Macroprudential Policy Interlinkages

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Abstract

Emerging markets are concerned about sudden stops in international capital flows, which may lead to severe recessions associated with vicious spirals of currency depreciations and tightening borrowing constraints. A common prescription is to impose macroprudential policies, including prudential capital controls, to limit international borrowing especially in foreign currency. This paper analyzes the supportive role of macroprudential policies geared toward the domestic financial market, suggesting that emerging markets should resort to a wide mix of policies, even when the domestic financial market is frictionless. A simple formula provides further insights: domestic and international macroprudential policies are imperfect complements rather than substitutes, due to distinctive characteristics of foreign and domestic currency bonds. Furthermore, the relative importance of domestic macroprudential regulation increases in the return of domestic bonds.

Keywords: Macroprudential Policies; Capital Controls; Emerging Markets; Welfare

JEL: F34, F41, E44, D62

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Emerging markets and developing economies are exposed to large fluctuations in foreign financial flows. Particularly sudden stops in international capital have been identified as a threat for financial and macroeconomic stability (Forbes and Warnock, 2012). Because emerging market crises are typically associated with currency depreciation, liabilities held in foreign currency (“liability dollarization”) can amplify risks. The aforementioned issues sparked interest in so called macroprudential policies. These policies are implemented ex-ante and are supposed to limit the build-up of systemic risk in the financial sector. In the context of emerging markets and developing economies (short: EMs), a large literature justifies prudential capital controls and international macroprudential policies to regulate the inflow of foreign currency debt. However, little is known on how EMs should regulate domestic debt in the presence of international financial cycles. Further, how do domestic and international regulatory initiatives interact? The purpose of this paper is to provide an integrated framework tailored to EMs that facilities an analysis of capital controls, international macroprudential policies and domestic macroprudential policies.

Figure 1 adds empirical significance to this endeavour. Panel (a) portrays the prevalence of domestic and international macroprudential policies, as well as capital controls in EMs. Capital controls are defined as general restrictions on international transactions. International macroprudential policies specifically refer to restrictions on foreign currency borrowings from either the domestic or international financial market. Domestic macroprudential policies represent rules that do not specifically target international flows or foreign currency loans. As apparent, domestic and international macroprudential policies have been steadily on the rise over the last two decades as more and more countries implement these policies. The usage of capital controls in contrast has been relative stable at an elevated level. Panel (b) highlights that an increasing share of EMs implement two (red area) or all three (blue area) of the aforementioned policies at the same time, emphasizing the urgency to provide a coherent unified framework.

The first contribution is a normative justification to regulate the domestic financial market in addition to policies geared towards international transactions, even when domestic markets are frictionless. The intuition is as follows: Regulating domestic

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1Galati and Moessner (2018) summarize recent research on macroprudential policies.

2See Rebucci and Ma (2019) and Erten et al. (2021) for surveys on the capital control literature. The IMF updated its negative view and now proposes capital controls under certain circumstances (Ostry et al., 2010; Ostry et al., 2011; Ostry et al., 2012).
currency bonds, traded in the domestic market, provides a margin to reallocate resources towards borrowers when they are limited in their ability to borrow internationally. This appreciates the exchange rate, since constrained borrowers have a higher marginal propensity to consume relative to domestic savers. Because available collateral increases with an appreciated exchange rate, sudden stops in international capital associated with a binding collateral constraint are muted.

The second contribution is a tractable formula that highlights the appropriate macroprudential regulatory mix. Two results stand out: First, domestic regulation increases with international regulation to prevent undesired portfolio adjustments as a consequence of the imperfect substitutability between domestic and foreign currency debt. Second, domestic regulatory efforts are more relevant when domestic assets have a high return, as more profitable investment opportunities support the reallocation of wealth over time. Because macroprudential policies by design reallocate resources toward crisis periods, regulators have an incentive to tax investments with a higher return.

**Figure 1:** Financial Regulation in Emerging Markets and Developing Economies

(a) Prevalence over Time

(b) Extent of Joint Implementation

Notes: Panel (a): The graph displays the prevalence of macroprudential policies and capital controls in emerging markets and developing economies. The vertical axis represents the implementation of each policy tool averaged across countries. Each index is normalized to range between [0,1], where a value of 1 would correspond to a hypothetical scenario in which all countries implemented the full range of each tool. Capital controls are defined as general restrictions on international transactions. Capital controls are averaged over various asset classes and inflow/outflow restrictions, but do not specifically target foreign currency borrowings. International macroprudential policies specifically refer to restrictions on foreign currency borrowings in either the domestic or international market. Domestic macroprudential policies represent rules that do not specifically target international flows or foreign currency loans. Panel (b): The chart presents the share of EMs that implement 0, 1, 2, 3 of the aforementioned different regulatory policies at the same time. Restrictions for each category must be above the 25% percentile across all countries and years, else a policy is considered “not implemented”. Data Sources: Cerutti et al. (2017) for macroprudential polices (domestic and international), Fernández et al. (2016) for capital controls. A list of included countries is available in Table A1 in the appendix.
I explore the interaction between macroprudential regulatory policies and capital controls by means of a stylized three-period small open economy model. The model consists of three agents: Domestic savers, domestic borrowers and risk-neutral international investors. Domestic households issue (purchase) bonds denominated in local or foreign currency. International investors only borrow (lend) in foreign currency. The model features an occasionally binding collateral constraint on foreign currency borrowings in the intermediate period. Access to domestic debt in contrast is unrestricted, subject to available resources.

The occasionally binding constraint on foreign currency debt is meant to capture sudden stops in international financing. The collateral is tied to available income expressed in foreign currency and therefore improves in the exchange rate. A depreciation consequently reduces the value of the collateral, which triggers a feedback loop of tightening constraints, capital outflows and further exchange rate depreciations. Domestic agents however do not internalize the dependency between investment decisions, the real exchange rate and hence the feedback loop, which introduces a pecuniary externality. This externality makes the competitive equilibrium inefficient and domestic agents overborrow.

The novel feature of this model is a frictionless domestic financial market in which agents trade distinctive domestic bonds. Households have hence access to two bonds to smooth consumption: An international foreign currency bond that is exchanged with foreign investors and a domestic currency bond which is exclusively traded between domestic borrowers and savers. Households are free to exchange domestic and foreign funds. Importantly, the pecuniary externality justifies domestic macroprudential policy: it is optimal to regulate the domestic financial market, in addition to capital controls and international macroprudential policies.

As is well known in the literature, a social planner would improve the external wealth position of domestic households via prudential capital controls (see, for example, Bianchi, 2011; Korinek, 2018). Capital controls are based on the residency principle. As such they drive a wedge ($\tau_{CC}$ in Figure 2, Panel (a)) between home and foreign investors. The idea is to impose inflow restrictions (subsidize outflows) to improve the external wealth position during benign times in order to limit the exposure to sudden stops during crisis periods. However, capital controls apply to all domestic agents equivalently, hence they cannot address the specific needs of heterogeneous agents in the economy. They are hence less targeted than macroprudential policies.

Only macroprudential policies are able to directly target vulnerable agents, that is, borrowers that are highly leveraged. Macroprudential policies can have a domestic or
international scope. International macroprudential policies specifically address debt instruments in foreign currency which, in this framework, relates to foreign currency borrowing between domestic borrowers and international savers ($\tau_{IM}$ in Figure 2, Panel (b)). As such, international macroprudential policies specifically improve the external net worth position of borrowers, who suffer the most during a recession. However unlike capital controls, they fail to internalize the positive general equilibrium effect of savers’ wealth on the severity of a recession. Due to this complementarity, it is generally desirable to implement both policies when dealing with international sudden stop dynamics (Korinek and Sandri, 2016).

The new insight in this paper pertains to purely domestic macroprudential policies. Domestic macroprudential policies affect transactions between savers and borrowers, however they do not specifically target foreign borrowings. In this paper, such policies refer to restrictions on domestic currency bonds traded between savers and borrowers at home ($\tau_{DP}$ in Figure 2, Panel (b)). On aggregate, domestic borrowing equals domestic savings. By borrowing less during good times, borrowers have more available resources for consumption during recessions when they have limited access to international funding. Savers reduce their consumption during recessions, but to a lesser extent as they are able to smooth consumption with savings. Therefore, domestic macroprudential policies, albeit redistributive, increase aggregate consumption and appreciate the exchange rate, which in turn relaxes the collateral constraint that borrowers face. The crucial point here is that, because of these general equilibrium implications, it is optimal to intervene in domestic markets, even when domestic financial markets are frictionless.

Related Literature: This paper relates to several strands of literature. First, the paper belongs to the literature on pecuniary externalities that emerge from endogenous prices in collateral constraints. In closed economy models, the collateral constraint usually depends on asset prices (see, for example, Bianchi and Mendoza, 2018; Jeanne and Korinek, 2019). I follow the small open economy literature (see, for example, Bianchi, 2011; Benigno et al., 2013; Korinek, 2018; Bengui and Bianchi, 2018) and introduce a constraint that depends on current income and hence the exchange rate. Relative to these papers, I specify a portfolio allocation problem in which households choose between domestic and foreign assets. This facilitates a joint study of prudential policies geared towards the international and domestic market.3

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3The theoretical literature is accompanied by a growing applied literature which generally documents desirable effects of macroprudential policies, and to a lesser extent of capital controls, on measures of financial vulnerability (see, for example, Forbes et al., 2015; Bruno et al., 2017; Cerutti et al., 2017; Ben Zeev, 2017; Ahnert et al., 2018; Akinci and Olmstead-Rumsey, 2018; Cesa-Bianchi et al., 2018; Ma et al., 2020).
Two papers are closely related to this one: Korinek and Sandri (2016) analyze the interaction between capital controls and international macroprudential policies. My paper builds on their framework, but adds a domestic currency bond. This creates a portfolio choice problem and provides several new insights, most prominently a justification to regulate domestic currency bonds due to different marginal propensities to consume even when trading with these bonds is frictionless.

Recent work by the IMF (Boz et al., 2020) finds that domestic macroprudential policies and capital controls are perfect substitutes, unless households have a way to circumvent domestic regulation, in which case households should be subject to capital controls. In contrast, I find that households should be subject to a mix of domestic and international regulatory policies. Two key features of my model explain these stark differences: First, I introduce two distinctive bonds in domestic and foreign currency. These bonds have unique characteristics in terms of availability and trading partners. They are therefore imperfect substitutes, whereas in Boz et al. (2020) households trade a domestic bond with either local banks or international investors. Second, I depart from a representative agent framework. As a consequence, savers and borrowers have a distinctive impact on exchange rates, which requires a more nuanced regulatory approach.

A few papers address portfolio choices in an international context. Salomao and Varela (2021) analyze the decision of firms to issue foreign debt. Foreign financing is cheaper, but leads to currency risk and potential default. More productive firms
are less likely to default and therefore choose more foreign currency debt. Gopinath and Stein (2020) study the interplay between trade invoicing and the demand for safe assets, which can lead to a single dominant currency. Bocola and Lorenzoni (2020) provide a theory in which dollarization endogenously arises from an insurance motive of domestic savers. Interestingly, ex-post policy intervention is able to reduce the riskiness of domestic bonds ex-ante and therefore stimulate domestic savings. Eren and Malamud (2019) build a model in which firms borrow in a currency that depreciates during global downturns, a description that fits the US dollar. Ize and Levy Yeyati (2003) explore how exchange rate and inflation regimes determine the extent of dollarization. This paper contributes to the literature by studying the implications of endogenous portfolio choices on optimal macroprudential ex-ante regulation.

2. Framework

2.1. Environment

A small open economy consists of three periods \( t = 0, 1, 2 \). There are three agents: domestic borrowers \((B)\), domestic savers \((S)\), each with measure one, and a large group of risk-neutral international investors. Both domestic households have identical preferences and derive utility from consuming tradable \((c^T_{i,t})\) and nontradable goods \((c^N_{i,t})\) with \( i \in \{B, S\} \). The only distinctive feature relates to different exogenous endowment/initial bond positions, which ensures that savers find it optimal to save, and borrowers optimal to borrow.

To inter-temporally relocate funds, households have access to two bonds: an international bond in units of tradable goods and a domestic bond in units of the nontradable good, which is meant to capture debt denominated in foreign and domestic currency respectively.\(^4\) I use the expressions international bond/debt and foreign currency bond/debt interchangeably. Similarly, domestic currency bonds are frequently referred to as domestic bonds. The international bond \((b^T_{i,t})\) is traded with inelastic international investors. Domestic bonds \((b^N_{i,t})\) are traded between borrowers and savers. The international financial market is subject to frictions, which limits international borrowings to a fraction of available income. The domestic financial market is frictionless.

\(^4\)Currency denomination can be modelled in other ways, for example by introducing nominal variables including monetary policy. The approach in this paper follows Bocola and Lorenzoni (2020). In the appendix, I show that one can derive identical results when tradable (nontradable) bonds are denominated in foreign (domestic) currency.
There is no production and both agents receive an exogenous endowment stream of tradable and nontradable goods \( \{y_{i,t}, y_{i,t}^N\} \). Further, I abstract from uncertainty, only to simplify the exposition. I subsequently characterize the environment in more detail. All proofs are delegated to the appendix.

**Household Maximization Problem**

Each household maximizes utility by choosing a path for consumption \( \{c_{i,t}, c_{i,t}^N\} \), and one-period international (\( b_{i,t+1}^T \)) or domestic bonds (\( b_{i,t+1}^N \)) where positive values indicate savings. The objective function is characterized as

\[
\max_{c_{i,t}, c_{i,t}^N, b_{i,t+1}^T, b_{i,t+1}^N} \left\{ u(c_{i,0}) + u(c_{i,1}) + u(c_{i,2}) \right\}. \tag{P}
\]

Preferences are characterized by log-utility, that is, \( u(\cdot) = \ln(\cdot) \). Consumption in period 2 is limited to tradables. The composite consumption index \( c_{i,t} \) at \( t = 0, 1 \) is defined as:

\[
c_{i,t} = (c_{i,t}^T)^\omega (c_{i,t}^N)^{1-\omega}
\]

The parameter \( \omega \) determines the expenditure share of tradable consumption and is frequently interpreted as a measure of home bias. If \( \omega < 0.5 \) households prefer to spend more resources on domestic or nontradable goods. Both agents receive exogenous endowments of \( \{y_{i,t}, y_{i,t}^N\} \) at date 0 and 1 as well as tradable endowment \( y_{i,t}^T \) in period 2.

The gross interest rate on international bonds is determined in world markets and hence exogenous to the small open economy. I normalize the gross return to one. The domestic bond pays a gross interest rate \( R \) which will be determined in equilibrium. Since agents do not value nontradable consumption in \( t = 2 \), domestic bonds are accessed in \( t = 0 \) only.

I denote the relative price of nontradables in terms of tradables as \( p_i \). As a standard feature of small open economy models, the relative price of nontradables can be equivalently characterized as a measure of the real exchange rate.\(^5\)

With the previous information, the budget constraints of savers and borrowers in \( t = 0, 1, 2 \) are

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\(^5\)The real exchange rate is defined as the aggregate price level at home divided by the aggregate price level abroad. Since I analyze a small open economy, the international price level as well as price of tradables is independent of domestic decisions. The real exchange rate therefore co-moves with the price of nontradables.
\[ c_{i,0}^T + b_{i,1}^T + p_0(c_{i,0}^N + b_{i,1}^N) = y_{i,0}^T + b_{i,0}^T + p_0(y_{i,0}^N + b_{i,0}^N) \]  
\[ c_{i,1}^T + b_{i,1}^T + p_1 c_{i,1}^N = y_{i,1}^T + b_{i,1}^T + p_1 (y_{i,1}^N + R b_{i,1}^N) \]  
\[ c_{i,2}^T = y_{i,2}^T + b_{i,2}^T. \]

The variables \( b_{i,0}^T \) and \( b_{i,0}^N \) refer to exogenous initial bond positions. I introduce these to ensure that savers (borrowers) save (borrow).

Borrowers are subject to an international financial constraint at date 1, which prevents them to borrow more than a fraction \( \phi \) of their current income.\(^6\) This could be inspired by more complex relationships with international lenders relative to domestic transactions. However, the point of this paper is that there is scope for purely domestic macroprudential regulation even in the absence of domestic borrowing limits.\(^7\)

\[-b_{B,2}^T \leq \phi (y_{B,1}^T + p_1 y_{B,1}^N)\]  

The specific functional form follows Mendoza (2002) and is common in the literature on financial crises and ex-ante regulation (see, for example, Bianchi, 2011; Bengui and Bianchi, 2018; Korinek, 2018).\(^8\)

The presence of \( p_1 \) in the borrowing constraint is crucial for the subsequent analysis. The real exchange rate is an equilibrium object and depends, among others, on aggregate consumption and saving decisions in \( t = 0 \). Individual households however do not internalize this dependency, which generates a pecuniary externality, and provides a justification for intervention via capital controls and macroprudential policies. Intuitively, households do not realize that savings in \( t = 0 \) improve the exchange rate and therefore relax the borrowing constraint if it binds. I make the following assumption on \( \phi \):

**Assumption 1** The collateralizable fraction of current income satisfies \( 0 < \phi < \frac{\omega}{1-\omega} \).

This assumption is standard in the literature on financial amplification and rules out multiple equilibria (see Korinek and Mendoza, 2014). On the contrary if this

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\(^6\)A binding constraint in period 0 would lead to a degenerate equilibrium in which international borrowing choices are dictated by constraints. This is not interesting to analyze. Hence, I do not impose a borrowing constraint at date 0.

\(^7\)If households in \( t = 2 \) valued nontradable consumption, a domestic bond market would naturally emerge in period 1. A domestic market would provide borrowers an option to access additional financing. However, nontradable funds are of limited supply, pinned down by nontradable endowment. The international borrowing constraint can therefore still bind.

\(^8\)The constraint can be endogenously derived based on limited commitment along the lines of Kiyotaki and Moore (1997). After borrowers received their loans in period 1, they have an opportunity to divert funds and default. In case of default lenders can at most claim a fraction \( \phi \) of period 1 income. To avoid losses, lenders are hence only willing to provide loans up to \( \phi \) times the current income.
condition was violated, an increase in $t = 1$ tradable consumption could relax the borrowing constraint by more than one unit; a coordinated increase in consumption would become self-financing.

In what follows, I solve this model via backward induction. Hence, I will first derive the continuation equilibrium for periods $t = 1, 2$ conditional on $t = 0$ borrowing and saving decisions (Section 2.2). The continuation equilibrium is characterized by two regimes depending on whether borrowers are constrained or not. I then proceed backward and solve for period 0 choices in the laissez-faire competitive equilibrium (Section 2.3). I contrast this solution with a social planner (Section 3.1), who internalizes the externality in this model. Subsequently I describe the implementation of the social planner allocation via capital controls and macroprudential policies (Section 3.2).

2.2. Period 1 Continuation Equilibrium

In period 1, savers and borrowers maximize period 1 and 2 utility. The state vector in $t = 1$ is fully described by date 0 international and domestic borrowing/saving decisions and exogenous endowments. I follow the convention in the literature and use capital letters to denote aggregate variables. The aggregate endogenous state vector is therefore described by $B = \{B^T_B, B^N_B, B^T_S, B^N_S\}$. Because households of each type are identical and of measure one, we have $b^T_i, b^N_i = b^T_B, b^N_B$ in equilibrium.

Each agent in period 1 solves

$$V_i(b^T_i, b^N_i; B) = \max_{c^T_i, c^N_i, b^T_{i,2}, c^T_{i,2}} \left\{ u \left( \left( c^T_{i,1} \right)^\omega (c^N_{i,1})^{1-\omega} \right) + u(c^T_{i,2}) \right\} \quad \text{(P1)}$$

subject to

$$c^T_{i,1} + b^T_{i,2} + p_1 c^N_{i,1} = y^T_{i,1} + b^T_{i,1} + p_1 (y^N_{i,1} + R b^N_{i,1}) \quad \text{(5)}$$
$$c^T_{i,2} = y^T_{i,2} + b^T_{i,2} \quad \text{(6)}$$
$$-b^T_{B,2} \leq \phi(y^T_{B,1} + p_1 y^N_{B,1}) \quad \text{(7)}$$

**Definition 1:** (Continuation Equilibrium) The continuation equilibrium is characterized by the real exchange rate $p_1$ and endogenous quantities $\{c^T_{i,1}, c^N_{i,1}, b^T_{i,2}, c^T_{i,2}\}$ with $i \in \{B, S\}$ such that

1. households maximize utility (P1) subject to the period 1 and 2 budget constraints (5), (6) and the borrowing constraint (7) (only borrowers) taking the real exchange rate as given;
2. The period 1 market for nontradables clears, \( \sum C_{i,1}^N = \sum Y_{i,1}^N \), which utilizes the observation that \( \sum B_{i,1}^N = 0 \).

Analysis

I define \( u_{i,1}^T = \frac{\partial u(c_{i,1})}{\partial c_{i,1}} \) as the marginal utility from tradable consumption in period 1. The same convention applies to \( u_{i,2}^T \) and \( u_{i,1}^N \). The first order conditions are then characterized as

\[
\begin{align*}
    u_{i,1}^T &= u_{i,2}^T + \lambda_i \\
    u_{i,1}^N &= p_1 u_{i,1}^T.
\end{align*}
\]

The first equation is a standard Euler equation augmented for a potentially binding borrowing constraint with Lagrange multiplier \( \lambda_i \). The multiplier is positive whenever the constraint binds and prevents borrowers from smoothing their tradable consumption between \( t = 1 \) and \( t = 2 \). Savers are by construction never constrained, hence \( \lambda_S = 0 \).

The second equation weights tradable and nontradable consumption. It can be aggregated over all agents and combined with the market clearing condition for nontradables. The resulting equation provides an expression for the real exchange rate:

\[
p_1 = 1 - \frac{\omega}{\sum Y_{i,1}^N} \sum C_{i,1}^T.
\]

The exchange rate improves with economy-wide tradable consumption. This is because tradable and nontradable consumption are complements. A rise in tradable consumption is accompanied by a higher demand for nontradable consumption. However, since available nontradable consumption equals endowment via the market clearing condition, the entire adjustment materializes in the price of nontradables.

In what follows, I characterize the exchange rate in the unconstrained and constrained regime. Specifically, I will show that the exchange rate is a function of economy-wide period 0 borrowing/saving decisions related to domestic and international bonds.

Unconstrained Period 1 Equilibrium

The unconstrained equilibrium emerges when borrowers have sufficient wealth and hence do not wish to borrow beyond their borrowing limit.
The unconstrained equilibrium is defined by $\lambda^B = \lambda^S = 0$. Both households perfectly smooth tradable consumption, that is, $u^T_{i,1} = u^T_{i,2}$. As a result, period 1 tradable consumption is given by

$$c^T_{i,1} = \frac{\omega}{2} \left( y^T_{i,1} + b^T_{i,1} + p_1(y^N_{i,1} + Rb^N_{i,1}) + y^T_{i,2} \right).$$

Agents spend half of their income in period 1 and a fraction $\omega$ thereof on tradable goods. The real exchange rate is therefore equal to

$$p_1(B^T_{B,1}, B^T_{S,1}) = \frac{(1 - \omega) \left[ \sum_i (Y^T_{i,1} + B^T_{i,1} + Y^T_{i,2}) \right]}{(1 + \omega) \left[ \sum_i Y^N_{i,1} \right]}.$$ 

The $t = 1$ real exchange rate is increasing in international savings. More wealth increases tradable consumption and the desire for nontradable consumption which must be offset by a higher real exchange rate. On the other hand, the equilibrium exchange rate is independent of the specific allocation of domestic bonds. Any ex-ante intervention that alters incentives to hold domestic bonds has hence no impact on the period 1 exchange rate if borrowers are unconstrained. This result emerges as borrowers and savers have the same marginal propensity to consume. Domestic savings and borrowings therefore cancel out. However, this is about to change once the international borrowing constraint binds.

**Constrained Period 1 Equilibrium**

The constrained or crisis equilibrium emerges when borrowers have insufficient borrowing capacity due to, for example, substantial initial debt.

In a constrained equilibrium, $\lambda^B > 0$, and via the Euler equation, $u^T_{B,1} > u^T_{B,2}$, that is, borrowers are no longer able to smooth tradable consumption. Period 1 tradable consumption for borrowers is therefore pinned down via the budget constraint:

$$c^T_{B,1} = \omega \left( y^T_{B,1} + b^T_{B,1} + p_1(y^N_{B,1} + Rb^N_{B,1}) + \phi(y^T_{B,1} + p_1y^N_{B,1}) \right).$$

Borrowers use all of their available resources and spend a fraction $\omega$ on tradable consumption. Savers are not constrained, hence their period 1 tradable consumption is still given by the consumption formula in the unconstrained equilibrium. The real exchange rate is consequently characterized by

$$p_1(B^T_{B,1}, B^T_{S,1}, B^N_{B,1}) = \frac{(1 - \omega) \left[ (1 + \phi)Y^T_{B,1} + B^T_{B,1} + \frac{Y^T_{S,1} + Rb^N_{B,1} + Y^T_{S,2}}{2} \right]}{\sum_i Y^N_{i,1} - (1 - \omega) \left[ Y^N_{B,1}(1 + \phi) + \frac{Y^N_{S,1}}{2} + Rb^N_{B,1} \right]}.$$
Since $B_{B,1}^N < 0$, Assumption 1 is a sufficient condition to ensure a positive denominator in the exchange rate formula. I summarize the impact of the date 0 borrowing/saving decisions on the exchange rate in the first Lemma.

**Lemma 1**

1. The real exchange rate is increasing in the international net worth position. If borrowers are constrained then $\frac{\partial p_1}{\partial B_{B,1}^T} > \frac{\partial p_1}{\partial B_{B,1}^N} > 0$, otherwise $\frac{\partial p_1}{\partial B_{B,1}^T} = \frac{\partial p_1}{\partial B_{B,1}^N} > 0$.

2. The exchange rate improves if borrowers (savers) hold less domestic debt (save less) in a constrained equilibrium, i.e., $\frac{\partial p_1}{\partial B_{B,1}^N} = -\frac{\partial p_1}{\partial B_{B,1}^N} > 0$, otherwise the exchange rate does not depend on the allocation of domestic savings/borrowings.

**Discussion**

A decrease in international debt by borrowers stabilizes the exchange rate by more than an increase in savings by savers, that is, $\frac{\partial p_1}{\partial B_{B,1}^T} > \frac{\partial p_1}{\partial B_{B,1}^N}$. This can be traced back to distinct marginal propensities to consume. A marginal increase in external net worth increases consumption expenditure for constrained borrowers by more than one unit due to feedback effects via the exchange rate in the collateral constraint. Contrary, savers prefer to smooth consumption between date 1 and 2 and hence save parts of the additional net worth. Borrowers therefore increase consumption by more than savers, which induces a comparably larger effect on the real exchange rate. Figure 3, Panel (a) further highlights a higher sensitivity of the exchange rate towards foreign currency debt in the constrained region.

Further, and this will be vital to justify domestic regulation, the exchange rate is a function of the domestic bond allocation. As illustrated in Figure 3, Panel (b), the exchange rate deteriorates with more domestic debt (higher $-B_{B,1}^N$) when borrowers are constrained. This effect stems from distributional considerations of nontradable net worth at the beginning of date 1. Borrowers have a higher marginal propensity to consume, so one unit of additional wealth in the hands of borrowers overcompensates the equivalent wealth loss of savers in terms of consumption demand. The domestic borrowings/savings decision hence materially affects tradable consumption, which in turn appreciates the real exchange rate and moderates the tightness of the borrowing constraint/severity of the recession.

2.3. Period 0 Laissez-faire Equilibrium

In period 0, savers and borrowers maximize date 0 utility and the value function from the continuation equilibrium $V_i(b_{i,1}^T, b_{i,1}^N, B)$ which depends on date 0 borrowing and
Figure 3: Exchange Rate: Comparative Statics

(a) International Debt

(b) Domestic Debt

Notes: The plots portray the effect of an increase in borrowers’ aggregate international debt (Panel (a)) and domestic debt (Panel (b)) on the real exchange rate. The area encompassing 1 marks the unconstrained and 2 the constrained region.

saving decisions. Further, borrowers are not constrained in terms of their access to international financial markets and households participate in a domestic bond market.

Each household solves

$$\max_{c_{i,0}^T, c_{i,0}^N, b_{i,1}^T, b_{i,1}^N} \left\{ u \left( (c_{i,0}^T)^{\omega} (c_{i,0}^N)^{1-\omega} \right) + V_i(b_{i,1}^T, b_{i,1}^N; B) \right\} \quad \text{(Po:CE)}$$

subject to

$$c_{i,0}^T + b_{i,1}^T + p_0(c_{i,0}^N + b_{i,1}^N) = y_{i,0} + b_{i,0}^T + p_0(y_{i,0}^N + b_{i,0}^N). \quad (8)$$

Definition 2: (Unregulated Equilibrium) The period 0 unregulated equilibrium is characterized by the real exchange rate $p_0$, the gross interest rate for domestic bonds $R$ and endogenous quantities $\{c_{i,0}^T, c_{i,0}^N, b_{i,1}^T, b_{i,1}^N\}$ with $i \in \{B, S\}$ such that

1. households maximize utility (Po:CE) subject to the period 0 budget constraint (8) taking the real exchange rates in $t = 0, 1$ and the gross interest rate as given;

2. the domestic bond market clears, that is, $\sum B_{i,1}^N = 0$;

3. the period 0 market for nontradables clears, i.e., $\sum_i C_{i,0}^T = \sum_i \{y_{i,0}^N + B_{i,0}^N\} = \sum_i Y_{i,0}^N$.

Households treat the exchange rates and the gross interest rate as given. The derivatives of the date 1 value function therefore correspond to $\frac{\partial V_i(\cdot)}{\partial b_{i,1}^T} = u_{i,1}^T$ and $\frac{\partial V_i(\cdot)}{\partial b_{i,1}^N} = Rp_1 u_{i,1}^T = Ru_{i,1}^N$. As a result:

$$u_{i,0}^T = u_{i,1}^T$$
$$u_{i,0}^N = Ru_{i,1}^T$$
$$u_{i,0}^N = p_0 u_{i,0}^T.$$
Both households smooth tradable and nontradable consumption. The third condition relates tradable and nontradable consumption.

3. Capital Controls and Macroprudential Policies

Can a benevolent regulator increase domestic welfare relative to the competitive equilibrium? In other words, would a planner distort the competitive allocation? The answer to these questions is affirmative. The pecuniary externality embedded in this model justifies prudential interventions in period 0. In order to characterize the optimal policy, I follow the common approach of a constrained social planner who achieves a second best solution by allocating date 0 resources efficiently given the set of markets operating (Stiglitz, 1982, Geanakoplos and Polemarchakis, 1986). The allocations in \( t = 1, 2 \) are determined by decentralized agents in competitive markets.\(^9\)

The solution is second best in the sense that the social planner is subject to the same financial constraint (4) at date 1. By choosing period 0 allocations on behalf of all domestic agents, the planner however internalizes the dependency between aggregate period 0 bond holdings, the real exchange rate at date 1 and the tightness of the borrowing constraint.

I adhere to the dynamic public finance literature and use the primal approach (Lucas and Stokey, 1983). That is, the social planner directly chooses the consumption path of domestic households as well as the allocation of domestic and international bonds (Section 3.1). Subsequently, I decentralize the allocation based on three distinctive policy instruments that are akin to capital controls and macroprudential policies (Section 3.2).

3.1. Period 0 Social Planner Equilibrium

The social planner maximizes the weighted utility (Pareto weights \( \gamma_i \)) of all borrowers and savers,

\[
\max_{C^T_{i,0}, C^N_{i,0}, B^T_{i,1}, B^N_{i,1}} \left\{ \sum_i \gamma_i \left\{ \nu \left( (C^T_{i,0})^{\alpha} (C^N_{i,0})^{1-\alpha} \right) + V_i(B) \right\} \right\} \quad \text{(Po:SP)}
\]

\(^9\)The approach rules out ex-post interventions. Benigno et al. (2016) show that a non-distortionary consumption subsidy on nontradables is able to appreciate the exchange rate until the collateral constraint does not bind. This result however does not generalize to distortionary financing. For related work on the interplay between ex-ante and ex-post policies, see, for example, Benigno et al. (2013), Bianchi (2016), and Jeanne and Korinek (2020).
subject to

\[
\sum_i \{ C_{i,0}^T + B_{i,1}^T \} = \sum_i \{ Y_{i,0}^T + B_{i,0}^T \} \tag{9}
\]

\[
\sum_i \{ C_{i,0}^N + B_{i,1}^N \} = \sum_i \{ Y_{i,0}^N + B_{i,0}^N \} \tag{10}
\]

\[
\sum_i B_{i,1}^N = 0. \tag{11}
\]

The first (second) equation is the period 0 resource constraint for tradable (nontradable) goods. The third constraint captures the market clearing condition for domestic bonds. The planner is hence able to freely allocate tradable and nontradable consumption goods across agents. However, domestic bonds must sum to zero, consistent with the functioning of the domestic financial market. Further, the planner directly chooses the endogenous period 1 aggregate state variables and hence internalizes that date 0 saving and borrowing decisions affect the real exchange rate at date 1.

**Definition 3**: (Social Planner Equilibrium)  *The period 0 social planner equilibrium is characterized by aggregate endogenous quantities \{C_{i,0}^T, C_{i,0}^N, B_{i,1}^T, B_{i,1}^N\} with \(i \in \{B, S\}\) such that*

1. *the social planner maximizes the weighed utility \((P_0:SP)\) subject to the tradable resource constraint (9), the nontradable resource constraint (10) and the constraint imposed by the domestic bond market (11);*

2. *the planner internalizes the impact of aggregate borrowing and saving decisions on the real exchange rate at date 1;*

3. *aggregate quantities are proportionally distributed, e.g., \(c_{i,0}^T = C_{i,0}^T\).*

The first order conditions of the social planner with respect to consumption provide two equations that balance the weighted marginal utility from tradables and nontradables across agents: \(\gamma_i u_{i,0}^T = \gamma_j u_{j,0}^T\) and \(\gamma_i u_{i,0}^N = \gamma_j u_{j,0}^N\) for \(i \neq j\). Further, combining the first order conditions for tradable and nontradable consumption implies \(u_{i,0}^N = \beta_2 u_{i,0}^T\). Households hence consume tradable and nontradables proportionally, determined by the Lagrange multiplier \(\beta_2\).\(^{10}\) The optimality conditions regarding the accumulation of foreign and domestic debt are summarized as

\(^{10}\)The Lagrange multipliers of equations (9), (10) and (11) are \(\beta_1, \beta_1 \beta_2\) and \(\beta_1 \beta_2 \beta_3\) respectively. With this definition, \(\beta_2\) reflects the value of nontradables in terms of tradables, similar to \(p_0\) in the decentralized equilibrium. \(\beta_3\) represents the tightness of the domestic bond resource constraint in units of nontradables.
The variable $\beta_3$ relates to the Lagrange multiplier of the domestic bond resource constraint (11). The multiplier plays a similar role as the interest rate R in the decentralized equilibrium.

Both Euler equations can be split into three parts. The first component is motivated by consumption smoothing and coincides with the Euler equations in the decentralized equilibrium. The second term captures distributional effects associated with movements in the real exchange rate. An appreciated exchange rate benefits sellers of nontraded goods. The sales are multiplied by the difference between the households’ marginal utility to balance the gains and cost from redistribution. The third term refers to the pecuniary externality of the model. Aggregate saving and borrowing decisions affect the real exchange rate and hence the tightness of the borrowing constraint.

Based on the above first order conditions it is possible to derive four wedges that account for the distinctive valuation of savings/borrowings relative to the competitive equilibrium. However, I have to make one additional assumption which ensures that transfers of tradables from savers to borrowers do not lead to a decline in borrowers wealth:

**Assumption 2** A transfer in tradables from savers to borrowers increases the wealth of borrowers, that is, $1 + \left( \frac{\partial p_1}{\partial B_{i,1}^T} - \frac{\partial p_1}{\partial B_{i,1}^N} \right) \left( Y_{i,1}^N + B_{i,1}^N - C_{i,1}^N \right) \equiv D > 0$.

A one unit transfer leads to an equivalent increase in borrowers’ period 1 tradable wealth, however it also leads to a change in the exchange rate which affects the net income from nontraded sales/purchases.

**Proposition 1** A constrained efficient solution satisfies:

$$\frac{u_{i,1}^T}{u_{i,0}^T} = 1 - \left( \frac{\lambda_B}{u_{i,0}^T} \frac{\partial p_1}{\partial B_{i,1}^T} \frac{\phi Y_{i,1}^N}{D} \right) \equiv \frac{\tau_B}{D}$$
The terms \( \{\tau_T^T, \tau_T^S, \tau_B^T, \tau_S^N\} \) represent wedges that characterize the difference between the social planner and decentralized equilibrium.

1. If the borrowing constraint is loose, that is, \( \lambda_B = 0 \), the planner has the same incentive to save/borrow as households in the decentralized equilibrium. Hence \( \tau_T^T = \tau_T^S = \tau_B^T = \tau_S^N = 0 \);

2. If the borrowing constraint binds, the planner introduces a wedge in the marginal rate of substitution and acts in a more precautionary manner on international bonds. The wedge is larger for borrowers than for savers (\( \tau_T^T > \tau_T^S > 0 \)) as \( \frac{\partial p_1^T}{\partial B_{T,1}} > \frac{\partial p_1^T}{\partial B_{T,1}} \);

3. In case of a binding borrowing constraint, the planner also introduces a wedge for domestic bonds. The wedge discourages domestic borrowing for borrowers (\( \tau_B^N > 0 \)) and discourages domestic savings for savers (\( \tau_S^N < 0 \)). The domestic wedges are equivalent in absolute terms, i.e., \( \tau_B^N = |\tau_S^N| \).

**Discussion**

The first Proposition characterizes the constrained efficient marginal rate of substitution, or equivalently the desire to inter-temporally relocate funds, for both households and bonds. The rate of substitutions differ from the competitive equilibrium as highlighted by four wedges. These wedges are however zero if borrowers are unconstrained (\( \lambda_B = 0 \)). Intuitively, the only justification for prudential intervention in this model relates to the pecuniary externality associated with the borrowing constraint. A social planner, unlike domestic households, internalizes the dependency between initial borrowing/saving decisions and the tightness of the constraint via general equilibrium exchange rate effects, which provides first order welfare gains relative to the unregulated competitive equilibrium. However, interventions are not warranted if the constraint is slack.
If the borrowing constraint binds \((\lambda_B > 0)\) all four wedges are nonzero. The wedges can be grouped into two sets. The first set pertains to the tradable Euler equations and hence the willingness to borrow (save) internationally in foreign currency. The planner introduces wedges \((\tau^T_B, \tau^T_S)\) that distort the competitive tradable marginal rate of substitution. Both wedges tilt consumption towards period 1. The wedge is larger for borrowers, precisely because borrowers have a higher marginal propensity to consume and hence a more pronounced impact on the period 1 exchange rate. Nevertheless, it is worth emphasising that a regulator finds it optimal to introduce a wedge for savers, even though they are not constrained in their ability to borrow international funds. This is because the planner maximizes the joint welfare of both agents and therefore internalizes the dependency between savers and the tightness of the borrowing constraint. Put differently, these wedges benefit borrowers, but since the planner solution is by construction constrained efficient, it provides a higher welfare overall relative to the unregulated competitive equilibrium. Thus, it would be possible to appropriately redistribute the welfare gains from internalizing the externality such that both households are better off.

The novel insight of this paper pertains to the remaining two wedges related to domestic currency bonds traded at home. Based on Lemma 1, the exchange rate appreciates with less domestic debt, precisely because borrowers have a higher marginal propensity to consume. The constrained efficient nontradable Euler equations reflect this observation. A planner finds it optimal to shift nontradable resources towards borrowers. This is mirrored by two wedges \((\tau^N_B, \tau^N_S)\). The wedge related to borrowers reduces borrowings in domestic currency \((\tau^N_B > 0)\), while the wedge for savers reduces savings \((\tau^N_S < 0)\). Since a reduction in domestic borrowings has the same effect as a decrease in savings, both wedges have the same size in absolute value. The crucial point here is that a planner finds it optimal to distort the allocation of domestic bonds, even when domestic financial markets are frictionless. The allocation of domestic wealth provides an additional margin to shift resources towards constrained households. Due to distinct marginal propensities to consume, this redistribution improves the real exchange rate and therefore relaxes the borrowing constraint.

3.2. Implementation

The constrained efficient social planner solution can be decentralized by date 0 taxes/subsidies on domestic and international bonds. The period 0 budget constraint becomes
\[ c_{i,0}^T + (1 - \tau_i^T)b_{i,1}^T + p_0(c_{i,0}^N + (1 - \tau_i^N)b_{i,1}^N) = y_{i,0}^T + b_{i,0}^T + p_0(y_{i,0}^N + b_{i,0}^N) + T_i. \]  

The term \( T_i = -\tau_i^T b_{i,1}^T - p_0 \tau_i^N b_{i,1}^N \) represents lump-sum transfers from tax revenues to avoid wealth effects. The variables \( \{\tau_i^T, \tau_i^N\} \) are distortionary policy instruments. If positive, the instrument represents a tax on borrowings or equivalently a subsidy on savings. Either way, a positive value induces households to save more or borrow less, which improves the net worth position in period 1. The Euler equations in the regulated competitive equilibrium are

\[
\frac{u_{i,1}^T}{u_{i,0}^T} = 1 - \tau_i^T \\
\frac{u_{i,1}^N}{u_{i,0}^N} R = 1 - \tau_i^N.
\]

**Lemma 2** The regulated equilibrium implements a constrained efficient allocation if the taxes/subsidies \( \{\tau_i^T, \tau_i^N\} \) with \( i \in \{B, S\} \) are set to their corresponding wedges as characterized in Proposition 1, where \( \frac{1}{1 - \beta_3} \) is replaced by \( R \).

If taxes/subsidies are set appropriately, the Euler equations of the decentralized regulated equilibrium and the social planner equilibrium coincide. In this case, the regulated competitive equilibrium implements one point of the constrained efficient Pareto frontier, which is fully described by the set of Pareto weights \( (\gamma_i) \). Crucially, these instruments are imposed ex-ante whenever a crisis materializes in \( t = 1 \), that is, when the borrowing constraint binds. The policies improve the international net worth position and reduce domestic debt during benign times, which dampens the subsequent recession. They are hence precautionary in nature and resemble prudential policies.

**Mapping to Capital Controls and Macroprudential Policies**

As a final building block, I map the taxes/subsidies to capital controls and macroprudential regulation. The model implies four distinctive wedges, however the borrowers’ tax on domestic debt equals the savers’ tax on savings, which reduces the effective number of distinctive interventions to three. As highlighted in the Introduction and Figure 2, capital controls are based on the residency principle: they drive a wedge between home and foreign investors to segment domestic from international financial markets. Macroprudential policies in contrast affect the relationship between savers and borrowers and specifically discourage borrowings in this setting. International macroprudential policies (imposed at home) influence foreign currency borrowings. Domestic macroprudential policies
are related to domestic currency arrangements between domestic savers and domestic borrowers. These considerations imply the following mapping between \( \{ \tau_B, \tau_S, \tau_N \} \) and \( \{ \tau_{CC}, \tau_{MP}^I, \tau_{MP}^D \} \):

**Corollary 1** The taxes/subsidies in the regulated decentralized equilibrium \( \{ \tau_B^T, \tau_S^T, \tau_N^T \} \) resemble capital controls (\( \tau_{CC} \)), international macroprudential regulation (\( \tau_{MP}^I \)) and domestic macroprudential regulation (\( \tau_{MP}^D \)). The mapping is as follows:

\[
\begin{align*}
\tau_{CC} &= \tau_S^T \\
\tau_{MP}^I &= \tau_B^T - \tau_S^T \\
\tau_{MP}^D &= \tau_N^T.
\end{align*}
\]

Domestic savers investing abroad are subject to capital controls, but not to international macroprudential policies. Hence, \( \tau_{CC} = \tau_S^T \). International macroprudential policies discourage international borrowings. Hence they account for the difference between the tax on international debt for borrowers and the level of capital controls. Domestic macroprudential policies in turn discourage domestic borrowings, therefore \( \tau_{MP}^D = \tau_N^T \).

**Relationship between Domestic and International Regulation**

The above mapping highlights that purely domestic macroprudential policies should accompany regulation geared towards international financial markets, even when domestic markets are frictionless. But to what extent? The strength of domestic regulation depends on four factors, which I summarize in Proposition 2.

**Proposition 2** Domestic regulation is characterized as follows:

\[
\begin{align*}
\tau_{MP}^D &= \frac{\gamma_I^T}{\Pi_N^T} + \tau_{CC} \\
\tau_{MP}^I &= \tau_B^T - \tau_S^T \\
\tau_{MP}^D &= \tau_N^T.
\end{align*}
\]

As a consequence, the extent of domestic regulation depends on

1. the level of international regulation. Domestic and international regulation are complements;
2. the relative cost of domestic regulation;
3. the rate of return on domestic currency bonds;
4. the relative inefficiency of domestic bonds.
Discussion

The second Proposition characterizes the relationship between domestic and international regulation by means of a simple formula. The level of domestic regulation depends on four factors. First, domestic regulation is increasing in international regulation, i.e., if a country decides to tighten the regulation of international financial flows, it should also tighten domestic regulation. The reason for this result relates to the substitutability of domestic and foreign assets. Households in period 0 can invest into either bond to transfer wealth between period 0 and 1. If a country imposes tight international regulations, domestic bonds become relatively more attractive and agents shift their investments. The substitutability is however imperfect, because domestic and foreign bonds are associated with distinctive costs, returns and externalities.

The second factor relates to the relative cost of domestic regulation as captured by the period 0 ratio of tradable and nontradable marginal utilities. If tradable consumption is scarce at date 0 (high $u^T_{B,0}$), it is more desirable to tax/regulate the domestic financial market or equivalently nontradable bonds. Contrary, if domestic households are subject to a home bias ($\omega$ small), it is appropriate to limit the degree of domestic regulation.

The strength of domestic regulation also depends on the investment profitability. Domestic regulation is more desirable, if domestic currency bonds provide a high return. The reason is that more profitable investment opportunities support the reallocation of wealth across periods. Since macroprudential policies by design reallocate resources towards distressed periods, regulators have an incentive to tax investments with a higher return.

The last term in the formula captures the impact on the real exchange rate. If the real exchange rate is more sensitive to domestic debt, a regulator should put more weight on domestic regulation. More generally, a regulator should focus on assets that are tied to the inefficiency in the economy. In this model, households do not internalize the impact of saving/borrowing decisions on the exchange rate, which generates welfare losses in combination with the collateral constraint.

4. Conclusion

Most emerging markets use a combination of macroprudential regulatory policies that affect international and domestic financial flows. Yet, there is limited knowledge how domestic macroprudential policies interact with their international counterparts.
and (prudential) capital controls. The contribution of this paper is to provide a simple framework that facilitates a comprehensive analysis.

The key result is that emerging markets rationally implement a wide mix of macroprudential policies including capital controls. Capital controls and international macroprudential policies limit foreign currency expose and improve the external wealth position of the domestic economy. As a consequence, they enhance financial stability and mitigate sudden stops when international creditors restrict access to foreign financing. This finding is extensively documented in the literature. What is new is that the aforementioned policies should be accompanied by purely domestic macroprudential policies aimed at regulating domestic currency debt. Intuitively, domestic currency bonds provide an additional margin to shift resources towards constrained agents. This reallocation of domestic wealth from savers to borrowers is welfare enhancing because it shifts resources towards agents who are limited in their ability to borrow internationally and as a consequence have a higher propensity to consume. Further, via positive general equilibrium effects on the international borrowing capacity, domestic macroprudential policies indirectly mute sudden stop dynamics in foreign capital.

The second contribution of this paper is a simple formula that characterizes the optimal level of domestic macroprudential regulation. Two results stand out: (i) Domestic regulation and international regulation are imperfect complements to prevent arbitrage by borrowers. (ii) The trade-offs between domestic regulation and international regulation are influenced by the relative cost of regulation, the rate of return on domestic currency bonds, and the extent to which investments affect the underlying inefficiency.

Lastly, I acknowledge the specific nature of the framework. I focus on macroprudential regulation that is imposed during good times to reduce the severity of a future crisis. In reality, macroprudential policies are accompanied by ex-post interventions. A complementary literature analyzes ex-ante versus ex-post interventions in models with a pecuniary externality (see, for example, Benigno et al., 2013, 2016; Bianchi, 2016; Jeanne and Korinek, 2020). The bottom line of this literature is that one should use a mix of ex-ante and ex-post policies since ex-post policies are likely to create moral hazard or entail efficiency losses stemming from distortionary financing.
Equivalence to setup with explicit nominal variables: The budget and borrowing constraints with nominal prices and explicit foreign and domestic currency debt are characterized as

\[
P_t^T c_{T,i,0} + E_0 P_{T,i,0}^T b_{T,i,0}^T + P_{N,i,0}^N (c_{N,i,0}^N + b_{N,i,0}^N) = P_t^T y_{T,i,0} + E_0 P_{T,i,0}^T b_{T,i,0}^T + P_{N,i,0}^N (y_{N,i,0} + b_{N,i,0}^N)
\]

\[
P_t^T c_{T,i,1} + E_1 P_{T,i,1}^T b_{T,i,1}^T + P_{N,i,1}^N c_{N,i,1}^N = P_t^T y_{T,i,1} + E_1 P_{T,i,1}^T b_{T,i,1}^T + P_{N,i,1}^N (y_{N,i,1} + R b_{N,i,1}^N)
\]

\[
P_t^T c_{T,i,2} = P_t^T y_{T,i,2} + E_2 P_{T,i,2}^T b_{T,i,2}^T
\]

\[-E_1 P_{T,i,1}^T b_{B,i,2}^T \leq \phi (P_{T,i,1}^T y_{B,i,1} + P_{N,i,1}^N y_{B,i,1}^N),\]

where \(P_t^T\) denotes the domestic currency price of tradables, \(P_{t,i}^N\) the domestic currency price of nontradables, \(P_t^{T,*}\) the foreign currency price of tradables, and \(E_t\) the nominal exchange rate defined as domestic over foreign currency. I assume the law of one price for tradables holds, hence \(P_{t,i}^T = E_t P_{t,i}^{T,*}\). One can then divide all budget constraints and the borrowing constraint by \(P_t^T\). This yields equations (1)-(4).

**Proof of Lemma 1:** I rewrite the date 1 exchange rate in the unconstrained and constrained equilibrium for convenience.

\[
p_{1,\text{uncon}}^T (B_{B,1}^T, B_{S,1}^T) = \frac{(1 - \omega) \left[ \sum_t (Y_{T,i,1} + B_{T,i,1}^T + Y_{T,i,2}^T) \right]}{(1 + \omega) \left[ \sum_t Y_{N,i,1}^N \right]}
\]

\[
p_{1,\text{con}}^T (B_{B,1}^T, B_{S,1}^T, B_{N,1}^T) = \frac{(1 - \omega) \left[ (1 + \phi) Y_{B,1}^T + B_{B,1}^T + \frac{Y_{S,1}^N + Y_{S,1}^T}{2} \right]}{\sum_t Y_{i,1}^N - (1 - \omega) \left[ Y_{B,1}^T (1 + \phi) + \frac{Y_{N,1}^N}{2} + R_{B,1}^N \right]}
\]
The gross interest rate on domestic bonds is endogenous and equals $R = p_0 / p_1$, following the first order conditions in $t = 0, 1$. The period 0 exchange rate is given by

$$p_0(B_{B,1}^T, B_{L,1}^T) = \frac{(1 - \omega) \left[ \sum_i (Y_{T,i,0} + B_{T,i,0} - B_{T,i,1}) \right]}{\omega \left[ \sum Y_{T,i,1} \right]}.$$

It is straightforward to verify that $\frac{\partial p_{\text{uncon}}}{\partial B_{B,1}^T} > 0$. I subsequently derive $\frac{\partial p_{\text{con}}}{\partial B_{i,1}^T} |_{\text{total}}$.

The term $\frac{\partial p_{\text{con}}}{\partial B_{i,1}^T}$ refers to the direct effect of $B_{i,1}^T$ on $p_{i,1}^{\text{con}}$ absent any feedback effects via $R$. It is straightforward to verify that $\frac{\partial p_{\text{con}}}{\partial B_{i,1}^T} > 0$. Further, $\frac{\partial p_{\text{con}}}{\partial \omega} < 0$, as $B_{B,1}^N < 0$. One also obtains $\frac{\partial p_0}{\partial B_{B,1}^T} < 0$. The numerator of $\frac{\partial p_{\text{con}}}{\partial B_{i,1}^T} |_{\text{total}}$ is therefore strictly positive. The denominator is also positive as I subsequently explain and corresponds to:

$$1 + \frac{\partial p_{\text{con}}}{\partial R} \frac{R p_{\text{con}}}{p_{i,1}^{\text{con}}} = 1 + \frac{(1 - \omega) R B_{B,1}^N}{\sum_i Y_{i,1} - (1 - \omega) \left[ Y_{B,1}^N (1 + \phi) + \frac{Y_{S,1}}{2} + R B_{B,1}^N \right]}.$$

Nontradable borrowing has a natural limit. Borrowers are not able to borrow more than the amount of available nontradable resources in $t = 0$, $\sum_i Y_{i,0}^N$ nor can claims in $t = 1$ exceed the discounted period 1 nontradable endowment, $\sum_i Y_{i,1}^N / R$. These results emerge as borrowers are free to sell their nontradables for tradables and subsequently borrow nontradables from savers. Nontradable borrowing is therefore limited by $-B_{B,1}^N \leq \min \{ \sum_i Y_{i,1}^N / R, \sum_i Y_{i,0}^N \}$. As a second observation, $\frac{\partial p_{\text{con}}}{\partial \omega} R / p_{i,1}^{\text{con}}$ is decreasing in domestic debt (increasing in $B_{B,1}^N$). A sufficient condition for $1 + \frac{\partial p_{\text{con}}}{\partial \omega} R / p_{i,1}^{\text{con}} > 0$ is therefore

$$1 - \frac{(1 - \omega) \left[ \sum_i Y_{i,1}^N \right]}{\sum_i Y_{i,1} - (1 - \omega) \left[ Y_{B,1}^N (1 + \phi) + \frac{Y_{S,1}}{2} - \sum_i Y_{i,1}^N \right]} > 0.$$

It is admissible to replace $\phi$ with its upper bound $\frac{\omega}{1 - \omega}$ since that value makes the left hand side as small as possible. Endowments subsequently cancel out and the above inequality simplifies to
\[ 1 > 0.5(1 - \omega), \]

which is always satisfied. I thus proved that \( 1 + \frac{\partial p^{\text{con}}}{\partial k} \frac{R}{p^{\text{con}}} > 0 \) and as a consequence, \( \frac{\partial p^{\text{con}}}{\partial B_{i,1}} \bigg|_{\text{total}} > 0 \). Further, it is straightforward to verify that \( \frac{\partial p^{\text{con}}}{\partial B_{b,1}} > \frac{\partial p^{\text{con}}}{\partial B_{s,1}} \), that is, the direct effect on the exchange rate is larger for borrowers. Hence, one obtains \( \frac{\partial p^{\text{con}}}{\partial B_{b,1}} \bigg|_{\text{total}} > 0 \) and as a consequence, \( \frac{\partial p^{\text{con}}}{\partial B_{b,1}} \bigg|_{\text{total}} > 0 \).

For the second part of the Lemma, it is obvious that \( \frac{\partial p^{\text{uncon}}}{\partial B_{N}} = 0 \). The formula for \( \frac{\partial p^{\text{con}}}{\partial B_{N}} \bigg|_{\text{total}} \) is

\[ \frac{\partial p^{\text{con}}}{\partial B_{N,1}} \bigg|_{\text{total}} = \frac{\partial p^{\text{con}}}{\partial B_{N,1}} \bigg|_{1 + \frac{\partial p^{\text{con}}}{\partial k} \frac{R}{p^{\text{con}}}}. \]

The direct effect in the numerator is positive. Further, I already verified that \( 1 + \frac{\partial p^{\text{con}}}{\partial k} \frac{R}{p^{\text{con}}} > 0 \). Therefore, \( \frac{\partial p^{\text{con}}}{\partial B_{b,1}} \bigg|_{\text{total}} > 0 \).

**Derivation of social planner Euler equations:** The first order condition with respect to international bonds is characterized as

\[ \gamma_i u^T_{i,0} = \gamma_i \frac{\partial V_i(B)}{\partial B_{i,1}^T} + \gamma_j \frac{\partial V_j(B)}{\partial B_{i,1}^T} \quad \text{(FOC: } B_{i,1}^T \text{)} \]

with \( i \neq j \). The expression on the left hand side makes use of the intratemporal first order condition of the social planner, \( \beta_1 = \gamma_i u^T_{i,0} \). An application of the Envelope Theorem yields

\[ \frac{\partial V_i(B)}{\partial B_{i,1}^T} = u^T_{i,1} \left( 1 + \frac{\partial p_1}{\partial B_{i,1}^T} \left( Y_{i,1}^N + B_{i,1}^N - C_{i,1}^N \right) \right) + \frac{\partial p_1}{\partial B_{i,1}^T} \lambda_i \phi Y_{i,1}^N \]

\[ \frac{\partial V_j(B)}{\partial B_{i,1}^T} = u^T_{j,1} \left( \frac{\partial p_1}{\partial B_{j,1}^T} \left( Y_{j,1}^N + B_{j,1}^N - C_{j,1}^N \right) \right) + \frac{\partial p_1}{\partial B_{j,1}^T} \lambda_j \phi Y_{j,1}^N. \]

Market clearing of nontradables implies \( Y_{i,1}^N + B_{i,1}^N - C_{i,1}^N = -(Y_{j,1}^N + B_{j,1}^N - C_{j,1}^N) \). Further, only borrowers face a collateral constraint. We can therefore write

\[ \gamma_i u^T_{i,0} = \gamma_i u^T_{i,1} + \frac{\partial p_1}{\partial B_{i,1}^T} \left( \gamma_i u^T_{i,1} - \gamma_j u^T_{j,1} \right) \left( Y_{i,1}^N + B_{i,1}^N - C_{i,1}^N \right) + \frac{\partial p_1}{\partial B_{i,1}^T} \gamma_B \lambda_B \phi Y_{B,1}^N. \]

The first order condition with respect to domestic bonds is
\[ \gamma_i u^N_{i,0} (1 - \beta_3) = \gamma_i \frac{\partial V_i(B)}{\partial B^N_{i,1}} + \gamma_j \frac{\partial V_j(B)}{\partial B^N_{i,1}} \] (FOC: \( B^N_{i,1} \))

with \( i \neq j \). The term on the left incorporates the intratemporal first order condition of the planner, \( \beta_1 \beta_2 = \gamma_i u^N_{i,0} \). The derivatives of the value functions are equal to

\[
\frac{\partial V_i(B)}{\partial B^N_{i,1}} = u^T_{i,1} \left( p_1 + \frac{\partial p_1}{\partial B^N_{i,1}} \left( Y^N_{i,1} + B^N_{i,1} - C^N_{i,1} \right) \right) + \frac{\partial p_1}{\partial B^N_{i,1}} \lambda_i \phi Y^N_{i,1} \\
\frac{\partial V_j(B)}{\partial B^N_{i,1}} = u^T_{j,1} \left( \frac{\partial p_1}{\partial B^N_{j,1}} \left( Y^N_{i,1} + B^N_{j,1} - C^N_{j,1} \right) \right) + \frac{\partial p_1}{\partial B^N_{j,1}} \lambda_j \phi Y^N_{j,1}.
\]

The intratemporal first order condition in period 1 implies \( u^N_{i,1} = p_1 u^T_{i,1} \). Imposing nontradable market clearing subsequently yields

\[ \gamma_i u^N_{i,0} (1 - \beta_3) = \gamma_i u^N_{i,1} + \frac{\partial p_1}{\partial B^N_{i,1}} \left( \gamma_i u^T_{i,1} - \gamma_j u^T_{j,1} \right) \left( Y^N_{i,1} + B^N_{i,1} - C^N_{i,1} \right) + \frac{\partial p_1}{\partial B^N_{i,1}} \gamma_B \lambda_B \phi Y^N_{B,1}. \]

**Proof of Proposition 1:** The first order condition for international bonds and household \( i \in \{ B, S \} \) corresponds to

\[ \gamma_i u^T_{i,0} = \gamma_i u^T_{i,1} + \frac{\partial p_1}{\partial B^N_{i,1}} \left( \gamma_i u^T_{i,1} - \gamma_j u^T_{j,1} \right) \left( Y^N_{i,1} + B^N_{i,1} - C^N_{i,1} \right) + \frac{\partial p_1}{\partial B^T_{i,1}} \gamma_B \lambda_B \phi Y^N_{B,1}. \]

Two equalities are useful for the subsequent manipulations: Based on the planner’s intratemporal risk sharing, the weighted marginal utilities of tradable period 0 consumption equal across agents, that is, \( \gamma_B u^T_{B,0} = \gamma_S u^T_{S,0} \). The nontradable market clearing condition implies \( \left( Y^N_{B,1} + B^N_{B,1} - C^N_{B,1} \right) = - \left( Y^N_{S,1} + B^N_{S,1} - C^N_{S,1} \right) \). Subtracting the tradable Euler equations for both households from each other therefore yields

\[ \gamma_B u^T_{B,1} - \gamma_S u^T_{S,1} = \frac{-\gamma_B \lambda_B \phi Y^N_{B,1} \left( \frac{\partial p_1}{\partial B^B_{B,1}} - \frac{\partial p_1}{\partial B^S_{B,1}} \right)}{1 + \left( \frac{\partial p_1}{\partial B^B_{B,1}} - \frac{\partial p_1}{\partial B^S_{B,1}} \right) \left( Y^N_{B,1} + B^N_{B,1} - C^N_{B,1} \right)}. \]
Plugging this expression back into the international bond first order condition yields

\[
\gamma_i u_{i,0}^T = \gamma_i u_{i,1}^T + \frac{\partial p_1}{\partial B_{i,1}^T} \left( \frac{\gamma_B \lambda_B \phi Y_{B,1}^N}{1 + \left( \frac{\partial p_1}{\partial B_{B,1}^T} - \frac{\partial p_1}{\partial B_{S,1}^T} \right) \left( Y_{B,1}^N + B_{B,1}^N - C_{B,1}^N \right)} \right) .
\]

The equation is subsequently divided by \( \gamma_i u_{i,0}^T \). Because \( \gamma_i u_{i,0}^T = \gamma_j u_{j,0}^T \), one obtains the first two equations of Proposition 1:

\[
\begin{align*}
\frac{u_{B,1}^T}{u_{B,0}^T} &= 1 - \frac{\lambda_B}{u_{B,0}^T} \frac{\partial p_1}{\partial B_{B,1}^T} \frac{\phi Y_{B,1}^N}{D} \\
\frac{u_{S,1}^T}{u_{S,0}^T} &= 1 - \frac{\lambda_B}{u_{S,0}^T} \frac{\partial p_1}{\partial B_{S,1}^T} \frac{\phi Y_{B,1}^N}{D} .
\end{align*}
\]

The domestic bond first order condition for household \( i \in \{B, S\} \) is

\[
\gamma_i u_{i,0}^N (1 - \beta_3) = \gamma_i u_{i,1}^N + \frac{\partial p_1}{\partial B_{i,1}^N} \left( \gamma_i u_{i,1}^T - \gamma_j u_{j,1}^T \right) \left( \gamma_i u_{i,1}^N + B_{i,1}^N - C_{i,1}^N \right) + \frac{\partial p_1}{\partial B_{i,1}^N} \gamma_B \lambda_B \phi Y_{B,1}^N .
\]

Following the same steps as with the international bond Euler equation implies

\[
\gamma_i u_{i,0}^N (1 - \beta_3) = \gamma_i u_{i,1}^N + \frac{\partial p_1}{\partial B_{i,1}^N} \left( \frac{\gamma_B \lambda_B \phi Y_{B,1}^N}{1 + \left( \frac{\partial p_1}{\partial B_{B,1}^N} - \frac{\partial p_1}{\partial B_{S,1}^N} \right) \left( Y_{B,1}^N + B_{B,1}^N - C_{B,1}^N \right)} \right) .
\]

The equation is divided by \( \gamma_i u_{i,0}^N (1 - \beta_3) \). Further, \( \gamma_i u_{i,0}^N = \gamma_j u_{j,0}^N \). Because \( \frac{\partial p_1}{\partial B_{B,1}^N} = -\frac{\partial p_1}{\partial B_{S,1}^N} \), one immediately derives the last two equations of Proposition 1:

\[
\begin{align*}
\frac{u_{B,1}^N}{u_{B,0}^N} \frac{1}{1 - \beta_3} &= 1 - \frac{1}{1 - \beta_3} \frac{\lambda_B}{u_{B,0}^N} \frac{\partial p_1}{\partial B_{B,1}^N} \frac{\phi Y_{B,1}^N}{D} \\
\frac{u_{S,1}^N}{u_{S,0}^N} \frac{1}{1 - \beta_3} &= 1 + \frac{1}{1 - \beta_3} \frac{\lambda_B}{u_{S,0}^N} \frac{\partial p_1}{\partial B_{B,1}^N} \frac{\phi Y_{B,1}^N}{D} .
\end{align*}
\]

\[\blacksquare\]
Proof of Lemma 2: The regulated equilibrium is constrained efficient and part of the social planner Pareto frontier if two conditions are met: First, the regulated allocation must be feasible for a social planner. Second, the regulated allocation must not violate the optimality conditions of a social planner. The latter is a sufficient condition for optimality due to the concavity of the optimization problem.

I summarize the allocation of the decentralized regulated equilibrium in vector $\mathbf{Y}^{CE}$. Because the social planner only intervenes in $t = 0$, it is sufficient to show that $\mathbf{Y}^{CE}$ does not violate the social planner constraints (9), (10) and (11). Bond market clearing and nontradable goods market clearing in the regulated equilibrium (see Definition 2 for an identical formulation in the unregulated equilibrium) are equivalent to the social planner constraints (10) and (11). Aggregating over the period 0 budget constraint (12), imposing the definition of lump-sum transfers, and nontradable bonds/goods market clearing immediately yields Equation (9). The allocation $\mathbf{Y}^{CE}$ in the regulated competitive equilibrium is therefore feasible for a social planner.

Suppose the social planner implements $\mathbf{Y}^{CE}$. I subsequently show that this allocation does not violate the optimality conditions of the planner. Following Lemma 2, the Euler equations in the regulated equilibrium are:

$$
\frac{u^{T}_{B,1}}{u^{T}_{B,0}} = 1 - \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{T}_{B,0} \partial B^{T}_{B,1} D}
$$

$$
\frac{u^{T}_{S,1}}{u^{T}_{S,0}} = 1 - \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{T}_{S,0} \partial B^{T}_{S,1} D}
$$

$$
\frac{u^{N}_{B,1}}{u^{N}_{B,0}} R = 1 - R \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{N}_{B,0} \partial B^{N}_{B,1} D}
$$

$$
\frac{u^{N}_{S,1}}{u^{N}_{S,0}} R = 1 + R \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{N}_{S,0} \partial B^{N}_{B,1} D}
$$

The social planner’s intertemporal first order conditions correspond to

$$
\frac{u^{T}_{B,1}}{u^{T}_{B,0}} = 1 - \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{T}_{B,0} \partial B^{T}_{B,1} D}
$$

$$
\frac{u^{T}_{S,1}}{u^{T}_{S,0}} = 1 - \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{T}_{S,0} \partial B^{T}_{S,1} D}
$$

$$
\frac{u^{N}_{B,1}}{u^{N}_{B,0}} \frac{1}{1 - \beta_{3}} = 1 - \frac{1}{1 - \beta_{3}} \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{N}_{B,0} \partial B^{N}_{B,1} D}
$$

$$
\frac{u^{N}_{S,1}}{u^{N}_{S,0}} \frac{1}{1 - \beta_{3}} = 1 + \frac{1}{1 - \beta_{3}} \frac{\lambda_{B} \partial p_{1} \phi Y_{B,1}^{N}}{u^{N}_{S,0} \partial B^{N}_{B,1} D}
$$

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None of the decentralized Euler equations violate the social planner’s Euler equations. Further, it immediately follows that \( \frac{1}{1-\beta_3} = R \). The intratemporal optimality condition in the regulated equilibrium is

\[ u^N_{i,0} = p_0 u^T_{i,0}. \]

For the social planner, the trade-off between tradable and nontradable consumption is characterized by

\[ u^N_{i,0} = \beta_2 u^T_{i,0}. \]

Hence, one obtains \( \beta_2 = p_0 \) and the intratemporal first order condition is satisfied. The allocation \( Y^{CE} \) is therefore consistent with the first order conditions of the social planner. The allocations in a planner equilibrium are pinned down by the Pareto weights \( \{\gamma_B, \gamma_S\} \). Specifically, \( C^T_{B,0} = \frac{\gamma_B}{\gamma_S} C^T_{S,0} \) and \( C^N_{B,0} = \frac{\gamma_B}{\gamma_S} C^N_{S,0} \). Hence, there must exist a ratio of \( \gamma_B \) and \( \gamma_S \) that implements the regulated equilibrium as a social planner equilibrium.

**Proof of Proposition 2**: Domestic regulation corresponds to

\[
\tau^D_{MP} = R \frac{\lambda_B}{u^N_{B,0}} \frac{\partial p_1}{\partial B^N_{B,1}} \frac{\phi Y^N_{B,1}}{D}. 
\]

International regulation is summarized by

\[
\tau^I_{MP} + \tau_{CC} = \frac{\lambda_B}{u^T_{B,0}} \frac{\partial p_1}{\partial B^T_{B,1}} \frac{\phi Y^N_{B,1}}{D}. 
\]

It is then straightforward to verify that

\[
\tau^D_{MP} = (\tau^I_{MP} + \tau_{CC}) \frac{u^T_{B,0}}{u^N_{B,0}} R \frac{\partial p_1}{\partial B^N_{B,1}} \frac{\partial p_1}{\partial B^T_{B,1}}. 
\]
References


