Attention Allocation and Heterogenous Consumption Responses

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Abstract

Recessions often have detrimental effects on both employment and equity returns, forcing individuals to make decisions about how to balance risks to their labor and capital income. In this paper, we study how individuals allocate their limited attention between capital income and labor income risks in a two-period consumption-saving model with recursive utility. Specifically, we examine how the optimal attention and consumption-saving decisions are influenced by individuals’ attention capacity, wealth endowments, income risks, and preferences for risk and time. We show that our model can generate results that are consistent with several novel facts regarding how differences in individuals’ wealth levels and beliefs about their unemployment risks influenced their consumption during the Great Recession. Furthermore, we find that the welfare loss due to limited attention is significantly larger for households with lower wealth; allowing these households to flexibly allocate their attention can significantly reduce this welfare loss.

Keywords: Capital Income and Labor Income Risks; Optimal Attention Allocation; Consumption and Saving Decisions.

JEL Classification Numbers: C61; D83; E21.

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1 Introduction

Recessions are usually associated with a significant rise in unemployment and a sharp decline in equity prices, putting both the labor income and capital income of individuals at risk. How individuals adjust their consumption in response to these shocks depends on the size of the shocks as well as their assessments of the associated risks. Intuitively, when the magnitudes of the shocks are large or when individuals are less prepared for them (due to limited information), their consumption responses are likely to be large. How well individuals prepare for these shocks in the face of limited information depends on their information-processing capacity as well as how they allocate that capacity to assessing various types of risks, such as labor and capital risks.

Although both empirical and theoretical research has examined how household consumption responds to income shocks, relatively few studies have explained consumption responses in the presence of both capital and labor income shocks under limited information capacity. In this paper, we therefore investigate, both empirically and theoretically, the implications of limited attention and attention allocation for the consumption and saving responses to both labor and capital income shocks. In particular, we investigate these responses during the 2007-09 Great Recession, a period that contains large labor and capital income shocks.

On the empirical side, we document three novel facts regarding households’ consumption changes around the 2007-09 Great Recession period using a unique dataset with detailed information about individuals’ prior beliefs about unemployment risks as well as ex-post changes in their employment status, consumption, and financial assets. First, individuals who have more uncertainty about their future labor income – that is, a larger prior variance – experience a smaller consumption decline in percentage terms when they become unemployed.\(^1\) Second, individuals who have a larger prior variance on their labor income experience a larger consumption decline in percentage terms when they experience the same loss in capital income (in percentage terms). Third, wealthier individuals experience a larger consumption drop in percentage terms when they become unemployed.

On the theoretical side, we develop a tractable two-period model with the following key elements: (i) rational inattention due to limited information capacity in the vein of Sims (2003), (ii) recursive utility, and (iii) multiple risks (additive labor risk and multiplicative capital risk) to study how individuals’ optimal consumption responses are jointly determined by their attention capacity, their endogenous attention allocation (to monitor each risk), their initial wealth endowments, and the magnitude of the shocks.\(^2\) To account for the different effects of individuals’ risk

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\(^1\)Our analysis uses data at both the individual and the household levels, with more details provided in the data section. Individual consumption and wealth are based on household levels, while unemployment and prior information are at the individual level. In the model, when we link changes in labor income at unemployment to consumption, we make corresponding adjustments to make consumption and income data comparable.

\(^2\)Leland (1968), Sandmo (1970), Kimball and Weil (2009), Seldon and Wei (2018), and Kubler, Selden, and Wei (2020) also adopt the two-period setting to examine the effects of income uncertainty on savings and/or investment.
and time preferences on the optimal attention and consumption allocation, we introduce a Kreps-Porteus-Seldon type recursive utility to fully separate the elasticity of intertemporal substitution (EIS) from the coefficient of relative risk aversion (CRRA). 3

Our central message is that the optimal attention allocation coupled with limited information capacity generate heterogeneous consumption responses among households, a pattern that is consistent with the data but is difficult to explain with full-information rational expectations (FI-RE) models. To the best of our knowledge, this is the first paper to use a framework beyond linear-quadratic preferences to explore how attention allocation influences households’ consumption responses to both labor and capital income shocks. This paper extends the literature on optimal consumption-saving decisions under rational inattention which focuses mainly on single income risk (see, for example, Sims (2003, 2006), Luo (2008), and Yin (2021)). It also complements the literature on consumption-saving under rational inattention, which studies multiple income risks but assumes (approximated) linear-quadratic or constant-absolute-risk-averse (CARA) preferences (see, for example, Mondria (2010), Van Nieuwerburgh and Veldkamp (2010), Maćkowiak and Wiederholt (2015)). 4

Specifically, our analysis contributes to the literature in three dimensions. First, we construct a model to study the joint optimal attention-consumption allocation problem and quantitatively show how households' optimal attention allocation and consumption-saving decisions are driven by their prior beliefs on risks, their attention capacity and wealth endowments, and their time and risk preferences. We find that an individual’s optimal attention allocation between labor and capital income risks depends on the relative size of their prior variance in these risks. For example, if an individual’s prior variance on labor income risks increases relative to their prior variance on capital income risks, they will allocate more attention to monitoring labor income risks; this behavior is unsurprising and in line with existing results in the rational inattention literature. What is more surprising is that individuals allocate more attention to capital income risk (which has a multiplicative form) than to labor income risk (which has an additive form) even when the prior variance of the labor income risk is higher than that of the capital income risk. 5 For the effects of wealth endowment and attention capacity, we find that richer households pay more attention to capital income risks and save at higher rates than less wealthy households; we find that the labor income risk becomes more important to households when they have a more limited attention capacity, and the average saving rate accordingly decreases due to the precautionary

3 Angeletos (2007) and Wang, Wang, and Yang (2016) find that these two parameters have different effects on the consumption-saving allocation.
4 Some empirical studies find that incomplete and noisy information about the state variable(s) plays an important role in affecting individual agents’ optimal decisions. For example, Andrade and Le Bihan (1997) and Cobion and Gorodnichenko (2008) find pervasive evidence consistent with Sims (2003)’s rational inattention theory using the U.S. and European surveys of professional forecasters and other agents, respectively.
5 Note that a common conclusion in the previous studies on rational inattention, such as Maćkowiak and Wiederholt (2009), is that when prior variances of different risks are the same, agents pay the same amount of attention to each risk.
saving motive. For the effects of risk preferences, we find that more patient individuals save more and pay more attention to the capital return risk. Furthermore, we find that an increase in the CRRA (the EIS) decreases (increases) the optimal attention allocated to the labor (capital) income risk and leads to higher (lower) savings.6

Second, we show how our model is consistent with the three novel facts empirically documented during the Great Recession period. The first fact – that individuals with a larger prior variance on their labor income experience a smaller consumption decline in percentage terms when they become unemployed – is actually the result of three distinct effects on an individual’s savings: i) a larger prior variance on the labor income that raises individuals’ savings due to the precautionary motive; ii) an increase in attention allocated to labor income risk which reduces the posterior variance of the labor-income risk and therefore reduces individuals’ saving; and iii) reduced attention allocated to capital risk, which increases the posterior variance of capital risk and thus reduces individuals’ savings.7 In our calibrated model, the first effect dominates the other two, leading to an overall increase in savings and a smaller decline in consumption when an individual becomes unemployed. In comparison, in the corresponding FI-RE model, a change in the prior variance of the unemployment shock has no effects on savings and thus consumption responses, which is inconsistent with the data.

The second fact documented during the Great Recession period – that a higher prior variance on labor income leads to a larger consumption decline in response to a financial loss – can be understood through the impact of the prior variance on savings. In particular, a larger prior variance on labor income is associated with higher savings (which has been explained in the previous paragraph), increasing an individual’s exposure to financial risks and thus leading them to make larger consumption cuts in response to a negative financial shock than individuals with a smaller prior variance on labor income.

The third fact documented during the Great Recession period – that wealthier individuals experience a larger consumption drop in percentage terms when they become unemployed – can be understood through two competing channels: i) individuals with a higher wealth endowment save more, and thus cut their consumption less after they become unemployed because they can draw on their savings; and ii) unemployment benefits are relatively less important to wealthier households, suggesting a larger decline in consumption when they become unemployed. We show that which effect dominates depends on households’ attention capacity. In particular, a larger attention capacity makes the first channel more important by increasing the sensitivity of the

6 Luo, Nie, Wang, and Young (2017) also discuss the effects of the EIS and the CRRA on the optimal attention amount. However, since they only consider one-type risk and have no attention allocation problem, their results cannot be compared with the results in this paper.

7 The increase in the posterior variance of the capital risk leads to a reduction in saving because the income effect dominates the substitution effect in our calibrated model. As shown in Weil (1990), when the EIS is small, a mean-preserving increase in the capital return risk (specifically, a lower certainty equivalent capital return) leads to increased saving for an FI-RE case.
saving rate to wealth changes. In other words, to explain the data, the model requires households to have a smaller attention capacity, which makes the second channel dominate the first one. When the information capacity is sufficiently large, such as in the FI-RE case, these two effects are exactly canceled out; thus, the consumption change is the same across wealth groups, contradicting the fact we document.

Third, our model also yields several important findings for households’ welfare. As highlighted in the existing literature, raising households’ information capacity improves their welfare (see, for example, Luo (2008) and Maćkowiak and Wiederholt (2015)). Our model shows that the welfare gain is heterogeneous across individuals with different wealth endowments. Specifically, we find that lower-wealth individuals experience larger welfare gains from increasing their information capacity. One possible explanation is that poor individuals consume less in absolute terms and can thus make more efficient consumption-saving plans than their wealthier counterparts.\(^8\) In addition, we find the welfare gains increase with the risk aversion coefficient: more risk-averse people benefit more from increasing their attention capacity, as they can use this increased attention capacity to reduce the uncertainty for their consumption and utility. Furthermore, we find the welfare gains decrease with the EIS: because people with a low EIS dislike consumption fluctuations across periods, a larger attention capacity allows them to more precisely predict their future income and thus smooth their consumption. One policy implication of these results is that providing additional information to households who have less wealth, a lower attention capacity, lower EIS, and who are more averse to risk is particularly welfare improving.

Our paper is related to three strands of literature. First, our paper is related to literature on the optimal consumption-saving theory. The idea that consumers smooth consumption and accumulate financial wealth when facing unexpected fluctuations in income dates back to Leland (1968) and Sandmo (1970). Later studies by Bhamra and Uppal (2006), Kimball and Weil (2009), and Seldon and Wei (2018) extend the expected utility specification to the recursive utility specification within the two-period framework and further explore how risk aversion and intertemporal substitution affect the optimal consumption and saving decisions. Skinner (1988), Zeldes (1989), Caballero (1990), Weil (1990), Kaplan and Violante (2014), and others extend the two-period framework to multiperiod models and further explore how consumption responds to anticipated and unanticipated income shocks, or how the amount of precautionary savings is related to an increase in the persistence and volatility of the income process. For example, Kaplan and Violante (2014) develop a structural model where households hold two assets (a low-return liquid asset and a high-return illiquid asset) and find that many households hold little liquid wealth despite owning sizable quantities of illiquid wealth. They then show that this model can be used to interpret the empirical fact of observed consumption responses to fiscal stimulus payments in the

\(^8\)Consider an extreme case, if a consumer has a sufficiently high wealth level such that the marginal utility is close to zero, limited attention or infinite attention makes no big difference.
Second, our paper is also related to the literature on the rational inattention theory proposed by \textcite{Sims(2003,2010)}. Most of the recent studies on optimal consumption-saving-investment under rational inattention (e.g., \textcite{Sims(2003,2006), Luo(2008), Mondria(2010), VanNieuwerburgh and Veldkamp(2010), Mackowiak and Wiederholt(2015), and Miao, Wu, and Young(2022)}) consider linear-quadratic preferences. In contrast, our paper is based on a recursive utility framework with constant elasticity of intertemporal substitution and constant relative risk aversion.

Third, our paper is also related to empirical studies on how consumption responds to income shocks. In the literature on consumption responses to income shocks, economists focus on different types of labor income shocks, such as the asymmetric responses to positive and negative shocks, or different responses to temporary and permanent shocks. (See \textcite{Jappelli and Pistaferri(2010)} for a survey and \textcite{Deaton(1993)} for a textbook treatment on this issue.) In addition, some empirical studies examine how consumption responds to capital income shocks. The results based on micro-data are mixed, with some studies finding large consumption responses to house and stock price shocks, and others finding smaller effects.\footnote{Sinai and Souleles(2005), Campbell and Cocco(2007), and Attanasio, Blow, Hamilton, and Leicester(2009) find that the consumption response to changes in capital income is quite heterogeneous across the population.}

However, very few studies have examined consumption responses to both capital and labor income shocks. One exception is \textcite{Christelis, Georgarakos, and Jappelli(2015)} who estimate the separate impacts of three different shocks, shocks to stocks, housing, and unemployment, on households’ expenditures during the Great Recession using recently available micro data (the 2009 Internet Survey of the Health and Retirement Study). In this paper, we use the same data set as in \textcite{Christelis, Georgarakos, and Jappelli(2015)} to examine how individuals with different wealth endowments and different assessments of unemployment risks respond differently to labor and financial shocks.

The remainder of this paper is organized as follows. Section 2 provides an empirical motivation for this paper. Section 3 describes our baseline model by introducing key elements step by step. Section 4 presents main results of our model with recursive utility and discusses the implications of the joint optimal attention-consumption/savings allocation. Section 5 discusses the testable implication on the heterogenous consumption responses to the two income shocks, and show how our model fits the data in these aspects. Section 6 examines the welfare implications of limited attention optimal attention allocation. Section 7 further discusses the implications for the relative dispersion of consumption to income as well as the role of the bequest motive. Section 8 concludes.

\footnote{In a recent paper by \textcite{Lian(2022)}, the author shows that inefficient responses of future consumption to saving changes lead to high marginal propensities to consume in current period although consumers have no liquidity constraints.}
2 Heterogeneous Consumption Responses: Empirical Evidence

In this section, we provide empirical evidence that the response of individuals’ consumption to labor and capital income shocks depends on their prior knowledge on the unemployment risk and their financial wealth.

2.1 Data

Our empirical analysis is based on two microdata surveys from Health and Retirement Study (HRS). The first one is the HRS main survey in 2006 and 2008, which is a longitudinal, nationally representative survey interviewing respondents aged 50 and above in the U.S. economy. The survey has been conducted on a biannual basis since 1992 and provides information on households demographic characteristics, income, and asset holdings. The second source is the HRS internet survey, which was conducted from March 2009 to August 2009, and contains 4,415 respondents belonging to 3,438 households. To reduce the possibility that estimates are affected by outliers, we delete observations for which the absolute value of the percentage change in consumption is larger than 0.8.

For the purpose of this paper, an important feature of the 2006 wave of the HRS main survey is that respondents are asked about their expectations regarding the likelihood that they will lose their jobs in the future. On the prior variance of the labor income, we follow Lusardi (1998) to define it as \( p(1 - p)(1 - \eta)^2 Y^2 \) where \( p \) is an individual’s subjective probability (i.e., prior) of losing the job, \( \eta \) is the replacement rate of unemployment benefits, and \( Y \) is the labor income. In the HRS, we have a direct measure of \( p \) that allows us to construct the prior variance. It is worth noting that a fundamental problem in the empirical studies on consumption and income is about how to measure the subjective uncertainty of future income fluctuations since this variable is unobservable, and the literature usually relies on indirect proxies for risk (or uncertainty). For example, Guiso, Jappelli, and Terlizzese (1992) use the 1989 Survey of Household Income and Wealth (SHIW) to infer information on the probability distribution of household earnings one year ahead. Hence, our assumption on using the likelihood of losing a job in the future as a proxy for the prior variance of the labor income shock is consistent with what is used in the existing literature.

An important feature of the 2009 internet survey is that respondents are asked to report percentage changes in their total spending compared to the previous year, i.e., 2008, changes in financial assets since September 2008, and detailed timing (year and month) of changes in their

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11 The details about the survey can be found in Hauser and Willis (2005).
12 In the HRS, the survey question is “On the same scale from 0 to 100, where 0 equals absolutely no chance and 100 equals absolutely certain, what are the chances that you will lose your job during the next year?” In addition, Guariglia (2001) measures earnings uncertainty by using a similar question in the British Household Panel Survey.
employment status. This helps us to identify whether respondents were hit by an unemployment shock between the survey date and a year before. Detailed definitions and descriptions about main variables and be found in Appendix 9.1 and Table 1 provides summary statistics on these variables.

2.2 Estimation Results

Our estimation approach is similar to that in Christelis, Georgarakos, and Jappelli (2015), but extends it to better focus on the prior variance of labor income and households’ financial wealth. Specifically, our benchmark estimation links households’ consumption to changes in financial wealth, changes in unemployment status, their interactions with individuals’ priors on labor income variance, as well as other control variables:

\[
\frac{\Delta C_{it}}{C_{i,t-1}} = \alpha + \beta \frac{\Delta FW_{it}}{FW_{i,t-1}} + \delta \Delta U_{it} + \gamma_1 pvar_i + \gamma_2 pvar_i \cdot \frac{\Delta FW_{it}}{FW_{i,t-1}} + \gamma_3 pvar_i \cdot \Delta U_{it} + \xi X_{it} + \epsilon_{it}, \tag{1}
\]

where \(i\) denotes individual households and the term on the left-hand side of the equation is percentage change in consumption. On the right side of the equation, the second term is the percentage change in the values of liquid financial wealth; \(\Delta U\) indicates whether an individual becomes unemployed between the survey date and a year before; \(pvar_i\) is individual \(i\)’s prior variance of the labor income (which is defined above); the next two terms measure the interactions between the prior variance and financial and unemployment shocks; \(X\) is a vector of demographic and economic variables, including gender, age, marital status, retirement status, percentage change in value of the main residences, and individuals’ expectation about the probability of an increase in Dow Jones Industrial Average; \(\epsilon_{it}\) is an error term.

Columns 1 and 2 of Table 2 report our benchmark estimation results. First, as the first two rows show, not surprisingly, household’s consumption declines at unemployment or when they experience a decline in financial wealth. Second, the positive coefficient of the interaction term of prior variance and unemployment shock implies that, on average, those who had higher prior variance in the labor income, experience a smaller decline in their spending conditional on a loss in financial wealth and labor income. From the positive coefficient of the interaction term of prior variance and the change in financial assets, we find that conditional on a certain drop in financial assets, those with higher prior variance of labor income experience a larger decline in spending. Together, these show that for those who have a larger prior variance on their labor income, their

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13 Respondents also report the amount of change in the value of their house compared to its value in the summer of 2006. For each assets owners of employer retirement saving plans (incl. 401k’s), individual retirement accounts (IRAs) or Keogh plans, investment trusts, mutual funds, directly held stocks, they are asked to report the percentage decline of the asset value since September 2008, which was the month in which Lehman Brothers collapsed. The discussion regarding biased estimation due to measurement error in Christelis, Georgarakos, and Jappelli (2015) also holds here in our analysis.
consumption declines less after becoming unemployed but declines more when they experience financial losses. In Column 2, we also find that adding more control variables (including the logarithm of income and household size) does not change the main results.\textsuperscript{14}

Columns 3 and 4 of Table 2 report another interesting heterogeneity in the consumption response to changes in the value of financial asset and the unemployment shock. In these exercises, we add an interaction term of net financial asset and the percentage drop in financial asset and an interaction term of net financial asset and the unemployment shock. From these results, we can see that rich people, i.e. those with high net financial asset (normalized by average income), tend to react more strongly while unemployed. This finding appears to be inconsistent with the finding in the literature of a positive correlation between the level of lifetime income and the saving rate (see Dynan, Skinner, and Zeldes (2004)) as a higher saving rate may lead to a smaller decline in consumption. However, as we will explain in section 5, our model can rationalize both facts by explaining how an increase in wealth can raise savings but also lead to a decline in consumption in the presence of attention allocation.

One thing we want to mention is that a standard life cycle consumption model with full information or no attention allocation cannot explain these empirical facts simultaneously. For example, in a two-period model with infinite attention, the saving rate and the consumption change do not react to various prior variances of income shocks. In addition, as shown later in this paper, a model with no attention (allocation) leads to the saving rate too large to match the empirical facts shown above. However, the following sections will show that, by introducing optimal attention allocation into an otherwise standard consumption-saving model, we can potentially explain these facts.

3 An Optimal Attention-Allocation Model with Both Labor and Capital Risks

To fully examine how the optimal attention allocation impacts consumption saving decisions with both labor and capital income shocks, we build a two period model that is rich enough to explore a series of fundamental factors in driving the optimal decisions but still tractable. In this section, we start with households’ preferences, budget constraints, and two fundamental shocks they face: shocks to capital income and labor income. We then discuss how to incorporate rational inattention due to information-processing constraints into an otherwise standard two-period consumption-saving model.

\footnote{Table 2 shows that changes in the values of their house have no significant effect on consumption changes. But Christelis, Georgarakos, and Jappelli (2015) find that the elasticity of consumption with respect to the value of their house is roughly equal to 0.056.}
3.1 Households’ Preferences and Budget Constraints

To capture how risk aversion coefficient and intertemporal elasticity of substitution affect the optimal attention-consumption/savings decisions, we follow Kimball and Weil (2009); Bommier and Le Grand (2019); Seldon and Wei (2018); and Kubler, Selden, and Wei (2020) in assuming a general KPS (Kreps and Porteus (1978) and Seldon (1978)) preference structure as well as the two-period specification. Specifically, in the model economy, households live for two periods: $t \in \{0, 1\}$, and have the following recursive utility:

$$U = u(C_0) + \beta u \left( v^{-1}(E v(C_1)) \right), \quad (2)$$

where $\beta \in (0, 1)$ denotes the households’ subjective discount factor and $C_0$ and $C_1$ are consumption in periods 0 and 1, respectively.\(^{15}\) The functions, $u(\cdot)$ and $V(\cdot)$, that govern the preferences for intertemporal substitution and risk aversion are characterized by the CES certainty and constant relative risk aversion risk preferences functional forms, respectively. Specifically, we assume that:

$$u(x) = \frac{x^{1-1/\psi}}{1-1/\psi} \quad \text{and} \quad v(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad (5)$$

where $\gamma$ is the CRRA, whereas $\psi$ is the EIS (i.e., $1/\psi$ is the relative resistance to intertemporal substitution.) This recursive utility specification rules out any possible time inconsistency problem. When $1/\psi = \gamma$, this specification reduces to the standard expected utility case. In this specification, $U$ represents the time preference over certain $(C_0, \hat{C}_1)$ pairs, where $\hat{C}_1$ is the period-2 certainty equivalent associated with the random period-2 consumption, $C_1$: $\hat{C}_1 = v^{-1}(E v(C_1))$.

We assume that households make consumption and saving decisions for a given initial endowment in period 0, and receive both capital income from this risky saving behavior and a risky labor income in period 1. Specifically, the households’ budget constraints in periods 0 and 1 can be written as:

$$C_0 + K_1 = Y_0, \quad (6)$$

$$C_1 = A_1 K_1 + Y_1, \quad (7)$$

respectively, where $Y_0$ is initial wealth which is strictly positive, and $K_1 > 0$ is the total savings/investment in period 0. It is worth noting that although for simplicity we do not consider\(^{15}\)It is worth noting that (2) is equivalent to the following recursions:

$$U = u(C_0) + \beta W^{-1}(E[W(U_1)]), \quad \text{or} \quad (3)$$

$$\tilde{U} = u^{-1} \left[ u(C_0) + \beta u \left( v^{-1}(E[v(U_1)]) \right) \right], \quad (4)$$

where $W = v \circ u^{-1}$, $U_1$ is future uncertainty utility, and $U = u \left( \tilde{U} \right)$. Using (3) or (4) is just a matter of normalization.

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the risk-free asset and optimal asset allocation between the risky asset and the risk-free asset in our model economy, the model with a risky portfolio is not unrealistic from a macroeconomic perspective. Some economists argue that the global economy could be faced with a shortage of safe assets. For example, during the 2007−2009 financial crisis, many of the private safe assets, perceived as safe because they were bestowed with a AAA rating, lost their quality and then disappeared.\footnote{During 2002−2007, the US and European financial markets created large amounts of private safe assets through the securitization of riskier assets.} As a result, the strains associated with the financial crisis quickly lead to concerns about the safety of sovereign debts, which leads to a further shrinkage in the global supply of safe assets.

### 3.2 Shocks and Information Structure

The capital return ($A_1$) and labor income ($Y_1$) processes are assumed as follows:

$$A_1 = \exp(\epsilon_a) \text{ and } Y_1 = \exp(\epsilon_y), \quad (8)$$

where $\epsilon_a$ and $\epsilon_y$ are exogenously \textit{i.i.d.} shocks. Households are endowed with prior beliefs about the distributions from which these shocks are drawn: $\epsilon_a \sim N(\mu_a - 0.5\sigma_a^2, \sigma_a^2)$ and $\epsilon_y \sim N(\mu_y - 0.5\sigma_y^2, \sigma_y^2)$.\footnote{This implies that the unconditional mean of capital return and labor income $E[Y_1] = \exp(\mu_y)$ and $E[A_1] = \exp(\mu_a)$.} However, the realizations of these two shocks are unobservable in period 0 due to households’ limited information-processing ability.

We then assume that households learn exogenous income shocks by observing the following noisy signals:\footnote{Sims (2010) provides two ways to solve models with limited information-processing capacity. The first way is to solve the optimal joint distribution of the control variable and the unobservable state variable. The second way is to assume a signal structure, and then solve for the optimal policy as a function of signal. However, as argued by Sims (2010), the optimal joint distribution can be characterized by many different combinations of signal structure and policy function.}

$$S_0 = \begin{bmatrix} S_a \\ S_y \end{bmatrix} = \begin{bmatrix} \epsilon_a + \zeta_a \\ \epsilon_y + \zeta_y \end{bmatrix}, \quad (9)$$

where the signals are noisy but unbiased. $\zeta_a \sim N(0, \eta_a^2)$ and $\zeta_y \sim N(0, \eta_y^2)$ are the endogenous noises induced by limited-information processing capacity. The variance of signal regarding capital income shock is $\sigma_a^2 + \eta_a^2$, and therefore, the precision of the signal is $1/(\sigma_a^2 + \eta_a^2)$. Similarly, the variance of signal regarding labor income shock is $\sigma_y^2 + \eta_y^2$, and therefore, the precision of the signal is $1/(\sigma_y^2 + \eta_y^2)$.

Households now use Bayes’ Law to combine their prior beliefs on the two shocks and the observed noisy signals in (9) to update their beliefs about the shocks such that $\epsilon_a | S_a \sim N(\hat{\epsilon}_a, \hat{\sigma}_a^2)$ and $\epsilon_y | S_y \sim N(\hat{\epsilon}_y, \hat{\sigma}_y^2)$, where $\hat{\epsilon}_a$ and $\hat{\epsilon}_y$ are the posterior means and $\hat{\sigma}_a^2$ and $\hat{\sigma}_y^2$ are the posterior
variances determined by the following updating rules:

\[ \hat{\epsilon}_a \equiv E[\epsilon_a | S_a = s_a] = \frac{(\mu_a - 0.5\sigma^2_a)\eta^2_a + \sigma^2_a s_a}{\sigma^2_a + \eta^2_a}, \quad (10) \]

\[ \hat{\sigma}^2_a \equiv V[\epsilon_a | S_a = s_a] = \frac{\sigma^2_a \eta^2_a}{\sigma^2_a + \eta^2_a}, \quad (11) \]

\[ \hat{\epsilon}_y \equiv E[\epsilon_y | S_y = s_y] = \frac{(\mu_y - 0.5\sigma^2_y)\eta^2_y + \sigma^2_y s_y}{\sigma^2_y + \eta^2_y}, \quad (12) \]

\[ \hat{\sigma}^2_y \equiv V[\epsilon_y | S_y = s_y] = \frac{\sigma^2_y \eta^2_y}{\sigma^2_y + \eta^2_y}, \quad (13) \]

Given the prior beliefs, (11) and (13) imply that the signal precision can be uniquely determined by the posterior variance. We can now define information sets before and after observing the signals, which are called Stages 1 and 2 of period 0.

**Definition.** \( I^1 \) and \( I^2 \) are the information sets in Stages 1 and 2, respectively:

\[ I^1 = \{ Y_0, \epsilon_a \sim N(\mu_a - 0.5\sigma^2_a, \sigma^2_a), \epsilon_y \sim N(\mu_y - 0.5\sigma^2_y, \sigma^2_y) \}, \]

\[ I^2 = I^1 \cup \{ S_a, S_y \}. \]

Following Sims (2003, 2010), we assume that households face a limited information-processing capacity, \( \kappa \):

\[ \kappa_a + \kappa_y \leq \kappa, \quad (14) \]

where \( 0 < \kappa < \infty \), \( \kappa_a \) and \( \kappa_y \) are capacity levels devoted to monitoring the capital and labor income shocks, respectively. For simplicity, we assume that the two signals are independent such that:

\[ I(\epsilon_a, S_a) = H(\epsilon_a) - H(\epsilon_a | S_a) = \frac{1}{2} \log \left( \frac{\sigma^2_a}{\sigma^2_a + \eta^2_a} \right) = \kappa_a, \quad (15) \]

\[ I(\epsilon_y, S_y) = H(\epsilon_y) - H(\epsilon_y | S_y) = \frac{1}{2} \log \left( \frac{\sigma^2_y}{\sigma^2_y + \eta^2_y} \right) = \kappa_y, \quad (16) \]

where \( \kappa_a \) and \( \kappa_y \) are measured in nats.\(^{19}\) \( H(\cdot) \) is the entropy of productivity shock, \( H(\cdot | \cdot) \) is the conditional entropy of productivity shock given signal observation, and \( I(\cdot, \cdot) \) is called the mutual information between the fundamental shock and signal observation and can be interpreted as how much information about the fundamental shock is contained in the corresponding noisy signal.

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\(^{19}\)Sims (2003) states that the logarithm in the formula can be to any base, because the base only determines a scale factor for the information measure, but conventionally it takes the logarithm to base 2, and as a result the entropy of a discrete distribution with equal weight on two points is \( 1 - 0.5 \log(0.5) - 0.5 \log(0.5) \), which is the unit of information called a “bit.” When the base is \( e \), the unit of information is a “nat.”
3.3 Households’ Optimization Problem

In this model, households need not only solve an optimal consumption-saving problem but also solve an optimal attention allocation problem. The whole optimization problem can be formalized as follows:

\[
V = \max_{(\kappa_a, \kappa_y)} \mathbb{E}_I \left[ u(C_0^*) + \beta u \left( v^{-1}(\mathbb{E}[v(C_1^*)|S_0]) \right) \right]
\]  
(17)

subject to:

\[
C_0^* = \arg \max_{C_0} \left\{ u(C_0) + \beta u \left( v^{-1}(\mathbb{E}_I[v(C_1)]) \right) \right\},
\]  
(18)

\[
C_1^* = A_1(Y_0 - C_0^*) + Y_1,
\]  
(19)

\[
\kappa_a + \kappa_y \leq \kappa,
\]  
(20)

where Equation (17) is the objective function for the household, \( \mathbb{E}_I \cdot \) is the expectation operator conditional on the information set \( I \), \( \mathbb{E}_I \cdot \) is the expectation over all possible signals, the budget constraints are incorporated into Equations (18) and (19), and Equation (20) displays the attention capacity constraint.

4 Model’s Implications for Attention Allocation and Consumption-Saving Decisions

In this section, we first briefly describe how we solve the model numerically and then explore intensively the model’s implications for households’ optimal consumption and saving decisions under the limited attention.

4.1 Solution Method

As illustrated in Figure 1, we decompose the optimization problem proposed above into two stages: (i) attention allocation and (ii) consumption-saving choice. In the first stage, before observing the noisy signals about capital return and labor income, households decide how much attention to allocate to learning capital return and labor income, respectively. This procedure determines how precise these two signals are. In the second stage, after observing the signals, households then decide how much to consume and how much to save out of the initial endowment. Following Maćkowiak and Wiederholt (2009), we solve these two sub-problems backward.

First, for any attention allocation strategy, we solve the following consumption-saving problem:

\[
U = \frac{(Y_0 - K_1)^{1-1/\psi}}{1 - 1/\psi} + \beta \left( \mathbb{E} \left[ (A_1 K_1 + Y_1)^{1-\gamma} | S_0 \right] \right)^{1-1/\psi} - \gamma \frac{1}{1 - 1/\psi}
\]
The first order condition for $K_1$ is:

$$\frac{\partial U}{\partial K_1} = -(Y_0 - K_1)^{-1/\psi} + \beta \left( \mathbb{E} \left[ (A_1 K_1 + Y_1)^{1-\gamma} | S_0 \right] \right)^{2-1/\psi} \mathbb{E} \left[ (A_1 K_1 + Y_1)^{-\gamma} A_1 | S_0 \right] = 0. \quad (21)$$

It is straightforward that the first order condition determines a unique solution to the consumption problem. Solving the condition yields the optimal choice of $K_1$ in period 0. Plugging $K_1^*(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)$ back to the utility function gives us the indirect utility. Taking the unconditional expectations by evaluating over $S_a$ and $S_y$ allows us to solve the first-stage attention allocation problem. The detailed procedure is provided in Appendix 9.2.

4.2 Parameterization

We provide details how we set parameter values in this section by going through each block of parameters. In the next section, we also illustrate how different key parameter values impact the attention allocation and consumption-saving decisions.

**Capital and labor income risks.** Following Campbell (2003), we set the prior variance of capital income risk $\sigma_a^2$ to 0.03 and assume that the ratio of prior variance of the labor income risk to that of the capital income risk $\sigma_y^2/\sigma_a^2 \in [1, 3, 5, 7, 9, 11, 13, 15]$. The benchmark value of labor income risk is 0.42. We set the unconditional mean of capital return to 1.03. According to HRS data, the average drop of financial asset compared to one-year ago is about 27%. Therefore, the true realization of capital return is 0.73. If households with one member become unemployed, the total households’ income becomes 1.475.21

**Initial endowment.** For the baseline parameterization, we set the unconditional expectation of labor income to 1 and the endowment in the initial period to $Y_0 = 7$. This value is calculated by using HRS data as follows. We first normalize each household’s net financial wealth by the mean of its income and calculate the average value, which is about 3.5. We set the unconditional mean of a representative agent’s labor income in the second period to 1 if employed and 0.5 if unemployed with unemployment probability 5%. We then obtain the individual’s unconditional expected income is 0.975. From our sample, the average household size is two, thus the expected income of a household with two workers is 1.95. To obtain a wealth-to-income ratio of 3.5, we need initial wealth to be about 7.23

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20This value is the average prior variance for individuals hit by the unemployment shock. The average prior variance for the whole sample is 0.36.
21We are not able to observe whether a spouse/partner was hit by the unemployment shock during the same period. Therefore, we calculate the household’s total income when the respondent becomes unemployed as the sum of the respondent’s unemployment benefit and the expected income of the partner: $0.5 + (1 \times 0.95 + 0.5 \times 0.05)$.
22The replacement ratio in the US was about 50% according to the information on the website of the US Department of Labor (Employment and Training Administration).
23The value of initial endowment ($Y_0$) may vary largely for different individuals, from 2 to 20 in the Survey of Consumer Finances (SCF) given that the mean income is 1.
Attention capacity. Luo (2008) shows that when $\kappa = 0.5$ nats, the otherwise standard permanent income model can generate the observed aggregate consumption smoothness. In addition, Coibion and Gorodnichenko (2008) use the SPF forecast survey data to test the degree of information rigidities governed by the degree of inattention and find that their model can fit the data well when $\kappa$ is close to 0.5 nat. Since our model considers two types of risks, we set the baseline value of $\kappa$ to be 1 nat. For the robustness check, we also consider the cases when $\kappa = 0.5$ nat and 2 nats.

The Discount factor. We set $\beta = 0.97$ as the baseline value. We also check the robustness of our main results by setting $\beta = 0.7$ and 0.8. 24

The CRRA. In macroeconomic studies, the value of $\gamma$ is between 1 and 6. We calibrate this value to match the overall consumption decline of 21% in the sample period, which results in a value of 5.

The EIS. We set $\psi = 1/3$. However, there is no consensus on the magnitude of the EIS ($\psi$), and the evidence is still mixed as the literature has found a very wide range of values. For example, Visising-Jorgensen and Attanasio (2003) estimate the EIS to be well in excess of 1, while Campbell and Cocco (2007) estimate a value well below 1 (and possibly 0). Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Best, Cloyne, Ilzetzki, and Kleven (2020) use U.K. mortgage data and find the EIS is close to 0.1. Havránek (2015) surveys the vast literature and suggests that a range around 0.3-0.4 is appropriate after correcting for selective reporting bias.

Next, we will show the comparative statics analysis for attention-consumption choice by varying one parameter while holding other parameters fixed at their baseline values. As we show below, our main results do not rely on the choice of these parameter values.

4.3 Optimal Joint Attention-Consumption/Savings Decisions

In this subsection, we study the effects on optimal attention-consumption/savings decisions of the following factors: the relative prior volatility of the labor income risk to the capital income risk, the endowments of wealth and attention, the risk and time preferences, the expected capital return, and expected labor income.

Before moving to the discussion regarding effects of limited attention on the joint attention-consumption/savings decisions, it is helpful to inspect the mechanism via which the capital return risk and the labor income risk affect the consumption and saving behavior under FI-RE. It is worth noting that within our RU framework with two income risks, both of them may increase the amount of precautionary savings. Specifically, as shown in Weil (1990) and Angeletos (2007), in an RU model with only capital income risk, the responses of consumption and savings to the

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24 Barsky, Juster, Kimball, and Shapiro (1997) use MSC data and show that the overall average slope of the desired consumption path at a zero interest rate is 0.78 percent per year.
capital return risk is theoretically indeterminate. The signs of the responses are determined by
the value of the EIS.²⁵ For example, when the EIS is small (i.e., the income effect is relatively
small), a mean-preserving increase in the capital return risk leads to a lower certainty equivalent
capital return, a lower marginal propensity to consume (MPC), and higher savings.²⁶ In addition,
the degree of risk aversion determines the magnitudes of the responses. In contrast, in a model
with only labor income risk, the presence of uncertain labor income interacts with the convexity
of the marginal utility (i.e., prudence) and leads to an additional demand for precautionary savings.
The intuition behind this result is that consumers increase their saving in order to better prepare
themselves to face future labor income risk, and is similar to that in Leland (1968), Caballero
(1990), and Wang, Wang, and Yang (2016).

4.3.1 The Effects of Relative Prior Variance

To examine the importance of the relative prior variance of labor and capital income, we first fix
other parameters at their benchmark values, and then vary the value of the relative prior variance
($\sigma^2_y / \sigma^2_a$). As shown in Figure 2, it is clear that agents allocate more attention to the labor income
shock as the relative importance of the prior variance of the labor income to that of the capital
income increases. This is in line with many previous studies on optimal attention allocation,
such as Ma`ckowiak and Wiederholt (2009). The intuition for this result is straightforward. When
labor income becomes more uncertain, monitoring the risk in the labor-income dimension becomes
relatively more important; as a result, agents pay more attention to the labor income risk relative
to the capital income risk.

However, different from most of the previous studies that usually assume symmetric risks,
the two risks (i.e., labor income risk and capital income risk) enter our model asymmetrically
meaning that the risk on labor income is additive while the risk on capital income is multiplicative
(as it is on the capital return). This asymmetry generates new implications for optimal attention
allocation. To be more specific, in previous studies with symmetric risks, we have often seen that
when the prior variances of two risks are the same, the agent allocates equal amounts of attention
to each risk. However, in our model and as shown in Figure 2, agents pay more attention to the
capital income risk when the variance of the labor income shock is the same as that of the capital
income shock. This is due to two opposite effects of increasing the prior variance of the labor
income shocks: (i) the direct effect makes agents pay more attention to the labor income risk and
(ii) the indirect effect makes agents pay more attention to the capital income risk because the

²⁵ Note that in an EU framework, there are two competing influences of the capital return risk at work: (i) the
riskiness of the capital return makes savings less attractive than saving at the risk-free rate with the same average
return, and (ii) the capital income risk will induce a precautionary saving motive to the prudent consumer. When
the degree of prudence is sufficiently high (i.e., is greater than the CRRA plus 1 in the CRRA utility case), the
precautionary motive dominates.

²⁶ This effect is due to the negative interest elasticity of savings.
increase in the prior variance of labor income increases the amount of savings and thus makes the
capital return more attractive. As shown in Figure 2, when $\sigma_y^2/\sigma_a^2$ is below approximately 1.5,
the indirect effect dominates, meaning that agents pay more attention to the capital income risk
than the labor income risk.

As savings in the model, $K_1$, depends on the signal received and is stochastic, we denote the
expected saving rate by the unconditional mean of the ratio of savings in period 0 over initial
wealth $E[s] = E[K_1/Y_0]$. Figure 7 illustrates how the expected saving rate increases with the
prior variance of the labor income shock, holding the prior variance of the capital income risk
fixed, as well as other model parameters.

4.3.2 The Effects of Initial Wealth
To examine how initial wealth ($Y_0$) affects optimal attention-wealth allocation, we first fix other
parameters at their benchmark values and then vary the value of $Y_0$. We can see from the
upper-left panel of Figure 3 that the optimal amount of attention devoted to monitoring the
capital income risk is rising with the level of initial wealth, suggesting that rich agents pay more
attention to the capital income risk compared to poor agents. One potential explanation is that
rich people have more risky assets in absolute amount than poor people and therefore have a
stronger incentive to pay more attention to the capital income risk. This attention allocation
mechanism makes the rich households’ posterior variance in capital income smaller and investing
in this risky asset becomes more attractive. In addition, as shown in Figure 3, the attention
amount devoted to the labor income risk, $\kappa_y$, is decreasing with initial wealth ($Y_0$), and rich
people save at a higher rate partially due to the amount of the precautionary savings from the
increased posterior variance of labor income.

We can see from the left-upper panel of Figure 7 that wealthier people indeed save at higher
rates on average. This is in line with many empirical studies that show heterogeneous saving
behavior across different wealth groups (see Dynan, Skinner, and Zeldes (2004).). For example,
increasing $Y_0$ from 6 to 8 leads to a rise of the expected saving rate by approximately 4%. Yin
(2021) also considers the impact of wealth inequality on the attention choice and the consumption-
saving behavior. However, Yin (2021) considers one income shock (a shock to capital income)
and assumes the information-processing cost to be fixed, whereas the present paper investigates
optimal attention allocation in a more general setting with both the labor and capital income
shocks.

4.3.3 The Effects of the Discount Factor
To conduct this comparative statics analysis, we first fix other parameters at their benchmark
values, and then vary the value of the discount factor ($\beta$). As shown in the upper-right panel of
Figure 3, the discount factor has significant effects on attention allocation. More patient agents (higher $\beta$) allocate more attention to the capital income shock than to the labor income shock. The intuition for this result is that patient agents save more and the larger amount of savings makes paying attention to the capital income risk more valuable.

The middle-left panel of Figure 7 shows that the average saving rate increases with the discount factor. As this parameter governs the degree of the agents’ patience, a higher value of $\beta$ means that agents care more about their future consumption and utility and thus leads agents to save at a higher rate in the current period. In addition, there is also a feedback effect of attention allocation on saving behavior: agents with higher discount factor pay more attention to the capital income risk and less attention to the labor income risk, and thus attention allocation behavior makes risky asset more attractive and leads to more precautionary savings due to higher perceived uncertainty in the labor income risk.

4.3.4 The Effects of Attention Capacity

The attention allocation also depends on the total attention capacity ($\kappa$). First, it is easy to see, from the right panel of Figure 4, that the amounts of attention allocated to each of the two shocks increase with the total amount of attention capacity. Second, fixing the labor and capital income risks (as measured by the two prior variances), we investigate the way how agents allocate the attention in these two dimensions depends on the total attention capacity. In general, as shown by the right two panels of the proportion of Figure 4, the less total capacity is, the larger share of total attention is allocated to the labor income dimension, especially when the labor income risk is large. This suggests that the labor income risk becomes relatively more important to agents when the total attention capacity is more limited.

The upper-right panel of Figure 7 shows that the average saving rate decreases with total attention capacity. To understand this, let us take an extreme case as an example. When total attention is zero, i.e., agents pay no attention to the income shocks, they face greater posterior uncertainty in future income than those who pay some attention, and consequently, they choose to save at a higher rate due to the precautionary saving motive. We also notice that the pattern of the expected saving rate becomes flatter and flatter when increasing the total attention capacity. This is intuitive because when agents have more capacity to process information, the difference in their posterior variance is not big no matter how large the prior variance is. In another extreme case, when $\kappa \to \infty$, our model becomes completely deterministic and the saving rate becomes flat.
4.3.5 The Effects of the CRRA

To do this comparative statics analysis, we first fix the EIS at its benchmark value of 1/3 and then vary the value of $\gamma$ from 4 to 6. The left panels of Figure 5 show that the results discussed above hold for each value of the CRRA: $\kappa_y$ increases with the prior variance of the labor income shock, and $\kappa_a$ decreases with the prior variance of the labor income shock. Furthermore, we can also see that the optimal amount of attention devoted to monitoring the capital income risk rises with the degree of risk aversion, meaning that more risk-averse agents pay more attention to the capital income shock and less attention to the labor income shock. This result is driven by the following two channels. First, though we do not explicitly model portfolio choice in our model, agents in our model can be viewed as investing 100% of their total savings in the risky asset. In this case, an increase in the CRRA leads agents to pay more attention to the capital income risk in order to reduce the uncertainty about the capital return. Note that from the theoretical perspective, the share invested in the risky asset decreases with both the CRRA and the variance of the perceived signal, and is irrelevant with the labor income risk because the two risks are assumed to be uncorrelated in this paper. Second, when the CRRA increases, the agent has a stronger incentive to save due to the precautionary motive, which leads the agent to pay more attention to the capital income risk. Note that in our two-income-risk specification, both income risks lead to precautionary savings.

From the middle-right panel of Figure 7, it is clear that increasing the CRRA raises the expected saving rate. This is partially due to the fact that more risk-averse agents have a stronger precautionary saving motive and partially because of their attention allocation behavior leads to higher perceived uncertainty in labor income and a more attractive risky asset to invest.

4.3.6 The Effects of the EIS and the Importance of RU

To do this comparative statics analysis, we first fix the CRRA at its benchmark value 5 and then vary the value of the EIS ($\psi$) from 1/3 to 0.75. From the right panel of Figure 5, we can see that a reduction in the EIS leads to a smaller amount of attention devoted to monitoring the capital income shock, and a larger amount of attention to the labor income shock. When the EIS is small, the change in consumption is less sensitive to the change of the capital return. As a result, consumers pay less attention to the capital income risk. However, as can be seen from Figure 5, this relationship between the EIS and attention allocation to the capital income risk may reverse if labor income is very uncertain. This is because a larger prior variance of the labor

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27 We can also view this specification as an equilibrium result of a typical consumption-based asset pricing model in which the optimizing investors choose to invest all of their total wealth in the risky portfolio and hold zero inside bonds.

28 The EIS governs the sensitivity of consumption change to the change of interest rate, i.e., the percent change in consumption in response to one percentage change of interest rate.
income risk leads to higher savings, and the EIS also governs how reluctant consumers change consumption across periods. In this situation, people with a low EIS have a stronger distaste for intertemporal substitution in utility and would like to pay more attention to the capital income risk due to the larger amount of savings. Note that under the expected utility specification, a lower EIS means a higher CRRA by a restriction, which means that a decrease in the EIS also leads to smaller attention to the capital income risk. However, with a recursive utility, this result may be misleading. That is, a lower EIS and a higher CRRA have opposite effects on attention allocation. As we discussed above, the main reason for this result is that the CRRA and the EIS affect the optimal attention allocation via distinct mechanisms.

When fixing $\gamma$ and varying $\psi$, we can see from the lower-left panel of Figure 7 that for each value of the EIS, agents, on average, save at higher rates for higher prior variances of the labor income shock. Furthermore, we can also see that the EIS has significant effects on the expected saving rate. Agents who are more reluctant to substitute consumption intertemporally (smaller $\psi$) have higher expected saving rates. It is worth noting that although increasing the CRRA and reducing the EIS have a similar effect on the expected saving rate, their economic intuitions are totally different. In our RU model with two income risks, both the CRRA and the EIS play roles in determining the demand of precautionary savings. Specifically, the EIS affects savings via the relative importance of the income and substitution effects (i.e., the sign of the interest elasticity of savings), while the CRRA determines the magnitude of the change in savings. The above analysis therefore shows the importance of introducing the recursive utility in our model.

4.3.7 The Effects of the Expected Capital Return and the Expected Labor Income

The left panels of Figure 6 show that the optimal amount of attention devoted to the capital income risk is increasing with the unconditional mean of the capital return ($\mathbb{E}[A_1]$) whereas the amount of attention allocated to the labor income risk is decreasing with it. The reason is that when the capital return is expected to be high, agents would think that the capital income will be more important for their consumption in period 1 and thus pay more attention to the capital income risk.

The lower-right panel of Figure 7 presents the negative effect of increasing the unconditional expected capital return on the saving behavior. However, compared to the effects of changing other parameters, the effect of an increase in the expected return is small because increasing the expected capital return has three distinct effects on the saving behavior: first, it makes saving in this asset more attractive and the substitution effect motivates agents to save more; second, a higher return leads to more capital income tomorrow and the income effect lowers the saving rate; third, it leads to reallocating more attention to the capital income risk, and less attention to the labor income risk, which leads agents to save more due to the precautionary saving motive. These three distinct effects work together on the saving behavior and weaken the total effect of
an increase in the expected capital return.

The right panels of Figure 6 show that the optimal amount of attention devoted to the capital income risk is decreasing with the unconditional mean of labor income ($E[Y_1]$) whereas the amount of attention allocated to the labor income risk is increasing with it. The intuition is similar to the one above. When labor income is expected to be high, agents think that this part of income will be more important for their consumption in period 1 and thus pay more attention to the labor income risk.

5 Confronting the Model with Data

In this section, we show how the mechanisms we highlighted in the previous section can help explain the data along three aspects.

5.1 Consumption Response to the Prior of the Labor Income Risk

The first aspect we want to explore is how individual consumption responds to the prior labor income risk. Column 1 in Table 2 reports that, on average, individuals with 1 more unit of the prior variance of labor income experienced a decline in consumption by about 11.6 percentage points after losing their jobs. Qualitatively, this finding is highly consistent with our model’s prediction.

As shown in the upper-right panel of Figure 8, our calibrated model predicts that the reduction in consumption after unemployment is smaller for agents with larger prior variances of labor income. Using the results regarding the optimal saving behavior, this empirical fact can be explained as follows. An agent with a larger prior variance of the labor income risk will save more due to the precautionary saving motive. We call this channel the direct precautionary saving effect. But at the same time the agent also pays more attention to the labor income risk, leading to a reduction in the posterior uncertainty about labor income and precautionary savings. Meanwhile, due to limited attention, the agent pays less attention to the capital income risk, making the risky asset riskier and less attractive and then reducing the saving rate. We call these two channels induced by limited attention the indirect precautionary saving effects. Figure 7 shows that the direct precautionary motive effect dominates the other two indirect effects, meaning a higher prior variance of the labor income risk causes a higher total saving rate, and thus the reduction of consumption is smaller for those who have higher posterior variance in labor income.

Our model can also be used to explain the heterogeneous consumption responses for households with different levels of prior uncertainty from a quantitative perspective. Specifically, as shown in Table 4, we divide individuals’ prior uncertainty into two groups: one group with prior variance below the average value (0.36), and the other one with prior variance above this value. We then set the low and high prior uncertainty as follows: the low prior, 0.06, is the average of individuals’
prior uncertainty in the first group, and 1.16 is that in the second group. From Table 4 we can see that the group with a low prior variance, on average, experiences a drop in consumption by 25.8% and the group with a high prior variance, on average, experiences a drop by 14.2%, which are close to the empirical counterparts of the change in consumption when being unemployed from the HRS data (25.2% and 12.9%, respectively).

5.2 Consumption Response to Financial Shocks

The second aspect we want to explore is how the consumption response to financial shocks depends on the prior labor income risk. As also shown in Column 1 of Table 2, when financial wealth drops, consumption declines more if the agent has a higher prior variance of the labor income risk.

To fully explore the mechanism, we rewrite the expression for the expected change in consumption as follows:

$$\frac{\Delta C_1}{C_0} = \frac{A_1 K_1 + Y_1 - (Y_0 - K_1)}{Y_0 - K_1} = \frac{s}{1-s} (A_1 - 1) + \frac{1}{1-s} \frac{Y_1}{Y_0} + \frac{s}{1-s} - 1, \quad (22)$$

where $s = K_1/Y_0$ is the saving rate (or the marginal propensity to save, the MPS) and $1-s$ is the marginal propensity to consume (the MPC). It is clear from this expression that the first term on the right hand side can be used to characterize the response of the consumption change to the loss of financial wealth. The empirical fact mentioned above can thus be explained as follows. When agents have a larger prior variance of the labor income risk, they save more, leading to an increase in $s/(1-s)$, i.e., the ratio of the MPS to the MPC. As a result, given the negative net capital return, $A_1 - 1$, the consumption declines more. In the upper-right panel of Figure 8, we plot the marginal effect of the loss in financial wealth for different values of the prior variance of labor income and different levels of initial wealth. It is clear that the curves in this figure are downward sloping and are thus consistent with the empirical facts shown in Table 2.

In addition, we can also compare the marginal effects of the prior variance of the labor income risk on the consumption change due to losses in financial wealth and human wealth in the model to the empirical counterparts. For the empirical part, the interaction terms in Table 2 show that increasing the prior variance of the labor income risk by 1 unit increases the marginal effect of one percentage point loss in financial wealth on the consumption drop by about 0.02 percentage points and decreases that of the unemployment shock by about 11.5 percentage points. In contrast, in the model, when $\kappa = 1$, the corresponding quantitative interaction effects are 0.1 and 10.7, respectively.\(^{29}\)

\(^{29}\)In the case with $\kappa = 0$, these effects become 0.2 and 18.8, which are too large to match the empirical results because these interaction effects increase with the saving rate and the saving rate is larger for smaller attention capacity.
5.3 Heterogeneous Consumption Responses Due to Different Wealth Levels

The third aspect we want to examine is the heterogeneous consumption response driven by different wealth levels. As shown in Columns 3 and 4 of Table 2, the coefficient for the interaction term of financial asset and the unemployment shock is negative, meaning that wealthier individuals experience larger consumption decline (in percent) at unemployment.

We can better explain this empirical result by rewriting (22) as follows:

\[
\frac{\Delta C_1}{C_0} = \frac{A_1 K_1 + Y_1 - (Y_0 - K_1)}{Y_0 - K_1} = \frac{s}{1 - s} A_1 + \frac{1}{1 - s} \frac{Y_1}{Y_0} - 1,
\]  

(23)

From (23), we can see that there are two opposite effects of an increase in initial wealth on the change in consumption. First, a higher level of initial wealth, \(Y_0\), leads to a higher savings rate, as explained in the previous section and Figure 7. Holding everything else equal, a higher savings rate would increase the first two terms on the right side of (23) and thus lead to a smaller decline in consumption.\(^{30}\) Second, a higher level of initial wealth lowers the gross growth rate of wealth, \(Y_1/Y_0\), or the relative importance of unemployment benefits to initial wealth. This tends to reduce \(\Delta C_1/C_0\) (i.e., leads to a larger percent decline in consumption). The left panel of Figure 9 plots these two effects and shows that in our calibrated exercise, the second effect dominates the first, suggesting that wealthier individuals experience a larger consumption decline at unemployment. This model’s prediction is also consistent with the empirical counterpart.

It is important to note that, quantitatively, these two effects are affected by the total attention capacity. As shown in the right panel of Figure 9, when attention capacity is increasing, the change in consumption becomes flatter and flatter. We can easily show that when attention capacity goes to infinity, the change in consumption in this deterministic scenario is \((\beta A_1)^{1/\gamma} - 1\). One potential explanation for this diminishing effect of initial wealth on the change in consumption is that increasing the attention capacity strengthens the effect of initial wealth on the saving behavior (the first effect mentioned above). Keeping all other parameters constant, the consumers’ saving decisions depend on initial wealth and the precautionary saving motive due to income risks. When attention capacity is small, agents face higher perceived uncertainty and the effect of initial wealth on total savings is smaller. However, when increasing attention capacity, agents have less and less perceived uncertainty in their income; as a result, the effect of initial wealth on saving decisions becomes stronger. As shown in Table 5, the positive effects of increasing initial wealth on the expected saving rate becomes larger for larger attention capacity.

\(^{30}\)Notice that in the calibration, the consumption change in (23) is negative, and thus a smaller decline means consumption change is less negative.
6 Welfare Implications

In this section, we compute the welfare gains if the inattentive agents are allowed to increase their channel capacity. Specifically, we follow Luo (2008) and Maćkowiak and Wiederholt (2015), and also conduct a welfare analysis. As shown in Table 6, we calculate the utility losses for three different values of $\kappa$ and four different values of $\sigma_y^2$, $Y_0$, $\gamma$, and $\psi$. Here is the procedure to conduct the welfare analysis. Our main purpose for this exercise is to investigate how an increase in attention capacity affects the expected lifetime utility. For example, we first choose $\kappa = 1$ as the starting value, and calculate the corresponding unconditional expected lifetime utility. Then we increase each starting value of attention capacity $\kappa$ by 100% and compute the corresponding unconditional expected lifetime utility for each $\kappa$. Finally, we can compute the percentage increase of the expected lifetime utility using this formula:

$$\frac{E[U(\kappa_{\text{new}})] - E[U(\kappa_{\text{baseline}})]}{E[U(\kappa_{\text{baseline}})]}.$$  \hspace{1cm} (24)

Next, we will present two exercises for the welfare analysis. In the first exercise, we choose two different starting points of attention capacity, namely, 1 and 2. And then at each point, we increase attention capacity by 100%. First, we can see from all the three panels of Table 6 that the utility gains are increasing with the level of attention capacity. This result is intuitive and in line with the findings in Luo (2008): agents with higher attention capacity can better predict their future income and in the extreme case when they have infinite capacity, it converges to the corresponding FI-RE scenario. Second, if we compare vertically for each panel, it clearly shows that the change in the expected utility is decreasing in $\kappa$. More precisely, as shown in the first column of Panel A where $Y_0 = 7$, increasing attention capacity from 1 to 2 leads to an increase of welfare by about 1.9%, but a rise of attention from 2 to 4 increases welfare by about 1%. Comparing Rows 1 and 2 of the table, these results suggest a heterogeneity in the welfare gain for agents with different levels of attention capacity.

We also show the effects of changing parameters values on welfare gains. Panel A of Table 6 shows that for a given attention capacity, the expected lifetime utility is decreasing with initial wealth. As shown in the first row where we increase $\kappa$ from 1 to 2, if $Y_0$ is increased from 6 to 8, we can see that the welfare gain decreases from about 2% to 1.8%. One potential explanation is that wealthier individuals already consume more than poor individuals, and therefore, increasing an additional unit of attention capacity is more beneficial for poor individuals as it can help them make more efficient consumption-saving plans. Panels B and C of Table 6 also show that the

---

31 Different from our two-period consumption model with two income shocks, Luo (2008) studies an infinite horizon permanent income model with a single labor income shock. He examines the welfare effects of income shocks under rational inattention by calculating how much utility agents will lose if the actual consumption path under rational inattention deviates from the first-best consumption path under full information.
welfare gain is increasing with the discount factor and the CRRA. However, the intuitions for these results are different. For Panel B, more patient agents care more about their future utility and increasing attention capacity can reduce their uncertainty in future income and consumption. In contrast, the intuition of the results in Panel C is that, for more risk-averse agents, the larger the attention capacity, the higher the lifetime expected utility because with more attention, they would face less uncertainty from labor and capital incomes. Finally, the results in Panel D show that the welfare gain is decreasing with the EIS. When the EIS becomes smaller, agents prefer the consumption profile to be smoother across periods and increasing attention can help reduce the fluctuations.

In the second exercise, we investigate the effect of optimal attention allocation on the welfare gain. More specifically, as shown in the third column of Table 7, we first repeat the welfare analysis for different levels of initial wealth by increasing total attention capacity. These results are subject to the attention allocation mechanism. In the second column, we first solve the optimal attention allocation strategy when $\kappa = 1$ for each value of initial wealth. Then we fix this attention allocation strategy and compute the welfare gain by increasing total attention capacity to 2, 3, and $\infty$, respectively. When comparing these two columns, we can see that the welfare gains are consistently larger in the optimal/flexible attention allocation case. This implies that allowing flexible attention allocation makes the agent better off for different values of initial wealth, and the welfare gains from the flexible adjustment are significant. For example, when $Y_0 = 7$ and $\kappa$ is increased from 1 to 2, the percentage change of the welfare gain from switching the fixed attention allocation mechanism to the flexible attention allocation mechanism is about 33.3%.

7 Further Discussions

7.1 Relative Consumption Dispersion to Income

The significant increase in household income inequality or dispersion for the U.S. in the 1980s and 1990s is a well-documented fact. Many studies have found that the dispersions of U.S. household earnings and incomes have a strong upward trend. In addition, the literature also documents that the recent increase in income inequality in the U.S. has not been accompanied by a corresponding rise in consumption inequality over the same period. In other words, over the sampling period, income and consumption inequality diverged and the relative dispersion/inequality of consumption to income has decreased, as discussed by Krueger and Perri (2006) and Blundell, Pistaferri, and Preston (2008). In the section, we show that the benchmark model proposed in Section 3 can also be used to study how attention allocation affects the relative dispersion of changes in consumption to income.\(^{32}\) The upper panel of Figure 10 shows the evolution

\(^{32}\)The relative consumption dispersion/inequality is measured as the ratio of the standard deviation of the change in consumption to the standard deviation of the change in income. Luo, Nie, Wang, and Young (2017) study how
of consumption and income dispersion as well as the relative dispersion of changes in consumption to income between 1980 and 2010. It clearly shows that the relative dispersion declines while the volatility of labor income increases.

The bottom two panels in Figure 10 show that our model generates a similar pattern under different parameters. That is, as the prior variance of the labor income risk rises, the relative consumption dispersion declines. It is worth noting that in the literature, the most appropriate empirical measure of labor income uncertainty is not obvious. Some previous studies have proxied income uncertainty with either the variability of a household’s income or the variability of its expenditures. However, as pointed out by Lusardi (1998) and Guiso, Jappelli, and Terlizzese (1992), the variability measures mentioned above may be poor proxies because they can contain large controllable elements. Here we follow Lusardi (1998) and Carroll, Dynan, and Krane (2003) and use the prior variance of labor income calculated using the probability of job loss to measure the uncertainty/volatility of labor income.

In addition, from the left panel of Figure 10, we can see that the relative dispersion is decreasing with $\kappa$. The intuition for this result is as follows. There are two factors that can affect the relative dispersion of consumption to income: one is attention capacity and the other one is the saving behavior. When agents have more attention capacity, the relative dispersion of consumption becomes smaller because they have more precise signals (i.e., smaller variances of the noises). When agents save at higher rates, their consumption becomes smoother and the relative dispersion becomes smaller. Therefore, a potential explanation for the positive correlation between the relative dispersion and the amount of attention is that when $\kappa$ is large, although agents save less, they have smaller perceived uncertainty in their future income and consumption; as a result, the change of consumption is less dispersed.

7.2 The Bequest Motive

We choose a two-period model so we can solve the optimal consumption-attention allocation problem with both capital and income risks. One potential inconsistency of the model is that consumers in the second period will consume all their resources which looks different from the HRS data we use: people in the second period do not spend all their resources in reality. In this subsection, we introduce a bequest motive into the model to overcome this inconsistency and show that the key implications of the model remain unchanged. In other words, introducing bequest motives helps the model to generate a realistic consumption level in the second period but the implication regarding relative changes in consumption across individuals remain largely unchanged.

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33See Online Appendix A in Luo, Nie, and Young (2020) for more details on how the panel is constructed from Panel Study of Income Dynamics (PSID).
For convenience, we follow Carroll (1998) and Dynan, Skinner, and Zeldes (2002), and consider an expected utility model (a special case of our benchmark model) with constant relative risk aversion preferences:

$$\max U = \mathbb{E} [u(C_0) + \beta \mathbb{E} [u(C_1) + b(W) | S_0]],$$

s.t. $C_0 = Y_0 - K_1$,

$C_1 = A_1 K_1 + Y_1 - W$,

$$\kappa_a + \kappa_u \leq \kappa,$$

where $W$ is the bequest left after period 1 and $b(\cdot)$ is the utility from leaving a bequest to their offsprings; signal structure and information-flow constraint are defined in Equation (9) and Equation (14).

Carroll (1998) assumes that $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}$ and $b(W) = \frac{(W+\omega)^{1-\alpha}}{1-\alpha}$. For simplicity, we consider a special case with $\omega = 0$ and $\gamma = \alpha$. In this case, we first obtain that:

$$C_1 = W = \frac{A_1 K_1 + Y_1}{2},$$

(25)

and that:

$$U = \mathbb{E} [u(C_0) + \beta \mathbb{E} [u(C_1) + b(W) | S_0]].$$

(26)

Plugging Equation (25) back to the utility function $u(C_0) + \beta \mathbb{E} [u(C_1) + b(W) | S_0]$, we can have the first order condition for optimal savings $K_1$ as:

$$u'(Y_0 - K_1) = \beta \mathbb{E} \left[ u' \left( \frac{A_1 K_1 + Y_1}{2} \right) \frac{A_1}{2} + b' \left( \frac{A_1 K_1 + Y_1}{2} \right) \frac{A_1}{2} | S_0 \right].$$

(27)

Finally, using the result of optimal savings in the $U$ function, we can find the optimal attention allocation that maximizes the expected utility.

We plot the optimal attention-consumption allocation and consumption responses in Figure 11. In these figures, we first notice that they are consistent with our main results obtained in our benchmark model: wealthier individuals pay more attention to the capital income risk than those with less initial wealth; people pay more attention to the labor income risk if the prior variance of the labor income shock increases; people with more initial wealth save at higher rate and have larger reduction in consumption when they experience unemployment shock and financial wealth loss. In addition, we also find that due to the bequest motive, consumers choose to save more than in the benchmark model (see the upper-left panel of Figure 7). This result is in line with the literature that studies the importance of the bequest motive in saving behavior, such as Carroll (1998) and Kopczuk and Lupton (2007). Moreover, the lower-right panel in Figure 11 shows that, after considering the bequest motive, the consumption response to income shocks is not
quantitatively different from the main results of our benchmark model. This is because although people with the bequest motive save at higher rates, their consumption in period 1 is also only a part of the total resource.

8 Conclusion

In this paper, we construct and solve a two-period consumption-saving model with recursive utility, capital income and labor income risks, and limited attention. The main feature of this model is that it allows agents with limited attention to choose how to allocate that attention in the presence of both labor and capital income risks. Specifically, our model allows us to quantitatively evaluate how the optimal attention-consumption allocation is affected by the relative prior variance of the two exogenous income risks (capital income and labor income risks), differences in individuals' endowments of wealth and attention, and differences in their risk and time preferences. We show that our simple model can capture some key aspects of household consumption behavior observed in the U.S. microdata. Finally, we show that the welfare loss due to limited attention is significantly larger for households with lower wealth; allowing households to flexibly allocate their attention can significantly reduce this welfare loss.

9 Appendix

9.1 Data Appendix

This appendix describes the variables used in our empirical analysis as well as the sample treatment.

- Consumption change \(\Delta C_t \) is the total spending change compared to the previous year. These data are from HRS 2009 internet survey, in which respondents were directly asked “By how much has your household spending [increased/reduced] compared to a year ago?” In our analysis, we follow Christelis, Georgarakos, and Jappelli (2015) and exclude observations with consumption change larger than 80% or smaller than -80%.

- Financial wealth change \(\Delta WF_t \) is the reduction of financial wealth since September of 2008. These data are from HRS 2009 internet survey, in which respondents were directly asked “How much has the total value of your retirement plans (IRA and Keogh plans, trust, mutual funds, stocks, and other investment) declined percentage-wise since September of 2008?”

- Likelihood of losing job \(p\) indicates the chance that the respondent believes that she/he will lose job during the next year. These data are from HRS 2006 core survey. We use this
information as the subjective belief of being unemployed in the future.

- Labor income is the total amount of income from all jobs. We use both 2006 and 2008 core surveys of HRS to obtain individuals’ labor income. Especially the labor income of 2006 is used to calculate the prior variance in labor income as shown below.

- Prior variance (\(pvar\)) is defined as \(p(1 - p)(1 - \eta)^2\tilde{Y}^2_{2006}\). Here we follow the measured introduced by Lusardi (1998). \(\tilde{Y}_{2006}\) is the labor income in 2006 normalized by the mean income in 2006. \(\eta\) is the replacement ratio of unemployment from the website of the US Department of Labor. In the empirical analysis, we exclude one outlier with prior variance larger than 10000.

- Unemployment shock (\(\Delta U_t\)) shows whether a respondent became unemployed between the date of the survey and the same date a year ago. Here we want to construct an unemployment shock that affects respondent’s current consumption, but not the consumption a year ago. These data are from HRS 2009 internet survey, in which respondents were asked to report their employment status, and the year and month of being unemployed.

- Net financial asset is the sum of respondent’s financial asset (including retirement plans, IRA and Keogh plans, trust, mutual funds, stocks, and other investment) and debts. These data are from HRS 2008 core survey.

- Subjective belief of stock market is measured by individual’s belief about the percentage chance that Dow Jones industrial average will increase next year. To keep the consistency with the prior belief about the labor income risk, we also use data from HRS 2006 core survey.

- Other personal characteristics (including, age, education, gender, marital status, retirement status, household size) are from HRS 2009 internet survey. Especially, education here is measured by years of education. Similar as the unemployment shock, retirement is a dummy variable indicates whether the respondent became retired between the data of interview and the same date a year ago.

9.2 Appendix: Solving the Recursive Utility Model

We adopt the outer optimization approach to solve the optimal attention allocation problem in the recursive utility case. For any given amount of attention to capital income shock \(\kappa_a\), we can obtain the distribution of the signal on capital income shock, \(S_a\). As the total amount of attention is fixed, we can also obtain the amount of attention to labor income shock, \(\kappa_y\), and the distribution of the signal on capital income shock, \(S_y\). Then, we can solve the optimal savings \(K_1^*\) for a combination of \((s_a, s_y)\) according to Equation (21).
Here we maximize the unconditional expected utility (evaluating over possible signals) by choosing the optimal attention allocation:

\[
U(K_1) = \frac{(Y_0 - K_1)^{1-1/\psi}}{1 - 1/\psi} + \beta \left( \frac{\mathbb{E} [(A_1 K_1 + Y_1)^{1-\gamma}|S_0]}{1 - 1/\psi} \right)^{1-1/\psi}
= U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1).
\]

Define \( V(\hat{\sigma}_a^2, \hat{\sigma}_y^2) = \mathbb{E}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2, K_1)] \), the attention allocation is to choose \( \kappa_a \):

\[
\max_{\kappa_a} V(\kappa_a),
\]

subject to

\[
\frac{1}{2} \log \left( \frac{\sigma_a^2}{\hat{\sigma}_a^2} \right) = \kappa_a, \quad \frac{1}{2} \log \left( \frac{\sigma_y^2}{\hat{\sigma}_y^2} \right) = \kappa_y
\]

(28)

\[
\kappa_a + \kappa_y = \kappa.
\]

(31)

Here we can easily show the mean and variance of the signals can be written as:

\[
\mathbb{E}[S_a] = \mu_a, \mathbb{V}(S_a) = \frac{(\sigma_a^2)^2}{\sigma_a^2 - \hat{\sigma}_a^2}; \quad \mathbb{E}[S_y] = \mu_y, \mathbb{V}(S_y) = \frac{(\sigma_y^2)^2}{\sigma_y^2 - \hat{\sigma}_y^2}.
\]

Their corresponding density functions are :

\[
f_{S_a} = \frac{1}{\sqrt{2\pi \mathbb{V}(S_a)}} \exp \left( - \frac{(s_a - \mu_a)^2}{2\mathbb{V}(S_a)} \right) \quad \text{and} \quad f_{S_y} = \frac{1}{\sqrt{2\pi \mathbb{V}(S_y)}} \exp \left( - \frac{(s_y - \mu_y)^2}{2\mathbb{V}(S_y)} \right).
\]

Define \( t_a = \frac{s_a - \mu_a}{\sqrt{2\mathbb{V}(S_a)}} \) and \( t_y = \frac{s_y - \mu_y}{\sqrt{2\mathbb{V}(S_y)}} \), we have

\[
s_a = \mu_a + \sqrt{2\mathbb{V}(S_a)} t_a = \mu_a + \frac{\sqrt{2}\sigma_a^2}{\sigma_a^2 - \hat{\sigma}_a^2} t_a,
\]

\[
s_y = \mu_y + \sqrt{2\mathbb{V}(S_y)} t_y = \mu_y + \frac{\sqrt{2}\sigma_y^2}{\sigma_y^2 - \hat{\sigma}_y^2} t_y.
\]

We apply the Gaussian quadrature approach to approximate the unconditional expectation of the utility and obtain the value for some \( \kappa_a \) (\( \kappa_y \)). In the second step, we adopt the inner optimization approach to solve the corresponding optimal consumption-saving problem. Specifically,
the RHS of the Euler equation (21) can be written as:

$$\beta \left( \mathbb{E} \left[ (A_1 K_1 + Y_1)^{1-\gamma} | S_0 \right] \right)^{\frac{1}{1-\gamma}} \mathbb{E} \left[ (A_1 K_1 + Y_1)^{-\gamma} A_1 | S_0 \right].$$

The conditional distributions of $\epsilon_a | S_a = s_a$ and $\epsilon_y | S_y = s_y$ can be written as:

$$f_1(\epsilon_a, \epsilon_y) = \begin{cases} \exp \left( \sqrt{2}\sigma_a x_a + \frac{\hat{\sigma}^2}{\sigma_a} \mu_a + \left( 1 - \frac{\hat{\sigma}^2}{\sigma_a^2} \right) s_a \right) K_1 + \exp \left( \sqrt{2}\sigma_y x_y + \frac{\hat{\sigma}^2}{\sigma_y} \mu_y + \left( 1 - \frac{\hat{\sigma}^2}{\sigma_y^2} \right) s_y \right) \end{cases} \left( 1 - \frac{\hat{\sigma}^2}{\sigma_a^2} \right)^{1-\gamma} \exp \left( \sqrt{2}\sigma_y x_y + \frac{\hat{\sigma}^2}{\sigma_y} \mu_y + \left( 1 - \frac{\hat{\sigma}^2}{\sigma_y^2} \right) s_y \right).$$

Define

$$x_a = \frac{\epsilon_a - \left( \mu_a + \sigma_a \rho_a \frac{s_a - \mathbb{E}[S_a]}{\sqrt{V(S_a)}} \right)}{\sigma_a \sqrt{1 - \rho_a^2} \sqrt{2}}, \quad x_y = \frac{\epsilon_y - \left( \mu_y + \sigma_y \rho_y \frac{s_y - \mathbb{E}[S_y]}{\sqrt{V(S_y)}} \right)}{\sigma_y \sqrt{1 - \rho_y^2} \sqrt{2}},$$

we have

$$\epsilon_a = \sigma_a \sqrt{1 - \rho_a^2} \sqrt{2} x_a + \mu_a + \sigma_a \rho_a \frac{s_a - \mathbb{E}[S_a]}{\sqrt{V(S_a)}},$$

$$\epsilon_y = \sigma_y \sqrt{1 - \rho_y^2} \sqrt{2} x_y + \mu_y + \sigma_y \rho_y \frac{s_y - \mathbb{E}[S_y]}{\sqrt{V(S_y)}},$$

where $\rho_a^2 = 1 - \sigma_a^2/\sigma_a^2$, $\sqrt{1 - \rho_a^2} = \sigma_a/\sigma_a$, $\mathbb{E}[S_a] = \mu_a$, $\mathbb{V}(S_a) = \sigma_a^2/\sigma_a^2$, $\rho_y = 1 - \sigma_y^2/\sigma_y^2$, $\sqrt{1 - \rho_y^2} = \sigma_y/\sigma_y$, $\mathbb{E}[S_y] = \mu_y$, and $\mathbb{V}(S_y) = (\sigma_y)^2/\sigma_y^2$. Finally, we have

$$\epsilon_a = \hat{\sigma}_a \sqrt{2} x_a + \frac{\hat{\sigma}^2}{\sigma_a^2} \mu_a + \left( 1 - \frac{\hat{\sigma}^2}{\sigma_a^2} \right) s_a,$$

$$\epsilon_y = \hat{\sigma}_y \sqrt{2} x_y + \frac{\hat{\sigma}^2}{\sigma_y^2} \mu_y + \left( 1 - \frac{\hat{\sigma}^2}{\sigma_y^2} \right) s_y.$$

Applying the Gaussian quadrature approach, we can approximate the RHS as follows:

$$\mathbb{E}[f_1(\epsilon_a, \epsilon_y)|S_a, S_y] = \int \int f_1(x_a, x_y) e^{-\frac{x_a^2}{2}} e^{-\frac{x_y^2}{2}} dx_a dx_y \approx \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\pi} \omega_{a,i} \omega_{y,j} f_1(\xi_{a,i}, \xi_{y,j}),$$

$$\mathbb{E}[f_2(\epsilon_a, \epsilon_y)|S_a, S_y] = \int \int f_2(x_a, x_y) e^{-\frac{x_a^2}{2}} e^{-\frac{x_y^2}{2}} dx_a dx_y \approx \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\pi} \omega_{a,i} \omega_{y,j} f_2(\xi_{a,i}, \xi_{y,j}),$$

where $\xi_a$ and $\xi_y$ are nodes and $\omega_a$ and $\omega_y$ are weights.
Next, we solve for the optimal attention allocation:

$$
\max_{\kappa_a, \kappa_y} V = \mathbb{E}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)],
$$

subject to (29)-(31). We then use the Gaussian-quadrature approach to approximate the indirect utility:

$$
\mathbb{E}[U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2)] = \int \int U(S_a, S_y, \hat{\sigma}_a^2, \hat{\sigma}_y^2) \, dt_a \, dt_y \approx \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\pi} \omega_{sa,i} \omega_{sy,j} \mathbb{E}[U(\xi_{sa,i}^{GH}, \xi_{sy,j}^{GH})],
$$

where $\xi_{sa}$ and $\xi_{sy}$ are nodes and $\omega_a$ and $\omega_y$ are weights.

In summary, we solve the model backwards. First, Solving $F(K_1) = -(Y_0 - K_1)^{-1/\psi} + \beta (\mathbb{E}[f_1(\epsilon_a, \epsilon_y)|S_a, S_y])^{-\psi/\psi'} \mathbb{E}[f_2(\epsilon_a, \epsilon_y)|S_a, S_y] = 0$ yields the optimal savings, $K_1^*$. Then plugging this result back into the utility function yields the indirect utility, $U(S_a, S_y, \kappa_a, \kappa_y)$. We can then compute the unconditional expected utility evaluated over signal observations and solve for the optimal attention allocation, $\kappa_a^*$ and $\kappa_y^*$, by maximizing the unconditional expected utility. The following is the detailed procedure of solving the model:

1. Set $\kappa_{a, min} = 0.0001$ and $\kappa_{a, max} = \kappa - 0.0001$, such that $\kappa_{a, max} = \kappa - 0.0001$ and $\kappa_{a, min} = 0.0001$.

2. For $\kappa_{a, min}$, use the Legendre-Gauss approach compute 7 nodes for $S_a$ and their corresponding weights. Similarly $S_y$ for $\kappa_{a, max}$. For each combination $(s_a, s_y)$, compute the optimal savings $K_1^*$, and then compute the value of $V(\kappa_{a, min})$.

3. For $\kappa_{a, max}$, use the Legendre-Gauss approach compute 7 nodes for $S_a$ and their corresponding weights. Similarly $S_y$ for $\kappa_{a, min}$. For each combination $(s_a, s_y)$, compute the optimal savings $K_1^*$, and then compute the value of $V(\kappa_{a, max})$.

4. Compute the slope $\left(V(\kappa_{a, max}) - V(\kappa_{a, min})\right) / (\kappa_{a, max} - \kappa_{a, min})$. If the slope is positive, set $\kappa_{a, min} = \left(\kappa_{a, min} + \kappa_{a, max}\right) / 2$; if the slope is negative, set $\kappa_{a, max} = \left(\kappa_{a, min} + \kappa_{a, max}\right) / 2$.

5. Iterate the steps above till the slope is close to zero, and we have $\kappa_a = \kappa_{a, max} = \kappa_{a, min}$.
References


Figure 1: **Timeline.** In this figure, we divide the optimization problem in period 0 into two stages to make the solution method clear; but in the model, the agent solves these two problems simultaneously.

Figure 2: **Attention Allocation and Relative Prior Variance**
Figure 3: Effects of Initial Wealth and Discount Factor on Attention Allocation

Figure 4: Optimal Attention Allocation and Total Attention Capacity
Figure 5: Effects of EIS and Risk Aversion on Attention Allocation

Figure 6: Effects of Capital Return and Labor Income on Attention Allocation
Figure 7: Comparative Analysis: Average Saving Rate
Figure 8: Consumption Response to Labor and Capital Income Shocks. Note: the left figure shows the response of consumption (in percentage change) to income shocks in the benchmark model; the right figure shows the consumption response (in percentage change) to financial losses in the benchmark.

Figure 9: Effects of Initial Wealth on Total Consumption Change and Its Components
Figure 10: **Relative Dispersion of Changes in Consumption and Income.** Data source for the left panel is PSID. Parameter are set at their benchmark values. We normalize the representative agent’s income to 1 and thus set the initial wealth to 3.
Figure 11: Attention Allocation, Saving, and Consumption Change with Bequest Motivations
Table 1: Summary Table

<table>
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<th>Mean</th>
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<tr>
<td>Consumption change (%)</td>
</tr>
<tr>
<td>Financial wealth change (%)</td>
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<tr>
<td>Prior variance</td>
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<tr>
<td>Unemployment shock</td>
</tr>
<tr>
<td>Age</td>
</tr>
<tr>
<td>Education</td>
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<tr>
<td>Sex</td>
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<tr>
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<tr>
<td>Retirement</td>
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<tr>
<td>Chances that Dow Jones will increase next year</td>
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<td>Income (thousand dollars)</td>
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Table 2: Elasticities of Consumption w.r.t. Financial Losses and Unemployment Shocks

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<th>(3)</th>
<th>(4)</th>
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<td>0.108**</td>
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<td>(2.34)</td>
<td>(2.72)</td>
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<td>-7.479**</td>
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<td>(-4.24)</td>
<td>(-4.20)</td>
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<td>(-2.15)</td>
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<td></td>
<td>(0.18)</td>
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<td>Prior variance* change in fin.</td>
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<td>0.0183*</td>
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<tr>
<td>asset(%)</td>
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<td>(1.66)</td>
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</tr>
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<td>Prior variance*unemployment</td>
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<td>11.52*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>shock</td>
<td>(1.89)</td>
<td>(1.88)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial asset (normalized</td>
<td>-0.0141</td>
<td>-0.0156</td>
<td>-0.0837</td>
<td>-0.0869</td>
</tr>
<tr>
<td>by average income)</td>
<td>(-0.56)</td>
<td>(-0.62)</td>
<td>(-1.12)</td>
<td>(-1.17)</td>
</tr>
<tr>
<td>Financial asset* change in fin.</td>
<td>-0.00326</td>
<td>-0.00335</td>
<td></td>
<td></td>
</tr>
<tr>
<td>asset(%)</td>
<td>(-1.00)</td>
<td>(-1.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial asset*unemployment</td>
<td>-0.300***</td>
<td>-0.296***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>shock</td>
<td>(-2.84)</td>
<td>(-2.72)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Household size</td>
<td>2.798</td>
<td>2.807</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.90)</td>
<td>(0.90)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log of income</td>
<td>0.190</td>
<td>0.129</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.30)</td>
<td>(0.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>747</td>
<td>747</td>
<td>747</td>
<td>747</td>
</tr>
</tbody>
</table>

Carried out are elasticities of consumption w.r.t. financial losses and unemployment shocks. The dependent variable is percentage change in consumption. Main explanatory variables are percentage change in financial assets, and whether respondent became unemployed in 2007 and 2008. Low prior means that in the 2006 survey, respondent answered that he or she has no chance to lose job next year, whereas high prior means that respondents answered that there is a positive chance of losing job in next year. Control variables include gender, age, marital status, retirement status, percentage change in value of the main residences, and individuals’ expectation about the probability of an increase in Dow Jones Industrial Average. Columns 1 and 3 show the baseline estimation. Columns 2 and 4 include two extra control variables (household size and log of income). t-statistics are shown in parentheses.

*p < 0.10, ** p < 0.05, *** p < 0.01
Table 3: Robustness checks

<table>
<thead>
<tr>
<th></th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in financial asset(%)</td>
<td>0.0992**</td>
<td>0.126**</td>
<td>0.0705</td>
<td>0.124**</td>
</tr>
<tr>
<td></td>
<td>(2.03)</td>
<td>(2.41)</td>
<td>(1.27)</td>
<td>(2.08)</td>
</tr>
<tr>
<td>Unemployment shock</td>
<td>-14.36***</td>
<td>-6.136*</td>
<td>-18.87***</td>
<td>-10.66***</td>
</tr>
<tr>
<td></td>
<td>(-3.93)</td>
<td>(-1.85)</td>
<td>(-4.38)</td>
<td>(-2.85)</td>
</tr>
<tr>
<td>Prior variance</td>
<td>-0.182</td>
<td>-0.800**</td>
<td>1.273</td>
<td>0.0821</td>
</tr>
<tr>
<td></td>
<td>(-0.34)</td>
<td>(-2.33)</td>
<td>(1.65)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>Prior variance* change in financial asset(%)</td>
<td>0.0224*</td>
<td></td>
<td>0.0408***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.83)</td>
<td></td>
<td>(3.50)</td>
<td></td>
</tr>
<tr>
<td>Prior variance*unemployment shock</td>
<td>14.90***</td>
<td></td>
<td>14.23*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.96)</td>
<td></td>
<td>(1.79)</td>
<td></td>
</tr>
<tr>
<td>Financial asset* change in financial asset(%)</td>
<td>-0.00348</td>
<td></td>
<td>-0.00516</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td></td>
<td>(-1.10)</td>
<td></td>
</tr>
<tr>
<td>Financial asset*unemployment shock</td>
<td>-0.316***</td>
<td></td>
<td>-0.408***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-2.94)</td>
<td></td>
<td>(-3.61)</td>
<td></td>
</tr>
<tr>
<td>Spouse/partner is working</td>
<td>0.748</td>
<td>0.774</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.47)</td>
<td>(0.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock to asset ratio</td>
<td>6.543***</td>
<td>6.373***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.85)</td>
<td>(2.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spouse/partner’s prior variance</td>
<td></td>
<td></td>
<td>-0.846</td>
<td>-0.601</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(-1.32)</td>
<td>(-0.94)</td>
</tr>
<tr>
<td>Observations</td>
<td>590</td>
<td>590</td>
<td>392</td>
<td>392</td>
</tr>
</tbody>
</table>

* t statistics in parentheses
* * p < 0.10, ** p < 0.05, *** p < 0.01

Dependent variable is percentage change in consumption. Main explanatory variables are percentage change in financial assets, and whether respondent became unemployed in 2007 and 2008. Low prior means that in the 2006 survey, respondent answered that he or she has no chance to lose job next year, whereas high prior means that respondents answered that there is a positive chance of losing job in next year. Control variables include gender, age, marital status, retirement status, percentage change in value of the main residences, and individuals’ expectation about the probability of an increase in Dow Jones Industrial Average. t-statistics are shown in parentheses. Columns 5 and 6 show results with controlling individual’s stock to asset ratio and the spouse of partner’s employment status. Columns 7 and 8 control spouse or partner’s prior variance in labor income.
Table 4: Consumption Response Comparisons

<table>
<thead>
<tr>
<th>Low prior</th>
<th>High prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>-25.2%</td>
<td>-25.8%</td>
</tr>
</tbody>
</table>

Note: we calibrate parameter values with $\beta = 0.97$, $\psi = 1/3$, $\gamma = 5$, $\kappa = 1$, $Y_0 = 7$, $\sigma^2_a = 0.03$, $\sigma^2_{y,\text{low}} = 0.06$, $\sigma^2_{y,\text{high}} = 1.16$. The prior uncertainty of labor income is computed according to the method in Lusardi (1998). Then we divide individuals’ prior uncertainty into 2 groups. The first group includes individuals with prior variance below the average value (0.42) and the second groups includes those above the average value. In the first and third columns, consumption responses are computed for unemployed individuals with prior uncertainty by using observations in these two groups respectively. In the second and fourth columns, we compute the consumption responses in the model, and the low and high prior is computed from HRS data: the low prior 0.06 is the average of individuals’ prior uncertainty in the first group, and 1.16 is that in the second group.

Table 5: Attention Capacity, Initial Wealth and Saving Rates

<table>
<thead>
<tr>
<th></th>
<th>$Y_0 = 6$</th>
<th>$Y_0 = 7$</th>
<th>$Y_0 = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = 0.5$</td>
<td>0.4326</td>
<td>0.4421</td>
<td>0.4490</td>
</tr>
<tr>
<td>$\kappa = 1$</td>
<td>0.4266</td>
<td>0.4367</td>
<td>0.4443</td>
</tr>
<tr>
<td>$\kappa = 2$</td>
<td>0.42</td>
<td>0.4309</td>
<td>0.4392</td>
</tr>
</tbody>
</table>

Table 6: Welfare Analysis

<table>
<thead>
<tr>
<th></th>
<th>$Y_0 = 6$</th>
<th>$Y_0 = 7$</th>
<th>$Y_0 = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A</td>
<td>$\beta = 0.7$</td>
<td>$\beta = 0.8$</td>
<td>$\beta = 0.97$</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>2.0013%</td>
<td>1.9146%</td>
<td>1.8284%</td>
</tr>
<tr>
<td>$\kappa$ from 2 to 4</td>
<td>1.0201%</td>
<td>0.9943%</td>
<td>0.9561%</td>
</tr>
<tr>
<td>Panel B</td>
<td>$\gamma = 4$</td>
<td>$\gamma = 5$</td>
<td>$\gamma = 6$</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.8875%</td>
<td>1.9003%</td>
<td>1.9146%</td>
</tr>
<tr>
<td>$\kappa$ from 2 to 4</td>
<td>0.9699%</td>
<td>0.9809%</td>
<td>0.9943%</td>
</tr>
<tr>
<td>Panel C</td>
<td>$\psi = 1/3$</td>
<td>$\psi = 0.5$</td>
<td>$\psi = 0.75$</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.2537%</td>
<td>1.9146%</td>
<td>2.5568%</td>
</tr>
<tr>
<td>$\kappa$ from 2 to 4</td>
<td>0.6451%</td>
<td>0.9943%</td>
<td>1.3366%</td>
</tr>
<tr>
<td>Panel D</td>
<td>$\beta = 0.7$</td>
<td>$\beta = 0.8$</td>
<td>$\beta = 0.97$</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.9146%</td>
<td>1.922%</td>
<td>0.4619%</td>
</tr>
<tr>
<td>$\kappa$ from 2 to 4</td>
<td>0.9943%</td>
<td>0.6201%</td>
<td>0.2411%</td>
</tr>
</tbody>
</table>

Note: This table reports the welfare change (in percent) when increasing attention capacity by 100% under different parameter values. The benchmark parameter values are set as follows $Y_0 = 7$, $\sigma^2_a = 0.03$, $\sigma^2_{y,\text{low}} = 0.42$, $\beta = 0.97$, $\gamma = 5$, $\psi = 1/3$. 
Table 7: Welfare Analysis (flexible attention allocation vs fixed attention allocation)

<table>
<thead>
<tr>
<th>$Y_0$ = 6</th>
<th>fixed attention allocation</th>
<th>flexible attention allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.5325%</td>
<td>2.0013%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 3</td>
<td>1.7534%</td>
<td>2.7320%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to $\infty$</td>
<td>1.8143%</td>
<td>3.1577%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Y_0$ = 7</th>
<th>fixed attention allocation</th>
<th>flexible attention allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.4363%</td>
<td>1.9146%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 3</td>
<td>1.7301%</td>
<td>2.6272%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to $\infty$</td>
<td>3.0185%</td>
<td>3.0427%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$Y_0$ = 8</th>
<th>fixed attention allocation</th>
<th>flexible attention allocation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$ from 1 to 2</td>
<td>1.5324%</td>
<td>1.8284%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to 3</td>
<td>1.9704%</td>
<td>2.5134%</td>
</tr>
<tr>
<td>$\kappa$ from 1 to $\infty$</td>
<td>2.9043%</td>
<td>2.9149%</td>
</tr>
</tbody>
</table>

Note: This table reports the comparison of welfare change (in percent) when increasing attention capacity between optimal attention allocation and a fixed attention allocation strategy for people with different wealth. Other parameters are set at their benchmark values.