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Wealth in The Utility Function and Consumption Inequality*

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Abstract

Wealth in the utility function (WIU) has been increasingly used in macroeconomic models and this specification can be justified by a few theories such as Max Weber's (1904-05, German; 1958) theory on "spirit of capitalism." We incorporate the WIU into a general equilibrium consumption-portfolio choice model to study the implications of the WIU for consumption inequality, equilibrium interest rate, and equity premium—an unexplored area in the literature. Our general equilibrium framework features recursive exponential utility, uninsurable labor risks, and multiple assets and can deliver closed-form solutions to help disentangle the effects of the WIU in driving the key results. We show a stronger preference for wealth lowers the risk-free rate but increases the consumption inequality and equity premium in the equilibrium. We show these properties improve the model's performance in explaining the data. We also compare the WIU with a closely related hypothesis, habit formation, and find that they have opposite effects on equilibrium asset returns and consumption inequality.

JEL Classification Numbers: *C61, D81, E21.*

Keywords: Wealth in the Utility Function, the Spirit of Capitalism, Precautionary Savings, Risk-free Rates, Risk Premia, Consumption and Wealth Inequality.

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1 Introduction

Wealth in the utility function (WIU) has been increasingly used in macroeconomic models to study various topics. This specification can be justified by several theories, such as Max Weber's (1904-1905, German; 1958) theory on "the spirit of capitalism"–the enjoyment of the accumulation of wealth regardless of its effect on consumption smoothing.[1] Based on recent research, one key channel for the WIU to affect the agent's decision is to allow the marginal rate of substitution between wealth and consumption to enter the household consumption Euler equation, which alters the consumption-saving decision (Luo et al. 2021; Michaillat and Saez 2021). However, how this may affect the consumption inequality, risk premium, and the equilibrium risk-free rate, which are key objectives in macroeconomics and finance, has not been explored.

Intuitively, the WIU may influence the consumption inequality through two channels: a direct channel through the effects on individuals' consumption and saving, and an indirect channel through its effects on the equilibrium interest rate and equity premium. To investigate these possible channels and how they are determined by various factors, we incorporate the WIU into a rich heterogeneous-agent general-equilibrium model (defined in the next paragraph) and derive analytical solutions to help disentangle different driving forces on consumption and savings, and illustrate how the WIU interacts with other conventional factors–such as patience, risk aversion, and intertemporal substitution–in determining the equilibrium asset returns and consumption inequality. When compared with a standard model that does not include the WIU, our model is more consistent with the micro data on the distribution of consumption, as well as the aggregate data on the equilibrium interest rate and equity premium.

To be more specific, our general equilibrium model features heterogenous agents, incomplete markets, multiple assets, and recursive preferences. The typical agent makes optimal consumption-saving and portfolio-choice decisions in an economy with uninsurable labor income risks. The incomplete markets hypothesis (Bewley 1986; Huggett 1993) leads agents to save for both consumption-smoothing and precautionary purposes. The multiple-asset setting (Merton 1971; Viceira 2001; Wang 2009) allows the agent to allocate wealth between risk-free and risky assets reflecting part

[1] Recent studies by Robert J. Barro and Rachel M. McCleary (2004, 2006, 2019) and by many other authors in QJE have emphasized the relationship among the Protestant (particularly Calvinist) ethic, human capital investment, and economic development. In Weber's view, the Protastant ethic such as hard work, thrift, discipline, patience, and efficiency in one's worldly calling are deemed signs of an individual's election, or eternal salvation. These religious beliefs have influenced large numbers of people to engage in work in the secular world, developing their own enterprises and engaging in trade and the accumulation of wealth for investment. Our WIU approach to the spirit of capitalism naturally captures the materialistic success through the Protestant ethic of work and the ascetic compulsion to save. In the Online Appendix A, from a long historical perspective, we provide detailed evidence to argue that the spirit of capitalism can be modeled as a direct preference for wealth, which means that an agent accumulates wealth not only for consumption but also for the sake of accumulation itself.

of the agent's risk attitude and uncertainty. The recursive preference (Epstein-Zin 1989) separates risk aversion from intertemporal substitution which play different roles in consumption and portfolio-choice decisions. The general equilibrium with heterogenous agents (Wang 2003) link individual savings to the equilibrium interest rate and also allow us to study the equity premium in the presence of risky assets. Incorporating the WIU into such a rich framework allows us to study how it interacts with each of these important factors in driving savings, asset returns, and consumption inequality in equilibrium.

We deliver three sets of results in this paper. First, we derive closed-form solutions for our consumption-portfolio choice model featuring the WIU, recursive exponential utility, uninsurable labor income, and multiple assets.[2] These closed-form solutions help us understand how the WIU interacts with the discount rate, the elasticity of intertemporal substitution (EIS), and constant absolute risk aversion (CARA) in determining the aggregate saving and equilibrium risk-free rate as well as the relative consumption inequality. At the individual level, we show that a stronger preference for wealth affects the optimal saving level through four distinct channels: (i) by increasing the agent's effective patience and raising the patience-induced saving; (ii) by driving up the agent's precautionary saving; (iii) by decreasing individual savings through lowering the certainty-equivalent wealth due to the existence of risky assets; and (iv) by reducing the amount of saving for "a rainy day." We then prove the existence and uniqueness of a general equilibrium in the vein of Bewley (1986), Huggett (1993), and Wang (2003), and show that a stronger preference for wealth lowers the equilibrium risk-free rate by raising the individual's aggregate saving through the first two channels while washing out the effects from the third and fourth channels.

Furthermore, we prove that the relative consumption inequality, defined as the relative dispersion of consumption to income, is an increasing function of the degree of the preference for wealth. This result can be understood using our derived consumption function, which shows an increased sensitivity of consumption to *unanticipated* income shocks in the presence of the WIU. In addition to the preference for wealth, we show the relative consumption inequality also depends on the equilibrium interest rate, the preference for wealth, and the persistence of the income process.

Second, we show quantitatively that introducing the WIU into the model helps explain the observed low risk-free rate, the high relative consumption inequality, and the large equity premium in the data. In particular, to explain the average real risk-free rate in the United States from 1980 to 2016, a rational expectations model without the WIU would require the coefficient of risk

[2] We use the negative exponential function (i.e., CARA functional form) to characterize the household's preference for risk. The main reason for adopting this specification in our paper is that incomplete markets generally imply that aggregate dynamics depend on the wealth distribution, and this "curse of dimensionality" can be overcome by adopting a CARA-Gaussian specification because it ensures that risk-taking and therefore investment is independent of wealth.

aversion parameter to be as high as 35 to 106 when the EIS takes reasonable values from 0.5 to 1.5.[3] However, when agents care even slightly about the WIU, the model can generate a low equilibrium interest rate with a reasonable level of risk aversion. Furthermore, even with a small amount of the WIU, the model can generate the realistic relative consumption inequality. In addition, the model's performance in generating a reasonable equity premium is greatly improved when incorporating the WIU. We also show that these results are robust to the more general income specification proposed in Wang (2004) and in the presence of rare disasters in the vein of Rietz (1988) and Barro (2009).

Third, we compare the implications of the WIU with the implications of the traditional internal habit formation model on the equilibrium risk-free rate and relative consumption inequality. We conduct this comparison because these two hypotheses both influence the agent's consumption-portfolio but from two different angles: one from the wealth angle and the other from the consumption angle. We show that the two models have opposite effects on the equilibrium consumption inequality and on asset returns. Habit formation increases the equilibrium risk-free rate and raises the relative consumption inequality, while the WIU does the opposite.

Our paper contributes to two branches of literature. First, our paper contributes to the broad literature on consumption inequality by examining the implication of the WIU on consumption inequality and asset returns. Krueger and Perri (2006) use a calibrated incomplete-markets model with limited commitment to explain the observed difference between consumption and income inequality. Blundell et al. (2008) show that consumption and income inequality diverged during the 1980-2004 period, driven by the degrees of income persistence and consumption insurance. Luo et al. (2020) show that a reasonable degree of ambiguity aversion can help explain both the low real interest rate and consumption dispersion. Our paper is the first to explore theoretically the implications of the WIU on consumption inequality. We show that the presence of the WIU increases the consumption inequality and help the model to better explain the data. This is particularly important as there is evidence that the consumption inequality might be underestimated due to some measurement issues (Attanasio and Pistaferri, 2016).

Second, our paper contributes to the theoretical literature on the WIU in two significant ways. Zou (1994, 1995), Fershtman et al. (1996), Smith (1999), and Corneo and Jeanne (2001) examine the importance of the WIU for capital accumulation and economic growth. Bakshi and Chen (1996), Smith (2001), and Gong and Zou (2002) study how incorporating the WIU into otherwise standard models affects asset prices. Luo et al. (2009) find that incorporating the WIU helps resolve the excess sensitivity puzzle and the excess smoothness puzzle in the literature on aggregate consumption. Karnizova (2010) studies the implications of the WIU for business cycle fluctuations.[4] We

[3] We normalize the mean consumption level to be 1, so the coefficient of relative risk aversion is equal to the coefficient of absolute risk aversion. While estimates of the EIS fluctuate wildly, risk aversion coefficients of above 5 are generally not considered reasonable in the macroeconomic and finance literature.

[4] In a related literature on the quest for status, Saez and Stantcheva (2018), Michaillat and Saez (2015, 2019)

expand this literature by offering a framework to study the implications on consumption inequality and the equilibrium asset returns.

The rest of paper is organized as follows. Section 2 introduces the WIU into a consumption-portfolio choice model with recursive exponential utility and incomplete markets, derives analytical optimal consumption-portfolio solutions, and shows key theoretical properties. Section 3 presents our quantitative results showing how our model with the WIU can account for both a lower equilibrium interest rate, high risk premia, and a higher relative consumption inequality. Section 4 provides further discussion on the presence of rare macroeconomic disasters and alternative income specifications, and finds that our main conclusions obtained in the benchmark model are robust to these alternative specifications. In addition, we also compare our model with the WIU with the internal habit formation model. Section 5 concludes.

2 An Equilibrium Consumption-Portfolio Model with the WIU

In this section, we lay out a general framework on consumption-portfolio choice with the WIU. The model is a general equilibrium model in continuous time with recursive utility, multiple assets, and uninsurable labor income. To help explain the key structure of the model, we will introduce each of the key elements one by one, starting with specifications of the recursive utility and the WIU, followed by investment opportunity and labor income.

2.1 Recursive Exponential Utility with A Direct Preference for Wealth

Although the expected power utility model has many attractive features, it implies that the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. Conceptually, risk aversion (attitudes towards atemporal risks) and intertemporal substitution (attitudes towards shifts in consumption over time) capture two distinct aspects of decision-making and need not be so tightly connected.[5] By contrast, the class of recursive utility functions (Epstein and Zin 1989) enable one to disentangle risk aversion from intertemporal substitution, which is important for us to better understand how they interact with the WIU and then affect the equilibrium dynamics of consumption, saving, and asset returns. In this paper, we assume that agents in our model economy have the Kreps-Porteus type preference with *recursive exponential utility* (REU): for every stochastic consumption-financial wealth stream, $\{c_t, w_t\}_{t=0}^{\infty}$, the utility stream,

introduce wealth in utility with the justification that wealth is a maker of social status and show it generates a few benefits, such as resolving several anomalies in New Keynesian models, generating results consistent with permanent liquidity traps, and helping address policy questions related to capital taxation.

[5] Risk aversion describes the agent's reluctance to substitute consumption across different states of the world and is meaningful even in a static setting. By contrast, intertemporal substitution describes the agent's willingness to substitute consumption over time and is meaningful even in a deterministic setting.

$\{V(U_t)\}_{t=0}^{\infty}$, is recursively defined by:[6]

$$V(U_t) = \left(1 - e^{-\beta\Delta t}\right) V(c_t, w_t) + e^{-\beta\Delta t} V\left(\mathbb{CE}_t\left[U_{t+\Delta t}\right]\right), \tag{1}$$

where Δt is the time interval, $\beta > 0$ is the agent's subjective discount rate,

$$V(U_t) = -\psi \exp\left(-\frac{1}{\psi} U_t\right), \tag{2}$$

$$V(c_t, w_t) = -\psi \exp\left(-\frac{1}{\psi}\left(c_t + bw_t\right)\right), \tag{3}$$

$$\mathbb{CE}_t\left[U_{t+\Delta t}\right] = g^{-1}\left(\mathbb{E}_t\left[g\left(U_{t+\Delta t}\right)\right]\right), \tag{4}$$

is the certainty equivalent of $U_{t+\Delta t}$ conditional on the period t information, and

$$g\left(U_{t+\Delta t}\right) = -\frac{1}{\gamma} \exp\left(-\gamma U_{t+\Delta t}\right). \tag{5}$$

In (1)-(5), $\psi > 0$ governs the elasticity of intertemporal substitution (EIS), $\gamma > 0$ governs the coefficient of CARA, and $b \geq 0$ governs the strength of the preference for wealth.[7] It is worth noting that here we assume that the utility function depends on absolute wealth, not relative wealth. The main justification for this specification is that the average amount of asset holdings and thus the corresponding (expected) average level of financial wealth are constant in general equilibrium. A high value of ψ corresponds to a strong willingness to substitute consumption over time, and a high value of γ implies a low willingness to substitute consumption across states of nature. Note that if $\psi = 1/\gamma$, the functions V and g are the same and the recursive utility reduces to the standard time-separable expected utility function used in Caballero (1990), Wang (2003), and Luo, et al. (2009).

The CARA-Gaussian specification has both advantages and disadvantages. It implies, for instance, that optimal investment in the risky asset is independent of individual wealth; our framework is thus not suitable to analyze the wealth distribution issues. The CARA-Gaussian specification, however, is probably not essential for the main insights of the paper and brings the benefit of tractability. As will be shown in this section, the cross-sectional distribution of consumption and wealth - an infinite dimensional object - will not be a relevant state variable for aggregate dynamics, and general equilibrium will be characterized in closed-form.

[6] Skiadas (Chapter 6, 2009) axiomatizes and systematically characterizes this type of recursive exponential utility (or *transition-invariant* recursive utility.) Skiadas (2009) also compares this type of recursive utility with the *scale-invariant* Kreps-Porteus recursive utility (e.g., the Epstein-Zin parametric utility form). See Angeletos and Calvet (2006) for an application of REU in a business cycles model and Luo et al. (2020) for an application in a robust control problem.

[7] It is well-known that the CARA utility specification is tractable for deriving optimal policies and constructing general equilibrium in different settings. See Caballero (1990), Wang (2003), and Angeletos and Calvet (2006).

It is worth noting again that the utility function we proposed in (1) to model the WIU depends on absolute wealth, w_t. In the literature, however, some studies document and examine the importance of concerns about relative wealth on individuals' well-being. For example, see Duesnberry (1949), Robson (1992), Clark and Oswald (1996), Bakshi and Chen (1996), Corneo and Jeanne (2001), and Roussanov (2010). They assume that individuals care about their WIUial status, which is in turn determined by their relative wealth. Within our recursive CARA setting, we may also model the WIU by assuming that the instantaneous utility function depends on relative wealth:[8]

$$V\left(c_t, w_t, \overline{w}_t\right) = -\psi \exp\left(-\frac{1}{\psi}\left[c_t + b\left(w_t - \vartheta \overline{w}_t\right)\right]\right), \tag{6}$$

where $\overline{w}_t$ is the per capita aggregate wealth, b governs the degree of the WIU, and ϑ measure the importance of average wealth in determining the WIU. It is not difficult to show that we can still analytically solve the model with relative wealth and construct a general equilibrium after imposing that the equilibrium net supply of risky assets is zero. The main results obtained in our benchmark model still hold in the model with relative wealth.[9] To keep our model tractable, we focus on our benchmark model in the main text.

2.2 Specifications of Financial Assets and Labor Income

Following Merton (1971), Viceira (2001), and Wang (2009), we assume that consumers can access two financial assets: one risk-free asset and one risky asset. Specifically, the typical consumer can purchase both a risk-free asset with a constant interest rate r and a risky asset (the market portfolio) with a risky return r_t^e. The instantaneous return dr_t^e of the risky market portfolio over dt is given by:

$$dr_t^e = (r + \pi)\, dt + \sigma_e dB_{e,t}, \tag{7}$$

where π is the market risk premium, σ_e is the standard deviation of the market return, and $B_{e,t}$ is a standard Brownian motion. Let ρ_{ye} be the contemporaneous correlation between the labor income process and the return of the risky asset. If $\rho_{ye} = 0$, then the labor income risk is purely idiosyncratic, so the risky asset does not provide a hedge against declines in labor income. The agent's financial wealth evolution is then given by:

$$dw_t = \left(rw_t + y_t - c_t\right) dt + \alpha_t \left(\pi dt + \sigma_e dB_{e,t}\right), \tag{8}$$

where α_t denotes the amount of wealth that the investor allocates to the market portfolio at time t.

[8] An alternative specification for the WIU can be written as a multiplicative form: $w_t/\overline{w}_t$. However, given the CARA additive setting adopted in this paper, the model with the multiplicative specification of the WIU is intractable.

[9] See Online Appendix XX for the detailed proof and deviation.

Furthermore, we assume that the uninsurable labor income (y_t) follows an Ornstein-Uhlenbeck process:[10]

$$dy_t = (\mu - \rho y_t)\, dt + \sigma_y dB_t, \tag{9}$$

where $\mu = \rho \overline{y}$, $\overline{y}$ is the unconditional mean of y_t, σ_y is the unconditional volatility of the income change over an incremental unit of time, $\sigma_y^2 / (2\rho)$ is the unconditional variance of y_t, the persistence coefficient ρ governs the speed of convergence or divergence from the steady state, and B_t is a standard Brownian motion.

2.3 The Optimization Problem

The optimization problem under rational expectations (RE) can thus be written as:

$$V(J_t) = \max_{(c_t, \alpha_t)} \left\{ \left(1 - e^{-\beta \Delta t}\right) V(c_t, w_t) + e^{-\beta \Delta t} V\left(\mathbb{CE}_t\left[J_{t+\Delta t}\right]\right) \right\}, \tag{10}$$

subject to (8)-(9). An educated guess is that $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t$. Following the standard procedure for solving the dynamic programming problem with recursive utility, we can obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\beta V(J_t) = \max_{\{c_t, \alpha_t\}} \left\{ \beta V(c_t, w_t) + \mathcal{D} V(s_t) \right\}, \tag{11}$$

where

$$\mathcal{D} V(J_t) = V'(J_t) \left((\partial J)^T \cdot \mathbb{E}_t[ds_t] + \upsilon_t^T \cdot \Sigma \cdot \partial J - \frac{\gamma}{2} \left[(\partial J)^T \cdot \Sigma \cdot \partial J \right] \right),$$

$s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}^T$, and $\Sigma = \begin{bmatrix} \alpha_t^2 \sigma_e^2 & \rho_{ye} \sigma_y \alpha_t \sigma_e \\ \rho_{ye} \sigma_y \alpha_t \sigma_e & \sigma_y^2 \end{bmatrix}$. Finally, the transversality condition, $\lim_{t \to \infty} \{\mathbb{E} \left| \exp(-\delta t) V_t \right|\} = 0$, holds at the optimum. (See Appendix 6.1 for the detailed derivation.)

2.4 Optimal Consumption-Portfolio Rules

We can now solve (11) and obtain the consumption and portfolio rules under the WIU. The following proposition summarizes the solution:

Proposition 1 *With the WIU, the decision rules for consumption and portfolio choices, the saving function, as well as the value function are given by:*

(i) The optimal consumption rule is:

$$c_t^* = r w_t + (r + b)\, h_t + \Psi - \Gamma + \Pi, \tag{12}$$

[10] In this paper, we abstract from income growth. It is worth noting that higher income growth generates higher risk-free rates. However, within our REU-OU framework, assuming constant income growth leads to time-varying risk-free rates, which greatly complicates our model.

where

$$h_t = \frac{1}{r+b+\rho}\left[y_t + \frac{\mu}{r+b} - \frac{\pi\rho_{ye}\sigma_y}{(r+b)\,\sigma_e}\right] \tag{13}$$

is the risk-adjusted human wealth;

$$\Psi = \left(\frac{\beta}{r+b} - 1\right)\psi \tag{14}$$

measures the effects of impatience on consumption and saving;

$$\Gamma \equiv \left(1-\rho_{ey}^2\right)\frac{(r+b)\,\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2 \tag{15}$$

is the investor's precautionary saving demand; and

$$\Pi = \frac{\pi^2}{2\,(r+b)\,\gamma\sigma_e^2} \tag{16}$$

is the additional increase in the investor's certainty equivalent wealth due to the presence of the risky asset.

(ii) The optimal portfolio rule is:

$$\alpha^* = \frac{\pi}{(r+b)\,\gamma\sigma_e^2} - \frac{\rho_{ye}\sigma_y}{(r+b+\rho)\,\sigma_e}, \tag{17}$$

where the first term is the standard speculation demand for the risky asset and the second term is the labor income-hedging demand.

(iii) The saving function is:

$$d_t^* = x_t - \Psi + \Gamma + \Pi, \tag{18}$$

respectively, where $x_t \equiv \rho\,(y_t - \overline{y})\,/\,(r+b+\rho)$ *is the demand for savings "for a rainy day".*

(iv) The associated value function is

$$V\,(w_t, y_t) = -\psi\exp\left(\frac{1}{\psi}\left[\alpha_0 + (r+b)\,w_t + \frac{r+b}{r+b+\rho}y_t\right]\right), \tag{19}$$

where

$$\alpha_0 = -\Psi + \Gamma - \Pi - \psi\ln\left(\frac{r+b}{\beta}\right) - \frac{\mu}{r+b+\rho} + \frac{\rho_{ye}\sigma_y\pi}{(r+b+\rho)\,\sigma_e}.$$

Proof. See Online Appendix B for the derivation. ■

Expression (12) clearly shows that current consumption is determined by the annuity value of total wealth, the sum of financial wealth (w_t) and human wealth (h) as well as three additional components, (14)-(16), governing the effects of relative impatience, the precautionary motive, and the presence of the risky asset, respectively. We can see from (12) that this solution is identical to that obtained in the model without the WIU but with an interest rate of $r+b$ rather than r. In other words, from a *partial equilibrium* perspective (i.e., for given r), the WIU increases the effective rate of interest.[11] As argued in Luo et al. (2009), in the presence of the WIU, investing additional dollars yields psychi returns because the additional increase in wealth raises utility; consequently, the effective interest rate, $r+b$, can thus be viewed as the "psychological" rate at which households used to discount future labor income when computing human wealth.

Using the effective rate, we can follow the literature (e.g., Wang 2004, 2009) and construct human wealth as the discount present value of the current and future labor incomes. Specifically, in our incomplete markets economy, there exists a unique stochastic discount factor ζ_t under the *minimal* martingale measure $\mathbb{Q}$ satisfying:

$$d\zeta_t = -\zeta_t \left[(r+b)\, dt + \frac{\pi \rho_{ye}}{\sigma_e} dB_{e,t} \right],$$

where $\zeta_0 = 1$. The human wealth under incomplete markets is then defined as follows:

$$h(y_t) = \mathbb{E}_t^{\mathbb{P}} \left[\int_t^\infty \frac{\zeta_s}{\zeta_t} y_s ds \middle| \mathcal{F}_t^y \right] = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^\infty e^{-(r+b)(s-t)} y_s ds \middle| \mathcal{F}_t^y \right], \tag{20}$$

where $\mathbb{Q}$ is the risk-neutral probability measure with respect to $\mathbb{P}$ for the original income process, (9). Using (9), the above expression of $h(y)$ can thus be simplified as (13).

From (12) and (13), it is also clear that the stronger the WIU, the more consumption responds initially to changes in current income (y_t) because the marginal propensity of consumption out of current income is increasing with the WIU, i.e., $\partial \left(\frac{r+b}{r+b+\rho} \right) / \partial b > 0$. That is, if we take the WIU into account, optimal consumption is more sensitive to unanticipated income shocks. More specifically, the responses of consumption to either an increase or a decrease in current income are stronger, which makes consumption more volatile and more dispersed. This response is referred to as "making hay while the sun shines" in the literature. See, e.g., Flavin (1981) and Jappelli and Pistaferri (2010).

Expression (14) shows that the presence of the WIU strengthens the relative importance of the interest rate to the discount rate, which is equivalent to increasing the degree of patience and thus makes saving and future consumption more attractive. It is worth mentioning that this equivalence result is consistent with a conclusion about the link between the degree of patience and the preference for wealth obtained in Doepke and Zilibotti (2008). They argued that families with a strong spirit of capitalism are much easier to develop patience and work ethic, which become the key determinants of the success of industrialization. From (14), it is also clear that if the household is impatient relative to the effective interest rate ($\beta > r + b$), the higher the EIS, the stronger the demand for consumption. If $\beta > r + b$, households want consumption to fall over time, and a

[11] In the next section, we will explore how the WIU affects the risk-free rate in a general equilibrium.

higher EIS implies that consumption will be allowed to fall faster for a given value of $\beta/(r+b)$; as a result, consumption must initially be high.

Expression (15) shows that the demand for precautionary saving is defined as the amount of saving induced by the combination of uninsurable labor income risk and risk aversion. It is clear from (15) that the precautionary saving demand is larger for a more volatile income innovation (higher σ_y) and a larger persistence coefficient (lower ρ).[12] In addition, since the risky asset can be used to hedge labor income risk (provided the correlation is not zero), it will reduce the precautionary saving demand arising from income uncertainty by a factor $1-\rho_{ye}^2 \in (0,1)$. It is straightforward to show that for given r, the WIU can increase the precautionary saving demand when $\rho > r+b$.[13]

From (16), we can see that stronger WIU reduces the increase in the household's certainty equivalent wealth due to the presence of risky assets because the effective risk-free rate makes investment in the risky asset less attractive. Expression (12) also shows that the presence of the risky asset in the agent's investment opportunity has two effects on current consumption. First, it reduces the risk-adjusted certainty equivalent human wealth by $\pi\rho_{ye}\sigma_y/(r(r+b+\rho)\sigma_e)$ because the agent faces more risk when holding the risky asset. Second, it increases current consumption because it offers a higher expected return. In general equilibrium, the second effect dominates the first effect. (See the next section for the general equilibrium analysis.)

Expression (17) clearly shows that the WIU reduces the standard speculation demand of the risky asset, while it increases the hedging demand for the risky asset. It is straightforward to show that for given asset returns, the WIU increases the overall demand for the risky asset if $\alpha^* > 0$.

Furthermore, from (18), we can see that the presence of the $\pi\alpha^*$ term has the potential to increase saving because it offers a higher expected return. Combining these two effects, it is clear from (18) that the net effect of the risky asset on current saving is governed by $\Pi > 0$ defined in (16). It is thus clear from (18) that there are four saving motives in our model. The first three saving motives–x_t, Ψ, and Γ–are the demand for savings "for a rainy day", the saving demand due to relative patience, and the precautionary saving demand respectively. The fourth term, Π, captures the additional saving demand due to the higher expected return of the risky asset.

In summary, we see the WIU affects each key component in the consumption-saving rule – it increases the sensitivity of consumption to income shocks; it increases the effective degree of patience and then interacts with the EIS in determining the saving demand due to relative patience; it raises precautionary demand; and it reduces the household's certainty equivalent wealth (which lowers consumption). In addition, the WIU lowers the standard speculation demand, while increasing

[12] As argued in Caballero (1990) and Wang (2003), a more persistent income shock takes a longer time to wear off and thus induces a stronger precautionary saving demand by a prudent forward-looking consumer.

[13] In the next section, we will show that this condition is always satisfied for our estimates of the income process.

the labor-hedging demand for risky assets, with a negative net effect on the risky asset demand if $\alpha^* > 0$.

2.5 General Equilibrium Implications

As in Bewley (1986), Huggett (1993), and Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous consumers, with each agent having the saving function, (18). In addition, we also assume that the risk-free asset in our model economy is a pure-consumption loan and is in zero net supply, and that the net supply of the risky asset is $\alpha_s \geq 0$. We first consider the equilibrium in the market for the risky asset. Assuming the equilibrium condition in the market for the risky asset is:[14]

$$\alpha_s = \frac{\pi}{(r+b)\,\gamma\sigma_e^2}, \tag{21}$$

for a given risk free rate, r.

In the model economy, the initial cross-sectional distribution of income is assumed to be its stationary distribution $\Phi(\cdot)$. By the law of large numbers (LLN), provided that the spaces of agents and the probability space are constructed appropriately, aggregate income and the cross-sectional distribution of permanent income $\Phi(\cdot)$ will be constant over time. Using the individual saving function (18), we have the following aggregation result on savings:

Proposition 2 *The total savings demand "for a rainy day" in our model equals zero for any positive interest rate. That is,* $F_t(r) = \int_{y_t} x_t(r)\, d\Phi(y_t) = 0$, *for* $r > 0$.

Proof. Given that labor income is a stationary OU process and the labor income risk is purely idiosyncratic, the LLN can be directly applied. The proof is the same as that in Wang (2003). ■

Using this result, from (18), after aggregating across all consumers, the expression for total savings can be written as:

$$D^{total}(r) \equiv \Gamma(r) - \Psi(r) - \Pi(r), \tag{22}$$

where $\Psi(r)$, $\Gamma(r)$, and $\Pi(r)$ are given in (14), (15), and (16), respectively. Note that compared with the original Huggett (1993), Calvet (1997), and Wang (2003) models, here $\Pi(r)$ is an additional term due to the positive net supply of the risky asset in this model. Since the net supply of the risk-free asset is zero in equilibrium, an equilibrium interest rate r^* satisfies:

$$D(r^*) \equiv \Gamma(r^*) - \Psi(r^*) = 0, \tag{23}$$

[14] It is worth noting that if $\rho_{ye} \neq 0$, the labor income risk might include a common component that is correlated with the equity return, which greatly complicates the aggregation mechanism. To keep our equilibrium analysis tractable, here we assume that the two risks, the labor income risk and the equity return risk, are uncorrelated, i.e., $\rho_{ye} = 0$.

where $D(r^*)$ denotes the amount of saving in the risk-free asset.

The following proposition proves that an equilibrium exists and that the PIH is satisfied.

Proposition 3 *In equilibrium, each consumer's optimal consumption-portfolio rules are described by:*

$$c_t^* = r^* w_t + (r^* + b) h_t, \tag{24}$$

and

$$\alpha^* = \alpha_s, \tag{25}$$

respectively, where h_t is defined in (13). Furthermore, in equilibrium, the evolution equations of c_t^* and w_t^* are:

$$dc_t^* = r^* dw_t + \frac{r^* + b}{r^* + b + \rho} dy_t, \tag{26}$$

$$dw_t^* = (x_t + \Pi) dt + \alpha_s \sigma_e dB_{e,t}, \tag{27}$$

respectively, and the risk premium is

$$\pi^* = (r^* + b) \gamma \alpha_s \sigma_e^2. \tag{28}$$

If $\rho > \beta$, the equilibrium, $r^* \in (-b, \beta - b)$ and $\pi^* \in (0, \beta\gamma\alpha_s\sigma_e^2)$, exists and is unique.

Proof. Appendix **??** for the proof. ■

The intuition behind this proposition is similar to that in Wang (2003) and Luo et al. (2020). With an individual's constant total precautionary savings demand $\Gamma(r)$, for any $r > 0$, the equilibrium interest rate r^* must be at a level with the property that individual's dissavings demand due to impatience is exactly balanced by their precautionary-savings in the risk-free asset, $\Gamma(r^*) = \Psi(r^*)$. It is clear from (23) that a high value of ψ would amplify the relative importance of the dissaving effect $\Psi(r)$ for the equilibrium interest rate. The intuition behind this result is simple. When ψ is higher, consumption growth responds more to changes in the interest rate. In order to clear the market, the consumer must be offered a higher equilibrium risk free rate in order to be induced to save more and make his consumption tomorrow even more in excess of what it is today (less smoothing). Figure 1 shows how the equilibrium risk-free rate is unique and is decreasing with the value of b when $\gamma = 6$, $\psi = 0.8$, $\delta = 0.048$, $\sigma_y = 0.162$, and $\rho = 0.102$.[15]

From the equilibrium condition, (23):

$$\frac{1}{2}(r^* + b)\gamma\left(\frac{\sigma_y}{r^* + b + \rho}\right)^2 - \left(\frac{\beta}{r^* + b} - 1\right)\psi = 0, \tag{29}$$

[15]In Section 3.1, we will discuss the choice of these preference parameters and provide more details about how to estimate the income process using the U.S. panel data. The main result here is robust to the choices of these parameter values.

it is straightforward to show that

$$\frac{dr^*}{db} = -1 < 0; \tag{30}$$

so that r^* is decreasing in the degree of the WIU, b. In other words, an increase in b has no effect on the values of Γ and Ψ in general equilibrium. In addition, it is straightforward to see that:

$$\frac{dr^*}{d\gamma} < 0 \text{ and } \frac{dr^*}{d\psi} > 0.$$

That is, the equilibrium interest rate decreases with the degree of risk aversion and increases with the degree of intertemporal substitution. From (26) and (27), we can conclude that although both the CARA model and the linear-quadratic (LQ) model lead to the PIH in general equilibrium, both risk aversion and intertemporal substitution play roles in affecting the dynamics of consumption and wealth in the CARA model via the equilibrium interest rate channel.

Note that mathematically, the cross-sectional dispersion of consumption and wealth (relative to income), our measures of consumption and wealth inequality, can be measured by the relative volatility of consumption and wealth to income, as our model satisfies a mixing condition in the steady state. In addition, as in the typical Bewley-Huggett type economy, the cross-sectional distribution of consumption and wealth in our model economy are the same as the long-run stationary distribution of individual household consumption and wealth. The following result is then immediate.

Proposition 4 *The relative inequality of consumption to income is:*

$$\mu \equiv \frac{\mathrm{sd}\left(dc_t^*\right)}{\mathrm{sd}\left(dy_t\right)} = \frac{r^* + b}{r^* + b + \rho}\sqrt{1 + \left(r^* + b + \rho\right)^2\left(\frac{\sigma_e}{\sigma_y}\alpha_s\right)^2}, \tag{31}$$

where $\mathrm{sd}\left(\cdot\right)$ *denotes standard deviation. Furthermore, the relative inequality of wealth to income is:*

$$\mu_{wy} \equiv \frac{\mathrm{sd}\left(dw_t^*\right)}{\mathrm{sd}\left(dy_t\right)} = \alpha_s\frac{\sigma_e}{\sigma_y}. \tag{32}$$

Proof. Using (26), the proof is straightforward. ■

Given the complexity of the expressions of the endogenous interest rate and (31), we will quantitatively evaluate the effects of the WIU on the equilibrium asset returns and the relative inequality of consumption to income in the next section. It is worth noting that it is clear from (31) that when $\alpha_s = 0$,

$$\mu = \frac{r^* + b}{r^* + b + \rho}, \tag{33}$$

which is clearly increasing with the degree of the WIU, b, for a given equilibrium risk-free rate. The main intuition of this result is that the presence of the WIU increases the "psychological" or effective rate of interest at which households discount future labor income when computing human wealth and compute the annuity value of human wealth. Furthermore, (26) clearly shows that the larger the value of b, the more consumption responds initially to changes in current income because $\partial\mu/\partial b > 0$. That is, the optimal consumption is more sensitive to unanticipated income shocks (either positive or negative) in the presence of the WIU; consequently, equilibrium consumption becomes more volatile and more dispersed. Figure 2 illustrates how the marginal propensity of consumption (the MPC) out of current income varies with the degree of the WIU. It is clear from the figure that this MPC is more volatile when the degree of the WIU becomes stronger.

As documented in many empirical studies in the PIH literature, consumption growth are robustly correlated with predictable changes in labor income at both the micro- and macro-levels. For example, Flavin (1981) estimates the joint consumption-income equations, and finds evidence of excess sensitivity of consumption to predicted income growth. Parker (1999) uses a psuedo-natural experiment provided by the pattern of WIUial Security tax withholding to test whether household consumption responds to expected changes in take-home pay. Souleles (1999) studies the relationship between consumption and the receipt of income tax funds. These studies find evidence of excess sensitivity and are thus interpreted as strong evidence against the PIH.[16] To examine how the WIU affects the excess sensitivity of equilibrium consumption to the *anticipated* change in income in our model, we rewrite (26) as follows:

$$dc_t^* = r^*\Pi dt + \frac{b}{r^*+b+\rho}\mathbb{E}_t\left[dy_t\right] + \left(\frac{r^*+b}{r^*+b+\rho}\sigma_y dB_t + r^*\alpha_s\sigma_e dB_{e,t}\right), \tag{34}$$

where $\mathbb{E}_t\left[dy_t\right]$ is the anticipated change in income and we use the fact that $dy_t = \mathbb{E}_t\left[dy_t\right] + \sigma_y dB_t$. It is clear from (34) that with the WIU, the anticipated growth of labor income can be used to predict changes in equilibrium consumption because of $b > 0$. That is, excess sensitivity can arise in equilibrium for consumers with the WIU even in the absence of borrowing constraints. Our WIU model can therefore be an alternative explanation for apparent excess sensitivity of consumption to anticipated changes in income that is consistent with the PIH in general equilibrium.

Finally, (32) clearly shows that once the net supply of the risky asset is exogenously set, the relative wealth inequality is determined by the interaction of the three parameters, α_s, σ_e, and σ_y, pinned down from the data. However, in the next section on quantitative analysis, we find that the implied relative wealth inequality is also consistent with the empirical evidence. Given these features, our model is not the ideal one to examine the effects of the WIU on the wealth inequality.

[16] In addition, in the literature following Hall (1978), excess sensitivity was generally held to result from the presence of borrowing constraints. See Jappelli and Pistaferri (2010) for a recent survey on the consumption response to income changes.

3 Model's Quantitative Implications

In this section, we evaluate the quantitative implications of the WIU in explaining the equilibrium interest rate, consumption inequality, and the risk premium. Our parameterization is based on the benchmark model. We provide a robustness check by allowing alternative specifications in the next section.

3.1 Parameterization and Data Moments

Labor Income Parameters. To estimate the labor-income process specified in Section **??**, we use the Panel Study of Income Dynamics (PSID) to first estimate a discrete AR(1) process and then calculate the key coefficients based on the estimated AR(1) process. Specifically, we follow Blundell et al.(2008) and Luo et al. (2020) to apply a two-step panel regression to estimate the following AR(1) process:[17]

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sigma\varepsilon_t, \ t \geq 1, \ \ |\phi_1| < 1, \tag{36}$$

where $\varepsilon_t \sim N(0,1)$, $\phi_0 = (1-\phi_1)\,\overline{y}$, $\overline{y}$ is the mean of y_t, and the initial level of labor income y_0 are given. Once we have estimates of ϕ_1 and σ, we can recover the drift and diffusion coefficients in the Ornstein-Uhlenbeck process specified in (9) by rewriting (36) in the time interval $[t, t+\Delta t]$ as:

$$y_{t+\Delta t} = \phi_0 + \phi_1 y_t + \sigma\sqrt{\Delta t}\varepsilon_{t+\Delta t}, \tag{37}$$

where $\phi_1 = \exp(-\rho\Delta t)$, $\sigma = \sigma_y\sqrt{(1-\exp(-2\rho\Delta t))\,/\,(2\rho\Delta t)}$, and $\varepsilon_{t+\Delta t}$ is the time-$(t+\Delta t)$ standard normal distributed innovation to income.[18] As the time interval, Δt, converges to 0, (37) reduces to the Ornstein-Uhlenbeck process, (9). The estimation results and the recovered persistence and volatility coefficients in (9) are reported in the top panel of Table 1. The detailed description on the data is in Online Appendix A.

Other Parameters. The rest of the key parameters in the benchmark model are most taken from the literature and reported in Table 2. Specifically, following the literature on consumption and asset pricing (e.g., Campbell 2003, Bansal and Yaron 2004), we set the coefficient of risk aversion (γ) and the discount factor (β) to be 6 and 0.048, respectively. The magnitude of the EIS (ψ) is a key issue in macroeconomics and asset pricing. For example, Vissing-Jorgensen and Attanasio (2003)

[17]Adopting a more general ARMA(1,1) specification,

$$y_t = \phi_0 + \phi_1 y_{t-1} + \sigma(\varepsilon_t + \varrho\varepsilon_{t-1}), \tag{35}$$

does not change the main quantitative conclusions. The results obtained using this ARMA(1,1) are available from the coressponding author by request.

[18]Note that here we use the fact that $\Delta B_t = \varepsilon_t\sqrt{\Delta t}$, where ΔB_t represents the increment of a Wiener process.

estimate the EIS to be well in excess of one. Bansal and Yaron (2004) show that a small, persistent component of consumption growth can have quantitatively important implications for asset prices if the representative agent has Epstein–Zin preferences with the EIS equaling 1.5. Campbell (2003), on the other hand, estimates its value to be well below one.[19] Crump et al. (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations. Here we choose $\psi = 0.8$ as the benchmark value.

In addition, estimating the correlation between individual labor income and the equity return is complicated by the lack of panel data on household portfolio choice, and we find several estimates in the literature: Viceira (2001) adopts $\rho_{ye} = 0.35$ when simulating a life-cycle consumption-portfolio choice model. Davis and Willen (2000) estimate that the correlation is between 0.1 and 0.3 for college-educated men, and is 0.25 or more for college-educated women. Here, we follow Viceira (2001) and set $\rho_{ye} = 0.35$ as a benchmark value. Following Campbell (2003), we measure the asset return volatility σ_e using stock market data between 1980 and 2016 and the resulting value is 0.17. The values of the risk premium (π) will be determined endogenously in equilibrium.

Key Data Moments. We examine the model's implications on the risk-free rate, consumption inequality, wealth inequality, and equity premium. These key moments are measured using various data. We follow Campbell (2003) to calculate the real risk-free rate based on the real 3-month Treasury yields. The equity premium is calculated as the difference between the average equity return and the average risk-free rate. The empirical counterpart of the relative consumption and wealth inequality is measured using the PSID data. Please see Online Appendix A for details in constructing a panel data with household-level consumption, income, and wealth. The values of these key moments are reported in the first row of Table 3.

3.2 Effects of the WIU in Matching the Key Moments

Figure 3 shows that the equilibrium interest rate decreases with the degree of the WIU for different plausible values of γ, ψ, β, and ρ_{ye} in the benchmark model. It is clear from the figure that a small degree of the WIU can have significant effects on the equilibrium interest rate. For example, if b is increased from 0 to 0.02, r^* falls from 4.1% to 2.1%.[20] In addition, the figure also shows that the

[19] Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1).

[20] Note that this value of the WIU parameter (b) is relatively small given that the coefficient attached to the consumption term is normalized to 1. In addition, this value is also well below that obtained in Bakshi and Chen (1996). In a model with the WIU and CRRA preferences, they find that when the relative weight of the degree of the WIU compared to the degree of risk aversion to consumption is about $1/3 - 1/2$, the model can explain the joint behavior of aggregate consumption and the equity return observed in the U.S. economy. He at al. (2020) show a growth model calibrated to the U.S. economy suggests a value of $b = 0.09$.

interest rate decreases with γ, and increases with ψ, β, and ρ_{ye} for different values of b. We can also see from the figure that the correlation between labor income risk and the equity return risk have only minor effects on the equilibrium rate of interest.

To put this into a statistical perspective, we compare the model's predictions on the key targets under different specifications. As reported in the left panel in Table 3, the average real risk-free rate in $1980-2016$ is 2.1%. Without the WIU (i.e., $b = 0$), it requires the degree of risk aversion to be as high as 57 to generate a risk-free rate as observed in the data when the EIS is 0.8. When the EIS is relatively lower, say, $\varphi = 0.5$, it still requires the degree of risk aversion to be 35 to generate the observed real risk-free rate.[21] In contrast, when there is a small degree of the WIU, the model can generate an equilibrium interest rate of 2.1% with much lower values of the coefficient of risk aversion. As reported in the third column of Table 3, with $\gamma = 6$, $\psi = 0.8$, and $b = 0.02$, the model generates the same interest rate as in the standard model with $\gamma = 57$. Note that $\gamma = 6$ is in the middle of the range of reasonable values for the coefficient of relative risk aversion used in most macro-asset pricing models. For example, Caballero (1990) and Wang (2004, 2009) set γ to be 2 or 3, while Bansal and Yaron (2004) set γ to be 10. More generally, Figure 4 shows the relationship between risk aversion parameter γ and the WIU parameter ϑ to generate an interest rate of 2.1% for different values of EIS parameter ψ.

Turning to the relative consumption inequality, as measured by ($\mu = \mathrm{sd}\left(dc_t^*\right) / \mathrm{sd}\left(dy_t\right)$), the average value in $1980 - 2016$ is 0.4 as reported in the middle panel of Table 3. Without the WIU, even the degree of risk aversion is set at 57 (the level which generates a real risk-free rate as in the data without the WIU), the model implied relative consumption inequality is only 0.22, which is about half of that in the data. In contrast, when there is a small degree of the WIU ($b = 0.02$), the model can generate a relative consumption inequality as in the data with a reasonable degree of risk aversion $\gamma = 6$. In other words, incorporating a small amount of the WIU could greatly improve the model's predictions on both the equilibrium real risk-free rate and the relative consumption inequality.

Furthermore, incorporating the WIU also helps produce more realistic equity premium. As reported in the right panel of Table 3, under the same degree of the WIU ($b = 0.02$), the model generates an equity premium of 4.3%, which is close to the actual equity premium in the data. In contrast, without the WIU, the model can only produce less than half of the observed equity premium for the same degree of risk aversion, 6.

Finally, our model implied relative wealth inequality is also consistent with the data. As also reported in the Table 3, at $b = 0.02$, our model's implied relative wealth inequality exactly matches the data. In our model, the relative wealth inequality is determined by the interaction of the three

[21] Note that since we set the mean income level to be 1, the coefficient of CRRA evaluated at this level is equal to the coefficient of CARA.

parameters, α_s, σ_e, and σ_y. It is worth noting that in our model, α_s, σ_e, and σ_y are exogenously calibrated or estimated using the data. (See Tables 1 and 2 for the details.) As a result, we cannot examine how the WIU affects the relative wealth inequality in our model.

4 Further Discussions

In this section we provide three extensions to the benchmark model as robustness checks. First, we introduce rare macroeconomic disasters into the model and check the robustness of the results. Second, we assume a more general labor-income process and examine whether our key results still hold. Third, we compare the WIU with habit formation, which represents an alternative attitude toward consumption and saving, and show they lead to completely different results on consumption inequality and the equilibrium interest rate. For all three discussions, we provide analytical solutions.

4.1 Incorporating Rare Macroeconomic Disasters

In our benchmark model, we assumed that the income process takes a diffusive form. In this subsection, we consider an important extension in which the households might experience rare macroeconomic disasters, and examine how the WIU affects the equilibrium risk-free rate and relative consumption inequality. Rietz (1988) first introduced rare disasters into a consumption-based asset pricing model and showed that it can help resolve the equity premium puzzle. Barro (2009) and Barro and Ursua (2011) showed that the presence of rare macroeconomic disasters has the potential to explain an array of macro-asset pricing puzzles. To model rare disasters (e.g., financial crisis), we assume that the labor income process facing the agents in our model economy not only include a diffusion component but also includes a Poisson jump process.[22] As argued in Barro (2009) and Barro and Ursua (2011), the probability and size distribution of rare disasters are difficult to quantify empirically because the relevant events are rare and possibly absent in short samples. In this paper, we therefore consider a simple model in which the rare event of a sharp decline in income is a Poisson process with a constant occurring probability per unit of time, λ (i.e., Poisson arrivals). When the event happens, labor income is reduced by a constant amount φ.[23] Specifically, we assume that the dynamics of labor income is now governed by:

$$dy_t = (\mu - \rho y_t)\,dt + \sigma_y dB_t - \varphi dq, \tag{38}$$

[22] Barro (2009) and Barro and Ursua (2011) considered the similiar output jump process within a discrete-time Lucas-type asset pricing setting.

[23] Note that here we can also interpret φ as a fraction of the mean of income because we have assumed that the mean level of income ($\overline{y}$) is 1.

where B_t reflects "normal" economic fluctuations and q is a pure jump process. That is, $dq = 1$ when the jump happens and $dq = 0$ otherwise. It is worth noting that we can use this Poisson process to model financial crises. When a financial crisis happens, the mean income of the economy is dropped by φ. The reason that we consider jumps with constant size is to keep our heterogeneous-agent incomplete markets model tractable. Note that allowing for a stochastic φ introduces an additional demand for precautionary savings, and does not change our main results. The HJB for the model with rare disasters can be written as:[24]

$$\delta f(J_t) = \max_{c_t} \left\{ \delta f(c_t, w_t) + f'(J_t) \left(\partial J \cdot \mathbb{E}[ds_t] - \frac{1}{2}\gamma \left[\partial J \cdot \sigma_y^2 \cdot (\partial J)^T \right] \right) + \lambda \left[f(J(w_t, y_t - \varphi)) - f(J(w_t, y_t)) \right] \right\}$$

where $f(J_t) = (-\psi)\exp(-J_t/\psi)$, $f'(J_t) = \exp(-J_t/\psi)$, $s_t = \begin{bmatrix} w_t & y_t \end{bmatrix}^T$, $ds_t = \begin{bmatrix} dw_t & dy_t \end{bmatrix}^T$, and $\partial J = \begin{bmatrix} J_w & J_y \end{bmatrix}$. For simplicity, here we assume that the supply of the risky asset is zero. The budget constraint for the typical consumer becomes: $dw_t = (rw_t + y_t - c_t)\,dt$. Following the same procedure, we can solve the above optimization problem with rare events and obtain the consumption and saving functions. The following proposition summarizes the solution:

Proposition 5 *Under rare events and the WIU, the consumption function is:*

$$c_t = rw_t + \frac{r+b}{r+b+\rho} y_t + \frac{1}{r+b+\rho} (\mu - \varphi\lambda) + \Psi - \Gamma - \Lambda, \tag{39}$$

and the saving function is:

$$d_t = x_t - \Psi + \Gamma + \Lambda, \tag{40}$$

where $x_t = \rho (y_t - \overline{y}) / (r+b+\rho)$ *is the demand for savings for a rainy day",* $\overline{y} = (\mu - \varphi\lambda)/\rho$ *is the mean income,* $\Psi = \psi(\beta/(r+b) - 1)$, $\Gamma = 0.5\gamma(r+b)\sigma_y^2/(r+b+\rho)^2$, *and*

$$\Lambda = \lambda \frac{\psi}{r+b} \left[\exp\left(\frac{r+b}{\psi} \frac{\varphi}{r+b+\rho} \right) - 1 \right] \tag{41}$$

is the additional saving demand due to the presence of rare disasters.

Proof. See Online Appendix C. ■

In the presence of rare events, Λ is the capitalized value of the expected future declines to labor income, capitalized at a higher rate than the risk-free rate reflecting the agents' degree of the WIU and aversion to intertemporal substitution.[25] It is clear from (41) that the presence of rare events increases individual saving via the interaction between frequency (λ) and size (φ) of the rare events.

[24] For simplicity, here we assume that the net supply of the risky asset is zero. It is worth noting that the assumption of a positive net supply of the risky asset does not change our main conclusions if the equity return is not correlated with the Poisson jump in the labor income process.

[25] Note that in the expected utility case this term is affected by the degree of risk aversion because we do not distinguish intertemporal substitution from risk aversion in this case.

Following the same procedure and the definition of general equilibrium adopted in the benchmark model, we can also obtain the general equilibrium implications of the WIU for the risk-free rate and consumption inequality in the presence of rare disasters. The following proposition summarizes the general equilibrium results:

Proposition 6 *There exists one equilibrium with an interest rate $r^* \in (0, \beta)$ such that*

$$D(r^*) = -\Psi + \Gamma + \Lambda = 0. \tag{42}$$

In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^* = r^* w_t + \frac{r^* + b}{r^* + b + \rho}\left(y_t + \frac{\mu}{r^*}\right). \tag{43}$$

Furthermore, in this equilibrium, the relative volatility of consumption growth to income growth is:

$$\mu \equiv \frac{\mathrm{sd}(dc_t^*)}{\mathrm{sd}(dy_t)} = \frac{r^* + b}{r^* + b + \rho}, \tag{44}$$

where $\mathrm{sd}(dy_t) = \sqrt{(\sigma_y^2 + \lambda\varphi^2)\,dt}$.

Proof. The proof is the same as that in the benchmark model in Section 2. ■

We make two comments on these theoretical results. First, from expressions (41) and (42), we can see that the presence of rare disasters has the potential to further drive down the risk-free rate because it generates an additional saving term Λ. In addition, (44) clearly shows that introducing rare disasters can indirectly lower the relative consumption inequality through generating a lower equilibrium interest rate. However, for a given equilibrium risk-free rate, introducing rare disasters has no effect on the relative consumption inequality.

To fully explore the general equilibrium effect of the WIU in the presence of rare disasters, we need to do a quantitative analysis. We adopt the disaster parameters estimated by Barro and Ursua (2011). When applying to disasters of size 10% or larger, they estimate that the disaster probability for GDP (i.e., the empirical frequency of entry into disaster states) is 3.7%. In addition, they estimate that the average disaster size, subject to the threshold of 10%, is 0.21 for GDP. We therefore set the probability (λ) and size (φ) to be 3.7% and 0.21, respectively, as the baseline values in our quantitative analysis. Figure 5 shows that how the equilibrium risk-free rate decreases with the degree of the WIU for different plausible values of λ, φ, and ψ when other parameters are set to their baseline values. It is clear from the figure that a small degree of the WIU can have significant effects on the equilibrium risk-free rate for plausible values of the probability (λ) and size (φ) as well as the EIS (ψ). That is, the main result in our benchmark model still holds in the presence of rare disasters. Furthermore, it is also clear from the figure that the probability and size do not have significant effects on the equilibrium risk-free rate. For example, if b is increased from 0 to

0.017, r^* falls from 3.9% to 2.1%, given $\gamma = 6$, $\psi = 0.8$, $\beta = 0.048$, $\lambda = 3.7\%$, and $\varphi = 0.21$. As is clear from (44), the quantitative implication of the WIU on the relative consumption inequality is the same as that obtained in the benchmark model once the equilibrium risk-free rate is reached to its empirical counterpart.

4.2 More General Income Specification

In our benchmark model, we assumed that labor income follows an AR(1) process. In this section, we consider an alternative realistic income specification. In the first case, we follow Wang (2004) and assume that labor income has two distinct components:

$$y_t = y_{1,t} + y_{2,t},$$

where

$$dy_{1,t} = (\mu_1 - \rho_1 y_{1,t})\, dt + \sigma_1 dB_{1,t}, \tag{45}$$

$$dy_{2,t} = (\mu_2 - \rho_2 y_{2,t})\, dt + \sigma_2 dB_{2,t}, \tag{46}$$

and the two Brownian innovations, $dB_{1,t}$ and $dB_{2,t}$, are independent, and are also independent of the innovation to the equity return. All the other notations are similar to those we used in our benchmark model. Without loss of generality, we assume that $\rho_1 < \rho_2$. For simplicity, here we assume that agents can distinguish the two individual components. Our main results still hold if we assume that the two individual components are indistinguishable.

Following the same procedure and the definition of general equilibrium adopted in the benchmark model, we can also obtain the general equilibrium implications of the WIU under this alternative income specification. The following proposition summarizes the general equilibrium results:

Proposition 7 *There exists one equilibrium with an interest rate $r^* \in (0, \beta)$ such that the amount of aggregate saving in the risk free asset is zero*

$$\pi^* = (r^* + b)\, \gamma \alpha_s \sigma_e^2. \tag{47}$$

In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^* = r^* w_t + \frac{r^* + b}{r^* + b + \rho_1} \left(y_{1,t} + \frac{\mu_1}{r^*} \right) + \frac{r^* + b}{r^* + b + \rho_2} \left(y_{2,t} + \frac{\mu_2}{r^*} \right). \tag{48}$$

Furthermore, in this equilibrium, the evolution equations of wealth and consumption are

$$dw_t^* = (f_t + \Pi)\, dt + \alpha_s \sigma_e dB_{e,t}, \tag{49}$$

$$dc_t^* = \left(\frac{r^* + b}{r^* + b + \rho_1} \sigma_1 + \frac{r^* + b}{r^* + b + \rho_2} \sigma_2 \right) dt + r^* \alpha_s \sigma_e dB_{e,t}, \tag{50}$$

respectively, where $f_t = \rho_1 (y_{1,t} - \overline{y}_1) / (r^* + \rho_1) + \rho_2 (y_{2,t} - \overline{y}_2) / (r^* + \rho_2)$ *and* $\Pi = \pi^2 / \left[2 (r + b) \gamma \sigma_e^2\right]$. *Finally, the relative volatility of consumption growth to income growth is*

$$\mu \equiv \frac{\mathrm{sd}(dc_t^*)}{\mathrm{sd}(dy_t)} = \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} \sqrt{\left(\frac{r^* + b}{r^* + b + \rho_1}\sigma_1\right)^2 + \left(\frac{r^* + b}{r^* + b + \rho_2}\sigma_2\right)^2 + (r^* \sigma_e \alpha_s)^2}. \quad (51)$$

Proof. The proof is the same as that in the benchmark model in Section 2. See Online Appendix D for the detailed derivation. ■

Based on these analytical solutions, we can evaluate the implications of the WIU on the risk-free rate and relative consumption inequality. We follow the approach in Blundell et al. (2008) to estimate the parameters which are reported in the bottom panel of Table 1. Using the above estimated parameter values, we can quantitatively evaluate how WIU affects the equilibrium risk-free rate. Figure 6 shows that the equilibrium interest rate decreases with the strength of the preference for wealth at different plausible values of ψ.[26] It is clear from the figure that similar to our benchmark model, a small degree of WIU can have significant effects on the equilibrium interest rate. For example, when $\gamma = 6$, $\psi = 0.8$, $\beta = 0.048$, we only need $b = 0.016$ for the model to generate an interest rate of 2.1%, which is the empirical counterpart. If we do not consider WIU, the risk aversion parameter γ need to be as high as 30 to generate the observed risk-free rate.

Using the formula (51), we can quantitatively evaluate the implications of the WIU for the relative consumption inequality. For example, without WIU, after setting $\gamma = 30$ to obtain that $r^* = 2.1\%$, we can then compute that $\mu = 0.35$, which is below the average value $\mu = 0.4$. In contrast, when we consider the WIU and set $b = 0.016$ and $\gamma = 6$, the same values we used above to generate a risk-free rate of 2.1%, the model implies a relative consumption inequality of $\mu = 0.42$, which is very close to the empirical counterpart.

4.3 Comparing with Internal Habit Formation

It has long been recognized by economists that preferences may not be intertemporally separable. In particular, high past consumption generates a desire for high current consumption and preferences may thus display habit formation (or intertemporal complementarity) features.[27] The notion that past consumption may affect current utility is very old. Deusenberry (1949) is probably the first to examine the implications of habit formation. Several papers have shown the importance of allowing for habit formation (HF) in utilities when examining consumption dynamics and equilibrium asset returns. For example, Sundaresan (1989), Constantinides (1990), and Ingersoll (1992) find that models with habit formation can obtain a high equity premium with low degrees of risk aversion

[26] The other parameter values are set as follows: $\psi = 0.8$ and $\beta = 0.048$.

[27] For simplicity, here we only discuss internal habit formation and do not consider the catch-up-with-Joneses effect (i.e., external habit formation).

and smooth consumption dynamics. As argued in Ingersoll (2011), the model with the WIU is close in spirit to the habit formation models mentioned above, but it is the interaction future consumption rather than past consumption that is of concern here because wealth is part of total resources that matters for financing future consumption.

With HF preferences, households try to smooth consumption growth (roughly speaking), rather than the level of consumption; the result is that consumption tends to respond slowly to changes in income shock. In this section, we compare the different implications of HF and the WIU with a general equilibrium framework. Following Sundaresan (1989), Ingersoll (1992), Alessie and Lusardi (1997), and Smith (1999), we assume that habit formation takes a subtractive form and the momentary utility function of households depends on the difference between current consumption and the habit stock. Specifically, we assume that the momentary utility function takes the following form:

$$V\left(c_t-\theta z_t\right)=\left(-\psi\right)\exp\left(-\frac{1}{\psi}\left(c_t-\theta z_t\right)\right), \tag{52}$$

where $\theta\in(0,1)$ governs the degree of habit formation, and z_t is the habit stock and is governed by the following equation:

$$z_t=z_0+\int_0^t\exp\left(-\kappa\left(t-s\right)\right)c_s ds,\text{ or } dh_t=\kappa\left(c_t-z_t\right)dt. \tag{53}$$

In other words, $V\left(c_t-\theta z_t\right)$ is dependent on not only current consumption at t, but also on the habit stock (i.e., the weighted average of past consumption). Here $\kappa>0$ is a smoothing constant. The higher κ is, the less weight is put on past consumption in determining the habit stock. We can then introduce HF into our REU model specified in Section **??** by assuming that the recursive utility takes the following form:

$$V\left(J_t\right)=\max_{\left(c_t,\alpha_t\right)}\left\{\left(1-e^{-\beta\Delta t}\right)V\left(c_t-\theta z_t\right)+e^{-\beta\Delta t}V\left(\mathbb{CE}_t\left[J_{t+\Delta t}\right]\right)\right\}, \tag{54}$$

where the definitions of $V\left(J_t\right)$ and $\mathbb{CE}_t\left[J_{t+\Delta t}\right]$ are the same as that in the benchmark model.

Following the same solution method and the definition of general equilibrium in our benchmark model, we can construct an general equilibrium under HF. The following proposition summarizes the solution and key properties:

Proposition 8 *There exists a unique equilibrium with an interest rate $r^*\in(0,\beta)$ such that $D\left(r^*\right)=0$, and*

$$\pi^*=\eta r^*\gamma\sigma_e\left(\frac{\rho_{ye}\sigma_y}{r^*+\rho}+\alpha_s\sigma_e\right). \tag{55}$$

(i) In any such equilibrium, each agent's optimal consumption-portfolio rules are described by:

$$c_t^*=\eta r^*\left(w_t+h_t\right), \tag{56}$$

and

$$\alpha^* = \alpha_s \geq 0, \tag{57}$$

respectively, where

$$\eta = 1 - \frac{\theta\kappa}{r+\kappa} \in (0,1), \tag{58}$$

governs the impact of HF on consumption and portfolio rules,

$$h_t = \frac{1}{r^*+\rho}\left(y_t + \frac{\mu}{r^*} - \frac{\pi\rho_{ye}\sigma_y}{r^*\sigma_e}\right)$$

is the risk-adjusted human wealth; and the equilibrium risk-free rate, r^*, is endogenously determined by:

$$\Gamma(r^*) = \Psi(r^*), \tag{59}$$

where

$$\Gamma(r^*) = \frac{1}{2}\eta^2 r^* \gamma \frac{\left(1-\rho_{ey}^2\right)\sigma_y^2}{\left(r^*+\rho\right)^2} \tag{60}$$

is the precautionary saving demand and $\Psi(r^*) = \psi(\beta/r^* - 1)$ is the dissaving effect due to relative impatience;

(ii) the equilibrium risk-free rate, r^*, increases with the strength of habit formation (measured by θ and κ). That is,

$$\frac{dr^*}{d\theta} > 0 \text{ and } \frac{dr^*}{d\kappa} > 0.$$

Proof. See Online Appendix E for the derivation. ■

We make a couple of comments here. First, from (59), it is clear that the only difference in the equilibrium condition between the HF model and the otherwise standard model is the presence of η^2 defined in (58). When $\theta = 0$ or $\kappa = 0$, the two models are identical. Second, we can see from (59) and (60) that HF affects the precautionary savings demand because $\eta^2 < 1$, while it has no effect on the dissaving effect due to relative impatience; consequently, HF drives up the equilibrium risk-free rate. That is, the stronger the habit persistence (higher θ), the higher the equilibrium interest rate.[28] Thus, we can conclude that the WIU and HF have opposite effects on the demand for precautionary savings, which then leads to opposite effects on the equilibrium interest rate. The WIU reduces the equilibrium interest rate, while HF increases it.

Figure 7 illustrates how the general equilibrium interest rates vary with θ for different values of κ.[29] We can see from the figure that r^* increases as the degree of habit persistence increases, i.e., either θ or κ increase. For example, for given $\kappa = 0.2$, if θ is raised from 0 to 0.5, r^* increases from

[28] In a partial equilibrium model, Alessie and Lusardi (1997) and Smith (2002) show that the stronger the habit, the smaller the effect of income uncertainty on the precautionary saving term.

[29] Here the other parameter values are set to be the same as in the benchmark model.

3.44 percent to 3.89 percent. In other words, the presence of habit formation actually exaggerates the risk-free rate puzzle we observed in the U.S. economy.

The following proposition summarizes the implications of habit formation for the relative volatility of consumption and wealth to income:

Proposition 9 *In equilibrium, the evolution equations of c_t^* and w_t^* are:*

$$dc_t^* = \eta r^* \left(dw_t + \frac{1}{r^* + \rho} dy_t \right), \tag{61}$$

$$dw_t^* = (x_t + \Pi)\, dt + \alpha_s \sigma_e dB_{e,t}, \tag{62}$$

respectively, where $x_t = \rho(y_t - \overline{y}) / (r^* + \rho)$ is the demand for savings "for a rainy day", and $\Pi = \pi^2 / \left(2r^* \gamma \sigma_e^2\right)$ is the additional increase in the agent's certainty equivalent wealth due to the presence of the risky asset. Using (61), the relative inequality of consumption to income is:

$$\mu \equiv \frac{\mathrm{sd}\,(dc_t^*)}{\mathrm{sd}\,(dy_t)} = \frac{\eta r^*}{r^* + \rho} \sqrt{1 + (r^* + \rho)^2 \left(\frac{\sigma_e}{\sigma_y} \alpha_s \right)^2}, \tag{63}$$

where $\mathrm{sd}\,(\cdot)$ denotes standard deviation. Furthermore, the relative inequality of wealth to income is the same as that obtained in the benchmark model.

Proof. Using (56), the derivation is straightforward. ■

As shown above, it is clear from (59) and (60) that habit formation has the potential to drive up the equilibrium risk-free rate, and thus cannot help resolve the risk-free rate puzzle. If we increase the degree of risk aversion to generate the observed r^* (2.1%), it is clear from (63) that habit formation will further drive down the relative consumption inequality given that $\eta < 1$, which makes the model fit the data worse in this dimension. For example, given $\gamma = 50$, $\theta = 0.2$, and $\kappa = 0.2$, using the equilibrium conditions for the risk-free rate and equity premium, we can pin down $r^* = 2.1\%$ and $\pi^* = 12.7\%$, respectively. However, these two values are obtained at the cost of setting γ to be as high as 50, which is much higher than the plausible values adopted in most macro-finance literature. Furthermore, if $r^* = 2.1\%$ is reached, the presence of HF (i.e., $\eta < 1$) makes μ be reduced to 0.16, which is even lower than that obtained in the model without HF and is also well below the empirical counterpart.

5 Conclusion

We construct a general framework to study the implications of the WIU on aggregate savings, the equilibrium interest rate, consumption inequality, and equity premium—an unexplored area in the literature. The model includes many key factors in determining consumption, saving, and portfolio choices, but can still be solved analytically. We use the closed-form solutions to help understand how the WIU interacts with other key factors in driving the equilibrium risk-free rate, consumption inequality, and equity premium. We show a small degree of the preference for wealth can significantly improve the model's predictions on all three key dimensions. In addition, we compare the WIU with a closely-related hypothesis, habit formation in consumption, by examining their equilibrium implications for asset returns and consumption inequality, and find that the WIU is more consistent than the habit formation in explaining the low risk-free rate and high consumption inequality in the data.

6 Appendix

6.1 Solving the Consumption-Portfolio Choice Model with the WIU

Given the optimization problem:

$$V(J_t) = \max_{(c_t,\alpha_t)} \left\{ \left(1 - e^{-\beta \Delta t}\right) V(c_t, w_t) + e^{-\beta \Delta t} V\left(\mathbb{CE}_t\left[J_{t+\Delta t}\right]\right) \right\}, \tag{64}$$

subject to

$$dw_t = (rw_t + y_t - c_t)\, dt + \alpha_t \left(\pi dt + \sigma_e dB_{e,t}\right), \tag{65}$$

$$dy_t = (\mu - \rho y_t)\, dt + \sigma_y dB_t, \tag{66}$$

we first guess that $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t$, and write $J_{t+\Delta t}$ as:[30]

$$\begin{aligned} J_{t+\Delta t} &\equiv J\left(w_{t+\Delta t}, y_{t+\Delta t}\right) = -\alpha_0 - \alpha_1 w_{t+\Delta t} - \alpha_2 y_{t+\Delta t} \\ &\approx -\alpha_0 - \left[\alpha_1 w_t + \alpha_1 \left(rw_t + y_t - c_t + \alpha_t \pi\right) \Delta t + \alpha_1 \sigma_e \alpha_t \Delta B_{e,t}\right] \\ &\quad - \left[\alpha_2 y_t + \alpha_2 \left(\mu - \rho y_t\right) \Delta t + \alpha_2 \rho_{ye} \sigma_y \Delta B_{e,t} + \alpha_2 \sqrt{1 - \rho_{ye}^2} \sigma_y \Delta B_{i,t}\right]. \end{aligned}$$

Using the above expression for $J_{t+\Delta t}$ and assume that the time interval Δt goes to infinitesimal dt, we can evaluate the certainty equivalent of $J_{t+\Delta t}$ as follows:

$$\begin{aligned} &\exp\left(-\gamma \mathbb{CE}_t\right) = \mathbb{E}_t\left[\exp\left(-\gamma J_{t+\Delta t}\right)\right] \\ &= \exp\left(-\gamma \mathbb{E}_t\left[-\alpha_1 w_{t+dt} - \alpha_2 y_{t+dt}\right] + \frac{1}{2}\gamma^2 \operatorname{var}_t\left[-\alpha_1 w_{t+dt} - \alpha_2 y_{t+dt}\right] + \gamma \alpha_0\right) \\ &= \exp\left(\gamma \alpha_0 - \gamma \left(\partial J\right)^T \cdot \left(s_t + \mathbb{E}_t\left[ds_t\right]\right) + \frac{\gamma^2}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right] dt\right) \\ &= \exp\left(-\gamma J_t\right) \exp\left(-\gamma \left(\partial J\right)^T \cdot \mathbb{E}_t\left[ds_t\right] + \frac{\gamma^2}{2}\left[\left(\partial J\right)^T \cdot \Sigma \cdot \partial J\right] dt\right), \end{aligned}$$

[30] Here $\Delta B_t = \sqrt{\Delta t}\epsilon$ and ϵ is a standard normal distributed variable.

where $s_t = \left[\begin{array}{cc} w_t & y_t \end{array} \right]^T$, $ds_t = \left[\begin{array}{cc} dw_t & dy_t \end{array} \right]^T$, $\partial J = \left[\begin{array}{cc} J_w & J_y \end{array} \right]^T$, and $\Sigma = \left[\begin{array}{cc} \alpha_t^2 \sigma_e^2 & \rho_{ye}\sigma_y \alpha_t \sigma_e \\ \rho_{ye}\sigma_y \alpha_t \sigma_e & \sigma_y^2 \end{array} \right]$.
Solving this equation yields:

$$\mathbb{CE}_t \left[J_{t+dt} \right] = J_t + \left((\partial J)^T \cdot \mathbb{E}_t \left[ds_t \right] - \frac{\gamma}{2} \left[(\partial J)^T \cdot \Sigma \cdot \partial J \right] dt \right).$$

Substituting the expression of $\mathbb{CE}_t$ into (64), we can obtain the following HJB equation:

$$\beta V \left(J_t \right) = \max_{\{c_t, \alpha_t\}} \left\{ \beta V \left(c_t, w_t \right) + \mathcal{D}V \left(s_t \right) \right\}, \tag{67}$$

where

$$\begin{aligned} \mathcal{D}V \left(J_t \right) &= V' \left(J_t \right) \left((\partial J)^T \cdot \mathbb{E}_t \left[ds_t \right] + \upsilon_t^T \cdot \Sigma \cdot \partial J - \frac{\gamma}{2} \left[(\partial J)^T \cdot \Sigma \cdot \partial J \right] \right) \\ &= V' \left(J_t \right) \left[\begin{array}{c} -\alpha_1 \left(r w_t + y_t - c_t + \alpha_t \pi \right) - \alpha_2 \left(\mu - \rho y_t \right) \\ -\frac{\gamma}{2} \left(\alpha_1^2 \sigma_e^2 \alpha_t^2 + 2\rho_{ey} \alpha_1 \alpha_2 \sigma_e \alpha_t \sigma_y + \alpha_2^2 \sigma_y^2 \right) \end{array} \right] \end{aligned}$$

The first-order condition (FOC) for consumption (c_t) is then:

$$\beta V_c \left(c_t, w_t \right) + \alpha_1 V' \left(J_t \right) = 0.$$

Using the facts that $V \left(c_t, w_t \right) = (-\psi) \exp \left(- \left(c_t + b w_t \right) / \psi \right)$ and $J_t = -\alpha_0 - \alpha_1 w_t - \alpha_2 y_t$, we have:

$$c_t = -\psi \ln \left(-\frac{\alpha_1}{\beta} \right) - \alpha_0 - \left(\alpha_1 + b \right) w_t - \alpha_2 y_t. \tag{68}$$

The FOC for portfolio choice (α_t) is:

$$\alpha_t = -\frac{\pi}{\alpha_1 \gamma \sigma_e^2} - \frac{\alpha_2 \rho_{ye} \sigma_y}{\alpha_1 \sigma_e}. \tag{69}$$

Matching the coefficients on the w_t, y_t, and constant terms yields:

$$\begin{aligned} \alpha_1 &= -\left(r + b \right), \alpha_2 = -\frac{r+b}{\rho + r + b}, \\ \alpha_0 &= \left(1 - \frac{\beta}{r+b} \right) \psi - \psi \ln \left(\frac{r+b}{\beta} \right) - \frac{\mu}{r+b+\rho} \\ &+ \frac{\rho_{ye} \sigma_y \pi}{\left(r+b+\rho \right) \sigma_e} - \frac{\pi^2}{2 \left(r+b \right) \gamma \sigma_e^2} + \left(1 - \rho_{ey}^2 \right) \frac{\left(r+b \right) \gamma}{2} \left(\frac{\sigma_y}{r+b+\rho} \right)^2 \end{aligned}$$

Substituting the expressions of α_1 and α_2 into (69), we can obtain the following optimal portfolio rule:

$$\alpha^* = \frac{\pi}{\left(r+b \right) \gamma \sigma_e^2} - \frac{\rho_{ye} \sigma_y}{\left(r+b+\rho \right) \sigma_e}. \tag{70}$$

Similarly, substituting the expressions of α_0, α_1, and α_2 into (68), we can obtain the following consumption function:

$$
\begin{aligned}
c_t^* = {} & rw_t + \frac{r+b}{r+b+\rho} y_t + \frac{\mu}{r+b+\rho} - \frac{\rho_{ye}\sigma_y \pi}{(r+b+\rho)\,\sigma_e} \\
& - \left(1 - \frac{\beta}{r+b}\right)\psi - \left(1-\rho_{ey}^2\right)\frac{(r+b)\,\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2 + \frac{\pi^2}{2\,(r+b)\,\gamma\sigma_e^2},
\end{aligned}
$$

which is just the consumption function in the main text after we using the definitions of $\Psi(r) \equiv \left(1-\frac{\beta}{r+b}\right)\psi$, $\Gamma(r) \equiv \left(1-\rho_{ey}^2\right)\frac{(r+b)\gamma}{2}\left(\frac{\sigma_y}{r+b+\rho}\right)^2$, and $\Pi(r) \equiv \frac{\pi^2}{2(r+b)\gamma\sigma_e^2}$.

Using the above optimal consumption-portfolio rules, the individual saving function can be written as:

$$
\begin{aligned}
d_t^* &= rw_t + y_t - c_t^* + \pi\alpha^* \\
&= \frac{\rho}{r+b+\rho}\left(y_t - \frac{\mu}{\rho}\right) + \left(1-\frac{\beta}{r+b}\right)\psi + \frac{1}{2}\left(1-\rho_{ey}^2\right)(r+b)\,\gamma\left(\frac{\sigma_y}{r+b+\rho}\right)^2 + \frac{\pi^2}{2\,(r+b)\,\gamma\sigma_e^2} \\
&= x_t - \Psi + \Gamma + \Pi,
\end{aligned}
$$

where $x_t = \rho\,(y_t - \overline{y})\,/\,(r+b+\rho)$ and $\overline{y} = \mu/\rho$.

6.2 Proof of the Existence and Uniqueness of the Equilibrium

Proof. The general equilibrium can be obtained by setting the saving amount in the risk-free asset$D(r) = \Gamma(r) - \Psi(r) = 0$, where $\Gamma(r)$ and $\Psi(r)$ are defined in (15) and (14) in the main text. We will show that there exists at least one equilibrium interest rate $r^* \in (-b, \beta - b)$ in the our model; and if $\beta < \rho$, the equilibrium interest rate is unique on $(-b, \beta - b)$.

If $r > \beta - b$, both $\Gamma(r)$ and $-\Psi(r)$ in the expression for $D(r) = \Gamma(r) - \Psi(r)$ are positive, which contradicts the equilibrium condition $D(r) = 0$. Since $\Gamma(r) - \Psi(r) < 0$ (> 0) when $r = -b$ ($r = \beta - b$), the continuity of the expression for savings implies that there exists at least one interest rate $r^* \in (-b, \beta - b)$ such that $D(r^*) = 0$.

To establish the conditions under which this equilibrium is unique, we take the derivative:

$$
\frac{\partial D(r)}{\partial r} = \gamma \frac{\sigma_y^2}{(r+b+\rho)^2}\left(\frac{1}{2} - \frac{r+b}{r+b+\rho}\right) + \frac{\beta\psi}{(r+b)^2}
$$

and note a sufficient condition for this derivative to be positive for any $r > 0$ is

$$
\frac{1}{2} - \frac{r+b}{r+b+\rho} > 0 \Leftrightarrow r < \rho - b.
$$

Therefore, if $\rho > \beta$, there is only one equilibrium in $(-b, \beta - b)$. Note that for plausible values of ρ and β in our model economy, the equilibrium is unique. ■

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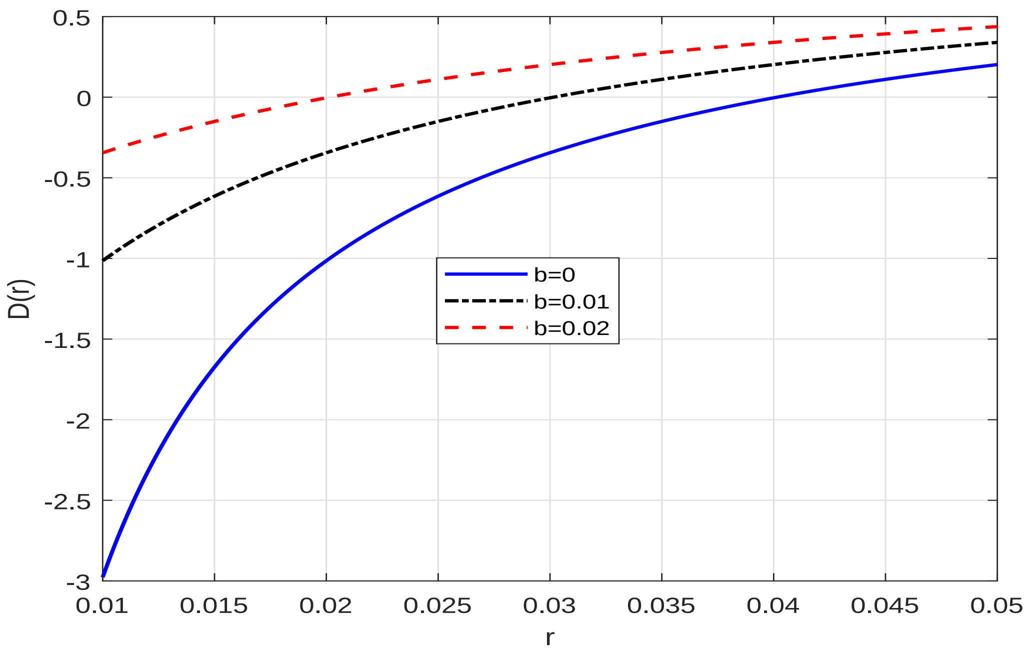

Figure 1: Effects of the WIU on Savings in the Risk-free Asset

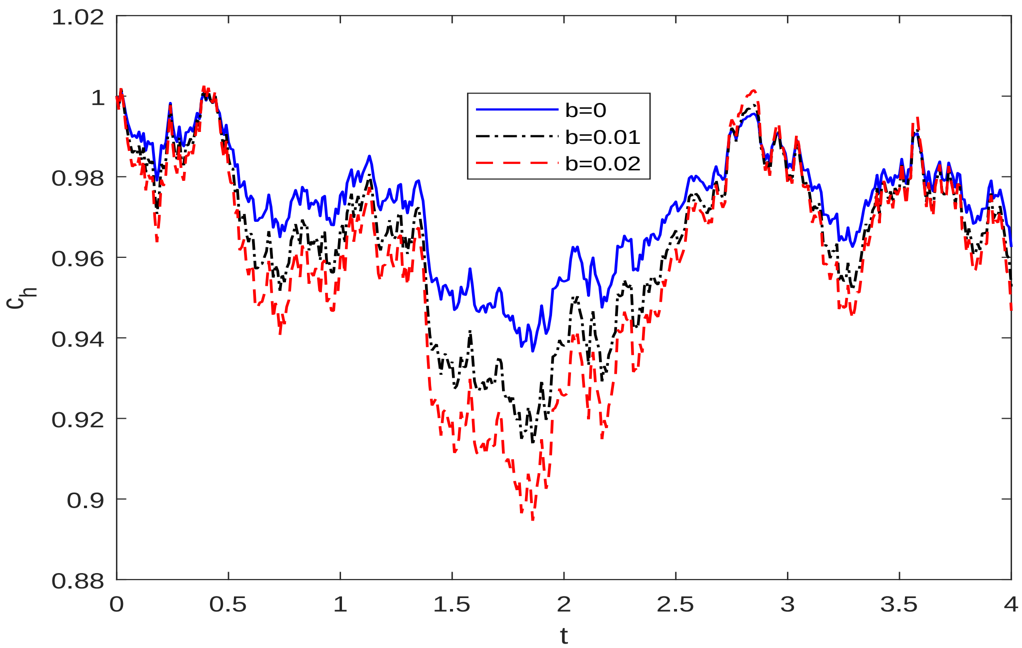

Figure 2: Effects of the WIU on the MPC out of Current Income

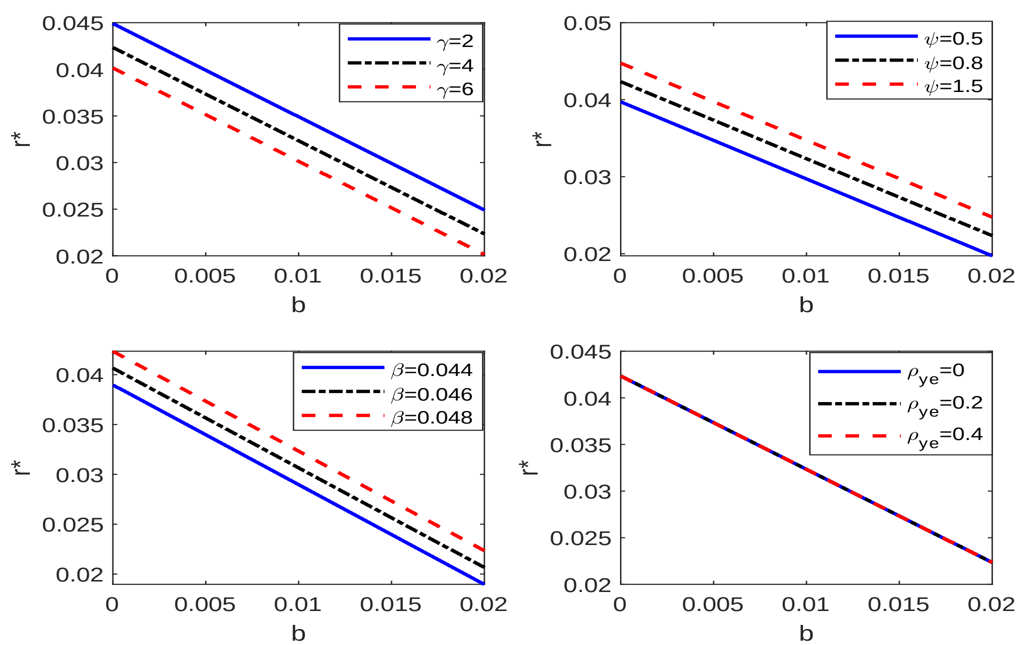

Figure 3: Effects of the WIU on the Equilibrium Risk-free Rate

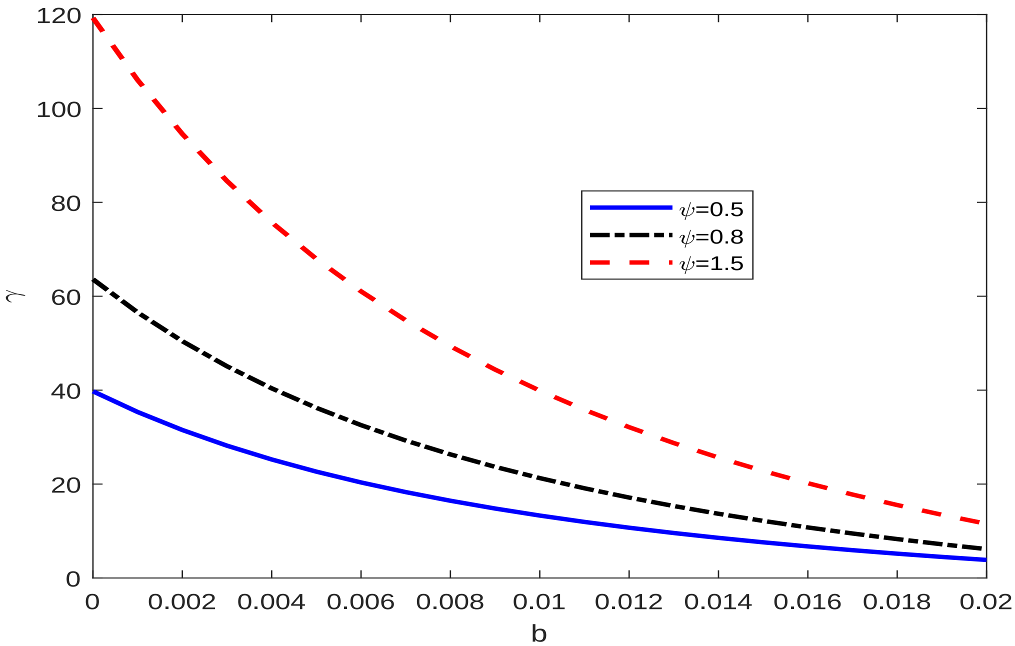

Figure 4: The Trade-off between b and γ

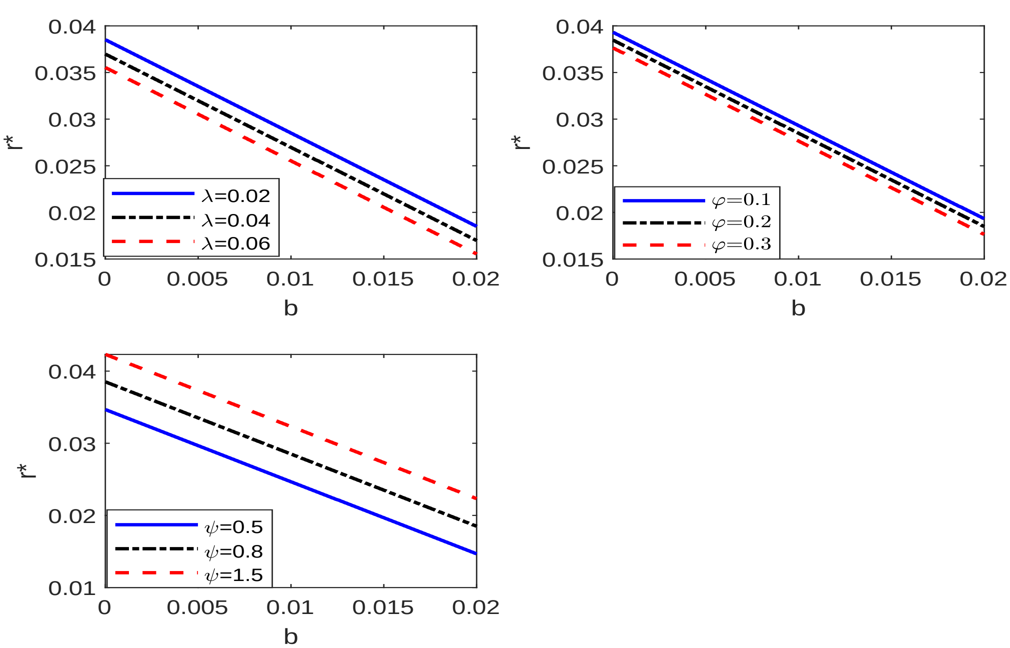

Figure 5: Effects of the WIU on the Equilibrium Risk-free Rate (with Rare Events)

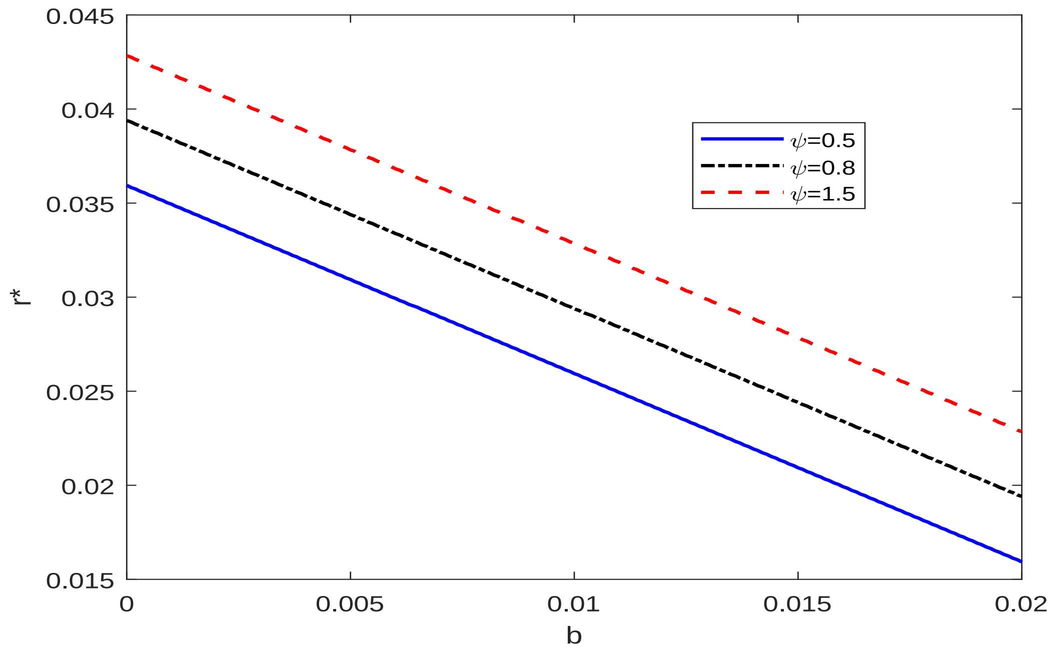

Figure 6: Effects of the WIU on the Equilibrium Risk-free Rate (2-Component Case)

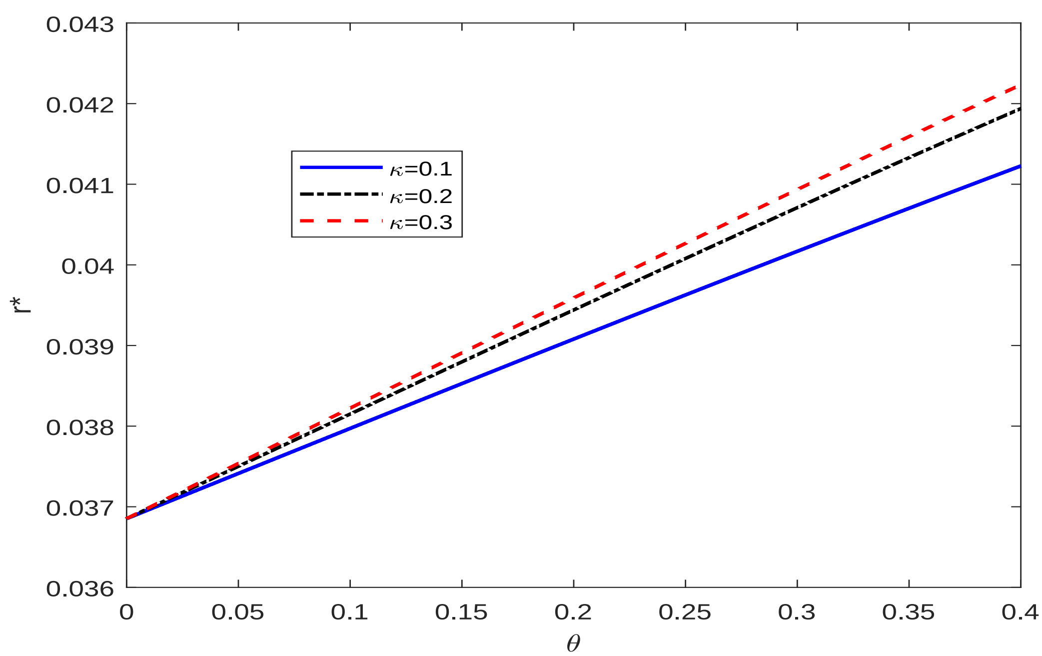

Figure 7: Effects of Habit Formation on the Equilibrium Risk-free Rate

Table 1: Estimation of Labor Income Parameters

	Parameter	Values
AR(1) Specification		
Discrete-time parameters		
Persistence	ϕ_1	0.903
Std. of shock	σ	0.154
Continuous-time parameters		
Persistence	ρ	0.102
Std. of income changes	σ_y	0.162
Two-Component Specification		
Discrete-time parameters		
Persistence	$\phi_{1,y}$	0.928
Persistence	$\phi_{2,y}$	0.361
Std. of shock	ω_1	0.133
Std. of shock	ω_2	0.194
Continuous-time parameters		
Persistence	ρ_1	0.075
Persistence	ρ_1	1.019
Std. of income changes	σ_1	0.138
Std. of income changes	σ_2	0.297

Table 2: Other Parameters

	Parameter	Values	Source
Benchmark Parameters			
Risk aversion	γ	6	Campbell (2003); Bansal and Yaron (2004)
Patience parameter	β	0.048	Campbell (2003); Bansal and Yaron (2004)
EIS parameter	ψ	0.8	Crump et al. (2015)
Correlation parameter	ρ_{ye}	0.35	Viceira (2001)
Std. of risky-asset return	σ_e	0.166	Data
Net supply of risky assets	α_s	5.03	Data
Rare-disaster Parameters			
Event probability	λ	0.037	Barro and Ursua (2011)
Event size	φ	0.21	Barro and Ursua (2011)
Habit Formation Model			
Degree of HF	ϑ	0.2	Constantinides (1990)
Smoothing parameter	κ	0.2	Constantinides (1990)

Table 3: Model's Quantitative Implications

	Real Risk-free Rate			Consumption Inequality		Wealth Inequality	Equity Premium	
Data	2.1%	2.1%	2.1%	0.40	0.40	5	5.8%	5.8%
Model	2.1%	2.1%	2.1%	0.22	0.40	5	4.3%	2.2%
b	0	0	0.02	0	0.02	0.02	0.02	0
γ	57	35	6	57	6	6	6	6
φ	0.8	0.5	0.8	0.8	0.8	0.8	0.8	0.8