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December 2021

RWP 21-18

<http://doi.org/10.18651/RWP2021-18>

FEDERAL RESERVE BANK *of* KANSAS CITY



The Income Share of Energy and Substitution: A Macroeconomic Approach*

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November 22, 2021

Abstract

As the atmospheric concentration of CO₂ emissions has grown to record levels, calls have grown for governments to make steeper emissions cuts, requiring to reduce an economy's use of fossil energy dramatically. Meanwhile, in the U.S., fossil energy still met 80 percent of the total energy demand as of 2019. This paper examines U.S. energy dependence, measured by its factor share, using a simple neoclassical framework in a systematic way. We find that with empirically plausible differences in substitution elasticities, particularly with a time-varying substitution elasticity between equipment capital and energy, changes in observed factor inputs can account for the variation in the income share of energy. Our analysis suggests that energy-saving technical change may simply be serving as a proxy for capital-energy substitutability. We also use our framework to think about the future. Substitution possibilities among different factor inputs can allow for a decline in the factor share of energy in the long-run.

JEL: E13, E25, J23, J3, J68, Q41, Q42, Q48

*We thank Christiane Baumeister, Lee E. Ohanian, and participants at the Federal Reserve Bank of Kansas City for helpful comments, and Sungil Kim for excellent research assistance. The views in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Kansas City, the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. First version: November 2021.

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1 Introduction

Burning fossil fuels, such as coal, natural gas and crude oil, creates carbon dioxide, the main greenhouse gas which contributes to global warming. As the atmospheric concentration of CO₂ emissions has grown to record levels, calls have grown for governments to make steeper emissions cuts. For example, President Biden issued its first batch of executive orders focusing on climate related issues, including rejoining the Paris climate agreement, a global effort to significantly reduce greenhouse gas emissions.

Meeting climate goals requires curbing an economy's use of fossil energy dramatically. However, as of 2019, fossil energy still met 80 percent of U.S. total primary energy demand, as high as a decade ago. In the meantime, U.S. income share of fossil energy doesn't seem to have an obvious trend. It posts large fluctuations, similar to the price of fossil energy: increasing considerably after the first oil shock, then falling dramatically in the 1980s, and staying low and relatively stable in the 1990s before increasing again in the 2000s. What can account for the changes in U.S. energy dependence, measured by its factor share?

In this paper, we examine the U.S. income share of energy, of which we have very little knowledge, using a neoclassical framework. Our goal is to provide a simple, explicit economic mechanism accounting for the historical trend in the income share of energy in terms of observed factor quantities. We also investigate whether our model can maintain consistency with other important characteristics of the U.S. economy, such as the declining labor share of income or the rise in the U.S. skill premium.

We consider an aggregate production function featuring equipment capital-energy substitutability and energy-skill complementarity, meaning energy is more substitutable with equipment capital than with skilled labor. We argue that these features may have offsetting effects and may be important for understanding the historical trend in the factor share of energy.

We quantitatively evaluate how much substitution elasticities have affected the energy share over 1963-2019. First, we modify the four-factor aggregate production function considered in [Krusell et al. \(2000\)](#) by adding fossil energy as a fifth factor, and allow for different elasticities of substitution among the factors. We then read factor prices off the marginal products using time series observations and compare the income share of energy in the model with that in the data.

We find that the key substitution elasticities are consistent with capital-energy substitutability and energy-skill complementarity, which seem to be important factors in accounting for the historical trend in the U.S. energy dependence. With empirically plausible differences in substitution elasticities, particularly with a time-varying elasticity of substitution between equipment capital and energy, changes in observed factor inputs can account for the movements in the

income share of energy over the last six decades. Our model with time-varying substitutability between equipment capital and energy provides a reasonable mechanism to interpret energy-saving technical change, as discussed in [Hassler et al. \(2021\)](#) for example. We argue that energy-saving technical change may simply be serving as a proxy for capital-energy substitutability, an important factor in understanding the variation in the energy share. Our five-factor production function also preserves the success of the previously studied four-factor production technology in maintaining consistency with the behavior of the U.S. labor market trends and the returns on physical capital over time.

We also use our theory to make projections into the future. We explore the implications of relatively higher fossil energy prices for the income share of energy and other factors of production, as the recent high energy prices due partly to energy shortages may be here to stay as we phase out fossil fuels. In doing so, we extend our five-factor baseline model to a six-factor production function with non-fossil energy as an alternative energy source. We find that energy dependence of the U.S. economy will likely decline significantly over the next few decades if the relative energy prices, the relative price of equipment capital prices, and labor and structural capital inputs continue their recent trends, and current aggregate technology is preserved.

The paper is organized as follows. In section 2, we discuss quantity and factor price data we use in the analysis. In Section 3, we present the basic model. In Section 4, we describe the quantitative methodology. In Section 5, we present our results. In section 6, we conclude by describing some implications of the results. In an Appendix, we present the construction of the data.

2 Data

We focus on the U.S. economy and review the changes in the prices and quantities of capital, labor, and energy inputs using annual data from 1963 to 2019.¹ Details on the data and construction of the variables can be found in the Appendix. Starting with fossil energy, we consider three types of fossil fuels: coal, crude oil and natural gas. We obtain an accompanying price index, P_t , and define the energy share as $E_t P_t / Y_t$, where E is quantity and Y is the real GDP net of the net export of fossil fuels.² As it can be seen in [Figure 2.1](#), there is no obvious trend in the income share of fossil energy. It increases considerably after the first oil shock, then it falls dramatically in the 1980s. The share stays low and relatively stable in the 1990s before increasing again in the 2000s. It fluctuates between 1 and 8 percent without a clear

¹Our time frame is constrained by the CPS data we use to construct the labor inputs.

²Our approach in constructing the energy series follows [Hassler et al. \(2021\)](#). Output here can be interpreted as GDP minus the value of energy use outside of domestic production. Due to lack of consistent data on household use of energy as a final good, which is small compared to the total, we set household use to zero.

long-run trend. On the other hand, real price of energy shows an increasing trend. In addition, the price and the energy share strongly, positively comove.

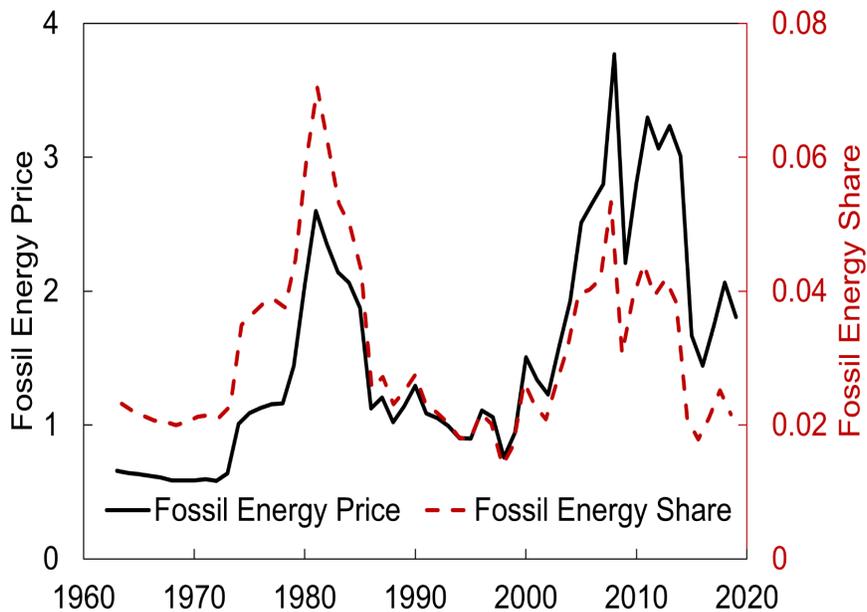


Figure 2.1: Price and income share of fossil energy

We consider two types of capital -structures and equipment- as they have grown at very different rates. Figure 2.2 top panel shows that, as of 2019, the stock of capital equipment is about 80 times of its level in 1963. The stock of structures, however, is only about 5 times of its 1963 level. In the meantime, the relative price of equipment capital (to consumption) falls significantly, while the relative price of structures (to consumption) is relatively stable over this period. Note that this long-run decline in the relative price of equipment capital is interpreted as the equipment-specific technological progress in the literature, see for example Greenwood et al. (1997).

We also look into the ratio of the quantity of capital equipment to the quantity of fossil energy input, because it is an important factor in our analysis. As we will show later, this ratio affects the energy share through capital-energy substitutability. This ratio shows a secular increase over the entire period as can be seen in Figure 2.2 middle panel.

We also distinguish between two types of labor -skilled and unskilled-, and define the level of labor skill on the basis of the level of workers' education, standard in the literature. We distinguish, because the two types of labor too show differing trends. There is a continuous increase in skilled labor input relative to unskilled labor input (not shown here), and the wage inequality -the wage of skilled labor relative to that of unskilled labor- increased by about 50%

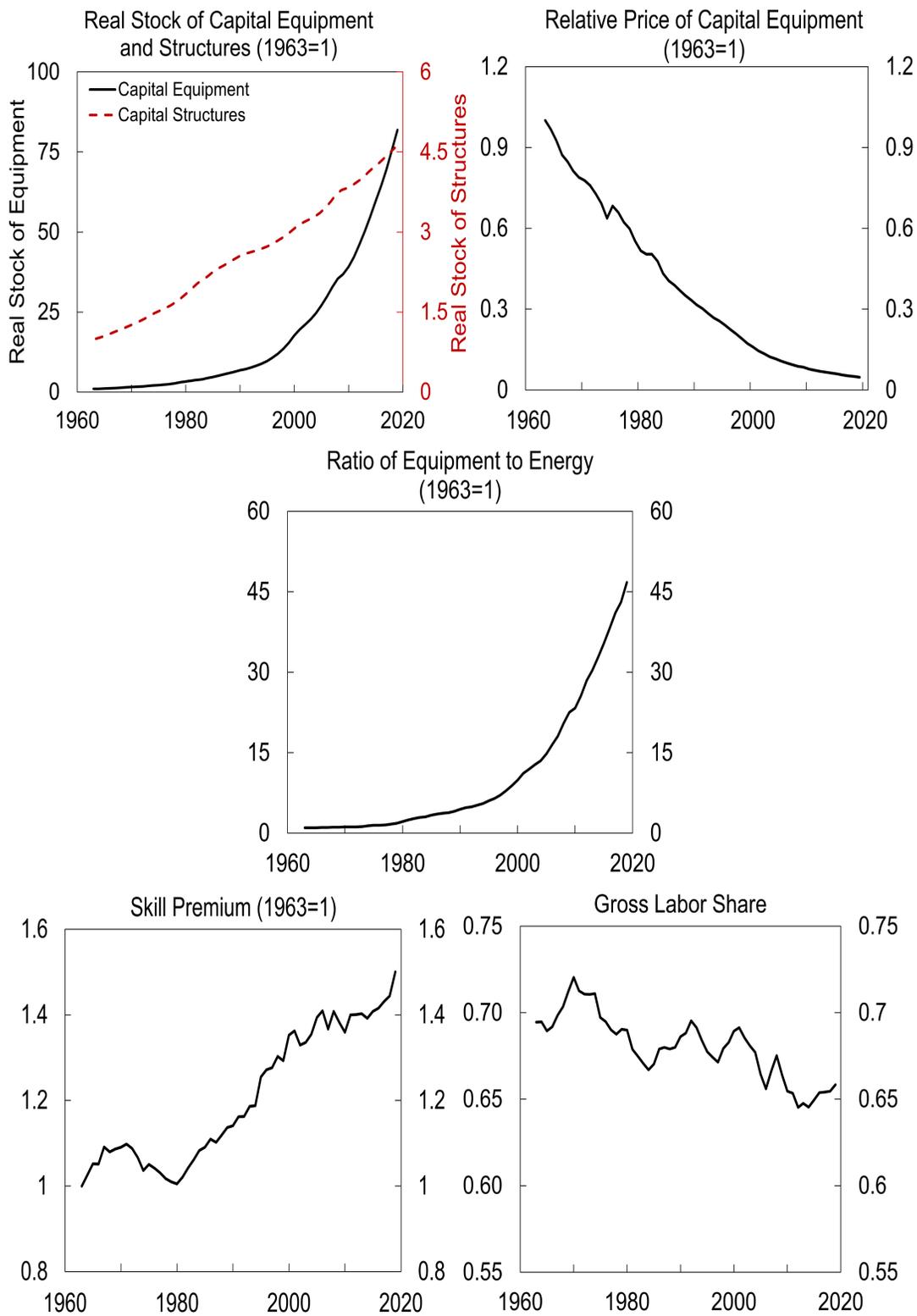


Figure 2.2: Stock and prices of capital equipment and structures, the ratio of capital equipment to energy, skill premium and gross labor share

since the early 1960s as shown in Figure 2.2 bottom left panel. Many studies have examined why the skill premium has risen during a significant growth in the relative supply of skilled labor and argued that skill-biased technological change must be an important factor. Krusell et al. (2000), on the other hand, address this question on the basis of observables and through capital-skill complementarity.

Finally, Figure 2.2 bottom right panel shows the share of income earned by aggregate labor, which is defined as the ratio of labor income (wages, salaries, and benefits) to the sum of labor income plus capital income (depreciation, corporate profits, net interest, and rental income of persons). It presents a declining trend, one of the striking features of the recent U.S. economy that has been widely discussed in the literature.

We interpret these data from the perspective of a production function for aggregate domestic output that has five inputs and develop such a framework in the next section before estimating the model on the U.S. data presented here.

3 Baseline Model

We extend the model in Krusell et al. (2000) by introducing fossil energy as a factor of production. We focus on the aggregate production function and abstract from modeling the household sector for simplicity. Our approach involves developing a five-factor production function, choosing values for the parameters of that function, and then obtaining factor prices given the time series data on inputs and marginal products from the production function. We form variables, such as the income share of energy, from our model and then compare model predictions with those in the data.

Our neoclassical production technology for the U.S. economy, Y_t , has constant returns to scale, and distinguishes between equipment capital ($K_{eq,t}$) and structures capital ($K_{s,t}$), skilled labor input ($L_{s,t}$) and unskilled labor input ($L_{u,t}$), and include fossil energy (E_t). The very different paths of the quantities and relative prices of these variables presented in the previous section motivate such distinction. Furthermore, we allow for different substitution possibilities between inputs.

There are three final goods in this economy: consumption, structures investment and equipment investment. Consumption and structures are produced with a constant returns to scale technology, and their prices are normalized to 1. Equipment is produced with the same technology scaled by equipment-specific productivity, q_t . With perfect competition, we have the relative price of equipment capital equal to $\frac{1}{q_t}$. So, we consider the relative price of equipment capital as the (inverse) proxy for technological progress, a common interpretation in literature. Under these assumptions, aggregate resource constraint ensures that aggregate output,

Y_t , equals total spending on consumption and structures and equipment investment.

We must choose a functional form for the production function. To simplify, we assume that it is Cobb-Douglas over capital structures and a CES function of the four remaining inputs. There are different ways of nesting $K_{eq,t}, L_{s,t}, L_{u,t}, E_t$ within a CES function. Our baseline specification combines the energy input with capital equipment, which we call *capital-energy services*. Capital-energy services are combined with skilled labor, which is then combined with the unskilled labor with different elasticities of substitution between equipment capital and energy, between skilled labor and capital-energy services, and between unskilled labor and combined output of capital-energy services and skilled labor.³ One can interpret this type of production technology along the lines of [Goldin and Katz \(1998\)](#) as follows. Production takes place in three stages. In the first stage, new technologies (or new equipment) adopt to work with energy efficiently in the firm. In the second stage, skilled workers adopt those new technologies and ensure that they work efficiently. And, the final stage involves maintenance and the more mechanical part of the production process that involves unskilled labor. Therefore, we choose the following baseline technology for our analysis:

$$\begin{aligned} Y_t &= A_t G(K_{s,t}, K_{eq,t}, E_t, L_{u,t}, L_{s,t}) \\ &= A_t K_{s,t}^\alpha \left[\mu L_{u,t}^\sigma + (1 - \mu) (\lambda [\xi K_{eq,t}^\nu + (1 - \xi) E_t^\nu]^\frac{\rho}{\nu} + (1 - \lambda) L_{s,t}^\rho)^\frac{\sigma}{\rho} \right]^\frac{1-\alpha}{\sigma}. \end{aligned} \quad (3.1)$$

where A_t is neutral technological change at time t .

In this specification, μ , λ , and ξ are parameters governing income shares. σ , ρ , and ν govern elasticities of substitution between different factors of production, and are restricted to lie in $(-\infty, 1)$ to maintain strict quasi-concavity of the production function. We define the elasticity of substitution between equipment capital and energy input as $\frac{1}{1-\nu}$, the elasticity of substitution between skilled labor and capital-energy services as $\frac{1}{1-\rho}$, and the elasticity of substitution between unskilled labor and composite output of capital-energy services and skilled labor as $\frac{1}{1-\sigma}$.⁴ Note that when either of σ, ρ, ν is zero, their corresponding nesting is Cobb-Douglas. Finally, $L_{s,t}$ and $L_{u,t}$ are skilled and unskilled labor inputs in efficiency units,

³We considered other nestings as well, however they yielded inconsistent elasticities and/or a bad model fit. We also compared baseline model results with a model where capital and labor are aggregate. In that case, not only model fit got worse too, but also we could no longer explain certain features of the U.S. economy such as the increasing wage inequality.

⁴There are other ways of defining elasticities between inputs. The definition we consider here assumes that no other factors change except the pair of factors considered.

respectively. They are combinations of raw labor hours, $h_{s,t}$, $h_{u,t}$, and efficiencies as follows:

$$\begin{aligned} L_{s,t} &= h_{s,t}e^{\varphi_{s,t}} \\ L_{u,t} &= h_{u,t}e^{\varphi_{u,t}}, \end{aligned}$$

where $\varphi_{s,t}$ and $\varphi_{u,t}$ denote (unmeasured) efficiencies of skilled and unskilled labor, respectively.

3.1 The Income Share of Energy

Now we examine how the energy share and factor inputs are related.

Given that factor prices are equal to marginal products, one can express energy share as a function of input ratios as follows. Taking the ratio of the first order conditions for skilled labor and energy implies

$$\frac{w_{s,t}h_{s,t}}{p_{E,t}E_t} = \frac{L_{s,share,t}}{E_{share,t}} = \frac{(1-\lambda)}{\lambda(1-\xi)} \frac{L_{s,t}^\rho}{E_t^\nu} (\xi K_{eq,t}^\nu + (1-\xi)E_t^\nu)^{\frac{\nu-\rho}{\nu}},$$

which yields

$$E_{share,t} = \frac{\lambda(1-\xi)}{(1-\lambda)} \left(\frac{E_t}{L_{s,t}} \right)^\rho L_{s,share,t} \left(\xi \left(\frac{K_{eq,t}}{E_t} \right)^\nu + (1-\xi) \right)^{\frac{\rho-\nu}{\nu}}. \quad (3.2)$$

The previous section presents that input ratios show significant trends over the past six decades. Equation 3.2 shows how changes in input ratios can affect the income share of energy, hence they are important objects. An increase in the skilled labor input, *ceteris paribus*, would increase the energy share if and only if $\rho < 0$, which implies complementarity between skilled labor input and capital-energy services. Moreover, for all admissible parameter values, the energy share is increasing in skilled labor share, which would work through the complementarity channel between skilled labor input and capital-energy services. Finally, an increase in equipment capital, *ceteris paribus*, would lead to a rise in the energy share if and only if $\rho > \nu$.

To further illustrate the implications of our model for the energy share, we log-linearize equation 3.2 and then differentiate with respect to time. After some algebra, we obtain

$$\begin{aligned} g_{E_{share,t}} &\simeq \underbrace{\rho(g_{E,t} - g_{\psi_{s,t}} - g_{h_{s,t}})}_{\text{relative quantity effect}} \\ &+ \underbrace{g_{L_{s,share,t}}}_{\text{energy-skill complementarity effect}} \\ &+ \underbrace{(\nu - \rho)\xi (g_{E,t} - g_{K_{eq,t}})}_{\text{capital-energy substitutability effect}}, \end{aligned} \quad (3.3)$$

where g_x denotes the growth rate of variable x . Equation 3.3 gives us a simple way to use our model to understand how changes in factor quantities and substitution elasticities affect the income share of energy. The first term, $\rho(g_{E,t} - g_{h_{s,t}} - g_{\psi_{s,t}})$, depends on the growth rate of skilled labor in efficiency units relative to the growth rate of energy use – the *relative quantity effect*. Relatively faster growth of skilled labor in efficiency units increases the energy share if and only if $\rho < 0$ (absolute complementarity). The second component, $g_{L_{s,share}}$, is the growth rate of skilled labor share. An increasing skilled labor share increases the energy share. We call this component the *energy-skill complementarity effect*. The final term, $(\nu - \rho)\xi(g_{E,t} - g_{K_{e,t}})$, is the *capital-energy substitutability effect*. It involves the growth in equipment capital relative to the growth of energy. If $\nu > \rho$, energy is more substitutable with equipment capital than is skilled labor. In this case, faster growth in equipment tends to lower the energy share as it lowers the relative demand for energy. Given the fast growth in capital equipment and the secular increase in skilled labor share we observe in the U.S. data, the latter two channels may have offsetting effects on the U.S. energy share if $\nu > 0 > \rho$.

4 Quantitative Analysis

4.1 An Exploration of the Importance of Substitution

We can use equation 3.3 to quantitatively assess how the relative input use and the substitution between energy and other inputs have affected the income share of energy. As an initial analysis, we present results from two experiments designed to address the importance of substitution in explaining the changes in the energy share. We will compare energy share in the data to the energy share in the model for particular values of the production function parameters. To examine the extent to which observable variables can account for changes in the energy share, we assume the unmeasured quality of the two types of labor (ψ_u, ψ_s) to be constant in these experiments.

The first experiment asks the following question: what would have happened to the energy share over 1963 and 2019 if there were no capital-energy substitutability and no changes in unmeasured labor quality? Such an experiment requires substitution elasticities between capital equipment and energy and between skilled labor and capital equipment to be the same. So, we consider the following simplified version of the production function.

$$Y_t = K_{e,t}^\alpha \left[\lambda E_t^\nu + (1 - \lambda) L_{s,t}^\nu \right]^{\frac{1-\alpha}{\nu}}. \quad (4.1)$$

We only need to assign a value to the curvature parameter ν , which governs the elasticity between skilled labor and energy, as the other parameters do not appear in the energy share

expression. Moreover, the values for the terms ψ_u, ψ_s can be normalized to one. One strategy is to use existing elasticity estimates. However, existing estimates of elasticities of substitution between energy and labor are based on measures of total labor. So, we consider estimates for aggregate labor from the literature to parametrize our model as a good starting point. Our goal is to determine whether capital-energy substitutability appears to be quantitatively important for understanding variations in the energy share based on substitution elasticities broadly in line with existing estimates.

We assume an elasticity of about 1.18 (i.e. $\nu = 0.15$) between energy and labor, which is consistent with the estimates presented in [Griffin and Gregory \(1976\)](#) or [Hassler et al. \(2021\)](#) on the long-run elasticity of substitution between energy and labor. Dotted line in [Figure 4.1](#) presents the energy share predicted by version 4.1 of our production function. The increase in the skilled labor causes an increasing trend in the energy share, which is in contrast to the actual energy share. This suggests that capital-energy substitutability can be an important mechanism in accounting for changes in the energy share.

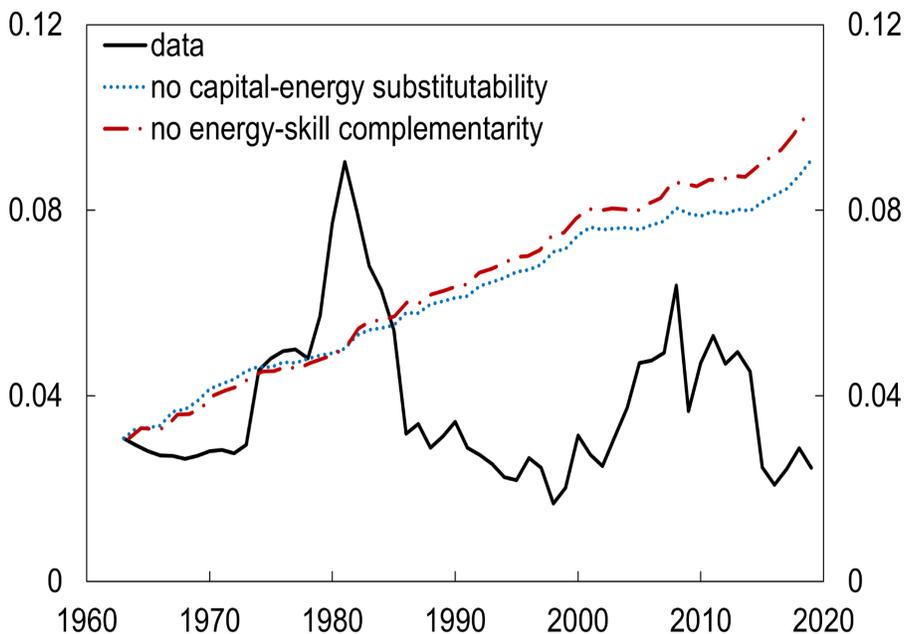


Figure 4.1: Energy Share: No capital-energy substitutability or No energy-skill complementarity

To explore the importance of the energy-skill complementarity mechanism, our second experiment asks: what would have happened to the energy share if there were no energy-skill complementarity? We continue to focus on the effects of the observables, hence maintain the assumption that unmeasured labor quality of the two types is constant. This experiment re-

quires substitution elasticities between skilled labor and energy and between skilled labor and capital equipment to be the same. So, we work with the following simplified version of the production function this time.

$$Y_t = L_{s,t}^\alpha \left[\lambda E_t^\nu + (1 - \lambda) K_{e,t}^\nu \right]^{\frac{1-\alpha}{\nu}} \quad (4.2)$$

Now, we need to choose values for two parameters: the elasticity of substitution between capital equipment and energy, ν , and the share parameter, λ . Again, existing estimates of elasticities of substitution between capital and energy are based on measures of total capital. So, we consider an estimate for the elasticity of substitution between aggregate capital and energy to parametrize our model.

We choose the share parameter λ to match the average energy share of income over the sample. For ν , we choose 0.01, a slight departure from the Cobb-Douglas specification. It implies that the elasticity of substitution between capital and energy is 1.01, which is overall consistent with the estimates on the long-run elasticity of substitution between capital and energy, see for example [Hassler et al. \(2021\)](#).⁵ Dash-dotted line in [Figure 4.1](#) presents the energy share predicted by version 4.2 of our production function. In contrast to the actual data, the energy share from the model show an upward trend over the sample, similar to the first experiment.

These two experiments suggest that changes in observables, operating through capital-energy substitutability and energy-skill complementarity mechanisms, can help account for the changes in the energy share. Note that the analysis presented here focuses only on the model's ability in explaining the energy share and it relies on estimates of elasticities based on aggregate capital and aggregate labor. In the next subsection, we examine whether our baseline production function can account for changes in the energy share while maintaining consistency with other important U.S. growth observations, which are frequently used to calibrate aggregate production functions in macroeconomics. Namely, a declining labor share, an increasing wage-bill ratio, and reasonable rates of return on capital. We estimate the parameters using the first order conditions of a profit-maximizing firm.

⁵Note that studies differ according to the functional form, estimation technique, data and time frame they consider. We consider values that are generally consistent with the estimates presented in the literature. We run these experiments with the short-run elasticities of substitution between aggregate labor and energy and between aggregate capital and energy as well. For short-run elasticities, studies generally suggest substitutability between the former pair, but strong complementarity between the latter pair (see for example [Berndt and Wood \(1975\)](#)). In that case, we find that the predicted path of the energy share too exhibit an increasing trend in both experiments, but the difference between the two series is more stark with a more muted increase in the first experiment and a sharper increase in the second experiment.

4.2 Estimation of the Baseline Model

We proceed in two steps for choosing parameter values of the (baseline) production function, presented in equation 3.1. First, we specify stochastic elements and model equations to be estimated, and then explain the estimation process.

The relative price of equipment capital is the first stochastic component, which affects the rate of return on equipment investment, described below. The second one is the (unobserved) efficiencies of the skilled and unskilled labor. We assume that $\varphi_{s,t}$ and $\varphi_{u,t}$ follow

$$\varphi_{i,t} = \varphi_{i,0} + \omega_{i,t}, \quad (4.3)$$

where $i = s, u$. $\varphi_{s,0}, \varphi_{u,0}$ correspond to average efficiency levels for skilled and unskilled labor, and $\omega_{s,t}, \omega_{u,t}$ are labor efficiency shocks for skilled and unskilled labor, respectively. We assume that these shocks have a multivariate normal distribution with zero mean and covariance matrix Ω with equal variances of η_ω^2 . Given that we focus on whether changes in observable variables can account for trend changes in the energy share, our baseline specification has no trend variation in labor quality of the two types.

We use the first-order conditions of a profit-maximizing firm, rewritten as income shares, to estimate the parameters.⁶ The first equation we use is

$$\frac{r_{eq,t}K_{e,t}}{p_{E,t}E_t} = \frac{\xi}{1-\xi} \left(\frac{K_{e,t}}{E_t} \right)^\nu, \quad (4.4)$$

which is based on income shares implied by the firm's first-order conditions for renting equipment capital and using energy, where $r_{eq,t} = A_t G_{K_{e,t}}$ and $p_{E,t} = A_t G_{E_t}$ with $G_{i,t}$ denoting the marginal product of input i at time t .

The other three equations we use are

$$\frac{w_{s,t}h_{s,t} + w_{u,t}h_{u,t}}{Y_t} = (1-\alpha) \frac{\mu L_{u,t}^\sigma + (1-\mu)(1-\lambda) \left(\lambda [\xi K_{e,t}^\nu + (1-\xi)E_t^\nu]^\frac{\rho}{\nu} + (1-\lambda)L_{s,t}^\rho \right)^\frac{\sigma-\rho}{\rho} L_{s,t}^\rho}{\left[\mu L_{u,t}^\sigma + (1-\mu) \left(\lambda [\xi K_{e,t}^\nu + (1-\xi)E_t^\nu]^\frac{\rho}{\nu} + (1-\lambda)L_{s,t}^\rho \right)^\frac{\sigma}{\rho} \right]}, \quad (4.5)$$

$$\frac{w_{s,t}h_{s,t}}{w_{u,t}h_{u,t}} = \frac{(1-\mu)(1-\lambda) \left(\lambda [\xi K_{e,t}^\nu + (1-\xi)E_t^\nu]^\frac{\rho}{\nu} + (1-\lambda)L_{s,t}^\rho \right)^\frac{\sigma-\rho}{\rho} L_{s,t}^\rho}{\mu L_{u,t}^\sigma}, \quad (4.6)$$

$$A_{t+1}G_{st,t+1} + (1-\delta_{st,t+1}) = q_t A_{t+1}G_{eq,t+1} + (1-\delta_{eq,t+1})E \left(\frac{q_t}{q_{t+1}} \right). \quad (4.7)$$

Equations 4.5 and 4.6 are again based on income shares. They are implied by the firm's first-

⁶Using income shares for choosing production technology parameters is a standard approach in macroeconomics, see for example Prescott (1986).

order conditions for hiring skilled and unskilled labor. Equation 4.5 specifies the total share of labor income defined by marginal products from the production function. Equation 4.6 is the ratio of earnings of skilled workers to unskilled workers. The last equation, 4.12, is a no-arbitrage condition that sets the expected net rate of return on investment in structures equal to that on investment in equipment. Structures and equipment capital depreciate at time-varying rates $\delta_{s,t}$ and $\delta_{eq,t}$, respectively, whose paths are assumed to be known by the firm.

The estimation is done in three steps. In the first step, we only estimate energy-related parameters: the weight parameter, ξ , and the parameter governing the substitution elasticity between energy use and equipment capital, ν . The reason is that these parameters can be estimated by ordinary least-squares (OLS) using a simple structural relationship. Namely, we take the ln of equation 4.4,

$$\ln \frac{r_{e,t}K_{e,t}}{p_{E,t}E_t} = \ln \frac{\xi}{1-\xi} + \nu \ln \frac{K_{e,t}}{E_t}, \quad (4.8)$$

and estimate equation 4.8 using OLS.⁷ The resulting estimated regression is as follows:

$$\ln \frac{\widehat{r_{e,t}K_{e,t}}}{p_{E,t}E_t} = 1.3721 + 0.1011 \ln \frac{K_{e,t}}{E_t}, \quad (4.9)$$

(.1152) (.0496)

where the values in parentheses are standard errors. It gives us $\xi = 0.7977$ and $\nu = 0.1011$ in the inner CES between equipment capital and energy use.⁸ In other words, the elasticity of substitution between equipment capital and energy is 1.11, which implies a slightly more substitutibility than the Cobb-Douglas case, consistent with the long-run estimates of the elasticity of substitution presented in the literature as noted in the previous subsection.

The next steps of the estimation process is more involved. We use the two-stage simulated pseudo maximum likelihood estimation (SPML) methodology employed by [Krusell et al. \(2000\)](#).⁹ In the first stage of this application, we regress both types of labor inputs and energy use on the current and lagged stocks of both types of capital, lagged relative equipment capital price, a time trend, lagged price of energy, and the lagged value of leading business cycle indicator of the Conference Board. The purpose of this stage is to control for the possible dependence of supplies of labor inputs and energy use on general macroeconomic shocks. We then use the

⁷[Polgreen and Silos \(2009\)](#) follow a similar approach. The construction of the series used in the regression are described in the appendix.

⁸With a p -value of 0.046, ν is statistically different from zero at 5 percent confidence level.

⁹A complete description of this estimation methodology is outside the scope of this paper. Interested readers are referred to [Ohanian et al. \(2000\)](#) for a detailed description of the methodology. For an alternative procedure, see [Polgreen and Silos \(2008\)](#).

fitted values from these regressions in the SPML stage. However, when assessing the model fit, we use actual (non-instrumented) labor and energy inputs to investigate how much of the changes in the energy share and other macroeconomic variables can be explained by observed variables.

The SPML stage relies on choosing a set of parameters that minimize the distance between data and model outcome for a number of targeted variables. We use three targets: wage-bill ratio, which is the ratio of income share of skilled labor to that of unskilled labor, the gross labor share, and the no-arbitrage condition, which ensures that expected rates of return on both types of capital are the same. We call this our baseline estimation. However, we consider two alternatives too. In the first alternative, in addition to those three targets, we use the income share of energy as the fourth target. However, using labor share and energy share as targets simultaneously causes a singularity problem, because the algorithm views two targets as very similar, not yielding meaningful results. In the second alternative, we replace the labor share target with the energy share and leave the other two targets the same. In this case, model fit worsened significantly and estimated elasticities deviated largely from the values reported in the literature.¹⁰ Facing these trade-offs, we choose to keep the labor share as one of the targets, as we also prefer to obtain energy share as a non-targeted outcome of the model. Furthermore, the production technology in equation 3.1 allows us to estimate energy-related parameters (ν and ξ) using a simple OLS regression as presented above.

Going back to the second stage of SPML methodology, the three targets we use are:

1. Wage-bill-ratio:

$$\underbrace{\frac{w_{s,t}h_{s,t}}{w_{u,t}h_{u,t}}}_{\text{data}} = \underbrace{wbr_t(X_t, \varphi_{u,t}, \varphi_{s,t}; \Upsilon)}_{\text{model}}, \quad (4.10)$$

2. Labor share:

$$\underbrace{\frac{w_{s,t}h_{s,t} + w_{u,t}h_{u,t}}{Y_t}}_{\text{data}} = \underbrace{lshare_t(X_t, \varphi_{u,t}, \varphi_{s,t}; \Upsilon)}_{\text{model}}, \quad (4.11)$$

3. No-arbitrage condition:

$$\underbrace{A_{t+1}G_{st,t+1} + (1 - \delta_{st,t+1})}_{\text{expected return on structures}} = \underbrace{q_t A_{t+1} G_{eq,t+1} + (1 - \delta_{eq,t+1}) E \left(\frac{q_t}{q_{t+1}} \right)}_{\text{expected return on equipment capital}}. \quad (4.12)$$

In equations 4.10 and 4.11, the left hand sides are data and right hand sides are their model equivalents, functions of $X_t, \varphi_{u,t}, \varphi_{s,t}$ and Υ . The goal is to choose the maximum value

¹⁰The results for this second alternative are reported in the Appendix

of the log-likelihood function that will minimize the distance between left and right sides, while also maintaining reasonable ex-post rates of returns on the two types of capital (as targeted in no-arbitrage condition in equation 4.12). Note that the vector Υ contains the parameters and X_t is the set of factor inputs. Following Krusell et al. (2000), we assume that there is no risk premium, tax treatments for these two types of capital are the same, and $(1 - \delta_{eq,t+1})E\left(\frac{q_t}{q_{t+1}}\right) = (1 - \delta_{eq,t+1})\frac{q_t}{q_{t+1}} + \varepsilon_t$ where ε_t is assumed to be normally distributed with mean zero and variance σ_ε^2 . Finally, from the production function in equation 3.1, we have $A_t = \frac{Y_t}{G(\cdot)}$, which can be identified simply as a residual given the estimated parameters and observed variables.

So, the baseline model is a non-linear state-space model with equations 4.10 - 4.12 as the measurement equations and 4.3 as the transition equations. We use the aforementioned two-stage SPML technique to estimate the parameters of this model.

The number of parameters to be estimated can be reduced by calibrating some of them. We calibrate time-varying depreciation rates using the NIPA tables for the capital stock and capital consumption. The average depreciation rate is 0.028 for structures and 0.148 for equipment capital. We assume that the future depreciation rates are known.¹¹ To calibrate σ_ε , we estimate an ARIMA model for the relative price of equipment capital and set σ_ε equal to $(1 - \bar{\delta}_{eq})$ times the standard error of the residuals of this ARIMA model. Given that we obtained ν and ξ using OLS above, we fix their values. Finally, given that μ , λ , ξ , φ_{u0} , and φ_{s0} can act as scaling parameters, we choose to normalize $\varphi_{s,0}$.

All these left us with seven parameters to be estimated: σ and ρ , parameters governing the substitution elasticities; μ and λ , parameters governing weights; α , income share of structures; $\varphi_{u,0}$, average efficiency of unskilled labor, and η_ω^2 , the variance of the labor efficiency shocks.

5 Results

We estimate the parameters using two-step SPML for the entire 1963–2019 period. Table 5.1 presents our estimates with asymptotic standard errors in parenthesis. We also include the estimate for ν , driving the elasticity of substitution between energy and capital equipment, obtained above by OLS.

These estimates imply that the elasticity of substitution between unskilled labor and the composite of skilled labor and capital-energy services, $\frac{1}{1-\sigma}$, is around 1.78. And, the elasticity of substitution between skilled labor and capital-energy services, $\frac{1}{1-\rho}$, is about 0.72. These estimates are in line with the ones reported in Ohanian et al. (2021), consistent with the theory

¹¹As reported in Ohanian et al. (2021), using constant depreciation rates will likely have no visible imprint on the results.

Table 5.1: Parameter Estimates, Baseline

	σ	ρ	α	η_ω	ν
Value	0.437	-0.389	0.090	0.241	0.101
(Std. Error)	(0.031)	(0.044)	(0.002)	(0.055)	(0.049)

of capital-skill complementarity. The income share of capital structures, α , is also close to the values presented in the literature. Our elasticities imply energy-skill complementarity as $\rho < 0$ and equipment capital-energy substitutability as $\nu > \rho$. The implied value for the elasticity of substitution between capital equipment and energy is around 1.1 as mentioned before, which is close to a long-run estimate discussed in [Hassler et al. \(2021\)](#).

Even though we use annual data, our elasticity estimates better to be interpreted more like long-run estimates. This is because the two-stage SPML methodology we employ relies on minimizing the distance between the model-implied trends and their data counterparts, rather than exploiting year-to-year fluctuations in data. This will be most clearly seen in the model results we present next, where the model misses large short-run fluctuations in the labor share and the energy share while capturing the long-run trends reasonably well. To be consistent with the rest of the parameters, the nature of our estimate for ν is also long-term, as we do not detrend data while running the OLS regression in equation 4.9. Should we use differenced data instead, we would obtain $\nu = -0.998$, which implies an elasticity of 0.5, a high degree of complementarity between equipment capital and energy (as opposed to an elasticity of 1.1 in the baseline model, which implies absolute substitutability between the two factors).¹²

Turning to the behavior of the estimated equations in our baseline model, Figure 5.1 presents the model and data equivalents of the share of labor income paid to skilled labor (the wage-bill ratio) and aggregate labor’s share of income, as well as ex post rates of return on equipment and structures computed from our model. The model statistics presented in these figures are generated by setting the i.i.d. shocks to labor quality to zero at every t. As a result, fluctuations in the model’s predictions are entirely due to changes in observable inputs.

The predictions of the estimated baseline model are broadly in line with the data. The model is able to capture the behavior of the wage-bill ratio. It is broadly consistent with the declining labor’s share of income, although it is a bit smoother than the data. Overall, our model is as consistent with the U.S. labor market trends as [Ohanian et al. \(2021\)](#). The model’s predictions for capital variables are also reasonable, see for example [Marx et al. \(2021\)](#).¹³ Note

¹²The short-run elasticity estimate we obtained using differenced data is consistent with the short run estimate of [Hassler et al. \(2021\)](#), who document close to perfect complementarity between energy and a composite of aggregate capital and labor; and with [Polgreen and Silos \(2009\)](#), who report strong complementarity between equipment capital and energy.

¹³They report an increasing return on U.S. productive capital from around 6 percent in the early 1980s to

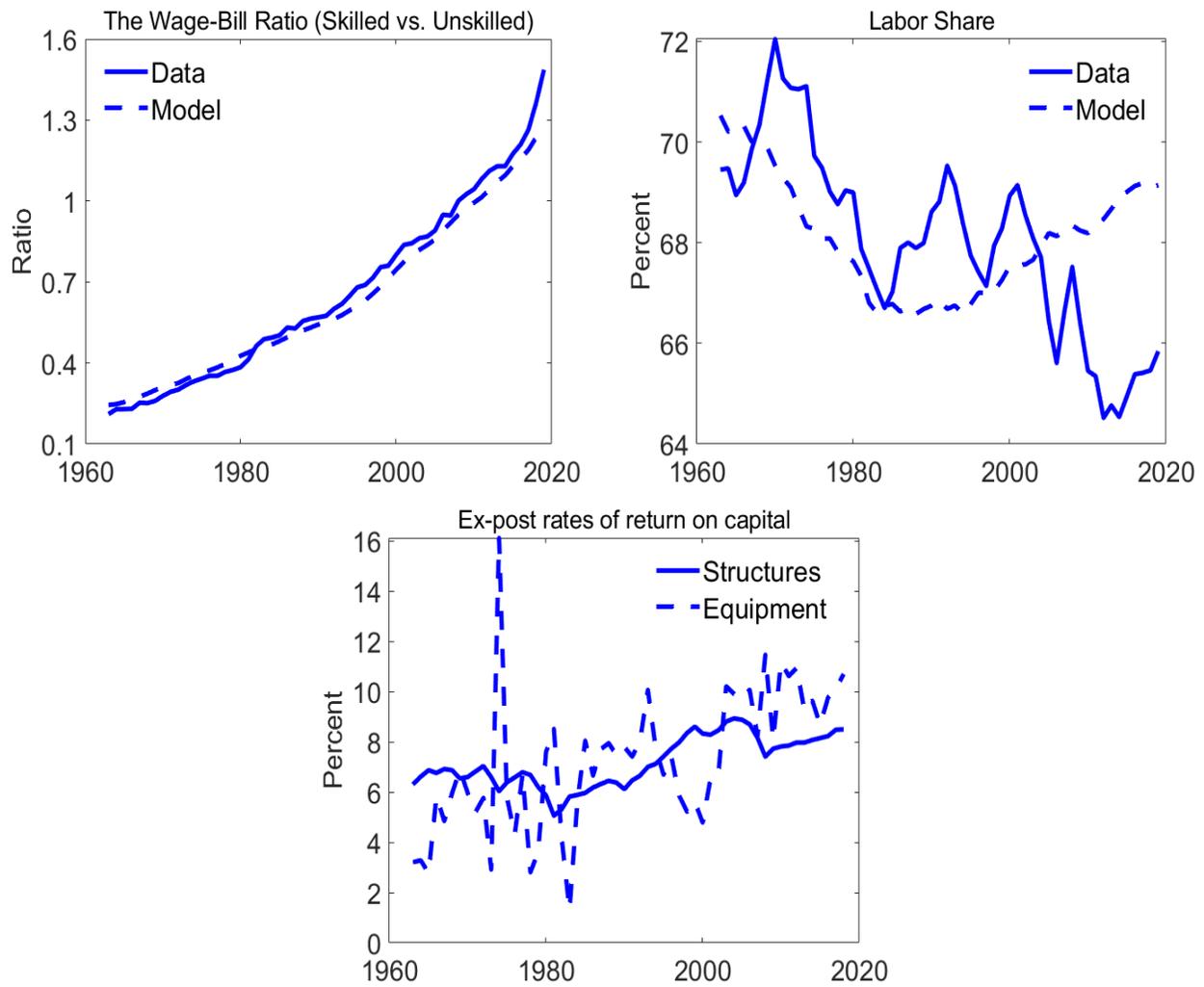


Figure 5.1: The baseline model's predictions for targeted series, 1963-2019

that the rate of return is more volatile for equipment than that of structures due to unexpected changes in the relative price of equipment.

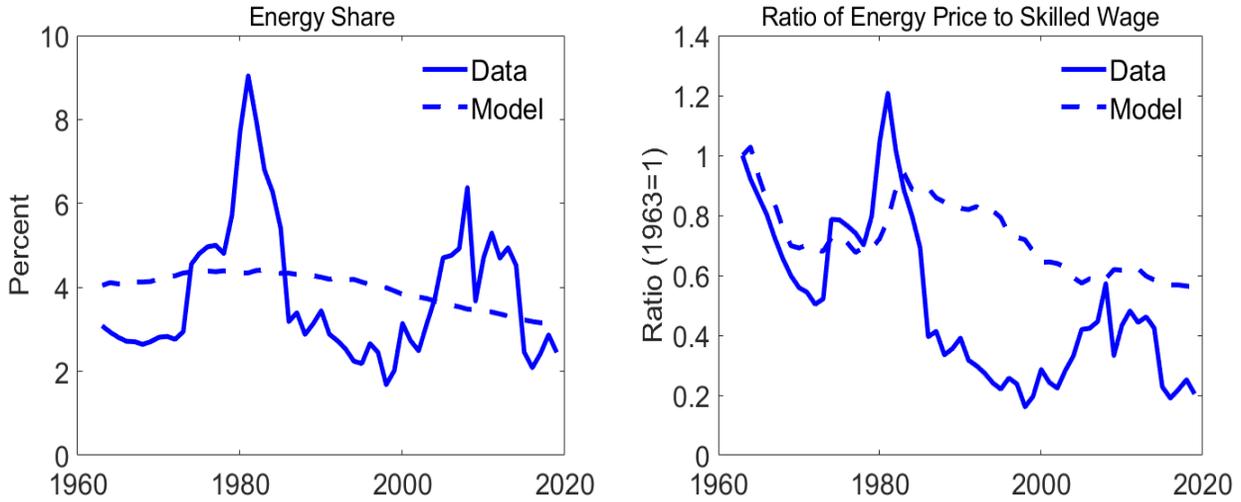


Figure 5.2: The baseline model’s predictions for non-targeted variables, 1963-2019

The income share of energy and the ratio of energy price to skilled wage in the data and that predicted by the baseline model are shown in Figure 5.2. Note that these are not used in our two-stage SPML estimation as targets. As seen on the left, driven entirely by changes in observed factor quantities, our model successfully captures the long-run trend of the energy share, but it fails to capture large short-run swings, which seem to be mostly driven by volatile energy prices (see Figure 2.1). In the model, the income share of energy share remains relatively stable until the early 1980s, and then it starts falling. Indeed, the model-implied energy share declines by around 1 percentage point, from around 4.4 percent to around 3.1 percent, between 1983 and 2019. The relationship between energy and skilled labor appear to play an important role in the course of the energy share. Figure 5.2 right panel presents the ratio of their prices. Even though the model does not target this ratio in the estimation stage, it successfully mimics the sharp decline until mid-1970s and subsequent rise until early 1980s. Although the model fails to generate the large drop seen in the second half 1980s, it captures the secular decline from then until the end of our sample period.

To gain insight into how our model helps understand the behavior of the energy share, we present a historical decomposition of the growth of the energy share in our baseline model into the three components defined in section 3: the capital-energy substitutability effect, the energy-skill complementarity effect, and the relative quantity effect. Based on this decomposition, we reconstruct how these three channels have affected the income share of energy over 1963-2019.

around 12 percent in post-2015.

Recall that there is no trend growth in labor quality. The historical decomposition is presented in log units in Figure 5.3. The sum of the three components gives the log of the model energy share shown by the black solid line.

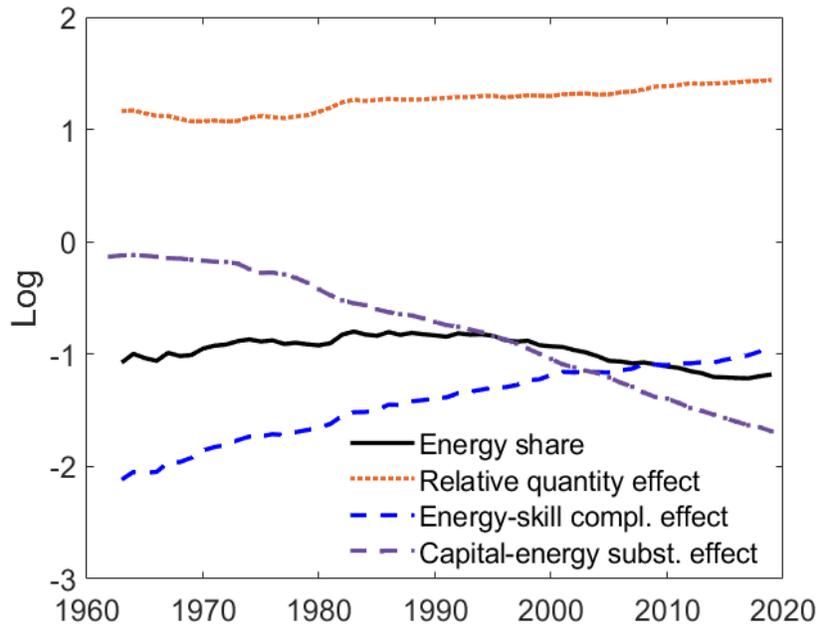


Figure 5.3: A decomposition of the baseline model’s income share of energy, 1963-2019, in logs

Starting with the dotted orange line, it shows that the relative quantity effect exerts a positive effect on the energy share throughout the sample. This is because the skilled labor input grew faster than the energy input and these two factors are complementary. However, this channel is relatively stable, hence it plays only a modest role on the long-run movements of the energy share. Meanwhile, the other two channels seem to have larger and offsetting effects on the energy share. On the one hand, given the secular trend rise in the income share of the skilled labor, energy share tends to rise (energy-skill complementarity effect, blue dashed line), because the skilled labor and energy are complementary ($\rho < 0$). On the other hand, given the absolute substitutability between equipment capital ($\nu > 0$) and energy, and the enormous equipment-specific technological progress, the rise in the income share of equipment capital depresses the income share of energy (capital-energy substitutability effect, purple dashed-dotted line). Although these two effects seem to largely offset each other, making the long-run trend of the energy share look relatively stable, capital-energy substitutability effect overall dominates, resulting in a slight decline in the income share of energy.

To further assess the importance of these channels in the evolution of the income share of energy, we conduct counterfactual exercises by shutting them off one by one. Figure 5.4 presents

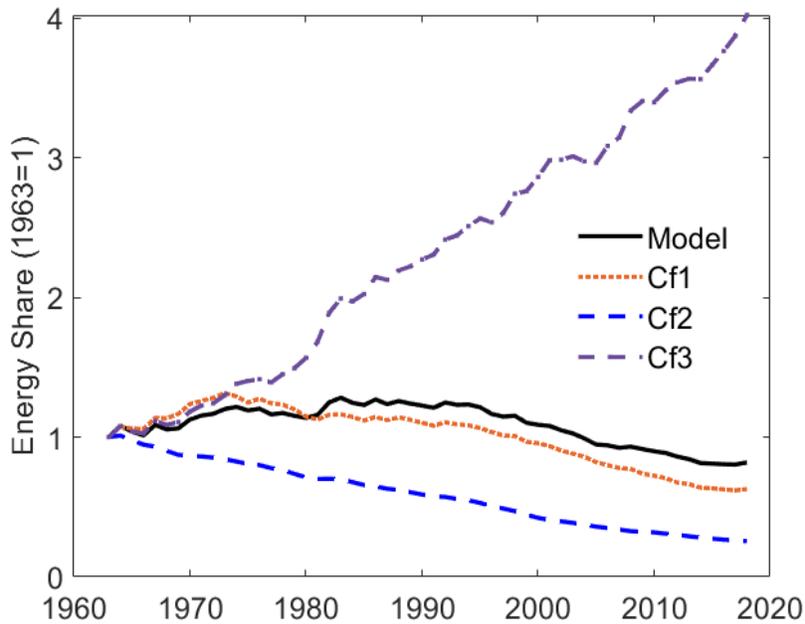


Figure 5.4: Counterfactual experiments: shutting three effects one by one, 1963-2019

The solid black line shows the baseline model’s prediction for the income share of energy using actual data for all factor inputs. Cf1 shows the model’s prediction under the assumption that energy input and skilled labor input grow at the same rate. Cf2 shows the model’s prediction with the income share of the skilled labor remaining unchanged at its 1963 level. Cf3 shows the model’s prediction with $\nu = \rho$ or equipment capital and energy input growing at the same rates.

the results. In the first counterfactual, we assume that energy and skilled labor inputs grow at the same rate (orange dotted line, shutting off the relative quantity effect). This channel appears to have been the least important one, as the resulting energy share trend is quite similar to the one the baseline model predicts. In counterfactual two, we assume that the income share of the skilled labor remains unchanged at its 1963 level (blue dashed line, shutting off energy-skill complementarity effect). This way, we can predict how energy share would evolve if we did not experience the secular decline in the aggregate labor share, which is partially attributed to change in composition of labor force in favor of the skilled ones as suggested by [Orak \(2017\)](#) and [Eden and Gaggl \(2019\)](#). This counterfactual suggests that without the rise in the income share of the skilled labor, the income share of energy would fall to about a quarter of its 1963 level. Finally, for the third counterfactual, one could either assume $\nu = \rho$ or assume equipment capital and energy input grow at the same rate, shutting off the capital-energy substitutability effect (purple dashed-dotted line). Without this effect, the income share of energy would have risen to more than four times its value in the early 1960s, mostly mimicking the rise in the income share of the skilled labor. This shows that the enormous rise in the

stock of equipment capital has prevented a larger share of U.S. income from being directed to energy input. With a less extreme assumption, the third counterfactual experiment suggests that a deceleration in equipment-specific technological progress could lead energy share to rise, everything else being equal. These exercises imply that the latter two factors are important in driving the historical trend in the income share of energy, but capital-energy substitutability seems particularly important, which motivates our next exploration.

5.1 Capital-energy substitution and energy-saving technical change

Our baseline model can account for the long-run trend in the income share of energy share, but it cannot explain the large short-run fluctuations (see Figure 5.2). Probably this is not surprising given the long-run nature of our estimation process and that we use only observed quantities in estimating the production technology with a fixed substitution elasticity; however, the large short-run fluctuations seem to be largely driven by changes in the price of energy (see Figure 2.1). One may wonder whether energy-saving technical change can help us explain the short-run fluctuations in the energy share, as higher energy prices can incentivize technical change, which may simply be serving as a proxy for omitted equipment capital and energy substitutability.

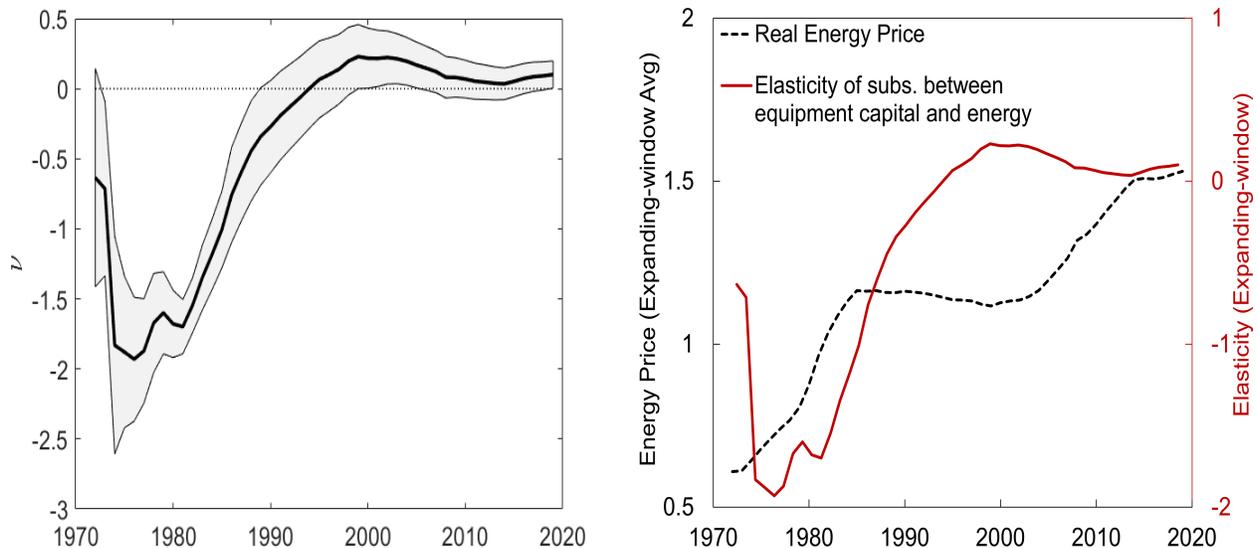


Figure 5.5: Expanding-window estimates of ν and real energy price

The values of ν , the parameter governing the elasticity of substitution between equipment capital and energy, are estimated by expanding-window OLS regressions of equation 4.8. The initial period covers 1963–1972. Each subsequent period expands by adding one more year to the previous period. Light shaded area on the left is the 95 percent confidence interval.

To explore this question, we allow for time-varying substitution elasticity between equipment capital and energy, ν . To do so, we run expanding-window OLS regressions to estimate ν starting from the 1963–1972 period, as energy prices start posting large fluctuations after the first oil shock, until the year 2019. The resulting estimates of ν are presented in the left panel of Figure 5.5 along with a 95 percent confidence interval. As seen in the figure, substitutability between equipment capital and energy have increased considerably since the mid-1970s, which coincides with higher average energy prices relative to the mid-1970s as shown on the right. This potentially points to energy-saving technological progress due to rising energy prices, supporting our interpretation that technical change may simply be serving as a proxy for capital-energy substitutability.

We then use those ν estimates presented in Figure 5.5, along with ξ estimates consistent with them, in our two-stage SPML methodology. Table 5.2 present the resulting parameters, which are very similar to those in our baseline using a fixed ν and ξ for the entire 1963–2019 period. As a result, the model fit for the labor market with time-varying ν 's are quite similar to those reported for the baseline, which can be seen in the top panels of Figure 5.6 (compared with Figure 5.1).

Table 5.2: Estimates of key model parameters with time-varying ν 's

	σ	ρ	α	ν
Baseline	0.437	-0.389	0.090	0.101
With time-varying ν 's	0.431	-0.404	0.084	—

While the model maintains its consistency with major U.S. labor market trends, it now better captures the large swings in the income share of energy, particularly in the earlier periods of study, as can be seen in the bottom panel. This suggests that when we take energy-saving technological progress as proxied by the varying capital-energy substitution into account, changes in observed quantities can do a better job in accounting for the variation in U.S. energy dependence as measured by the income share of energy.

5.2 Alternative energy sources

Our baseline production function presented in equation 3.1 considers fossil energy. This is partly because fossil fuels have been the major source of primary energy consumption in the U.S. for many decades, Figure 5.7.¹⁴ Hence, in terms of thinking about the U.S. energy dependence over the period of our study, non-fossil energy seems not really relevant. In addition, given its

¹⁴Moreover, emissions from fossil fuels are the dominant cause of global warming (Intergovernmental Panel on Climate Change).

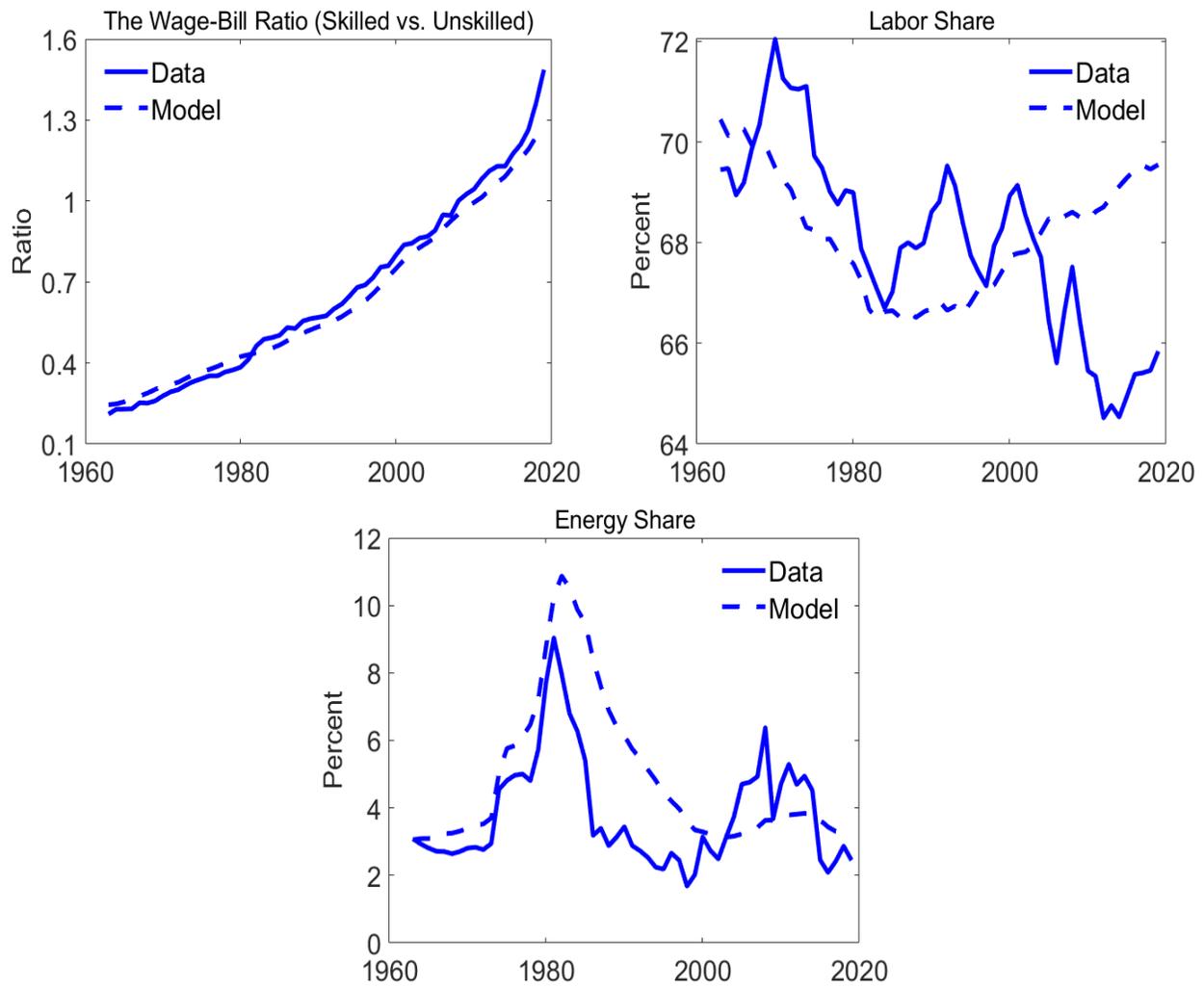


Figure 5.6: The model's predictions with time-varying ν 's, 1963-2019

very small share -compared to fossil but especially compared to other factors of production-, we wouldn't expect our baseline results to change with the inclusion of non-fossil energy.¹⁵ Still, one may wonder about the substitutability between the two types and what including alternative sources may mean for the U.S. energy dependence. Hence, using available data on non-fossil energy use, we estimate a version of the model where the energy composite consists of two types of energy, fossil and non-fossil.

We find that fossil and non-fossil energy are substitutable, with an elasticity of substitution of 1.53.¹⁶ As expected, including non-fossil energy didn't have much of an impact on our baseline results. Both our estimates of substitution elasticities and model predictions have remained largely the same.

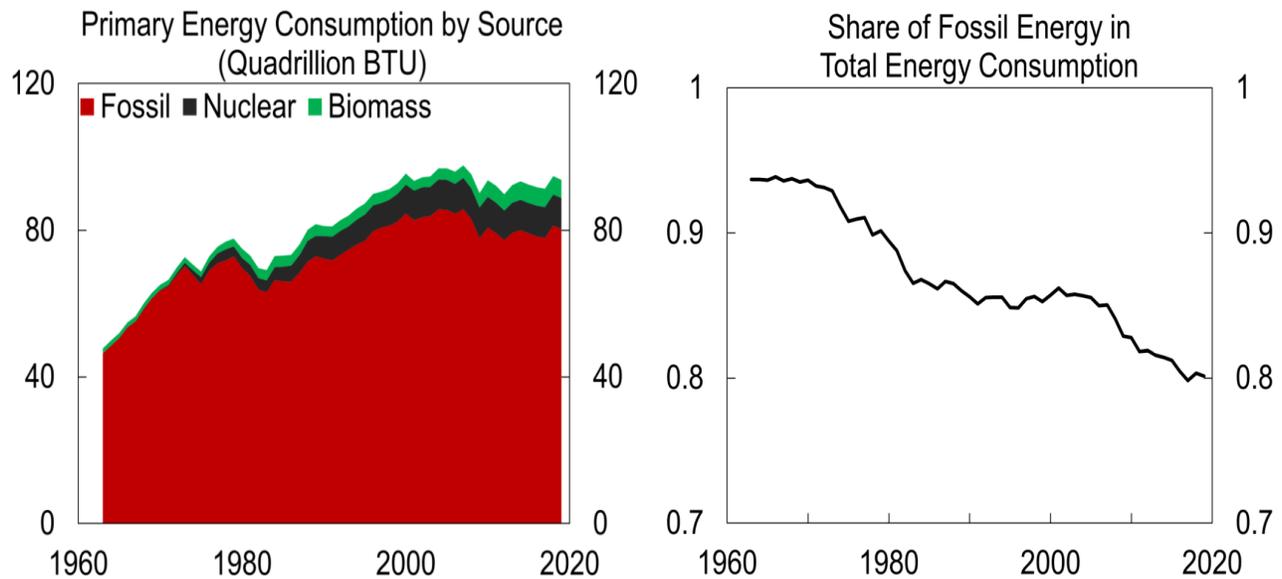


Figure 5.7: Fossil & non-fossil energy use and fossil energy share in total energy consumption the U.S., 1963-2019

Now, using this version of the model, we can address some questions which are especially relevant in today's high energy price environment, which is partly driven by energy shortages. As we phase out fossil fuels, it is likely that the relative price of energy will remain high, which has implications not only for the income share of energy but also for the factor shares of other inputs.

¹⁵Another caveat is that there is no consistent, long time series data on different non-fossil energy types, especially on their prices.

¹⁶Our non-fossil energy sources are biomass and nuclear. Details on the model and estimation results are available upon request.

5.2.1 Implications of high relative energy prices for the future income share of energy

To evaluate the implications of increasing relative energy prices for future U.S. energy dependence and for the shares of other factors of production, we conduct a prediction exercise until the year 2050 with the following assumptions.

We assume that the three non-energy factors of production -structural capital and two types of labor- and the total factor productivity, A_t , grow at their post-global financial crisis (post-GFC) average growth rates through 2050. We also assume that the model-implied ratio of the price of fossil energy to the price of non-fossil energy rises at its post-GFC average. This implies that fossil energy should get about 11 percent more expensive relative to non-fossil energy in 2050 than in 2019 (figure 5.8, left panel). Finally, we assume that the relative price of equipment capital continue to fall at its pre-GFC average growth rate of 5.5 percent between 2020 and 2050 (figure 5.8, right panel). We feed those series into the estimated model, and look at the future predictions for the factors of production.

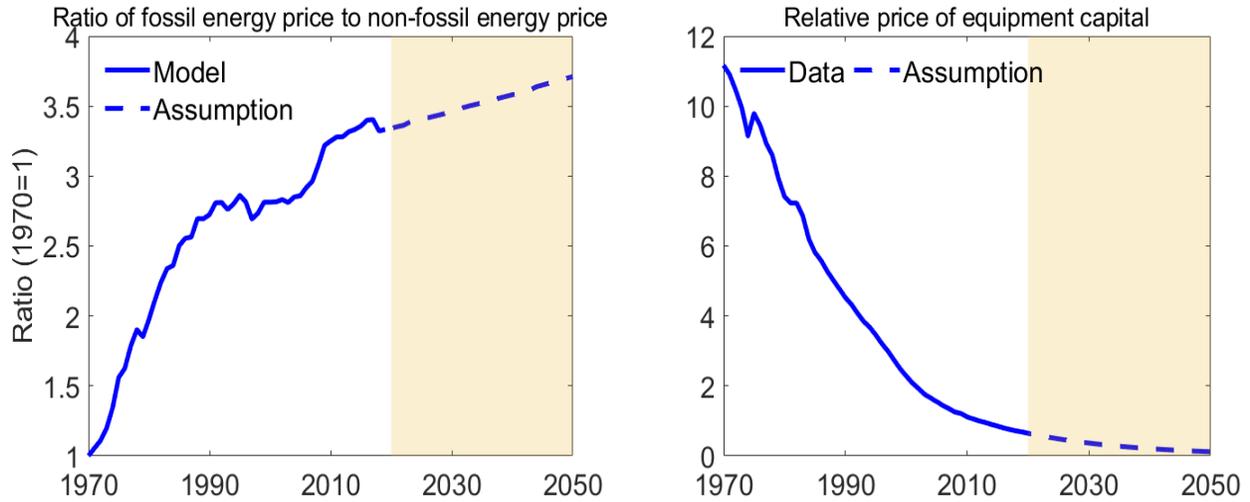


Figure 5.8: Assumed price paths

In the left panel, the solid line in the non-shaded area shows the ratio of fossil energy price to non-fossil energy price implied by the baseline model. We rely on model-implied price ratio, as there is no time series data to construct a non-fossil energy price series. In the right panel, the solid line in the non-shaded area shows the relative price of equipment capital in the data. In both panels, the dashed lines in the shaded areas show our assumed price paths post-2019.

Our model predicts that, under those assumptions, the energy use in the U.S. will be directed slightly towards non-fossil energy. As the left panel of Figure 5.9 shows, the share of fossil energy in total energy use will continue its downward trend, declining by another 2 percentage points over the next three decades. Despite this modest change in the composition of energy use, the

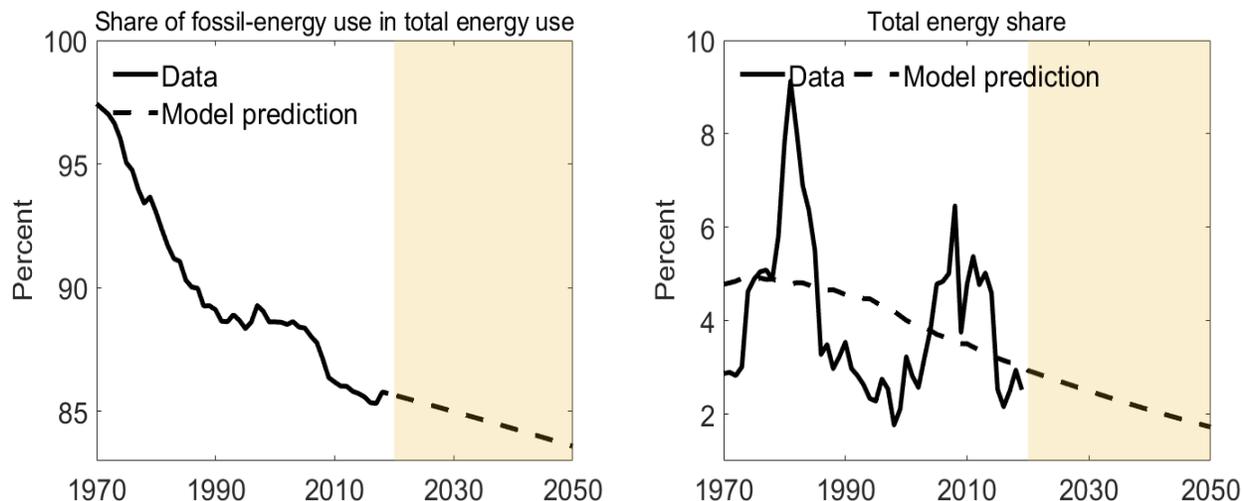


Figure 5.9: Model predictions for future energy uses and income share of energy share

Shaded areas show the model projections under the aforementioned assumptions.

right panel of Figure 5.9 shows that the model predicts the income share of energy (fossil and non-fossil combined) to almost halve by 2050. In other words, better and cheaper equipment capital can help reduce the economy's dependence on energy. On the other hand, the model predicts a significant increase in the gross labor share and a decline in the skill premium (not shown).

This exercise suggests that energy dependence of the U.S. economy will likely decline significantly over the next few decades if the relative energy and equipment capital prices and labor and structural capital inputs continue their recent trends, and current aggregate technology is preserved. It is important to note that these predictions are sensitive to assumed path of factor prices and technology. For instance, if we assume the relative price of equipment capital to remain constant at its 2019 level through 2050, as opposed to declining at 5.5 percent annually, the model predicts the income share of energy to remain relatively stable at around 2.8 percent over the next three decades. This is because, under such scenario, capital-energy services would be relatively much more valuable compared with the skilled labor input, so a larger fraction of U.S. income would be directed towards capital-energy services. As such, when designing policies to alleviate U.S. energy dependence, the interaction between all factors of production, particularly between capital-energy services and skilled labor, should be taken into account.

6 Conclusion

In this paper, we propose a simple, explicit economic mechanism for thinking about an economy’s dependence on energy. We estimate an aggregate production function in two types of capital, two types of labor, and energy using U.S. data over the last six decades. We find that capital-energy substitutability and energy-skill complementarity are important factors in understanding the long-run trend in the U.S. fossil energy dependence. With empirically plausible differences in substitution elasticities, particularly with a time-varying elasticity of substitution between equipment capital and energy, changes in observed factor inputs can account for the movements in the income share of energy over the last six decades. We argue that energy-saving technical change may simply be serving as a proxy for capital-energy substitutability. Our findings suggest that the development of better and cheaper equipment capital can help reduce an economy’s dependence on energy. Finally, a motivating factor behind this paper has been an interest in climate change as fossil energy is critical for related studies. Our analysis provides critical inputs, substitution elasticities, into the model of economics and climate.

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A Appendix

A.1 Additional Results

Baseline Model vs. Alternative Model

In the baseline model, we consider three targets: wage-bill-ratio, no-arbitrage condition, and aggregate labor share. In an alternative scenario, we replace aggregate labor share with the income share of energy, and keep the other two targets the same. That is,

1. Baseline: no-arbitrage, wbr, labor share
2. Alternative: no-arbitrage, wbr, energy share

Then, we estimate the parameters using the two-stage SPML. Note that ν is estimated using an OLS in both cases, resulting in 0.101. The estimated parameters for the alternative case are presented along with the baseline values in Table A.1. The alternative model’s predictions are presented in Figure A.1.

Table A.1: Parameters estimated for the 1963–2019 period

	σ	ρ	α
Baseline	0.437	−0.389	0.090
Alternative	0.828	−1.126	0.100

A.2 Construction of energy-related series

We construct the energy variables following Hassler et al. (2021), using data from the EIA, as follows.

Our data covering 1949-2019 suggests that average price for crude oil in 2005 dollars per Btu is 3.75 times as high as coal price, while average price for natural gas in 2005 dollars per Btu is 1.57 as high as coal price. So, we compute fossil energy use as $E_t = E_t^c + 3.75E_t^o + 1.57E_t^g$, where E_t^c , E_t^o , and E_t^g stand for Coal Consumption, Crude Oil Consumption (Excluding Biofuels), and Natural Gas Consumption (Excluding Supplemental Gaseous Fuels) in million Btu, respectively. These series are available for the period 1949-2019 from Table 1.3: “Primary Energy Consumption by Source”, Monthly Energy Review.

We obtain fossil energy price variable as $P_t = (P^c E_t^c + P^o E_t^o + P^g E_t^g)/E_t$. For that, we first create individual price series P^c , P^o , and P^g of coal, crude oil, and natural gas measured in

dollars per million Btu. For the crude oil price, we use the "Domestic Crude Oil First Purchase Prices (dollars per barrel)", which can be found at https://www.eia.gov/dnav/pet/pet_pri_dfp1_k_a.htm. We divide this series by "Crude Oil Production Heat Content (million Btu per barrel)" to create a series of crude oil price in dollars per million Btu. The Crude Oil Production Heat Content series is available in Table A2: "Approximate Heat Content of Petroleum Production, Imports, and Exports" of Monthly Energy Review. For the natural gas price, we use two different series due to discontinuity of a series. We use "U.S. Natural Gas Wellhead Price (dollars per thousand cubic feet)" from 1949 to 2012 and use "U.S. Natural Gas Electric Power Price (dollars per thousand cubic feet)" from 2013 to 2019. The former can be found at <https://www.eia.gov/dnav/ng/hist/n9190us3a.htm> and the latter can be found at <https://www.eia.gov/dnav/ng/hist/n3045us3a.htm>. We alter the unit of the combined series to dollars per cubic foot and then divide it by "Natural Gas Production, Marketed Heat Content (million Btu per cubic foot)" to construct the natural gas price in dollars per million Btu. The Natural Gas Production Heat Content series is available in Table A4: "Approximate Heat Content of Natural Gas", Monthly Energy Review. For the coal price, we use "Nominal Coal Price, Total" for the period 1949-2019 available in Table ES-4: "Nominal Coal Prices", Annual Coal Report. We divide the coal price series by "Coal Production Heat Content (million Btu per short ton)" to construct a coal price series in dollars per million Btu. After constructing individual price series in dollars per million Btu, we deflate them with the GDP deflator to express the price series in thousands of 2005 dollars per million Btu.

We calculate the output measure as $Y = GDP - (\text{net exports of fossil fuel})$. For GDP we use real GDP in 2005 dollars. We use data from Table 1.4c: "Primary Energy Net Imports by Source (million Btu)" in Monthly Energy Review and multiply net exports by price to express the net export in dollars. Then we sum the net export values of three fuel types to construct the Fossil Fuel Net Exports series. Here we use GDP deflator to express the series in 2005 dollars. We then construct our fossil energy share as $E_t P_t / Y_t$, where Y_t is measured net of the net export of fossil fuel as mentioned.

A.3 Construction of labor inputs and wages

We follow [Krusell et al. \(2000\)](#) and [Ohanian et al. \(2021\)](#) in constructing the labor inputs and wage rates. Here we summarize our steps briefly, and refer interested readers to those papers for more details.

Labor inputs and wage rates are constructed using the individual level data from the Current Population Surveys (CPS) for the years between 1963 and 2019. Also similar to [Domeij and Ljungqvist \(2019\)](#), we have both a labor input sample and wage sample. In the former, we

drop those younger than 16 or older than 70, those reported as unpaid family workers or in the military, those that were not in the labor force in the previous year, and those, who have missing qualifications, such as their educational attainment. In the latter, we also drop the self-employed, agents that reported working less than full time (i.e. 40 weeks a year and 35 hours a week). Finally, observations with allocated income, those with hourly wages below half of the minimum federal wage rate, and those with weekly pay less than \$62 in 1980 dollars are all dropped.

Individuals are first divided into 264 groups based on their age (16–20, 21–25, 26–30, 31–35, 36–40, 41–45, 46–50, 51–55, 56–60, 61–65, and 66–70); sex (male and female); race (white, black, others); and educational attainment (less than high school, high school, some college, and a college degree and/or more). For each individual, we also record their employment status, class, weekly hours worked (usual hour worked per week for the post-1975 period and weeks worked last year prior to that), weeks worked a year before, and total wage and income a year before. Using these, and CPS personal supplement weights, we calculate income and total hours worked for each individual for each year in the sample. In doing so, we do not make any adjustments for topcodes, this is because, as [Ohanian et al. \(2021\)](#) report, results are not affected much by the treatment of topcoded incomes.

Once we have the total income and hours worked for each individual, we calculate their wage rates simply by dividing their total income to total hours worked. We then aggregate these individual hours to calculate total hours for each 264 groups, and also calculate average wage rates for each group by taking the weighted average of wages of each individual in the relevant group. Finally, these 264 groups are aggregated into two skill categories: skilled and unskilled, based on the educational attainment of each group. We define skilled labor only as those with a college degree or a graduate degree. We use the group wages of 1980 as the weights when aggregating the groups into skilled and unskilled categories.

A.4 Construction of the income shares of labor and equipment capital

To maintain comparability, we construct gross labor share as described in [Krusell et al. \(2000\)](#). As such, labor share is constructed as “ $1 - \text{aggregate capital share} - \text{energy share}$ ”, where capital share is the ratio of the sum of net interest and miscellaneous payments, rental income of persons with capital consumption adjustment, corporate profits with inventory valuation and capital consumption adjustments, and depreciation to the difference between gross domestic income and proprietors’ income from the BEA’s NIPA Tables 1.10 and 1.17.5.

To construct a series for the income share of equipment capital, we first assign a constant

share of income to structures capital, consistent with the Cobb-Douglas technology we assumed for it. That is, we take the α estimate of [Ohanian et al. \(2021\)](#) as given. Then, we obtain *equipment capital share* as the remaining part of the aggregate capital share, which is used in estimating energy-related parameters ν and ξ employing equation [4.9](#).

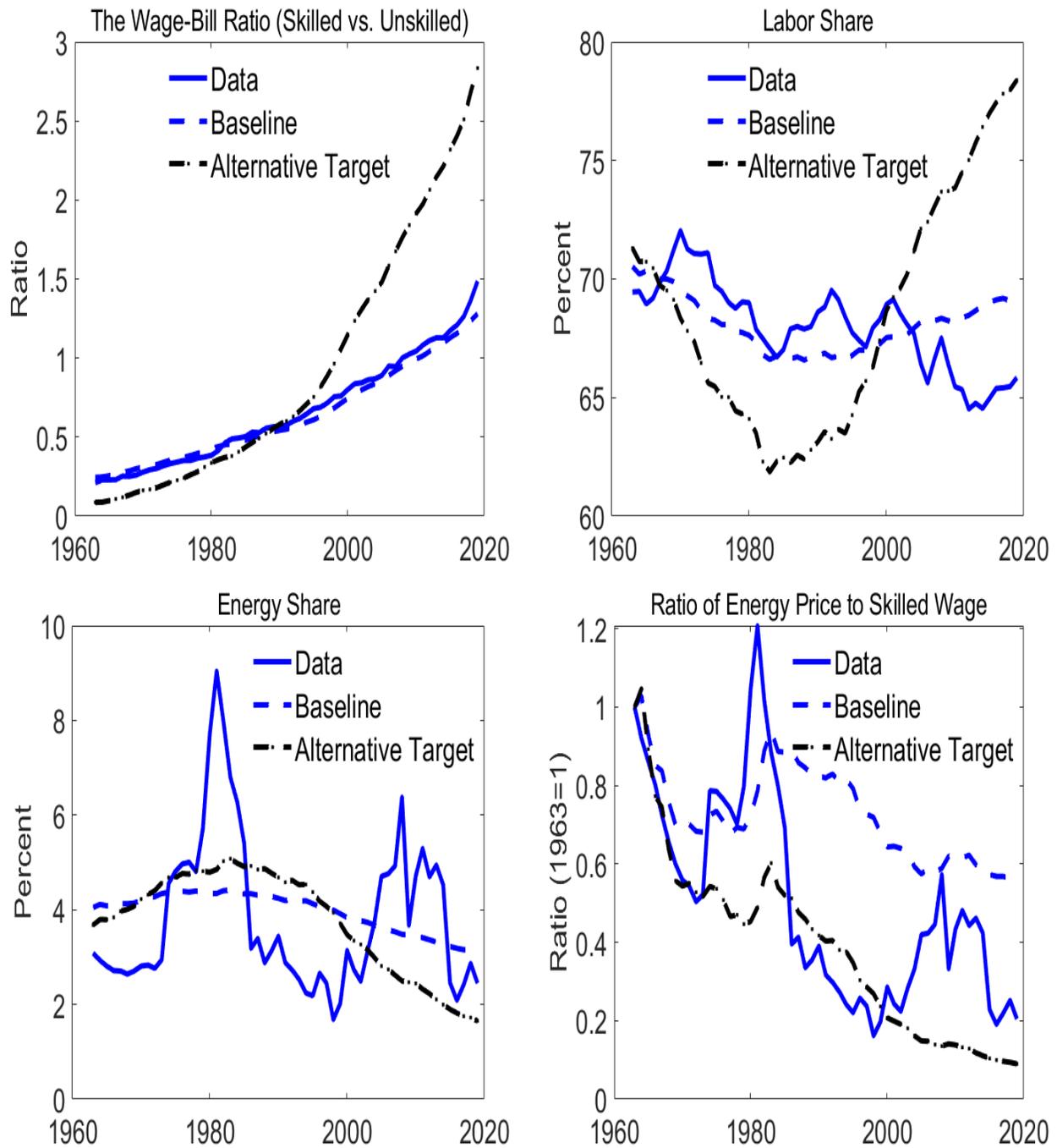


Figure A.1: Baseline Model vs. Alternative Model