

# Did the Federal Reserve Break the Phillips Curve? Theory & Evidence of Anchoring Inflation Expectations Technical Appendix\*

Brent Bundick<sup>†</sup>      A. Lee Smith<sup>‡</sup>

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<sup>†</sup>Federal Reserve Bank of Kansas City.      Email: [brent.bundick@kc.frb.org](mailto:brent.bundick@kc.frb.org)

<sup>‡</sup>Federal Reserve Bank of Kansas City.      Email: [andrew.smith@kc.frb.org](mailto:andrew.smith@kc.frb.org)

# A Derivation & Calibration of Theoretical Model

This section provides further details on the derivation and calibration of our theoretical model in Section 3.2 of the main text. Our model combines features from the previous works of Ireland (2007) and Leduc and Liu (2016). The key agents in our model are a representative household, a retail goods sector which produces differentiated products subject to nominal rigidities, an aggregation sector which aggregates the differentiated products into the final output, intermediate goods producers which hire labor in a frictional labor market, and a government which sets the short-term nominal interest rate and sets lump-sum taxes to finance unemployment benefits.

## A.1 Households

The model features a representative household populated by a continuum of worker members which maximize utility from consumption and leisure:

$$\max E_t \sum_{s=0}^{\infty} a_{t+s} \beta^s \left\{ \log(C_{t+s}) - \chi N_{t+s} \right\}$$

where  $C_t$  denotes consumption,  $N_t$  is the fraction of employed household members,  $\chi$  denotes the disutility from working,  $\beta$  is the household's discount factor, and  $a_t$  is an exogenous preference shock which triggers unexpected fluctuations in household demand. The representative household chooses its consumption and bond holdings to maximize its utility subject to its budget constraint each period:

$$C_t + \frac{B_t}{P_t R_t} = \frac{B_{t-1}}{P_t} + W_t N_t + \phi_u (1 - N_t) + D_t - T_t, \quad \forall t \geq 0,$$

where  $P_t$  denotes the aggregate price level,  $B_t$  denotes holdings of a nominal risk-free bond,  $R_t$  denotes the nominal interest rate,  $W_t$  denotes the real wage rate,  $\phi_u$  denotes an unemployment benefit (the replacement ratio),  $D_t$  denotes profit income from ownership of intermediate goods producers and of retailers, and  $T_t$  denotes a lump-sum tax paid to the government. The household's optimal choices of consumption and bond holdings satisfy the following first-order conditions:

$$\lambda_t = \frac{a_t}{C_t}, \tag{1}$$

$$1 = E_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{R_t}{\Pi_{t+1}} \right) \right\}, \tag{2}$$

where  $\Pi_t = P_t/P_{t-1}$  denotes the gross rate of inflation and  $\lambda_t$  denotes the nonnegative Lagrange multiplier on the household's budget constraint.

The discount factor of the household  $\beta$  is subject to shocks via the stochastic process  $a_t$ . We interpret these fluctuations as demand shocks since an increase in  $a_t$  induces households to consume more today for no technological reason. The stochastic process for these fluctuations is as follows:

$$\log(a_t) = \rho_a \log(a_{t-1}) + \sigma^a \varepsilon_t^a, \quad (3)$$

where  $\varepsilon_t^a$  is an independent and standard normal random variable.

## A.2 Aggregation Sector

The aggregation sector uses  $Y_t(i)$  units of each retail good produced by the retail goods-producing firm  $i \in [0, 1]$  to create the final output  $Y_t$  using the following constant returns to scale technology:

$$\left[ \int_0^1 Y_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} \geq Y_t,$$

where  $\eta > 1$  is the elasticity of substitution between differentiated products. Each intermediate good  $Y_t(i)$  sells at nominal price  $P_t(i)$  and each final good sells at nominal price  $P_t$ . The representative producer in the aggregation sector chooses  $Y_t$  and  $Y_t(i)$  for all  $i \in [0, 1]$  to maximize the following expression of firm profits:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di.$$

subject to the constant returns to scale production function. Optimization results in the following first-order condition:

$$Y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\eta} Y_t. \quad (4)$$

The market for final output is perfectly competitive, and thus the aggregation earns zero profits in equilibrium. Using the zero-profit condition, the first-order condition for profit maximization, and the objective function, the aggregate price index  $P_t$  can be written as follows:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \quad (5)$$

## A.3 Retail Goods-Producing Firms

There exists a continuum of retail goods-producing firms, each producing a differentiated product using a homogeneous intermediate good as input. The production function of a retail good of type  $i \in [0, 1]$  is given by

$$Y_t(j) = X_t(i), \quad (6)$$

where  $X_t(i)$  is the input of intermediate goods used by retailer  $j$  and  $Y_t(i)$  is the output. The retail goods producers are price takers in the input market and monopolistic competitors in the product markets, where they set prices for their products, taking as given the demand schedule in Equation (4) and the price index in Equation (5).

Firm  $i$  faces a quadratic cost to adjusting its nominal price  $P_t(i)$ :

$$\frac{\phi_P}{2} \left[ \frac{P_t(i)}{\Pi_t^{LT} P_{t-1}(i)} - 1 \right]^2 Y_t$$

where  $\phi_P$  governs the magnitude of the adjustment costs.  $\Pi_t^{LT} = \exp(\pi_t^{LT})$  is the gross rate of long-term inflation expectations which are determined by the following equation:

$$\pi_t^{LT} = \pi_{t-1}^{LT} + \delta^\pi (\pi_t - \pi_{t-1}^{LT}), \quad (7)$$

where  $\pi_t = \log(\Pi_t)$ . The coefficient  $\delta^\pi$  determines the degree to which long-term inflation expectations are anchored. In the extreme case, if  $\delta^\pi = 0$ , then long-term inflation expectations are fully anchored in the sense that they are invariant to realized inflation. On the other extreme, if  $\delta^\pi > 0$ , then inflation expectations are unanchored and drift with realized inflation.

Each retail firm producing good  $i$  chooses  $P_t(i)$  to maximize its discounted present-value of profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left( \frac{\beta^s \lambda_{t+s}}{\lambda_t} \right) \left[ \left( \frac{P_{t+s}(j)}{P_{t+s}} - q_{t+s} \right) Y_{t+i}(i) - \frac{\phi_P}{2} \left( \frac{P_{t+s}(i)}{\Pi_{t+s}^{LT} P_{t+s-1}(i)} - 1 \right)^2 Y_{t+s} \right], \quad (8)$$

where  $q_t$  denotes the relative price of the intermediate good. In a symmetric equilibrium with  $P_t(i) = P_t$  for all  $i$ , the optimal price-setting decision implies:

$$q_t = \frac{\eta - 1}{\eta} + \frac{\phi_P}{\eta} \left\{ \left( \frac{\Pi_t}{\Pi_t^{LT}} \right) \left( \frac{\Pi_t}{\Pi_t^{LT}} - 1 \right) - \mathbb{E}_t \left[ \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right) \left( \frac{\Pi_{t+1}}{\Pi_{t+1}^{LT}} \right) \left( \frac{\Pi_{t+1}}{\Pi_{t+1}^{LT}} - 1 \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] \right\}. \quad (9)$$

## A.4 The Labor Market

Our formulation of the labor market in our model closely follows [Leduc and Liu \(2016\)](#). At the beginning of each period, there exist  $N_{t-1}$  employed workers,  $u_t$  unemployed workers searching for jobs, and  $v_t$  vacancies posted by firms. Matches between unemployed workers and vacancies are created using a Cobb-Douglas matching function:

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (10)$$

where  $m_t$  is the number of successful matches, the parameter  $\alpha \in (0, 1)$  denotes the elasticity of job matches with respect to the number of searching workers, and the parameter  $\mu$  scales the matching efficiency. We define the job filling rate, the probability that an open vacancy is matched with a searching worker, as follows:

$$q_t^u = \frac{m_t}{v_t}. \quad (11)$$

We define the job finding rate, the probability that an unemployed and searching worker is matched with an open vacancy, as follows:

$$q_t^v = \frac{m_t}{u_t} \quad (12)$$

There exist  $N_{t-1}$  workers in the beginning of period  $t$ . Each period, a fraction  $\rho$  of these workers lose their jobs. Thus, the number of workers who survive the job separation is  $(1-\rho)N_{t-1}$ . At the same time,  $m_t$  new matches are formed. Following the timing assumption in Blanchard and Galí (2010), we assume that new hires start working in the period they are hired. Thus, aggregate employment in period  $t$  evolves according to

$$N_t = (1 - \rho)N_{t-1} + m_t. \quad (13)$$

With a fraction  $\rho$  of employed workers separated from their jobs, the number of unemployed workers searching for jobs in period  $t$  is given by

$$u_t = 1 - (1 - \rho)N_{t-1}. \quad (14)$$

We assume full participation and define the unemployment rate as the fraction of the population who are left without a job after hiring takes place. Thus, we can write the unemployment rate as follows:

$$U_t = u_t - m_t = 1 - N_t. \quad (15)$$

## A.5 Intermediate Goods Producers

Each intermediate goods firm produces a homogenous intermediate good and hires at most one worker subject to search and matching frictions in the labor market. Since our model abstracts from changes in productivity, each firm employs a single worker produces one unit of the intermediate good each period. If a firm finds a match, the firm obtains a flow profit in the current period after paying the worker. In the next period, the match may survive with probability probability  $1 - \rho$  or dissolve with probability  $\rho$ . If the match dissolves, the

firm posts a new job vacancy at a fixed cost  $\kappa$  units of the final good with the value  $V_{t+1}$ . Thus, the following Bellman equation captures the value of the firm:

$$J_t^F = q_t - W_t + \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( (1 - \rho) J_{t+1}^F + \rho V_{t+1} \right) \right\}. \quad (16)$$

where  $q_t$  denotes the relative price of the intermediate good,  $W_t$  denotes the real wage, and  $\lambda_t$  is the representative household's marginal utility from consumption.

$\kappa$  denotes the cost of posting a new vacancy in terms of final goods. The vacancy is filled with probability  $q_t^v$ , in which case the firm obtains the value of the match. Otherwise, the vacancy remains unfilled and the firm goes into the next period with the value  $V_{t+1}$ . Thus, the value of an open vacancy is given by

$$V_t = -\kappa + q_t^v J_t^F + \mathbb{E}_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} (1 - q_t^v) V_{t+1} \right]. \quad (17)$$

Free entry implies that  $V_t = 0$  which implies:

$$\frac{\kappa}{q_t^v} = J_t^F, \quad (18)$$

which describes the optimal job creation decisions. The benefit of creating a new job is the match value  $J_t^F$  while the expected cost of creating a new job is the flow cost of posting a vacancy  $\kappa$  multiplied by the expected duration of an unfilled vacancy  $1/q_t^v$ .

If a worker is employed, he obtains wage income but pays a utility cost of working. In period  $t + 1$ , the match is separated with probability  $\rho$  and the separated worker can find a new match with probability  $q_{t+1}^u$ . Thus, a separated worker failed to find a new job in period  $t + 1$  and enters the unemployment pool with probability  $\rho(1 - q_{t+1}^u)$ . Otherwise, the worker continues to be employed. The marginal value of an employed worker (denoted by  $J_t^W$ ) therefore satisfies the Bellman equation

$$J_t^W = W_t - \frac{\chi}{\lambda_t} + \mathbb{E}_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} \left[ (1 - \rho(1 - q_{t+1}^u)) J_{t+1}^W + \rho(1 - q_{t+1}^u) J_{t+1}^u \right] \right\}, \quad (19)$$

where  $J_t^U$  denotes the value of an unemployed worker. An unemployed worker obtains the flow unemployment benefit  $\phi_u$  and can find a new job in period  $t + 1$  with probability  $q_{t+1}^u$ . Thus, the value of an unemployed worker satisfies the Bellman equation

$$J_t^U = \phi_u + \mathbb{E}_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} \left[ q_{t+1}^u J_{t+1}^W + (1 - q_{t+1}^u) J_{t+1}^U \right] \right\}. \quad (20)$$

Firms and workers bargain over wages in which the parameter  $b$  determines the bargaining weight. Leduc and Liu (2016) derive the following expression for the Nash bargaining wage.

$$W_t^N = (1 - b) \left[ \frac{\chi}{\lambda_t} + \phi_u \right] + b \left\{ q_t Z_t + \beta(1 - \rho) \mathbb{E}_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}} \right] \right\}. \quad (21)$$

The Nash bargaining wage is a weighted average of the worker's reservation value and the firm's productive value of a job match. By forming a match, the worker incurs a utility cost of working a forgoes unemployment benefits. By employing a worker, the firm receives the marginal product from labor in the current period and saves the vacancy cost from the next period.

Following Hall (2005) and Blanchard and Galí (2010), we assume actual wages adjust slowly to changing economic conditions:

$$W_t = W_{t-1}^\gamma (W_t^N)^{1-\gamma} \quad (22)$$

where  $W_t^N$  is the wage under Nash bargaining and  $\gamma \in (0, 1)$  represents the degree of real wage rigidity.

### A.5.1 Monetary Policy

The central bank in the model sets its short-term nominal policy rate  $R_t$  to minimize fluctuations in inflation in deviation from its long-term expectations:

$$\log(R_t) = \log(R_{t-1}) + \phi_\pi \log(\Pi_t / \Pi_t^{LT}) \quad (23)$$

where  $\phi_\pi$  denotes the central banks response to inflation deviations and changes in output growth.

## A.6 Government Policy

The government finances transfer payments for unemployment benefits through lump-sum taxes. We assume that the government balances the budget in each period so that

$$\phi_u(1 - N_t) = T_t. \quad (24)$$

## A.7 Equilibrium

In equilibrium, the markets for final consumption goods, intermediate goods, and the zero net-supply bonds ( $B_t = 0$ ) all clear. Therefore, we can write the aggregate resource constraint:

$$C_t + \kappa V_t + \frac{\phi_P}{2} \left( \frac{\Pi_t}{\Pi_{t+1}^{LT}} - 1 \right)^2 Y_t = Y_t. \quad (25)$$

## A.8 Complete Model

We can write down the complete model as follows:

$$\lambda_t = \frac{a_t}{C_t}, \quad (26)$$

$$1 = \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{R_t}{\Pi_{t+1}} \right) \right\}, \quad (27)$$

$$q_t = \frac{\eta - 1}{\eta} + \frac{\phi_P}{\eta} \left\{ \left( \frac{\Pi_t}{\Pi_t^{LT}} \right) \left( \frac{\Pi_t}{\Pi_t^{LT}} - 1 \right) - \mathbb{E}_t \left[ \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right) \left( \frac{\Pi_{t+1}}{\Pi_{t+1}^{LT}} \right) \left( \frac{\Pi_{t+1}}{\Pi_{t+1}^{LT}} - 1 \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] \right\}, \quad (28)$$

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (29)$$

$$q_t^u = \frac{m_t}{v_t}, \quad (30)$$

$$q_t^v = \frac{m_t}{u_t}, \quad (31)$$

$$N_t = (1 - \rho)N_{t-1} + m_t, \quad (32)$$

$$u_t = 1 - (1 - \rho)N_{t-1}, \quad (33)$$

$$U_t = 1 - N_t, \quad (34)$$

$$Y_t = N_t, \quad (35)$$

$$\log(R_t) = \log(R_{t-1}) + \phi_\pi \log(\Pi_t / \Pi_t^{LT}) + \phi_y \log(Y_t / Y_{t-1}), \quad (36)$$

$$C_t + \kappa V_t + \frac{\phi_P}{2} \left( \frac{\Pi_t}{\Pi_{t+1}^{LT}} - 1 \right)^2 Y_t = Y_t, \quad (37)$$

$$J_t^F = q_t - W_t + \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( (1 - \rho)J_{t+1}^F + \rho V_{t+1} \right) \right\}, \quad (38)$$



$$\frac{\kappa}{q_t^v} = J_t^F, \quad (39)$$

$$W_t^N = (1 - b) \left[ \frac{\chi}{\lambda_t} + \phi_u \right] + b \left\{ q_t Z_t + \beta(1 - \rho) \mathbb{E}_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}} \right] \right\}, \quad (40)$$

$$W_t = W_{t-1}^\gamma (W_t^N)^{1-\gamma}, \quad (41)$$

$$\log(a_t) = \rho_a \log(a_{t-1}) + \sigma^a \varepsilon_t^a, \quad (42)$$

$$\log(\Pi_t^{LT}) = \log(\Pi_{t-1}^{LT}) + \delta^\pi \left( \log(\Pi_t) - \log(\Pi_{t-1}^{LT}) \right). \quad (43)$$

To keep track of the model's predictions for inflation and the nominal interest rate, we also include the growth rates of inflation and the nominal interest rate as equations in our model:

$$g_t^\pi = \Pi_t / \Pi_{t-1}, \quad (44)$$

$$g_t^r = R_t / R_{t-1}. \quad (45)$$

## A.9 Stationary Model

Due to our random-walk specification for long-term inflation expectations, the nominal variables in our model feature a unit root. To write our model in stationary form, we define the following variables:  $\tilde{\pi}_t = \Pi_t / \Pi_t^{LT}$ ,  $\tilde{\pi}_t^{LT} = \Pi_t^{LT} / \Pi_{t-1}^{LT}$ , and  $\tilde{r}_t = R_t / \Pi_t^{LT}$ . Then, we can re-write our model in terms of these stationary variables as follows:

$$\lambda_t = \frac{a_t}{C_t}, \quad (46)$$

$$1 = \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( \frac{\tilde{r}_t}{\tilde{\pi}_{t+1}} \right) \left( \frac{1}{\tilde{\pi}_{t+1}^{LT}} \right) \right\}, \quad (47)$$

$$q_t = \frac{\eta - 1}{\eta} + \frac{\phi_P}{\eta} \left\{ \left( \tilde{\pi}_t \right) \left( \tilde{\pi}_t - 1 \right) - \mathbb{E}_t \left[ \left( \frac{\beta \lambda_{t+1}}{\lambda_t} \right) \left( \tilde{\pi}_{t+1} \right) \left( \tilde{\pi}_{t+1} - 1 \right) \left( \frac{Y_{t+1}}{Y_t} \right) \right] \right\}, \quad (48)$$

$$m_t = \mu u_t^\alpha v_t^{1-\alpha}, \quad (49)$$

$$q_t^u = \frac{m_t}{v_t}, \quad (50)$$

$$q_t^v = \frac{m_t}{u_t}, \quad (51)$$

$$N_t = (1 - \rho)N_{t-1} + m_t, \quad (52)$$

$$u_t = 1 - (1 - \rho)N_{t-1}, \quad (53)$$

$$U_t = 1 - N_t, \quad (54)$$

$$Y_t = N_t, \quad (55)$$

$$\log(\tilde{r}_t) = \log(\tilde{r}_{t-1}) + \phi_\pi \log(\tilde{\pi}_t) + \phi_y \log(Y_t/Y_{t-1}) - \log(\tilde{\pi}_t^{LT}), \quad (56)$$

$$C_t + \kappa V_t + \frac{\phi_P}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 Y_t = Y_t, \quad (57)$$

$$J_t^F = q_t - W_t + \mathbb{E}_t \left\{ \left( \beta \frac{\lambda_{t+1}}{\lambda_t} \right) \left( (1 - \rho)J_{t+1}^F + \rho V_{t+1} \right) \right\}, \quad (58)$$

$$\frac{\kappa}{q_t^v} = J_t^F, \quad (59)$$

$$W_t^N = (1 - b) \left[ \frac{\chi}{\lambda_t} + \phi \right] + b \left\{ q_t Z_t + \beta(1 - \rho) \mathbb{E}_t \left[ \frac{\beta \lambda_{t+1}}{\lambda_t} \frac{\kappa v_{t+1}}{u_{t+1}} \right] \right\}, \quad (60)$$

$$W_t = W_{t-1}^\gamma (W_t^N)^{1-\gamma}, \quad (61)$$

$$\log(a_t) = \rho_a \log(a_{t-1}) + \sigma^a \varepsilon_t^a, \quad (62)$$

$$\log(\tilde{\pi}_t^{LT}) = \delta^\pi \left( \log(\tilde{\pi}_t) - \log(\tilde{\pi}_t^{LT}) \right). \quad (63)$$

$$g_t^\pi = (\tilde{\pi}_t / \tilde{\pi}_{t-1}) \tilde{\pi}_t^{LT}, \quad (64)$$

$$g_t^r = (\tilde{r}_t / \tilde{r}_{t-1}) \tilde{\pi}_t^{LT}. \quad (65)$$

## A.10 Calibration and Solution Method

After writing the model in stationary form, we calibrate the parameters of the model and solve the model using a first-order approximation around the deterministic steady state. Table A.1 contains the calibrated parameters of our model. Since our model combines the frictional market specification of Leduc and Liu (2016) with the estimated macroeconomic model of Ireland (2007), we almost exclusively use the parameters from those papers in calibrating our model. Following Blanchard and Galí (2010), we set the matching elasticity parameter  $\alpha$  and the wage bargaining parameter  $b$  equal to 0.5. Our calibration of job separations  $\rho = 0.1$  implies a monthly job separation rate of roughly 3.5%. Consistent with calibration of Hall and Milgrom (2008), the replacement ratio of unemployment is set such that  $\phi = 0.25$ . The remaining labor market parameters are calibrated using the strategy in Section 4.1 of Leduc and Liu (2016) which implies a steady-state unemployment rate of  $U = 0.064$  and a total cost of posting vacancies at roughly 2 percent of gross output. The real wage rigidity parameter  $\gamma = 0.8$ , which is in line with Gertler and Trigari (2009).

With a couple of exceptions, the remaining parameters are calibrated match the calibrated or estimated values in Ireland (2007). We set the household discount factor  $\beta = 0.9995$  and the elasticity of substitution across intermediate goods  $\eta$  equal to 6. For the persistence of the preference shock process, we calibrate  $\rho_a = 0.9097$  to the estimated value of Ireland (2007). We set the volatility of the preference shock process  $\sigma_a = 0.01$  such that a one standard deviation shock moves the demand shock process by one percent. For the monetary policy rule, Ireland’s assumption of full interest rate smoothing (a coefficient of one on lagged interest rates in the policy rule), allows us to write the policy rule in a stationary form. We calibrate the policy response of inflation deviations  $\phi_\pi = 0.8594$ , the estimated value of Ireland (2007). With respect to the policy response to changes in the real economy, Ireland (2007) includes a nontrivial response of policymakers to changes in output growth. However, we find that including a response to output growth generates much larger (and likely counterfactual) fluctuations in inflation when we incorporate frictions in the labor market, so we remove this feature to generate more sensible inflation dynamics.

For the remaining parameters of the model ( $\delta^\pi$  and  $\phi_P$ ), we calibrate them based on our empirical evidence in Tables 1 and 5 of the main text which we discuss in Section 4.2.

## B Additional High-Frequency Empirical Exercises

### B.1 Break in $\delta^\pi$ , Not Change in Nature of CPI Surprises

We now show that the break in the estimate of  $\delta^\pi$  we document appears to reflect a change in the reaction of inflation expectations to CPI surprises rather than a change in the nature of CPI surprises. Table B.1 shows the summary statistics for CPI surprises both before and after 2012. The standard deviation of the Bloomberg core inflation surprises is equal to 0.07 prior to 2012 and 0.06 thereafter. Furthermore, the surprises in both samples are not significantly skewed nor do we find evidence that they are non-normal as the Jarque-Bera statistic falls below its critical value. The most notable difference between the two samples is the presence of an average downside inflation surprise after 2012. From the viewpoint of our regression model, this change in the distribution of inflation surprises has the potential to impact the intercept  $\delta^0$ , but not the slope coefficient  $\delta^\pi$ . However, Table 1 of the main text shows that we find no statistically significant evidence of a change in the regression intercept across the two sample periods.

### B.2 Robustness to Dropping the Global Financial Crisis

In Section 4.1, we discuss that, if we drop the precipice of the global financial crisis, we find evidence indicating the presence of a single structural break in early 2010. For our baseline inflation compensation model, Table B.2 shows that we estimate the exact same break date of May 2010 for the core inflation coefficient if we drop the fourth quarter of 2008 and first quarter of 2009 from the estimation. The blue dotted lines in Panel's A and B of Figure 3 of the main text plot the time series of the Chow statistics for samples that exclude the financial crisis. For both the inflation compensation and forward rate models, the presence of a peak in the time series of the break statistics in 2010 is insensitive to the inclusion or exclusion of the financial crisis. After excluding the precipice of the financial crisis, Table B.3 shows that the estimated break date for the forward rate model is February of 2010 and that break is estimated to be statistically significant using the Andrews-Quandt test and the Andrews-Ploberger test. This finding suggests that the source of instability in the response of forward bond yields to inflation surprises occurring around 2010 is not simply a reflection of financial market volatility but, instead, is likely due to deeper structural change.

## C SVAR Model of Household Inflation Expectations

In this section, we provide further details and results from our VAR model of household's inflation expectations that is presented in Section 4.2 of the main text.

### C.1 Data and VAR Model

We analyze impulse responses from a four variable structural VAR model to detect a potential change in the degree to which household's longer-term inflation expectations are anchored since 2012. We include m/m annualized CPI energy inflation and m/m annualized CPI food inflation in addition to m/m annualized overall CPI inflation to account for the documented fact that some consumer prices, such as gasoline and grocery prices, are more important than others in shaping household inflation expectations (Coibion and Gorodnichenko, 2015; Cavallo, Cruces and Perez-Truglia, 2017). We add to these CPI inflation measures the median 5- to 10-year inflation expectation from the University of Michigan's Survey of Consumers.

We model these series as a VAR(3) over two distinct samples: January 1999 - December 2011 and January 2012 - December 2019 on the basis of lag-selection criteria which recommend 1 or 2 lags depending on the sample and the criteria. The start date of the recent sample period, January 2012 - December 2019, is dictated by the Federal Reserve's January 2012 adoption of an inflation target. It is precisely this change in FOMC communication that we wish to analyze. The start date of the early sample period, January 1999 - December 2011, is selected to align with the regression samples used for other specifications in the main text. Finally, to make the results comparable across samples, we scale the shocks to have the same impact effect on the driving variable (i.e. energy inflation, food inflation, or overall inflation) in both samples.

We conduct inference on the estimated VAR models from a Bayesian perspective. In particular, error bands are calculated by assuming a non-informative conjugate (Normal-Inverse Wishart) prior over the VAR lag coefficients and covariance matrix. Our exact implementation follows Koop and Korobilis (2010) closely. 68% error bands are calculated based on 10,000 draws from the posterior distribution of the identified impulse responses.

### C.2 Shock Identification

The timing of CPI data releases relative to when the household survey data are collected naturally lends itself to identification by zero short-run restrictions. In particular, we use a

recursive identification strategy with the following ordering: CPI energy inflation, CPI food inflation, the median of households' expectations for inflation over the next 5 to 10 years, and CPI inflation. This ordering allows us to identify 3 inflation shocks: an energy inflation shock, a food inflation shock, and a core inflation shock. Our recursive identification strategy aims to identify these shocks based on the following sequence of surveys and data releases.

We assume that households are aware of energy and food inflation in the current month when they respond to the University of Michigan survey. After all, the typical household frequents gasoline stations, grocery stores, and restaurants at least once a week. Therefore, our shock identification allows for inflation in salient goods, namely gasoline and food, to influence their inflation expectations within the current month. This argues for ordering energy and food inflation ahead of households' expectations for inflation over the next 5 to 10 years in our recursively identified VAR model. We distinguish energy from food inflation shocks by ordering energy inflation ahead of food inflation. Our assumption is that an exogenous increase in energy prices can spillover to food prices within the month, perhaps through transportation and delivery costs. Conversely, we assume that an exogenous increase in food inflation can't spillover to energy inflation within the month.

Finally, we identify a core inflation shock on the basis of the timing of the consumer interviews conducted by the University of Michigan. Every month, the University of Michigan calls more than 500 households to conduct interviews. These interviews are conducted beginning either late in the previous month or early in the current month. Importantly, the final interviews are always completed before the end of the current month. So, in March for example, the first interview is conducted on February 26 and the last interview is completed by March 24. Importantly for our identification strategy, the CPI report for the reference month is always released the following month. For example, the March CPI report is released sometime in early April. Therefore, the assumption we make is that households' long-term inflation expectations can't respond within the month to the CPI release. After all, even a consumer that is eagerly awaiting the latest BLS report on price inflation in the current month won't be able to acquire this information until the following month. This argues for ordering CPI inflation after households' expectations for inflation over the next 5 to 10 years in a recursively identified VAR model.<sup>1</sup> Since we order energy and food inflation ahead of inflation expectations and CPI inflation, we interpret the orthogonalized innovation to CPI inflation as a core inflation shock.

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<sup>1</sup>Leduc, Sill and Stark (2007); Clark and Davig (2011); Leduc and Sill (2013) use a similar justification for their recursive identification schemes.

### C.3 Additional Results

Figure 6 in the main text shows a meaningful change in the impulse responses of longer-term inflation expectations in response to each of the three shocks we identify. In particular, these impulse responses suggest that energy inflation, food inflation, nor core inflation surprises no longer feed through to household's longer-term inflation expectations after in the 2012-2019 sample period.

In this section, we show that, in contrast to longer-term inflation expectations, household's near-term inflation expectations continued to respond to these inflation surprises after 2012. In particular, we now repeat the above analysis but replace households' expectations for inflation over the next 5 to 10 years with households' expectations for inflation over the next 1 year. In Figure C.1, we observe statistically significant revisions of household's 1-year inflation expectations in response to both energy and food inflation shocks.

These results serve two purposes. First, they show that anchoring longer-term inflation expectations need not eliminate fluctuations in near-term inflation expectations. Recall that, in response to energy inflation shocks, which Coibion and Gorodnichenko (2015) demonstrate as a primary driver of household inflation expectations, longer-term inflation expectations show little response after 2012. Therefore, these results help to further reconcile our conclusions with those in Coibion and Gorodnichenko (2015) since they illustrate the disparate behavior of households' near-term and longer-term inflation expectations in response to salient price changes in the post-2012 sample. Second, these results demonstrate that the nature of inflation shocks didn't fundamentally change over the two sample. In particular, households continued to revise their near-term inflation expectations after 2012 in much the same way they did prior to 2012. Instead, what changed, is the degree to which these revisions in near-term inflation expectations led to corresponding revisions in their longer-term inflation expectations.

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Table A.1: Calibrated Model Parameters

Parameter	Description	Value	Source
$\beta$	Household Discount Factor	0.9995	Ireland (2007)
$\chi$	Disutility of Working Scalar	0.476	Leduc and Liu (2016)
$\theta$	Elasticity of Substitution Intermediates	6.0	Ireland (2003)
$\alpha$	Share Parameter in Matching Function	0.5	Blanchard and Galí (2010)
$\mu$	Matching Efficiency	0.645	Leduc and Liu (2016)
$\rho$	Job Separation Rate	0.1	Monthly Separation Rate of 3.5%
$\phi_u$	Flow Benefit of Unemployment	0.25	Hall and Milgrom (2008)
$\kappa$	Vacancy Cost	0.14	Leduc and Liu (2016)
$b$	Nash Bargaining Parameter	0.5	Blanchard and Galí (2010)
$\gamma$	Real Wage Rigidity	0.8	Gertler and Trigari (2009)
$\phi_P$	Cost of Adjusting Nominal Prices	280	Calibrated to Match Phillips Curve
$\phi_\pi$	Central Bank Response to Inflation	0.8594	Ireland (2007)
$\rho_a$	Preference Shock Persistence	0.9097	Ireland (2007)
$\sigma_a$	Preference Shock Volatility	0.01	Implies 1% Demand Shock

Table B.1: Summary Statistics of US Core CPI Inflation Surprises

	1997-2011	2012-2019
Mean	0.00 [0.63]	-0.02 [0.01]
Standard Deviation	0.07	0.06
Skewness	0.16 [0.39]	-0.23 [0.37]
Kurtosis	-0.33 [0.38]	0.21 [0.69]
Jarque-Bera	1.58 [0.45]	1.00 [0.61]
Observations	179	96

Note: p-values in brackets.

Table B.2: Structural Break Tests at an Unknown Date: US Inflation Compensation Model Excluding Financial Crisis

$\Delta$ 1-Year, 9-Year Fwd Breakeven Inflation	Structural Break Test		
	Break Date	Andrews-Quandt Test Statistic	Andrews-Ploberger Test Statistic
Constant	2003:05	1.33 [0.95]	0.11 [1.00]
Core CPI surprise	2010:05	9.61** [0.03]	2.40** [0.03]
Food & Energy CPI surprise	2011:08	3.40 [0.48]	0.45 [0.48]
All Coefficients	2010:05	12.46* [0.09]	3.61* [0.09]
Residual Variance	2003:04	3.28 [0.50]	1.06 [0.18]

Note: Approximate asymptotic p-values from Hansen (1997) in brackets.

Observations: 245. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$

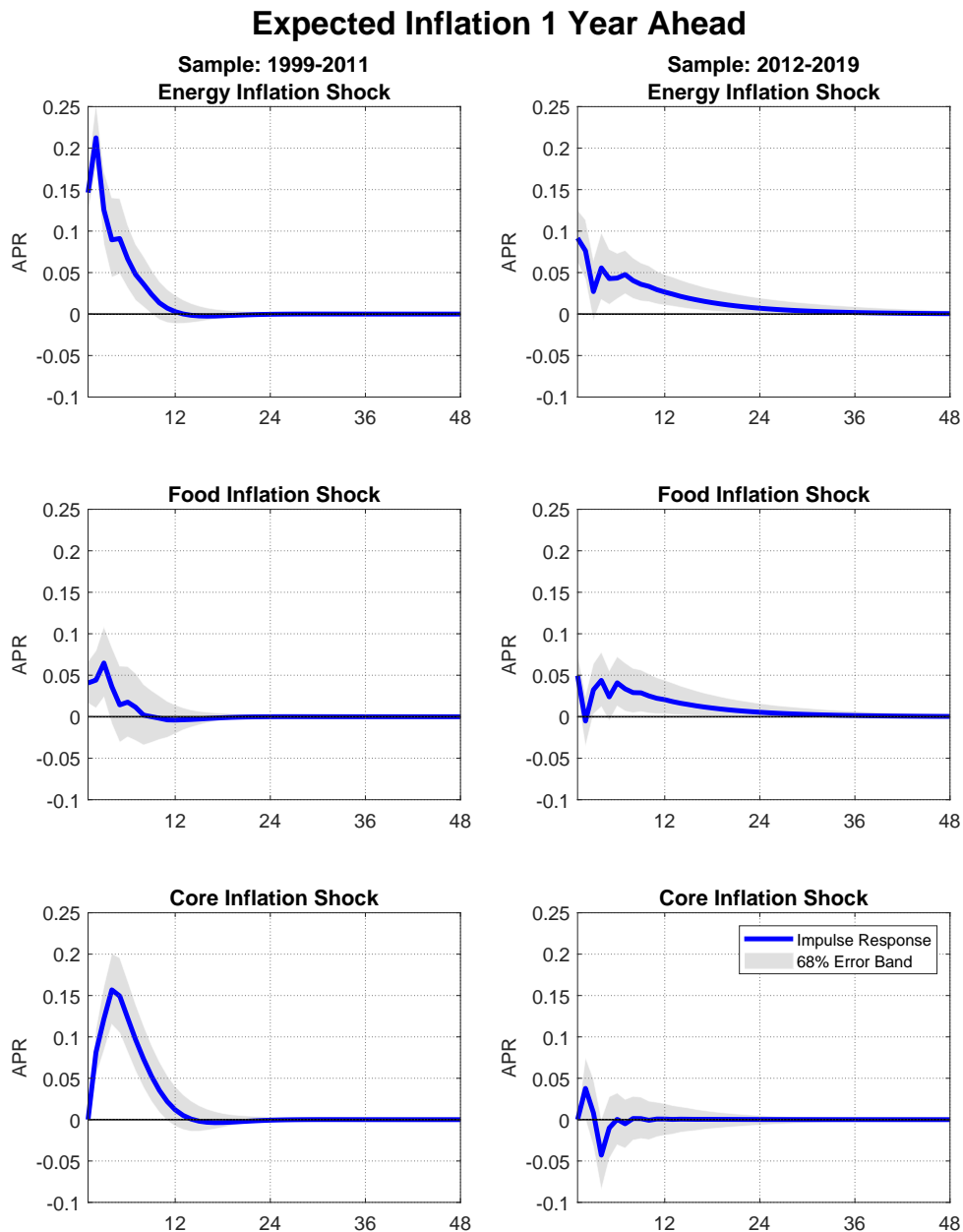
Table B.3: Structural Break Tests at an Unknown Date: US Forward Rate Model Excluding Financial Crisis

$\Delta$ 1-Year, 9-Year Fwd Nominal Rate	Structural Break Test		
	Break Date	Andrews-Quandt Test Statistic	Andrews-Ploberger Test Statistic
Constant	2013:11	2.81 [0.60]	0.53 [0.42]
Core CPI surprise	2010:02	8.45* [0.05]	1.96* [0.06]
GS Agriculture Price Index	2003:09	5.20 [0.23]	1.26 [0.14]
GS Energy Price Index	2011:05	4.19 [0.35]	0.63 [0.35]
All Coefficients	2010:02	12.96 [0.16]	3.96 [0.14]
Residual Variance	2013:11	2.66 [0.63]	0.67 [0.33]

Note: Approximate asymptotic p-values from Hansen (1997) in brackets.

Observations: 268. \* $p < 0.10$ , \*\* $p < 0.05$ , \*\*\* $p < 0.01$

Figure C.1: VAR Impulse Responses of Household's Near-Term Inflation Expectations



Note: The figure shows VAR-estimated impulse responses of household's near-term inflation expectations in response to various inflationary impulses. The impulse responses in the left column are estimated from 1999-2011. The impulse responses in the right column are estimated from 2012-2019. Each VAR model is estimated on monthly data comprised of CPI energy inflation, CPI food inflation, near-term inflation expectations collected from the University of Michigan Survey of Consumers, and CPI inflation. The CPI inflation series enter the VAR in month-over-month inflation rates, annualized.