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Wealth Distribution with State-dependent Risk Aversion^{*}

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Abstract

A growing body of literature has suggested that agents' risk attitudes may not be constant and are correlated with factors such as wealth. We introduce state-dependent risk aversion into Aiyagari's (1994) heterogenous-agent version of standard neoclassical growth model with uninsurable idiosyncratic shocks to earning. We first quantitatively show the relationships among risk aversion, saving rate, equilibrium interest rate and wealth distribution. In particular, we show that if agent's risk aversion increases with wealth, the model predicts a larger wealth inequality, while assuming risk decreases with wealth leads to a smaller wealth inequality. We then use experimental data to estimate how risk aversion is correlated with an individual's wealth. We found that the relationship between risk aversion and wealth is hump-shaped. That is, risk aversion first increases with an individual's wealth and then decreases with it. Using the same model, we quantify the implication of this relationship between risk aversion and wealth on wealth inequality. The results show that the overall wealth inequality changes very little compared to the case with constant risk aversion. This is because, though the poorest agents save less, which increases wealth inequality, the richest agents also save less, which reduces the wealth inequality. Putting these two together, it leaves the overall wealth inequality implied by the model similar to that of the case of constant risk aversion.

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1 Introduction

How much agents save depends on the uncertainty agents face as well as the agents' attitude toward risks. In most models which study the wealth distribution (saving is one due option), such as the Aiyagari (1994) type of heterogeneous-agent models, agents' risk attitude are assumed to be constant. Although the assumption of constant relative risk aversion (CRRA) is commonly used and empirically supported in economics (Brunnermeier and Nagel (2006); Chiappori and Paiella (2011)) recent studies have found that agents risk aversion may not be constant (Arrow (1965); Dohmen et al. (2011); Eckel and Grossman (2008); Halek and Eisenhauer (2001); Laffont (1989); Pålsson (1996)). In particular, a growing body of literature has suggested agents' risk attitudes are correlated with their wealth (such as Bucciol and Miniaci (2011); Dohmen et al. (2011); Wik et al. (2004)). Besides these empirical studies in economics, psychologists also add evidences for such speculation. For example, Anderson and Galinsky (2006) shows that more wealth will bring more power and generate more optimistic risk perceptions which results in greater preference for risk. Griskevicius et al. (2011) shows that, for individuals who grew up relatively wealthy, mortality cues led them to value the future and avoid risky gambles. There is no consistent result regarding the relationship between wealth and risk aversion. Based on both intuition and observations of rational behavior, we can assume that absolute risk aversion decreases with wealth Laffont (1989). However, Arrow (1965) argues that RRA should increase with wealth over the whole domain of the utility function. In some special context, Padmanabhan and Rao (1993) find that higher-income consumers are more likely to buy extended service contracts in the automobile category. Experimental studies, like Bosch-Domènech and Silvestre (1999), find the possible moderation variables which will change the relation between wealth and risk aversion.

Motivated by these findings, we study the implications of various specifications on agents' risk aversion to wealth distribution. To help understand how changing risk aversion influence wealth distribution, we first increase the constant risk aversion coefficient of *all* the agents and see how the wealth distribution changes. The purpose is to explore the relationships among risk aversion, saving behaviors, equilibrium interest rate and wealth distribution. We find that increasing the risk aversion coefficient makes the wealth distribution more equal. The intuition is as follows. As agents become more risk averse they save more to insure against earning uncertainty. The increase of aggregate asset holding pushes the interest rate down to clear the asset market. The low interest rate reduces the rich agents' earning from the asset market but has smaller effect on poor agents since they do not hold much asset. On the other hand, the expansion of capital used in production raises the marginal production of labor and hence raises wage. The increase of wage benefits both rich agents and poor agents by the same amount. This shifts the wealth

from the asset market to labor market and therefore shifts the wealth from the rich to the poor. Thus, although both rich and poor agents's *absolute* wealth increases, the *percent* of wealth held by the rich decreases.

Next we introduce the state-dependent risk aversion into this model and assume that agents' risk aversion changes with their wealth level. We discuss both pro-cyclical risk aversion case and counter-cyclical risk aversion case. Generally speaking, the pro-cyclical risk aversion specifications make the wealth distributions more unequal (i.e, the Gini wealth coefficient is larger) than the constant risk aversion case while the counter-cyclical risk aversion specifications make the wealth distributions more equal (i.e, the Gini wealth coefficient is smaller) than the constant risk aversion case. The intuition is as follows. The pro-cyclical risk aversion assumption makes rich agents more risk averse thus precautionary saving motivation lets them save even more than before. Similarly, since poor agents become less risk averse they will decrease their saving rates. This makes the rich accumulate their wealth at a higher rate and the poor accumulate their wealth at a lower rate than before, resulting in a less equal wealth distribution. The contrary holds for the counter-cyclical risk aversion case. In that situation, rich agents become less risk averse than before while the poor agents' attitude of risk averse increases. This makes the poor save relatively more and the rich save relatively less than before thus the wealth distribution becomes more equal.

After theoretically exploring how different specifications on risk aversion change wealth distribution, we proceed to empirically examine the relationship between risk aversion and wealth. We use experimental data to estimate how risk aversion is related to wealth. Then, assuming agents' risk aversion varies with wealth in the way consistent with the empirical relationship, we re-examine the wealth distribution. The main results are summarized as follows. First, consistent with the literature, we found agents' risk aversion is correlated with agents' wealth, education attainment, and gender. Second, after controlling other factors, agents' risk aversion varies with the wealth level in a non-monotonic way. It is hump-shaped: it first increases with the wealth level and then declines with it. Third, using a risk aversion function consistent with the empirical relationship, we re-examine the wealth distribution. We found the induced wealth inequality is similar to the one generated from the economy with constant risk aversion. This is a result of two competing forces. On the one hand, the poorest part of agents in the economy will save less (as their risk aversion is low) which increases wealth inequality. On the other hand, the richest part of agents also save less for the same reason, which reduces the wealth inequality.¹ Overall, these results suggest that even if agents' risk aversion is not constant, it won't change the wealth inequality at the aggregate level for the reasons we just explained. In other words,

¹In order to increase the wealth inequality, we need rich people to save more so the implied wealth distribution will be skewed to right.

in terms of wealth distribution, the CRRA assumption could be used a good approximation in macroeconomic models.

Our paper is related to three branches of literature. First, it is related to papers using heterogeneous-agent models to study wealth distribution (Díaz-Giménez et al. (1997); Krusell and Smith (1998); Quadrini (1999); De Nardi (2004); Cagetti and De Nardi (2006)). Second, it is related to the literature which empirically examines how risk aversion is related to other factors. One group of factors are demographics. For example, women are more risk averse than men (Eckel and Grossman (2008); Dohmen et al. (2011); Pålsson (1996)). The older people tend to take fewer risks than the younger (Dohmen et al. (2011); Pålsson (1996)), and education level significantly reduces risk aversion (Guiso and Paiella (2008); Halek and Eisenhauer (2001)). Other factors include affect (Isen and Patrick (1983); Isen (2008); Loewenstein et al. (2001)) and task characteristics and decision contexts (Harrison et al., 2007). Third, it is related to experimental literature using field studies to measure agents' risk attitude. A recent experimental and survey study by Falk et al. (2011) measures risk attitudes on a carefully constructed representative sample of the adult population living in Germany. They find risk attitudes to be significantly positively correlated with income, and to a lesser extent, with wealth. Another survey study among Indonesian families by Ng (2013) finds wealthier respondents appear to be less risk averse in survey-elicited risk preferences. We also measured individual risk attitudes in the lab and explore their correlation with wealth. One might be concerned about the endogeneity of wealth, that is, the wealthier subjects have more cushion and hence is willing to take more risk in the experiment or survey. But this problem may not be as severe as shown in Heinemann (2008). He found evidence from various experimental studies that initial wealth from outside the laboratory is not fully integrated when individual make decisions and laboratory-measured risk attitude is free of the effect of the individual's current wealth.

This paper is organized as follows. Section 2 uses a standard heterogeneous-agent model to explain how various specifications on risk aversion change the wealth distribution. Section 3 describes the experiment we designed to measure the risk aversion and provides empirical analysis to quantify the relationship between wealth and risk attitudes. Section 4 quantifies risk aversion and reports the effects on wealth distribution using an estimated risk aversion function. Section 5 concludes with some suggestions for future work.

2 The economies investigated

To highlight the role played by the risk aversion coefficient, in the benchmark model we follow as closely as possible the environment introduced by Aiyagari (1994) and only change the

specification of agents' degree of risk aversion. In this section we describe the basic fixed elements of this economy. All the economies that will be investigated in the following sections are either special cases or simple modifications of the economy described in this section.

2.1 Production

There is a neoclassical aggregate production function, $Y_t = A_t K_t^\alpha L_t^{1-\alpha}$, where Y_t is per capita output, K_t is per capita capital, L_t is per capita labor input. Capital is assumed to depreciate at the constant rate δ . Product and factor markets are assumed to be competitive, so the wage rate w_t and the interest rate r_t satisfy

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha) \frac{Y_t}{L_t} \quad (1)$$

$$r_t = \frac{\partial Y_t}{\partial K_t} - \delta = \alpha \frac{Y_t}{K_t} - \delta \quad (2)$$

2.2 Agent's problem

We now describe agent's problem. The economy considered here has a continuum of infinitely lived agents of measure unity who receive idiosyncratic shocks to their labor productivities and supply one unit of labor inelastically. Let l_t denote an individual's labor productivity, and suppose that this productivity is i.i.d. across agents and follows some Markov process over time. We normalize per capita labor productivity to unity so that $E(l_t) = 1$. We also assume that there is no aggregate uncertainty. Let c_t , a_t and l_t denote period t consumption, assets, and the labor endowment. Let $U(c)$ be the period utility function, β be the utility discount factor, r be the return on assets, and w be the wage. The household's problem is to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(c_t) \right\} \quad (3)$$

subject to

$$c_t + a_{t+1} = w_t l_t + (1 + r) a_t \quad (4)$$

$$a_t \geq -\phi \quad (5)$$

where ϕ is the limit on borrowing. As explained by Aiyagari (1994), this borrowing limit can be either the natural borrowing limit or an ad hoc one. Therefore, without loss of generality, we may specify the limit on borrowing as

$$\phi \equiv \min\{b, w l_{\min}/r\} \quad (6)$$

where b is the ad hoc borrowing limit.

2.3 Asset markets

There is only one asset in the asset market which is the private capital hold by the agents. Since agents receive uninsurable idiosyncratic shocks on their labor productivity, they will trade with their private capital (subject to the borrowing constraint) to smooth their consumption.

2.4 Equilibrium

We can define the stationary equilibrium as follows:

Definition 1. A stationary competitive equilibrium for this economy consists of decision rules $c_t(a_t, l_t)$, $a_{t+1}(a_t, l_t)$ for consumption and asset holding, together with the value function, V ; an interest rate r , a wage rate w and aggregate capital K ; a stationary distribution of assets and labor endowment, $\omega(a, l)$; such that

- the interest rate and wage rate satisfy (1) and (2)
- given the interest rate r , the decision rule solves the household's problem in (3) – (5)
- the asset market clears:

$$\int \int a_{t+1}(a_t, l_t) d\omega = K \quad (7)$$

- $\omega(a, l)$ is a stationary distribution, that is, it satisfies

$$\omega(B) = \int_X P(x, B) d\omega \quad (8)$$

2.5 Specify the preference

After specifying the basic elements of the model economy we are now in the place to specify the agent's preference carefully. We begin with the economy in which the degree of risk aversion is constant for all the agents. Then, we introduce the idea of state-dependent risk aversion. Specially, we assume agent's attitude of risk aversion varies with his wealth level. This is the main part of this paper.

The utility function we use here is very *similar* to the Constant Relatively Risk Aversion (CRRA) utility function. However, the risk aversion coefficient can be either constant or a function of wealth. So, the general form of the utility function we propose here is

$$u(c, w) = \frac{c^{1-\gamma(w)}}{1-\gamma(w)} \quad (9)$$

where the risk aversion coefficient $\gamma(w)$ is a function of wealth. $\gamma'(w) = 0$ means the risk aversion coefficient is constant; $\gamma'(w) > 0$ means agent's risk aversion coefficient increases with his wealth,

which implies agents are more risk aversion when they become richer; $\gamma'(w) < 0$ means agent's risk aversion coefficient decreases with his wealth, which implies agents are more risk aversion when they become poorer.

2.5.1 Constant risk aversion

Constant risk aversion is the most commonly used assumption in the macroeconomics papers. It plays an important role on determining the risk free rate and aggregate saving in Aiyagri's heterogenous-agent model. But how the degree of risk aversion affects the wealth distribution remains unclear in this framework. So with this question in mind, my first experiment is conducted as follows. We still keep the assumption that the agent's risk aversion coefficient is constant but assign different values to this parameter. Then we observe how the resulting wealth distribution changes with it. The result is reported in the next section.

2.5.2 State-dependent Risk aversion

Recent evidence from experimental psychology shows that agents in different status show different risk aversion attitudes.(Some examples to be added here.) So, in this section we assume the risk aversion coefficient is function of wealth, $\gamma(w)$.

The main problem is how to specify this function form. First, most literature agree that this function should be monotonic. But there is still a debate on whether this function is increasing (pro-cyclical risk aversion) or decreasing (counter-cyclical risk aversion). (More examples to be added here.) In addition, no literature shows *how* people's risk aversion coefficient increases or decreases with the wealth level. Thus, to make the analysis as complete as possible, in the benchmark model we propose six possible risk aversion functions as shown in Figure 1. The first three panels describe the cases that agent's risk aversion attitude is pro-cyclical. For case 1, the risk aversion function $\gamma(a)$ increases rapidly in the middle part and slowly in two sides. This implies that agent's degree of risk aversion is more sensitive to the change of wealth level in the middle part than in the two sides. In case 2, agent's degree of risk aversion γ increases proportionally with their wealth level a . For case 3, the risk aversion function $\gamma(a)$ increases slowly in the middle part but rapidly in two sides. This implies that agent's degree of risk aversion is less sensitive to the change of the wealth level in the middle part than in the two sides. Similarly, case 4, 5, 6 describe the three situations in which agent's risk aversion attitude is counter-cyclical.

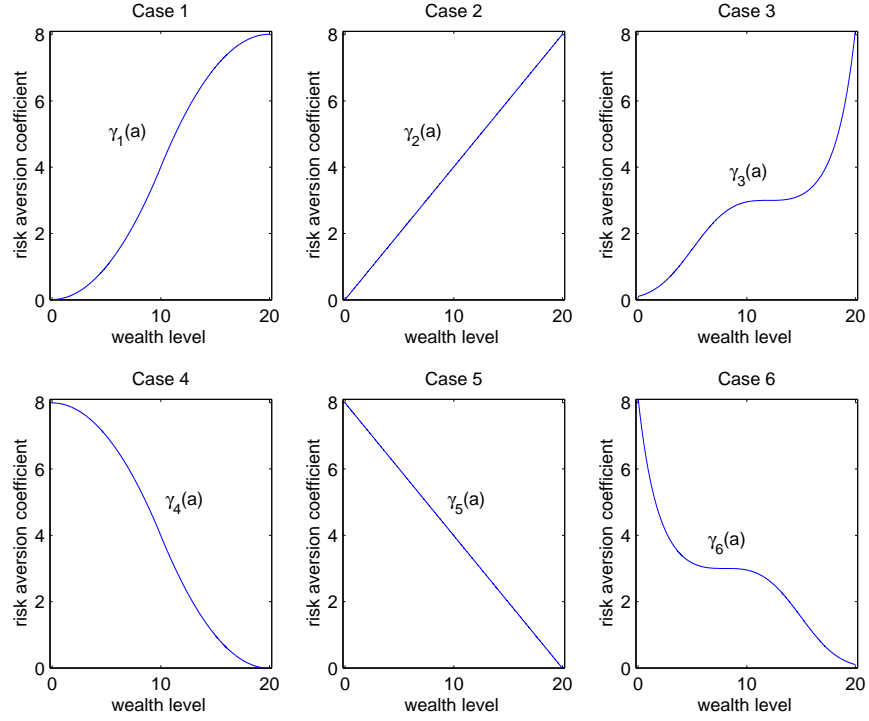


Figure 1: This graph shows six possible shapes of the risk aversion function. The upper three panels show the situations in which agent's risk aversion attitude increases with his wealth level (pro-cyclical risk aversion). The bottom three panels show the situations in which agent's risk aversion attitude decreases with his wealth level (counter-cyclical risk aversion).

2.6 Parametrization and Computation

Since the results of this paper mainly rely on quantitative analysis, we first calibrate the model economies. Except for the specification of the risk aversion function, all the other parameters values are taken from the recent literature.

As in Aiyagari’s model, the model period is taken to be one year. The utility discount factor β is chosen to be 0.95 and the depreciation rate of capital δ is set at 0.08.

The natural logarithm of the idiosyncratic labor endowment shock l_t is assumed to be a first-order autoregressive process:

$$\log(l_t) = \bar{l}_t + \rho \log(l_{t-1}) + e_t \quad (10)$$

where e_t is i.i.d normal with mean zero and standard deviation σ . ρ is the correlation coefficient of the first-order autoregressive process. In the benchmark model, ρ and σ are set to be 0.53 and 0.24 which are the estimated values from Heaton and Lucas (1996).

After specifying these two parameters, we use the procedure of Tauchen (1986) to approximate the autoregression of $\log(e_t)$ with a first-order Markov chain that has N states.² Thus the Markov process are characterized by two states and a probability transition matrix.

All the parameters values are listed in the following table.

Table 1

depreciation rate	δ	0.08
labor endowment correlation	ρ	0.53
labor endowment standard deviation	σ	0.24
capital share	α	0.36
discount factor	β	0.95
number of states	N	2
borrowing limit	b	0

The computation algorithm is based on Aiyagari’s method (QJE, 1994) and very similar with that in Ahmet Akyol’s paper (JME, 2004). First for a given interest rate r_1 we employ smooth approximation methods to obtain the agents’ policy function based on the first-order equation. Then we simulate 5000 agents at the same time. We calculate the percentiles in 20 points evenly distributed between 0% and 100% for each period. The simulation will stop until the percentiles converge under the criterion of 0.2%. The sample mean of agents’ asset holding in the stopping period is taken to be Ea . Using the production product function we then calculate r_2 such

² N is set to be 2 in the benchmark model.

that $K(r_2)$ equals Ea .³(Without loss of generality we assume that $r_1 < r_2$.) Then we define $r_3 = (r_1 + r_2)/2$ and calculate Ea corresponding to r_3 . If $Ea > K(r_3)$ then we replace r_2 by r_3 . If $Ea < K(r_3)$ then r_1 is replaced by r_3 . We continue this process until we find some r_3 such that $Ea = K(r_3)$. Then this interest rate is the equilibrium interest rate which clears the asset market. This method is called bisection method. It provides an excellent approximation to the steady state within few iterations. But note that by using this method we have assumed the aggregate asset holding of the agents monotonically increases with the interest rate r .⁴ We leave a detailed description on how to approximate agents' policy function and how to simulate the policy function in Appendix B.

2.7 Theoretic results and discussions

2.7.1 Constant risk aversion case

We shall now show the numerical results. In the first exercise all the agents are assumed to have the same *constant* degree of risk aversion. We then increase this risk aversion coefficient from 2 to 3, 4, 5, 8 one by one. For each case we calculate the Aggregate Supply Curve (AS) which shows the aggregate amount of asset held by agents at different interest rates. These curves are displayed in Figure 2. The Aggregate Demand Curve (AD) is also drawn in this figure. This curve graphs the amount of capital demanded by the production side at different interest rates. We can see the AS curve shifts to the right as γ increases. Since the equilibrium interest rate is determined by the intersection of the AS curve with AD curve and AD curve doesn't change, the equilibrium interest rate becomes lower as γ increases. As Table 2 shows, the interest rate drops from 3.66% to 2.21% as risk aversion coefficient increases from 2 to 8. This has a good intuition. As agents become more risk averse, they will increase their asset holding to insure their earning uncertainty. In equilibrium the aggregate amount of asset held by agents equals to the aggregate amount of capital used in production. Since the production side doesn't change, the interest rate has to go down to meet the demand side with the supply side.

To see how saving rates change with the agent's risk aversion coefficient, we draw all the saving rate curves associated with the high labor productivity shock in Panel *b* of Figure 3. First we see the saving rate curves are very close to each other. But we can still observe the saving rates of the rich agents slightly go up and the poor agents' saving rates slightly go down as risk aversion coefficient increases. To see more clearly this property, Panel *c* and *d* in Figure 3 draw

³ $K(r)$ is the amount of capital used in production when the interest rate is r .

⁴As in Aiyagari's paper (1994) we also did not provide a formal proof for this property. But numerical results show this is true in most cases.

the differences between the saving rates at $\gamma = 8$ and that at $\gamma = 2$. It shows the saving rates of the rich agents increase but the saving rates of the poor agents decrease in both high shock and low shock situations as agent's risk aversion coefficient increases from 2 to 8.

Table 2 and Panel *a* in Figure 3 summarize the wealth distributions in these economies. As Panel *a* displays, the steady-state wealth distributions of the economies with different degrees of risk aversion are also very close to each other. Actually, the Gini coefficient only decreases from 0.3425 to 0.3205 as risk aversion coefficient increases from 2 to 8.

Now careful readers may raise two questions concerning on these results:

- That the saving rates of rich agents increase and that of the poor agents decrease implies the rich should accumulate more wealth and the poor should become even poorer. But why does the Gini wealth coefficient go down rather than go up?
- Why the saving rate of the poorest agents decreases as the risk aversion coefficient increases?

To answer the first question we need to find out all the potential factors which are able to affect the wealth distribution. Let's first look at the wealth of one special agent. In the current framework, his wealth (W) is composed of his current asset holding(a), his interest rate earning (ra) and his labor earning (lw), where w is the wage rate and l is the current labor productivity endowment. Thus,

$$W = a + r \cdot a + l \cdot w \quad (11)$$

And remember the asset in this period is what he saved last period. So,

$$a = s \cdot W_{-1} \equiv a(s) \quad (12)$$

where s is the saving rate and W_{-1} is the wealth in previous period. And in steady state the wage can be expressed as a function of interest rate r (see Appendix C),

$$w = (1 - \alpha) \left(\frac{\alpha}{r + \delta} \right)^{\alpha/1-\alpha} \equiv w(r) \quad (13)$$

Note $a'(s) > 0$ and $w'(r) < 0$. So, the increase of saving rate will increase the asset holding⁵ and the decrease of interest rate will increase the wage. As a result, for a typical rich agent in the economy, the increase of saving rate ($s \uparrow$) and decrease of interest rate ($r \downarrow$) lead to his asset holding increasing ($a \uparrow$), labor income increasing ($lw \uparrow$) and interest rate earning either increasing or decreasing ($ra \uparrow$ or \downarrow);⁶ for a typical poor agent, the decrease of saving rate ($s \downarrow$) and decrease

⁵This also requires W_{-1} do not decrease too much. In the computation result, as γ increases from 2 to 8, the wealth held by agents increases.

⁶Since r decreases and a increases, the interest rate earning ra can either increase or decrease.

of interest rate ($r \downarrow$) lead to his asset holding either decreasing or increasing ($a \downarrow$ or \uparrow), labor income increasing ($lw \uparrow$) and interest rate decreasing or increasing (\downarrow or \uparrow).⁷ In addition, rich agent hold much more asset than the poor agent (in other words, the asset holding for the poor agent is very small) while the labor earning of the rich agent and that of the poor agent are the same. This implies that for the poor agent the increase of labor earning (due to the increase of wage) constitutes the main part of his wealth changing. But for the rich agent the changing in asset market earning may dominate the changing in labor market earning. We summarize these results in following two equations

$$W_{rich} = a \uparrow + r \cdot a(\uparrow \text{ or } \downarrow) + l \cdot w \uparrow$$

$$W_{poor} = a(\uparrow \text{ or } \downarrow) + r \cdot a(\uparrow \text{ or } \downarrow) + l \cdot w \uparrow$$

Now we can understand why the wealth distribution looks more equal (i.e, Gini coefficient falls) as agents become more risk averse. As we see, the increase of risk aversion coefficient lowers the interest rate and increases the wage. Since the poor hold relatively less asset than the rich, the drop in asset return will have a relatively *less* effect on the poor. On the other hand, and also most importantly, the increase of wage in labor market will increase the earning of the agents (both the rich and the poor) by the *same* amount. The latter effect make it possible that the *percent* of wealth held by the rich decreases. We provide a simple example here to help us understand this possibility. Suppose there are two agents in the economy, agent 1 and agent 2. At beginning, agent 1 owns 20 units of asset and agent 2 owns 1 unit of asset. Both agent 1 and 2 have one unit of labor endowment. Assume the wage is 1 and interest rate is 0.04 at the beginning. So at the beginning the wealth of agent 1 and 2 are 21.8 and 2.04. Suppose after the agent's risk aversion increases, the interest rate falls to 0.02 and wage increases to 1.5. And assume agent 1 hold 1 more unit of asset so have 21 unit of asset but agent 2 reduce saving and only have 0.9 unit of asset.⁸ So the wealth of agent 1 and 2 are 22.94 and 2.418 now. Although agent 1 increase saving and agent 2 reduce saving, the percent of wealth held by agent 1 decreases from 91.44% to the 90.46%.

We summarize the intuition is as follows. As agents become more risk averse they save more to insure the earning uncertainty. The increase of aggregate asset holding push the interest rate

⁷If his wealth decreases, the asset holding decreases and interest rate earning decreases. If the his wealth increases (due to the increase in labor market earning), the asset holding may increase although his saving rate decrease and similarly the interest rate earning may increase.

⁸Here we let agent 2 decrease his asset holding to show the percent of wealth held by agent 1 may also decrease in this case. For the case agent 2 increase his asset holding the percent of wealth held by agent 1 decreases even more. In computation result, the asset held by poor agents increases as the risk aversion coefficient increases. Their saving rates decreases because their labor income increases.

down to clear the asset market. The low interest rate reduces the rich agents' earning from the asset market but has smaller effect on poor agents since they do not hold much asset. On the other hand, the expansion of capital used in production raises the marginal production of labor hence raises the wage rate in the labor market. The increase of wage benefits both rich agents and poor agents by the same amount. This shifts the wealth from the asset market to the labor market and therefore shifts the wealth from the rich to the poor. Thus, although both rich and poor agents's *absolute* wealth increases the *percent* of wealth held by the rich decreases.

As to the second question we need to look at the agent's Euler equation

$$u'(c_t) = \beta(1 + r)E_t u'(c_{t+1}) \quad (14)$$

On the one hand, the decrease of the interest rate r lowers the marginal utility of consuming tomorrow therefore make the agents to consume relatively more today. On the other hand, the increasing attitude of risk aversion make agents more concerned about the earning fluctuations thus induces them to save more (this part is called precautionary saving). These two effects together determine the saving behavior of the agents. For the rich agents, the second effect dominates the first one so their saving rates increase; for the poor agents, the first effect dominates the second one thus their saving rates decrease.⁹

⁹Although saving rate decreases their absolute saving increases.

Table 2

Wealth Distribution and Equilibrium Interest Rates

Constant Risk Aversion Case

Model	Wealth% Held					Wealth% Held	Equilibrium	GINI
	by Top					by Bottom	Interest	Coeffi
	1%	5%	10%	20%	30%	40%	rate(%)	
Risk aversion								
coefficient								
$\gamma = 2$	2.4	11	21	39	54	15	3.66	.353
$\gamma = 3$	2.3	11	21	38	53	16	3.34	.348
$\gamma = 4$	2.3	11	20	38	53	16	3.02	.343
$\gamma = 5$	2.3	11	20	38	53	16	2.67	.343
$\gamma = 8$	2.3	11	20	37	52	17	2.21	.330
Borrowing limit								
$b = 0$	2.3	11	21	38	53	16	3.34	.348
$b = 0.5$	2.3	11	21	38	53	15	3.35	.352
$b = 1$	2.3	11	21	38	54	15	3.36	.362

2.7.2 State-dependent risk aversion case

The previous section says if we increase (or decrease) the degree of risk aversion of all the agents by the same amount, the saving rate curve and Lorenz curve of wealth holding won't change much. This leaves us a question: will the Lorenz curve of wealth holding change much if we assume that agent's risk aversion is state-dependent? For example, what do the wealth distribution and Lorenz curve look like in an economy in which people becomes more (less) risk averse when they become richer (poorer)? Will the Lorenz curve change much under this assumption? This section will give an answer to these questions.

To make the analysis as complete as possible, we divide the state-dependent risk aversion function into 6 possible cases as shown in Figure 1. The first 3 cases represent the pro-cyclical cases of risk aversion in which the agent's degree of risk aversion increases with his wealth level. Case 4 to case 6 represent the counter-cyclical cases of risk aversion which means the agent's degree of risk aversion decreases with his wealth level. We choose some general functions to represent them (see Appendix A). The computed Lorenz curves for wealth distribution are drawn in Figure 4. To make a comparison, we also draw the Lorenz curves for the constant risk aversion case and representative-agent economy case in the same figure. The 45-degree straight line represents

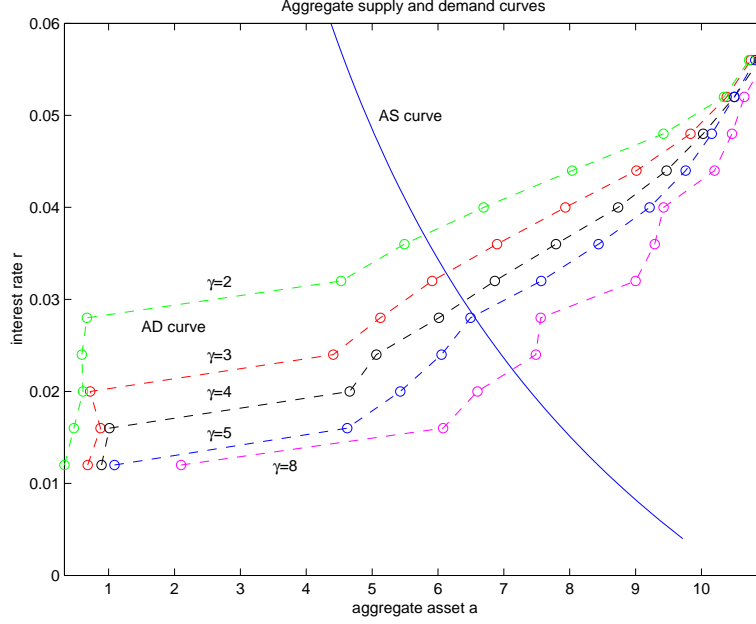


Figure 2: This graph shows the aggregate supply and demand curves of the asset. The aggregate demand curve (AD) describes the amount of asset demanded by the aggregate production side at different interest rate. The aggregate supply curve (AS) shows the aggregate amount of asset held by the agents at different interest rate. γ is the constant risk aversion coefficient faced by all the agents in the economy. This graph lays out 5 AS curves corresponding to γ equals to 2, 3, 4, 5, 8. When the risk aversion coefficient increases, the graph shows that the AS curve shifts to the right. The equilibrium interest rate which is determined by the intersection of the AS and AD curve then becomes lower when the degree of risk aversion increases.

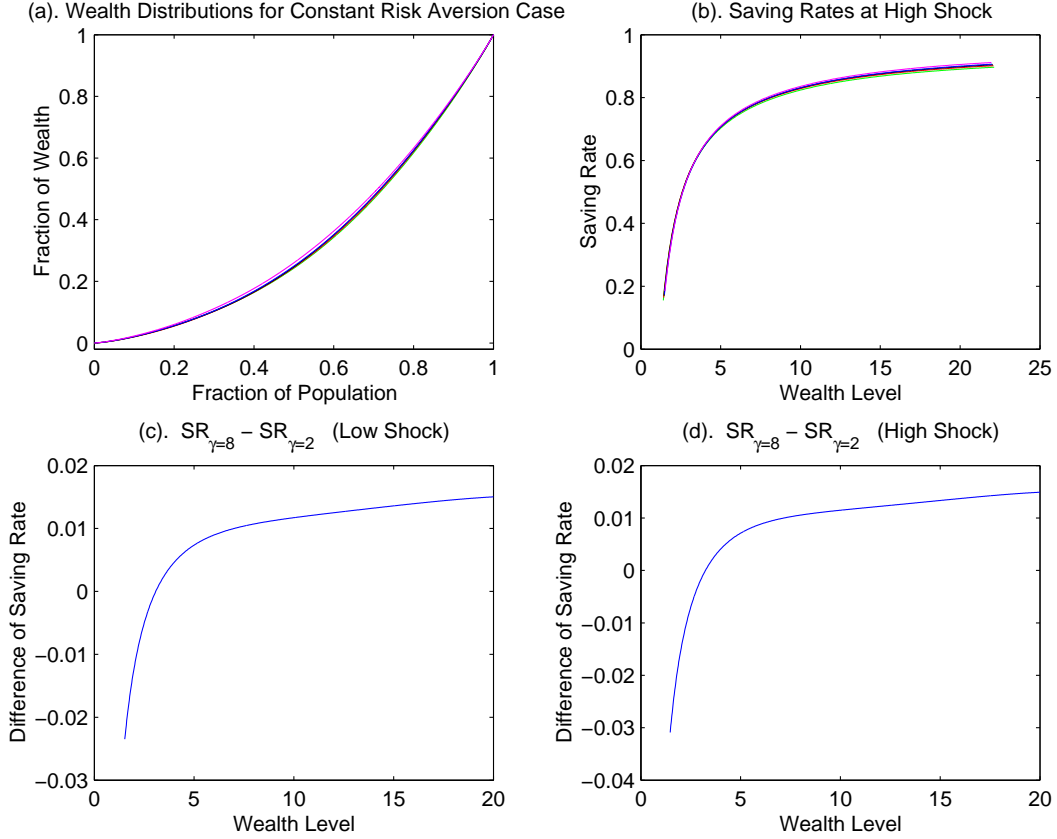


Figure 3: Panel *a* shows the steady-state wealth distributions of the economies when risk aversion coefficient γ equals to 2, 3, 4, 5, 8 separately. As we can see, the wealth distributions do not change much as the risk aversion coefficient increases. Actually, the Gini coefficient only decreases from 0.3425 to 0.3205 as risk aversion coefficient increases from 2 to 8. Panel *b* plots the saving rate curves in these economies. To see more clear how saving rate changes with the risk aversion coefficient, Panel *c* and *d* draw the difference between the saving rates at $\gamma = 8$ and that at $\gamma = 2$. It shows that the saving rates of the rich agents increase but that of the poor agents decrease as agent's risk aversion coefficient increases from 2 to 8.

the Lorenz curve of wealth holding in a representative-agent economy. Since in this economy all the agents are identical thus the Lorenz curve is a straight line. The dashed curve is the Lorenz curve of wealth holding for the economy with constant risk aversion ($\gamma = 3$ case in Figure 2). The curves labeled 1, 2, 3, 4, 5, 6 are the Lorenz curves for wealth holding corresponding to the state-dependent risk aversion case 1, 2, 3, 4, 5, 6.

As we can see from figure 4, the three Lorenz curves corresponding to the pro-cyclical risk aversion cases lie on the right side of the Lorenz curve of the constant risk aversion case and the other three Lorenz curves corresponding to the counter-cyclical risk aversion cases lie on the left side of the Lorenz curve of the constant risk aversion case.

As Table 3 shows, when assuming agents have constant risk aversions the Gini wealth coefficient is about 0.34 and the bottom 40% of the population hold 16% of total wealth.¹⁰ However, in the data, the Gini coefficient is 0.79 and the bottom 40% of population only hold 1% of the total wealth. If we assume agents' degree of risk aversion increases linearly with their wealth level (i.e, case 2), the model will predict a Gini coefficient of 0.43 and 11% of wealth held by the bottom 40% of population. Although this is still far way from the data it is much closer than the constant risk aversion case. Now if we assume the risk aversion function looks like case 3, the Gini wealth coefficient will be increased to 0.49 and the percentage of wealth held by the bottom 40% population decreases to 6.9% which is even closer to that in the U.S. data. For the risk aversion function of case 2, the Gini coefficient is also 0.51 and bottom 40% of population hold 6.8% of total wealth. Thus, even within the group of pro-cyclical risk aversion specifications (case 1, 2 and 3), the wealth distributions are also very different.

For the counter-cyclical risk aversion cases, the Gini wealth coefficients are smaller than the constant risk aversion case and the percentages of wealth held by the bottom 40% of population are larger than the constant risk aversion case.

So, generally speaking, the pro-cyclical risk aversion specifications make the wealth distributions more unequal (i.e, the Gini wealth coefficient is larger) than the constant risk aversion case while the counter-cyclical risk aversion specifications make the wealth distributions more equal (i.e, the Gini wealth coefficient is smaller) than the constant risk aversion case. The intuition is as follows. The pro-cyclical risk aversion assumption makes rich agents more risk averse thus precautionary saving motivation lets them save even more than before. Similarly, since poor agents become less risk averse they will decrease their saving rates. This makes the rich accumulate their wealth at a higher rate and the poor accumulate their wealth at a lower rate than before. As a result, the wealth distribution looks more unequal. The contrary holds for the counter-cyclical

¹⁰From last section we see when the agent's risk aversion coefficient γ varies from 2 to 8, the Gini coefficient doesn't change much.

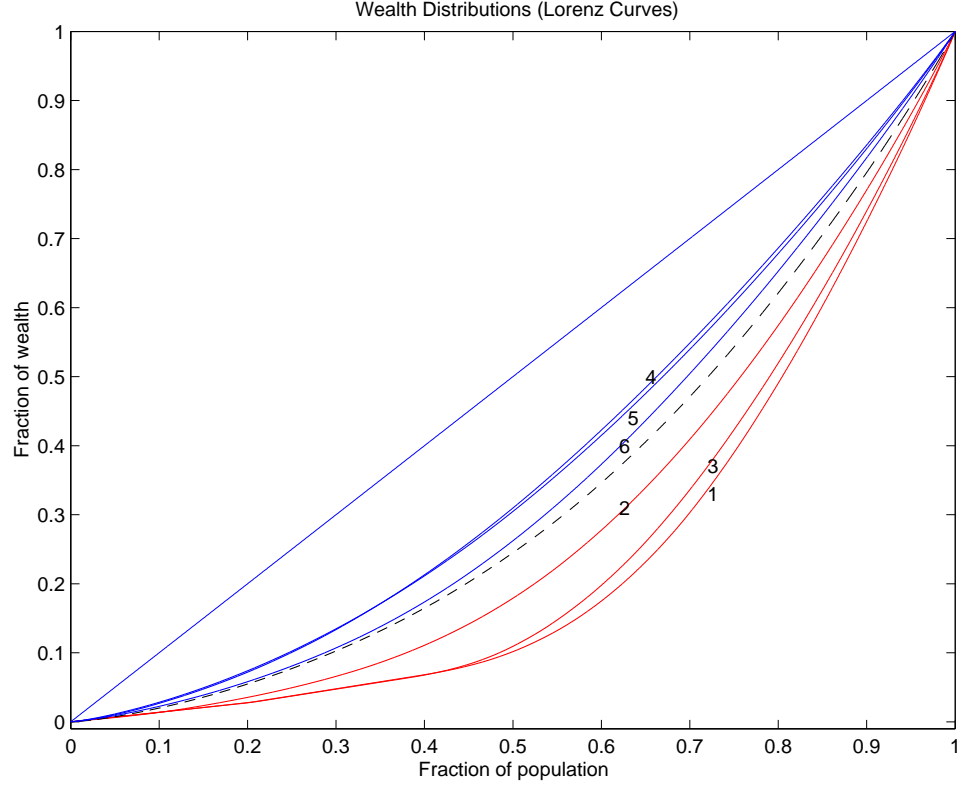


Figure 4: This graph shows the steady-state accumulated wealth distributions (Lorenz curves) from the economies with different specifications of risk aversion functions. The 45-degree straight line represents the accumulated wealth distribution in a representative-agent economy. Since in this economy all the agents are identical thus have the same wealth level the accumulated wealth distribution is a straight line. The dashed curve is the accumulated wealth distribution of the economy with constant risk aversion ($\gamma = 3$ case). The curves labeled 1, 2, 3, 4, 5, 6 are the accumulated wealth distributions corresponding to the state-dependent risk aversion case 1, 2, 3, 4, 5, 6.

risk aversion case. In that situation, rich agents become less risk averse than before while the poor agents' attitude of risk averse increases. This makes the poor save relatively more and the rich save relatively less than before thus the wealth distribution becomes more equal.

To see this more clearly we compare the saving rates in these cases in Figure 5. The upper two panels plot the difference between the saving-rate curve in the economy with pro-cyclical risk aversion case 1 (denote this economy by A below) and that in the economy with constant risk aversion case $\gamma = 3$ (denote this economy by B below). The x -axis is the wealth level of the agents and y -axis measures the difference between the agent's saving rate in economy A and economy B , $SR_A - SR_B$. As the first two panels show, this value is negative for the poor agents and positive for the rich agents. In high shock case, this difference is more than 15% for those poorest agents. For the low shock case, this difference is smaller. Similarly, the bottom two panels plot the difference between the saving-rate curve in the economy with counter-cyclical risk aversion case 4 (denote this economy by C below) and that in economy B . They show this difference $SR_C - SR_B$ is positive for the poor agents and negative for the rich agents. And this difference curve is not monotonic in the low wealth level area. The reason is as follows. On the one hand, as Table 3 shows, the interest rate in the economy C is lower than that in the economy B . Thus from equation (14) we know the wage in the economy C , is higher than that in the economy B . The high labor income makes the poorest agents in the economy C richer than those in the economy B . On the other hand, the poorest agents are in short of consumption thus have a higher consumption tendency as shown in Panel c and d of Figure 9. So, as their wealth increases, they will spend more on consumption and less on saving. Thus, the saving rate of those poorest agents increase less than that of *relatively* rich agents, which means the saving-rate curve increases at the lowest wealth area.

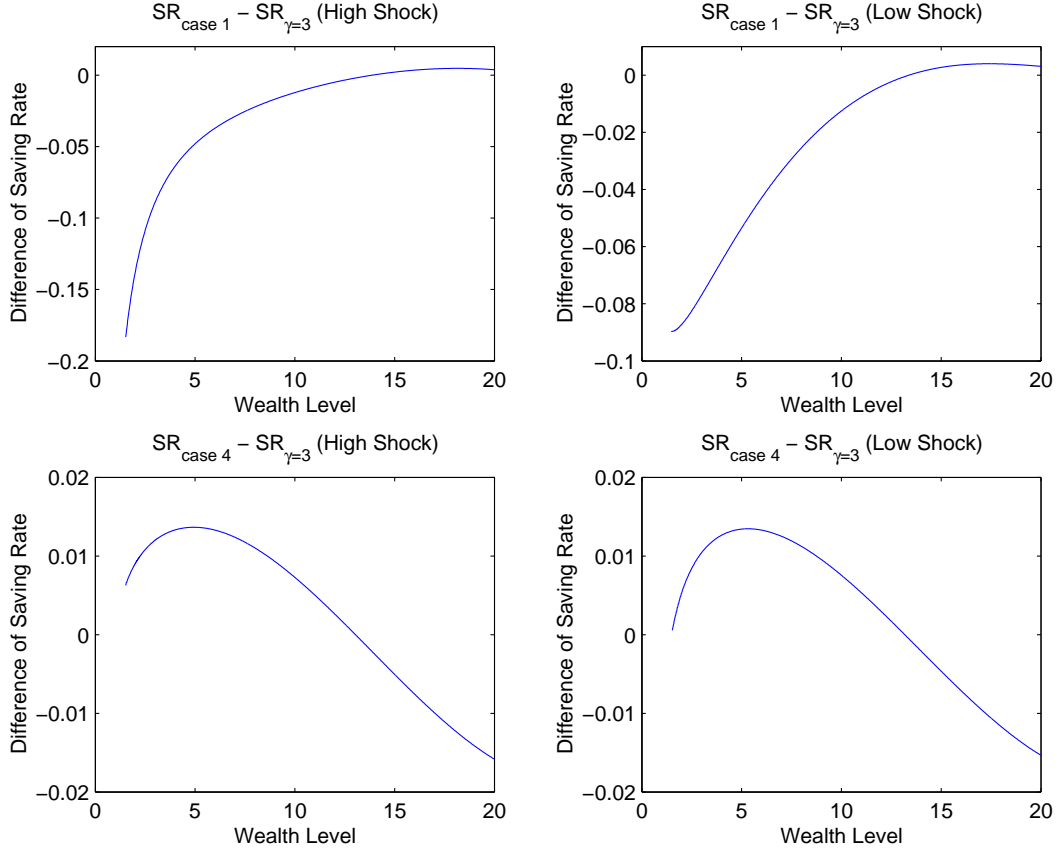


Figure 5: This graph compares the saving rate curves for pro-cyclical risk aversion case (case 1), constant risk aversion case ($\gamma = 3$) and counter-cyclical risk aversion case (case 4). The upper two panels show the difference of the saving rates in the economy with pro-cyclical risk aversion case 1 and the economy with constant risk aversion case $\gamma = 3$. The x -axis is the wealth level of the agents and y -axis measures the difference of the saving rates in two economies. Similarly, the bottom two panels show the difference of the saving rates in the economy with counter-cyclical risk aversion case 4 and the economy with constant risk aversion case $\gamma = 3$.

Table 3. Wealth Distributions: Models and Data

State-dependent Risk Aversion Cases

Model	Wealth% Held					Wealth % Held	Equilibrium	GINI
	by Top					by Bottom	Interest	Coeffi
	1%	5%	10%	20%	30%	40%	Rate	
Data	30	51	64	79	88	1		.79
Const. risk avers.								
$\gamma = 3$	2.3	11	21	38	53	16	3.34	.34
Pro-cyc. risk avers.								
Case 1	3.0	14	27	51	69	6.8	4.12	.51
Case 2	2.5	12	23	42	59	11	3.60	.43
Case 3	2.9	14	26	48	66	6.9	4.17	.49
Counter-cyc. risk avers.								
Case 4	1.8	9	16	31	45	21	2.91	.25
Case 5	2.0	9	18	34	48	20	2.99	.28
Case 6	2.0	9	18	35	50	17	3.49	.31
Est. risk avers.								
Average $\gamma = 3$	2.3	11	20	38	53	16		.34

3 Estimate the Relationship Between Risk Aversion and Wealth

In this section we use experimental data to measure risk aversion and quantify the relationship between risk aversion and wealth.

3.1 Measuring Risk Aversion through Experimental Methods

To figure out the relationship between risk aversion and wealth level, we use data gathered by the experimental study in Potamites and Zhang (2012), where the risk attitudes and wealth levels of individual stock traders are measured. We reproduce Table 2 in Potamites and Zhang (2012) here as Table 4, from which we can see that the sample of investors that participated in the experiment is roughly comparable to the general stock trading population in China. For each subject of this unique sample, we get a direct measure of risk attitude from incentive-compatible experimental questions, as well as detailed data on wealth, income and other demographic characteristics.

Table 4. A comparison of our investors with the average Chinese investor

Our subjects	Mean	Std. Dev.	Min.	Max.	N
Age	43.4	13.4	21	80	292
% Female	0.48				293
% College+	0.70				291
Annual Income (US\$)	3,023	2,624	188	11,250	291
Financial Wealth (US\$)	13,797	7,713	5,000	25,000	293
% Fin.Wealth in Stocks	0.41	0.23	0.1	1	287
Years Investing	8.45	3.5	0	16	277
Watch Market Daily	0.80				293
Survey of Chinese Investors*					
% College+	0.55				2587
% Ann.Inc.<US\$2500**	0.56				2587
% Wealth in Stocks	0.50				2587
Years Investing	5.4				2587
Use Trading Floors	0.70				2587

*Source: Shenzhen Stock Exchange Research Institute Report No. 0055, December 2005.

**Note: Chinese GNI per capita in 2005 was approximately US\$1100.

Table 5 summarized relevant questions in the experiment study that are useful to our current question. A background survey is distributed to the subjects to answer before they participate in the experiment. The survey includes questions on basic demographic information, income and wealth levels, investment history, and their daily and yearly trading performance. In order to facilitate truthful response and minimize non-response, they are only asked to pick the ranges of wealth and income to which they belong. During the experiment, subjects make a series of individual investment decisions in risk for which the pay-out were given in cash. In every risk decision question, subjects decide how much of their endowment of 20 points they want to invest. If their investment is successful then they are paid out 2.5 times the amount invested and if it is unsuccessful then they lose the amount invested. Similar investment decision problems are used by Hopfensitz and van Winden (2008) and Charness and Gneezy (2003). To be more specific, subjects are asked 6 questions about how much they would like to invest in the “Huanghe company” when the probability of success is known to be 50%, 30%, 60%, 20%, 40%, or 70%. At the end of the experiment, one question was chosen at random to be paid out. Before choosing a card to determine which question their payoff would depend upon, subjects were asked to pick their “success color”, either black or white. At the beginning of the experiment it was explained that their investment would be considered successful if they drew a token of the same color as their “success color” out of the appropriate pouch.¹¹ For example if they picked white as their “success color” and the question with $p = 0.4$ from the risk section was chosen to be played out for real, then they would draw one token out of a pouch that they knew had 40 white tokens and 60 black tokens.

¹¹A pouch contains 100 Chinese chess pieces as tokens, either white or black.

Table 5. Relevant Questions in the Experimental Design

Section	Description
Background Survey	7 questions on demographics 5 questions on income, wealth and financial wealth 15 questions on investing history, style and performances
Individual Investment Decisions in Risk	6 risk scenarios, $p \in \{0.2, 0.3, 0.4, 0.5, 0.6, 0.7\}$ in each scenario, endowment is 20, rate of return is $1 + r = 2.5$ one randomly chosen scenario will pay off, subject should optimize in each case

In each risk investment decision, the investor in the experiment is endowed with wealth W , and she has to decide how much to invest in a risky project. There are two outcomes of each project, success and failure. In the case of failure, the funds invested in the project are lost. In the case of success, the investor receives back $1 + r$ times the amount invested, I_p , where p indicates the probability of success of the risky project. In order to gauge the risk aversion attitude of each subject, we model the subject's behavior using a classic CRRA utility function:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (15)$$

We assume that the decision maker is an expected utility maximizer, so the investment amount I_p the agent choose should maximize:

$$p \frac{(W + rI_p)^{1-\gamma}}{1-\gamma} + (1-p) \frac{(W - I_p)^{1-\gamma}}{1-\gamma}$$

and the first order condition, assuming an interior solution, is:

$$p(1-\gamma)(W + rI_p)^{-\gamma}r = (1-p)(1-\gamma)(W - I_p)^{-\gamma}$$

Rearranging terms, we obtain

$$\frac{pr}{1-p} = \left(\frac{W - I_p}{W + rI_p} \right)^{-\gamma}$$

Since $p \in (0, 1)$ both sides are positive and we can take logs:

$$\log \frac{pr}{1-p} = (\rho - 1) \log \frac{W - I_p}{W + rI_p}$$

Since we observe a sample $\{(p, I_p) | p = 0.2, 0.3, \dots, 0.7\}$ in our data for each individual, i , we can take a linear projection of $\log(pr/(1-p))$ on $\log((W - I_p)/(W + rI_p))$ to get a point estimate for $-\gamma_i$.¹²

3.2 The Relationship between Wealth and Risk Aversion

With the measured risk aversion and data on other individual characteristics including individual wealth levels, we can further quantify the relationship between risk aversion and wealth. To do this, we regress the measured risk aversion coefficient on factors that are likely to be correlated with agent risk aversion, such as age, education, gender and wealth. In general, we found that risk aversion coefficient is significantly correlated with an individual's wealth level and its higher-order term of wealth. In addition, risk aversion is positively correlated with educational attainment. It is negatively correlated with age, but age becomes insignificant when educational attainment is included in the regression. The results also show that females are more risk averse than males.

The detailed results are reported in Table 6. W refers to normalized wealth and W^3 refers to its third power.¹³ Across different regressions, the relationship between wealth (especially its higher-order term) and risk aversion is significant. And the coefficients associated with the wealth terms are relatively stable across different regressions. In the next section we will construct the risk aversion function based on this relationship. Before going to the next section, some explanations on this relationship. First, this suggests the relationship between risk aversion and wealth is hump-shaped. Risk aversion first increases with wealth and then decreases with it. Second, this relationship suggests the poorest group of people and the richest group of people save less (relative to their total wealth levels) than those people in the middle. There may exist many explanations on why this is the case. But one explanation is as follows. For the poorest people, they can rely on the welfare system thus are less afraid of losing their relatively less wealth in making financial decisions. For those richest people, since they have enough savings and they may own better resources and business opportunities than common people, thus they are also less risk averse than the people with the middle-level wealth.

¹²For each individual we only use their (p, I_p) pairs if $I_p \neq 0$ or W .

¹³We normalize individual wealth level by dividing it by the average wealth level. In other words, W is the ratio of individual wealth to the average wealth level.

Table 6. Regression Results (Dependent Variable: Measured Risk Aversion)

	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5
constant	0.23***	0.29***	0.27***	0.10	0.09
W	0.12	0.16**	0.16*	0.14*	0.13*
W^3	-0.038*	-0.48**	-0.47**	-0.47**	-0.45*
Age		-0.0019*	-0.0020*	-0.0005	-0.0003
Gender			0.050*		0.050*
Education				0.050***	0.050***

4 Implications of the Estimated Risk Aversion Function for Wealth Inequality

The analysis in Section 2 shows how different specifications on risk aversion functions influence the wealth distribution. In particular, a pro-cyclical risk aversion function leads to more wealth inequality while a counter-cyclical one leads to less wealth inequality. However, the previous section shows that in the data the relationship between wealth and risk aversion is neither pro-cyclical nor counter-cyclical. It is actually hump-shaped. This section will quantify the implication of this empirically consistent risk aversion function to the wealth distribution.

This is achieved in the following way. We adopt the function form and the estimated coefficients from the regression analysis. In particular, the risk aversion function used in the model is as follows.

$$\mu(W) = b_0 + b_1 \cdot W + b_2 \cdot W^3 \quad (16)$$

where we set $b_1 = 0.13$ and $b_2 = -0.45$. Consistent with the empirical specification in the previous section, W is the ratio of individual wealth to the average wealth level. We calibrate the model to match the average wealth level. To be more comparable to the results shown in Table 3, we chose b_0 such that the average risk aversion coefficient is 3.

The wealth distribution for such an economy is reported in the last row in Table 3. It shows that the wealth distribution is almost identical to the one with constant risk aversion. With all the analysis in Section 2 on how different risk aversion functions may have changed the wealth distribution, it is very easy to see why using the empirically estimated risk aversion leads to the same wealth distribution as in the constant risk aversion case. On the one hand, like in the economy with pro-cyclical risk aversion, the poor agent in this economy save less because they are less risk averse. This pushes the wealth distribution to the left; thus increases the wealth inequality (all else held constant). On the other hand, like in the economy with counter-cyclical

risk aversion, those rich people now save less as they are less risk averse (than the case in which risk aversion does not change). This also pushes the wealth distribution to the left which *reduces* the wealth inequality. Thus, these two competing forces result in an unchanged wealth inequality. In addition, in terms of the total savings in the economy, they are also unchanged because the declines in the savings by the poor and by the rich are offset by the increases in savings by the people in the middle.

5 Conclusions

In this paper we introduce state-dependent risk aversion into Aiyagari (1994) heterogeneous-agent version of standard neoclassical growth model with uninsurable idiosyncratic shocks to earning. The relationships among risk aversion, saving rate, equilibrium interest rate and wealth distribution are explored. In the current framework, different attitudes toward risk lead agents to have different saving behaviors. Their aggregate saving behaviors affect the equilibrium interest rate and the wage rate hence affect agents income differently.¹⁴ The changing wealth level will affect agents' risk aversion attitude and thus affect their saving behavior again. We find that by assuming that agent's risk aversion increases with wealth, the model generates more unequal wealth distribution. However, assuming risk aversion decreases with wealth leads to a more equal wealth distribution.

Using experimental data, we have measured the risk aversion and quantified the relationship between risk aversion and wealth. We found this relationship is hump-shaped, meaning risk aversion first increases with wealth and then decreases with wealth. Putting the estimated risk aversion function into the model suggests an almost unchanged wealth distribution as in the economy with constant risk aversion. This is due to the fact that both the poorest agents and the richest agents in the economy save less compared with the case of constant risk aversion. The former increases the wealth inequality while the latter reduces it. Thus, overall, wealth distribution is unchanged. This result suggests that the assumption of constant risk aversion may be a good approximation in macroeconomic models to study wealth distribution. However, in terms of policy implications, the risk-dependent risk aversion may generate some distortion to the policies which influence people with different wealth level differently. This falls in the scope of our future research.

¹⁴Since rich agents hold more wealth the changing in asset return has a larger effect on them. But the changing in wage rate affects the income of both poor and rich agents (with same labor productivity) by the same amount.

6 Appendix

6.1 Risk aversion function

In this appendix we describe the risk aversion function we used in the computation part. The risk aversion function is chosen according to two criterions. First, it should be able to cover all the cases in Figure 1 and be as general as possible. Second, the risk aversion function should be as convenient as permissible for computation.¹⁵

Basing on these two criteria, we assume the risk aversion function is monotonic, which is consistent with the literature. But there is a debate on whether the risk aversion is pro-cyclical or counter-cyclical. Thus, to make the analysis complete in this paper, we propose several risk aversion functions to cover both the pro-cyclical and the counter-cyclical cases.

Let the individual's wealth level be defined in $[0, 20]$ and risk aversion coefficient vary among $[0, 8]$. The first risk aversion function we construct is as follows,

$$\gamma(a) = \begin{cases} \gamma_0(\frac{a}{10})^\kappa & \text{if } 0 \leq a < 10 \\ 2\gamma_0 - \gamma_0(\frac{20-a}{10})^\kappa & \text{if } 10 \leq a \leq 20 \end{cases}$$

where γ_0 controls the range of this function. In the benchmark model we assume agent's risk aversion varies between 0 and 8. So we let $\gamma_0 = 4$. The parameter κ controls the shape of this function. For example, $\kappa = 3$ generates the case 1 in figure 1 and $\kappa = 1$ generates the case 2 in figure 1. Thus, for $\kappa = 1$, the risk aversion function $\gamma(a)$ is linear which implies that agents' degree of risk aversion increases proportionally with their wealth level. For $\kappa = 3$, the risk aversion function $\gamma(a)$ increases rapidly in the middle part and slowly in two sides. This implies that agents' degree of risk aversion are more sensitive to the change of wealth level in the middle part than in the two sides.

Next we define $\gamma^*(a) \equiv \gamma(20 - a)$. Then case 4 in Figure 1 is generated by $\gamma^*(a)$ by when $\kappa = 3$ and case 5 is generated by $\gamma^*(a)$ by when $\kappa = 1$. Thus, for $\kappa = 1$, the risk aversion function $\gamma^*(a)$ is linear which implies that agents' degree of risk aversion decreases proportionally with their wealth level. For $\kappa = 3$, the risk aversion function $\gamma^*(a)$ decreases rapidly in the middle part and slowly in two sides. This implies that agents' degree of risk aversion are more sensitive to the change of wealth level in the middle part than in the two sides.

To generate the graphs in case 3 and 6 we construct another risk aversion functions $\mu(a)$ and $\bar{\mu}(a)$ similarly:

$$\mu(a) = \mu_0 \exp(\frac{a - \eta}{\lambda})^g \tag{17}$$

¹⁵For example, for some specifications of risk aversion function, the computation result doesn't converge when approximating the agent's policy function.

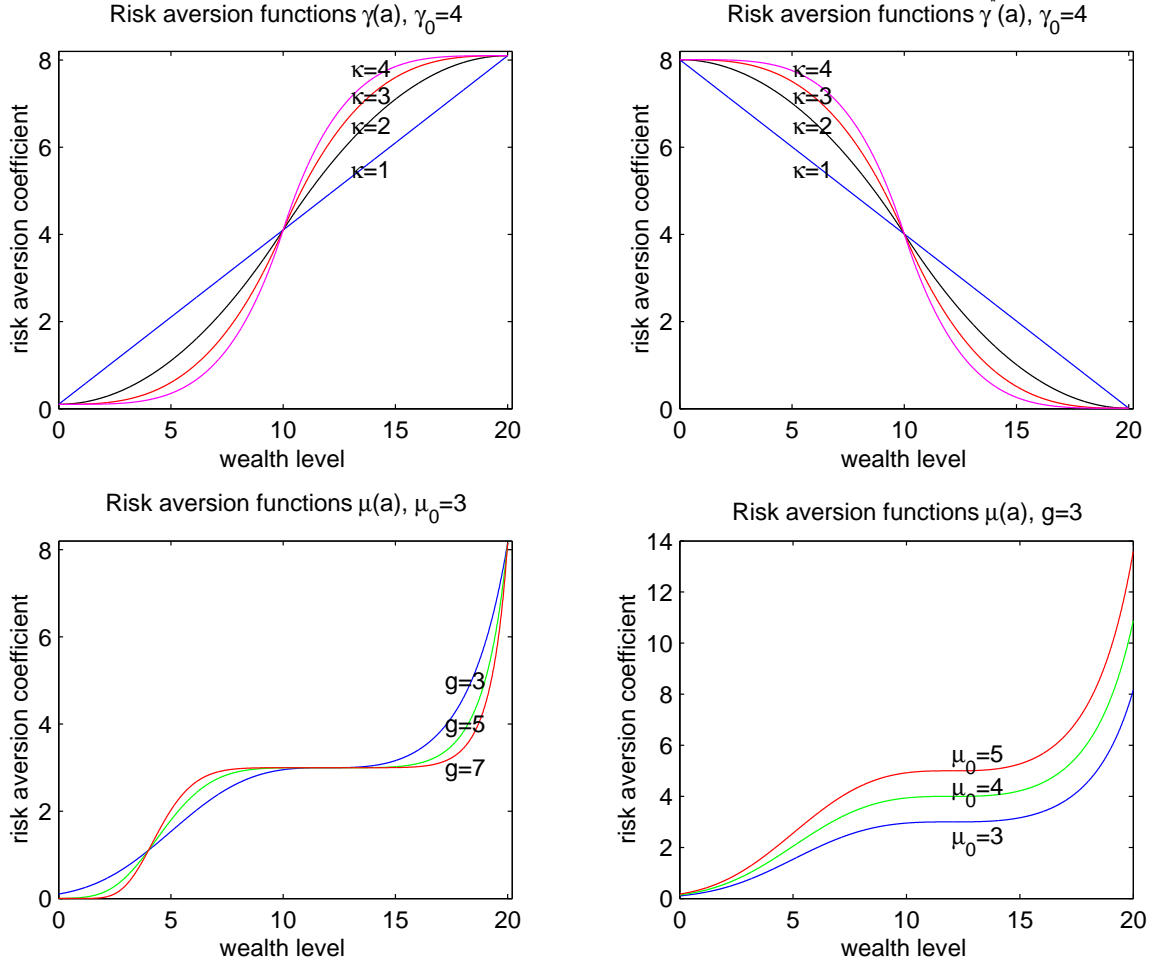


Figure 6: This figure shows the shapes of different risk aversion functions.

and

$$\bar{\mu}(a) \equiv \mu(20 - a) \quad (18)$$

where parameters μ_0, η, λ and g together control the range, shape and the curvature of this function. For example, if we let $\mu_0 = 3, \eta = 12, \lambda = 8$ and $g = 3$, $\mu(a)$ generates case 3 in Figure 1 and $\bar{\mu}(a)$ generates case 6 in the same figure.

In order to familiarize the reader with these risk aversion functions, Figure 7 exhibits the shape of these functions at different parameter values.

6.2 Numerical solution method for agent's problem

To solve the agent's problem we can iterate on either the value function (agent's Bellman equation) or the agent's first-order conditions. And for each case we can choose discrete approximation method or continuous approximation method. To use the discrete approximation method we first discretize the asset space and then restrict agents to choose the asset amount only in those discretized points. This method simplifies the computation process, making it much easier to find the steady state asset and wealth distribution.¹⁶ However, this convenience comes at the cost of a less accurate solution to the agent's policy function. Therefore in this paper we employ smooth approximation methods to obtain the policy function based on the first-order condition equations of the agent's problem. More accurately speaking, we approximate the policy function using Chebychev polynomials of degree 5 and with 30 grid points.¹⁷

In the baseline model, the asset space is set to be $[0, 20]$. The borrowing constrain is 0 in the benchmark model. The tolerance criterion for computing the agent's asset policy function is 0.002.

¹⁶This is because we do not need to worry about how to choose assets in those infinite number of points between the finite discretized asset points.

¹⁷To have a comparison, we also conducted the linear interpolation method to approximate the policy function.

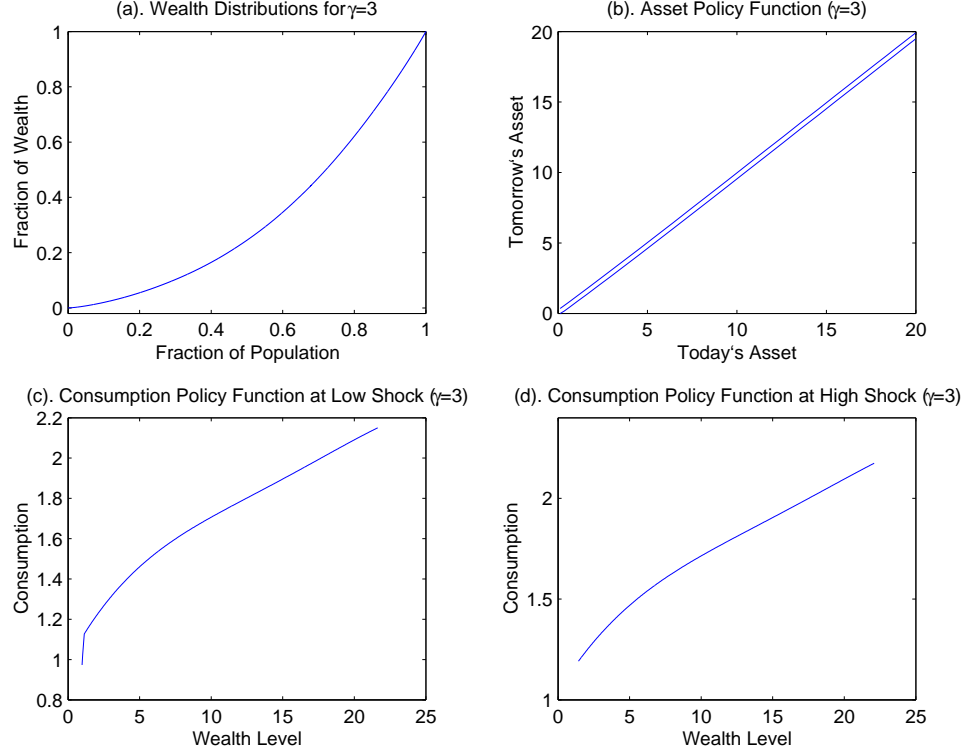


Figure 7: This figure shows the shape of wealth distribution, asset policy function, consumption policy function for constant risk aversion coefficient case $\gamma = 3$.

6.3 Algebra for finding steady state wage rate

In steady state the average of labor supply is normalized to be 1 and technology parameter A is also set to be 1. From equation (1) and (2) we get

$$w = (1 - \alpha)K^\alpha \quad (19)$$

$$r = \alpha K^{\alpha-1} - \delta \quad (20)$$

Combining these two equation to eliminate variable K we can express w as a function of r

$$w = (1 - \alpha) \left(\frac{\alpha}{r + \delta} \right)^{\alpha/1-\alpha} \quad (21)$$

6.4 U.S Wealth Distribution in 2001

Here we report some evidence on the wealth distribution of the U.S. households in 2001. Using data from 2001 Survey of Consumer Finances (SCF), figure 8 shows the wealth distribution of

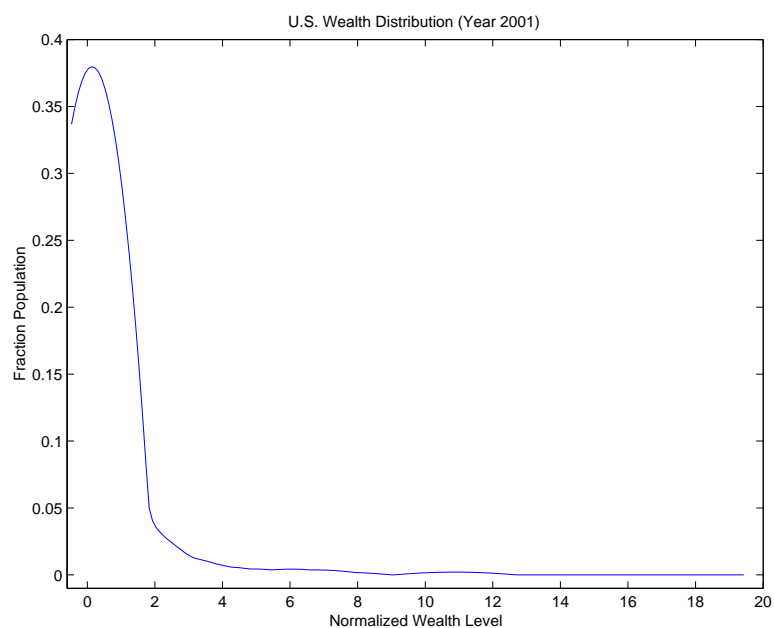


Figure 8: This figure shows the wealth distribution of the U.S. households at 2001.

the U.S. households in 2001. Figure 9 draws the Lorenz curve of the U.S. households' wealth at 2001. Table 5 reports some statistics on the wealth distribution.

Table 5						
U.S. Wealth Distribution of 2001						
%Wealth Held					%Wealth Held	GINI
by Top					by Bottom	
1%	5%	10%	20%	30%	40%	
32	57	70	83	90	1.1	.81

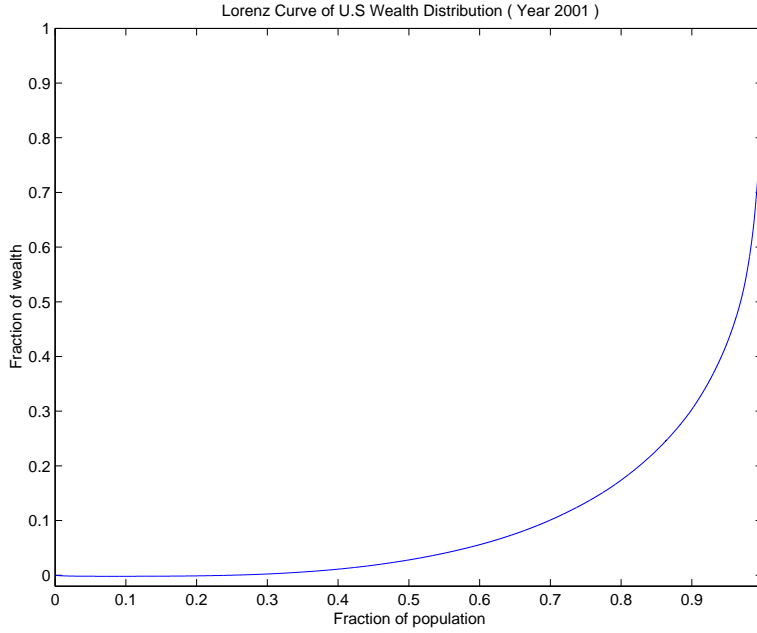


Figure 9: This figure draws the Lorenz curve of the U.S. households' wealth at 2001.

6.5 Robustness Check to Section 3

In this section we vary the parameter values which govern the range of the risk aversion, the correlation and variability of the labor endowment shocks and the borrowing limit and compute again the wealth distributions. There are two purposes for these experiments. First, we want to check the robustness of the results we got in the previous section. Second, by checking the role played by each parameter on affecting the wealth distribution we want to explore the direction along which we can further improve the performance of the model on matching the theoretical wealth distribution with data. To make things simple, we choose the case 2 of the pro-cyclical risk aversion (see $\gamma_2(a)$ in Figure 1) as an example. This straightforward risk aversion function says that agents' risk aversion attitudes increase linearly with their wealth level.

6.5.1 The range of risk aversion coefficient

From the results in section 3 (see Table 3) we know that pro-cyclical risk aversion specification can improve the model's prediction on matching the wealth distribution of the U.S. economy. The reason why this specification works is that they let the rich agents be more risk averse than the poor agents so the rich will save relatively more than the poor. This makes the rich accumulate their wealth at an even higher rate than the poor and therefore the wealth gap between the rich and poor increases. This seems to say the inequality of wealth distribution will increase with

difference of the risk aversion attitudes between the rich and poor. Thus, some may think that if we increase the range of risk aversion functions, the wealth distribution should look more unequal. In this section we investigate this possibility.

Recall that in the benchmark model we assume the agent's risk aversion coefficient varies from 0 to 8. Now we conduct two experiments that increase this range. Specifically we assume the risk aversion varies from 0 to 12 in one experiment and from 0 to 16 in the other one. In both cases we still assume the risk aversion function is linear.

The results are reported in Table 7. We find that when the range of the risk aversion increases, the Gini wealth coefficient decreases. This seems a little surprising at the first glance: why does the wealth distribution look more equal when the difference of risk aversion attitudes increases? We can give an answer to this question by comparing the saving rates, interest rates and wages in these economies. Actually increasing the range of risk aversion effects the wealth distribution through the following three aspects. First, increasing the range of risk aversion makes all the agents more risk averse than before.¹⁸ As a result, the aggregate amount of capital in equilibrium increases and the equilibrium interest rate falls as reported in Table 7. The decreasing of the interest rate affects the rich agent more than the poor agents because the main source of income of the rich agents comes from the asset market. Second, the decreasing of equilibrium interest rate increases the equilibrium wage. The higher wage increases the wealth of the poor agents *relatively* more than that of the rich agents since the labor market is the main source of income for the poor. Third, as panels *c* and *d* of Figure 10 show, increasing the range of risk aversion increases the saving rate of poor agents by more than that of the rich agents. This is because the agent's saving rate increases faster when his risk aversion coefficient increases from relatively low levels.¹⁹ These three effects make the wealth distribution looks more equal. As Table 7 shows, the Gini coefficient decreases from 0.43 to 0.38 as the maximum of the risk aversion coefficient increases from 8 to 16.

6.5.2 Labor endowment shock

The main idea of this paper is to differentiate agents' attitudes of risk aversion to affect the wealth distribution through the channel of affecting agents' saving behaviors. In other words, any factor which plays an important role in affecting the agent's saving decision may be sensitive

¹⁸Since we assume agent's risk aversion increases *linearly* with his wealth level, increases the range of risk aversion makes the risk aversion function more steeper as shown in panel *a* of Figure 6.

¹⁹By the pro-cyclical risk aversion specification poor agents have low risk aversion attitudes in these economies. For why the agent's saving rate increase faster when his risk aversion coefficient increases from relatively low levels, see Appendix D for a detailed discussion.

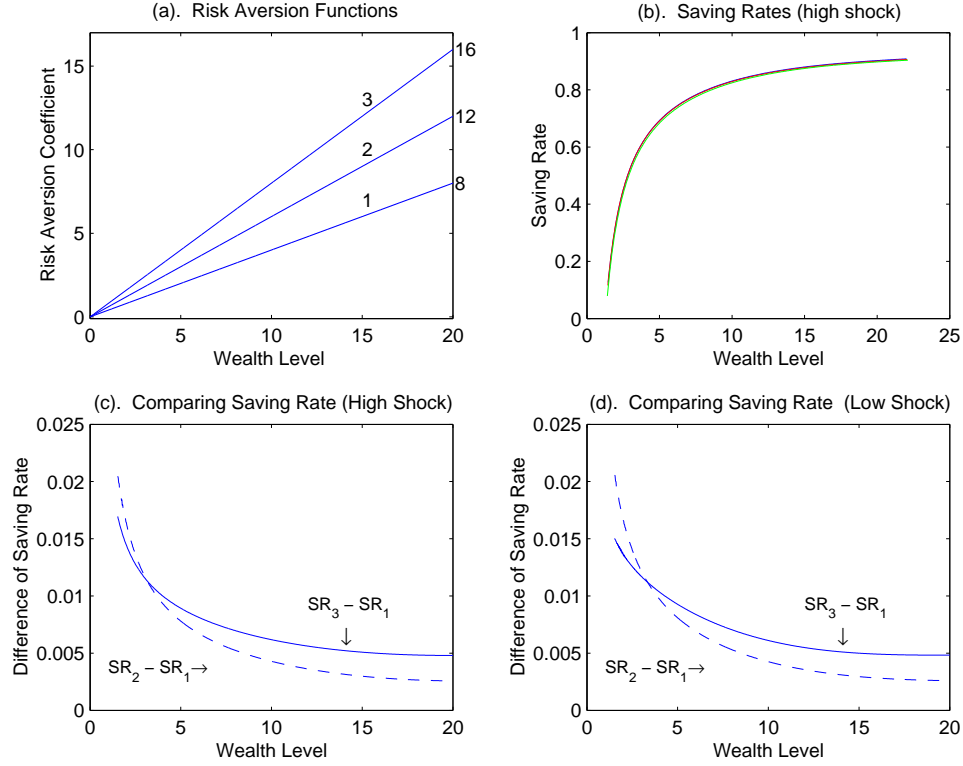


Figure 10: Robustness Experiment 1: increasing the range of risk aversion function. Panel *a* shows the agent's risk aversion functions in three economies. In these economies the risk aversion coefficient increases linearly with agent's wealth. But in economy 1 the range of agent's risk aversion coefficient is $[0, 8]$ and in economy 2 and 3 the risk aversion coefficient's ranges are $[0, 12]$ and $[0, 16]$ separately. Panel *b* plots the saving rates of the agents with high labor productivity in three economies. Panel *c* and *d* plot the differences of the agents' saving rates in these economies with high shocks and low shocks separately. The real line plots the difference between the saving rates in the economy with range $[0, 16]$ and the economy with range $[0, 8]$. The dashed line plots the difference between the saving rates in the economy with range $[0, 12]$ and the economy with range $[0, 8]$.

to the results. Since one important saving motivation for the agents in the current framework comes from the uncertainty of earning brought by idiosyncratic labor endowment shocks, the specification of the labor productivity shock deserves more discussions. The two parameters governing the process of the labor shock is the AR(1) correlation coefficient ρ and the standard deviation of the shock σ . In the benchmark model, ρ and σ are set to be 0.53 and 0.24 separately. Now we change these parameter values and see how the result changes.

In the first experiment we fix the σ at 0.24 and increase ρ from 0.5 to 0.6, 0.7, 0.8. Then we fix ρ at 0.5 and increase σ from 0.24 to 0.4, 0.6 and 0.8 separately. Finally we consider several extreme cases: $\rho = 0.1$ and $\sigma = 0.1$, $\rho = 0.1$ and $\sigma = 0.8$, $\rho = 0.1$ and $\sigma = 0.8$, $\rho = 0.8$ and $\sigma = 0.8$. The wealth distribution and equilibrium interest rates are reported in the middle panel of Table 7. As it shows, the wealth distribution is not very sensitive to the parameter ρ . The Gini wealth coefficient decreases slightly from 0.43 to 0.41 as ρ increases from 0.5 to 0.8 (σ is fixed at 0.24). On the contrary, changing the parameter value of σ has relatively larger effect on the wealth distribution. For example, when σ increases from 0.24 to 0.8 (ρ is fixed at 0.5) the Gini wealth coefficient decreases from 0.43 to 0.38; when σ decreases from 0.24 to 0.1 (ρ is fixed at 0.5) the Gini wealth coefficient increases from 0.43 to 0.48.

The following analysis will help us understand these results. Firstly, in the current framework, saving has two purposes for the agents: to insure the earning uncertainty and to gain from the interest rate (and then used for future consumption). Let S , S_p , S_c denote total saving, precautionary saving and saving for future consumption ($S = S_p + S_c$). We know S_p is determined by the variance of the labor productivity shock $Var(l_t)$. And S_c is affected by the interest rate r (given the preference). When $Var(l_t)$ decreases, S_p decreases, which reduces the aggregate saving and hence push the interest rate r up. The increase of r will then increase S_c . Given that most of the S_c is held by the rich agents, this process makes the rich hold *relatively* more wealth and therefore the Gini wealth coefficient increases.²⁰ Secondly, from equation (9) we know that the variance of the logarithm of labor productivity shock, $Var(log l_t)$, equals to $\frac{\sigma^2}{1-\rho^2}$. This implies when σ and ρ decreases (increases), $Var(log l_t)$ decreases (increases) and therefore the Gini wealth coefficient increases (decreases). Lastly, since the variance $Var(log l_t)$ is more sensitive to the parameter σ than to ρ , changing the value of σ will have a larger effect on wealth distribution than changing ρ .²¹

²⁰Note the *absolute* amount of saving of rich agents may decrease since their precautionary saving part also decreases.

²¹For example, let $\sigma = 0.24$ and $\rho = 0.5$ (the parameter values in the benchmark model). Then decreasing σ from 0.24 to 0.1 decreases $Var(log l_t)$ from 0.0768 to 0.0133. But decreasing ρ from 0.5 to 0.1 only decreases $Var(log l_t)$ from 0.0768 to 0.0582.

Simply speaking, the intuition is as follows. Decreasing the variety of labor productivity shock makes *all* the agents fear less for the uncertainty and therefore reduce their precautionary saving. This will push the interest rate up. Since most assets are hold by the rich agents, the high interest rate benefits the rich agents relatively more than the poor agents. On the other hand, the decreasing of supplied capital also decreases the marginal productivity of labor therefore lower the wage rate. Since the labor income is the main source of the poor agent's earning, the low wage rate hurts them more than the rich agents. Thus the wealth distribution looks more unequal and Gini coefficient increases.

Table 7

Robustness Results: Pro-cyclical risk aversion case

Model	Wealth% Held					Wealth% Held	Equilibrium	GINI
	by Top					by Bottom	Interest	Coeffi
	1%	5%	10%	20%	30%	40%	rate(%)	
Data	30	51	64	79	88	1		.79
Range of the risk aversion								
[0, 8]	2.5	12	23	42	59	11	3.60	.43
[0, 12]	2.5	12	22	41	57	12	3.14	.40
[0, 16]	2.4	11	22	40	56	13	2.77	.38
Shock specification								
$\rho = 0.5, \sigma = 0.24$	2.5	12	23	42	59	11	3.60	.43
$\rho = 0.6, \sigma = 0.24$	2.7	12	24	43	60	10	3.48	.43
$\rho = 0.7, \sigma = 0.24$	2.7	12	23	42	59	11	3.41	.42
$\rho = 0.8, \sigma = 0.24$	2.7	13	23	43	58	11	3.36	.41
$\rho = 0.5, \sigma = 0.40$	2.7	12	23	43	59	12	3.20	.43
$\rho = 0.5, \sigma = 0.60$	2.7	12	23	42	57	13	3.05	.41
$\rho = 0.5, \sigma = 0.80$	2.7	13	23	42	57	14	2.78	.38
$\rho = 0.8, \sigma = 0.80$	2.6	13	24	42	57	13	2.42	.40
$\rho = 0.1, \sigma = 0.80$	2.7	13	24	43	59	14	3.05	.39
$\rho = 0.8, \sigma = 0.10$	2.7	13	24	45	62	7	3.67	.47
$\rho = 0.5, \sigma = 0.10$	2.6	13	24	45	62	7	3.68	.48
$\rho = 0.1, \sigma = 0.10$	2.6	13	25	46	64	5	3.75	.49
Borrowing limit								
$b = 0$	2.5	12	23	42	59	11	3.60	.43
$b = 0.5$	2.7	13	24	45	63	6.7	3.61	.47
$b = 1$	2.8	13	25	47	66	3.9	3.82	.51
$b = 1.5$	3.0	13	28	51	71	-1	4.12	.58

6.5.3 Borrowing constraint

As analyzed in Aiyagari's paper, the borrowing constraint plays an very important role in agent's asset holding decision since the private capital is the only asset agents can trade to insure against earning fluctuation. In the benchmark model we set the borrowing limit to 0 which means

agents can not borrow. Now we let agents can borrow some amount of asset.

As shown by the bottom panel of Table 7, increasing the borrowing limit significantly increases the Gini wealth coefficient and equilibrium interest rate. For example, increasing borrowing limit, b , from 0 to 1.5 raises Gini wealth coefficient from 0.43 to 0.58 and increases the interest rate from 3.53% to 4.12%. Also, the percent of wealth held by bottom 40% of population decreases from 10% to -1% as the borrow limit increases from 0 to 1.5. This has a good intuition. That is, when agents are permitted to borrow, poor agents need not rely solely on holding asset to buffer earning fluctuations, so their saving rate will decrease. But for rich agents the increase of the borrow limit has less effect on them since they have relatively more saving to insure against the earning variation. In addition, the decrease of saving will push up the interest rate which incentivizes agents to save more. Combining these two effects the saving rate of the rich agents do not change much but the poor agents' saving rates decrease a lot as shown in Figure 11. This change in saving rates enhances the wealth gap between the rich agents and poor agents and hence increases the inequality of the wealth distribution.

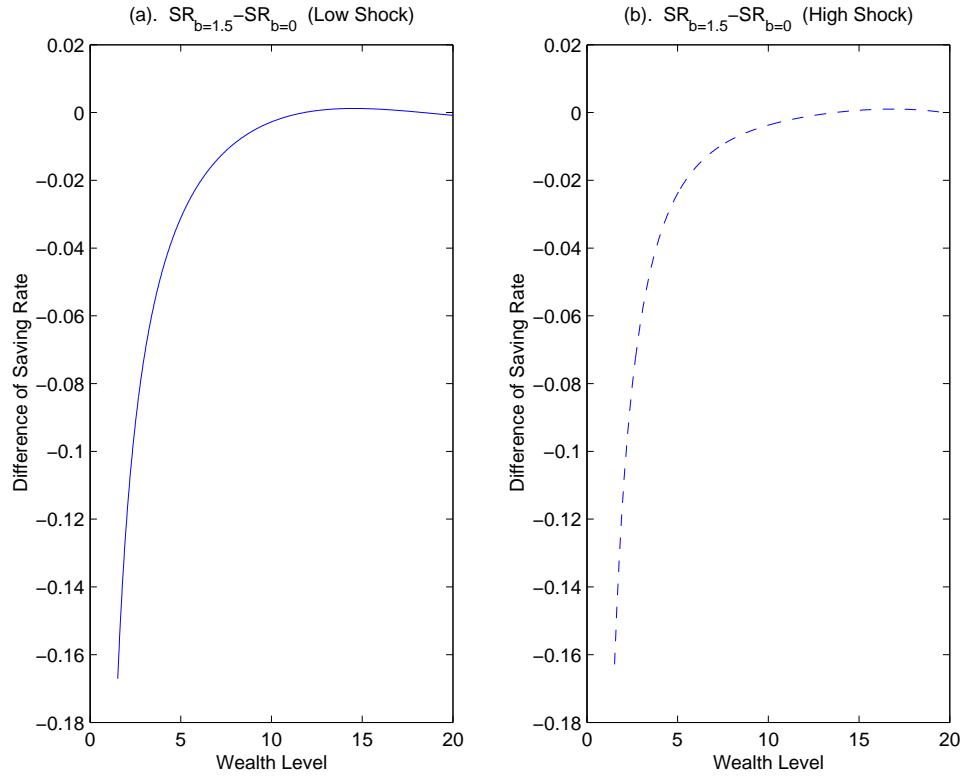


Figure 11: Robustness Experiment 3: Comparing the saving rates in the economy with borrow limit $b = 1.5$ and the economy with the borrowing limit $b = 0$. Panel *a* plots the saving-rate difference curve at low shock and Panel *b* plots the saving-rate difference curve at high shock.

References

- Aliyagari, S.R. (1994) “Uninsured idiosyncratic risk and aggregate saving,” *Quarterly journal of economics*, Vol. 109, pp. 659–684.
- Anderson, Cameron and Adam D Galinsky (2006) “Power, optimism, and risk-taking,” *European journal of social psychology*, Vol. 36, pp. 511–536.
- Arrow, Kenneth Joseph (1965) *Aspects of the theory of risk-bearing*: Yrjö Jahnssonin Säätiö.
- Bosch-Domènech, Antoni and Joaquim Silvestre (1999) “Does risk aversion or attraction depend on income? An experiment,” *Economics Letters*, Vol. 65, pp. 265–273.
- Brunnermeier, Markus K and Stefan Nagel (2006) “Do wealth fluctuations generate time-varying risk aversion? Micro-evidence on individuals’ asset allocation,” Technical report, National Bureau of Economic Research.

- Buccioli, Alessandro and Raffaele Miniaci (2011) "Household portfolios and implicit risk preference," *Review of Economics and Statistics*, Vol. 93, pp. 1235–1250.
- Cagetti, Marco and Mariacristina De Nardi (2006) "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, Vol. 114, pp. 835–870.
- Charness, Gary and Uri Gneezy (2003) "Portfolio Choice and Risk Attitudes: An Experiment," *Economic Inquiry*, Vol. 48, pp. 133–146.
- Chiappori, Pierre-André and Monica Paiella (2011) "Relative risk aversion is constant: Evidence from panel data," *Journal of the European Economic Association*, Vol. 9, pp. 1021–1052.
- De Nardi, Mariacristina (2004) "Wealth Inequality and Intergenerational Links," *Review of Economic Studies*, Vol. 71, pp. 743–768.
- Díaz-Giménez, Javier, Vincenzo Quadrini, and José-Víctor Ríos-Rull (1997) "Dimensions of Inequality: Facts on the U.S. Distribution of Earnings, Income, and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, Vol. 21, pp. 3–21.
- Dohmen, Thomas, Armin Falk, David Huffman, Uwe Sunde, Jürgen Schupp, and Gert G Wagner (2011) "Individual risk attitudes: Measurement, determinants, and behavioral consequences," *Journal of the European Economic Association*, Vol. 9, pp. 522–550.
- Eckel, Catherine C and Philip J Grossman (2008) "Men, women and risk aversion: Experimental evidence," *Handbook of experimental economics results*, Vol. 1, pp. 1061–1073.
- Faff, Robert, Terrence Hallahan, and Michael McKenzie (2009) "Nonlinear linkages between financial risk tolerance and demographic characteristics," *Applied Economics Letters*, Vol. 16, pp. 1329–1332.
- Griskevicius, Vladas, Joshua M Tybur, Andrew W Delton, and Theresa E Robertson (2011) "The influence of mortality and socioeconomic status on risk and delayed rewards: a life history theory approach," *Journal of personality and social psychology*, Vol. 100, p. 1015.
- Guiso, Luigi and Monica Paiella (2008) "Risk aversion, wealth, and background risk," *Journal of the European Economic Association*, Vol. 6, pp. 1109–1150.
- Halek, Martin and Joseph G Eisenhauer (2001) "Demography of risk aversion," *Journal of Risk and Insurance*, pp. 1–24.

- Hartog, Joop, Ada Ferrer-i Carbonell, and Nicole Jonker (2002) "Linking measured risk aversion to individual characteristics," *Kyklos*, Vol. 55, pp. 3–26.
- Heinemann, Frank (2008) "Measuring risk aversion and the wealth effect," in James C. Cox and Genn W. Harrison eds. *Risk aversion in experiments*: Emerald Group Publishing Limited.
- Hopfensitz, Astrid and Frans van Winden (2008) "Dynamic Choice, Independence And Emotions," *Theory and Decision*, Vol. 64, pp. 249–300.
- Iglesias, José-Roberto, Sebastian Gonçalves, Guillermo Abramson, and José-Luis Vega (2004) "Correlation between risk aversion and wealth distribution," *Physica A: Statistical Mechanics and its Applications*, Vol. 342, pp. 186–192.
- Isen, Alice M (2008) "Some ways in which positive affect influences decision making and problem solving," *Handbook of emotions*, pp. 548–573.
- Isen, Alice M and Robert Patrick (1983) "The effect of positive feelings on risk taking: When the chips are down," *Organizational Behavior and Human Performance*, Vol. 31, pp. 194–202.
- Krusell, Per. and Anthony. A. Smith (1998) "Income and wealth heterogeneity in the macroeconomy," *Journal of Political Economy*, Vol. 106, pp. 867–896.
- Laffont, Jean-Jacques (1989) "The economics of uncertainty and information."
- Loewenstein, George F, Elke U Weber, Christopher K Hsee, and Ned Welch (2001) "Risk as feelings.," *Psychological bulletin*, Vol. 127, p. 267.
- Moscardi, Edgardo and Alain de Janvry (1977) "Attitudes toward risk among peasants: An econometric approach," *American Journal of Agricultural Economics*, Vol. 59, pp. 710–716.
- Mossin, Jan (1968) "Aspects of rational insurance purchasing," *The Journal of Political Economy*, Vol. 76, pp. 553–568.
- Ng, James (2013) "Elicited risk and time preferences: the role of demographics, cognition, and interviewers," *University of Southern California Job Market Paper*.
- Ogaki, Masao and Qiang Zhang (2001) "Decreasing relative risk aversion and tests of risk sharing," *Econometrica*, Vol. 69, pp. 515–526.
- Padmanabhan, V. and R.C. Rao (1993) "Warranty policy and extended service contracts: theory and an application to automobiles," *Marketing Science*, Vol. 12, pp. 230–248.

- Pålsson, Anne-Marie (1996) “Does the degree of relative risk aversion vary with household characteristics?” *Journal of economic psychology*, Vol. 17, pp. 771–787.
- Paravisini, Daniel, Veronica Rappoport, and Enrichetta Ravina (2010) “Risk aversion and wealth: Evidence from person-to-person lending portfolios,” Technical report, National Bureau of Economic Research.
- Potamites, Elizabeth and Bei Zhang (2012) “Heterogeneous ambiguity attitudes: a field experiment among small-scale stock investors in China,” *Review of Economic Design*, Vol. 16, pp. 193–213.
- Quadrini, Vincenzo (1999) “The importance of entrepreneurship for wealth concentration and mobility,” *Review of Income and Wealth*, Vol. 45, pp. 1–19.
- Sahm, Claudia R (2012) “How much does risk tolerance change?” *The Quarterly Journal of Finance*, Vol. 2.
- Spears, Dean E (2010) *Economic decision-making in poverty depletes behavioral control*: Center for Economic Policy Studies, Princeton University.
- Wik, Mette, Tewodros Aragie Kebede, Olvar Bergland, and Stein T Holden (2004) “On the measurement of risk aversion from experimental data,” *Applied Economics*, Vol. 36, pp. 2443–2451.
- Yesuf, Mahmud and Randall A Bluffstone (2009) “Poverty, risk aversion, and path dependence in low-income countries: Experimental evidence from Ethiopia,” *American Journal of Agricultural Economics*, Vol. 91, pp. 1022–1037.