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# Janus's Money Demand and Time Inconsistency: A New Impossibility Theorem? *

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**Abstract**

We derive a very general ("Janus") money demand function, which reflects backward and forward-looking habit formation. This generality offers an explanation for the breakdown of money-demand functions and policy relevance of monetary aggregates. Integrating our Janus money demand into a Barro-Gordon framework reveals new insights for time inconsistency in monetary policy, and a new impossibility theorem.

**Keywords:** Money, Sidrauski, Habits, Anticipation of Future Consumption, Optimal Policy, Rules, Discretion.

**JEL:** E41, E5, E61, E71.

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*Janus is believed to have had two faces and so could see before him and behind his back – a reference, no doubt, to the foresight and shrewdness of the King, as one who not only knew the past but would also foresee the future* **Macrobius (1969)**

# 1 Introduction

It is now well established that time-separable formulations of utility can be highly restrictive: for example, in terms of capturing the stylized facts of consumption, as well as in addressing various anomalies in standard discounted utility theory and financial decisions (e.g., Loewenstein and Thaler, 1989). This has led to a vibrant literature in augmenting utility with reference measures of consumption, be they backward or anticipatory.

In that vein, we study a novel utility formulation that takes into account the past and expected future habits and examine its implication for money demand and monetary policy. This new specification combines habit formation (**HF**: Abel, 1990; Fuhrer, 2000) and anticipation of future consumption (**AFC**: Loewenstein, 1987; Faria and McAdam, 2013; Monteiro and Turnovsky, 2016; Gómez and Monteiro, 2022) into a unified framework to derive an optimal money demand. Since it is able to look simultaneously to the past and future, we dub it the *Janus* money demand (after the Roman God with two faces).

To derive this money demand, we assume a representative agent in a Sidrauski (1967a,b) neoclassical framework that uses past habits $\eta \in (0,1)$ units of time, formed through HF, to demand money balances. For the remaining $(1-\eta)$ units, she uses future habits, formed through AFC, to demand money. We consider $\eta$ (hereafter, the *Janus parameter*) to be unknown to the policymaker (potentially even time-varying).

Despite its simplicity, this formulation yields three main insights:

(1) **Money is super-neutral**: real quantities are not affected by money and inflation, neither by time inconsistent preferences. This standard Sidrauskian outcome, though, is shown to be a special case related to whether the agent's expected money issuance rule matches the government money supply rule.

(2) **Money is neither neutral nor super-neutral if** the government's money supply departs from the agent's expectations of money issuance. In such cases, money can have real effects.

(3) In all cases, though, and in contrast to real variables, **equilibrium money demand depends on the HF and AFC parameters**.

One important consequence of (3) is that, depending on $\eta$, the agent can be attached to the past, present, future, or all horizons implying that money demand may vary over time. These characteristics make the *Janus* money demand the most general of the theoretical money-demand formulations.

They also provide a new take on why money demand functions broke down in recent decades, in turn weakening the policy relevance of monetary aggregates (Lown et al., 1999; Judson et al., 2014). In our framework, money demand became unstable as an *endogenous* response (i.e., via changing preferences) to events external to the agent (such as technology shocks), Bernanke (2004).

However, in an interesting twist, these characteristics *also* offer a new perspective on the issue of time inconsistency in monetary policy (Kydland and Prescott, 1977; Calvo, 1978; Barro and Gordon, 1983; Walsh, 1995; Levine et al., 2008; Davis et al., 2018). Time inconsistency refers to a preferred plan, decided upon at time $t$, for the future period $t + j$, which is no longer optimal when $t + j$ arrives. The logic being that if the policy is believed, and is used to form expectations by private agents, the government may have an incentive to subsequently deviate from announcements, thereby inducing policy "surprises". The outcome, in a rational-expectations equilibrium where the government cannot credibly pre-commit, results in lower welfare than otherwise.

The concept has proved hugely influential in policymaking[1] and modeling consumer choice, e.g., Hoch and Loewenstein (1991). In the former, time inconsistency derives from the sequential nature of the game played between policymaker and agent. In the latter it arises from the specification of the agent's preferences (Strotz, 1955; Tirole, 2002). Our analysis of time inconsistency will in fact straddle both approaches.

The representative agent has a weighted utility function that looks to future consumption streams, and one based on backward-looking consumption habits. The resulting optimal money demand inherits both elements. This in turn highlights a neglected source of time inconsistency. Not only can optimal plans prove to be time inconsistent because of the potential reaction of agents to anticipated events, but the inconsistency of optimal plans may also reflect the anchor of habit formation in consumption and money demand.

A consequence of this modeling strategy is that we can investigate the well-known literature on discretion-versus-rules in monetary policy through new lenses. Models in the Barro and Gordon (1983) tradition focus on money supply, taking as given money demand. In our setup, the derivation of the optimal money demand when agents are time inconsistent adds a new layer to the study of monetary policy and time inconsistency. We in fact unveil a surprising result

[1] See Nobel Prize Committee Lecture (2004). Finn Kydland and Edward Prescott's Contribution to Dynamic Macroeconomics: The Time Consistency of Economic Policy and the Driving Forces Behind Business Cycles, available at https://ideas.repec.org/p/ris/nobelp/2004_001.html.

that the central bank (**CB**) is incapable of setting the inflation rate either at zero or at some other desired value. This happens because the CB cannot control, or influence, the way individuals set their preferences relating to past and future habits. This *impossibility result* leads us to a paradox, namely: there will be inflation, but not at the level that satisfies the CB. In short, the uncertainty over consumer preferences means that the central bank cannot know the outcome of surprise monetary policies.

Here, however, the innovation is that the origin of the inconsistency arises from the agent's choice to balance different habit forms, not from some reneging, calculating government. Moreover, the heart of the time inconsistency comes from the agent's model of money demand rather than solely from the policy choices of the government. This provides a richer foundation to time inconsistency analyses in a monetary context, and to the neutrality predictions of the Sidrauski monetary model.

**Organization** Section 2 incorporates HF and AFC utility preferences into a Sidrauski money-in-the-utility function formulation and derives the optimality conditions. It finishes by deriving the full generalized money demand and compares our specification to the existing literature. Section 3 reveals the implication of such a general money demand function for the issue of time consistency. This leads to the impossibility theorem such that the government will neither know the resulting inflation rate, nor the nature of monetary surprises. Next, we focus on the issue that the agent's model of money departs from the government's money supply. Their possible discrepancy strengthens our impossibility theorem. In Section 4 we extend the framework to the case where the agent additionally chooses the optimal Janus parameter, $\eta$. This allows us to resolve the impossibility result (in the sense that policy making can be done without knowledge of $\eta$) but retains the Barro-Gordon time inconsistency outcome. Section 5 concludes.

## 2 The Model

**Utility Modeling with HF and AFC** A key milestone in modeling utility has been in integrating the effect of past consumption (Abel, 1990). This 'habit-formation' hypothesis has a long and distinguished history in addressing various economic questions: e.g., consumption patterns (Duesenberry, 1967); monetary policy transmission (Fuhrer, 2000); asset pricing (Cochrane, 2017); addiction patterns (Dockner and Feichtinger, 1993); growth (Overland et al., 2000); and job satisfaction (Clark, 1999).

Under HF, the stock of habits is (Ryder and Heal, 1973):

$$H = \rho_H e^{-\rho_H t} \int\limits_{-\infty}^{t} c(\tau) e^{\rho_H \tau} d\tau \tag{1}$$

where $H$ is a weighted average of past per-capita consumption levels, $c$. The larger is parameter $\rho_H > 0$, the less weight given to past consumption in determining habit stock $H$. Empirically, we might assume that $\rho_H$ can be estimated. Indeed, its value precisely plays a pivotal role in finance, growth, and macroeconomic policy models (Chetty and Szeidl, 2016; Cochrane, 2017; Gómez, 2021).

However, there are also several important life events that involve *anticipation* of future consumption: savings for retirement, the final payment of a mortgage etc. In his seminal paper Loewenstein (1987) formalized the insight that the anticipation of future consumption, like consumption itself, yields utility, or disutility. It seems intuitive that pessimism or optimism regarding the future affects a person's present motivation and decisions: agents may, for instance, prolong a desirable experience to 'savor' it, or expedite an undesirable one to shorten the period of 'dread'. In the AFC framework, therefore, utility is a function of both current consumption and a reference consumption level based on expected future consumption. This 'anticipal-utility' concept has proved influential in some behavioral fields to address anomalies in standard discounted utility theory and financial decisions (e.g., Hoch and Loewenstein, 1991; Kőszegi and Rabin, 2006). Kuznitz et al. (2008) for instance show that AFC preferences reduce the mean allocation to stocks, and that agents save more and invest less in risky assets (relative to the absence of such preferences). Like habit formation, AFC represents a comparative benchmark for agents. In assessing their well-being, they may compare current consumption to their expected future consumption profile, economic status, or prospects (Elster and Loewensten, 1992; Kőszegi and Rabin, 2006).

The adaptation of the AFC model consists of two steps. The first, based on Loewenstein (1987), assumes the representative agent derives utility from AFC. The second step follows Kuznitz et al. (2008) in assuming that all effects of future consumption on current well-being are captured by a single variable, the "stock of future consumption", analogous to habit formation models. Defining the stock of future per-capita consumption, $A$, as a function of future consumption:[2]

$$A = \rho_A e^{\rho_A t} \int\limits_{\tau}^{\infty} e^{-\rho_A \tau} c(\tau) d\tau \tag{2}$$

[2] Note, unlike habit formation with its backward-looking, status-quo setting, here the relevant reference point is an endogenously determined expectation, Kőszegi and Rabin (2006).

where $\rho_A > 0$ represents the speed of adjustment of the stock of future consumption (the relative weights of future consumption at different times). There is thus a positive but exponentially declining weight to consumption in future periods. The larger is $\rho_A$, the less weight is given to future consumption in $A$. For expositional convenience, and without loss of generality, we shall hereafter assume that $\rho_A = \rho_H = \rho$. Appendix A for completeness relaxes this. This stock $A$, defined as the expected discounted value of the future consumption stream, can be considered akin to a wealth component in utility, or a measure of consumer confidence (see the discussions in Merella and Satchell, 2022).[3]

Since there is no particular reason to restrict the agent's utility to exclusively incorporate one but not the other, we take the novel approach of accommodating HF *and* AFC formulations. We then examine the case for money neutrality à la Sidrauski and discuss whether the resulting money demand specification leads to new insights regarding time inconsistency.

**Maximization** We can write the representative agent's intertemporal problem as,

$$\max_{c,m} \int_0^\infty \Big\{ \eta U(c,H,m) + (1-\eta) V(c,A,m) \Big\} e^{-\theta t} dt \quad \textbf{s.t.} \tag{3}$$

$$\dot{a} = ra + w + x - \big(c + (\pi + r)\, m\big) \tag{4}$$

$$\dot{A} = \rho\,(A - c) \tag{5}$$

$$\dot{H} = \rho\,(c - H) \tag{6}$$

$$\lim_{t\to\infty} a_t e^{-\int_0^t r_\tau d\tau} \geq 0 \tag{7}$$

where $U$ and $V$ distinguish the utility forms associated, respectively, with the inclusion of HF and AFC formulations; $m = M/PN$ is real per-capita money balances ($P$, $M$ and $N$ are the price level, quantity of money and population[4]); $\theta > 0$ is the rate of time preference; $a = m + k$ is real wealth; $w$ is the real wage; $\pi = \dot{P}/P = (dP/dt)/P$ is the inflation rate; $r$ is the real interest rate (the sum of inflation and the real interest rate is given by the nominal rate, $i$); and $x$ is government lump-sum transfers.

Equation (4) shows that wealth evolves as a function of past balances accumulated at the real interest rate, wages and transfers, and decumulates with consumption and money balances (the

[3] In that respect, Ludvigson (2004) finds empirical evidence that consumer confidence helps forecast consumer expenditure growth.

[4] Without loss of generality, we assume population growth is zero, $n = \dot{N}/N = 0$.

opportunity cost of which is the nominal interest foregone).[5] Constraints (5) and (6) are the dynamic HF and AFC conditions (namely, the time differentials of (1) and (2)). Equation (7) is the no-Ponzi-game condition.

The optimality conditions are as follows:

$$\eta U_c(c, H, m) + (1-\eta)V_c(c, A, m) = \lambda + \rho(\mu_A - \mu_H) \tag{8}$$

$$\eta U_m(c, H, m) + (1-\eta)V_m(c, A, m) = \lambda(\pi + r) \tag{9}$$

$$\dot{\lambda} = \lambda\,(\theta - r) \tag{10}$$

$$\dot{\mu}_A = \mu_A(\theta - \rho) - (1-\eta)V_A(c, A, m) \tag{11}$$

$$\dot{\mu}_H = \mu_H(\theta - \rho) - \eta U_H(c, H, m) \tag{12}$$

where $\lambda$, $\mu_A$ and $\mu_H$ are the costate variables associated with the state variables through conditions (4), (5) and (6), respectively. Plus, the transversality conditions,

$$\lim_{t\to\infty} a_t \lambda(t) e^{-\theta t} = \lim_{t\to\infty} A_t \mu_A(t) e^{-\theta t} = \lim_{t\to\infty} H_t \mu_H(t) e^{-\theta t} = 0 \tag{13}$$

Condition (8) states that, for the agent to be in equilibrium, the marginal utility of consumption must equal the marginal utility of wealth plus the difference between the marginal utility of anticipated and habit consumption references (weighted by their speed of adjustment, $\rho$). In other words, at the margin the agent will consume up to the point where the additional utility from an extra unit of consumption matches the "cost" (or shadow price) of foregone wealth accumulation, plus the difference between the effective shadow price of alternate habit forms.

Condition (9) equates the marginal utility of real money balances to the product of the nominal rate of interest and the shadow price of wealth. Equation (10) is the familiar Keynes-Ramsey Rule. Finally, equations (11) and (12) are the laws of motion for the shadow value of the dynamic AFC and HF constraints, respectively.

To close the model, the agent assumes that lump sum transfers equal the (expected) seigniorage from money issue:

$$x = \frac{\dot{M}}{M}\left(\frac{M}{PN}\right) = \Phi m \tag{14}$$

where $\Phi = \dot{M}/M$ is the growth rate of nominal money balances (assumed to be constant), and

[5] Term $(\pi + r)m$ therefore represents the opportunity cost of placing part of the financial wealth in money rather than in interest-bearing assets.

$m = M/(PN)$. In other words, the agent assumes that monetary policy is set by an exogenous (i.e., non-discretionary) constant money growth rule – and conditions her maximization problem on that basis.

Finally, factor markets are assumed to be competitive and that firms use a constant returns technology $f(k)$:

$$r = f_k(k) \tag{15}$$

$$w = f(k) - f_k(k)\,k \tag{16}$$

where $f_k > 0$, $f_{kk} < 0 \;\; \forall k > 0$.

**Steady State Equilibrium** The model above is block recursive and the steady state solution is as follows: setting $\dot{a} = \dot{\lambda} = 0$ implies

$$\Phi = \pi \tag{17}$$

$$f_k(k^*) = \theta \tag{18}$$

Equation (17) shows us that inflation occurs in the steady state with the growth of nominal balances, and (18) determines the steady state capital stock, $k^*$. Given that $a = m + k$, (4) yields the condition that determines $c^*$

$$c^* = f(k^*) \tag{19}$$

From (19), (5) and (6), given $c^*$, we determine the steady-state $A$ and $H$:

$$c^* = A^* = H^* \tag{20}$$

**Proposition 1** (18)–(20) *demonstrates that* ***it does not matter how often*** *(or with what intensity) the agent uses HF or AFC; the Janus parameter $\eta$ has no impact on steady-state income, capital stock and consumption.*

Now, we determine steady-state money demand. In the Sidrauski model, the marginal rate of substitution between consumption and money equals the nominal interest rate. This is because the opportunity cost of holding wealth in the form of real money balances is the nominal interest rate evaluated at the marginal utility of consumption. Here, however, this is no longer true since the term $\left(1 - \rho\frac{\mu_A - \mu_H}{\mathbb{U}_c}\right)$ can be shown to drive an intervening wedge (reflecting additional benefits/costs from holding money):[6]

$$\frac{\mathbb{U}_m}{\mathbb{U}_c} = (\pi + r)\left(1 - \rho\frac{\mu_A - \mu_H}{\mathbb{U}_c}\right) \tag{21}$$

Equation (21) can be rearranged as

$$\mathbb{U}_m = \mathbb{U}_c(\pi + r) - \underbrace{\rho(\pi + r)(\mu_A - \mu_H)} \tag{22}$$

where $\mathbb{U}_z \equiv \eta U_z + (1-\eta)V_z$ denotes total marginal utility with respect to $z = \{c, m\}$. The left side of (22) measures the total marginal utility from holding an extra unit of real money balances. The first right element represents costs: the loss of interest-bearing assets evaluated at the total marginal utility of consumption. Regarding the final bracketed term, an additional benefit of holding real money balances is that consumption is necessarily deferred and deferred consumption yields utility through "anticipal" channels. However, looking backwards, deferred consumption is a cost since it thwarts attaining a desired habit-consistent consumption path. These benefits/costs accrue at the marginal utility of anticipated and habit-formation consumption, $\mu_A$ and $\mu_H$, weighted by $\rho \times (\pi + r)$.

**Proposition 2** *Habit formation and AFC **only** matters for money demand which depends on $\eta$ and $\rho$.*

**Proof:** Condition (22) thus generates the equilibrium steady-state money demand. Using equilibrium conditions $c^* = A^* = H^*$ and $\dot{\mu}_A = \dot{\mu}_H = \dot{\lambda} = 0 \Rightarrow \mu_A^* = \frac{1-\eta}{\theta-\rho}V_A$ and $\mu_H^* = \frac{\eta}{\theta-\rho}U_H$, yields,

$$m^* = m(c^*, i;\ \theta, \rho, \eta) \tag{23}$$

■

Outwardly, money demand (23) looks standard: it depends on a measure of real activity[7] and the nominal interest rate. However, $m^*$ also now depends on time preference $\theta = r$; dynamic parameter $\rho$; and the Janus parameter $\eta$. Equilibrium money balances thus has elements that reflect the agent's decision to look forward, backward, and contemporaneously.

## 2.1 Optimal Money Demand: An Illustration

For illustrative purposes, we can parameterize the $U$ and $V$ utility functions to derive a closed-form equilibrium money demand consistent with our framework. Assume the following two utility

[6] The standard tradeoff between money balances and consumption spending is a special case of (21) where $\rho = 0$.

[7] Although here it is consumption rather than income, see Mankiw and Summers (1986).

functions:[8]

$$U(c, H, m) = \log\left(\left(\frac{c}{H^\gamma}\right)^q m^{1-q}\right) \tag{24}$$

$$V(c, A, m) = \log\left(\left(cA^b\right)^a m^{1-a}\right) \tag{25}$$

where $0 < a, q < 1$. Using these utility functions, manipulating expression (22), and using Proposition 2, we can derive the following parametric equivalent to equilibrium money demand (23):

$$m^* = \left(\frac{c}{\pi + r}\right)\left(\frac{\kappa - 1}{\kappa + \xi}\right) \tag{26}$$

where $\kappa = \eta a + (1 - \eta)q$ and $\xi = \frac{\rho\left(ab(\eta-1)-\eta\gamma q\right)}{\rho-\theta}$

Thus, money demand depends as before on a measure of activity and negatively on the nominal interest rate. However, what is interesting is that our parameters of interest (the Janus parameter $\eta$; and the dynamic parameter of AFC and HF processes, $\rho$) directly affect the mapping of these two variables to money demand. Since (23) constitutes a *new* model of money demand, it is useful to briefly compare and contrast it with the existing literature. We do so next.

**Money Demand In Retrospect** There is some consensus that money demand functions have become unstable (since at least the mid-2000s and possibly back to the 1980s) and in turn weakening the information content and policy relevance of monetary aggregates (Hendry and Ericsson, 1991b; Lown et al., 1999). That instability – often ascribed to financial innovation, institutional changes, financial crises (Judson et al., 2014) – would be attributable in our model to the Janus preference parameter. Indeed, empirical studies have suggested that a key channel of money demand volatility (i.e., the interest semi-elasticity) has declined since at least the 1980s, reflecting changing household preferences brought upon by financial innovation, e.g., Benchimol and Qureshi (2020).

What is more, whilst some (e.g., Hendry and Ericsson, 1991a; Jung, 2016) argue that money demand can be rendered stable if additional variables (like wealth) are included, or by employing long-run or low frequency estimation methods (e.g., cointegration), in our framework money demand will always be unstable if preference parameter $\eta$ changes. Thus, targeting the money supply rather than interest rates (wherein money passively adjusts) would be destabilizing.[9]

Finally, let us reflect on how our framework connects to the commonly understood motives for holding money: Transactions, Precautionary, Speculative, and Portfolio. The first motive is clearly

[8] Utility functions (24) and (25) can be thought of as originating with a constant relative risk-aversion utility function with a unit coefficient of constant relative risk-aversion.

[9] From around 1982, there was a shift away from targeting `M1` by the Federal Reserve.

embodied in any type of Sidrauski framework, but perhaps strengthened if the bulk of transactions are of a regular habit-forming variety. The other motives touch upon but are nonetheless distinct from the AFC motive since they do not capture behavioral explanations related to anticipation of future consumption. For instance, if money demand functions and the information content of money indeed broke down in the mid-1980s and 1990s (the 'Great Moderation'), one explanation might be that the promise of financial innovation and technologically-induced higher growth rates allied to a more stable macroeconomic environment shifted otherwise stable preferences from HF towards AFC, thus undermining the stability of money demand.

In the next section, some of these themes reappear: namely the instability of money demand per se, and associated issues of time inconsistency in a monetary policy setting.

## 3 Time Inconsistency

Having introduced the Janus money demand, we now consider its importance using the perspective of time inconsistency in monetary policy.

Let us now introduce a central bank which takes into account the optimal money demand implicitly derived in (21) or (23). The CB can use rules, or discretion, à la Barro and Gordon (1983). Let us rewrite (21) to solve for the public's inflation expectations:

$$\pi^e = \frac{\mathbb{U}_m}{\mathbb{U}_c - \rho(\mu_A - \mu_H)} - r \tag{27}$$

**Rules** First assume the CB uses ***rules*** and wants to set zero inflation: $\pi^e = 0$. Accordingly, this implies that it sets money supply at a level that satisfies the condition:

$$\underbrace{\frac{\mathbb{U}_m}{\mathbb{U}_c - \rho(\mu_A - \mu_H)}}_{} = r \tag{28}$$

**Discretion** For a ***discretionary*** CB, though, we can embed our framework directly into the Barro-Gordon setup with the Lucas supply function:

$$y = y_n + \alpha\left(\pi - \pi^e\right) + \upsilon \tag{29}$$

where $y$ is the log of output, $y_n$ is the log of natural output, and $\upsilon$ is a stochastic error. The slope coefficient $\alpha$ is expected to be positive and captures the relationship between the inflation forecast error and the output gap.

The CB's objective is to maximize the expected value (Walsh, 2017, p 324),

$$\Omega = \omega(y - y_n) - \frac{\pi^2}{2} \tag{30}$$

More output is thus preferred to less output, with constant marginal utility (and so output enters linearly), while inflation is assumed to generate increasing marginal disutility (and thus enters quadratically). Parameter $\omega > 0$ reflects the weight on output expansion relative to inflation stabilization.

In the standard Sidrauski framework monetary policy, as in (14), is assumed to be set by a deterministic rule. Now however monetary policy is assumed to link inflation with the actual money supply through,

$$\pi = g(m) \tag{31}$$

Function $g(m)$ thus represents the policy instrument of the government or central bank. Specifying this rule is natural in the Barro-Gordon context since it grants the policy maker an independent instrument – rather than the passive Sidrauskian money issuance term (14) – allied to their policy objective function (30). Its specification also intriguingly opens up the possibility that conditions (31) and (14) could differ from one another. That this could be so, conforms to the time-inconsistency theme of policy surprises. Accordingly, we can consider two cases

$$\texttt{The Sidrauskian Equilibrium}: g(m) = \Phi \tag{32}$$

$$\texttt{The Undetermined Outcome}: \quad g(m) \neq \Phi \tag{33}$$

**The Sidrauskian Equilibrium** The first case, (32), is solved as follows. Inserting (27), (29) and (31) into (30):

$$\Omega = \omega\left\{\alpha\left(g(m) - \frac{\mathbb{U}_m}{\mathbb{U}_c - \rho(\mu_A - \mu_H)} - r\right) + \upsilon\right\} - \frac{g(m)^2}{2} \tag{30$'$}$$

The optimal choice of $m$ is then,

$$m^* = \frac{\omega\alpha}{g'(m)^2}\left(g'(m) - \underbrace{\frac{\mathbb{U}_{mm}}{\mathbb{U}_c - \rho(\mu_A - \mu_H)} - \frac{\mathbb{U}_m\left(\mathbb{U}_{cm} - \rho(\mu_{Am} - \mu_{Hm})\right)}{\left(\mathbb{U}_c - \rho(\mu_A - \mu_H)\right)^2}}\right) \tag{34}$$

Consideration of the rules vs. discretion outcomes relating to the CB – given respectively by (27) and (34) – yields an *impossibility theorem* with regards to the optimal monetary policy: the bracketed components in (28) and (34) contains terms in $\rho$ (the dynamic parameter of the habit processes) and $\eta$ (the consumer's preference parameter for HF and/or AFC).[10] Such terms (especially $\eta$) are beyond the policy maker's control, knowledge and influence. This effectively makes it impossible for the CB to set $m$ so as to satisfy either (28) or (34).

An additional unexpected result ensues: since $m$ need not satisfy (28) and (34), then, monetary policy set by the CB will generate inflation, however at a level different from the desired rate intended by the CB. In sum, there will be inflation, but not at the level desired by the monetary authority; both public and CB are worse off in a one-shot game.

**The Undetermined Outcome** of condition (33), can be summarized as follows: equation (14) establishes how the agent assumes the government supplies money. This in turn is used to condition her optimization problem, (17). Then, when we introduce the CB, it supplies money according to (31) which may or may not be the same as in (14). Therefore, in this case, we call the inflation rate from the representative agent "expected", not the actual inflation rate.

The importance of this is that the agent relies upon a potentially incorrect model of money issuance. Accordingly, the agent can no longer close the model in a block recursive manner, and all endogenous variables ($Y, c, K$ etc) are determined simultaneously, implying that money ceases to be neutral and super neutral. This result is a serious blow to the typical Sidrauskian approach, because *neither* the agent nor the government is able to find an equilibrium. This makes our impossibility theorem even more serious.

# 4 Revisiting The Impossibility Theorem

One could argue that the whole problem with this impossibility result stems from the fact that the consumers are irrational in the sense that they do not optimize their money demand controlling for their preferences for past or future habits, i.e., they do not maximize (3) with respect to the Janus parameter, $\eta$. Actually, they are unable to do that since the problem is linear in $\eta$.

One way to find an optimal solution is to include convex costs associated to changes in the Janus parameter $\eta$ wherein the agent incurs a penalty whenever she shifts habit preferences across time. Accordingly, we replace (3) by,

$$\max_{c,m,\eta} \int_0^{\infty} \left\{ \eta U(c,H,m) + (1-\eta)V(c,A,m) - \frac{\varphi}{2}(\eta - \overline{\eta})^2 \right\} e^{-\theta t} dt \tag{3'}$$

**s.t.** the same constraints and steady-state solution to problem (3)-(7) (recall Proposition 2), where $\varphi > 0$, and $\overline{\eta} > 0$ is some reference level of $\eta$. Now, we have two equations to simultaneously

[10] Recalling that $\mathbb{U}_z \equiv \eta U_z + (1-\eta)V_z$ for $z = \{c, m\}$.

determine optimal money demand $m^*$ and weight $\eta^*$:

$$\frac{U_m}{U_c - \rho(V_A - \eta(V_A + U_H))} = r + \pi \tag{21$'$}$$

$$\eta = \frac{U - V}{\varphi} + \overline{\eta} \tag{35}$$

Substituting (35) into (21′) yields an equation that determines optimal money demand which is *independent from* $\eta$:

$$\frac{U_m}{U_c - \rho\Upsilon} = r + \pi \tag{36}$$

with the composite term $\Upsilon = V_A - \big(\frac{U-V}{\varphi} + \overline{\eta}\big)(V_A + U_H)$ (also independent of $\rho, \eta$).

We can rewrite (36) to yield an expression for the public's expectation of inflation:

$$\pi^e = \frac{U_m}{U_c - \rho\Upsilon} - r \tag{37}$$

If the CB uses ***rules*** and wants to set inflation to zero, $\pi = \pi^e = 0$, we have:

$$\frac{U_m}{U_c - \rho\Upsilon} = r \tag{38}$$

For a ***discretionary*** CB, however, we have the policy objective,

$$\Omega = \omega\left(\alpha\left(g(m) - \frac{U_m}{U_c - \rho\Upsilon} - r\right) + \upsilon\right) - \frac{g(m)^2}{2} \tag{30$''$}$$

The optimal choice of $m$ is then,

$$m = \frac{\omega\alpha}{g(m)^2}\left\{g'(m) - \frac{U_{mm}}{U_c - \rho\Upsilon} - \frac{U_m\Big(U_{cm} - \rho\big(V_{Am} - \frac{U_m - V_m}{\varphi}(V_A + U_H) - \frac{U-V}{\varphi}(V_{Am} + U_{Hm})\big)\Big)}{(U_C - \rho\Upsilon)^2}\right\} \tag{34$'$}$$

Comparing (29') with (29) reveals that the previous impossibility issue is no longer present since $\eta$ has been substituted out. The Barro and Gordon (1983) result of multiple equilibria between a combination of rules and discretion is retained.

# 5 Conclusions

We introduced habit and anticipation of future consumption into the Sidrauski model. This yields a very general (Janus) money demand reflecting past and future habits. Equilibrium money demand remains neutral and super-neutral only when the expected model of money issuance between agents and government is the same. In contrast to real variables, equilibrium money demand depends on the HF/AFC parameters. Foremost, our framework provides an endogenous explanation for the breakdown of money demand functions and the policy relevance of monetary aggregates (namely, by dint of variations in the Janus parameter, and thus changing preferences).

Interestingly, the Janus money demand framework adds an additional dimension to the issue of time inconsistency in monetary policy. The central bank cannot influence inflation, thus shedding light on neglected channels of time inconsistency. This reveals a type of impossibility theorem. When preferences for alternative utility specifications themselves have a reference component, time inconsistency remains, but the impossibility result no longer pertains.

Finally, a notable feature of our treatment is that we stay within a representative agent framework. Departing from that, though, would be an interesting extension since then Janus term $\eta$ would no longer be a parameter but a distribution over heterogeneous individuals. This would make the knowledge requirements for policy makers even more complex and heroic since – as we have demonstrated – uncertainty over $\eta$ has real effects on the economy. Tackling these sets of issues in a heterogeneous framework could be a direction for future work. More generally applying a maximization framework admitting both backward and anticipatory habits as done here could prove fruitful in addressing consumption and asset-pricing puzzles on which the habit formation literature has traditionally focused.

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# A Model Conditions When $\rho_{_H} \neq \rho_{_A}$

Following the same structure as the main text, we can write the representative agent's intertemporal problem as,

$$\max_{c,m} \int_0^\infty \Big\{ \eta U(c,H,m) + (1-\eta)V(c,A,m) \Big\} e^{-\theta t} dt \quad \textbf{s.t.} \tag{A.1}$$

$$\dot{a} = ra + w + x - \big(c + (\pi + r)\, m\big) \tag{A.2}$$

$$\dot{A} = \rho_{_A}\,(A - c) \tag{A.3}$$

$$\dot{H} = \rho_{_H}\,(c - H) \tag{A.4}$$

$$\lim_{t\to\infty} a_t e^{-\int_0^t r_\tau d\tau} \geq 0 \tag{A.5}$$

The optimality conditions are:

$$\eta U_c(c,H,m) + (1-\eta)V_c(c,A,m) = \lambda + \mu_A \rho_{_A} - \mu_H \rho_{_H} \tag{A.6}$$

$$\eta U_m(c,H,m) + (1-\eta)V_m(c,A,m) = \lambda(\pi + r) \tag{A.7}$$

$$\dot{\lambda} = \lambda\,(\theta - r) \tag{A.8}$$

$$\dot{\mu}_{_A} = \mu_A(\theta - \rho_{_A}) - (1-\eta)V_{_A}(c,A,m) \tag{A.9}$$

$$\dot{\mu}_{_H} = \mu_H(\theta - \rho_{_H}) - \eta U_{_H}(c,H,m) \tag{A.10}$$

with

$$\mathbb{U}_m = \mathbb{U}_c(\pi + r) + (\pi + r)(\mu_H \rho_{_H} - \mu_A \rho_{_A}) \tag{A.11}$$

and so on.