Employer Credit Checks: Poverty Traps versus Matching Efficiency

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January 2023
RWP 23-01
http://doi.org/10.18651/RWP2023-01
Employer Credit Checks:
Poverty Traps versus Matching Efficiency

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January 6, 2023

Abstract
We develop a framework to understand the effects of pre-employment credit screening in both labor and credit markets. People differ in both their propensity to default on debt and the profits they create for firms that employ them. In our calibrated economy, workers with a low default probability are highly productive and therefore generate more profits for their employers; thus, firms create more jobs for those with good credit. However, using credit reports to screen job applicants creates a poverty trap: an unemployed worker with poor credit has a low job-finding rate and cannot improve their credit without a job. In the calibrated economy, this manifests as an endogenous loss in the present value of lifetime wages that is roughly half of the amount widely used in quantitative models of consumer default. Banning employer credit checks eliminates the poverty trap, but job seekers with good and bad credit now apply to the same jobs, which reduces matching efficiency. As a result, average job-finding rates fall 1.3 percent for high-productivity workers and rise by 1.7 percent for low-productivity workers.

Keywords: Pre-Employment Credit Screening, Consumer Default, Adverse Selection
JEL: E2

*E-mail: dean.corbae@wisc.edu, andrew.glover@kc.frb.org. We wish to thank Briana Chang, Hal Cole, Christoffer Koch, Rasmus Lentz, and Igor Livshits for very useful comments on an earlier version of this paper. We also thank participants at Goethe University, University of Pennsylvania, Ohio State University, University of Colorado - Boulder, FRB of Richmond, NC State University, as well as the NBER Microeconomic Data and Macro Models Group, Texas Monetary Conference, North American Econometric Society, Society for Economic Dynamics, Society for the Advancement of Economic Theory, Human Capital and Economic Opportunity Markets Group, Consumer Finance: Micro and Macro Approaches Conference at Chicago Booth, and the 2018 Consumer Financial Protection Bureau Research conference. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of Kansas City or the Federal Reserve System. This paper uses data from the NYFed/Equifax Consumer Credit Panel, but the authors are not associated with Equifax.
1 Introduction

The three largest consumer credit agencies (Equifax Persona, Experian Employment Insight, and TransUnion PEER) market credit reports to employers, which include credit histories and public records (such as bankruptcy, liens and judgments). According to a Survey by the Society for Human Resource Management (2010), 60% of human resource representatives who were interviewed in 2009 indicated that their companies checked the credit of potential employees. Furthermore, a report by the policy think tank DEMOS found that 1 in 7 job applicants with bad credit had been denied employment because of their credit history (Traub [45]).

Until recently, pre-employment credit screening (PECS) was largely unregulated and remains so at the federal level – the FTC writes “As an employer, you may use consumer reports when you hire new employees and when you evaluate employees for promotion, reassignment, and retention as long as you comply with the Fair Credit Reporting Act (FCRA).”[1] However, since 2005, numerous state and federal laws have been introduced with the goal of limiting or banning employer credit checks and, as of 2018, eleven states have enacted such laws.[2] Legislators often express concern of a “poverty trap” arising due to employer credit checks: a worker loses her job and cannot pay her debts, which negatively impacts her credit report and thereby makes her unable to find a job. We build a model of unsecured credit and labor market search with adverse selection in which such poverty traps arise endogenously, which we use to assess the welfare consequences of policies to ban PECS.

A growing empirical literature seeks to estimate the effects of PECS on labor market outcomes. Most directly related to this paper, Cortes, Glover and Tasci [14] estimate a fall in posted vacancies for affected occupations following the implementation of employer credit check bans, but not in occupations that are exempted (jobs with access to financial or personal information). We reproduce Figure 1 from their paper in Figure 1a. This plot shows the difference between vacancies posted by employers in occupations who are forbidden from using credit checks relative to occupations that are exempt in employer credit check bans (which means they retain the ability to check the credit reports of job applicants). Figure 1a shows that vacancies in affected and exempt occupations follow a similar path before a ban goes into effect (since the difference is approximately zero on average before the ban goes into effect at $t = 0$) while affected occupations experience a significant decline in posted vacancies following the ban, which persists even after a year[3] Their labor market estimates are directly related to the labor demand effect of our theory.

An additional feature of our theory that has not yet been studied empirically is the effect of employer credit check bans on consumer credit markets. Specifically, in our model, consumers are incentivized to repay debts by the effect of their future credit score on their job finding rate and expected earnings. Therefore, banning the use of credit checks by employers removes an incentive to repay and increases the rate of strategic default. Importantly, our model predicts that people with higher credit scores are the most affected by this reduction in dynamic incentives, since they are more patient on average and are therefore more responsive to future labor market outcomes more than people with low scores.

Figure 1b provides support for this mechanism by plotting regression coefficients of a linear

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[2] The states with bans are CA, CO, CT, DE, HI, IL, MD, NV, OR, VT, WA.
[3] The difference between affected and exempt post ban is $-5.5\%$ and is statistically significant at the 5% levels.
Notes: Regression for vacancies is reproduced from Cortes, Glover, and Tasci [14]. Vacancies are classified by \( o \in \{ \text{exempt}, \text{affected} \} \). The estimated equation is

\[
\log \text{vacancies}_{c,o,t} = \sum_{k=-4}^{5} \beta_k^{\text{Affected}} o_{c,t-k} \times \text{BAN}_{c,t-k} + \text{FE}_{c,t} + \text{FE}_{o,t} + \epsilon_{c,o,t},
\]

where \( \text{BAN}_{c,o,t} = 1 \) if county \( c \) has a PECS ban in quarter \( t \) and occupation \( o \) is affected. Lead-lags are in quarters, with \(-5\) representing one year before the ban (normalized to zero) and \(5\) representing more than one year post ban. Blue boxes are 90% confidence intervals. Exempt occupations are two-digit SOC codes representing Business and Financial (SOC-13), Legal (SOC-23), and Protective Services (SOC-33).

Regression for delinquencies uses the NYFed/Equifax Consumer Credit Panel to estimate the differential change in delinquent status of individual consumers as an employer credit check ban is implemented. The estimated equation is

\[
\text{D}_{i,s,t} = \sum_{k=-4}^{5} \beta_k^{\text{BAN}} \text{RiskScore}_{i,t-k} \times \text{BAN}_{s,t-k} + \gamma \text{RiskScore}_{i,t} + \text{FE}_{s,t} + \text{FE}_{i} + \epsilon_{i,s,t},
\]

where \( \text{D}_{i,s,t} \) is an indicator of whether the consumer \( i \) living in state \( s \) has a delinquent credit account at the end of quarter \( t \) and \( \text{RiskScore}_{i,t} \) is the standardized Equifax Risk Score for that consumer.

Figure 1: Effect of PECS Ban on Vacancies and Delinquencies
labor productivities (which creates adverse selection); labor search frictions; and unsecured credit with endogenous default. Employers value the PECS process because credit records are an externally verifiable and inexpensive signal about a residual component of labor productivity that is not observable before the worker is hired. We infer that worker types with high patience also have high residual labor productivity from the negative cross-sectional correlation between credit delinquency and residual earnings. We model the underlying correlation between productivity and repayment rates through an ongoing costly investment in human capital. Since patient people are more willing to incur the disutility of investment in exchange for the expected future benefit of higher wages, they invest, on average, more than impatient people.

We make assumptions on matching and wage determination to keep the labor market model tractable and rationalizes the observed use of PECS by employers. First, we assume that all matches have positive surplus, so low-score matches generate low, but still positive, expected profits. Since our results depend on the job finding rate’s sensitivity to the score rather than the exact point in the matching process at which the job finding rate is determined, we find this assumption innocuous. Second, we assume that productivity is immediately learned by employers once a match occurs as in Jarosch and Pilossof [26]. This is partially a technical assumption to retain tractability by avoiding asymmetric information during wage bargaining, but also guarantees that the effect of an individual’s credit score on her post-employment earnings is small, which is the case empirically. A slower learning process post-match based on changes in the individual’s credit report would generate large swings in an individual’s earnings when she defaults, which is inconsistent with the small effect on individual earnings estimated in [17]. Finally, we use post-match Nash-Bargaining rather than a competitive search model with pre-match contracts requiring commitment designed to perfectly separate types since that would obviate the use of costly PECS in the first place. The fact that we observe PECS conditioning on credit scores suggest that such perfectly separating contracts are hard to design in the real world.

Given the above mentioned empirical evidence that there appear to be interactions between labor and credit markets, we develop a rich model of credit markets with adverse selection. We model the credit market as a sequence of short-term loans, linked by the worker’s score, which enters as a state variable representing the market belief that a worker is the high type (and therefore low risk) given her history of repayments. Our short-term credit market equilibrium concept borrows from Netzer and Scheuer [38], which determines both interest rates and credit supply as the unique equilibrium of an extensive form game played between lenders competing to make loans to borrowers with private information about their default rates. This framework allows us to rationalize the credit market effects of credit scores and to study how the credit

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6In our model, a credit record contains the borrower’s history of debt repayment. This will map into a worker’s ex-ante probability of being a high-patience/productivity type, which coincides with a higher ex-ante probability of repaying debt. We will therefore refer to the worker’s “score” rather than report, since it is this probability of being high-patience/productivity that is relevant for employers and lenders.

7If the surplus from an low-patience/productivity worker was negative, then they would not be hired at all. With a positive surplus, they simply face a longer expected duration of unemployment.

8Netzer and Scheuer apply their extensive form game to the Rothschild and Stiglitz [40] insurance model rather than a borrowing and lending model like our own. Fundamentally, however, our models are within the same class of principal-agent problems with adverse selection in which the principal’s marginal rate of transformation between contract terms is higher whenever the low-risk agent takes the contract and there is a single-crossing property on agent preferences over the contract terms. We detail how to alter their proofs for our model in Appendix C.1.
market responds to a PECS ban. First, the equilibrium contracts posted by lenders depend on the borrower’s score because high-risk borrowers may be cross subsidized through lower interest rates, while higher scores relax credit constraints for low-risk borrowers. Second, the PECS ban affects individual repayment incentives and therefore the equilibrium credit market contracts (both interest rates and supply of credit).

We then use this model as a laboratory to assess the effect of a policy that bans PECS (i.e. forces employers to ignore applicants’ credit histories in the hiring decision). A PECS ban has both direct and indirect effects on the equilibrium. First, as expected by policy makers, there is a redistribution of labor market opportunity (and therefore welfare) from high to low credit score workers, which in equilibrium also translates into a redistribution from high to low productivity workers. This directly reduces matching efficiency by eliminating the ability of employers to recruit from a less adversely selected pool of applicants. Furthermore, there is also an indirect effect on repayment that lowers welfare for everyone. When credit scores are not used in the labor market, workers lose some of their incentives to repay debts. This leads to higher interest rates and less borrowing. This general equilibrium cost of PECS bans has not been considered by policy makers, even by those who advocate on behalf of lower income households with bad credit.

We proceed as follows. In Section 2, we place our paper in the context of the literatures on private information in both credit and labor markets. In Section 3 we describe the economic environment and in Section 4 we define, prove existence, and characterize equilibrium for our adverse selection environment as well as compare it to a full information version. In Section 5 we calibrate the economy and describe properties of the adverse selection equilibrium such as a poverty trap and quantify labor market inefficiencies. In Section 6 we study the welfare consequences of a ban on using credit checks in the labor market.

2 Related Literature

Almost all of the previous work focusing on the use of credit market information to screen job applicants (i.e. PECS) is on the empirical side. Bartik and Nelson [3] use a statistical discrimination model to study the impact of PECS bans on different racial groups. They find that bans significantly reduce job-finding rates for Blacks. Similarly, Ballance, Clifford and Shoag [2] find that employment falls for younger workers and Blacks in states that ban PECS. These findings are consistent with PECS bans reducing the match quality of newly hired job applicants in affected groups (more high match-quality applicants are rejected and more low match-quality applicants are hired after the ban). Friedberg, Hynes and Pattison [18] estimate an increase in job-finding rates for financially distressed households following PECS bans, which highlights the distributional effect of these laws and provides a key elasticity that our quantitative model matches.

While there is a growing structural literature on asymmetric information in unsecured consumer credit markets with default, ours is the first which includes its interaction with the labor market. Specifically, we include labor market search frictions and endogenous wages

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9The first key feature of this game is that an equilibrium always exists. This would not be the case for low scores (i.e. when there are few high risk borrowers) in the competitive framework of Rothschild and Stiglitz [40].

10Some closely related papers that deal with private information in the credit market only include Athreya, Tam and Young [1], Chatterjee, et. al. [9], Livshits, MacGee and Tertilt [31], and Narajabad [37].
via bargaining, as in Mortensen and Pissarides [35], along with information revelation in the match as in Jarosch and Pilossoph [26] [11]. This endogenizes potential income losses from default (via a lower credit score) which is taken as exogenous in the earlier structural default literature. [12] Second, we employ a different equilibrium concept in the credit market. This equilibrium, studied by Netzer and Scheuer [38], is the robust sub-game perfect equilibrium of a sequential game between firms competing to make short term loans to borrowers with private information about their default propensities. The salient assumption is that competitive lenders endogenously choose both the level of debt and the price at which it is offered as opposed to offering a risk adjusted competitive (break even) price for each given level of debt as in, for instance, Chatterjee, et. al. [9]. The equilibrium allocation of this game solves a constrained optimization problem with incentive compatibility constraints and the equilibria may feature cross-subsidization or even pooling. [13] We make a methodological contribution to the static model of Netzer and Scheuer by introducing a dynamic Bayesian type score upon which contracts are conditioned every period so that an individual’s credit access varies over time in response to past behavior. With the assumption that the only information from the credit market which can be passed through time is one’s credit score, the credit allocation is constrained efficient. Our use of the Netzer and Scheuer equilibrium concept allows us to tractably solve for credit market equilibria with adverse selection, which lets us make a general contribution to the literature on signals from one market incentivizing behavior in other markets. [14]

While we model the effect of credit scores on labor demand, a related literature uses changes in an individual’s credit history to instrument for credit access in order to estimate the labor supply response to credit. [15] Along this dimension, Herkenhoff et. al. (24, 25) show that increased credit access leads workers to become more selective in their job search (accept longer unemployment durations in order to obtain higher post-employment wages). We do not model the search decision of unemployed workers, but note that in our model an unemployed worker with bad credit would have a strong incentive to find a job in order to begin rebuilding her credit history. Furthermore, a worker with bad credit has a weaker bargaining position, which is reflected in lower equilibrium wages (although we find quantitatively this effect is small). [16]

[11] Specifically, Jarosch and Pilossoph [26] build a labor search model with ex-ante private information about worker productivity that is correlated with unemployment duration and therefore used to screen job seekers ex-ante. Since all matches have positive surplus in our model, duration provides no additional information about type beyond the credit report. Fundamentally, what differs is that our signal is directly affected by a worker’s credit market decisions and the information context of the signal endogenously responds to labor market policies.

[12] For example, default in Chatterjee, et. al. [9] incurs an exogenous loss proportional to the household’s income.

[13] We discuss the relationship between our allocations and the fully separating equilibria in Guerrieri, Shimer and Wright [20] in Section 4.3 where we present the programming problem.

[14] An important early paper tying cross-market incentives is the reputation based model of Cole and Kehoe [11] which demonstrated how an exogenous utility loss in the labor market can incentivize sovereigns not to default in the credit market. This is important since credit scores are used to screen for insurance, housing, etc. For example, Chatterjee et. al. [8] explore the link between insurance and credit scores.

[15] A related literature studies how financial status (i.e. ability to borrow or dis-save to fund current consumption) affect job-finding rates. Relevant contributions include Chaumont and Shi [15], Krusell et. al. [27], and Lentz and Tranaes [29].

[16] Relatedly, while not focusing on PECS, Dobbie, et al. [17] estimate that annual earnings do not change when a person who filed for chapter 13 bankruptcy has that flag fall off of her credit report after seven years, relative to a person who filed for chapter 7 bankruptcy (whose flag remains for ten years), although they do
3 Environment

Time is discrete and infinite. Each period is split into two subperiods (i.e. a beginning and end of the month). The economy is composed of a large number of workers, firms, lenders, and the credit reporting agency.

A newborn starts life unemployed and draws a discount factor in \( \{ \beta_H, \beta_L \} \), which determines her type \( i \in \{ H, L \} \). The probability the agent initially draws \( \beta_H > \beta_L \) is given by \( \pi_H \), while she remains of a given type \( i \) from period to period with probability \( \rho \). A worker’s type is private information; type cannot be observed by lenders, credit scorers, and can be observed by the firm only after the worker is hired (i.e. in the production process the firm can observe the workers productivity).

In any period \( t \), workers have one unit of time in the first subperiod and zero in the second subperiod. They can either be unemployed \( (n_t = 0) \) or employed \( (n_t = 1) \), which means they work for a firm. Worker preferences are represented by the function \( U(c_{1,t}, c_{2,t}, n_t) = c_{1,t} + z(1 - n_t) + \psi c_{2,t} \) with the unemployed getting \( U(0, 0, 0) \) and the employed getting \( U(c_{1,t}, c_{2,t}, 1) \) (i.e. the employed derive disutility from work). We assume that \( \psi < 1 \) so that workers prefer consumption in the first subperiod to the second (i.e. end-of-month consumption is discounted). Since an unemployed worker does not receive income with which to repay debt, she cannot borrow, and hence her flow utility is simply \( z \).

At the end of a period, the unemployed worker knows whether or not she has found a job for the next period. At this point, she must decide whether or not to invest in her human capital by choosing \( h_{i,t+1} \in \{ h, 0 \} \). Choosing \( h_{i,t+1} = 0 \) incurs a cost of \( \phi \) utils. Similarly, an employed worker who retains her job also chooses \( h_{i,t+1} \in \{ h, 0 \} \).

An employed worker’s residual productivity, \( h_{i,t} \) is observable to the firm. Production takes place in two stages: the worker inelastically supplies labor \( (n_t = 1) \) in the first subperiod which generates output \( y_{i,t} = h_{i,t} \) in the second subperiod. The worker and firm Nash bargain over her wage \( w_{i,t} \) to be paid when her effort yields output in the second subperiod. The worker’s bargaining weight is \( \lambda \) and her outside option is to walk away, receive \( z \) utility from leisure in this period, and to search for another match tomorrow. The outside option for the firm is to produce nothing this period and post another vacancy at cost \( \kappa \) (in equilibrium the firm’s outside option will be zero due to free entry). The firm sells its second subperiod output, yielding period \( t \) profits of the firm given by \( h_{i,t} - w_{i,t} \), which are valued as \( \psi (h_{i,t} - w_{i,t}) \) in the first subperiod of \( t \). After production, the worker and firm may exogenously separate with probability \( \sigma \).

Since an employed worker is paid at the end of the period, if she wants to consume at the beginning of the period and has no savings, she can borrow \( Q_t \) from a lender. When an employed worker borrows in the first subperiod, she is expected to repay the unsecured debt \( b_t \) once she is paid in the second subperiod, provided she does not default. In the second subperiod, however, an employed worker receives an expenditure shock, \( \tau_t \), drawn from a distribution with CDF \( F(\tau_t) \), which is i.i.d. across agents and time. The expenditure shock is unobservable to anyone but the worker. Her choice of whether to repay in the second subperiod \( d_t \in \{ 0, 1 \} \) is recorded by a credit reporting agency. If the worker does not repay (i.e. \( d_t = 1 \)) we say she is delinquent at time \( t \) and defaults at \( t + 1 \). Default bears a bankruptcy cost \( \epsilon \) in

\[\text{estimate a statistically significant increase in the probability of being employed after flag removal. We show in Section 5.5 that our calibrated model is consistent with the empirical results in Dobbie, et al.}\]

\[\text{17 The worker defaults on both debt and her expenditure shock. In our model, the worker has no incentive to}\]
the second subperiod at \( t + 1 \), which corresponds to both direct costs (legal fees), but is also a reduced form for higher costs borne in other markets due to bad credit (for example, higher insurance premiums, as explored in Chatterjee, Corbae and Rios-Rull \[8\]).

A credit reporting agency records the history of repayments by a worker, which is summarized by a score \( s_t \). This score is the probability that a given worker is type \( i = H \) with discount factor \( \beta_H \) at the beginning of any period \( t \). Given the prior \( s_t \) and the repayment decision \( d_t \), the credit reporting agency updates the assessment of a worker’s type \( s_{t+1} \) via Bayes Rule. \[19\]. Since a high-type worker cares about their future ability to borrow more than a low-type worker, repayment is a signal to a scorer that the worker is more likely to be a high type. Our type score \( s_t \) is therefore not directly comparable to empirical credit scores such as FICO, which orders repayment likelihood on an index from 300 to 850. However, we can rank people by their expected repayment rate within the model, which allows us to group them into credit ratings (subprime, prime, and super prime) based on their ordering in the population, as in the data. \[20\]

Since a worker’s type influences her productivity and default decisions, but is only observable after she is hired by a firm and is never observed by lenders, a worker’s score may be used in hiring and lending decisions. We assume that matches between unemployed job seekers with score \( s_t \), denoted \( u(s_t) \), and firms posting vacancies for such workers, denoted \( v(s_t) \), are governed by a constant returns to scale matching function, \( M(u(s_t), v(s_t)) \). Therefore, an unemployed worker with score \( s_t \) matches with a firm with probability \( f(\theta_t(s_t)) = \frac{M(u(s_t), v(s_t))}{u(s_t)} = M(1, \frac{v(s_t)}{u(s_t)}) \). We will assume that a tighter labor market (higher \( \theta(s_t) \)) increases the job finding rate for workers (i.e. \( f'(\theta_t(s_t)) > 0 \)). The cost to a firm of posting a vacancy for workers with score \( s_t \) is denoted \( \kappa \) and the job filling rate is denoted \( q(\theta_t(s_t)) \), which is decreasing in tightness (i.e. \( q'(\theta_t(s_t)) < 0 \)). Future profits of the firm are discounted at rate \( R^{-1} \). Importantly, this matching process is how we model pre-employment credit screening (PECS).

There are a large number of competitive lenders who have access to consumption goods in the first subperiod, for which they must pay an exogenously given worldwide interest rate of \( R \) in the second subperiod. \[21\] At the beginning of any period, lenders observe each potential borrower’s type score \( s_t \) but not the history of their actions. \[22\] Lenders post a menu of contracts \( C_t(s_t) = \{(Q_{j,t}(s_t), b_{j,t}(s_t))\}_{j=1}^J \), each of which specifies an amount to be lent in the first period. The repayment decision \( q(s_t, k, x) \) is made after the expenditure shock once she has defaulted on debt.

\[18\] We focus on defaults due to expenditure shocks rather than unemployment shocks for two reasons. First, we want to highlight how incentives to repay debt from the labor market can affect strategic defaults, whereas an unemployment shock between the first and second subperiods in our model would lead to a non-strategic default. Second, Chakravorty and Rhee \[10\] report that job loss is the direct cause of only 12.2% of bankruptcy filings, whereas reasons that are more akin to expenditure shocks are reported for a much higher share of bankruptcies.

\[19\] We assume that unemployed workers do not receive the expenditure shock since they have no income with which to pay it. If an unemployed worker received an i.i.d. expenditure shock, she would default with probability one, which would not provide any new information and their score would remain the same.

\[20\] Credit rankings are also employed in Chatterjee, et. al. \[9\].

\[21\] For notational simplicity, we will develop the model without intertemporal savings, but will assume that \( \beta_H \leq R^{-1} \) which, along with the linearity of preferences, ensures that households do not want to save. We focus on the borrowing decision since credit score agencies base their assessments on the basis of borrowing not saving.

\[22\] This anonymity assumption, as in Bernanke, et. al. \[11\] and Carlstrom and Fuerst \[6\], is analogous to assuming that atomistic borrowers are matched with atomistic lenders at random each period, so that there is zero probability of any given lender meeting the same borrower multiple times.
subperiod (i.e. at the beginning of the month), \( Q_{j,t} \), and a promised repayment in the second subperiod (i.e. at the end of the month), \( b_{j,t} \). Lenders realize that households may default on their debt and the probability may differ by worker type, which affects their expected profits for a given contract. As in Netzer and Scheuer [38], after posting these menus the lenders observe all other menus posted and then may withdraw from the market at a cost \( k \).

Lenders play a game against one another by posting menus of contracts (including \((0, 0)\) so that a worker need not borrow) for each observable credit score \( C_t(s_t) \). The game has three stages, all of which occur in the beginning of the first subperiod of \( t \):

Stage 1: Lenders simultaneously post menus of contracts.

Stage 2: Each lender observes all other menus from stage 1. Lenders simultaneously decide whether to withdraw from the market or remain. Withdrawal entails removing the lender’s entire menu of contracts with a payoff of \(-k\) (i.e. it is costly to withdraw).

Stage 3: Workers simultaneously choose the contract they most prefer.

To summarize the information structure, workers observe everything \((i_t, s_t, h_{i,t}, \tau_t)\). Before hiring a worker, a firm only observes the worker’s score \( s_t \), which we refer to as pre-employment credit screening (PECS). After hiring a worker, a firm observes her residual productivity, \( h_{i,t} \), and current type \( i_t \). Lenders only observe the worker’s score \( s_t \); not the broader history of their previous credit market behavior and nothing from their labor market history (such as past wages or human capital choices). The credit reporting agency observes a worker’s current score \( s_t \) and default decision \( d_t \). Credit and labor markets are segmented in the sense that lenders and scorers cannot communicate with firms who know the worker’s type after the hiring decision.

Having described the environment for workers, firms, lenders, and credit reporting agencies, we now describe the timing of actions.

- For an unemployed worker who is currently type \( i \), has score \( s_t \), and productivity \( h_{i,t} \) all determined in the previous period:
  1. Enjoy utility \( z_t \) from leisure \( n_t = 0 \).
  2. Die with probability \( \delta \).
  3. Type score updated, \( s_{t+1}(s_t, \emptyset) \).
  4. Remain type \( i \) with probability \( \rho \).
  5. Surviving workers with score \( s_t \) are matched with a firm in labor sub-market \( s_t \) with probability \( f(\theta_t(s_t)) \).
  6. Choose \( h_{i,t+1} \in \{h, \overline{h}\} \) at cost \( \phi \) if \( h_{i,t+1} = \overline{h} \).

In theory, \( J \) is a choice and any finite number of contracts can be included in the menu. In the equilibrium of our model, \( J = 2 \) is enough since there are two unobservable types with each \( s_t \).

The ability to withdraw contracts after observing all others posted is key to ensuring that an equilibrium exists, counter to purely competitive models with adverse selection (this idea is used in Wilson [44] and Miyazaki [44] for labor and insurance markets, while Livshits, MacGee, and Tertilt [32] extend the game-theoretic argument of Hellwig [22] to unsecured credit markets). That the withdrawal of contracts is costly ensures that the equilibrium is unique.
For an employed worker who is currently type $i$, has score $s_t$, and productivity $h_{i,t}$ all determined in the previous period:

1. First Subperiod:
   1.1 Determine earnings $w_{i,t}$ via Nash Bargaining and work $n_{i,t} = 1$.
   1.2 Choose debt contract $(Q_{j,t}(s_t), b_{j,t}(s_t))$ and consume $Q_{j,t}$.

2. Second Subperiod:
   2.1 Output $y_{i,t} = h_{i,t}$ is created, from which earnings $w_{i,t}$ are paid.
   2.2 Draw expenditure shock $\tau_t$ from CDF $F(\tau_t)$
   2.3 Choose whether to default $d_t \in \{0, 1\}$ and pay $(1 - d_t)(b_{j,t} + \tau_t)$.
   2.4 Type score updated $s_{t+1}(s_t, d_t)$.
   2.5 Separate from employer exogenously with probability $\sigma$ and die with probability $\delta$.
   2.6 Remain type $i$ with probability $\rho$.
   2.7 Choose $h_{i,t+1} \in \{\underline{h}, \bar{h}\}$ at cost $\phi$ if $h_{i,t+1} = \bar{h}$.

4 Equilibrium

We now provide the decision problems for all agents in recursive form. To that end, we let variable $x_t$ be denoted $x$ and $x_{t+1}$ be denoted $x'$. Further, to save on notation we denote $s_{t+1}(s_t, d_t)$ as $s'_d$, with $d = 0$ denoting somebody who was unemployed, $d = 0$ denoting somebody who repaid their debt, and $d = 1$ denoting somebody who defaulted on their debt.

4.1 Worker Decisions

Note that given our timing assumption that the human capital choice is made at the end of the period, this implies that an unmatched worker will always choose $h_{i,t+1} = \underline{h}$ in order to avoid incurring $\phi$. Hence, the value function for an unemployed worker of type $i$ and score $s$ is independent of $h$ and given by

\[
U_i(s) = z + \psi(1 - \delta) \left[ f(\theta(s)) W_i(s'_0) + \left(1 - f(\theta(s))\right) U_i(s'_0) \right],
\]

where the intermediate value functions $W$ and $U$ are defined as

\[
W_i(s') = \rho \max_{h' \in \{\underline{h}, \bar{h}\}} \left( \beta_{i} W_i^{*}_{-i,h'}(s') - \phi_{(h' = \bar{h})} \right)
\]

\[+ (1 - \rho) \max_{h' \in \{\underline{h}, \bar{h}\}} \left( \beta_{-i} W_{-i,h'}(s') - \phi_{(h' = \bar{h})} \right),
\]

\[
U_i(s') = \rho \beta_{i} U_i(s') + (1 - \rho) \beta_{-i} U_{-i}(s'),
\]

9
where $W_{i,h}^*(s')$ is the value function evaluated at equilibrium credit contracts and wages, as described below and the notation $-i$ refers to the other worker type. The unemployed worker receives current flow utility $z$ and survives until the next period with probability $1 - \delta$. She then transits to employment next period with probability $f(\theta(s))$ and remains unemployed with probability $1 - f(\theta(s))$. Note that, with no credit market activity, the unemployed worker’s score changes only through the Markov process on type. Furthermore, since job-finding rates are identical for both worker types conditional on score and all matches have positive surplus, scores are independent of the length of an unemployment spell or total number of spells. Furthermore, the choice of future human capital is independent of the current value.

The value function for an employed worker of type $i$ with current human capital $h$ and score $s$ who has chosen contract $(Q, b)$ and faces wage $w$ is given by

$$W_{i,h}(Q, b, w, s) = Q + \psi w + \int_0^{\infty} \max_{d \in \{0, 1\}} \left[(1 - \delta) \left(\psi V_i(s_d') - d\psi \mathbb{E}[\beta'|\beta_i']\right) - (1 - d)(b + \tau)\right) dF(\tau),$$

where we have introduced the intermediate value function:

$$V_i(s_d') = \left[(1 - \sigma) V_i(s_d') + \sigma \mathcal{U}_i(s_d')\right].$$

and $\mathbb{E}[\beta'|\beta_i] = \rho \beta_i + (1 - \rho) \beta_{-i}$. The first line in (4) reflects borrowing $Q(s)$ to pay for first subperiod consumption and the second subperiod wage $w$ payment. The second line in (4) reflects the strategic decision of whether to go delinquent to avoid paying off $b + \tau$ in the second subperiod followed by default which bears bankruptcy cost $\epsilon$ the following period. Note that the scorer updates his assessment $s_d'$ of the agent’s type given the worker’s default decision $d$. Working backwards, we start by noting that workers know their future type, employment status, and score when they choose $h'$. Furthermore, since human capital only pays off when employed, a person who knows that she will be unemployed will always set $h' = \emptyset$, since she will be able to optimize again before starting any future job. We denote the optimal choice for somebody who is employed as $h^*_i(s')$.

The next decision, working backwards, is the worker’s default choice, taking all other objects (in particular their contract choice) as given. The worker defaults if and only if:

$$\tau > \tau^*_i(s, b) \equiv (1 - \delta) \left[\psi e \mathbb{E}[\beta'|\beta_i'] + V_i(s_0') - V_i(s_1')\right] - b,$$

Thus, higher debt and higher expenditure shocks make default more likely. Furthermore, a lower current discount factor or a lower value from a good reputation make default more likely. We note that the value of a good reputation is given by $V_i(s_0') - V_i(s_1')$ and can be large or small based on the size of $V_i'(s)$ or the gain in score from repaying, $s_0' = s_{t+1}(s_t, 0)$, relative to defaulting, $s_1' = s_{t+1}(s_t, 1)$. Using $\tau^*_i(s, b)$, after integrating by parts and some cancelation, this allows us to evaluate the integral in $W_{i,h}$ for given values of $(Q, b, w)$:

$$W_{i,h}(Q, b, w, s) = Q + \psi w + \psi \int_0^{\tau^*_i(s, b)} F(\tau) d\tau + \psi (1 - \delta) \left[V_i(s_1') - \psi e \mathbb{E}[\beta'|\beta_i']\right]$$

\footnote{Since we assume that discount factors are persistent, we have that $\beta_H > \mathbb{E}[\beta'|\beta_H] > \mathbb{E}[\beta'|\beta_L] > \beta_L$. This means that the term $\psi e \mathbb{E}[\beta'|\beta_i']$ is lower for people who currently have discount factor $\beta_L$.}
We can then write the worker’s surplus (i.e. utility when employed versus unemployed) evaluated at the equilibrium contracts \((Q_i^*(s), b_i^*(s))\) as the difference:

\[
W_{i,h}(Q_i^*(s), b_i^*(s), w, s) - U_i(s),
\]

where the value of unemployment has the same human capital as the value of employment since this difference is meant to capture the off-equilibrium hypothetical of walking away from a match during negotiation. We use this surplus in bargaining below to determine \(w_{i,h}^*(s)\), thereby allowing us to define the equilibrium value functions from above, \(W_{i,h}^*(s)\) as

\[
W_{i,h}^*(s) = W_{i,h}(Q_i^*(s), b_i^*(s), w, s, b).
\]  

4.2 Firm’s Problem and Wage Determination

Recall that after a firm and worker are matched, the worker’s type and human capital choice is observed by the firm. The value function for a firm matched with a worker of type \(i\) and current type score \(s\) who owes \(b\), for a given wage \(w\) is:

\[
J_{i,h}(w, s, b) = \psi \left[ h - w + R^{-1} (1 - \sigma)(1 - \delta) F(\tau_i^*(s, b), J_i(s)) \right] + R^{-1} (1 - \sigma)(1 - \delta) [1 - F(\tau_i^*(s, b)) J_i(s)]
\]

where the intermediate value function \(J_i(s)\) is defined as

\[
J_i(s') = \rho J_{i,h_i^*(s')} \left( w_{i,h_i^*(s')}^*(s'), s', b_i^*(s') \right) + (1 - \rho) J_{-i,h_{-i}^*(s')} \left( w_{-i,h_{-i}^*(s')}^*(s'), s', b_{-i}^*(s') \right)
\]

While \(s\) does not add information for the firm’s inference about worker type, it influences the worker’s bargaining position since it determines their credit contract and hence the worker’s flow surplus from being employed. Since Nash Bargaining ensures that the firm receives a constant fraction of the match surplus as in [13] below, the firm’s surplus will also depend on \(s\) even though the firm knows \(i\) and \(h\) during bargaining since \(s\) affects the worker’s probability of finding another job, should the bargaining process break down. Since free entry ensures that the firm’s value of posting a vacancy is zero, the firm’s surplus from a match is simply \(J_{i,h}(w, s)\).

The wage is then determined by generalized Nash Bargaining in which the worker’s bargaining weight is \(\lambda\). The wage solves:

\[
w_{i,h}^*(s) = \arg\max_w \left[ W_{i,h}(Q_i^*(s), b_i^*(s), w, s) - U_i(s) \right]^{\lambda} J_{i,h}(w, s)^{1-\lambda}
\]  

Given that worker utility and firm profits are linear in earnings, expression [12] amounts to a simple splitting rule for the total surplus so that firms receive fraction \(1 - \lambda\), i.e.

\[
J_{i,h}(w, s) = (1 - \lambda) \left( W_{i,h}(Q_i^*(s), b_i^*(s), w, s) + J_i(w, s) - U_i^*(s) \right),
\]

and the worker’s surplus is fraction \(\lambda\) of the total. Note that the current wage does not directly affect the repayment decision or optimal debt choice of a household due to the linearity of
preferences. If these choices were to depend on the wage, then the wage would affect both the size of the worker’s surplus and the split of the total surplus, creating a nonconvexity that would complicate the analysis.

Firms post vacancies in labor “sub-markets” indexed by an unemployed worker’s score \( s \) so that labor “sub-market” tightness is given by \( \theta(s) \)\(^{26}\). The expected profits from posting a vacancy must be equal to the cost of the vacancy in equilibrium:

\[
\kappa = R^{-1} q(\theta(s)) \left[ s \mathcal{J}_H(s) + (1 - s) \mathcal{J}_L(s) \right]. \tag{14}
\]

This means that market tightness will be higher for workers with higher scores as long as the discounted expected profits of employing an \( H \)-type worker is larger than an \( L \)-type. As a result, workers with higher scores will experience higher job finding rates.

### 4.3 Lender’s Problem and Credit Contract Determination

Lending markets are segmented by \( s \) and are open to people with those scores. Since \( s \) corresponds to the share of \( H \)-type borrowers with that score, it is equivalent to the exogenous fraction of the \( H \)-type from the static model studied by Netzer and Scheuer \[38\]. Invoking their Proposition 2, for sufficiently small \( k > 0 \) (i.e. \( k \to 0 \)), the unique equilibrium to the lending game for credit sub-markets with score \( s \) is the two-contract menu \( \{(Q_H(s), b_H(s)), (Q_L(s), b_L(s))\} \) that solves the following constrained optimization problem (which we will reference as the “Miyazaki-Wilson” problem):

\[
\max_{(Q_H, b_H, Q_L, b_L)} Q_H + \psi \int_0^{\tau^*_H(s, b_H)} F(\tau) d\tau \tag{15}
\]

s.t.

\[
s \left[ -Q_H + R^{-1} F(\tau^*_H(s, b_H)) b_H \right] + (1 - s) \left[ -Q_L + R^{-1} F(\tau^*_L(s, b_L)) b_L \right] \geq 0 \tag{16}
\]

\[
Q_L + \psi \int_0^{\tau^*_L(s, b_L)} F(\tau) d\tau \geq Q_H + \psi \int_0^{\tau^*_H(s, b_H)} F(\tau) d\tau \tag{17}
\]

\[
Q_H + \psi \int_0^{\tau^*_H(s, b_H)} F(\tau) d\tau \geq Q_L + \psi \int_0^{\tau^*_L(s, b_L)} F(\tau) d\tau \tag{18}
\]

\[
Q_H + \psi \int_0^{\tau^*_H(s, b_H)} F(\tau) d\tau \geq Q_L + \psi \int_0^{\tau^*_L(s, b_L)} F(\tau) d\tau \tag{19}
\]

\[
\max_b R^{-1} F(\tau^*_L(s, b)) b + \psi \int_0^{\tau^*_L(s, b)} F(\tau) d\tau.
\]

\(^{26}\)Our sub-markets are indexed by score rather than contract terms as in the models of directed search. A form of block recursivity, as in Menzio and Shi \[33\], exists when firms can screen using scores because the score corresponds to the fraction of good types with that score and hence firms do not need to know the entire distribution of workers over scores to evaluate the expected value of posting a vacancy in that sub-market.
The Miyazaki-Wilson problem says that the credit contract for a worker whose score is \( s \) is designed to maximize the utility of the type \( H \) (low-risk) borrower subject to profitability, incentive compatibility, and participation constraints. The first constraint (16) says that the lender must make non-negative profits on the contract for each score. The first term is the profit (or loss) per type \( H \) borrowers’ contract times the number of high-type borrowers with score \( s \). The second term is profit (or loss) for type \( L \) borrowers’ contract times the number of low-type borrowers with score \( s \). Note that (16) does not rule out cross-subsidization. The second and third inequalities ((17) and (18)) are incentive compatibility constraints. For instance, (17) says that low-type borrowers must choose the contract designed for them rather than the one designed for high-type borrowers. The final constraint (19) says that a low-type borrower must get at least the utility from a credit contract that breaks even and maximizes her utility. That is, the equilibrium contract must give the low-type borrower at least her utility from her least cost separating contract, and will deliver strictly more utility if the contract cross subsidizes low-type borrowers.

We note some special properties of the Miyazaki-Wilson problem and its solution, under the assumption that \( H \)-types have a lower default probability than \( L \)-types (which arises in equilibrium for our calibrated model). First, we need a well defined solution for all credit scores, which would not be the case in purely competitive models (as in Rothschild and Stiglitz [40]). In a competitive model there would be no equilibrium for a score close enough to one, whereas in this model an equilibrium always exists. The allocation arising from the Miyazaki-Wilson problem (corresponding to the Netzer-Scheuer equilibrium) can be one of three types: least cost separating (denoted LCS), cross-subsidized separating (denoted CSS), or pooling (denoted PC). Unlike a purely competitive equilibrium, cross-subsidization can occur in a Netzer-Scheuer equilibrium because lenders can withdraw their contracts. If another lender posted a contract that cream-skimmed (ie, attracted only high-type borrowers) then the lender posting the cross-subsidizing contract would make losses and withdraw for sufficiently low \( k \). Low-type households would then choose the cream-skimming contract, which would then cease to make profits. Second, we want a model in which workers care about their future scores because their score improves credit contract terms (lower rates or looser constraints) and the fact that credit contracts are cross-subsidizing or pooling for high scores ensures this. This would not be the case in a model in which the credit contracts were always least-cost separating, such as the competitive search model of Guerrieri, Shimer and Wright [20]. In that case, an individual’s future credit contracts would be independent of their score, which means that credit scores provide no independent incentive to repay current debts. Finally, the Netzer-Scheuer equilibrium concept ensures that credit market allocations are always statically constrained efficient. In our calibration, most workers are of the high type and have scores in the region where the LCS contract is dominated by either the CSS or PC contracts, so the...

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27 Non-existence follows from the standard argument of Rothschild and Stiglitz: the competitive equilibrium cannot include a pooling contract, since lenders could “cream skim” the high-type borrowers by posting a contract with a slightly tighter borrowing constraint but lower interest rate. On the other hand, if there were very few low-type borrowers and all other lenders were offering separating contracts with borrowing limits then a lender could post a pooling contract and attract the entire market at a profit. Hence, there would be no competitive equilibrium.

28 Their equilibrium concept also has search frictions and contract posting in the credit market and hence an extra endogenous variable. Their framework is directly comparable with the least-cost separating contracts in our work if the cost of posting credit contracts was taken to zero.

29 For instance, in states with PECS bans, interest rates on debt would be independent of credit scores.
welfare gains from using the Netzer-Scheuer equilibrium can be substantial.

In order to understand how type score $s$ affects the credit contract, we first consider the full-information allocation and then demonstrate the general form of optimal constrained allocations that arise for different scores. The full-information allocation is shown in Figure 2. The high-type worker chooses more debt and receives a lower interest rate on this debt since she is less likely to default. But then, if type was private information, a low-type worker would choose the high-type worker’s contract, violating incentive compatibility in (17).

Figure 3 compares two different types of allocations under private information. In this case the low-type worker’s incentive compatibility constraint (17) is binding (as well as their participation constraint (19)). The least cost separating (LCS) contracts are shown in the left box Figure 3. These types of contracts typically arise for low scores (in our calibrated model, they arise for $s < 0.28$, whereas the median score is 0.69). The low-type borrower receives the same amount of debt as under full information and pays the risk-adjusted break-even interest rate. On the other hand, the high-type borrower’s contract is distorted because of the binding incentive compatibility constraint of the low-type worker. In particular, the high-type borrower receives less debt than the low-type borrower, although her interest rate is still equal to the risk-adjusted break even rate on her loan. This puts the high-type borrower on a lower indifference curve than in Figure 2.

The full information contract maximizes an employed borrower type $i$’s utility subject to zero expected profits on the type $i$ contract. This corresponds to maximizing $Q_i + \psi \int_0^{\tau^*_i(s,b_i)} F(\tau) d\tau$ (as in (15)) for each type $i$, subject to $Q_i \leq R^{-1} F(\tau^*_i(s,b_i)) b_i$ (as in (16)). Graphically, this gives us indifference curves with slopes $\frac{dQ}{db} = \psi F(\tau^*_i(s,b_i)) \geq 0$ and isoprofit curves with slopes $\frac{dQ}{d\tau} = R^{-1} [F(\tau^*_i(s,b_i)) - F'(\tau^*_i(s,b_i)) b_i]$. Since for a given $(s,b)$, $\tau^*_L(s,b) < \tau^*_H(s,b)$, the slope of the type $H$ indifference curve is greater than the slope of the type $L$. Furthermore, since the interest rate on these contracts is given by $\frac{\tau}{\tau^*_i}$, the interest rate can be seen as the inverse of the slope of a ray from the origin to the contract point. This is analogous to the continuous asset version of Chatterjee, et. al. [7].
Figure 3: Least Cost (LCS) vs. Cross-Subsidized Separating (CSS) Contracts

As a worker’s score rises the optimal contract typically switches from LCS to CSS. For CSS contracts, the low-type worker’s participation constraint is slack, because she still receives the full-information level of debt but pays a lower interest rate (illustrated by $Q_L$ being above the low-type zero profit curve in Figure 3b). This moves the low-type borrower to a higher indifference curve, while shifting the effective zero-profit curve for high-type borrowers downward by the total subsidy to low-type borrowers. The high-type borrower’s contract is given by the intersection of the low-type borrower’s new indifference curve and the high-type borrower’s effective zero-profit curve. The CSS contract delivers more debt to the high-type borrower than the LCS contract for the same score, but carries a higher interest rate than the LCS contract. The CSS contract dominates the LCS for intermediate scores ($0.28 \leq s < 0.42$ in our calibration) because the extra interest paid per high-type borrower to subsidize low-type workers is more than offset by the high type’s utility gains from receiving more debt (e.g. loosening her credit limit).

The third contract type is pooling (PC), which can arise as $s$ increases further (above 0.42 in our calibrated model) as the interest rate cross-subsidy to low-type workers becomes extremely generous. In this case, unlike the previous two, the high-type household’s incentive constraint binds. That this constraint binds can be seen in Figure 4a, where the interest rate paid by a low-type borrower in the CSS is so low that a high-type borrower would prefer the low-type contract to the one prescribed to her. With so few low-type borrowers with a high score,

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31 We say typically because we cannot prove this in general because a higher score both changes the lender’s participation constraint and the default thresholds. When we compute equilibria we verify which contract type is optimal and these examples are illustrative of how the contracts change with score.

32 In some settings, such as the constant risk insurance model in Netzer and Scheuer, the $h$-type incentive compatibility constraint never binds. This is not the case in our model because of our interaction of adverse selection and moral hazard, which means that default rates (and therefore the indifference curves and zero-profit curves) depend on debt for each borrower. In Appendix 3.4, we algebraically show the $h$-type incentive compatibility constraint can bind and why it is more likely for higher $s$. 

15
the subsidy per low-type contract is too generous and the high-type borrower would rather have the low-type borrower’s subsidized rate, even though this gives her less credit. Therefore both incentive compatibility constraints bind, which means that the contract must be pooling (i.e. each type receives the same debt and interest rate). We find this contract by maximizing the utility of the high-type borrower subject to the pooled zero-profit condition. Graphically, this is given by the tangency between the high-type worker’s indifference curve and the pooled zero-profit curve, as in Figure 4b.

4.4 Type Scoring

Given the prior probability \( s \) that a worker is type \( H \), the credit reporting agency forms a Bayesian posterior \( s' \) the worker is type \( H \) conditional on seeing whether she repays \( d \):

\[
s'_0(s) = \frac{\rho F\left(\tau^*_H(s, b^*_H(s))\right) s + (1 - \rho) F\left(\tau^*_L(s, b^*_L(s))\right) (1 - s)}{F\left(\tau^*_H(s, b^*_H(s))\right) s + F\left(\tau^*_L(s, b^*_L(s))\right) (1 - s)},
\]

\[
s'_1(s) = \frac{\rho \left[ 1 - F\left(\tau^*_H(s, b^*_H(s))\right) \right] s + (1 - \rho) \left[ 1 - F\left(\tau^*_L(s, b^*_L(s))\right) \right] (1 - s)}{\left[ 1 - F\left(\tau^*_H(s, b^*_H(s))\right) \right] s + \left[ 1 - F\left(\tau^*_L(s, b^*_L(s))\right) \right] (1 - s)}.
\]

For an unemployed person, we have

\[
s'_0(s) = \rho s + (1 - \rho)(1 - s).
\]

The formula for the high-type borrower’s indifference curve is the same as before. The slope of the pooled zero-profit curve is given by

\[
\frac{dg}{db} = \frac{d}{db} \left\{ R^{-1} \left[ sF(\tau^*_H(s, b)) + (1 - s)F(\tau^*_L(s, b)) \right] b \right\}.
\]

Figure 4: Violation of Incentive Compatibility in CSS Vs. Pooling Contracts
Typically a credit score is a measure of how likely the borrower is to repay. In the context of our model, \( s \) is a “type” score. In equilibrium we can map \( s \) to a credit score (i.e. the probability of repayment given \( s \)) as follows:

\[
\Pr(d = 0|s) = F\left(\tau^*_H(s, b^*_H(s))\right) s + F\left(\tau^*_L(s, b^*_L(s))\right)(1 - s). \tag{23}
\]

### 4.5 Distributions

We denote the measure of workers of type \( i \) over employment status \( e \in \{0, 1\} \) (where 1 denotes employed and 0 denotes unemployed) and with score no greater than \( s \) by his history. Conditioning on other variables observed by an econometrician; since unobservable type does not change across an agent’s lifetime, all that matters is that the score encapsulates something about the worker’s type revealed by his history.

\[
\mu'_{i,1} (s') = (1 - \delta) \int_0^1 f(\theta(s)) \left[ \rho d\mu_{i,0}(s) + (1 - \rho) d\mu_{-i,0}(s) \right] \mathbb{I}_{\{s'_0(s) \leq s'\}} \\
+ \rho (1 - \delta) \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F(\tau^*_s (s, b^*_s (s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) \right\} d\mu_{i,1}(s) \\
+ (1 - \rho) (1 - \delta) \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) \right\} d\mu_{-i,1}(s)
\]

where \( \mathbb{I}_{\{s'_d(s) \leq s'\}} \) is an indicator function which takes the value one if \( s'_d(s) \leq s' \) and zero otherwise.

For the unemployed we have two regions. For scores lower than the population share of high-types (i.e., for \( s < \pi_H \)):

\[
\mu'_{i,0} (s') = (1 - \delta) \int_0^1 [1 - f(\theta(s))] \left[ \rho d\mu_{i,0}(s) + (1 - \rho) d\mu_{-i,0}(s) \right] \mathbb{I}_{\{s'_0(s) \leq s'\}} \\
+ \rho (1 - \delta) \sigma \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F(\tau^*_s (s, b^*_s (s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) \right\} d\mu_{i,1}(s) \\
+ (1 - \rho) (1 - \delta) \sigma \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F(\tau^*_1 (s, b^*_1 (s))) \right\} d\mu_{-i,1}(s).
\]

For scores above \( \pi_H \) we must add the newborns who start unemployed with \( s = \pi_H \). That is,

\[\text{Our score is consistent with credit scoring in reality, in that past actions in the credit market are used to forecast the likelihood of an individual defaulting on her debt (though her type). In our model, this is reflected by interest rates falling with credit rating, which we calibrate to be consistent with the data, as seen in Figure 6a. This is true even if the score is not highly predictive of a borrower’s future likelihood of default after conditioning on other variables observed by an econometrician; since unobservable type does not change across an agent’s lifetime, all that matters is that the score encapsulates something about the borrower's type revealed by his history.}\]
for \( s \geq \pi_H \):

\[
\mu'_{i,0}(s') = \delta + (1 - \delta) \int_0^1 \left[ 1 - f(\theta(s)) \right] \left[ \rho d\mu_{i,0}(s) + (1 - \rho)d\mu_{-i,0}(s) \right] \mathbb{I}\{s'_0(s) \leq s'\} 
\]

\[
+ (1 - \delta) \sigma \int_0^1 \left\{ \mathbb{I}\{s'_0(s) \leq s'\} F(\tau^*_i(s, b^*_i(s))) + \mathbb{I}\{s'_1(s) \leq s'\} F(\tau^*_i(s, b^*_1(s))) \right\} d\mu_{i,1}(s)
\]

\[
+ (1 - \delta) \sigma \int_0^1 \left\{ \mathbb{I}\{s'_0(s) \leq s'\} F(\tau^*_{-i}(s, b^*_i(s))) + \mathbb{I}\{s'_1(s) \leq s'\} F(\tau^*_{-i}(s, b^*_1(s))) \right\} d\mu_{-i,1}(s).
\]

### 4.6 Definition of Equilibrium

A steady-state Markov equilibrium consists of the following functions:

1. Worker value functions, \( U_i(s), W^*_i,h(s) \), satisfy (1) and (4).
2. Default threshold functions, \( \tau^*_i(s, b) \), satisfy (6).
3. Firm value functions, \( J_{i,h}(s) \), satisfy (10).
4. Wage functions, \( w^*_i,h(s) \), satisfy (12).
5. Market tightness functions, \( \theta(s) \), satisfy the free entry condition (14).
6. Credit market contracts, \( \{(Q^*_i(s), b^*_i(s))\}_{i \in \{H,L\}} \), satisfy (15)-(19).
7. The updating functions, \( s'_d \), satisfy (69) and (71).
8. Stationary measures of each worker type over scores, \( \mu^*_i,1(s), \mu^*_i,0(s) \) that satisfy (24) through (26) with \( \mu'_{i,e}(s) = \mu_{i,e}(s) = \mu^*_i,e(s) \) for \( e \in \{0,1\} \) and \( i \in \{L,H\} \).

### 4.7 Full Information Equilibrium Characterization

Since we will define matching efficiency relative to the equilibrium outcomes of a full information model, we provide a characterization for that case. We first make parametric assumptions to guarantee that workers borrow within a period and do not save across periods (A.1), that the match surplus of both workers is positive (A.2), and that credit contracts are unique (A.3). We also ensure that all workers would repay some positive level of debt (A.4) and that all workers default with positive probability (A.5). Finally, we set the cost of investing in human capital relative to the future wage gain so that the \( H \)-type invests but the \( L \)-type does not (A.6).

**Assumption 1 .**

A.1 \( \psi < (\omega R)^{-1}, \beta_L < \beta_H \leq R^{-1} \)

A.2 \( z < h_L \)

A.3 \( F''(\tau) \leq 0 \)
A.4 $F(\beta_L(1-\delta)\psi \epsilon) > 0$

A.5 The support of $\tau$ is unbounded above.

A.6 $\beta_H \psi \lambda(\bar{h} - \underline{h}) \geq \phi > \beta_L \psi \lambda(\bar{h} - \underline{h})$.

In Appendix A we define a full-information equilibrium and prove the following:

**Theorem 1** Under Assumption 1, there exists a full information steady-state Markov equilibrium where $i$ is publicly observable that is characterized by the following equations:

$$
\theta_H > \theta_L \implies f(\theta_H) > f(\theta_L),
$$

(27)

$$
w_H > w_L,
$$

(28)

$$
F_0(\tau^*_H(b^*_H)) > F_0(\tau^*_L(b^*_L)),
$$

(29)

$$
h^*_H = \bar{h}, h^*_L = \underline{h}.
$$

(30)

Importantly, with full information under the parametric restrictions in Assumption 1, high-type workers have higher job finding rates (in (27)), have higher wages (in (28)), have lower default rates (29), and have higher human capital (30).

5 Quantitative Exercise

To demonstrate how a poverty trap may arise and how markets respond to a policy banning PECS, we compute an equilibrium of the economy and then change the determination of market tightness so that it is independent of type score (consistent with a ban).

5.1 Computing a Private Information Equilibrium

Existence of equilibrium with adverse selection is complicated by the scoring functions, which are not contractions, and the Miyazaki-Wilson programming problem generating credit contracts. In Appendix B we define a computationally feasible version of the private information equilibrium, prove existence for a set of parameters, and provide the algorithm we use to compute the equilibrium. 35

5.2 Calibration

A model period is taken to be a month. We use a Cobb-Douglas matching technology so that the job-finding and filling rates are given by $f(\theta) = \theta^\alpha$ and $q(\theta) = \theta^{\alpha-1}$. We assume that expenditure shocks have an exponential CDF: $F(\tau) = 1 - e^{-\gamma \tau}$ 36 Once these functional forms are set, we must choose parameter values. Some values we set externally, while the remainder we choose to match data with model moments. The parameter values are listed in Table 1.

---

35 Given our focus on computable equilibria, we discretize the support of the Bayesian forecast as in Chatterjee, et al. 9 in our definitions and proofs. One can use the existence proof for the given parameter space as an initial guess in computing equilibria for other regions of the parameter space.

36 In order to guarantee model convergence, we include a small fixed probability of a shock that is too large to pay for any borrower. See the computational appendix for details.

37 Hall 21 uses a value of $\alpha = 0.24$. Shimer 41 uses $\alpha = 0.72$. Other authors have used values in between, with many settling on 0.5. See Gertler and Trigari 19.
Many of our parameters are taken from previous papers or otherwise calibrated externally. We set $\beta_H R = 1$ to ensure that $H$–type workers do not save (so neither will $L$–type workers). The remainder are chosen to match data in the initial steady state, with the exception of the worker’s bargaining parameter, which we choose to match the change in unemployment duration for workers with bad credit following the ban on PECS.

We have chosen moments on credit card debt from various sources, some of which are new to the quantitative household credit literature (to our knowledge). The average credit card rate and share of borrowers in each credit bracket are from the Consumer Financial Protection Bureau’s “Consumer Credit Card Market” report [12]. The interest rates are “total costs of credit” for each credit bracket in 2015, less 2% for inflation, and reported as monthly rates. These are the most comparable numbers to the model interest rates, since some people pay all balances monthly in the data (and therefore do not pay interest) whereas everyone pays interest in the model.

We also use the CFPB’s data to compute credit card debt to income and the share of debt that is defaulted upon. Total credit card debt was $779$ Billion in 2015, which we divide by labor’s share of average monthly GDP, which was $0.60 \times $6.108 Trillion. Finally, we use the CFPB’s reported share of accounts that are more than three months past due as our measure of the delinquency rate.

Our moments on labor market outcomes are taken from economy wide reports since we do not have merged data with credit scores and earnings or job-finding rates. For the residual earnings $50 – 10$ ratio, we use the log of median earnings minus the log of the earnings of the tenth percentile, which is reported by Lemieux [28]. We choose $\lambda$ so that unemployment duration for workers in the bottom quartile of credit scores falls by 28% following a ban on

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source or Informative Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_H$</td>
<td>0.997</td>
<td>No inter-temporal savings condition</td>
</tr>
<tr>
<td>$R - 1$</td>
<td>0.17%</td>
<td>Risk free rate 4%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.21%</td>
<td>45 Years in Market</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.50</td>
<td>Matching Elasticity</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.6%</td>
<td>Separation Rate, Shimer (2005)</td>
</tr>
<tr>
<td>$h_H$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>$z$</td>
<td>0.4</td>
<td>Shimer (2005)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source or Informative Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_H$</td>
<td>62.0%</td>
<td>Subprime through super prime rates, CFPB (2015)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.521</td>
<td>Subprime through super prime rates, CFPB (2015)</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>0.742</td>
<td>Subprime through super prime rates, CFPB (2015)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.976</td>
<td>Debt to Labor Income, CFPB (2015)</td>
</tr>
<tr>
<td>$h_L$</td>
<td>0.572</td>
<td>Residual Earnings 50 – 10, Lemieux (2006)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.49</td>
<td>28% Drop in UE Duration for Bottom 26% Post Ban</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.68</td>
<td>Job-finding rate, Shimer (2005)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>15.25</td>
<td>Delinq. debt share, CFPB (2015)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.999</td>
<td>Persistence of Super Prime Status</td>
</tr>
</tbody>
</table>
Table 2: Model Fit

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data Value</th>
<th>Model Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Super Prime CC Rate, top 49%</td>
<td>0.87%</td>
<td>0.84%</td>
</tr>
<tr>
<td>Prime CC Rate, 34 – 50%</td>
<td>1.17%</td>
<td>1.19%</td>
</tr>
<tr>
<td>Subprime CC Rate, 0 – 33%</td>
<td>1.60%</td>
<td>1.61%</td>
</tr>
<tr>
<td>Debt to Labor Income</td>
<td>21.24%</td>
<td>21.23%</td>
</tr>
<tr>
<td>Delinq. Rate</td>
<td>0.95%</td>
<td>0.96%</td>
</tr>
<tr>
<td>Residual Earnings 50 – 10</td>
<td>0.57%</td>
<td>0.57%</td>
</tr>
<tr>
<td>Monthly Job Finding Rate</td>
<td>45.0%</td>
<td>45.0%</td>
</tr>
<tr>
<td>Persistence of Super Prime Status</td>
<td>85%</td>
<td>87.4%</td>
</tr>
</tbody>
</table>

Note: Appendix 2 has definitions of model moments.

PECS\(^{35}\) This is in line with estimates by Friedberg, Hynes, and Pattison \(^{18}\). Using the Survey of Income and Program Participation, they estimate a 28% decline in unemployment duration for the financially distressed, who represent 26% of the population, living in states that ban PECS. For the job finding rate we use the monthly rate implied by Shimer \(^{41}\).

As a consequence of our calibrated parameters \(\pi_H, \delta,\) and \(\rho,\) the stationary share of \(H-\text{types}\) satisfies \(P_H = (1 - \delta)(\rho P_H + (1 - \rho)(1 - P_H)) + \delta \pi_H.\) For our calibration this gives \(P_H = 56.2\%\), which in turn affects the aggregate default rate and debt-to-income ratios.

5.3 Properties of Stationary Equilibrium

The equilibrium stationary distribution of workers over “type” scores and employment status is determined by the relative solvency and default rates of high-type versus low-type workers, as well as job-finding rates. Since type scores are not directly observable, we construct a data comparable distribution by sorting borrowers by their default probability and then assigning credit ratings consistent with the empirical shares of households within each rating. This means that as in the data, the bottom third are labeled “subprime”, the next 15% are “prime” and the top 50% are “super prime”. Figure 5a plots the histogram of workers over credit ratings constructed in this way. While the population shares over credit ratings are defined to match the data, the share of workers of each type within each credit rating is endogenous – it depends on the relative default rates of each worker type in equilibrium. We plot these distributions in Figure 5b, where it is clear that the most low-type workers have subprime credit, while less than 10% of high-type workers have such poor credit since they only default due to extremely large expenditure shocks. Likewise, nearly 90% of high-type workers have scores in the super prime range.

The composition of types over ratings determines the gradient of interest rates, default rates, and debt-to-income ratios with respect to credit rating. This can be understood by considering the average and type-specific default rates by credit rating, which we report in red text in Figures 5a and 5b. The average default rate is falling with credit rating, from 1.42% to 0.66%, but this is because the composition of borrowers in each group is changing, not because...\(^{35}\) A common alternative calibration strategy for workers’ bargaining parameter is to impose the Hosios condition, which in our model would be \(\lambda = 1 - \alpha.\) With type switching, there is no way to ensure that workers and firms share the same discount factor for the duration of a match, so the Hosios condition does not guarantee that market tightnesses are efficient even under full information.
an individual always defaults less when her score is higher. For example, the average super prime high-type borrower actually defaults roughly three times more than high-type borrowers with subprime credit. This is because she receives much less credit when subprime and because she has a strong incentive to repay. In fact, a high-type borrower in the prime category has the strongest incentive to repay and therefore the lowest average default rate because default generates the largest drop in score in the updating function in Figure 7b.

Our calibration is also consistent with dimensions of the data not used to fit the model. Figure 6a reproduces the fit of the model’s interest rates with data, while Figure 6b shows the shares of debt held by borrowers with each credit rating, both in the data and our model. The fact that credit shares are increasing with rating is a success of the Netzer and Scheuer equilibrium concept and would not be generated by models in which credit contracts were least cost separating for all scores (since high-type households would always have less debt than low-type households in such a model to maintain incentive compatibility as is clear in Figure 3a).

Notes: Unconditional shares are constructed to match the data, type-conditional are inferred from model. Red numbers are average default rates for workers in each rating, unconditional on type in Figure 5a and conditional on type in Figure 5b.

Figure 5: Histograms over Credit Ratings

---

We plot all theoretical functions over the score range 0.01 – 0.99 because these scores are never reached in theory.

The data is from the Consumer Financial Protection Bureau’s 2017 credit card report [13].
The stationary distribution is derived from the law of motion for a worker’s employment status and score, which depends on the job-finding rate for unemployed and the average change in score for employed workers. Figure 7a plots the job-finding rate \( f(\theta(s)) \), which is bounded below by the low-type worker’s full information rate and above by the high-type worker’s. The finding rate rises monotonically for scores between zero and one, reflecting the rising surplus associated with high-type workers. Since most high types have superprime credit, while most low types are subprime, high types find jobs at a substantially higher rate than low types, on average. Of course, some unlucky high-type workers have substantially lower scores than average and therefore experience lower job-finding rates due to being pooled with the low-type. The median unemployed worker, marked by \( p_{U_{50}} \) on the graph, has a score of 0.62 and therefore a job finding rate of roughly 47\%\(^{41}\).

The score updating functions are plotted in Figure 7b, the shape of which can be understood by the relative solvency and default rates of the two worker types. Because both worker types repay with a high probability at all scores, there is very little information revealed by repayment\(^{42}\). The score therefore updates very slowly in the positive direction, with \( s'_0(s) \) just slightly above the forty-five degree line. However, the default rate for low-type workers is up to ten times that of high types. Therefore, observing a borrower default leads to a dramatic downward update of her score, thus \( s'_1(s) \) is much lower than \( s \) for most scores. The median employed borrower has a score of 0.67, implying that a default would reduce her score to 0.08 (which would make her a subprime borrower).

\(^{41}\)Throughout, we use \( p^x \) to denote the \( x^{th} \) percentile of scores. If we condition on type or status then we use a subscript, so that the notation \( p_{U}^x \) is the score held by \( x^{th} \) percentile of the unemployed and \( p_{H}^x \) is the score held by the \( x^{th} \) percentile of high types. Likewise, \( p_{HU}^x \) is the score held by the \( x^{th} \) percentile of the high-type unemployed.

\(^{42}\)These rates are implied by the interest rate targets, which are relatively low relative to the risk-free rate.
5.4 Covariance Between Earnings and Credit History

Our model generates a positive covariance between earnings and credit histories through two channels. First, unobservable heterogeneity in underlying types cause differences in both average credit rating and earnings. High-type workers have higher earnings than low types for a given credit history and have better credit histories on average, which creates a positive correlation between credit score and earnings “across” types. Second, a worker of a given type with better credit has a larger threat point, since she knows that she can walk away from a match and find another job with high probability. This means that a better credit score causes higher wages “within” each worker type.

Figure 8 demonstrates these two covariances for our model calibration. On average, prime borrowers earn 22.5% more than subprime and super prime earn an additional 31.4% than prime. Over 98% of this total covariance is driven by the “across” component, since high-type workers earn roughly 75% more than low-type workers for each credit rating and the ratio of earnings by type is essentially constant over scores.

While there is no direct empirical counterpart to these numbers, there is a strong negative association between adverse credit events and residual earnings. We demonstrate this by estimating an earnings regression from the 2016 Survey of Consumer Finance, in which respondents answered three questions: Q1) whether they were ever delinquent on debt in 2015, Q2) whether they were ever delinquent on debt by more than two months, and Q3) whether they were ever turned down for a loan. We use the answers to these questions (1 = “yes”) to estimate the cross-sectional regression

$$\log \text{earnings}_i = \beta_1 Q1_i + \beta_2 Q2_i + \beta_3 Q3_i + \text{controls}_i + \varepsilon_i,$$

(31)
where controls include a quadratic function of age as well as dummies for years of education, gender, race, industry, and occupation. Table 3 reports our estimated $\beta$ coefficients across various specifications. We consistently find a significantly large negative coefficient on adverse credit terms, with a magnitude ranging from 20.3% lower earnings for delinquency alone to 36.7% lower earnings for all three adverse events. These numbers are of similar magnitudes as our model’s overall covariance between credit rating and earnings, although we do not know exactly how much these events would move someone’s credit rating.

<table>
<thead>
<tr>
<th>Specification</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1</td>
<td>$-20.3^{***}$</td>
<td>$-14.7^{***}$</td>
<td>$-13.6^{***}$</td>
</tr>
<tr>
<td></td>
<td>(4.9)</td>
<td>(2.8)</td>
<td>(2.6)</td>
</tr>
<tr>
<td>Q2</td>
<td>$-13.9^*$</td>
<td>$-12.7^*$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.9)</td>
<td>(1.7)</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td>$-10.4^{**}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.332</td>
<td>0.333</td>
<td>0.333</td>
</tr>
<tr>
<td>Obs</td>
<td>4451</td>
<td>4451</td>
<td>4451</td>
</tr>
</tbody>
</table>

Notes: Estimates from equation $\log \text{earnings}_i = \beta_1 Q_1 + \beta_2 Q_2 + \beta_3 Q_3 + \text{CONTROLS}_i$, where column (1) restricts $\beta_2 = \beta_3 = 0$ and column (2) restricts $\beta_3 = 0$. Questions are 1) were you ever delinquent on debt payments, Q2) were you ever delinquent by more than two months, and Q3) were you ever turned down for a loan. Parenthesis report absolute values of t-statistics. Significance levels represented as $^{***} = 1\%$, $^{**} = 5\%$, $^* = 10\%$.

Table 3: Cross-Sectional Regression of Earnings on Credit Events

Finally, the fact that our “within” covariance is small is supported by estimates in Herkenhoff, Phillips, and Cohen-Cole [25], who report the average change in annual earnings for an individual one year before and after the removal of a bankruptcy flag from their credit report. This effectively isolates the effect of credit above and beyond any permanent worker type and turns out to be roughly 1% in their panel data (similar to our model finding that moving from subprime to super prime increases earnings by 0.9% on average).
5.5 Effect of Default on Credit and Earnings

Another way of seeing that the “within” component is small is to estimate regressions on data simulated from our calibrated model that are in line with Dobbie, et al. [17]. They use chapter-7 bankruptcy filers as a control group to look at credit and labor market outcomes for chapter-13 filers in the years after the bankruptcy flag is removed from their credit reports. This occurs seven years after default, whereas chapter-7 filers must wait ten years. They find a large improvement in credit outcomes but very little change in labor markets. While we cannot perform their exact exercise because we have only one type of default, we now show that defaults have large effects on an individual’s credit access but limited effect on their earnings.

We estimate linear regressions of earnings and credit balances on lagged default using a panel of 10,000 individuals simulated from our calibrated model. That is, we regress log-earnings and borrowing on lagged default after subtracting individual and time fixed effects:

\[ 100 \times \log(w_{i,t} + 1) = FE_{i}^{w} + FE_{t}^{w} + \beta^{w}(D_{i,t}^{7}) + \epsilon_{i,t}^{w}, \]

\[ 100 \times \frac{Q_{i,t}}{Q} = FE_{i}^{Q} + FE_{t}^{Q} + \beta^{Q}(D_{i,t}^{7}) + \epsilon_{i,t}^{Q}, \]

where \( w_{i,t} \) is simulated earnings for individual \( i \) in period \( t \), \( Q_{i,t} \) is the amount borrowed by that individual which we normalize by mean borrowing \( Q \). The \( FE \) terms are fixed effects in each regression. Note that we negate the default indicators so that coefficients have the same sign as in Dobbie et al, who compare people who have a default flag fall off relative to those who retain the flag. We estimate these regression models on thirty years of simulated data.

\[ \text{Notes: Average earnings by credit rating and worker type. Left vertical axis corresponds to high-type workers and right vertical axis to low-type workers.} \]

Figure 8: Credit Ratings and Wages by Type
for people with twenty to fifty years of access to credit markets, which is in line with the age restrictions used by Dobbie, et al.

Table 4 shows our point estimates and 95% confidence intervals along with the empirical estimates of Dobbie, et al.’s. While our effects are larger than theirs for both earnings and credit, our coefficients are within their 95% confidence intervals, so cannot be rejected.

Table 4: Panel Regressions of Earnings and Credit on Default

<table>
<thead>
<tr>
<th>Model Estimate</th>
<th>Earnings</th>
<th>Credit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model C.I.</td>
<td>[1.39%, 1.47%], [8.1%, 8.9%]</td>
<td></td>
</tr>
<tr>
<td>Dobbie, et al. Estimate</td>
<td>0.00%</td>
<td>6.93%</td>
</tr>
<tr>
<td>Dobbie, et al. C.I.</td>
<td>[-1.95%, 1.95%], [4.80%, 9.06%]</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Row labeled “Model” presents \( \beta \) estimates from equations 100 \( \times \) log\( (w_{i,t} + 1) = FE^{w} + FE^{w} + \beta^{w} D^{w}_{i,t} + \epsilon^{w}_{i,t} \) and 100 \( \times \) Qi,t = FE^{Q} + FE^{Q} + \beta^{Q} D^{Q}_{i,t} + \epsilon^{Q}_{i,t} using panel of 10,000 simulated individuals over thirty years from calibrated model. Row labeled “Dobbie, et al. Estimate” is the negation of one-year ahead estimates of Chapter 13 filers versus Chapter 7 filers in Dobbie, et al. Rows labeled as C.I. are the 95% confidence intervals of each point estimate.

5.6 Poverty Traps

The definition of a poverty trap is not universally agreed upon, so we discuss two possible definitions. The first is a situation in which a worker’s experience is made worse due to her credit score relative to an otherwise identical worker. In our case, this happens for the high-type households. A high-type worker who becomes unemployed with a bad score has a harder time finding a job than one who becomes unemployed with a good score. This leads to further divergence between the two, since the worker with good credit will find a job sooner and therefore have an even better credit score in the future. This is because employed high-type workers experience an increase in their credit score on average while the unemployed do not borrow and therefore are unable to improve their score through repayment. We say that the high-type household is subject to a poverty trap because, on average, she experiences a decrease in her score (relative to being employed) and the decrease in score makes it harder to find a job in the next period.

We use two figures to understand how such a poverty trap may arise. Figure 9a uses the job-finding rates (as in Figure 7a) to compute the expected unemployment duration of an unemployed high-type household as a function of her score \( s \). It is falling with score, reflecting the fact that high-type workers are more productive in equilibrium and tend to have higher scores. Note that there are some high-type workers who end up with low scores, illustrated by the vertical bar at the tenth percentile. This is the first part of the poverty trap: an unlucky high-type worker with a bad credit history has a hard time finding a job and therefore expects longer unemployment spells than if her score was higher.

---

44Since our estimates from model simulations are of the effect of a default appearing on somebody’s credit report, whereas their research design uses the default being removed, we have multiplied our estimates by \(-1\) so that the signs match. For Dobbie, et al’s effect of removing default on earnings, we use the estimate from their Table V, column (2) and for the effect on credit we use their estimate from Table III column (2) normalized by the mean in Table III column (1).
Figure 9: Poverty Trap for High Types

We next look at the average change in a worker’s score when unemployed relative to when she is employed. Figure 9b plots this function for high-type workers. On average, an employed high-type worker experiences a rising score, while her score mean reverts while unemployed. It is evident from the figure that an unlucky high-type worker with a low score therefore experiences a deterioration in her score relative to if she was employed, which reinforces the longer unemployment duration.

Another way of defining the poverty trap is relative to the full information equilibrium. The idea is that the job-finding rate for a worker with a low score may be strictly lower than if her human capital was observable. Again, consider Figure 9a and compare the finding rates between the private and full information economies. The high-type worker experiences a lower job-finding rate for all $s < 1$ while the opposite is true for the low-type worker. For example, the bottom quintile of unemployed high-type workers have scores below 0.62 and a job-finding rate below 47%, which is 4.4% below the full information rate of high-type workers. Private information has the opposite effect for the low-type workers, 10% of whom have scores above 0.62 and therefore finding rates above 47%, which is 10% above their full information rate.

The extent of the poverty trap relative to full information depends on the high-type worker’s score. Using the score percentiles in Figure 9a we can say that the poverty trap adds roughly one day to the median high-type worker’s unemployment duration, almost five days for the

The average relative change in score is defined as:

$$
\Delta(s) = s - F_0 \left( \tau_{\bar{H}} \left( s, b | \bar{H}(s) \right) \right) s_0(s) - F_1 \left( \tau_{\bar{H}} \left( s, b | \bar{H}(s) \right) \right) s_1(s)
$$

The change while unemployed is 0 while the average change while employed is the negative of the above expression. Thus, the relative average change is $\Delta(s)$. 

---

45 The average relative change in score is defined as:
25th percentile, and just under ten days for the lowest decile of high-type job seekers. The bottom one percent of these workers have a poverty trap of nearly three weeks.

A useful summary of the labor market impact of default can be computed as the present value of wages conditional on repayment minus the same value conditional on default. We compute these measures for each worker type and employment status, as well as the unconditional average, amortize them over 10 years, and report this measure relative to the average wage in Table 5. Our model generates expected wage losses from default through two mechanisms. First, the job-finding rate falls due to a lower score. Second, the worker’s bargaining position becomes weaker and therefore their wages fall even conditional on being employed. The average across all worker types, scores, and employment statuses amounts to 0.89% of earnings in each month for ten years, with high-type workers suffering 1.34% and low-type only 0.32%. Interestingly, our endogenous estimate of 0.89% is roughly half of the small loss imposed following bankruptcy for an average of 10 years in Chatterjee, et. al. [7] who consider a proportional loss (denoted γ in their paper) of 1.9%.

Table 5: Present Value of Wage Losses From Default

<table>
<thead>
<tr>
<th></th>
<th>Employed</th>
<th>Unemployed</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>High type (β_H)</td>
<td>1.32%</td>
<td>1.75%</td>
<td>1.34%</td>
</tr>
<tr>
<td>Low type (β_L)</td>
<td>0.31%</td>
<td>0.48%</td>
<td>0.32%</td>
</tr>
<tr>
<td>Overall</td>
<td>0.97%</td>
<td>1.25%</td>
<td>0.89%</td>
</tr>
</tbody>
</table>

5.7 Labor Market Efficiency

We define a measure of labor market efficiency by considering the average difference between each worker type’s average finding rate in the economy with private information relative to the full information economy. For the high-type households in the calibrated economy, the monthly job-finding rate averages 49.7%, which is 1.3 percentage points lower than the efficient 50.9%. On the other hand, Low-type households have an inefficiently high job-finding rate. In the calibrated economy their monthly job-finding rate is 40.5%, which is 1.7 percentage points higher than the efficient rate.

6 Policy Experiment: Banning Credit Checks

We now solve the economy with the same parameters, except that vacancies cannot be conditioned on a worker’s score which implies market tightness \( \theta \) is independent of \( s \). That is, we substitute \( q(\theta) \) for \( q(\theta(s)) \) in the free entry condition in (14). While market tightness and the job-finding rate are therefore independent of \( s \) (and independent of \( \beta_i \) as before), match surplus and therefore bargained wages still depend on \( s \) since the worker’s score affects her bargaining position post match. Credit markets operate as before the ban, except that the workers’ incentives to repay endogenously fall: since default (which lowers a worker’s credit score) does not affect the worker’s job finding rate, there is less punishment associated with default.
6.1 Changes in Labor and Credit Market Variables

The ban’s effect on aggregate variables can be seen in Table 6. The average job-finding rate rises as we move from the PECS stationary equilibrium to the one without it (from 45.0% to 45.7%). This occurs for three reasons. First, the finding function is concave in scores, which means that the finding rate rises mechanically from pooling, keeping all other equilibrium variables constant. Second, the equilibrium unemployment pool’s composition shifts towards higher productivity workers following the ban. This shift occurs because high-score workers find jobs at a higher rate in the baseline economy and the high-types are disproportionately represented in the upper credit ratings. Therefore, the high-types have shorter unemployment durations and make up a smaller fraction of the unemployed pool than they do in the population as a whole. Once the ban goes into effect, they have the same job-finding rate as everyone else, and therefore their share of the unemployed is the same as their share of the population. Third, and most interestingly, eliminating PECS weakens the threat point of $H$—type workers since they can no longer leverage their high scores to find a job quickly if the bargaining process was to break down. This means that employing a $H$—type worker with a high score (which represents the vast majority of them) is more profitable without scores.

At first glance, this result may seem counter to the empirical results in Figure 1a. The model can be reconciled with the data by noting that the data is unlikely to represent a new stationary equilibrium, but instead represents the incentive of firms to post vacancies for a distribution of unemployed workers that reflects the PECS equilibrium and wages that are unlikely to have fully adjusted. We therefore calculated the number of vacancies that firms would post if they 1) could no longer use scores to screen, 2) drew from the initial stationary distribution of unemployed when posting a vacancy, and 3) could only bargain for new wages each month with probability $\frac{1}{12}$ (so wages remained fixed for a year on average). In this situation, firms post 2% fewer vacancies, and the job finding rate falls from 45% to 44.9%.

The effects on job-finding rates differ substantially across the score distribution, as seen...
in Figure 10a. We find that the job finding rate for workers with very low scores rises substantially, which causes the average duration of unemployment for the bottom quintile of workers to decline by 27%. This is consistent with Friedberg, Hynes, and Pattison [18]. They estimate that workers in the bottom quintile of financial health enjoy a 25% decline in expected unemployment duration when PECS bans are enacted at the state level, while our calibrated model predicts that the bottom quintile of borrowers would enjoy a 27.8% reduction in unemployment duration. While the bottom quintile in our model is not precisely the same as the bottom quintile of Friedberg, Hynes, and Pattison [18], we are encouraged that our calibration predicts similar labor market effects of the PECS ban for financially distressed workers.

The ban also affects the credit market through the repayment decisions of borrowers, as seen in both Figure 10b and Table 6. The average interest rate rises from 1.16% to 1.24% as the average default rate rises from 0.92% to 1.16% as borrowers are no longer incentivized to repay in order to find jobs faster in the future. However, these incentive effects differ across worker types and scores. Specifically, the high-type worker’s repayment rate falls more than the low type’s, since they respond to dynamic incentives more in the first place. Since high-type workers have higher scores on average, Figure 10b shows larger declines in repayment as scores rise. This is consistent with the empirical evidence from Figure 1b, where we found the largest declines in repayment rates for people with higher scores. The new stationary equilibrium therefore features less separation of worker types by credit score (i.e. more workers of each type in the prime rating rather than low types in subprime and high-type in super prime).

The ban affects workers by changing equilibrium labor and credit market functions, which in turn affect dynamics of credit ratings. Since the dynamic incentive to repay falls, especially for high types, there is less information generated by observing a default. Figure 11a demonstrates the effect on scoring - the default curve moves closer to the 45 degree line. This reinforces the lower incentive to repay and helps drive default rates upward. The equilibrium effect is less separation of types by scores, as demonstrated in the histograms plotted in Figure 11b. There are now more high types with prime and subprime credit than in the baseline, while the opposite is true for low types.

46 We plot changes in the expected unemployment duration in Figure 10a since it is in more easily interpreted units (weeks). The relationship with the job finding rate is monotone - a higher finding rate implies a lower duration.

47 Note that the change in default rate is zero at both \( s = 0 \) and \( s = 1 \) since these are absorbing scores and therefore the dynamic incentives to repay are zero for both types in both the baseline economy and the one with PECS bans.
Repay Update, Post Ban
Default Update, Post Ban
Repay Update, Baseline
Default Update, Baseline

(a)

Notes: Scoring functions before (green) and after (red) employers are restricted from using scores.

(b)

Figure 11: Effect of Ban on Updating
Notes: Wages and post-match expected discounted profits by worker type and credit rating before PECS ban and after.

Figure 12: Effect of PECS Ban on Bargaining

6.2 Changes in Wages and Profits

Banning pre-employment credit screening also affects the size and split of rents after a match has occurred by affecting a worker’s bargaining position. We demonstrate this in Figures 12a-12d. Prior to the ban, there is a clear positive effect of credit rating on wages for both worker types and, likewise, a downward effect on profits. Wages depend on the score because it affects the job-finding rate of unemployed workers. A higher credit score means that the worker would find a job faster if she was to walk away from her current match. One one hand, this means that the match surplus is smaller overall. However, it also means that the worker has a better bargaining position and therefore captures a larger share of the surplus as her credit score rises. The net effect causes wages to rise with credit score for a given worker type. Of course, the unconditional wage rises even faster with credit rating since high-type workers have higher wages at all scores. The opposite profile appears in profits - conditional on worker
type, profits are highest for workers with bad credit ratings. On the other hand, the level of profits is strictly higher for high-type workers than for low types, due to their higher labor productivities, which generates the positive profile of vacancies with respect to score.

Once the ban goes into place, job finding rates are no longer score specific, which means that a worker’s outside option is less affected by her score. This leads to a near complete flattening of the wage profiles in Figures 12a and 12b and profit profiles in Figures 12a and 12b. Relative to the baseline, this causes a decline in wages for workers with high scores but a rise in wages for subprime and prime, while profits move in the opposite direction.

Finally, post-match expected discounted profits rise on average after the ban because workers’ threat points change. As shown in Figures 12c and 12d, the post-match profitability of employing a worker of either type with super prime credit rises, since these workers experience a deterioration in their threat points. On the other hand, the post-match profitability of employing a worker with prime or subprime credit falls since these workers experience an increase in their threat points (i.e. they no longer suffer from low job finding rates due to their bad credit). On net, however, post-match profits rise after the ban, since almost all high-type households have excellent credit (post-match profits rise in 58.4% of matches overall, which is driven by an increase in 82.3% of matches with high-type workers).

Note that since there is no change in the cost of posting a vacancy, ex-ante expected profits from posting a vacancy is zero in both environments. The above increase in average post-match profits occurs after the ban goes into effect through changes in the equilibrium threat point of workers, which are taken as given during bargaining. The result does not mean that firms would choose to ignore PECS in an environment where they are not banned. In particular, if equilibrium threat points are consistent with all other firms choosing to ignore PECS, then it is individually rational for an atomistic firm to conduct PECS in order to raise post-match expected discounted profits. We demonstrate this point in Figure 13 where calculate the expected profit in excess of the cost of posting a vacancy for a firm that is allowed to restrict job applicants to their vacancy to those with scores above some threshold value when all other firms are posting unconditional vacancies. It is clear that firms would like to use PECS if given the option.

### Table 6: Effect of Employer Credit Ban

<table>
<thead>
<tr>
<th>heightMoment</th>
<th>Baseline</th>
<th>After Ban</th>
<th>Full Info.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>β_H</td>
<td>β_L</td>
<td>Avg.</td>
</tr>
<tr>
<td>Job Finding Rate</td>
<td>49.0%</td>
<td>40.7%</td>
<td>45.0%</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>0.84%</td>
<td>1.57%</td>
<td>1.16%</td>
</tr>
<tr>
<td>Debt to Income</td>
<td>21.7%</td>
<td>20.15%</td>
<td>21.34%</td>
</tr>
</tbody>
</table>

### 6.3 Changes in Matching Efficiency

It is important to note that, while banning PECS eliminates the poverty trap, most of the people with inefficiently low finding rates (i.e. unlucky, high-type workers) experience lower...
job-finding rates. The pre-PECS ban equilibrium is nearly separating, with only 12.4% of low-type workers carrying scores below $s = 0.51$, which is the threshold for which durations rise post-ban (as seen in Figure 10a). On average, high-type workers experience 3.8 days more unemployment following the ban, which is large when compared to the effects of much broader labor market policies. For example, Card and Levine [5] estimate that a thirteen week extension of unemployment benefits increases average unemployment duration by roughly one week.

This exercise shows that banning PECS may actually increase the average job-finding rate, but still does so at the cost of labor market efficiency measured relative to the full information job-finding rate. This can be seen by the small fall in the median job-finding rate, which is due to a decline in the job-finding rate for almost all of the high-type workers (who are 62% of the population in our baseline economy and tend to have high scores). In fact, the unemployment rate for high-type workers rises from 5.5% to 5.8% following the ban. Relative to the efficient job-finding rate, the low-type worker’s finding rate is 8.7% higher after the ban (in levels, it rises from 40.7% to 45.7%). On the other hand, high-type workers are now pooled with more low-patience/productivity workers and therefore experience a more inefficiently low finding rate than in the economy with pre-employment credit screening. Their finding rates falls from 49.0% to 45.7%, which is 5.8% lower than the efficient level. When we average over these absolute changes, then the ban moves job-finding rates away from efficiency by 7.2%.

6.4 Changes in Welfare

We now study the net effect of the ban on welfare. For the unemployed, Figure 14a shows how the direct change on market tightness and finding rates affects these workers. Workers with low type scores experience a gain in welfare, since they experience a higher job finding

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50 We make this comparison to put the magnitude into context, not because they are directly comparable policies. Specifically, unemployment benefits likely work through labor supply rather than demand, as in our model.

51 See the appendix for the definition of these welfare measures.
rate than when firms can discriminate based on score. Furthermore, high-type workers gain more since they put a higher weight on finding a job due to their higher \( \beta \). The welfare gains are falling for both worker types as scores rise, eventually becoming negative for those with high scores. Likewise the welfare effect is positive but falling for employed workers, as seen in Figure 14b. On average, low-type workers gain from the ban and high-type workers lose. The effects (both positive and negative) are magnified for unemployed workers since any change in job finding rates affects them immediately.

We summarize these conditional averages in Table 7. In aggregate, only 43% of the population have a positive gain from banning PECS. However, the distributional effects are substantial, with high-type workers losing slightly on average (equivalent to 0.62% of consumption each month), but low-type workers gaining a lot, especially the unemployed (3.70% of consumption). If we consider the ex-ante lifetime utility of a worker before her type is realized (i.e. who has a \( \pi_H \) probability of being high-type and will enter the economy as unemployed), then there is a welfare loss of 0.09% of monthly consumption for a worker born into the economy without PECS, relative to being born into an economy that allows them.

Even within a worker type and employment status, there is substantial heterogeneity in the welfare effect of banning PECS. We illustrate this in Figure 15, which shows that subprime

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52We can evaluate the welfare effects for workers at each score, even if the theoretical measure of them is zero. For example, we calculate the value function of high-type workers at \( s = 0 \) and low-type at \( s = 1 \) when we solve the model. However, we omit these points from our plots because there are no workers who actually experience them in equilibrium.

53If private information persisted after hiring, then we would expect reduced expected profits due to overpaying the low-productivity type. This would make scores more valuable than in our baseline model. So, getting rid of PECS would have bigger negative effects on matching and welfare losses would be larger than what we are estimating.

54So the ban would be voted down.
Table 7: Avg. Welfare Effects

<table>
<thead>
<tr>
<th></th>
<th>High-Type</th>
<th>Low-Type</th>
<th>Ex Ante</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>-0.61%</td>
<td>0.38%</td>
<td></td>
</tr>
<tr>
<td>Unemployed</td>
<td>-0.74%</td>
<td>3.70%</td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>-0.62%</td>
<td>0.59%</td>
<td>-0.09%</td>
</tr>
</tbody>
</table>

workers gain from banning PECS no matter the worker’s type or employment status, while the opposite is true for super prime workers, who lose from the ban regardless of type or status. In each case, the unemployed gains/losses are larger than the employed because they are immediately affected by changes in the job-finding rate. Furthermore, the high-type employed have muted welfare changes since they greatly discount the effect of the ban on their future job finding rates when they become unemployed.

7 Conclusion

As the difference-in-difference empirical results in Figure 1 document, a ban on PECS leads to a decline in vacancies in those affected occupations and a relative rise in delinquencies for those with better credit ratings. We provide a framework to link labor and credit markets to understand such facts by extending the workhorse Diamond-Mortensen-Pissarides model to include ex-ante private information about worker productivity, while also building a novel framework for including credit scores when borrowers have private information about their repayment rates. The model provides a theoretical foundation for why employers may use credit histories in the hiring process and how this practice can create a poverty trap. Combining these two microeconomic models highlights the connection across markets in the presence of private information and overcomes the Lucas Critique in our policy analysis. The basic idea of using information pertinent to credit markets to screen people whose unobservable type is correlated with outcomes in the labor market can be extended to cover many other markets. Our model also provides a channel to endogenize the income losses associated with default which is typically taken as exogenous in models of consumer default like Chatterjee, et. al. [9]. Finally, we find that a unified theory of labor and credit markets under adverse selection is important since the direct effect of PECS bans in the labor market spill over to the credit market.

Our model complements the empirical literature on the effect of banning PECS by address-
ing the effect on unmeasurable outcomes – labor market efficiency and welfare. We show that these effects can be large even when the aggregate effects of banning PECS on measurable outcomes may appear small (see Table [6]). Banning PECS increase the job finding rate of low-score workers, but these workers are predominantly low productivity. The opposite is true for high-score workers, who mostly have high productivity: they experience an increase in average unemployment duration of 3.8 days following the ban. While efficiency is unequivocally reduced, the welfare effects are more nuanced. Low-type workers, who tend to have relatively bad credit, gain from the ban, the equivalent of 0.59% of monthly consumption. High-type workers, who are the majority, lose 0.62% of monthly consumption, so that only 43% of the population gains from the ban. We conclude that policy makers should consider the trade-off between equity and efficiency when considering PECS bans.

References


A Full Info Equilibrium

The full information equilibrium consists of \((w_i^{fi}, q_i^{fi}, b_i^{fi}, Q_i^{fi}, h_i^{fi})\)_{i \in \{L,H\}} and associated values \(W_i^{fi}, U_i^{fi}, J_i^{fi}\). This gives the following system of equations:

\[
\begin{align*}
  b_i^{fi} &= \arg\max_b \left[ R^{-1} F \left( \beta_i (1 - \delta) \psi \epsilon - b \right) b + \psi \int_{0}^{\delta_i (1 - \delta) \psi \epsilon - b} F(\tau) \, d\tau \right] \quad (34) \\
  Q_i^{fi} &= R^{-1} F \left( \beta_i (1 - \delta) \psi - b_i^{fi} \right) b_i^{fi} \quad (35) \\
  J_i^{fi} &= \psi \left[ h - w_i^{fi} + R^{-1} (1 - \delta) (1 - \sigma) J_i^{fi} \right] \quad (36) \\
  \kappa &= R^{-1} q \left( \theta_i^{fi} \right) J_i^{fi} \quad (37) \\
  W_i^{fi} &= Q_i^{fi} + \psi \left[ w_i^{fi} + \int_{0}^{\delta_i (1 - \delta) \psi \epsilon - b_i^{fi}} F(\tau) \, d\tau + \delta_i (1 - \delta) (V_i^{fi} - \psi \epsilon) \right] \quad (38) \\
  U_i^{fi} &= z + \psi \beta_i (1 - \delta) \psi \left[ f(\theta_i^{fi}) W_i^{fi} + \left[ 1 - f(\theta_i^{fi}) \right] U_i^{fi} \right] \quad (39) \\
  V_i^{fi} &= \sigma U_i^{fi} + (1 - \sigma) W_i^{fi} \quad (40) \\
  W_i^{fi} - U_i^{fi} &= \lambda [W_i^{fi} + J_i^{fi} - U_i^{fi}] \quad (41)
\end{align*}
\]

A first useful result is that \(b_i^{fi}\) and \(Q_i^{fi}\) are independent of \(h\) and that the problem is concave whenever \(F''(\tau) \leq 0\). This can be seen from the first order condition on \(b\):

\[
R \psi F'(\beta_i (1 - \delta) \psi - b_i^{fi}) = F'(\beta_i (1 - \delta) \psi - b_i^{fi}) - F'(\beta_i (1 - \delta) \psi - b_i^{fi}) b_i^{fi}. \quad (42)
\]

The left hand side has intercept \(\psi R < 1\) when \(b = 0\) while the right hand side has an intercept of one. The slope of the left-hand side is \(-R \psi F'(\beta_i (1 - \delta) \psi - b_i^{fi})\) while the slope of the right-hand side is \(-2F''(\beta_i (1 - \delta) \psi - b_i^{fi}) + F'''(\beta_i (1 - \delta) \psi - b_i^{fi})\). We can guarantee that the right-hand side is steeper than the left-hand side by assuming \(F''(\tau) \leq 0\), so the intersection will have \(b_i^{fi} > 0\). Furthermore, this condition shows that \(b\) is increasing in \(\beta_i (1 - \delta) \psi\) (and therefore in \(\beta_i\)) since implicit differentiation gives:

\[
\frac{\partial b}{\partial (\beta_i (1 - \delta) \psi)} = \frac{(1 - R \psi) F'(\beta_i (1 - \delta) \psi - b) - F''(\beta_i (1 - \delta) \psi - b) b}{(2 - R \psi) F''(\beta_i (1 - \delta) \psi - b) - F''(\beta_i (1 - \delta) \psi - b) b} > 0 \quad (43)
\]

and from this we can conclude that \(F(\beta_H (1 - \delta) \psi - b_H^{fi}) \geq F(\beta_L (1 - \delta) \psi - b_L^{fi})\) by considering \(\frac{\partial x}{\partial b}\) in

\[
\frac{\partial x}{\partial b} \left[ (1 - R \psi) F'(x) - F'(x) b \right] = F'(x), \quad (44)
\]

which means that the optimal \(b\) from this first order condition can only increase if the term inside of \(F(\beta_i (1 - \delta) \psi - b)\) increases.

We now consider a single worker of type \(i\) who chooses \(h\) freely for the next period, given the equilibrium. That is, the worker’s type is public, but the deviation to \(h\) rather than \(b_i^{fi}\) is
not, so that $\theta^f_i$ is fixed. Using that $Q$ and $b$ are independent of $h$, we can write the resulting deviating match surplus as

$$S_i(h) = S^f_i + \psi(h - h^f_i).$$  \hfill (45)

Then after bargaining, the worker will receive an additional surplus of $\lambda \psi(h - h^f_i)$ from a given deviation. In order to ensure that the $H-$type chooses $h$ we need

$$\beta_H \left[ S_H(h) - S_H(h^f) \right] \geq \phi$$  \hfill (46)

which is true if

$$\beta_H \psi \lambda (h - h^f) \geq \phi,$$  \hfill (47)

and likewise we will have

$$\beta_L \left[ S_L(h) - S_L(h^f) \right] < \phi$$  \hfill (48)

if

$$\beta_L \psi \lambda (h - h^f) < \phi.$$  \hfill (49)

### B Private Information Equilibrium

We now define a computationally feasible version of the private information equilibrium, prove existence for a set of parameters, and provide the algorithm we use to compute the equilibrium.

Fix a grid $\mathcal{S} = \{s_1, s_2, ... s_N\}$ with each $s_i \in [0, 1]$ and $s_i < s_{i+1}$. We equilibrium functions as vectors of length $N$, and list them in a matrix

$$v = (W_{H,j}, W_{L,j}, U_{H,j}, U_{L,j}, J_{H,j}, J_{L,j}, Q_{H,j}, Q_{L,j}, b_{H,j}, b_{L,j}, w_{H,j}, w_{L,j}, \theta_j, s^0_{0,j}, s^1_{1,j}, s^\emptyset_{\emptyset,j})_{j=1}^N.$$  

Note that the equilibrium value functions, debt contracts, wages, and market tightnesses are independent of the measures of people over scores, so we have omitted $\mu$ from this matrix. We will also fix human capitals by type, so that everybody with $\beta_H$ has $h$ and everybody with $\beta_L$ has $h$.

Since the updated values, $s'$, represent a Bayesian update of type, they may not lie on the grid $\mathcal{S}$. A vital step in the approximation is to assign future scores with the following probabilities. For any $m \in \{0, 1, \emptyset\}$, we define

$$s'_{m,j}(s^t; v) = \begin{cases} 
    s_{m,j}^t - s_{t-1} & \text{if } s^t = \inf_{x \in \mathcal{S}} \{ x \geq s_{m,j}' \} \\
    s_{m,j}' - s^t & \text{if } s^t = \sup_{x \in \mathcal{S}} \{ x \leq s_{m,j}' \} \\
    0 & \text{otherwise.}
\end{cases}$$

These probabilities are then used to create expected values of all continuations based on $m \in \{0, 1, \emptyset\}$, which we denote by writing with an over-tilde. For example,

$$\tilde{W}_{H,m,j} = \sum_{t=1}^N s'_{m,j}(s^t; v) \times W_{H,t},$$

and similar expressions for each of the remaining continuation values.
We can now begin defining the mapping that takes a $v$ and returns updated continuation values, debt contracts, wages, and market tightness. We denote the updated matrix by $T[v]$, with an element $T_{j,v}[v]$. Starting with the value of being employed, we have

$$T_{j,1}[v] = Q_{H,j} + \psi w_{H,j} + \psi \int_0^{\bar{\tau}_H(s_j,b_{H,j})} F(\tau) d\tau + \psi(1-\delta)\left[\bar{V}_{H,1,j} - \psi eE[\beta'|\beta_H]\right] \quad (52)$$

$$T_{j,2}[v] = Q_{L,j} + \psi w_{L,j} + \psi \int_0^{\bar{\tau}_L(s_j,b_{L,j})} F(\tau) d\tau + \psi(1-\delta)\left[\bar{V}_{L,1,j} - \psi eE[\beta'|\beta_L]\right], \quad (53)$$

where we define

$$\bar{V}_{i,m,j} = \left[(1-\sigma)\bar{W}_{i,m,j} + \sigma\bar{U}_{i,m,j}\right], \quad (54)$$

$$\bar{W}_{i,m,j} = \rho \bar{W}_{i,m,j} + (1-\rho)\bar{W}_{-i,m,j}, \quad (55)$$

$$\bar{U}_{i,m,j} = \rho \bar{U}_{i,m,j} + (1-\rho)\bar{U}_{-i,m,j}. \quad (56)$$

$$\bar{\tau}_i(s_j,b) = (1-\delta)\left[\psi eE[\beta'|\beta_i] + \bar{V}_{i,0,j} - \bar{V}_{i,1,j}\right] - b. \quad (57)$$

Similarly, unemployed values are updated as

$$T_{j,3}[v] = z + \psi(1-\delta)\left[f(\theta_j)\bar{W}_{H,m,j} + (1-f(\theta_j))\bar{U}_{H,m,j}\right], \quad (58)$$

$$T_{j,4}[v] = z + \psi(1-\delta)\left[f(\theta_j)\bar{W}_{L,m,j} + (1-f(\theta_j))\bar{U}_{L,m,j}\right]. \quad (59)$$

The updated firm values are given by

$$T_{j,5}[v] = \psi \left[\bar{h} - w_{H,j} + R^{-1}(1-\delta)(1-\sigma)\left(F(\tau_{H,j}^*),\bar{f}_{H,j0} + [1-F(\tau_{H,j}^*)]\bar{f}_{H,j1}\right)\right] \quad (60)$$

$$T_{j,6}[v] = \psi \left[\bar{h} - w_{L,j} + R^{-1}(1-\delta)(1-\sigma)\left(F(\tau_{L,j}^*),\bar{f}_{L,j0} + [1-F(\tau_{L,j}^*)]\bar{f}_{L,j1}\right)\right]. \quad (61)$$

For credit contracts, we solve the maximization problem

$$(T_{j,7},T_{j,8},T_{j,9},T_{j,10})[v] = \arg\max_{Q_H,b_H,Q_L,b_L} Q_H + \int_0^{\bar{\tau}_H(s_j,b_H)} F(\tau) d\tau \quad \text{s.t.}$$

$$s_j \left[-Q_H + R^{-1}F(\bar{\tau}_H(s_j,b_H))b_H\right] + (1-s_j) \left[-Q_L + R^{-1}F(\bar{\tau}_L(s_j,b_L))b_L\right] \geq 0 \quad (62)$$

$$Q_H + \psi \int_0^{\bar{\tau}_H(s_j,b_H)} F(\tau) d\tau \geq Q_H + \psi \int_0^{\bar{\tau}_H(s_j,b_H)} F(\tau) d\tau \quad (63)$$

$$Q_L + \psi \int_0^{\bar{\tau}_H(s_j,b_H)} F(\tau) d\tau \geq Q_L + \psi \int_0^{\bar{\tau}_H(s_j,b_H)} F(\tau) d\tau \quad (64)$$

$$Q_L + \psi \int_0^{\bar{\tau}_L(s_j,b_L)} F(\tau) d\tau \geq \max_b R^{-1}F(\bar{\tau}_L(s_j,b))b + \psi \int_0^{\bar{\tau}_L(s_j,b_L)} F(\tau) d\tau. \quad (65)$$
Wages are then updated via Nash Bargaining, so that
\[
\psi T_{j,11}[v] = \lambda \left[ W_{H,j} - U_{H,j} + J_{H,j} \right] - Q_{H,j} - \psi \int_0^{\tau_{H,j}} F(\tau)d\tau - \psi(1 - \delta) \left[ \tilde{V}_{H,1,j} - \psi \mathbb{E}[\beta'|\beta_H] \right]
\]
\[
\psi T_{j,12}[v] = \lambda \left[ W_{L,j} - U_{L,j} + J_{L,j} \right] - Q_{L,j} - \psi \int_0^{\tau_{L,j}} F(\tau)d\tau - \psi(1 - \delta) \left[ \tilde{V}_{L,1,j} - \psi \mathbb{E}[\beta'|\beta_L] \right]
\]
And market tightness comes from the zero profit condition
\[
T_{j,13}[v] = q^{-1} \left( \frac{R\kappa}{s_j \mathcal{J}_{H,j} + (1 - s_j) \mathcal{J}_{L,j}} \right).
\] (68)
And finally the laws of motion are updated as
\[
T_{j,14}[v] = \frac{\rho F(\tau_{H,j} s_j + (1 - \rho) F(\tau_{L,j}^*(1 - s_j))}{F(\tau_{H,j} s_j + F(\tau_{L,j}^*) (1 - s_j)},
\] (69)
\[
T_{j,15}[v] = \frac{\rho \left[ 1 - F(\tau_{H,j}) \right] s_j + (1 - \rho) \left[ 1 - F(\tau_{L,j}^*) \right] (1 - s_j)}{1 - F(\tau_{H,j})} s_j + \left[ 1 - F(\tau_{L,j}^*) \right] (1 - s_j),
\] (70)
\[
T_{j,16}[v] = \rho s_j + (1 - \rho)(1 - s_j).
\] (71)

In order to prove existence, we must find a compact domain, \( \mathcal{M} \subset \mathbb{M} \), such that \( T : \mathcal{M} \to \mathcal{M} \) continuously. We then apply Brouwer’s Fixed Point Theorem. We must then provide a condition so that \( \beta_H \) workers always choose \( \bar{n} \) and \( \beta_L \) workers choose \( \bar{h} \) and we will have established existence of the type of equilibrium studied in our quantitative model. We now prove that there is a non-empty set of expenditure shock distributions and preference parameters for which the above mapping has a fixed point and the assumption that \( \beta_H \)-type people choose \( \bar{n} \) while \( \beta_L \)-type people choose \( \bar{h} \) is individually rational.

First observe that type-switching ensures that scores will always fall between \( 1 - \rho \) and \( \rho \), so we set a grid \( \mathcal{S} \subset [1 - \rho, \rho] \). To find the appropriate compact space, we first that the match surplus is always positive since \( \bar{h} > \bar{z} \). We can therefore bound the discounted utilities below by zero and above by the surplus achieved from delivering all consumption to the worker in the first subperiod (i.e. the surplus delivered if the net interest rate was zero). Denote these values by \( (W_{H}^*, W_{L}^*, U_{H}^*, U_{L}^*, J_{H}^*, J_{L}^*) \), which means that \( \theta_j \in [0, q^{-1}(\frac{R}{s_j \mathcal{J}_{H,j} + (1 - s_j) \mathcal{J}_{L,j}})] \). We put \( Q_{H,j}, b_{H,j} \) in \( [0, R^{-1} \bar{h}] \times [0, \bar{n}] \) and \( Q_{L,j}, b_{L,j} \) in \( [0, R^{-1} \bar{h}] \times [0, \bar{h}] \). Likewise, we put \( w_{H,j} \in [0, \bar{h}] \) and \( w_{L,j} \in [0, \bar{h}] \) and the updated scores each lie in \([0, 1]\). The resulting space, \( \mathcal{M} \), is compact and \( T : \mathcal{M} \to \mathcal{M} \) by the definitions above and the fact that we have taken extreme values of each variable to bound \( \mathcal{M} \).

**Theorem 2** If \( \beta_L, R, \rho, \psi, \) and \( F(\tau) \) are such that the Miyazaki-Wilson programming problem in lines [64] through [65] has unique maximal arguments that are continuous in \( v \) then there exists \( v^* \in \mathcal{M} \) such that \( v^* = T[v^*] \).
Continuity of \( T \) is immediate for everything except for columns 7 through 10, which represent the credit market contracts. If these columns are continuous, as assumed in the above theorem, then the entire mapping is continuous and we can apply Brouwer’s Fixed Point theorem to guarantee a \( v^* = T[v^*] \). We now prove that the set of parameters and shock distributions that satisfy this assumption is non empty.

**Lemma 1** If \( \beta_L = 0, \ 0 < \rho < 1, R^{-1}(1 - \rho) - \psi > 0, F'(\tau) \geq 0, \text{ and } F''(\tau) < 0 \) then there exists a fixed point to \( T \) in a compact space \( \mathcal{M} \) and, for a sufficiently small cost of investing in human capital, people with \( \beta_H \) always prefer human capital \( h \) over \( h^55 \).

For \( \beta_L = 0 \), the impatient type always defaults on any debt, no matter the \( v \). This means that the last constraint is always slack. The term \( \int_0^{\tau_L(s_j, b_L)} F(\tau)d\tau = 0 \), therefore \( T_{j,9}[v] \geq T_{j,7}[v] \) from the impatient type’s incentive compatibility constraint. Furthermore, any value of \( T_{j,10}[v] \) is consistent with the constraints, since \( \tau_L(s_j, b_L) = 0 \). Finally, we can conclude that \( T_{j,7}[v] = T_{j,9}[v] \), since we can increase the objective by increasing \( Q_H \) whenever \( Q_H < Q_L \). The problem can therefore be simplified to

\[
T_{j,8}[v] = \arg\max_{0 \leq b_H \leq h} R^{-1}s_j F(\tau_H(s_j, b_H))b_H + \psi \int_0^{\tau_H(s_j, b_H)} F(\tau)d\tau, \tag{72}
\]

with

\[
T_{j,7}[v] = R^{-1}s_j F(\tau_H(s_j, T_{j,8}[v])) T_{j,8}[v] \tag{73}
\]
\[
T_{j,9}[v] = T_{j,7}[v] \tag{74}
\]
\[
T_{j,10}[v] = T_{j,8}[v]. \tag{75}
\]

Since the maximization problem determining \( T_{j,8} \) is over a compact and continuous constraint correspondence (a constant interval) and the objective is continuous, we can apply the Theorem of the Maximum. Furthermore, the second derivative of the objective being maximized in equation \( 72 \) is given by

\[
R^{-1}s_j F''(\tau_H(s_j, b)) - F'(\tau_H(s_j, b))(R^{-1}s_j - \psi) \tag{76}
\]

which is strictly negative since we have assumed that \( F''(\tau) \leq 0 \) and \( R^{-1}s_j - \psi \geq R^{-1}(1 - \rho) - \psi > 0 \). That means that \( T_{j,8}[v] \) is single-valued and therefore a continuous function of \( v \), as are \( T_{j,7}[v], T_{j,9}[v], \text{ and } T_{j,10}[v] \). Finally, the human capital selection is rational since \( \beta_L = 0 \) means that impatient people choose \( h \) for any \( \kappa > 0 \) whereas for \( \kappa \) sufficiently small, patient people will choose \( \tilde{h} \) since it generates higher earnings in the next period.

## C Credit Allocation

In this appendix we argue that the Miyazaki-Wilson programming problem gives payoffs that are equivalent to the robust subgame perfect equilibrium of the three stage game in which lenders post contracts in stage 1, observe which others have been posted in stage 2 and then may withdraw at cost \( k > 0 \), and then borrowers apply to their preferred contracts.

\[55\] Alternatively, we could have \( F'(\tau) > 0 \) and \( F''(\tau) \leq 0 \).
C.1 Equivalence Between Miyazaki-Wilson Problem and Robust SPE

We derive indirect utilities for households of each type that take the form
\[ u_i(Q, b) = Q + \psi \int_0^{\tau^*_i(s,b)} F(\tau) d\tau \]  
and profit functions for lenders of the form
\[ p_i(Q, b) = -Q + R^{-1} F(\tau^*_i(s,b)) b. \]

Note that these payoffs are both quasi-linear in \( Q \).

This differs from the problem as written in Netzer and Scheuer [38] who have consumer preferences of the form
\[ \nu_i(u_0, u_1) = q_i u_0 + (1 - q_i) u_1, \]
where \( q_i \) is the probability that the consumer avoids an accident, so that a high type is less likely to have one. The profits for an insurance agent who delivers these utils in each state of the world are then
\[ \pi_i(u_0, u_1) = q_i u^{-1}(u_0) + (1 - q_i) u^{-1}(u_1), \]
where \( u^{-1}(u) \) is the inverse of the felicity function and is convex. Note that the consumer’s payoff is linear in both arguments, while the insurer’s is convex.

We now argue that the equivalence result in Netzer and Scheuer’s (we will shorten to NS below) Proposition 2 applies in our model. Rather than build the notation for the extensive form game, we refer the reader to their appendix A.4 to point out where they use their functional form and how their arguments need to be altered in our case.

They first prove that any robust SPE, if it exists, must solve the Miyazaki-Wilson problem. Starting on page 27, there are three places where a variational argument is used in their proof:

1. In Step 1 on the bottom of page 27 and top of page 28, NS argue that there must be at least one principal who is making zero profits from the contracts that they post. Suppose not (i.e. all principals make strictly positive profit). Their variational argument adds \( \epsilon \) to each state of the world for one \( j \)'s contract, which attracts all agents, is incentive compatible (since \( \epsilon \) is being added to both the high and low risk in each state) and still makes profits. In our case, we can simply add \( \epsilon \) to \( Q \) for each borrower type. This would attract all agents and be profitable for sufficiently small \( \epsilon \).

2. NS then argue that a SPE must satisfy the constraints of the Miyazaki-Wilson problem. The only part that references the functional forms is the proof that the \( \ell \)-type receives at least as much utility as she would in her full information contract (constraint 5 in the problem). NS suppose that the SPE values violate 5, then add \( \epsilon > 0 \) to the \( \ell \)-type contract in each state. Since \( u^{-1}(u) \) is continuous, a sufficiently small \( \epsilon \) can be found so that constraint 5 is still violated and a firm who attracts \( \ell \)-types to such a contract would make profits. In our model, if constraint 5 was violated at the SPE values, then we could introduce a contract that keeps \( b_\ell \) constant but raises \( Q_\ell \) by \( \epsilon > 0 \). This would attract all \( \ell \)-types and be profitable for sufficiently small \( \epsilon \).
3. The final variational argument is in the proof that the SPE values must maximize the objective in the Miyazaki-Wilson problem. NS wedge a new contract between the SPE and Miyazaki-Wilson by reducing the Miyazaki-Wilson values by $\epsilon$ for both types. We simply reduce $Q$ for each type and have a firm that previously made zero profits introduce contracts $\left\{ (Q^M_{MW} \omega - \epsilon, b^M_{MW}), (Q^M_{MW} \omega - \epsilon, b^M_{MW}) \right\}$. For $\epsilon$ sufficiently small, the $h$–type prefers this contract to the supposed SPE one. The $\ell$–types may or may not choose it, but even if they do it will be profitable for deviating lender since it just reducing payments to both types relative to the Miyazaki-Wilson contract, which makes zero profit if it attracts everyone.

These are the only points in the entire proof of Netzer and Scheuer’s Proposition 2 that use the functional forms for payoffs. As we discuss next, the functional forms matter much more for characterizing the solution to the Miyazaki-Wilson problem.

C.2 When and Why Does the $h$–type IC Bind?

While we use the equilibrium concept from Netzer and Scheuer [38], the allocations that solve the Miyazaki-Wilson programming problem in [15]-[16] have regions where $h$–type whereas pooling allocations are never optimal for Netzer and Scheuer. We now show why the variational argument that Netzer and Scheuer use to eliminate pooling allocations fails in our setting and why the endogeneity of the default probability is key to this difference.

To check if a pooling allocation can be dominated by a separating one, we start with some pooling allocation $(Q, b)$ and consider a deviation to $(Q^h, b^h, Q^\ell, b^\ell) = (Q^h - \epsilon, b^h - \delta, Q, b)$ where we put $\delta > 0$ and find $\epsilon$ to keep the $\ell$–type indifferent between the two allocations. This requires:

$$\epsilon(\delta, s) = \psi \int_{\tau^*_h(s, b)}^{\tau^*_h(s, b)} F(\tau) d\tau > 0$$

(81)

We now show that, while there is some $\delta > 0$ that raises the utility of $h$–type borrowers, this can create negative profits for the lender, thereby making this deviation impossible. First, we define the gain for the $h$–type as

$$\Delta U(\delta) = \psi \left[ \int_{\tau^*_h(s, b)}^{\tau^*_h(s, b)} F(\tau) d\tau - \int_{\tau^*_h(s, b)}^{\tau^*_h(s, b)} F(\tau) d\tau \right]$$

(82)

which has the property that $\Delta U(0) = 0$ and

$$\Delta U'(0) = \psi \left[ F(\tau^*_h(s, b)) - F(\tau^*_h(s, b)) \right] > 0.$$  

(83)

This means that there is some $\delta > 0$ that would raise the utility of $h$–types without affecting $\ell$–types, but would this deviation be profitable? The change in profits is given by

$$\Delta \Pi(\delta) = \epsilon(\delta, s) + s R^{-1} \left[ F(\tau^*_h(s, b - \delta)) (b - \delta) - F(\tau^*_h(s, b)) b \right]$$

(84)

which is zero at $\delta = 0$ but is actually decreasing at zero whenever

$$\Delta \Pi'(0) = \psi F(\tau^*_h(s, b)) - s R^{-1} \left[ F(\tau^*_h(s, b)) - F'(\tau^*_h(s, b)) b \right] < 0.$$  

(85)
Here $\psi F(\tau^*_h(s, b)) > 0$ but so is $F(\tau^*_h(s, b)) - F'(\tau^*_h(s, b))b$, since it is the slope of the zero-profit curve for the $h$–type borrower. As $s$ increases, there is more weight put on a negative term, so $\Delta \Pi'(0)$ can fall below zero, thereby making the deviation that eliminates a pooling allocation unprofitable.

C.3 Algorithm

In practice, we use the above as an initial guess to solve for the private information equilibrium for a given set of parameters. The algorithm involves solving each market’s equilibrium iteratively as follows:

H Assume that all $H$–types choose $\overline{h}$ and $L$–types choose $\underline{h}$.

C Update credit contracts holding labor market functions fixed:

C.1 Use guesses of worker’s value functions and score updating function to find credit market allocations for each score. Interpolation of value functions for updated scores off of the grid is required to calculate expected future values from default and repayment.

C.2 Use new contracts to update value functions and default decision rules. If new credit contracts are sufficiently close to old, then continue, otherwise return to Step C.1.

(a) Use default decision rules to update scoring functions. If new scoring functions are similar to old, then continue, otherwise return to C.1.

L With newly converged worker value functions and scoring updating functions, we now update labor market functions.

L.1 Use Nash Bargaining to update firm value functions.

L.2 Use firm value function and score updating functions to update wage functions.

L.3 Use free entry condition to update market tightness and job finding rate functions. If the new finding rate is sufficiently similar to old, then continue, otherwise return to C.1.

Once this algorithm has converged, set the the cost of investing in human capital to support the assumption made in Step H. Since the investment choice is made at the end of each period when employment is known for the next and only affects the wage through the direct change in surplus via $h$, any value of $\phi \in (\beta_L \psi \lambda(\overline{h} - \underline{h}), \beta_H \psi \lambda(\overline{h} - \underline{h}))$ will suffice.
D Definitions of Moments

For model moments we use the stationary distribution. For the average value of an endogenous variable \( x_{i\ell}(s) \) where \( i \) is worker type, \( \ell \) is worker employment status, and \( s \) is score we compute:

\[
\bar{x} = \int_0^1 \sum_{i \in \{L,H\}} \sum_{\ell \in \{U,E\}} x_{i\ell}(s) d\mu^*_i(s)
\]

\[
\bar{x}_i = \int_0^1 \sum_{\ell \in \{U,E\}} x_{i\ell}(s) d\mu^*_i(s) \quad \sum_{\ell} \mu^*_i(1)
\]

\[
\bar{x}_{\ell} = \int_0^1 \sum_{i \in \{L,H\}} x_{i\ell}(s) d\mu^*_i(s) \quad \sum_i \mu^*_i(1)
\]

So, for example, the quarterly repayment rate is conditional on employment and is therefore defined as:

\[
\int_0^1 \sum_{i \in \{L,H\}} G_0(\tau^*_i(s, b^*_i(s))) d\mu^*_i(s) \quad \sum_i \mu^*_i(1)
\]

In order to compute percentiles of the score distribution, we first define the cumulative distribution for the level of aggregation of interest. For unconditional percentiles, we use CDF:

\[
\mu^*(s) \equiv \sum_{i \in \{L,H\}} \sum_{\ell \in \{U,E\}} \mu^*_{i\ell}(s)
\]

Unconditional percentiles are then found by first solving for the type score of that percentile. For percentile \( x \in [0,1] \) we solve:

\[
x = \mu^*(p^x)
\]

Likewise we define conditional percentiles using the conditional cumulative distributions. So, for example, the \( x^{th} \) percentile of unemployed uses the CDF of unemployed households defined by:

\[
\mu^*_U(s) \equiv \sum_{i \in \{L,H\}} \mu^*_{iU}(s) \quad \sum_{i \in \{L,H\}} \mu^*_i(1)
\]

which is then used to solve for \( p^x_U \):

\[
x = \mu^*_U(p^x_U)
\]

These percentiles are used to report conditional means. We also use the stationary distribution to create distributions over other endogenous variables. For example, to compute a percentile of earnings we create a grid \( W \equiv \{w^*_i(s) \mid s \in \{s_0, s_1, \ldots, s_N\}, i \in \{L, H\} \} \) and create the approximate probability distribution:

\[
PDF^w(w^*_i(s)) \equiv \frac{\mu^*_E(s_{j+1}) - \mu^*_E(s_j)}{\sum_{i \in \{L,H\}} \mu^*_E(1)}
\]

We then arrange \( W \) in ascending order and for any \( w \in W \) create:

\[
CDF^w(w) = \sum_{m \in W, m \leq w} PDF^w(m)
\]
And finally we use these approximate cumulative densities to compute percentiles of the earnings distribution.

For our welfare measures, we use the consumption equivalent concept. Since our preferences are linear, this corresponds to the percentage change in welfare. We ask “what fraction of total consumption in each period of the economy with employer credit checks would the worker exchange in order to switch to the economy without employer credit checks?” When this number is negative the household gains from the ban and when it is positive the household loses. We scale consumption in each sub-period in each date by a number $1 + \gamma_{ij}(s)$, where $i$ is worker type and $j$ is employment status. Denoting $W_{nc}$ and $U_{nc}$ as the value functions without employer credit checks, we define $\gamma_{ij}(s)$ by:

$$W_{iH_i}(s)[1 + \gamma_{iE}(s)] = W_{iH_i}^{nc}(s)$$  \hspace{1cm} (92)

$$U_{iH_i}(s)[1 + \gamma_{iU}(s)] = U_{iH_i}^{nc}(s)$$  \hspace{1cm} (93)

### E Human Capital Investment

Suppose that workers can choose whether to expend effort in accumulating human capital in each period $t$. The choice is discrete and doing it costs $\phi$ utils in the present period. Expending effort means that labor productivity will be $h_H$ in the next period rather than $h_L$. An employed worker chooses to invest or not by solving

$$\max \left\{ -\phi + \beta_i(1 - \delta)\bar{W}_{i,H}(s), \beta_i(1 - \delta)\bar{W}_{i,L}(s) \right\},$$

where

$$\bar{W}_{i,H}(s) = \max_t \left\{ Q_t^*(s) + \psi w_H^*(s) + \psi \int_0^{\tau_{ih}^*(s)} f_W(s, \tau) d\tau + \psi \beta_i(1 - \delta) \left[ V_{ih}(s_1^{1}) - \psi \epsilon \right] \right\},$$  \hspace{1cm} (95)

$$\bar{W}_{i,L}(s) = \max_t \left\{ Q_t^*(s) + \psi w_H^*(s) + \psi \int_0^{\tau_{ih}^*(s)} f_W(s, \tau) d\tau + \psi \beta_i(1 - \delta) \left[ V_{ih}(s_1^{1}) - \psi \epsilon \right] \right\}.$$  \hspace{1cm} (96)

An unemployed worker’s problem is similar, except she discounts the gain by the probability of being employed in the next period.

In our calibrated economy, the discount factor for low-type workers is $\beta_L = 0.742$ versus $\beta_H = 0.997$ for high-type, which generates a large difference in their valuations of future gains from current investments. Figure 16 plots $\bar{W}_{i,H}(s) - \bar{W}_{i,L}(s)$ for both types, which demonstrates that any value of $\phi$ between 2 and 11 would rationalize our assumption that high-type workers have high labor productivity and low-type workers have low productivity.
Figure 16: Gains From Exerting Effort