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Labor Market Dynamics with Endogenous Labor Force Participation and On-the-Job Search[*]

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Abstract

Studies that incorporate endogenous labor force participation, and search and matching frictions in a real business cycle model find that this three-state model generates counterfactual results: labor force participation is very volatile, unemployment is acyclical and highly positively correlated with vacancies. Based on the evidence that job-to-job flows are large in the U.S. labor market, this paper enriches the three-state model with an on-the-job search mechanism which leads to job-to-job flows. The modified model successfully generates countercyclical unemployment and the Beveridge Curve relationship. Quantitatively, business cycle statistics reproduced by the modified model are more in line with their empirical counterparts.

Keywords: Search and matching, endogenous participation, on-the-job search, job-to-job flows, business cycle

JEL Classification: E24, E32, J63, J64

1   Introduction

A major shortcoming of most existing real business cycle models with labor market search and matching frictions is that all agents in the economy are assumed to be part of the labor force. Studies that extend this structure by incorporating an out-of-the-labor-force state through allowing for a market work vs. home production decision find that this three-state model generates counterfactual labor market dynamics: labor force participation is very volatile, while unemployment is weakly procyclical, or acyclical, and has a high positive correlation with vacancies. This paper presents an alternative general equilibrium business cycle model that also features labor market frictions and endogenous labor force participation. Based on the empirical evidence that job-to-job flows are large in the U.S. labor market, an on-the-job search mechanism is introduced. This mechanism serves as an additional margin along which labor market participation adjustments can take place. The proposed model with on-the-job search generates labor market dynamics that are significantly different from those presented in previous studies. More specifically, the model successfully generates countercyclical unemployment and the Beveridge Curve relationship.

Incorporating labor market search and matching frictions a la Mortensen and Pissarides (1994), and Pissarides (2000) into the basic real business cycle model has become a common exercise for macroeconomists. Merz (1995), Andolfatto (1996), Den Haan et al. (2000), and others replace the standard Walrasian labor market with a search-theoretic one, which has been shown to improve the quantitative properties of the real business cycle model. These studies assume that all agents in the economy are in the labor force (or, equivalently, labor force participation decision is exogenous). There are only two possible labor market states: unemployment and employment. One interpretation of this structure is that the out-of-the-labor-force state is completely ignored. In this environment only the dynamics of and flows between employment and unemployment can be studied. However, empirical evidence shows that flows into and out of the labor force are quantitatively as important as flows between employment and unemployment in the U.S. labor market. It is also important to consider the participation margin in models focusing on other aspects of labor markets, for example the effects of policies, such as unemployment benefits and minimum wages on labor market outcomes and dynamics. Another interpretation of the two-state model is that unemployment and out-of-the-labor-force states are lumped into a single state. This

is problematic as well, since these two states do not have the same business cycle properties. Unemployment is highly countercyclical, whereas participation is weakly procyclical. Unemployment is seven times more volatile than output, while the volatility of participation is much lower than that of output.

In an attempt to address the above criticisms, several studies have included being out of the labor force as a third state. Shi and Wen (1999) develop a three-state model in order to examine the dynamic effects of taxes and subsidies. Tripier (2004) considers a real business cycle version of the Mortensen and Pissarides search and matching model with three states in order to investigate the business cycle properties of the major labor market variables. His results indicate that the model can match the behavior of employment; however, it cannot match the empirical properties of unemployment and labor force participation. When the economy is subject to only aggregate technology shocks, the model fails to generate observed strong countercyclicality of unemployment and the strong negative relationship between unemployment and vacancies. More recently, Veracierto (2008) extends the Lucas and Prescott (1974) islands model by adding an out-of-the-labor-force state, as well as endogenous job acceptance and job separation decisions. As in Tripier (2004), Veracierto (2008) investigates the dynamic properties of the labor market variables, and concludes that the model fails in many directions when the third state is introduced. The volatility of unemployment turns out to be very low and unemployment becomes weakly procyclical, while labor force participation becomes strongly procyclical and turns out to be as volatile as employment.

The main reasons why these three-state models fail to match the data are that labor force participation follows employment too closely and search decisions respond too much to aggregate productivity shocks. When the economy is hit by a positive productivity shock, labor force participation increases and more workers begin to search for jobs since it is a bad time to be out of the labor force, whether engaging in home production or enjoying leisure. In turn, labor force participation becomes strongly procyclical. Since forming matches takes time, not all agents searching for jobs get placed at jobs initially. Unemployment increases sharply at first and follows an acyclical (or a weakly procyclical) pattern overall. As firms open more vacancies employment increases and unemployment decreases. The decrease in unemployment results in less incentive for firms to open vacancies, and vacancy creation decreases as well. Since both vacancies and unemployment increase

on impact with the positive and persistent technology shock, and then fall quickly to levels around their steady states, the model cannot generate the downward-sloping Beveridge curve. This observation suggests that, in order to bring the three-state model closer to the data there needs to be a mechanism that dampens the movements along the participation margin and breaks the close relationship between labor force participation and employment.

Empirical evidence shows that job-to-job transitions are a crucial part of U.S. labor market dynamics. These flows are as large as flows between out of the labor force and employment, and they are twice as large as flows between unemployment and employment. In light of this evidence, this paper enriches the basic three-state model with an on-the-job search mechanism. The intuition for why on-the-job search affects the performance of the three-state model is as follows. In the simpler model without on-the-job search adjustments to aggregate economic conditions mainly take place at the participation margin. In response to a positive productivity shock the representative household increases labor market participation by assigning more members to search for jobs. However, when an on-the-job search mechanism is introduced there is a second margin along which the household's labor market adjustments can take place. In addition to the unemployed, the employed workers can also search for better jobs. As long as job finding and wage rates remain high the overall utility of the household can be increased without large adjustments at the participation margin.

The proposed model is an extension of the real business cycle model with labor market search and matching frictions as in Merz (1995) and Andolfatto (1996). It differs from these models by allowing agents to be out of the labor force, as well as being employed, or being unemployed and searching for jobs. In introducing endogenous labor force participation the model in Tripier (2004) is followed. There is also on-the-job search, which gives the employed agents the ability to switch to better-paying jobs. With the introduction of the on-the-job search mechanism, the three-state model performs better in matching the dynamic properties of the major labor market variables. The business cycle statistics reproduced by the model are more in line with their empirical counterparts. Most importantly, the model can generate countercyclical unemployment and the negative correlation between unemployment and vacancies observed in the data.

2   Related Literature

This study is related to three strands of literature. First, it builds on the existing studies that integrate the canonical two-state Mortensen and Pissarides search and matching model into the real business cycle environment, such as Merz (1995), Andolfatto (1996) and Den Haan et al. (2000).

Second, it is related to papers that model three employment states. Andolfatto and Gomme (1996), Alvarez and Veracierto (2000), Garibaldi and Wasmer (2005), Veracierto (2007), Kim (2008), Pries and Rogerson (2009), and Krusell et al. (2011) consider the three-state labor market structure, but do not analyze business cycle dynamics. As discussed earlier, Tripier (2004) and Veracierto (2008) analyze business cycle dynamics and conclude that when an out-of-the-labor-force state is added to the basic two-state model, the model generates counterfactual results, which is puzzling, since the two-state model is reasonably successful in generating the observed labor market dynamics. This study attempts to solve this puzzle by proposing an alternative three-state model with on-the-job search, which allows for job-to-job flows. Another paper which studies this puzzle is Ebell (2011). Different from this paper's approach, she relies solely on an alternative parametrization to improve the results of the three-state model. Shimer (2011) proposes that rigid wages may help in resolving the shortcomings of the three-state model. More recently, Krusell et al. (2012) develop a general equilibrium model with three employment states and heterogeneity to study the role of various shocks in accounting for business cycle fluctuations in the labor market.

Lastly, this study is related to the literature that studies on-the-job search and job-to-job transitions observed in the U.S. labor market. In an early study, Burdett (1978) constructs a model where both the employed and the unemployed engage in search activity. He shows that workers move to better-paying jobs when possible with the help of on-the-job search, and that the probability of a separation from a job is negatively related to its wage rate. He uses this framework to explain the wage-tenure relationship observed in the data. Pissarides (1994), and Burdett and Mortensen (1998) integrate a similar on-the-job search mechanism in a general equilibrium setting. The former study argues that on-the-job search influences the composition of jobs, which leads firms to open relatively more jobs for the employed job seekers. He suggests that this mechanism can amplify the response of vacancies, while muting that of unemployment in response to changes in aggregate economic conditions. The latter study uses a version of the basic job-ladder model

to explain wage differentials across ex-ante identical workers. In a more recent study, Nagypal (2005) shows that a basic job-ladder model is unable to account quantitatively for the features of observed job-to-job transitions. She proposes an alternative theoretical framework, where job-switching is used as a way to escape from unemployment. Nagypal (2004), Tasci (2007) and Krause and Lubik (2010) both address the Shimer (2005) puzzle by using on-the-job search. The former study shows that the preference of firms to hire the employed workers can help explain the large fluctuations in the vacancy-unemployment ratio, which cannot be generated by the two-state search and matching model. Krause and Lubik (2010) take a similar approach, but integrate on-the-job search in a real business cycle model with labor market frictions. They find that adding an on-the-job search mechanism helps to increase the volatility of vacancies and unemployment, and it enhances the overall amplification and propagation properties of the basic two-state model.

The theoretical framework presented in Section 4 follows closely Krause and Lubik (2010) in developing a two-sector economy where workers are allowed to flow between jobs. The main difference is that, while they build on a two-state labor market framework, this study includes an out-of-the-labor-state and endogenizes the labor market participation decision.

3   Aggregate Flows in the U.S. Labor Market

Empirical evidence shows that flows in the U.S. labor market are large. Previous studies have mainly focused on worker flows between employment, unemployment, and out-of-the-labor-force states. Efforts to measure job-to-job flows have been limited. Fallick and Fleischman (2004) is the first study that provides reliable empirical measures of employer-to-employer flows using data from the Current Population Survey.[1] Their dataset is used to construct Table 1, which summarizes average monthly aggregate flows.

On average, 2.56 percent of the employed workers change employers each month. These flows constitute almost 40 percent of all separations from employment, they are twice as large as flows from employment to unemployment, and comparable to flows from employment to out of the labor force.

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[1]For data download: http://www.federalreserve.gov/pubs/feds/2004/200434/200434abs.html. Employer-to-employer flows correspond to job-to-job flows in this study; therefore, the two terms are used interchangeably.

Table 1: Aggregate Flows in the U.S. Labor Market: 1994-2005 (Monthly)

State in the 1st Period	State in the 2nd Period			
	Percentage of Population			
State in the 1st Period	Same Employer	New Employer	Unemployed	Out of the Labor Force
Employed	58.81	1.61	0.81	1.70
Unemployed	-	0.97	1.69	0.81
Out of the Labor Force	-	1.58	0.81	31.20
	Percentage of State in the 1st Period			
State in the 1st Period	Same Employer	New Employer	Unemployed	Out of the Labor Force
Employed	93.45	2.56	1.29	2.70
Unemployed	-	27.95	48.70	23.34
Out of the Labor Force	-	4.70	2.41	92.88

The number of individuals changing employers is almost equal to the number of unemployed staying unemployed (1.69 percent of the total population) and is more than the number of unemployed finding jobs (0.97 percent of the total population), or dropping out of the labor force (0.81 percent of the total population). Flows into and out of the labor force are at least as large as flows between employment and unemployment. More specifically, while 2.70 percent of the employed leave the labor force each month, only 1.29 percent of the employed become unemployed. Similarly, flows from unemployment to out of the labor force (23.34 percent of the unemployed) are almost as large as flows from unemployment into employment (27.95 percent of the unemployed). As for flows into the labor force, each month 2.39 percent of the total population move into the labor force, of which 66 percent become employed and the rest becomes unemployed.

Given the empirical evidence, it is highly misleading to assume only two states (employment and unemployment) in a model focusing on labor market dynamics. Employer-to-employer flows, as well as flows into and out of the labor force, are quantitatively as important as flows between employment and unemployment.

4 The Model

The model is a real business cycle model with labor market search and matching frictions which has two non-standard features: on-the-job search, which

leads to job-to-job transitions, and endogenous labor force participation. The economy consists of a continuum of measure one of identical households, and heterogenous firms owned by them. Acemoglu (2001), and Krause and Lubik (2010) are followed in introducing two types of firms, which open vacancies for high-wage and low-wage jobs for workers. Vacancy creation is costly, and this cost is higher for a high-wage firm than a low-wage firm. Heterogeneity in wages provides the motive for on-the-job search. Heterogenous firms produce two types of intermediate goods. These goods are then used by the final goods sector that produces the single consumption good in the economy.

4.1 The Representative Household and Labor Markets

The representative household consists of a continuum of homogenous household members with a total measure of one. A household member can be employed, unemployed and searching for a job, or out of the labor force. The household members who are not participating in the labor force engage in home production, which increases the utility of the whole household. The resources of the household are pooled by its members, and there is complete risk-sharing within the household. In each period, the household decides how many of its members will work, how many will search for a job, and how many will stay out of the labor force. The employed household members work at two possible types of jobs: high-wage and low-wage jobs. A measure n_t^g of household members work at a high-wage job, where the wage rate is w_t^g in period t. A measure n_t^b of household members work at a low-wage job at the wage rate of w_t^b. From workers' point of view, the two jobs differ only in wages. All members working at low-wage jobs search for high-wage jobs with an endogenous search intensity, s_t, which is subject to time cost of $\kappa(s_t)$.[2] A measure u_t^g of the unemployed household members search for high-wage jobs, while measure u_t^b search for low-wage jobs in period t. That is, job search is directed. All unemployed members search with intensity one and there is no direct utility cost associated with their search.

Given the above specification, the degree of search activity by the unemployed is equal to $u_t^g + u_t^b$. Each unemployed member receives a fixed unemployment benefit of d. These benefits are financed by the government via collecting a lump-sum tax from the household. Those out of the labor

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[2]This is a real world interpretation of the on-the-job search in the model. Literally, the search activity results in less members engaging in home production, which will become clearer later.

force provide $l_t = 1 - u_t^g - u_t^b - n_t^g - [1 + \kappa(s_t)]n_t^b$ units of home production. While the intensive margin of hours per worker is not modeled, the interpretation of this equation is similar to a time allocation story. The household allocates some members to work, some to search for jobs, and the rest contributes to home production. With the on-the-job search cost added, the time cost of a household member working at a low-wage job becomes $[1 + \kappa(s_t)]$, resulting in less household members engaging in home production. This can be interpreted as on-the-job searchers spending time to look for jobs in addition to their work hours.[3]

The consumption good of the household is purchased from the final goods sector and its price is normalized to unity. The household owns a capital stock of k_t at the beginning of period t, rents its capital to firms at the competitive rental rate of r_t, and decides the level of capital investment i_t. The capital stock depreciates at rate δ. The household also owns the firms and receives profits of π_t in the form of lump-sum payments. Finally, the transfer between the household and the government is in the form of lump-sum taxation T.

The frictional matchings between workers and firms are represented by the matching functions for each type of job. High-wage firms open v_t^g measure of vacancies for high-wage jobs in period t. Unemployed workers who are looking for high-wage jobs are matched with these vacancies with an endogenous probability equal to p_t^g. This probability is equal to $s_t p_t^g$ for job-switchers. All matches (including the newly formed matches) are destroyed with an exogenous probability of ψ at the end of each period, while all surviving matches become productive in the next period. The matching function, evolution of employment, workers' job finding probability, firms' vacancy filling probability, and labor market tightness in the market for high-wage jobs are represented as:

$$m_t^g = m(v_t^g, u_t^g + s_t n_t^b), \quad n_{t+1}^g = (1 - \psi)[n_t^g + p_t^g(u_t^g + s_t n_t^b)] \tag{1}$$

$$p_t^g = \frac{m(v_t^g, u_t^g + s_t n_t^b)}{u_t^g + s_t n_t^b}, \quad q_t^g = \frac{m(v_t^g, u_t^g + s_t n_t^b)}{v_t^g}, \quad \theta_t^g = \frac{v_t^g}{u_t^g + s_t n_t^b} \tag{2}$$

Similarly, low-wage firms open v_t^b measure of vacancies for low-wage jobs in period t. Unemployed workers who are looking for low-wage jobs get matched with these vacancies with an endogenous probability equal to p_t^b. Moreover,

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[3]In an alternative version, the cost of on-the-job search is modeled as a pecuniary cost in the household's budget constraint. The results are not affected by this change.

some working at low-wage jobs get exogenously separated, or move to high-wage jobs at the end of each period. The matching function, evolution of employment, workers' job finding probability, firms' vacancy filling probability, and labor market tightness in the market for low-wage jobs are:

$$m_t^b = m(v_t^b, u_t^b), \quad n_{t+1}^b = (1 - \psi) \left[(1 - p_t^g s_t) n_t^b + p_t^b u_t^b \right] \qquad (3)$$

$$p_t^b = \frac{m(v_t^b, u_t^b)}{u_t^b}, \quad q_t^b = \frac{m(v_t^b, u_t^b)}{v_t^b}, \quad \theta_t^b = \frac{v_t^b}{u_t^b} \qquad (4)$$

The optimization problem of a representative household is to choose c_t, u_t^g, u_t^b, i_t and s_t, taking as given $\left\{ w_t^g, w_t^b, p_t^g, p_t^b, r_t \right\}$, to maximize the value function:

$$V_t^h(n_t^g, n_t^b, k_t) = max \left\{ u(c_t) + h(l_t) + \beta E_t[V_{t+1}^h(n_{t+1}^g, n_{t+1}^b, k_{t+1})] \right\} \qquad (5)$$

subject to the budget constraint, the laws of motion for capital and employment, and the definition of home production:

$$w_t^g n_t^g + w_t^b n_t^b + r_t k_t + d(u_t^g + u_t^b) + \pi_t = c_t + i_t + T_t \qquad (6)$$

$$i_t = k_{t+1} - (1 - \delta) k_t \qquad (7)$$

$$n_{t+1}^g = (1 - \psi) \left[n_t^g + p_t^g(u_t^g + s_t n_t^b) \right] \qquad (8)$$

$$n_{t+1}^b = (1 - \psi) \left[(1 - p_t^g s_t) n_t^b + p_t^b u_t^b \right] \qquad (9)$$

$$l_t = 1 - u_t^g - u_t^b - n_t^g - [1 + \kappa(s_t)] n_t^b \qquad (10)$$

where $u(c_t)$ is the utility derived from the consumption of goods and $h(l_t)$ is the utility derived from home production. $0 < \beta < 1$, $u_c(c_t) > 0$, and $h_l(l_t) > 0$ are the assumptions.

Combining the FOCs with respect to c_t and i_t yields the standard Euler equation for consumption $u_c(c_t) = \beta E_t\{u_c(c_{t+1})(r_{t+1} + 1 - \delta)\}$, where $u_c(c_t)$ is the derivative of the household's utility function with respect to consumption at time t. Using this Euler equation household's stochastic discount factor is derived as $\Xi_{t+1|t} = \beta \frac{u_c(c_{t+1})}{u_c(c_t)}$.

The FOC with respect to u_t^g yields to the optimality condition representing the household's participation decision in the market for high-wage

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jobs:

$$(11)$$

$$\frac{h_l(l_t) - u_c(c_t)d}{p_t^g} = (1-\psi)\beta E_t \left\{ u_c(c_{t+1})w_{t+1}^g - h_l(l_{t+1}) + \frac{h_l(l_{t+1}) - u_c(c_{t+1})d}{p_{t+1}^g} \right\}$$

Once the household allocates a member to look for a high-wage job, she joins the pool of the unemployed in the market for good jobs. In this state, the household has one less member engaging in home production, but one more member receiving unemployment benefits. The left hand side of the above equation represents this marginal cost of searching for a good job, which is the disutility due to reduced home production net of the unemployment benefits received by the searcher. Note that this cost is scaled by the endogenous job finding rate, since with probability p_t^g the unemployed worker gets matched with a high-wage job. The right hand side stands for the household's expected marginal benefit from having one more member with a high-wage job in the next period. Since production takes place one period after the matches take place, the wage income and the marginal disutility of work are inside the expectations sign. The last term on the right hand side is the asset value of having one less household member search for a job in the next period. In order to write down all expressions in terms of goods, the wage rate and the unemployment benefits are multiplied by $u_c(c_t)$, which is the value of the Lagrange multiplier associated with the budget constraint of the household.

Similarly, the FOC with respect to u_t^b yields to the optimality condition representing the household's participation decision in the market for low-wage jobs:

$$\frac{h_l(l_t) - u_c(c_t)d}{p_t^b} = (1-\psi)\beta E_t \left\{ u_c(c_{t+1})w_{t+1}^b - [1+\kappa(s_{t+1})]h_l(l_{t+1}) \right\}$$
$$+ (1-\psi)\beta E_t \left\{ p_{t+1}^g s_{t+1} \frac{h_l(l_{t+1}) - u_c(c_{t+1})d}{p_{t+1}^g} \right\}$$
$$+ (1-\psi)\beta E_t \left\{ (1 - p_{t+1}^g s_{t+1}) \frac{h_l(l_{t+1}) - u_c(c_{t+1})d}{p_{t+1}^b} \right\} \quad (12)$$

Again, the left hand side of this equation shows the household's marginal cost of having a member search in the market for low-wage jobs. The right hand side stands for the household's expected marginal benefit of having a member work at a low-wage job next period. The first part on the right hand side is

the marginal gain from having a member work at a low-wage job, which is the wage income net of cost for search effort and disutility of work. All household members working at low-wage jobs search with the same endogenous search intensity, so they all suffer from the search cost. The last two terms on the right hand side represent the asset value of the employed worker participating in the market for low-wage jobs. With probability $p_{t+1}^g s_{t+1}$ the worker will switch to a high-wage job, otherwise she will stay in the current low-wage job. As long as the worker is not separated exogenously, she will continue to work at one of the two types of jobs. Again, the terms representing the marginal cost of participation are scaled by the endogenous job finding rate, and all monetary terms are expressed in terms of goods. Lastly, the FOC with respect to the search intensity s_t yields to the following optimality condition:

$$\kappa_s(s_t) = \left(1 - \frac{p_t^g}{p_t^b}\right) \left[1 - \frac{u_c(c_t)d}{h_l(l_t)}\right] \tag{13}$$

This equation determines the optimal level of search intensity for the workers who are currently employed at low-wage jobs. $\kappa(s_t)$ is increasing and convex in s_t. This optimality condition states that search intensity increases with the difference between the asset values of high-wage and low-wage jobs. Also note that, as long as the probability of finding a high-wage job is less than that of a low-wage job ($p_t^g < p_t^b$), and as long as being unemployed carries a net utility cost ($h_l(l_t) > u_c(c_t)d$), it is always optimal for the household to choose a positive search intensity for on-the-job search.

4.2   Firms

There are two types of firms, which offer high-wage and low-wage jobs. All firms use the same constant returns to scale production function and are subject to an aggregate technology shock z_t. Labor and capital are the inputs of production. Firms differ only in the cost they face when opening new vacancies. The vacancy creation cost for high-wage firms is assumed to be higher than that for low-wage firms, that is $\gamma^g > \gamma^b$. The two outputs are imperfect substitutes in the final goods production. The production functions of the heterogenous intermediate goods and the final goods firms are $Y_t^g = e^{z_t}(N_t^g)^\alpha(K_t^g)^{(1-\alpha)}$, $Y_t^b = e^{z_t}(N_t^b)^\alpha(K_t^b)^{(1-\alpha)}$, and $Y_t = (Y_t^b)^\varrho(Y_t^g)^{1-\varrho}$. The price of the final consumption good is normalized to unity. The prices of the goods produced by the intermediate goods firms

are Pr_t^g and Pr_t^b, which can be further expressed as $Pr_t^g = (1 - \varrho)\left(\frac{Y_t}{Y_t^g}\right)$ and $Pr_t^b = \varrho\left(\frac{Y_t}{Y_t^b}\right)$.

Firms' vacancy filling rates depend on the matching functions for each type of job. High-wage firms fill their vacancies with an endogenous probability of $q_t^g = \frac{m_t^g}{v_t^g}$, whereas this probability is $q_t^b = \frac{m_t^b}{v_t^b}$ for low-wage firms.

The optimization problem of a high-wage firm is to choose v_t^g and K_t^g, taking $\{w_t^g, q_t^g, Pr_t^g, r_t, z_t\}$ as given, to maximize:

$$V_t^{fg}(N_t^g, z_t) = \tag{14}$$
$$max\left\{Pr_t^g e^{z_t}(N_t^g)^\alpha(K_t^g)^{(1-\alpha)} - w_t^g N_t^g - r_t K_t^g - \gamma^g v_t^g + E_t[\Xi_{t+1|t}V_{t+1}^{fg}(N_{t+1}^g, z_{t+1})]\right\}$$

subject to the law of motion for employment in the market for high-wage jobs $N_{t+1}^g = (1 - \psi)\left[N_t^g + q_t^g v_t^g\right]$. $\Xi_{t+1|t}$ is the stochastic discount factor defined earlier. The market for capital is competitive; therefore, the rental price of capital is equal to the marginal product of capital $r_t = (1 - \alpha)Pr_t^g \frac{Y_t^g}{K_t^g}$.

Using the FOC with respect to vacancies the job creation equation for a high-wage firm is derived as:

$$\frac{\gamma^g}{q_t^g} = (1 - \psi)\beta E_t\left\{\frac{u_c(c_{t+1})}{u_c(c_t)}\left[\alpha Pr_{t+1}^g \frac{Y_{t+1}^g}{N_{t+1}^g} - w_{t+1}^g + \frac{\gamma^g}{q_{t+1}^g}\right]\right\} \tag{15}$$

The left hand size of this job creation equation represents the firm's average cost of opening a high-wage vacancy and searching for a worker. The vacancy creation cost is divided by q_t^g, the endogenous probability of filling the vacancy. The right hand side represents the discounted expected benefit of hiring a worker. The first term is the expected marginal benefit of having a worker engaged in the job, the second one is the expected wage rate paid to the worker, and the last term is the asset value of having the vacancy filled in the next period. All terms on the right hand side are multiplied with $(1 - \psi)$ to account for the exogenous separation probability.

Similarly, the optimization problem of a low-wage firm is to choose v_t^b and K_t^b, taking $\{w_t^b, q_t^b, p_t^g, s_t, Pr_t^b, r_t, z_t\}$ as given, to maximize the the value function:

$$V_t^{fb}(N_t^b, z_t) = \tag{16}$$
$$max\left\{Pr_t^b e^{z_t}(N_t^b)^\alpha(K_t^b)^{(1-\alpha)} - w_t^b N_t^b - r_t K_t^b - \gamma^b v_t^b + E_t[\Xi_{t+1|t}V_{t+1}^{fb}(N_{t+1}^b, z_{t+1})]\right\}$$

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subject to the law of motion for employment in the market for low-wage jobs $N_{t+1}^b = (1 - \psi) \left[(1 - p_t^g s_t) N_t^b + q_t^b v_t^b \right]$. Again, the rental price of capital is equal to the marginal product of capital $r_t = (1 - \alpha) Pr_t^b \frac{Y_t^b}{K_t^b}$.

Solving the firm's problem yields to the following job creation equation:

$$\frac{\gamma^b}{q_t^b} = (1 - \psi)\beta E_t \left\{ \frac{u_c(c_{t+1})}{u_c(c_t)} \left[\alpha Pr_{t+1}^b \frac{Y_{t+1}^b}{N_{t+1}^b} - w_{t+1}^b + (1 - p_{t+1}^g s_{t+1}) \frac{\gamma^b}{q_{t+1}^b} \right] \right\} \tag{17}$$

The interpretation of this job creation equation is similar to the one above. The only difference is that now the firm takes into account the probability that the worker currently engaged in a low-wage job may switch to a high-wage job. The left hand side shows the average cost of opening a vacancy in the current period. Since the worker becomes productive in the next period, the gains from hiring the worker are expressed in expectations. The first two terms represent the expected gain of hiring the worker net of wages paid. The last term is the expected gain of keeping the vacancy occupied in the next period, accounting for the probability that the worker leaves for a high-wage job. All terms on the right hand side are multiplied by $(1 - \psi)$, since there is also the possibility of exogenous separation.

Wage rates in the markets for high-wage and low-wage jobs are determined through Nash bargaining between matched worker and firm pairs. μ and $(1 - \mu)$ represent the bargaining weights for the worker and the firm, respectively. The Nash bargaining problem in both markets is to choose the corresponding wage rates that solve:

$$max_{w_t^i} \left[\frac{V_{n^i}^h(n_t^g, n_t^b, k_t)}{u_c(c_t)} \right]^{\mu} \left[V_{N^i}^{fi}(N_t^i, z_t) \right]^{(1-\mu)} \tag{18}$$

where $i = g, b$ stands for the type of the job, $V_{n^i}^h(n_t^g, n_t^b, k_{t+1})$ is the marginal value for the household of having a member working at a type i job, and $V_{N^i}^{fi}(N_t^i, z_t)$ is the marginal value for the firm of hiring a worker for a type i job. The corresponding wage rates are:

$$w_t^g = \mu \left[\alpha Pr_t^g \frac{Y_t^g}{N_t^g} \right] + (1 - \mu) \left[\frac{h_l(l_t)}{u_c(c_t)} \right] \tag{19}$$

$$w_t^b = \mu \left[\alpha Pr_t^b \frac{Y_t^b}{N_t^b} - p_t^g s_t \frac{\gamma^g}{q_t^g} \right] + (1 - \mu) \left[[1 + \kappa(s_t)] \frac{h_l(l_t)}{u_c(c_t)} \right] \tag{20}$$

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For high-wage jobs, the wage rate is the weighted sum of the threshold values for the worker and the firm. On the one hand, the firm's marginal benefit from hiring a worker for a high-wage job is $\alpha Pr_t^g \frac{Y_g^t}{N_t^g}$, which is the firm's threshold level. The firm would not agree to pay a wage rate higher than this amount. On the other hand, the threshold level for the worker to accept a good job is the disutility from giving up home production $\frac{h_l(l_t)}{u_c(c_t)}$, expressed in terms of goods. The worker would not agree to work for a wage rate below this value. Note that the weights that multiply the threshold levels correspond to the bargaining weights for the worker and the firm. Similarly, the wage rate for a low-wage job is the weighted sum of the threshold values for working and hiring. When the worker and the firm are engaged in a low-wage job relationship, they both incur some additional costs. By assumption, the worker searches for a high-wage job; therefore, she pays for the cost of her search activity. Also, the firm faces the risk of having an unfilled vacancy due to the possibility of the worker leaving to work at a high-wage job. The threshold values are adjusted to compensate for the costs of a possible endogenous termination of the employment relationship.

4.3 Closing the Model and the Competitive Equilibrium

There is no government consumption. The government collects taxes to finance the unemployment benefits. The aggregate resource constraint for the economy reads as $Y_t = c_t + \gamma^g v_t^g + \gamma^b v_t^b + i_t$. The logarithm of the aggregate technology shock follows an AR(1) process of the form $z_{t+1} = \rho^z z_t + \epsilon_{t+1}^z$ with $\epsilon^z \sim (0, \sigma_z^2)$ and $0 < \rho^z < 1$. This subsection ends with the definition of a competitive equilibrium of the model.

$\underline{\textit{Definition 1.}}$ A competitive equilibrium of the model is defined as the decision rules for the stochastic processes coming from the household's problem $\left(c_t, n_t^g, n_t^b, u_t^g, u_t^b, i_t, s_t\right)$, and from the firms' problems $\left(N_t^g, N_t^b, K_t^g, K_t^b, v_t^g, v_t^b\right)$, given the factor prices $\{w_t^g, w_t^b, r_t, Pr_t^g, Pr_t^b\}$, the probabilities of matching $\{p_t^g, p_t^b, q_t^g, q_t^b\}$, and the stochastic technology shock $\{z_t\}$, such that: Labor markets clear $n_t^b = N_t^b$, $n_t^g = N_t^g$; the market for capital clears $k_t = K_t^g + K_t^b$; the household's choices, firms' choices, the laws of motion for capital and employment, the definitions of matching functions, labor market tightness, job finding and vacancy filling probabilities, wages, the rental rate of capital,

and the aggregate resource constraint are satisfied.[4]

5   Calibration and Impulse Responses

The model is calibrated to match the dynamics of the U.S. data for the period 1951-2005. Period length is one quarter. First, the parameters that can be set without solving the model are chosen. Next, the remaining parameters are determined by targeting some relevant first-order and second-order moments calculated from the data.

Household preferences are taken as $u(c_t) + h(l_t) = ln(c_t) + \bar{H}\left(\frac{l_t^{(1-\phi)}}{1-\phi}\right)$, where $\bar{H}$ is a constant, and ϕ is the elasticity parameter. The matching functions for both types of jobs have the usual Cobb-Douglas specifications of $m_t^b = \bar{m}[u_t^b]^\epsilon[v_t^b]^{(1-\epsilon)}$ and $m_t^g = \bar{m}[u_t^g + s_t n_t^b]^\epsilon[v_t^g]^{(1-\epsilon)}$. $\kappa(s_t) = Bs_t^\sigma$ is the functional form for the search cost, where B is a positive constant, and $\sigma > 1$, so that the cost of on-the-job search is strictly increasing and convex in search intensity s_t.

The household discount factor β is 0.99, which corresponds to a 4 percent annual interest rate. Veracierto (2008) calculates the steady state level of monthly investment-to-capital ratio as 0.006. This implies a quarterly capital depreciation rate of 0.018; therefore, $\delta = 0.018$. Following earlier studies in the real business cycle literature, the elasticity of output with respect to capital in the intermediate goods production function $(1 - \alpha)$ is 0.36, and the persistence parameter in the AR(1) process of the logarithm of the aggregate technology shock is 0.95. The elasticity parameter ϵ in the matching functions is 0.40, which is the estimated value in Blanchard et al. (1989). In order to satisfy the Hosios (1990) condition, the bargaining power of workers in the Nash bargaining problem μ is also set equal to 0.40.

In the model, total separations consist of both exogenous and endogenous separations. The data compiled by Fallick and Fleischman (2004) show that employer-to-employer flows correspond to almost 40 percent of total separations from employment. Davis, Faberman, Haltiwanger and Rucker (2008) examine the JOLTS data, adjust the worker flow rates, and report a monthly total separation rate of 4.96. This corresponds to a quarterly total separation rate of 14.9. Using this information exogenous and endogenous separation

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[4]Also, see Appendix A1 for the derivation of workers' indifference condition as an equilibrium result.

rates are set as 0.095 and 0.059, respectively. All parameter values chosen a priori are reported in Table 2.

Table 2: Parameters Chosen Without Solving the Model

	Value	Description	Source
β	0.99	Household Discount Factor	Annual Interest Rate = 0.04
δ	0.018	Depreciation Rate	Veracierto (2008)
α	0.64	Elasticity of Output wrt. Labor	RBC Literature
ρ^z	0.95	Persistence of the Agg. Tech. Shock	RBC Literature
ϵ	0.4	Match Elasticity	Blanchard and Diamond (1989)
μ	0.4	Nash Bargaining Share	Hosios (1990)
ψ	0.095	Exogenous Separation Rate	Based on JOLTS and CPS Data

The remaining parameters are set so that the model solution matches certain data moments. These parameters are reported in Table 3. The elasticity of home production in the household preferences ϕ is 0.22, which is calibrated to match the volatility of employment relative to output in the data, as in Tripier (2004) and Veracierto (2008).[5] The preference constant $\bar{H}$ is calibrated to generate an employment to population ratio of 0.59, which is the average quarterly ratio in the U.S. labor market for the period 1951-2005. The resulting value is 1.05.

To set the elasticity parameter in the search cost function σ, the volatility of employer-to-employer flows relative to output is used as the target. From the dataset provided by Fallick and Fleischman (2004) the volatility of employer-to-employer flows relative to output is calculated as 12.25 for 1994-2005 at quarterly frequency.[6] Using this target, the calibrated value

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[5]Both Tripier (2004) and Veracierto (2008) calibrate of the preference parameter in the household's utility function in the same way. Veracierto (2008) uses three different utility functions, one linear as in Merz (1997), and two non-linear as in Shi and Wen (1999) and Hornstein and Yuan (1998), to evaluate the performance of the basic three-state model. In all three calibrations, he determines the curvature of home production in the utility function by targeting the relative standard deviation of employment calculated from the U.S. data. Veracierto (2008) uses the same utility function specified earlier in this paper and repeats Tripier's calibration strategy.

[6]The series are logged and HP-filtered (smoothing paramater is 10^5). This is of course an imperfect way to measure the volatility of job-to-job flows, however no employer-to-employer flow data exists before 1994. Krause and Lubik (2010) use quits from the BLS labor turnover series for the manufacturing sector for 1950-1981 and calculate the relative volatility of quits as 10.06.

Table 3: Parameters Chosen by Solving the Model

	Value	Description	Targets
ϕ	0.22	Elasticity of Home Production	Relative Volatility of Emp.
H	1.05	Constant in the HH Pref. for Home Prod.	Emp./Pop. = 0.59
σ	1.2	Elasticity of Search Cost	Relative Volatility of E-E Flows
B	0.09	Constant in the Search Cost Function	Endogenous Separation Rate = 0.059
$\bar{m}$	0.75	Matching Function Constant	Avg. Firm Matching Probability = 0.78
γ^g	0.6	High-Wage Vacancy Creation Cost	Ratio of Vacancy Creation Costs = 3
γ^b	0.2	Low-Wage Vacancy Creation Cost	Total Vacancy Cost/Output = 0.05
ϱ	0.4	Low-Wage Firms' Weight in Production	Emp. Share of Low-Wage Firms = 0.4
d	0.6	Unemployment Benefit	Benefit to Wage Ratio = 0.7
ϵ^z	0.0074	Std. of Log of Agg. Tech. Shock	Volatility of Output

for the elasticity parameter in the search cost function becomes 1.2. The constant in the search cost function B is 0.09, calibrated to match a steady state endogenous separation rate of 0.059, as was calculated above.

Since the matching technologies are the same in the two sectors, the constants in the matching functions are the same as well. Targeting an average quarterly vacancy filling probability of 0.78, the constant becomes 0.75. Vacancy creation costs γ_b and γ_g are calibrated to generate a total vacancy creation cost of 5 percent of total output as in Krause and Lubik (2010). The vacancy creation cost for high-wage firms is assumed to be three times the cost for low-wage firms.[7] Using these two targets, γ_b and γ_g become 0.2 and 0.6, respectively.

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[7]A brief explanation on the ratio of the vacancy creation costs is necessary. Davis, Faberman and Haltiwanger (2010) report that not all firms post vacancies in order to attract workers. More specifically, they find that 67.2 percent of hiring occurs without vacancy posting at establishments in the construction sector. In terms of employment, 73.7 percent of employment in the construction sector is at establishments that do not report vacancies. These numbers are 57.8 and 59.2 percent in natural resources and mining, 49.1 and 59.3 percent in retail trade, 47.7 and 54.2 percent in leisure and hospitality, 41.5 and 51.2 percent in transport, wholesales and utilities, and lastly, 54.5 and 70.6 percent in other service sectors. On the contrary, some other sectors, such as education and health seem to attract workers primarily by posting vacancies. In light of this evidence, the firms in these sectors are linked to high-wage firms in the model, while the rest is linked to low-wage firms. Krause and Lubik (2010) set the vacancy creation cost ratio $\frac{\gamma_g}{\gamma_b}$ to 4, arguing that vacancy creation costs are related to the capital intensities of sectors, and that the difference between the capital intensity of average high-wage and low-wage jobs is around

The total production share of low-wage firms ϱ is set to 40 percent based on the sectoral classification and employment shares reported in Davis, Faberman and Haltiwanger (2010).[8] The unemployment benefit level d is 0.6, which is calibrated to match a ratio of benefits to average wages of 0.7. This ratio lies between the two extreme calibration targets used in Shimer (2005) and Hagedorn and Manovskii (2008). Lastly, the standard deviation of the log of the aggregate technology shock is 0.0074, which is calibrated to replicate the observed standard deviation of the total output in the sample.

5.1   Impulse Responses

First, consider the three-state model without on-the-job search. The proposed on-the-job search model converges to this simpler model when there is no employed search, that is, $s = 0$, and firms are homogenous.[9] Figure 1 shows the dynamics of the model in response to a 1 standard deviation positive productivity shock. Impulse responses are reported as percentage deviations from the steady state values.

On impact, the positive productivity shock leads to an increase in the output level. Firms open more vacancies and the job finding probability of workers increases, which results in higher overall employment. But, note that unemployment also increases sharply. This is due to the fact that the household sends more members to participate in the labor market since it is a good time to engage in market work rather than home production. The number of searchers increases as labor force participation goes up. Since it takes time to form matches, not all searchers can find jobs; therefore,

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this level. They do not include capital in their model. However, capital is modeled here; therefore, vacancy creation costs are assumed to represent explicit costs of recruitment, such as job advertising, hiring recruiters, screening, interviewing, etc. There is no data source that reports actual vacancy creation costs in different sectors. In the baseline calibration the vacancy creation cost ratio is set to 3 in order to generate a moderate wage differential across the sectors. Sensitivity analysis is provided in Appendix A3.

[8]This corresponds to the total employment share of sectors with lower propensities to post vacancies. Employment shares are 5.3 percent in construction, 0.5 percent in natural resources and mining, 11.4 percent in retail trade, 9.3 percent in leisure and hospitality, 8 percent in transport, wholesales and utilities, and 4.1 percent in other service sectors.

[9]The calibration strategy used to determine the parameters for this simplified model is identical to the calibration strategy used to set the parameters of the on-the-job search model. There is no endogenous separation and exogenous separation rate is set to 0.1 as in the earlier studies.
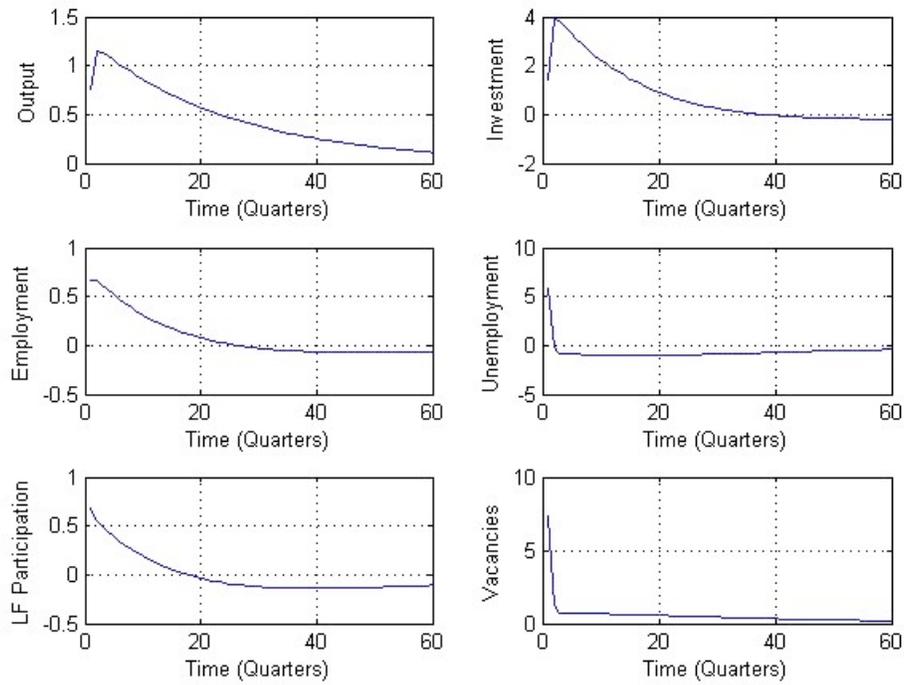
Figure 1: Impulse Responses for the Economy Without On-the-Job Search: Percentage Deviations from the Steady State

unemployment increases. Over time, as the newly-opened vacancies get filled by workers, unemployment falls quickly to a level around its steady state value. This results in a lower incentive for firms to open vacancies, so vacancy creation goes down quickly as well. Both investment and employment follow the output level, increasing on impact and then slowly returning to their steady state levels.

Next, consider the on-the-job search model. Job searchers now include not only the unemployed, but also the employed agents who would like to work at better-paying jobs. As shown in Figure 2 aggregate output and investment both increase on impact in response to the positive productivity shock. This leads to higher aggregate vacancy creation by firms. As in the model without on-the-job search, the household sends more members to search for jobs. The increased labor force participation with new searchers and the time lag for match formations result in an initial increase in aggregate unemployment. However, the subsequent evolution of unemployment is very different compared to the previous case. As firms open more vacancies and workers' job finding probabilities increase, job searchers become employed, and the aggregate unemployment level falls quickly below its steady state level. Unemployment stays below its steady state level for a long time because vacancy creation remains high due to on-the-job search. In the model without on-the-job search, the household's only adjustment mechanism to a favorable shock is changing the number of members participating in the labor market. However, with on-the-job search there is a second margin for labor market adjustments to take place. As job finding rates increase workers in low-wage jobs get matched with better jobs that pay higher wages. This is why job-to-job transitions increase substantially. Also, vacancy creation remains high, as will become more clear below.

Figure 3 shows the responses in the two sectors separately. The increases in output, employment, and wages are higher at high-wage jobs than at low-wage jobs. As workers flow from low-wage jobs to high-wage jobs the relative output in high-wage firms increases, and in turn, the relative price of the intermediate good produced by low-wage firms increases. This leads to a higher incentive for low-wage firms to keep posting vacancies. Within a few periods after the shock, vacancy creation by high-wage firms falls sharply (similar to the evolution of vacancy creation in the model without on-the-job search), but vacancy creation by low-wage firms remains high.

Almost all the variation in aggregate unemployment is due to the evolution of search effort among workers searching for high-wage jobs. On impact,
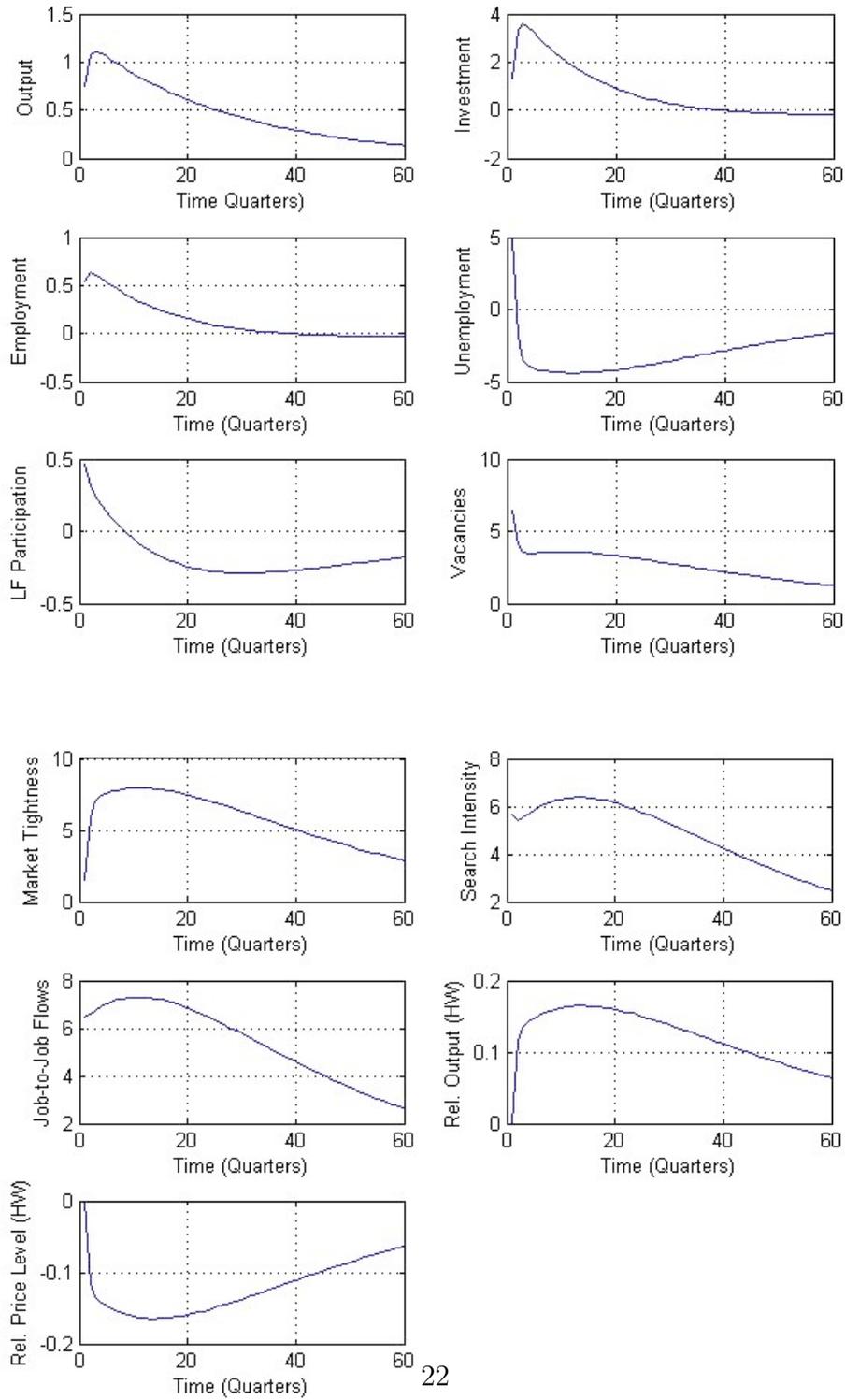
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Figure 2: Impulse Responses for the On-the-Job Search Economy - Aggregate
Variables: Percentage Deviations from the Steady State

as output and vacancy creation increase in the high-wage sector, the employed searchers increase their search intensity and move to high-wage jobs. Note that this search activity by the employed leads to congestion in the market for high-wage jobs. Therefore, the unemployed searchers direct their search towards low-wage jobs, and this results in a pronounced fall in the number of unemployed searching for high-wage jobs compared to the steady state level. In the same time, with the increased flow of workers from low-wage jobs to high-wage jobs, more job opportunities become available in the market for low-wage jobs. In turn, the unemployed search in the market for low-wage jobs, where competition is low. This keeps the demand for jobs, vacancy creation, and workers' job finding rate high in this market.
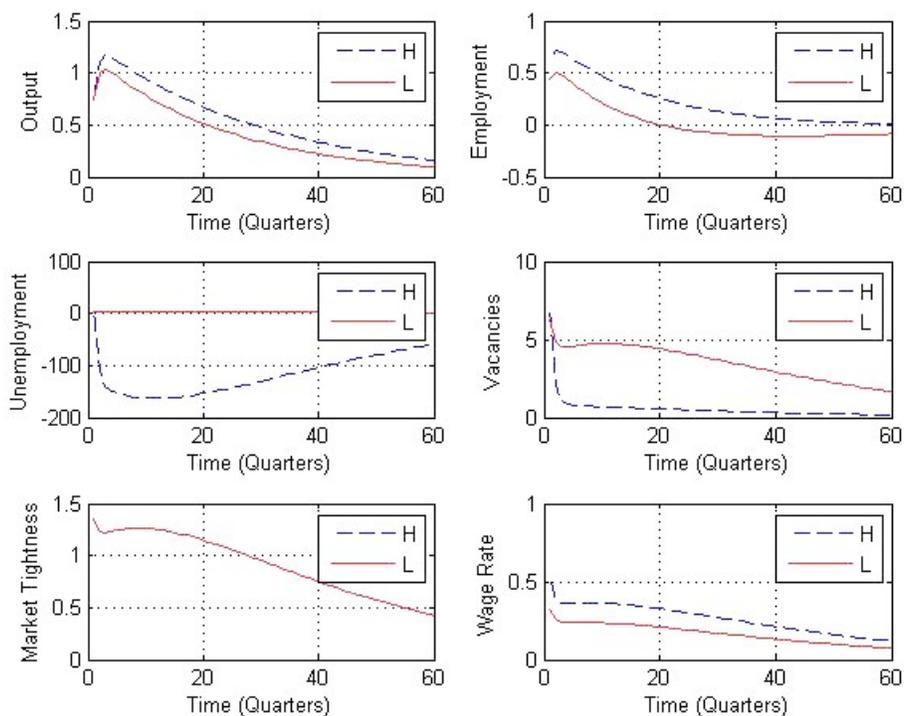
Figure 3: Impulse Responses for the On-the-Job Search Economy - Comparison of the Two Sectors: Percentage Deviations from the Steady State

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6 Results and Discussion

6.1 Simulation Results

This section compares the business cycle statistics reproduced by the model to the observed values in the data. The first column in Table 4 reports quarterly U.S. business cycle statistics for the period 1951-2005.[10] Other columns present the simulation results for the basic three-state model without on-the-job search, and the proposed on-the-job search economy. These statistics correspond to averages across 100 simulations of 220 periods.[11] All series are logged and detrended with the Hodrick-Prescott filter before the statistics are computed.[12]

To begin with, consider the simulation results for the model without on-the-job search. The calculated relative standard deviations and the correlations of all major variables with output are reported in Table 4, while cross-correlations are reported in Table 5. Table 4 shows that the relative volatilities of unemployment and labor market tightness are small, whereas the relative volatility of labor force participation is large compared to the data. The relative standard deviation of unemployment is 2.78, which is 65 percent lower than the empirical value. Similarly, the relative volatility of labor market tightness is only 1.62, which is 1/10 of the actual value. The relative standard deviation of labor force participation is 0.65, which is almost twice the actual level. The gap between the model generated and the actual statistics is even more pronounced when out of the labor force is considered (1.19 vs. 0.48). The model correlation of unemployment with output is only -0.12, compared to -0.76 in the data. The correlation of unemployment with vacancies is high at 0.85, but has the opposite sign of its empirical counterpart. The correlations of unemployment and vacancies with workers' job finding probability are both small in absolute value, corresponding to -0.13 and 0.41, respectively. The correlations of unemployment and vacancies with firms' vacancy filling probability are 0.13 and -0.41, respectively; these numbers are again very small in absolute value compared to the data values. Note that in the data unemployment is highly negatively correlated with workers' job finding probability (correlation is -0.95), while it is highly

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[10]See Appendix A2 for the data used to calculate these statistics.

[11]In order to match the data for the period 1951-2005. 320 periods of data are generated in each simulation, the first 100 periods discarded and the rest is used for calculations.

[12]The smoothing parameter is 10^5 given the criticism in Shimer (2005).

Table 4: Business Cycle Statistics

a. Relative Standard Deviation $(\frac{\sigma_x}{\sigma_Y})$

	U.S. BC Stats.	Without OJS	OJS Model
Output (Y)	1	1	1
Consumption (C)	0.56	0.39	0.31
Investment (I)	3.89	3.34	3.17
Employment (N)	0.56	0.56	0.56
Unemployment (U)	7.92	2.78	5.63
Labor Force (L)	0.35	0.65	0.39
Out of LF (OLF)	0.48	1.19	0.74
Wage Rate (W)	0.68	0.48	0.25 & 0.39
Market Tightness (θ)	15.89	1.62	14.57

b. Correlation with Output $(\rho_{x,Y})$

	U.S. BC Stats.	Without OJS	OJS Model
Output (Y)	1	1	1
Consumption (C)	0.85	0.79	0.73
Investment (I)	0.92	0.97	0.96
Employment (N)	0.74	0.92	0.95
Unemployment (U)	-0.76	-0.12	-0.59
Labor Force (L)	0.14	0.65	0.47
Out of LF (OLF)	-0.23	-0.82	-0.48
Wage Rate (W)	0.58	0.92	0.95 & 0.93
Market Tightness (θ)	0.83	0.91	0.72

positively correlated with firms' vacancy filling probability (correlation is 0.96). The opposite is true for vacancies (corresponding statistics are 0.92 and -0.98). Finally, the correlations of labor force participation and out of the labor force with output are too high in absolute value compared to the data values (0.65 vs. 0.14 for participation, and -0.82 vs. -0.23 for out of the labor force, respectively).

These results support the conclusions of both Tripier (2004) and Veracierto (2008). The simple three-state model fails to generate strongly countercyclical unemployment and the Beveridge curve relationship between vacancies and unemployment. Moreover, the model-generated relative standard deviations of unemployment and labor market tightness are too low, while the correlations of labor force participation and out of the labor force with output are too high.

Table 5: Cross-Correlations of the Major Labor Market Variables

a. U.S. Data

	U	V	θ	p	q
U	1	-0.90	-0.97	-0.95	0.96
V	-	1	0.98	0.92	-0.98
θ	-	-	1	0.96	-0.99
p	-	-	-	1	-0.92
q	-	-	-	-	1

b. Without On-the-Job Search

	U	V	θ	p	q
U	1	0.85	-0.13	-0.13	0.13
V	-	1	0.41	0.41	-0.41
θ	-	-	1	0.99	-0.98
p	-	-	-	1	-0.99
q	-	-	-	-	1

c. On-the-Job Search Model

	U	V	θ	p	q	s	jtj flows
U	1	-0.34	-0.63	-0.58	0.58	-0.54	-0.51
V	-	1	0.65	0.95	-0.94	0.89	0.84
θ	-	-	1	0.74	-0.73	0.82	0.87
p	-	-	-	1	-0.99	0.90	0.85
q	-	-	-	-	1	-0.89	-0.84
s	-	-	-	-	-	1	0.93
$jtj\ flows$	-	-	-	-	-	-	1

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Next, consider the proposed on-the-job search model. Compared to the previous model, the relative standard deviations of unemployment and labor force participation implied by the on-the-job search model are much more in line with their empirical counterparts. The relative standard deviation of unemployment with respect to output is 5.63, which is a major improvement compared to the generated value in the model without on-the-job search (2.78). The relative volatility of labor force participation is 0.39, which is very close to the observed value of 0.35. Another major improvement is seen in the relative volatility of aggregate labor market tightness, which is 14.57 (compared to 1.62 previously), close to the data value of 15.89.

As shown in Table 4, the model with on-the-job search predicts the correlation of output and unemployment as -0.59. Although this is not as high as its empirical counterpart of -0.76, the fact that the correlation is negative and large is very important, since the three-state model without on-the-job search fails in this dimension. The correlations of unemployment with workers' job finding probability and firms' vacancy filling probability are much higher at -0.58 and 0.58, which are more in line with the data. The results are even more satisfactory when the correlations of vacancies with job finding (0.95) and vacancy filling (-0.94) probabilities are considered. The model successfully reproduces the negative correlation between unemployment and vacancies, also known as the Beveridge Curve relationship. This correlation is -0.34, which is a major improvement relative to the correlation of 0.85 in the model without on-the-job search. The correlation of labor force participation with output is lower compared to the previous model (0.47 vs. 0.65 earlier). A similar conclusion applies for the correlations between out of the labor force and output (-0.48 and -0.82 for the model with and without on-the-job search, respectively). Finally, the model predicts search intensity and job-to-job flows to be highly positively correlated with vacancies (0.89 and 0.84), labor market tightness (0.82 and 0.87) and workers' job finding probability (0.90 and 0.85). The correlations of search intensity and job-to-job flows with unemployment (-0.54 and -0.51) and firms' vacancy filling probability (-0.89 and -0.84) are highly negative as expected.

Robustness checks are presented in Appendix A3.

6.2 Discussion

As mentioned earlier, the failure of the basic three-state model without on-the-job search is mainly due to high responsiveness of participation to ag-

gregate technology shocks. To address this problem, an on-the-job search mechanism is introduced as an additional adjustment margin for the household. The proposed model can generate countercyclical unemployment, and it is more successful in matching the relative volatilities of unemployment and labor market participation. Moreover, the model predicts highly volatile aggregate labor market tightness, which is in line with the U.S. data.

How does on-the-job search contribute to the model's success? When a positive productivity shock hits the economy, the incentive of the household to send more workers to search for jobs increases. The incentive for higher labor force participation increases because the return from market activities increases with the favorable shock. However, on-the-job search works as an alternative margin for adjusting the labor market activities of the household. With the positive productivity shock, the household allocates more time for on-the-job search. The increase in the on-the-job search activity dampens the movements along the labor force participation margin. The magnitude of the dampening effect depends on the time cost of on-the-job search, which depends on the elasticity parameter in the search cost function. As the elasticity σ goes from one to infinity, the cost of on-the-job search increases as well. An extremely high level of σ corresponds to shutting down the on-the-job search margin, so that the model becomes similar to the basic three-state model. Overall, the relative volatilities of labor force participation and unemployment become closer to their empirical counterparts due to the labor market movements generated by on-the-job search.

Another important contribution of the on-the-job search mechanism is its impact on the volatility of aggregate labor market tightness. The mechanism generates highly volatile aggregate labor market tightness ($\theta_t = v_t/u_t$) due to the high responsiveness of vacancy creation by low-wage firms. In order to further investigate this point, consider labor market tightness in the market for high-wage jobs $\theta_t^g = \frac{v_t^g}{u_t^g + s_t n_t^b}$. Compared to the model without on-the-job search, this equation has the additional term for the employed job seekers $s_t n_t^b$. On-the-job search reduces the responsiveness of labor market tightness in the high-wage sector to productivity shocks. Recall the wage equations in

the two sectors:

$$w_t^g = \mu \left[\alpha Pr_t^g \frac{Y_t^g}{N_t^g} \right] + (1 - \mu) \left[\frac{h_l(l_t)}{u_c(c_t)} \right] \tag{21}$$

$$w_t^b = \mu \left[\alpha Pr_t^b \frac{Y_t^b}{N_t^b} - s_t \gamma^g \theta_t^g \right] + (1 - \mu) \left[[1 + \kappa(s_t)] \frac{h_l(l_t)}{u_c(c_t)} \right] \tag{22}$$

Labor market tightness in the high-wage sector appears in the wage equation for the low-wage jobs. Any increase in labor market tightness in the high-wage sector results in a reduction of wages in the low-wage sector, since low-wage firms take into account the possibility of losing a worker to a high-wage firm when bargaining for wages. On-the-job search dampens the movements of labor market tightness in the high-wage sector, which also corresponds to dampened movements of wages in the low-wage sector. Since wages are more stable, low-wage firms continue to post vacancies. Moreover, firms have diminishing marginal product of labor, therefore losing a worker raises the marginal benefit of hiring for low-wage firms, which also explains why vacancy creation remains high in this sector. Additionally, there is high competition in the high-wage sector due to congestion caused by the employed searchers. This leads to increased search activity by the unemployed in the low-wage sector following a favorable shock. As more workers search for jobs in the low-wage sector, firms continue to post vacancies. In turn, aggregate vacancy creation and volatility of vacancies in the economy remain high, resulting in a highly responsive aggregate labor market tightness. This helps all searchers in finding jobs. Therefore, a few periods after the positive aggregate technology shock hits the economy, unemployment falls quickly below its steady state level and follows a countercyclical pattern. The persistence of high vacancy creation also explains why the model no longer generates a high positive correlation between unemployment and vacancies. In fact, the correlation now has the correct negative sign. Lastly, it must be noted that the on-the-job search mechanism also affects relative wages. Given that the vacancy creation cost is higher for the high-wage firms, and that both types of firms face downward-sloping demand, the wage rate has to be higher in equilibrium in the high-cost sector than in the low-cost sector. This wage difference provides the incentive for on-the-job search. Additionally, on-the-job search has an effect on the difference between the wage rates. As long as the search cost is elastic ($\sigma > 1$), for any positive search intensity $s_t \gamma^g \theta_t^g > \kappa(s_t) \frac{h_l(l_t)}{u_c(c_t)}$. The maximum wage that a low-wage firm is willing to

pay is lower, while the minimum wage that the worker would accept is higher due to costly on-the-job search. The decrease in the firm's match value dominates the increase in the worker's match value. This is why the wage gap between the two sectors increases as the search intensity increases.

7 Conclusion

This paper develops a general equilibrium business cycle model with labor market search and matching frictions, endogenous labor force participation, and on-the-job search. Previous studies that incorporate endogenous labor force participation and labor market search and matching frictions in a real business cycle framework find that the model fails to replicate the labor market dynamics observed in the U.S. data. In order to improve the shortcomings of this three-state model, the model is enriched with an on-the-job search mechanism that leads to job-to-job flows, which are important flows in the U.S. labor market. On-the-job search mechanism helps the model in generating countercyclical unemployment and the negative correlation between unemployment and vacancies observed in the data. Quantitatively, the business cycle statistics reproduced by the proposed model are more in line with their empirical counterparts.

Previous studies had pointed out the importance of considering the participation margin in models focusing on labor markets dynamics. However, the failure of the earlier attempts to incorporate the participation margin in a real business cycle framework with search frictions had been discouraging. This study serves as a promising step. It shows that incorporating an on-the-job search mechanism into the simple three-state model can significantly improve the model's performance in matching the key quantitative facts on the cyclical properties of major labor market variables. Therefore, it suggests that it is worthwhile to build richer models in this direction.

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A The Workers' Indifference Condition

Since both jobs coexist in the equilibrium, it must be the case that an unemployed household member is indifferent between looking for a high-wage and a low-wage job. In other words, the asset values of both types of unemployment have to be equal. This leads to the following equilibrium result.

Proposition 1. In the equilibrium, the relative labor market tightness for the two types of jobs is inversely proportional to their relative vacancy creation costs. $\frac{\theta_t^b}{\theta_t^g} = \frac{\gamma^g}{\gamma^b}$. Since it is assumed that $\gamma^g > \gamma^b$, labor market tightness must be higher in the market for low-wage jobs in the equilibrium.

Proof. Setting the asset values of unemployment in the two sectors equal to each other gives $p_t^b \frac{\gamma^b}{q_t^b} = p_t^g \frac{\gamma^g}{q_t^g}$. Note that labor market tightness in the two sectors can be defined as $\theta_t^b = \frac{p_t^b}{q_t^b}$ and $\theta_t^g = \frac{p_t^g}{q_t^g}$. Then, the derived arbitrage condition simplifies to $\frac{\theta_t^b}{\theta_t^g} = \frac{\gamma_t^g}{\gamma_t^b}$.

B Calculating the U.S. Business Cycle Statistics

Seasonally adjusted quarterly data on consumption and investment are taken from the U.S. Bureau of Economic Analysis (BEA).[13] Real aggregate consumption is calculated as the sum of real personal consumption expenditures on non-durable goods and services. Real aggregate investment corresponds to real gross private domestic investment. To be consistent with the model, which omits government purchases and trade, real aggregate GDP is calculated as the sum of these investment and consumption measures. Monthly data on the levels of employment, unemployment and labor force participation are taken from the website of the Bureau of Labor Statistics (BLS).[14] The monthly series are transformed to quarterly frequency by taking the average value for a quarter. Quarterly per person wage rates are calculated as $wage = \frac{compensation \times output}{employment \times current\$output}$ using non-farm business series from the BLS.

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[13]Available from the BEA website at http://www.bea.gov. The real series are in billions of chained (2005) dollars.

[14]See http://www.bls.gov/bls/employment.htm for more details.

The vacancy data come from the Conference Board's Help Wanted Index.[15] Quarterly averages of job finding and vacancy filling rates are taken from the data set constructed by Robert Shimer.[16] The corresponding probabilities are then calculated as $probability = 1 - e^{-rate}$. All data series are logged and detrended with the Hodrick-Prescott (HP) filter before the business cycle statistics are calculated.[17]

The usual properties of the U.S. business cycle statistics are observed. Consumption is half as volatile as output, while investment is 4 times as volatile as output. Of the labor market variables, unemployment and labor market tightness have the highest standard deviations. The volatility of unemployment is more than 7 times that of output, while labor market tightness is 15 times more volatile than output. The least volatile variables are labor force participation and out of the labor force, with relative standard deviations of 0.35 and 0.48, respectively. Finally, the relative standard deviation of employment is 56 percent.

The cross-correlations show that consumption, investment, employment and labor market tightness are highly positively correlated with output, while unemployment is highly negatively correlated with output. Labor force participation has a small positive correlation with output, while the correlation between out of the labor force and output is small and negative. Numerically, the cross-correlations of output with employment, unemployment, labor force participation and out of the labor force are 0.74, -0.76, 0.14 and -0.23, respectively. Unemployment is also highly negatively correlated with vacancies, labor market tightness and workers' job finding probability, with cross-correlation values of -0.90, -0.97 and -0.95, respectively. Vacancies are highly positively correlated with labor market tightness (0.98) and workers' job finding probability (0.92). Finally, firms' vacancy filling probability has a high positive correlation with unemployment (0.96) and a high negative correlation with vacancies (-0.98).

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[15]This data is borrowed from the dataset used in Den Haan and Kaltenbrunner (2009), which is publicly available at http://www1.feb.uva.nl/mint/wdenhaan/data.htm.

[16]For additional details, see Shimer (2012) and his webpage at the following address: http://sites.google.com/site/robertshimer/research/flows.

[17]The smoothing parameter is chosen as 10^5 given the criticism in Shimer (2005).

C   Sensitivity Analysis

First, consider the elasticity parameter in the household's preferences for home production ϕ. As explained in the text, this parameter is calibrated to match the model-generated relative volatility of employment to its empirical value. This is the calibration method used in Veracierto (2008) and Tripier (2004), and this method is followed to make the results fully comparable to theirs.

Similar to this study, Ebell (2011) also aims to improve the puzzling counterfactual results generated by the three-state model. She argues that an alternative calibration strategy allows the model to generate countercyclical unemployment and a negative correlation between unemployment and vacancies observed in the data. She proposes three alternative calibration techniques: calibrating the elasticity of labor supply to match the relative volatility of labor force participation, the small surplus calibration strategy used in Hagedorn and Manovskii (2008), and correcting for a possible time aggregation problem by calibrating to weekly frequency. Since the only common calibration element between this study and hers is the first one, the elasticity parameter ϕ is calibrated by targeting the relative volatility of labor force participation. The resulting statistics are reported in Table A1.

Note that, the on-the-job search mechanism introduced in this study already keeps the participation margin stable, resulting in a low relative volatility of labor force participation. Matching the exact data moment has almost no effect on the results of the baseline calibration.

	Current Calibration $\phi =0.22$	Ebell's Calibration $\phi =0.26$
σ_U/σ_Y	5.63	5.62
σ_N/σ_Y	0.56	0.54
σ_{LF}/σ_Y	0.39	0.35
$\rho(U,V)$	-0.34	-0.36
$\rho(U,Y)$	-0.59	-0.61
$\rho(N,Y)$	0.95	0.96
$\rho(LF,Y)$	0.47	0.46

Table 6: Varying the Elasticity of Home Production

Next, consider the elasticity of search cost σ . The target for this parameter is the relative volatility of employer-to-employer flows. Any increase in

this elasticity parameter results mainly in reductions in the relative volatilities of job-to-job flows and labor market tightness. On the contrary, as σ approaches one from above, these variables become highly volatile.

Finally, vary the only free parameter of the calibration, which is the ratio of the vacancy creation costs $\frac{\gamma^g}{\gamma^b}$. In the current calibration the vacancy creation cost for high-wage firms is three times the cost for low-wage firms. This value is chosen to be high enough to guarantee a moderate wage difference across the two sectors. On the one hand, a reduction in the ratio of the vacancy creation costs leads to a decrease in the wage difference across the two sectors and dampens the volatilities of job-to-job flows, unemployment and labor market tightness. On the other hand, an increase in this ratio increases the responsiveness of job-to-job flows, unemployment and labor market tightness without leading to any significant changes in the degree of countercyclicality of unemployment or the Beveridge Curve relationship.

A better way to choose this parameter would be to use micro level data to obtain a wage differential estimate by running a regression of wages of all workers on sectoral dummies and control variables. This regression would give a ratio between the wages paid in the sectors that heavily depend on vacancy creation (such as health, education, etc.) and the wages paid in the sectors that depend less on vacancy creation (such as construction, retail trade, etc.). Then, the ratio of vacancy creation costs can be calibrated by targeting the estimated wage differential.