) - v],$$

where $\zeta^S > 0$ is the multiplier associated with constraint (12). The first-order condition is given by

$$U'(m) + \zeta^S V'(m) = 0$$  \hfill (13)
Because $\zeta^S V'(m) \geq 0$, it must be the case that $U'(m) \leq 0$ which implies

$$m \geq \frac{kr_f}{2(r_f - \lambda)} (p(y - \lambda) - b) \equiv m^S.$$

Clearly, $V(m)$ achieves its minimum at $m = m^S$. Therefore,

$$v \geq V(m^S) = \frac{1}{8} \left( \frac{kr_f (p(y - \lambda) - b)}{r_f - \lambda} \right)^2 - kr_f \equiv v^S.$$

Denote by $\phi^S_i(\lambda, k, v)$ the partial derivative of the value function with respect to the $i$-th argument where $i = 1, 2, 3$. From the Envelope theorem, it follows that

$$\phi^S_3(\lambda, k, v) = \frac{\partial L}{\partial v} = -\zeta^S < 0,$$

i.e., $\phi^S(\lambda, k, v)$ is strictly decreasing in $v$. Therefore,

$$\phi^S(\lambda, k, v) \geq \phi^S(\lambda, k, v^S) = \left( p(y - \lambda) - \frac{r_f - \lambda}{kr_f} \right) (1 + k) \equiv u^S,$$

$$\phi^S(\lambda, k, v) \leq \phi^S(\lambda, k, v^S) = \left( b + \frac{kr_f}{4(r_f - \lambda)} (p(y - \lambda) - b)^2 \right) (1 + k) \equiv w^S.$$

Finally, the optimal loan and deposit rates associated with the borrower-lender pair $(\lambda, k)$ are respectively given by

$$R^S(\lambda, k, v) = R(m^S(\lambda, k, v)),$$
$$r^S(\lambda, k, v) = r(R(m^S(\lambda, k, v))).$$

**Proof of Lemma 2**

Under insured deposits, each depositor receives $r = r_f$. The deposit insurance premium, $\pi$ is determined from (4). Substituting the values of $r$ and $\pi$ into $U(R, m)$ and $V(R, r, m)$,
we obtain

\[ U(R, m) = mp(y - R)(1 + k) + (1 - m)b(1 + k), \]
\[ V(R, r, m) = mp(R - \lambda)(1 + k) - \frac{1}{2}m^2 - krf - \left[ \alpha + r_f - \lambda(1 + k) \right]. \]

From the monitoring incentive compatibility constraint, we obtain

\[ m(R) = p(R - \lambda)(1 + k) \iff R(m) = \lambda + \frac{m}{p(1 + k)}. \]

Substituting \( R \equiv R(m) \) into the expressions of expected utility, the maximization problem of the borrower reduces to

\[
\begin{align*}
\max_m U(m) &\equiv \{ b + m(p(y - \lambda) - b) \} (1 + k) - m^2, \\
\text{subject to } V(m) &\equiv \frac{1}{2} m^2 - krf - \mu(l, k) = v.
\end{align*}
\] (12)

The optimal monitoring, \( m^T(\lambda, k, v) \) solves (13). The result of the proof is similar to that of Lemma 1. Hence, we only provide the expressions of the relevant bounds on utility.

For future reference, let \( \zeta^T \) be the Lagrange multiplier associated with constraint (13). We have

\[
\begin{align*}
\underline{v}^T &\equiv \frac{1}{8}[(p(y - \lambda) - b)(1 + k)]^2 - krf - \mu(l, k), & \overline{v}^T &\equiv \frac{1}{2} - krf - \mu(l, k), \\
\underline{u}^T &\equiv p(y - \lambda)(1 + k) - 1, & \overline{u}^T &\equiv [b + \frac{1}{4}(p(y - \lambda) - b)^2(1 + k)](1 + k).
\end{align*}
\]

Finally, the optimal loan and deposit rates associated with the borrower-lender pair \((\lambda, k)\) are respectively given by

\[
\begin{align*}
R^T(\lambda, k, v) &\equiv R(m^T(\lambda, k, v)), \\
r^T(\lambda, k, v) &\equiv r(R(m^T(\lambda, k, v))).
\end{align*}
\]
Proof of Proposition 1
The result follows from the fact that $\mu(l, k) > 0$ for all $(l, k)$.

Proof of Proposition 2
Note that $\mu(\lambda, k) > 0$ implies that $\nu^S > \nu^T$. On the other hand, $\nu^S > \nu^T$ because the inequality is equivalent to $r_f > \lambda(1 + k)$ which is Assumption A3. Hence, because both frontiers, $\phi^S(\lambda, k, v)$ and $\phi^T(\lambda, k, v)$ are downward-sloping with respect to $v$, the two frontiers intersect at least once. We now show that $\phi^S(\cdot, \cdot, v)$ steeper than $\phi^T(\cdot, \cdot, v)$ for all $v$ so that the intersection at $\hat{v}$ is unique. From the proofs of Lemmas 1 and 2, recall that

\[ \zeta^S = -\frac{U'(m)}{V'(m)} = \frac{2m \left( \frac{r_f - \lambda}{kr_f} \right) (1 + k) - (p(y - \lambda) - b)(1 + k)}{m}, \]

\[ \zeta^T = -\frac{U'(m)}{V'(m)} = \frac{2m - (p(y - \lambda) - b)(1 + k)}{m}. \]

Assumption A3, i.e., $r_f > \lambda(1 + k)$ implies that $\zeta^S > \zeta^T$. Because $\phi^S_3(\lambda, k, v) = -\zeta^S$ and $\phi^T_3(\lambda, k, v) = -\zeta^T$, the result follows.

Proof of Proposition 3
For each $(\lambda, k_0)$, there is a cut-off value $\hat{v}$ of lender’s outside option that determine the choice of funding modes for this given pair. When we vary $\lambda$, $\hat{v}$ becomes a function of the liquidation values, i.e., $\hat{v}(\lambda)$ which solves

\[ \phi^S(\lambda, k_0, \hat{v}(\lambda)) = \phi^T(\lambda, k_0, \hat{v}(\lambda)). \]

Thus,

\[ \frac{d\hat{v}}{d\lambda} = -\frac{\phi^T_1(\lambda, k_0, \hat{v}(\lambda)) - \phi^S_1(\lambda, k_0, \hat{v}(\lambda))}{\phi^S_3(\lambda, k_0, \hat{v}(\lambda)) - \phi^T_3(\lambda, k_0, \hat{v}(\lambda))}. \]

We have already shown that $\phi^T_3(\lambda, k_0, \hat{v}(\lambda)) - \phi^S_3(\lambda, k_0, \hat{v}(\lambda)) < 0$. It is easy to show that $\phi^T_1(\lambda, k_0, \hat{v}(\lambda)) - \phi^S_1(\lambda, k_0, \hat{v}(\lambda)) > 0$, and hence, $d\hat{v}/d\lambda < 0$. In order not to make the
problem trivial, let us assume that \( \hat{v}(\lambda_{\min}) < v_0 < \hat{v}(\lambda_{\max}) \), and hence, the intersection between \( v_0 \) and \( \hat{v}(\lambda) \) is unique. In other words, \( \lambda^* \) is unique.

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