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Bank Competition and Risk-Taking under Market Integration*

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Abstract

Linkages between bank competition and stability are analyzed in a generalized theoretical framework where market integration is the principal driver of increased competition. Risk implications of across-market competition under banking market integration are significantly different from that of within-market competition. While both modes of analyzing competition increase the number of competitor banks, any relation between competition and risk-taking under within-market competition can be shown to reverse with across-market competition under market integration. Robust to different settings, this result suggests that the lack of consensus in the bank competition-financial stability literature is not an anomaly but an inherent feature of the problem.

JEL codes: D82, G21, L13.
Keywords: Economic integration; loan rate; risk-shifting.

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1. Introduction

The Global Financial Crisis has rekindled interest about the much-studied but unresolved relation between bank competition and financial stability.[1] Over the past five decades, *market integration* has transformed banking from a within-market local phenomenon to span multiple erstwhile segmented markets.[2] Empirical studies have analyzed this progressive evolution of bank competition through events of geographic deregulation and the expansion of banking *across* markets. Meanwhile, theory on bank competition and risk-taking has largely defined competition as increases in the number of competitor banks *within* an individual market. While both modes of analyzing competition increase the number of competitor banks, does competition from across-markets under banking market integration have the same implications for risk-taking as within-market increases in the number of competitor banks?

In this paper, we model how increased banking competition affects risk-taking when market integration is the principal driver of increased competition. Previous theory has analyzed the effect of increased competition on risk-taking by modeling competition as a within-market increase (or the threat of increase) of competitor banks (Besanko and Thakor, 1992; Allen and Gale, 2004; Boyd and De Nicoló, 2005).[3] In practice, however, market integration has been the key driver of increased bank competition, and understanding its effect on risk-taking is critical to understanding its financial stability implications. Market integration not only increases the number of banks, but also depositors and borrowers—the potential customer base of each bank—in the integrated market. We present a generalized theoretical framework for analyzing bank competition and stability linkages between integrating economies or regions.

Our results show that risk implications of across-market competition under banking market integration are significantly different from that of increases in the number of within-market competitor banks. We find that any relation between competition and risk-taking that prevails under within-market competition can be reversed with across-market competition under market integration. This result is robust to a variety of settings, namely, allowing for bank mergers (relaxing assumptions about entry and exit), the presence of an interbank market, and benefits of geographic diversification from integrating previously segmented markets. Moreover, the richer set of results can also help understand why the weight of empirical evidence regarding banking market structure and risk-taking is mixed, with no clear consensus.

This paper distinguishes between two risk-incentive mechanisms from an increase in competition under market integration that operate through loan rates. Increased competition, tra-

[1] Carletti and Hartmann (2003); OECD (2011); Zigraiova and Havranek (2016); Vives (2016) and Degryse et al. (2019) provide comprehensive reviews. However, the lack of consensus is pervasive and reflected even in surveys of this literature. For example, Vives (2016) and Corbae and Levine (2018) argue that there is a significant trade-off between competition and financial stability and the OECD report OECD (2011, p. 10), finds that " "[T]he pre-crisis regulatory landscape has set in motion changes in business models and activities in response to competition that proved not to be conducive to financial stability." In contrast, Carletti and Hartmann (2003) and Beck, Coyle, Dewatripont, Freixas, and Seabright (2010) argue that the aforementioned tradeoff does not generally hold and that competition is important for financial stability.

[2] The term "market integration" is used as a shorthand to refer to the evolution of bank competition through deregulation episodes that led to the removal of geographic restrictions on banking. For example, branching restrictions both within and across state borders in the United States until the 1970s created local monopolies in banking (Kroszner and Strahan, 1999). Thereafter, deregulation within states allowed state-wide branching and deregulation in banking across state lines occurred through bilateral, regional, and even national reciprocal arrangements (Amel, 1993; Strahan, 2003). Reciprocity in these agreements helped the geographic expansion of bank operations and enabled bank competition to span multiple local markets (Radecki, 1998; Dick, 2006; Aguirregabiria, Clark, and Wang, 2016), culminating in the Dodd-Frank Act, which permitted de novo branching across state lines (Congress of the United States of America, 2010). Vives (2016) describes how the European experience has followed a similar trajectory.

[3] Theory has long argued that competitiveness cannot be measured by market structure indicators alone, such as number of institutions, or Herfindahl and other concentration indexes (Novshek, 1980; Baumol, 1982). The threat of entry is no less an important determinant of the behavior of market participants (Besanko and Thakor, 1992).

ditionally defined as a within-market increase in the number of competing banks, reduces loan rates. We term this negative relationship the *bank-competitor effect* of integration. However, when markets integrate, the potential expansion of banks' customer base (borrowers and depositors) generates an additional mechanism of increased competition across markets. As more markets integrate, market expansion makes both deposit supply and loan demand schedules more elastic, and the resulting increase in competition induces banks to behave more like price-takers (Novshek, 1980).[4] This *bank-customer effect* results from the asymmetric expansion in sizes of the deposit and loan markets under market integration. Unlike an increase in the number of banks, the bank-customer effect is not unambiguously negative and can even increase loan rates. Moreover, when the bank-customer effect is positive and sufficiently strong, it dominates the negative bank-competitor effect. As a result, this new channel of market integration can reverse the traditional negative association between competition and risk-taking.

Our results underscore the importance of incorporating market integration into any examination of the competition-stability linkages. In our tractable setting with homogenous agents, the equilibria under within-market competition are isomorphic to that with across-market competition under market integration.[5] Despite this similarity, we demonstrate that within- and across-market competition have different implications for risk-taking as long as the integrating markets are heterogeneous in size. Size heterogeneity yields a nonzero bank-customer effect under across-market competition that can potentially reverse the relation between within-market competition and risk-taking. Evidently, the novel bank-customer effect highlighted here is, by construction, absent under within-market competition.

Main results. We begin with an examination of the association between loan rates and risk-shifting.[6] Banks lend to entrepreneurs (borrowers) who invest in risky projects but have limited liability. Banks face entrepreneurial moral hazard because the borrowers' choice of project risk is unverifiable and cannot be contracted upon. Typically, models of moral hazard in banking yield a positive relation between loan rates and risk-taking. Raising the loan rate decreases the net return on successful projects, incentivizing borrowers to seek projects less likely to succeed but with higher returns when successful (Stiglitz and Weiss, 1981). In a generalized version of this basic model, where the project output exhibits diminishing marginal productivity of investment, we find that the relation between loan rates and risk-taking can also be negative. Under decreasing marginal productivity, we show that risk-taking increases (decreases) with loan rates according as the output elasticity decreases (increases) with investment.

Prior research has shown that the bank-competitor effect is negative (Boyd and De Nicoló, 2005). Although formulated in the context of competition within an individual market, we show that this result extends to competition across markets under integration. As more markets integrate and each bank faces more competitors, the increased competition tends to lower loan rates charged by banks. Combining the effect of competition on loan rates with that of loan rates on risk-taking generates implications for risk-taking in our framework. Our baseline

[4] Novshek (1980, p. 473) argues "Firms (banks) may become small relative to the market in two ways: through changes in technology, absolute firm size (the smallest output at which minimum average cost is attained) may become small, or, through shifts in demand, the absolute size of the market (the market demand at competitive price) may become large."

[5] This follows from our assumption that agents (banks, borrowers and depositors) are homogenous across markets. Our approach retains the tractability and parsimony of previous work because it not only demonstrates how predictions of prior theory prevail under specific conditions (but are not the only outcomes of our model) but also helps isolate the key mechanism by which the financial stability implications of within-market bank competition can be different from across-market competition.

[6] We use the terms risk-taking and risk-shifting interchangeably. In debt-financed firms, owners and managers have incentives to take excessive risks because they benefit from the upside potential while debt-holders bear the downside risks. This well-known risk-shifting problem is particularly acute in banks where a substantial share of liabilities is insured deposits.

model assumes that project risks are perfectly correlated across borrowers so that risk-taking by borrowers coincides with risk-taking by banks (Boyd and De Nicoló, 2005). In the *segmented market equilibrium* (SME), the negative bank-competitor effect combines with the effect of decreased loan rates on risk-taking to yield the effect of increased competition on risk-taking. However, unlike previous studies that have shown this effect to be unambiguously negative, we find that increased competition in the SME can also increase risk-taking. In an environment where risk-taking increases with loan rates, the bank-competitor effect of increased competition decreases risk-taking. The opposite is true when risk-taking decreases with loan rates.

Market integration not only increases the number of banks, but also the number of potential customers (depositors and borrowers) available to each bank—thereby expanding deposit supply and loan demand. Competition increases with this expansion in market size because both deposit supply and loan demand become more elastic and individual banks become small relative to the market. In expanding the customer base, this bank-customer effect of market integration induces banks to behave more like price takers. We show that, if integrating markets are heterogenous in size (measure of depositors and borrowers), increased competition under market integration can raise or lower loan rates. When markets of different sizes integrate, the rate of expansion in the measure of depositors is not always equal to that of borrowers. This unequal rate of expansion between deposit supply and loan demand is sufficient to generate a nonzero bank-customer effect. If the expansion in deposit supply is greater than the expansion in loan demand, the increase in the supply of loanable funds, relative to loan demand, tends to lower loan rates (a *negative* bank-customer effect). Conversely, when integration increases the measure of borrowers more than that of depositors, the relative increase in loan demand tends to increase the loan rate (a *positive* bank-customer effect). Through its effect on the loan rate, the bank-customer effect can increase or decrease risk-taking depending on the relative magnitudes of changes in the deposit supply and loan demand schedules. The overall effect of market integration depends on the relative strength of both bank-competitor and the bank-customer effects. A negative bank-customer effect reinforces the negative bank-competitor effect and the aggregate effect of market integration on loan rates is unambiguously negative. In contrast, when the bank-customer effect is positive and sufficiently large, it dominates the negative bank-competitor effect, increasing loan rates charged by banks.

We show that any association between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the *integrated market equilibrium* (IME). First, consider the case where risk-taking increases with the loan rate. With the bank-competitor effect always negative, risk-taking decreases with competition. However, a positive and sufficiently strong bank-customer effect in the IME increases risk-taking when it outweighs the negative bank-competitor effect. Next, consider the case where risk-taking decreases with the loan rate. Here, risk-taking tends to increase with competition because of the negative bank-competitor effect. This positive relationship can also reverse with a positive and sufficiently strong bank-customer effect in the IME. With a sufficiently large bank-customer effect, increased competition in the IME increases loan rates and thereby reduces risk-taking. To summarize, our paper demonstrates how increased competition under market integration affects loan rates in ways beyond a simple increase in the number of rival banks.

Extensions. We present three extensions of the model to demonstrate that our baseline results are robust to different modeling choices. The first extension allows for consolidation as has been the dominant trend under market integration in the banking industry.[7] Market segmentation and the ensuing lack of competitive pressures have been viewed as the source of inefficiencies among banks (Koetter, Kolari, and Spierdijk, 2012). As a result, regulatory

[7]This relaxes our assumption of no entry and exit of banks, which was intended to ensure that the number of competitor banks increase with the number of integrating markets.

barriers to competition can generate heterogeneity so that banks across different segmented markets operate at different efficiencies.[8] In Section 6.1, we model this heterogeneity in terms of differences in bank-specific operating costs (non-interest expenses) across different markets. Focusing attention on pairwise mergers, we analyze an extensive-form game wherein banks decide whether to merge and a planner or antitrust authority decides whether to allow mergers. We find that mergers between like banks (a pair of high-cost banks or a pair of low-cost banks) are almost never profitable.[9] In contrast, mergers between pairs of unlike banks (a high-cost bank and a low-cost bank) are always profitable and yield efficiency gains, as has been documented in numerous empirical studies (Berger, Demsetz, and Strahan, 1999; DeYoung, Evanoff, and Molyneux, 2009). In particular, we model how efficient banks merge with relatively inefficient, less profitable, banks as markets integrate.[10] The model also reflects the dominant pattern following market integration of across-market mergers as opposed to within-market mergers (Berger et al., 1995; Dick, 2006). We show that even when they are privately optimal, bank mergers may not be socially optimal, thereby presenting a rationale for merger reviews. On the other hand, when mergers increase aggregate welfare in our setting, they can also be accompanied by higher risk-taking. Accordingly, our framework presents scenarios in which the implications for welfare and risk-taking can generate conflicting recommendations for merger reviews. The financial stability implications of this trade-off present a rationale behind the inclusion of the *financial stability factor* for merger reviews in the United States.[11]

In a second extension, we allow for the *imperfect correlation* of risks across the individual (integrating) markets (Section 6.2). The integration of markets with default risks that are imperfectly correlated across markets introduces diversification benefits for banks.[12] As the correlation decreases, the gains from diversification increase allowing banks to diversify risks at lower cost and thereby set lower loan rates. In this way, we find that greater diversification benefits tend to reinforce the negative bank competitor effect, requiring a stronger bank-customer effect in the IME to reverse any association between competition and risk-taking in the SME.

Finally, in the third extension, we show that results of the baseline model are robust to the introduction of an interbank market (Section 6.3). We assume that banks take the interbank rate, the policy rate set by the central bank, as given. With interbank lending, the bank-competitor effect on loan rates comprises a negative direct effect (reduced market power from an increase in competitor banks) and a positive indirect effect that operates through deposit

[8] For example, Kroszner (2001, p. 38) argues that, "…branching restrictions tend to reduce the efficiency and consumer convenience of the banking system, and small banks tend to be particularly inefficient in states where branching restrictions offer them the most protection."

[9] In a result similar to that obtained in Salant et al. (1983), we find that mergers between like banks are profitable only if the market consist of the same type of banks (all high-cost or all low-cost banks) and if the resulting post-merger market structure is a monopoly.

[10] Berger et al. (1999, p. 150) presents empirical evidence in support of such merger activity: "The prior geographic restrictions on competition may have allowed some inefficient banks to survive. The removal of these constraints allowed some previously prohibited M&As to occur, which may have forced inefficient banks to become more efficient by acquiring other institutions, by being acquired, or by improving management practices internally."

[11] Regulatory reviews of bank merger applications in the United States have avoided consolidation where excessive increases in risks would be expected. More recently, the inclusion of the financial stability factor in Section 604(d) of the Dodd-Frank Act has replaced Section 3(c) of the Bank Holding Company Act of 1956 (Congress of the United States of America, 2010). "[T]he addition of a financial stability factor …contrasts with an antitrust pre-merger review, in which the focus is solely on whether the transaction would substantially lessen competition" (Tarullo, 2014). When evaluating a proposed bank acquisition or merger, the Federal Reserve Board is now required to consider "the extent to which [the] proposed acquisition, merger, or consolidation would result in greater or more concentrated risks to the stability of the United States banking or financial system".

[12] Indeed, geographic risk diversification has been one of the stated goals of banking deregulation in the United States (see Aguirregabiria et al., 2016, for details).

rates (reduced market power tends to raise deposit rates).[13] At low interbank rates, the negative direct effect dominates and the overall bank-competitor effect remains negative. However, the positive indirect effect dominates at high interbank rates resulting in a bank-competitor effect that is positive. Such a positive association between the number of banks and loan rate implies that, just as with the bank-customer effect, the sign on the bank-competitor effect is no longer unambiguous in the presence of interbank lending. Nevertheless, the result of the baseline model still holds, that is, the risk-incentive mechanism of increased competition in the SME can be reversed in the IME.

Related literature. We discuss our contribution to the theoretical literature here and examine the implications of our model for empirical studies in Section 2. The theoretical literature on the effect of increased competition on risk-taking can be viewed as comprising two segments. The first set of studies emphasize the effect of increased competition on interest rates (Allen and Gale, 2004; Repullo, 2004; Boyd and De Nicoló, 2005), while the second set examines the effect of interest rates on risk-taking incentives of banks and borrowers(Stiglitz and Weiss, 1981; Martinez-Miera and Repullo, 2010; Wagner, 2010; Dell'Ariccia and Marquez, 2013; González-Aguado and Suárez, 2015). This paper contributes to both segments of the literature.

Our contribution to the first set of studies lies in the bank-customer effect, a previously unexplored risk-incentive mechanism, which is distinct from the bank-competitor effect explored in prior studies (Boyd and De Nicoló, 2005; Martinez-Miera and Repullo, 2010). We demonstrate that any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME. This has a couple of implications. First, unlike increased competition from an increase in the number of banks, increased competition under market integration is not necessarily rate-reducing.[14] Second, the effect of increased competition from market integration depends on the underlying conditions, namely, loan demand and deposit supply in the integrating markets. Therefore, as long as the integrating markets are asymmetric, no two deregulation (market integration) episodes necessarily yield the same outcome in terms of loan rates (see discussion in Section 2).

With respect to the second set of studies, our contribution lies in extending to a more generalized version, wherein risk-taking can increase or decrease with loan rates. Models of borrower moral hazard and limited liability have shown that risk-taking increases with loan rates (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). However, risk-taking can decrease with loan rates in moral hazard settings where banks monitor borrower actions (Besanko and Kanatas, 1993; Dell'Ariccia and Marquez, 2013; Martinez-Miera and Repullo, 2017). We show that both effects are possible, even in the absence of two-sided moral hazard in bank and borrower actions (such as bank monitoring) as long as borrowers' project returns exhibit diminishing marginal productivity.

The composite effect of increased competition on risk-taking under market integration produces a richer set of results that can help explain some observed patterns in the evolution of competition. For example, empirical studies have documented that market integration yields pro-competitive gains such as lower loan rates even without a concomitant increase in the number of competitor banks (Jayaratne and Strahan, 1996; Dick, 2006). In our model, the negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior, or from efficiency gains (Section 6.1) and diversification benefits (Section 6.2). Importantly, a more parsimonious setting that defines

[13]Because the probability of default is endogenous in our framework, loan and deposit rates are no longer independent with interbank lending.

[14]Under strategic default and non-exclusive contracts, wherein lenders cannot prevent borrowers from taking multiple loans, increased competition among lenders can also lead to higher loan rates relative to the competitive level (Parlour and Rajan, 2001).

increased competition in terms of the number of banks within an individual market cannot explain this pattern in the data. In addition to the much explored diversification benefits and efficiency gains from deregulation episodes, our model offers a third mechanism by which integration yields competitive outcomes.[15] Notably, this mechanism is not only independent of efficiency gains and diversification benefits but also of market concentration.

In addition to the theories of bank competition and risk-taking, our paper also contributes to theories examining the integration of banking markets. Morgan, Rime, and Strahan (2004) extend the model of monitored financing in and borrower moral hazard in Holmström and Tirole (1997) to show how integration made "state business cycles smaller, but more alike." In a manner similar to that study, we compare between equilibria wherein banks are immobile across markets (intrastate banking in Morgan et al., 2004) and wherein banks are mobile across markets (interstate banking).[16] However, while they focus on the convergence of business cycles, we study how banking market integration affects risk-taking incentives.

When the integrating markets are homogenous, market integration becomes equivalent to *market replication*. The effect of increased competition on loan demand and deposit supply is symmetric under market replication and the (net) bank-customer effect is zero. For this special case, the overall effect of competition on risk-taking is composed solely of the negative bank-customer effect. In this way, the predictions in Boyd and De Nicoló (2005) emerge as the special case of market replication in our general model.

Our work builds on the traditional definition of competition because market integration presents us with seemingly *broader* meanings of "increased competition". The first is an increase in the number of banks, and the second is a convergence towards banks' price-taking behavior in deposit and loan markets. The notion that increased competition goes beyond a simple increase in the number of banks (firms) has its roots in Novshek (1980). Hermalin (1994, p. 526) summarizes the argument as follows:

> "[t]his requires defining the phrase 'more competitive'. As Novshek (1980) points out, this is not a straightforward task, because more competitive has at least two meanings. It suggests (1) more firms and (2) a closer approximation to perfect competition. This second meaning is not fully captured by the first—simply adding firms does not yield a closer approximation of perfect competition because the firms do not become price takers in the limit. Following Novshek, a definition that is consistent with this second meaning is one in which the slope of the inverse demand curve tends to zero with the increase in the number of firms in such a way that, in the limit, each firm produces zero, earns zero profits, and faces a flat demand curve."

[15] Diversification benefits from integration have been known to lower the cost of making loans (Demsetz and Strahan, 1997; Aguirregabiria, Clark, and Wang, 2016; Goetz, Laeven, and Levine, 2016; Levine, Lin, and Xie, 2021). Additionally, integration has also resulted in efficiency gains in the banking industry (Berger et al., 1999; Berger and Mester, 2003; Koetter et al., 2012). Both factors contribute to competitive outcomes such as reduced loan rates in the absence of decreased market concentration.

[16] Our model also extends to market integration *within* state borders. For example, the SME in our setting is general in that it extends to situations where banking markets within a state are segmented, e.g. unit banking laws in the United States (Kroszner and Strahan, 1999). Relative to unit banking, statewide (intrastate) banking would be captured by the IME in our model. In this way, our model captures integration of banking markets both within and across state lines.

2. Empirical implications

While theory has modeled competition as an increase in the number of banks, empirical evidence for the same remains scant.[17] For example, empirical studies have documented that market integration yields pro-competitive outcomes such as lower loan rates even without a concomitant increase in the number of competitor banks (Jayaratne and Strahan, 1996; Dick, 2006). In our model, the negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior, or from efficiency gains (Section 6.1) and diversification benefits (Section 6.2).[18] Importantly, a more parsimonious setting that defines increased competition in terms of a within-market increase of competitor banks cannot explain this pattern in the data. In addition to the much explored diversification benefits and efficiency gains from deregulation episodes, our model offers a third mechanism by which integration yields competitive outcomes. Notably, this mechanism is not only independent of efficiency gains and diversification benefits but also of market concentration.

Our theoretical contribution dwells on how increased competition can affect deposit and loan markets asymmetrically—the source of the bank-customer effect. Park and Pennacchi (2009) find that increased market integration (in their case, with large banks entering local markets populated by small banks) tends to "promote competition in retail loan markets but also tends to harm competition in retail deposits." While their paper does not directly examine the effects on risk-taking, it does demonstrate the differential impact of integration on deposit and loan markets. In doing so, it underscores the relevance of the bank-customer effect of increased competition under market integration.

Notably, our results do not contradict previous findings. We present a generalized framework in which prior predictions in the literature emerge under specific conditions but are not the only outcomes of the model. For example, our model also predicts scenarios under which the negative bank-competitor effect lowers risk-taking as in Boyd and De Nicoló (2005). However, we find that increased competition can also increase risk-taking under within-market competition if the relation between loan rates and risk-taking is negative. Although this result departs from predictions in Boyd and De Nicoló (2005), recent empirical work on the National Banking Era – a natural experiment that is arguably close to their setting (the SME in Section 4) – finds that risk-taking increases with competition (Carlson, Correia, and Luck, 2022).

Despite a vast literature on the effects of bank competition, there is yet no consensus about its effects on financial stability. Empirical studies have been classified as belonging to two opposing views: the "competition-stability" view and the the "competition-fragility" view (Berger, Klapper, and Turk-Ariss, 2009; Beck, De Jonghe, and Schepens, 2013; Berger, Klapper, and Turk-Ariss, 2017; Akins, Li, Ng, and Rusticus, 2016; Corbae and Levine, 2018).[19] Differ-

[17] Overall, empirical work on this topic have adopted one of two strategies: (1) examine cross-country variation spanning several countries or (2) analyze various deregulation episodes spanning local markets in the United States and other advanced nations (see Degryse et al., 2019, and references therein). The majority of studies on the topic have adopted the second approach, which is the focus of the discussion here.

[18] Diversification benefits from integration have been known to lower the cost of making loans (Demsetz and Strahan, 1997; Aguirregabiria, Clark, and Wang, 2016; Goetz, Laeven, and Levine, 2016; Levine, Lin, and Xie, 2021). Additionally, integration has also resulted in efficiency gains in the banking industry (Berger et al., 1999; Berger and Mester, 2003; Koetter et al., 2012). Both factors contribute to competitive outcomes such as reduced loan rates in the absence of decreased market concentration.

[19] The competition-fragility view argues that an increase in competition increases risk-taking incentives by reducing bank profit margins and lowering franchise values (Keeley, 1990; Chong, 1991; Gan, 2004; Dick, 2006; Beck et al., 2006; Berger et al., 2009; Beck et al., 2013; Corbae and Levine, 2018; Corbae and D'Erasmo, 2021; Carlson et al., 2022). In contrast, the competition-stability view argues that competition is critical to financial stability and increased concentration has been shown to increase risk-taking using different empirical measures (Jayaratne and Strahan, 1996; Schaeck, Cihak, and Wolfe, 2009; Houston, Lin, Lin, and Ma, 2010; Fiordelisi and Mare, 2014; Goetz, Laeven, and Levine, 2016; Jiang, Levine, and Lin, 2017; Goetz, 2018; Jiang, Levine, Lin, and Wei, 2020).

ences in findings have often been attributed to the challenges of measuring competition, defining the relevant geographic market, and determining the accurate measure of risk-taking (Beck et al., 2010). Indeed, the empirical relationship between competition and risk-taking has been shown to vary over time and location. In particular, studies have arrived at opposite conclusions while examining the same markets over different periods and using similar measures of competition and risk-taking. For example, Jayaratne and Strahan (1996) document a decrease in risk in the loan portfolio (measured by provisions for loan losses or charge-offs) following earlier stages of deregulation in the United States, while Dick (2006) finds an increase in risk examining data from later stages. On the other hand, differences in the relationship between bank competition and risk-taking in different markets within the same period are not uncommon (Jiménez et al., 2013; Liu et al., 2013). Our setup of integrating markets presents examples wherein the relationship between competition and risk-taking under market integration can be U-shaped (Section 5.2).[20] We show that risk-taking decreases with competition when the number of integrating markets is small, but increases with competition when the number of integrated markets is large.

A final contribution in guiding empirical work is methodological. Our model suggests that no two cases of market integration are necessarily alike. Proposition 5 shows how, depending on the bank-customer effect, any relation between competition and risk-taking in the segmented markets can be reversed upon integration. The bank-customer effect, in turn, depends on the relative sizes of deposit supply and loan demand among the integrating markets. When the set of integrating markets varies with each episode of deregulation, it calls for exploring the possibility that treatment is heterogeneous and deregulation events be treated separately. Studies exploring this possibility and conducting separate assessments for each deregulation event (or group of deregulation events) have shown that the effects to be quite heterogeneous between the various episodes (Jayaratne and Strahan, 1996; Wall, 2004; Huang, 2008). The evidence against the imposition of homogeneity restrictions on different deregulation events has been substantial. Perhaps more importantly, we should not ignore the hypothesis that the lack of consensus about the linkages between bank competition and financial stability described above is not an anomaly but an inherent feature of the problem.

3. Microfoundations

There are $J = \{1, 2, \ldots\}$ segmented banking markets.[21] Each market $j \in J$ consists of three distinct groups of risk-neutral agents—a continuum of depositors of measure $a_j > 0$, a continuum of borrowers or entrepreneurs of measure $b_j > 0$, and n_j (≥ 1) banks. Within each group, all agents are identical—we assume that borrowers, depositors, and banks are homogenous both within and across markets. However, markets vary in size so that no two markets necessarily have the same number of agents in each group. We refer to depositors and borrowers collectively as *customers* of banks. The basic framework below follows Allen and Gale (2004) and Boyd and De Nicoló (2005).

3.1. Deposit supply

Let $R_j \geq 1$ be the deposit rate offered by banks in market j. Each depositor in market j solves

$$d(R_j) = \underset{d}{\operatorname{argmax}}\ R_j d - V(d), \tag{1}$$

[20] In an extension to Boyd and De Nicoló (2005) that allow for imperfect correlation in loan defaults within a single market, Martinez-Miera and Repullo (2010) show that the relationship between competition and risk can also be U-shaped.

[21] We consider markets segmented due to institutional, non-economic barriers such as geography, legislation, or regulation. The terms markets, regions, and economies are used interchangeably.

where $V(\cdot)$ is the foregone utility associated with making deposits d, which is assumed to be strictly increasing and strictly convex. The first-order condition associated with (1) is given by $V'(d(R_j)) = R_j$ from which we obtain that individual deposit supply $d(R_j)$ is strictly increasing in the deposit rate R_j because $V''(d) > 0$. Given identical depositors, aggregate deposit supply in market j is $D_j = a_j d(R_j)$, and the inverse deposit supply is

$$R_j = d^{-1}(D_j/a_j) \equiv R(D_j/a_j) \quad \text{with } R'(D_j/a_j) > 0.$$

Note that the above maximization problem of identical borrowers allows us to write the inverse deposit supply as a function of supply-of-funds per depositor, D_j/a_j.

3.2. Loan demand under borrower moral hazard

Loan demand in market j is obtained from a simple model of lending under borrower moral hazard. We assume a contractual environment where entrepreneurs have access to a set of risky projects indexed by θ whose returns are random and perfectly correlated.[22] Entrepreneurs with zero net worth must borrow to invest in the project. If k dollars are invested in a given project, it yields

$$\tilde{y}(\theta, k) = \begin{cases} y(\theta)f(k) & \text{with probability } p(\theta), \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

We assume that (i) the return, $y(\theta)$, is strictly increasing and strictly concave on $[0, \bar{\theta}]$, (ii) the probability of success, $p(\theta)$, is strictly decreasing and strictly concave on $[0, \bar{\theta}]$ with $p(0) = 1$ and $p(\bar{\theta}) = 0$, and (iii) the production function, $f(k)$, is strictly increasing and strictly concave on $[0, \bar{k}]$ with $f(0) \geq 0$. The variable θ represents the "riskiness" of the project—for a given k, the higher the θ, the higher is the return $y(\theta)$, but the lower is the probability of success, $p(\theta)$. Borrowers' choice of risk is not publicly verifiable, and therefore, not contractible.

There are two decision stages. In stage 1, each borrower chooses the level of investment, k, which generates individual demand for loans as a function of the loan rate $r_j \geq 1$.[23] In stage 2, taking the stage-1 borrowing decision as given, the borrower privately chooses project riskiness θ after which, project returns are realized, and payoffs are made. Borrowers have *limited liability* and the conjunction of moral hazard and limited liability affects terms of the loan contract. In granting loans, banks cannot write contracts that are contingent on project riskiness θ because this is private information of the borrower. However, banks correctly anticipate the risk-shifting incentives of borrowers, imposing a sequential rationality constraint on the equilibrium (Brander and Spencer, 1989). In stage 2, given k and r_j, each borrower in market j solves

$$\theta(k; r_j) = \underset{\theta}{\operatorname{argmax}}\ p(\theta)\left\{y(\theta)f(k) - r_j k\right\}. \tag{3}$$

The first-order condition associated with (3) is

$$h(\theta) = \frac{r_j}{f(k)/k}, \tag{4}$$

where $h(\theta) \equiv y(\theta) + y'(\theta)(p(\theta)/p'(\theta))$ is strictly increasing in θ. The above condition is the

[22] Section 6.2 extends the model to imperfectly correlated risks. The assumption that entrepreneurs' returns are perfectly correlated follows from Boyd and De Nicoló (2005) and is equivalent to the assumption of bank portfolios comprising perfectly correlated risks in (Allen and Gale, 2004). The risk associated with each project can in general be decomposed into a systemic and idiosyncratic component. With a large number of projects, the idiosyncratic component can be perfectly diversified away. This assumption helps focus the baseline analysis on the common component representing systemic risks.

[23] More formally, borrower i in market j chooses k_{ij}. Again, we drop the individual subscripts for ease of notation.

equality between the expected marginal revenue of risk-shifting and its expected marginal cost.[24] The borrower's risk-shifting choice, θ, in (4) depends on the average product of capital, $f(k)/k$, and the loan rate, r_j . We obtain $\theta_r(k; r_j) > 0$ because $h'(\theta) > 0$, so that risk-shifting is increasing in the loan rate r_j. In other words, higher cost of investment increases risk-shifting. We also obtain $\theta_k(k; r_j) > 0$ because $f(k)/k$ decreases with k as $f(\cdot)$ is concave and $f(0) \geq 0$, so that risk-shifting is also increasing in investment k. Therefore, when investment k increases, capital is less productive, and this incentivizes the borrower to take on more risk. In short, those who invest more are more liable to moral hazard (Banerjee, 2003).

Individual loan demand is determined at stage 1 where the borrower chooses investment k so that

$$k(r_j) = \underset{k}{\operatorname{argmax}}\ p(\theta(k; r_j)) \left\{ y(\theta(k; r_j)) f(k) - r_j k \right\}. \tag{5}$$

It follows that $k'(r_j) \leq 0$, so that individual borrower's loan demand, $k(r_j)$, is downward sloping. Given identical borrowers, aggregate loan demand in market j is $L_j(r_j) = b_j k(r_j)$, and the inverse loan demand is

$$r_j = k^{-1}(L_j/b_j) \equiv r(L_j/b_j) \quad \text{with } r'(L_j/b_j) < 0.$$

As in the case of deposit supply, the maximization problem of identical borrowers allows us to express the inverse demand for loans as a function of the loan volume per borrower, L_j/b_j.

3.3. Loan rates and risk-taking

There are two ways in which loan rates affect risk-taking in this framework. In addition to the direct effect on entrepreneurs' optimal choice of risk, loan rates also affect risk-taking through its effect on borrower's loan demand, so that $\hat{\theta}(r_j) \equiv \theta(k(r_j); r_j)$. Taken together,

$$\frac{d\hat{\theta}}{dr_j} = \theta_r(k; r_j) + \theta_k(k; r_j) k'(r_j) \tag{6}$$

From (4), the *direct effect*, denoted by the first term on the right-hand side of (6), is positive. However, with $\theta_k(k; r_j) > 0$ from (4) and $k'(r_j) \leq 0$ from (5), the *indirect effect* of the loan rate on risk-taking—denoted by the second term on the right-hand side of (6)—is negative. Overall, the relation between loan rates and risk-taking is summarized by the following proposition.

Proposition 1 *Borrowers' optimal risk choice in market j, $\hat{\theta}(r_j) \equiv \theta(k(r_j); r_j)$, depends on the loan rate, r_j, and borrower investment, $k(r_j)$ which is a decreasing function of the loan rate. If the output elasticity of investment, $\varepsilon(k) \equiv kf'(k)/f(k)$ is decreasing (increasing) in k, then optimal risk-taking $\theta^*(r_j)$ is increasing (decreasing) in r_j.*

The effect of loan rate changes on risk-taking, $d\hat{\theta}/dr_j$, depends on the relative magnitudes of the direct and indirect effects because they point in opposite directions. We show that the average productivity, $f(k)/k$ is more (less) responsive to a change in investment caused by a change in the loan rate according as the output elasticity of investment, $\varepsilon(k)$,[25] is decreasing

[24]Condition (4) can be written as

$$[p'(\theta)y(\theta) + p(\theta)y'(\theta)]f(k) = p'(\theta)r_j k.$$

The left-hand-side and the right-hand side of this equation are the expected marginal revenue and the expected marginal cost of risk-shifting, respectively.

[25]In particular, $kf'(k)/f(k)$ is the ratio of the marginal product to the average product of capital and is often defined as the output elasticity of input for any given factor of production (see Jehle and Reny, 2011, p. 133). It has also been referred to as the scale elasticity or the elasticity of production.

(increasing) in k (see the proof of Proposition 1 in Appendix A). When average productivity is more responsive to changes in investment due to a change in r_j, the indirect effect in (6) outweigh the direct effect. The converse occurs when the average productivity is less responsive. In sum, the relation between loan rate and risk-taking depends on the behavior of the output elasticity with respect to investment.

	Functional form of $f(k)$	$\varepsilon'(k)$	Risk-taking ___ with loan rate
Example 1	$f(k) = k(1-k);\ 0 \le k \le 1/2$	negative	increases
Example 2	$f(k) = \sqrt{k_0 + k};\ \ k, k_0 > 0$	positive	decreases
Example 3	$f(k) = k^{\delta};\ k > 0, 0 < \delta < 1$	zero	does not change

Table 1: *The relationship between loan rate and the optimal risk-taking under different functional forms with $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$.*

Table 1 illustrates how different functional forms of $f(k)$ yield differences in the relation between loan rates and risk-taking. Loan rates affect risk-taking through two channels—a positive direct effect, $\theta_r(k; r_j)$, and a negative indirect effect, $\theta_k(k; r_j)k'(r_j)$, which works through changes in optimal investment, $k(r_j)$.

The possibility of a negative relation between loan rates and risk-taking deviates from conventional models of moral hazard with limited liability which obtain an unambiguously positive relation between loan rates and risk-taking (Stiglitz and Weiss, 1981). We show that risk-taking can increase or decrease with loan rates in settings where the production technology exhibits diminishing returns. Notably, the indirect effect is null for constant elasticity production functions as shown by Example 3 in Table 1. For models using these functions, the indirect effect is absent and the relation between loan rates and risk-taking is unambiguously positive (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). In Section 5, we present a linear model where the technology exhibits constant returns to scale, $f(k) = k$. In terms of (6), it follows from (4) that $\theta_k(k; r_j) = 0$ and the effect of loan rates on risk-taking is positive and comprised entirely of the direct effect $\theta_r(k; r_j)$.

Before concluding this section, it is important to point out that, in our framework with perfectly correlated risks, entrepreneurial risk-taking is synonymous with risk-taking by banks. In this context, banks' optimal asset allocation is determined by an optimal contracting problem as discussed in Boyd and De Nicoló (2005) instead of a portfolio choice problem as modeled in Allen and Gale (2004).

4. Market Equilibrium

We begin by characterizing equilibrium loan rates and risk-shifting for each segmented market. We refer to this equilibrium as the *segmented market equilibrium* (SME) and show how competition affects loan rates and risk-taking in the SME. Next, we solve for the equilibrium when $m(\ge 2)$ segmented markets are integrated into a single banking market. We refer to this equilibrium as the *integrated market equilibrium* (IME) and analyze the effects of competition on loan rates and risk-shifting in the IME. Finally, we compare the effect of competition on loan rates and risk-shifting between the SME and the IME.

In solving the model, we make several simplifying assumptions. First, banks compete in a Cournot fashion. Second, there is no exit or entry of banks. We relax this assumption in Section 6.1 where we allow for bank mergers. Third, banks are funded entirely by customer deposits—banks have no equity and there is no interbank market for deposits. Again, we relax this assumption when we introduce interbank markets in Section 6.3. Fourth, all bank deposits are insured for which each bank pays a flat premium that is normalized to zero. Lastly, all customers (borrowers and depositors) can switch costlessly between banks. Given our assumptions that agents are homogenous within each group, this is a fairly innocuous (zero transaction cost) assumption.[26]

4.1. The segmented market equilibrium

4.1.1. *Equilibrium*

With no equity and no interbank market, the balance sheet identity of bank i in market j implies $L_{ij} = D_{ij}$. Consequently, aggregate loan demand equals aggregate deposit supply in market j so that $L_j = \sum_{i=1}^{n_j} L_{ij} = \sum_{i=1}^{n_j} D_{ij} = D_j$. Bank i in market j chooses the volume of loans, L_{ij}, to maximize expected profits, taking into account choices made by its competitors and the entrepreneurs' choice of risk. In the segmented market, each bank in market j solves

$$\max_{L_{ij}} P(L_j/b_j)[r(L_j/b_j) - R(L_j/a_j)]L_{ij}, \tag{7}$$

where $P(L_j/b_j) \equiv p(\theta^*(r(L_j/b_j)))$, $R(0) \geq 0$, $R'(D_j/a_j) > 0$, $R''(D_j/a_j) \geq 0$, $r(0) \geq R(0)$, $r'(L_j/b_j) < 0$ and $r''(L_j/b_j) \leq 0$.

Given that all banks are identical, and that they face the same aggregate deposit supply and loan demand schedules, in the segmented market, there are no asymmetric equilibria (see the proof of Lemma 2 in the Appendix). Moreover, the symmetric equilibrium is unique. In the symmetric Cournot equilibrium, we have $L_{ij} = L_j/n_j$ for all i. The following lemma characterizes the (symmetric) SME in market j.

Lemma 1 *The symmetric SME is characterized by loan rate, $r_j = r(L_j/b_j)$, deposit rate, $R_j = R(L_j/a_j)$, risk-shifting, $\theta_j = \theta^*(r(L_j/b_j))$, and intermediation margin*

$$r(L_j/b_j) - R(L_j/a_j) = \frac{\left[b_j R'(L_j/a_j) - a_j r'(L_j/b_j)\right] P(L_j/b_j) L_j}{a_j \left[n_j b_j P(L_j/b_j) + P'(L_j/b_j) L_j\right]}, \tag{8}$$

where L_j denotes the aggregate loans in market j.

Equation (8) determines bank loan volumes in the symmetric Cournot equilibrium. We obtain the loan rate in the SME from the inverse loan demand function, $r(L_j/b_j)$. Likewise, we obtain aggregate deposits and the deposit rate in the SME from $D_j = L_j$, and the inverse deposit supply function, $R(D_j/a_j)$, respectively.

4.1.2. *Competition and risk-taking in the SME*

Increased competition in the SME is defined as an increase in the number of banks. Competition can increase in any of the segmented markets if local banking authorities lower fixed set-up costs (Mankiw and Whinston, 1986). Such costs include charter fees or capital requirements at the founding of the bank as was required during the National Banking Era (Carlson et al., 2022). From Lemma 1, comparative statics reveal that the SME loan rate, r_j, decreases

[26] As banks obtain information on borrower quality during the course of a lending relationship, switching between banks is beset with problems of information asymmetry. A large literature examines how competition in credit markets affects the screening problem banks face in granting loans (Broecker, 1990; Sharpe, 1990). It is important to mention that such models of switching costs are generally bank-specific (i.e., they apply to customers switching from one bank to another, even in the absence of market integration) and not necessarily market-specific (i.e., they do not generally apply to customers switching from local to non-local banks when markets integrate). We abstract from these considerations both for the SME and the IME.

as the number of banks, n_j, increases. It follows from Proposition 1 that entrepreneurial risk-shifting in the SME may increase or decrease with n_j.

Proposition 2 *In the SME of each market j, loan rate r_j is strictly decreasing in n_j. As a result, risk-shifting θ_j decreases (increases) according as the output elasticity of investment, $\varepsilon(k)$ is increasing (decreasing) in k.*

While increased competition in the SME unambiguously decreases loan rates, the effect on risk-taking is not unambiguous. Increased competition from an increase in the number of banks decreases risk-taking if and only if risk-taking increases with loan rates. Proposition 2 asserts that this risk-incentive mechanism, first shown in Boyd and De Nicoló (2005), is obtained in the SME when the production technology exhibits decreasing elasticity of investment. However, in situations where the production technology exhibits increasing elasticity of investment, we find that increased competition can increase risk-taking by borrowers. Using a natural experiment that has similarities to the SME in our model, Carlson et al. (2022) find that banks operating in markets with lower entry barriers in the National Banking Era increased riskiness in lending and were more likely to default. This empirical finding lends support for our result that increased competition within segmented markets can also increase risk-taking.

4.2. The integrated market equilibrium

4.2.1. Deposit supply and loan demand in the integrated market

Now, suppose that a subset of segmented markets, $J_m \subset J$, are integrated to form a single banking market, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Given the assumption that there is no entry or exit of banks, the number of banks in the integrated market is equal to the total number of banks across the m markets combined, so that

$$n(m) = \sum_{j=1}^{m} n_j.$$

Although the aggregate number of banks remains unaltered after market integration, each bank faces new rivals in the integrated market. In a similar vein, the measure of depositors and borrowers in the integrated market equals the aggregate of the measures of depositors and borrowers in each of the m individual markets, respectively. Therefore

$$a(m) \equiv \sum_{j=1}^{m} a_j \quad \text{and} \quad b(m) \equiv \sum_{j=1}^{m} b_j. \tag{9}$$

Under market integration, depositors and borrowers solve the same maximization problems as given in (1) and (5), respectively. This follows directly from our assumption that depositors and borrowers are homogenous across the m integrating markets. Customer homogeneity before and after integration implies that, for given deposit and loan rates, deposit supply, $d(R)$, and loan demand, $k(r)$, in the segmented and the integrated markets are the same.

From (9), aggregate deposit supply and loan demand in the integrated market are $D = a(m)d(R)$ and $L = b(m)k(r)$, respectively. It follows that the inverse deposit supply and loan demand functions in the integrated market are

$$R = R(D/a(m)) \quad \text{where} \quad R'(D/a(m)) > 0, \tag{10}$$

$$r = r(L/b(m)) \quad \text{where} \quad r'(L/b(m)) < 0, \tag{11}$$

respectively. The inverse deposit supply of the integrated market, $R(D/a(m))$, is the horizontal sum of the m individual inverse deposit supply schedules, and consequently, more elastic than $R(D_j)$ for any $j = 1, \ldots, m$. Likewise, the inverse loan demand function, $r(L/b(m))$, which is the horizontal sum of the m individual inverse loan demand schedules, is more elastic than $r(L_j)$ for any $j = 1, \ldots, m$. Being the horizontal sum of convex deposit supply (concave loan demand) schedules, aggregate deposit supply (loan demand) is also convex (concave), that is, $R''(D/a(m)) \geq 0$ ($r''(L/b(m)) \leq 0$). Figure 1 shows the inverse loan demand function for $m = 2$ and $b_1 < b_2$.

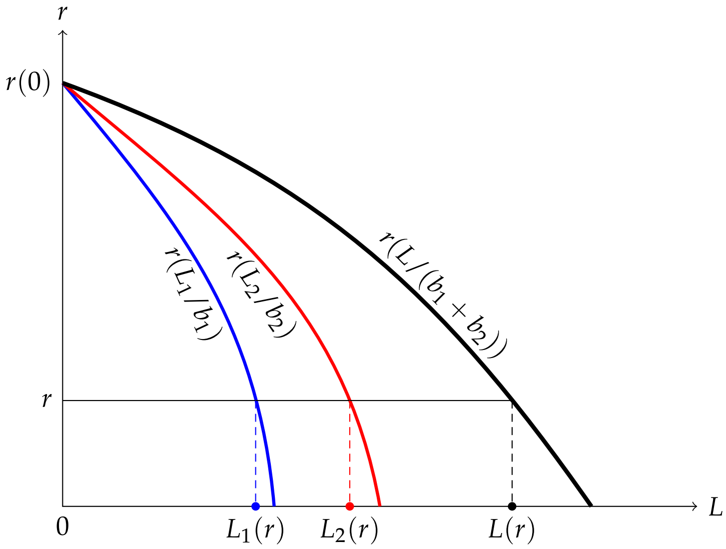

Figure 1: *Loan demand when two markets integrate. Loan demand in the segmented market i is* $L_j(r) = b_j k(r)$, $j = 1, 2$. *If* $b_1 < b_2$, $r_1(L_1)$ *lies below* $r_2(L_2)$. *Loan demand in the integrated market,* $L(r) = L_1(r) + L_2(r) = (b_1 + b_2)k(r)$, *is more elastic than that in any of the segmented markets.*

4.2.2. *The integrated market equilibrium*

In the integrated market, J_m, all $n(m)$ banks are identical and they face the same inverse deposit supply (10) and inverse loan demand (11). Given that banks are exclusively deposit-financed, $L_i = D_i$ holds for all i. This yields the market clearing condition, $L = D$. Bank i solves

$$\max_{L_i} P(L/b(m))[r(L/b(m)) - R(L/a(m))]L_i. \tag{12}$$

It follows that $L_i = L/n(m)$ for all i, and the symmetric IME is described by the following proposition.

Lemma 2 *The symmetric IME is characterized by loan rate,* $r = r(L/b(m))$, *deposit rate,* $R = R(L/a(m))$, *optimal risk-shifting* $\theta = \theta^*(r(L/b(m)))$, *and the intermediation margin*

$$r(L/b(m)) - R(L/a(m)) = \frac{[b(m)R'(L/a(m)) - a(m)r'(L/b(m))]\, P(L/b(m))L}{a(m)\, [n(m)b(m)P(L/b(m)) + P'(L/b(m))L]}, \tag{13}$$

where L denotes the aggregate loans in the integrated market and $a(m)$ *and* $b(m)$ *are given by* (9).

Evidently, the expressions for equilibrium loan rate and risk-taking for the IME in Lemma 2 are isomorphic to those obtained for the SME in Lemma 1. In spite of this similarity, we show below how the integration of heterogenous markets can alter the relation between competition and loan rates, and consequently, competition and risk-taking in the SME.

4.2.3. Effect of increased competition on loan rates

Increased competition in the IME is defined as an increase in the number of integrating markets. Given that integrating markets are heterogenous in our setting, we take an increase in m to imply that additional markets are integrated. Formally, if the set of integrated markets expands from J_m to $J_{m'}$, where $|J_m| = m$, $|J_{m'}| = m'$, then $J_m \subseteq J_{m'} \subset J$. In other words, we compare loan rates and risk-shifting between the IME in market J_m (shorthand for the smaller integrated set of m markets) and that in market $J_{m'}$ (shorthand for the larger integrated set of m' markets). Agents (banks and customers) are homogenous across the two sets of markets. But, their measures are different in each market, and with integration, deposit supply and loan demand schedules are more elastic in the larger (integrated) market.

Market integration intensifies bank competition in two ways. First, an increase in m increases the number of competitor banks so that each bank faces competition from more rivals in $J_{m'}$ relative to J_m . Second, integration of additional markets increases competition by expanding the size of deposit and loan markets. The deposit supply schedules for J_m and $J_{m'}$ are given by $D(R, m) \equiv a(m)d(R)$ and $D(R, m') \equiv a(m')d(R)$, respectively. With $a(m) < a(m')$, the inverse supply function for deposits in the larger market, $J_{m'}$, is more elastic than that in J_m. In this way, market integration makes the deposit market, and by the same logic, the loan market, more competitive as individual banks become small relative to these markets.[27]

The bank-competitor and the bank-customer effects. We explore the comparative static properties of the IME described in Lemma 2 to disentangle the effects of $a(m)$, $b(m)$, and $n(m)$ on loan rates.[28] Formally,

$$\frac{dr}{dm} = \underbrace{\frac{\partial r}{\partial n} \cdot n'(m)}_{\text{bank-competitor effect}} + \underbrace{\frac{\partial r}{\partial a} \cdot a'(m) + \frac{\partial r}{\partial b} \cdot b'(m)}_{\text{bank-customer effect}} . \tag{14}$$

Increasing m increases the number of competitor banks in the integrated market $n(m)$ and so, the first term on the right-hand-side of (14) denotes the *bank-competitor effect*. The second and third terms together constitute the *bank-customer effect*—the effect of an increase in the measure of customers (depositors and borrowers) under market integration. Because $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing in m, the sign of each term on the right-hand side of (14) is determined by the sign of the partial derivative in each term. We summarize our findings in terms of the following proposition.

Proposition 3 *In the IME, the effect on an increase in m on the loan rate r is given by*

$$\frac{dr}{dm} = Z_n \cdot \hat{n}(m) + Z_c \cdot \left\{ \hat{b}(m) - \hat{a}(m) \right\}, \tag{15}$$

where $Z_n < 0$, $Z_c > 0$, and $\hat{n}(m) \equiv n'(m)/n(m)$, $\hat{b}(m) \equiv b'(m)/b(m)$ and $\hat{a}(m) \equiv a'(m)/a(m)$ are the expansion rates of banks, borrowers and depositors, respectively. A negative bank-customer effect, that is, $\hat{b}(m) < \hat{a}(m)$ implies a negative association between competition and loan rates. On the other hand, a positive and sufficiently strong bank-customer effect, that is, $\hat{b}(m) > \hat{a}(m)$, induces a positive association between competition and loan rates.

[27] Integration of economies in models of intra-industry trade not only increases the measure of consumers but also expands their choice (Krugman, 1979). Although each consumer can potentially transact with more firms upon integration, consumers spend less on each variety because products are horizontally differentiated. This prompts some firms to exit the integrated market. Such exits do not occur in our setting because we assume that products of banks, namely deposits and loans, are not differentiated.

[28] We treat m as a continuous variable. Clearly, $n(m)$, $a(m)$, and $b(m)$ are all strictly increasing functions of m.

Equation (15) is isomorphic to (14). Just as in (14), the first term on the right-hand-side of (15) captures the bank-competitor effect. Moreover, the bank-competitor effect is negative (i.e., $Z_n < 0$) in the IME, just as that in the SME. All else equal, an increase in the number of competitor banks with more integration reduces the market power of the banks in the loan market by reducing their market share, which in turn lowers the loan rate.

Next, we show that $Z_c > 0$ in (15). Therefore, the bank-customer effect is positive, zero, or negative according as $\hat{b}(m) \gtreqless \hat{a}(m)$. All else equal, an increase in the number of depositors, $a(m)$ tends to lower the deposit rate as $R'(D/a) > 0$. Lower deposit rates means lower cost of loanable funds, which increases the amount lent by each bank. Consequently, increases in the number of depositors tend to increase aggregate loan volume and decrease the loan rate. On the other hand, an increase in $b(m)$ tends to increase loan rates, other things being equal, because $r'(L/b) < 0$. In short, the bank-customer effect comprises two terms: a negative effect associated with deposit supply and a positive effect associated with loan demand. Because they point in opposite directions, the sign of the bank-customer effect depends on the relative magnitudes of the increases in deposit supply and loan demand. When $\hat{b}(m) > \hat{a}(m)$, an increase in m expands the loan market more than the deposit market and this tends to increase loan rates. Conversely, when $\hat{b}(m) < \hat{a}(m)$, increasing m expands the deposit market more than the loan market, which tends to reduce loan rates.

The overall effect of market integration on the equilibrium loan rate comprises the bank-competitor and bank-customer effects. A negative bank-customer effect that lowers the equilibrium loan rate reinforces the negative bank-competitor effect. Accordingly, a negative bank-customer effect is a *sufficient condition* for a negative association between competition and loan rates. In contrast, a positive bank-customer effect tends to increase the equilibrium loan rate. Therefore, a positive bank-customer effect is a *necessary condition* for a positive association between competition and loan rates. If this positive effect is sufficiently strong, it can outweigh the negative bank-competitor effect. In sum, the effect of market integration on loan rates is not unambiguous.

The contrast between the two effects of competition under market integration is noteworthy. While we can make unambiguous predictions about the bank-competitor effect, we are unable to do so for the bank-customer effect. The intuition is straightforward. As banks operate in both deposit and loan markets, an increase in $n(m)$ affects both the deposit and loan markets symmetrically. Therefore, in scenarios where the effect of increased competition is comprised only of the bank-competitor effect, loan rates decrease as m increases. However, integrating dissimilar markets implies that increases in $a(m)$ and $b(m)$ affect deposit and loan markets asymmetrically. Therefore, in scenarios where the effect of increased competition is comprised only of the bank-customer effect, the effect on loan rates is indeterminate and depends on the relative expansion rates of each market.

However, if market integration expands deposit supply and loan demand at the same rate, i.e., $\hat{a}(m) = \hat{b}(m)$ for all m, the bank-customer effect is null. As a result, the effect of increased competition under market integration is comprised entirely of the bank-competitor effect and is unambiguously negative. A notable special case in this context is one where the integrating markets are identical. Understanding the implications of the integration of homogeneous markets in our setting underscores the significance of market heterogeneity in the IME. When integrating markets are homogeneous, the overall effect of an increase in m is comprised entirely of the bank-competitor effect and is unambiguously negative. In this way, the predictions in Boyd and De Nicoló (2005) emerge as a special case of our model. In sum, heterogeneity of the segmented markets is a necessary condition for a non-null bank-customer effect in the IME.

Bank-customer effect and the reference market .

It turns out that the reference market is important in determining the direction of the bank-customer effect. Consider two segmented markets, market 1 and market 2.[29] It follows from (8) that the difference in loan rates in the SME depends on the number of banks, n_j, the number of depositors, a_j, and the number of borrowers, b_j where $j = 1, 2$.[30] From (15), we know that the bank-customer effect with respect to market 1 is given by

$$\frac{b_1 + b_2 - b_1}{b_1} - \frac{a_1 + a_2 - a_1}{a_1} = \frac{b_2}{b_1} - \frac{a_2}{a_1} \tag{16}$$

Likewise, the bank-customer effect with respect to market 2 is given by

$$\frac{b_1 + b_2 - b_2}{b_2} - \frac{a_1 + a_2 - a_2}{a_2} = \frac{b_1}{b_2} - \frac{a_1}{a_2} \tag{17}$$

From (16) and (17), the bank-customer effects with respect to markets 1 and 2 must have opposite sign. This important caveat helps describe loan rates in the IME and their relation to loan rates in the SME.

Proposition 4 *Let r_1 and r_2 denote the SME loan rates of markets 1 and 2, respectively, and r^* denote the IME loan rate when markets 1 and 2 integrate. We must have $r^* \leq \max\{r_1, r_2\}$. Without loss of generality, assume that the bank-customer effect with respect to market 1 is positive (i.e., the bank-customer effect with respect to market 2 is negative). Then, we obtain $r_1 < r^* < r_2$ if the bank-customer effect with respect to market 1 is sufficiently strong, and $r^* < r_1 < r_2$, otherwise.*

Intuitively, the loan rate in the integrated market cannot exceed that of both the markets prior to integration because the bank-customer effect with respect to one of the markets is always negative. If the bank-customer effect with respect to one market, say market 1, is positive and sufficiently strong, then we have $r_1 < r^*$. At the same time, because the bank-customer effect is negative with respect to market 2, we have $r^* < r_2$. It is possible that the bank-customer effect with respect to one market is positive (i.e., it is negative with respect to the other), but not sufficiently strong. Consequently, we have the IME loan rate lower than that in the SMEs of both markets. Proposition 4 also underscores the importance of analyzing the effect of increased competition via banking market integration as opposed to the effect of an independent increase in either the number of banks or that of bank customers within an individual market. Measuring the intensity of competition by market integration allows us to compare the loan rates and risk levels of all markets prior to integration with those of the integrated economy.

4.2.4. Competition and risk-taking in the IME

We turn now to analyze the effect of competition on risk-taking incentives. The effect of increased competition on risk-taking in the IME comprises two effects: first, the effect of increased competition on the loan rate as shown above and second, the effect of the loan rate on risk-taking incentives of borrowers as analyzed in Section 3.3. Taken together, the effect on increased competition on risk-taking in the IME is non-trivial.

First, consider the case where risk-taking increases with the loan rate, that is, $\varepsilon'(k) < 0$. As described in Section 3.3, this is a standard result in most models of borrower moral hazard and limited liability (Stiglitz and Weiss, 1981; Boyd and De Nicoló, 2005). With the bank-competitor effect always negative, we have shown above that risk-taking decreases with competition in

[29] Without loss of generality, we may follow the discussion above and assume that market 1 is market J_m, a market created by integrating m previously segmented markets, and market 2 integrates with J_m to form the market, $J_{m+1} = J_m \cup \{2\}$.

[30] For example, if $n_1 = n_2 = n \geq 1$, $a_1 = a_2 = a > 0$ and $b_1 \neq b_2$. It follows from (8) that $r_2 > r_1$ if and only if $b_2 > b_1$.

the SME. However, increased competition under market integration can reverse the relation between competition and risk-taking. A sufficiently strong and positive bank-customer effect can lead to increased risk-taking in the IME, through its effect of increasing loan rates.

Next, consider the case where risk-taking decreases with the loan rate. In Section 3.3, this result is obtained when the production technology exhibits increasing elasticity of investment, that is, $\varepsilon'(k) > 0$. In this case, risk-taking increases with competition in the SME because of the negative bank-competitor effect—an increase in the number of banks lowers loan rates, and that leads to increased risk-taking. This relationship can also change with a sufficiently strong and positive bank-customer effect in the IME. With a sufficiently large bank-customer effect, increased competition in the IME increases loan rates and thereby reduces risk-taking. Overall, these results can be summarized as follows.

Proposition 5 *Any relationship between competition and risk-taking in the SME can be reversed by a positive and sufficiently strong bank-customer effect in the IME.*

Proposition 5 summarizes the impact of the bank-customer effect on risk-taking. As the generalized model here shows, the bank-customer effect exists even if bank customers (borrowers and depositors) are homogenous across all markets. As long as markets vary in terms of size (measures of the customers in each market), the bank-customer effect has the potential to alter the relationship between competition and risk-taking as more markets integrate.

5. A Linear Model

In this section, we show that the results of the general model in Sections 3 and 4 can be obtained in a simple linear setting. The linear model described here is used in the next section to demonstrate the robustness of our results to alternative assumptions.

Linear deposit supply is obtained from a simple version of the maxmization problem in (1). However, obtaining a linear loan demand function using the borrowers' maximization problem in (5) is involved. A somewhat cumbersome approach would be to pin down the exact functional forms of $p(\theta)$, $y(\theta)$ and $f(k)$ in (3) that yields a linear loan demand function. Instead, we prove the following existence result.

Lemma 3 *Let $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$. Then, there exists a unique $f(k)$ on $(0, \bar{k}) \subset (0, \lambda)$ such that $k(r)$ is a linear function of the form $k(r) = \lambda - r$.*

Lemma 3 motivates the use of linear functions in the general model.[31] Below, we present an alternative formulation (microfoundation) with linear deposit supply and loan demand functions, which differs from the general model in that bank customers are heterogenous in terms of their reservation utilities (e.g. Martinez-Miera and Repullo, 2010). The linear model below demonstrates that the results of the general model are also applicable to the case where bank customers are heterogenous.

5.1. Deposit supply, loan demand, and heterogenous customers

There are m distinct segmented markets. The endowment of each (risk-neutral) depositor is normalized to \$1. Depositors are heterogeneous in their reservation utility v. Let $G_j(v)$ denote the measure of depositors in market j that have reservation utility less than or equal to v with $G_j'(v) > 0$ for all v. We assume that the reservation utilities of depositors are distributed

[31] When considering the analysis in Section3.2, we note that there are other functional forms of $p(\theta)$ and $y(\theta)$ for which Lemma 3 holds.

uniformly on the support $[0, 1/a_j]$. A depositor would deposit \$1 only if $R \geq v$, where R is the deposit rate.[32] Therefore, the deposit supply in market j is given by $D_j(R) = G_j(R) = a_j R$. Markets are heterogeneous in their measure of depositors, so that $a_j \neq a_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the supply of deposits is $D(R, m) = a(m)R$ where $a(m) \equiv \sum_{j=1}^{m} a_j$ and the inverse supply function is

$$R(D, m) = \frac{D}{a(m)}. \tag{18}$$

In loan market j, each entrepreneur requires an investment $k \in [0, 1]$ which yields θk when the project succeeds (with probability $p(\theta) = 1 - \theta/\lambda$ with $\lambda > 1$), and zero when it fails (with probability $1 - p(\theta)$). Given loan rate r in market j, each entrepreneur in stage 2 solves

$$\theta(k; r) = \operatorname*{argmax}_{\theta} \ (1 - \theta/\lambda)\{\theta k - rk\}. \tag{19}$$

The optimal risk-shifting is given by $\theta = (\lambda + r)/2$. Unlike the expression for θ obtained in (4), optimal risk does not depend on the level of investment here because the production function is linear (i.e., $f(k) = k$). So, we denote the optimal risk-taking by $\theta(r)$ instead of $\theta(k; r)$. In stage 1, each entrepreneur solves

$$k(r) = \operatorname*{argmax}_{k \in [0,1]} \ (1 - \theta(r)/\lambda)\{\theta(r)k - rk\}.$$

The objective function is increasing in k and the first-order condition of the above maximization problem yields $k(r) = 1$. In this simplified setting, the relation between loan rates and risk-taking is always positive. In terms of (6), $d\theta^*/dr = \theta_r > 0$ because $\theta_k = 0$.

Let $u(r)$ denote the value function of the entrepreneurs' maximization problem (19). Also, let $H_j(u)$ denote the measure of entrepreneurs in market j that have reservation utility less than or equal to u with $H_j'(u) > 0$ for all u. An entrepreneur would participate in the loan market only if $u(r) \geq u$. Therefore, loan demand in market j is given by $L_j(r) = H_j(u(r))$. We assume $H_j(u) = 2b_j\sqrt{\lambda u}$ defined on the support $[0, 1/4\lambda b_j^2]$, so that $L_j(r) = H_j(u(r)) = b_j(\lambda - r)$.

Markets are heterogeneous in their measure of borrowers, so that $b_j \neq b_{j'}$ for any $j \neq j'$. When m distinct markets integrate, the demand for loans is $L(r, m) = b(m)(\lambda - r)$ where $b(m) \equiv \sum_{j=1}^{m} b_j$ and the inverse demand function is

$$r(L, m) = \lambda - \frac{L}{b(m)}. \tag{20}$$

Optimal risk-shifting equals $\theta(r(L/b(m))) = \lambda - L/2b(m)$ so that the probability of success is given by $P(L/b(m)) \equiv 1 - \theta(L/b(m))/\lambda = L/2\lambda b(m)$.

5.2. Competition and risk-taking in the linear model

In our simple linear setting, the association between loan rates and risk taking is positive. Because the effect of competition on loan rates is analogous to that on risk-taking, the discussion below on the effect of increased competition under market integration focuses almost exclusively on equilibrium risk-taking. The discussion on loan rates is omitted here for the sake of brevity.

[32] For a deposit of \$1, the bank pays R with probability $p(\theta)$. Assuming that deposit insurer pays the reservation utility, v, with probability $1 - p(\theta)$ when the project fails, a depositor deposits only if $p(\theta)R + (1 - p(\theta))v \geq v$ which translates to $R \geq v$ because $p(\theta) > 0$.

We assume that $a(m) = m^{\alpha}$ and $b(m) = m^{\beta}$ with $\alpha, \beta \in [0, 1]$. When $a_j = b_j = 1/m$ for all j, it follows that $\alpha = \beta = 0$. On the other hand, when $a_j = b_j = 1$ for all j, $\alpha = \beta = 1$. For any other distribution of $\{a_j\}_{j=1}^m$ and $\{b_j\}_{j=1}^m$, we have $\alpha, \beta \in (0, 1)$. Note that $\hat{a}(m) = \alpha/m$ and $\hat{b}(m) = \beta/m$, and therefore, the bank-customer effect is positive, zero, or negative according as $\alpha \gtreqless \beta$. We also assume that $n_j = j$, so that

$$n(m) = \sum_{j=1}^{m} j = \frac{m(m+1)}{2}\,.$$

Moreover, using (13) and $\theta(L/b(m)) = \lambda - L/2b(m)$, aggregate loan volume and risk-shifting in the IME are obtained as

$$L(m) = \frac{\lambda a(m) b(m)[n(m)+1]}{[a(m)+b(m)][n(m)+2]}\,, \tag{21}$$

$$\theta(m) = \lambda \left\{ 1 - \frac{1}{2} \cdot \frac{m(m+1)+2}{m(m+1)+4} \cdot \frac{1}{1+m^{\beta-\alpha}} \right\}. \tag{22}$$

The following result describes the IME in the linear model.

Proposition 6 *In the IME with linear deposit supply and loan demand schedules, where risk-shifting is given by (22), the following relations between competition and risk-shifting hold.*

(i) *If $\beta \leq \alpha$, the bank-customer effect is negative and risk-shifting is monotonically decreasing in m.*

(ii) *If $\alpha < \beta < \alpha + x$ where $x \approx 0.435$, the bank-customer effect is positive but weak, and risk-shifting is U-shaped with respect to m. In particular, there is a unique $\widehat{m} > 2$ such that risk-shifting is monotonically decreasing (increasing) in m for all $m < (>)\widehat{m}$.*

(iii) *If $\beta \geq \alpha + x$, the bank-customer effect is positive and sufficiently strong so that risk-shifting is monotonically increasing in m.*

Figure 2 describes the effect of increased competition on risk-shifting for the IME in the α-β space.

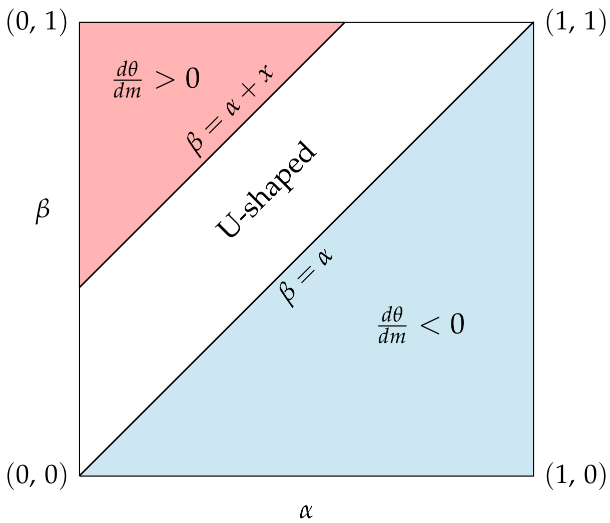

Figure 2: *Equilibrium association between risk-shifting and the number of integrated markets when $n(m) = \frac{1}{2}m(m+1)$, $a(m) = m^{\alpha}$ and $b(m) = m^{\beta}$.*

The simple linear model allows us to describe the differences in the relationship between competition, as measured by m, and risk-taking in terms of the parameters α and β. If $\beta < \alpha$, the negative bank-customer effect is a sufficient condition for the negative association between competition and risk-shifting (blue region in Figure 2). On the other hand, if $\beta > \alpha$, the bank-customer effect is strictly positive and leans against the negative bank-competitor effect. We show that when the difference $\beta - \alpha$ is sufficiently large, that is, $\beta - \alpha \geq x$, the positive bank-customer effect dominates the negative bank-competitor effect for all values of m and the association between risk-shifting and competition is positive (red region in Figure 2). However, if $0 < \beta - \alpha < x$, the positive bank-customer effect is not strong enough to outweigh the negative bank-competitor effect for all m. If fewer markets are integrated, (i.e., m is small), the bank-competitor effect dominates the bank-customer effect, and consequently, risk-shifting is decreasing in m. In contrast, when a greater number of markets are integrated, (i.e., m is sufficiently large), the bank-customer effect dominates the bank-competitor effect, and risk-shifting is increasing in m. Put differently, equilibrium risk-shifting first decreases and then increases with the number of integrated markets generating a non-monotonic (U-shaped) relationship between risk-shifting and market integration (white region in Figure 2).

If the number of banks is invariant to market integration, that is, $n(m) = \bar{n}$, then the effect of increased competition on risk-taking in the IME is composed entirely of the bank-customer effect.[33] In this case, Proposition 6 implies that entrepreneurial risk-taking is decreasing (increasing) under market integration according as $\alpha > (<) \beta$. In other words, the region in Figure 2 over which risk-shifting is U-shaped with respect to m would disappear. Notably, competition from market integration can yield lower (or higher) loan rates and risk-taking without any change in the number of competitor banks. This feature is discussed further in Section 6.1 below.

5.3. Welfare implication of market integration

We use the linear model above to examine the welfare effects of market integration. To begin, we derive the expressions for aggregate welfare in any market j. Following this, we consider a planner's decision to integrate two segmented banking markets.

The (inverse) deposit supply and loan demand functions in any market j are

$$R_j(D_j) = \frac{D_j}{a_j} \quad \text{and} \quad r_j(L_j) = \lambda - \frac{L_j}{b_j} \quad j = 1, 2, \tag{23}$$

respectively, where L_j and D_j are total loans and total deposits in market j. We use the shorthand $P_j(L_j) \equiv p(\theta(r_j(L_j)))$. Using the linear loan demand function in (23), we get $P_j(L_j) = L_j/2\lambda b_j$. Also, in equilibrium we have $L_j = D_j$.

At any given deposit rate $R_j(D_j)$ and depositor reservation utility v, the expected gross depositor surplus is

$$GDS_j(L_j) \equiv P_j(L_j)L_jR_j(L_j) + (1 - P_j(L_j)) \int_0^{R_j(L_j)} vG_j'(v)dv - \int_0^{R_j(L_j)} vG_j'(v)dv \tag{24}$$

The second term in (24) is covered by deposit insurance. As a result, deposit surplus net of

[33] This stylized setting has an interesting parallel in U.S. banking history. Expansion of multibank holding companies since the 1960s provided banks with an alternative means to expand beyond their local market where branching was prohibited or limited. With the removal of branching barriers, subsidiary banks were often consolidated as branches under a single charter. In the stylized example where integrating markets are populated by the same holding companies, the post-integration consolidation of subsidiaries would leave the number of rival banks invariant to market integration.

insurance is given by[34]

$$DS_j(L_j) = P_j(L_j)L_jR_j(L_j) - \int_0^{R_j(L_j)} vG_j'(v)dv. \tag{25}$$

At any given loan rate $r_j(L_j)$ and borrower reservation utility u, the borrower surplus is

$$BS_j(L_j) = L_ju(r_j(L_j)) - \int_0^{u(r_j(L_j))} uH_j'(u)du. \tag{26}$$

Bank surplus, $\Pi_j(L_j)$, is the sum of profits of individual banks in the symmetric equilibrium

$$\Pi(L_j) = P_j(L_j)(r_j(L_j) - R_j(L_j))L_j. \tag{27}$$

The aggregate welfare in market j is the sum of (25), (26) and (27):[35]

$$W_j(L_j) = P_j(L_j)L_jr_j(L_j) - \int_0^{R_j(L_j)} vG_j'(v)dv + L_ju(r_j(L_j)) - \int_0^{u(r_j(L_j))} uH_j'(u)du. \tag{28}$$

The distribution functions of the reservation utilities, v and u, imply that

$$\int_0^{R_j(L_j)} vG_j'(v)dv = \frac{L_j^2}{2a_j} \quad \text{and} \quad L_ju(r_j(L_j)) - \int_0^{u(r_j(L_j))} uH_j'(u)du = \frac{L_j^3}{6\lambda b_j^2}.$$

From (28), aggregate welfare in the linear model is

$$W_j(L_j) = \frac{L_j^2}{2\lambda b_j}\left(\lambda - \frac{L_j}{b_j}\right) - \frac{L_j^2}{2a_j} + \frac{L_j^3}{6\lambda b_j^2} = \frac{(a_j - b_j)L_j^2}{2a_jb_j} - \frac{L_j^3}{3\lambda b_j^2}. \tag{29}$$

A necessary condition for $W_j(L_j)$ to be positive is that $a_j > b_j$. We analyze the integration of two economies, $j = 1, 2$, that are otherwise identical but vary in the size of their loan markets. In particular, let $a_1 = a_2 = a > 0$, $b_1 = b > 0$, $b_2 = \beta b$ with $\beta > 0$ and $n_1 = n_2 = n \geq 1$. Using this example, we consider a situation wherein a planner (equivalently, a regulator or antitrust authority) decides whether to integrate the two markets or keep them segmented. In its decision to integrate the two economies, we assume that the planner simply compares aggregate welfare under market integration with aggregate welfare when markets are segmented. Importantly, the planner makes no other decision such as setting rates or taxes to influence the optimization behavior of market participants. Therefore, all market participants optimize as described in the model above and the planner decides to integrate the two markets if

$$\Delta W \equiv W(L^*) - [W_1(L_1^0) + W_2(L_2^0)] > 0,$$

where L^* denotes aggregate loans in the IME and L_j^0 denotes aggregate loans in the SME for market $j = 1, 2$, respectively.

We use a numerical exercise to illustrate how the change in aggregate welfare from market integration, ΔW, varies with model parameters. In particular, Figure 3 plots ΔW as a function of the measure of the bank-customer effect, β, for a given set of fixed parameters λ, a, b, and n in the linear model. It is worth noting that the SME risk-shifting in market 1 is higher than that

[34]Following Martinez-Miera and Repullo (2010), we use depositor surplus net of deposit insurance in our calculation of aggregate welfare.

[35]We ignore social costs and other negative externalities associated with bank liquidation. Our results are qualitatively similar if we assume a fixed social cost, $C > 0$ so that the expected social loss associated with bank failure in market j is $(1 - P_j(L_j))C$.

in market 2 if and only if $\beta > 1$. We set $\lambda = 1.5$, $a = 4$, $b = 1.4$, $n = 2$, and vary β on $[0, 4]$. In addition to ΔW, Figure 3 also plots $\Delta\theta_j \equiv \theta(L^*) - \theta_j(L_j^0)$ where $\Delta\theta_j$ measures change in risk-taking due to market integration. In particular, $\Delta\theta_j$ is the difference in risk-taking between the IME and the SME for individual market j, $j = 1,2$.

From (16) and (17), we obtain that the bank-customer effect with respect to market 2 is decreasing in β and is positive for $\beta < 1$ (and negative for $\beta > 1$). In contrast, the bank-customer effect with respect to market 1 is increasing in β and is positive for $\beta > 1$ (and negative for $\beta < 1$). For sufficiently low β (i.e., $\beta < \beta_2$), we find that $\Delta\theta_2 > 0$ as shown by the orange curve in Figure 3. This happens when the positive bank-customer effect with respect to market 2 is sufficiently strong to outweigh the negative bank-competitor effect, so that loan rate and risk-taking are higher in the IME compared to their SME values in market 2. Also, $\Delta\theta_1 < 0$ for $\beta < \beta_2$ because bank-customer effect with respect to market 1 is negative and reinforces the bank-competitor effect. The situation is reversed at sufficiently high values of β (i.e., $\beta > \beta_1$), where we obtain $\Delta\theta_1 > 0$ as shown by the red curve in Figure 3. This occurs when the positive bank-customer effect with respect to market 1 is sufficiently strong to outweigh the negative bank-competitor effect, so that loan rate and risk-taking are higher in the IME compared to their SME values in market 1. Moreover, $\Delta\theta_2 < 0$ for $\beta > \beta_1$ because the bank-customer effect with respect to market 2 is negative and reinforces the bank-competitor effect. Finally, for all intermediate values of β, where $\beta \in (\beta_2, \beta_1)$, loan rate and risk-taking in the IME are lower compared to their SME values in both markets 1 and 2.

The change in aggregate welfare, ΔW, is shown by the blue curve in Figure 3. For most moderate values of β (i.e., $0 \leq \beta < \beta^*$), market integration increases aggregate welfare (i.e., $\Delta W > 0$), irrespective of whether loan rates and risk-taking are increasing or decreasing. In particular, we find that $\Delta W > 0$ even if risk-taking is increasing for market 1 (for $\beta_1 < \beta < \beta^*$) or for market 2 (for $\beta < \beta_2$). Only when β is substantially high (i.e., $\beta > \beta^*$), we find that $\Delta W < 0$, where a substantial increase in loan rates and risk-taking with respect to market 1 is accompanied by a decrease in aggregate welfare due to market integration.

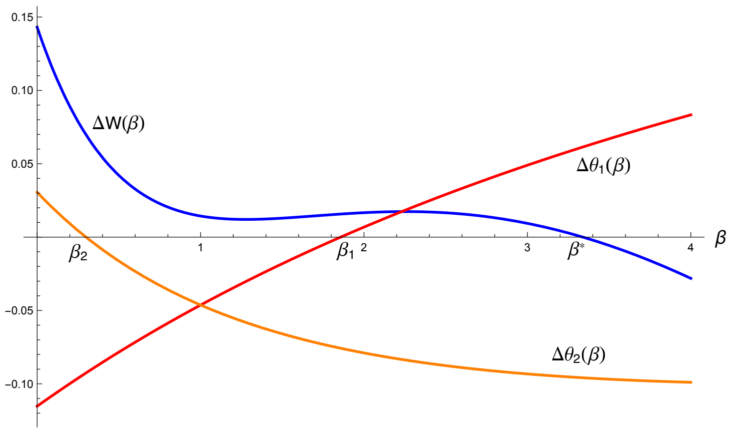

Figure 3: *Welfare comparison between the SME and the IME.*

The numerical example here yields results that are important in the context of the large empirical literature on competition and risk-taking. First, the effect of competition on risk-taking are not necessarily universal across all integrating markets. Increased risk-taking with respect to one market can be accompanied with reduced risk-taking with respect to another, as shown in Figure 3. Moreover, as the example shows, increased competition under integration can also lower risk-taking in both markets. Second, market integration can increase both risk-taking and aggregate welfare. Restricting attention to only risk-taking implications of competition often ignores the possibility that aggregate welfare can increase with risk-taking.

Lastly, market integration can be also decrease aggregate welfare as shown for the case with substantial increases in loan rates and risk-taking in the example above.

6. Extensions

In this section, we show that the results of the linear model developed in Section 5 are robust to different modeling choices. In doing so, we point to other factors that affect loan rates and risk-taking when markets integrate. For the IME described above, we assumed that (1) there is no exit and entry of banks, (2) loan default risks are perfectly correlated, and (3) banks do not transact in the interbank market.[36] We relax each of these assumptions in turn to demonstrate that the baseline results are robust to different settings.

Notably, the (negative) indirect effect of loan rates on risk-taking described in (6) is set to zero in the linear model. Consequently, the relation between loan rates and risk-shifting is positive and any effect of competition on the loan rate, r, translates directly to risk-shifting, θ, in the comparative statics below. This allows us to simplify the discussion below exclusively in terms of equilibrium risk-taking. Alternatively, it allows us to omit the discussion in terms of loan rates.

6.1. Bank mergers

In this extension, we show that market integration creates incentives for bank mergers. Mergers increase the market power of surviving banks thereby increasing equilibrium loan rates and risk-taking because of the negative bank-competitor effect. In this way, consolidation in the integrated market reinforces any positive bank-customer effect.

6.1.1. Heterogenous bank operating costs and the SME

We start with the linear model presented in Section 5.1. To this model, we introduce market-specific bank operating costs so that operating costs of bank i in market j are

$$C_{ij}(D_{ij}) = c_j D_{ij},$$

where $c_j \geq 0$ is the constant marginal operating cost of funds.[37] While all banks operating within market j have identical (marginal) operating cost of funds, we assume that markets are heterogeneous in terms of bank operating costs, so that $c_j \neq c_{j'}$ for any $j \neq j'$. We denote market-j banks with cost of operations c_j as a type-j banks. The assumption of heterogeneity in operating costs (non-interest expenses) captures differences in efficiencies, with more efficient banks having lower operating costs. Differences in efficiency have been attributed to the lack of competition due to restrictions on geographic expansion that provide banks with local market power. Kroszner (2001) argues that, prior to the relaxation of branching restrictions in the United States, geographic variation in banks' cost efficiencies could be linked to variations in the degree of protection across the different banking jurisdictions (cf. footnote 8).

Following Section 5.1, $p(\theta) = 1 - \theta/\lambda$ so that the inverse deposit supply and loan demand functions in any segmented market j are given by (23). We also assume that (i) there are only two markets, market 1 and market 2, (ii) banks in market 1 are more efficient than those in

[36] Fecht, Inderst, and Pfeil (2017) treat interbank lending and bank mergers as alternatives in the process of market integration. However, we model them separately.

[37] Costs are typically a function of both deposits and loans in the Monti-Klein model (Klein, 1971). From the linear model, we retain the assumption that each bank is financed entirely by deposits—there is no equity-financing and no interbank market. Given that banks in our model do not hold equity, it is fairly innocuous to assume that costs depend on the volume of deposits.

market 2, so that $c_1 = 0 < c = c_2$, (iii) deposit supply and the number of banks are the same in both markets, so that, $a_1 = a_2 = a > 0$ and $n_1 = n_2 = n \geq 2$, but the loan demand in market 2 is larger than that in market 1, $b_1 = b > 0$ and $b_2 = \beta b$ with $\beta \geq 1$.

Lemma 4 *Risk-shifting in markets 1 and 2 in the SME are given by*

$$\theta_1^0 = \lambda - \frac{\lambda a(n+1)}{2(a+b)(n+2)} \quad \textit{and} \quad \theta_2^0 = \lambda - \frac{(\lambda - c)a(n+1)}{2(a+\beta b)(n+2)},$$

respectively. The SME risk-taking is lower in market 1, that is, $\theta_1^0 < \theta_2^0$.

All else equal, higher loan demand in market 2 yields a higher loan rate and more risk-taking in market 2, so that the SME loan rate in market 2 increases with β. At the same time, greater inefficiency in market 2 raises operating costs and loan rates for banks in market 2, so that the SME loan rate and risk-taking in market 2 increases with c. By construction, both effects point in the same direction and the extent to which loan rates and risk-taking is higher in market 2 depends on both β and c.[38]

6.1.2. Heterogenous bank operating costs in the integrated market

In Section 5.3, we derived the conditions under which market integration increases welfare in the linear model. In this section, we assume that these conditions have been met and that market integration is welfare-enhancing. Given that market integration has been approved, a planner or antitrust authority is now faced with the decision of whether to allow bank mergers. Mergers occur if the planner permits banks to merge and banks find it profitable to merge. On the other hand, if mergers are disallowed or unprofitable, both high- and low-cost banks compete in a Cournot fashion in the integrated market. We continue to assume that there is no entry of banks but that bank exits can occur through bank mergers.

The timeline is as follows. At $t = 0$, the planner decides whether to allow mergers in the integrated market. To simplify, we assume that the planner allows only pairwise mergers—that is mergers between any two banks are allowed, but mergers of more than two banks are not. With pairwise mergers and $2n$ banks in the integrated market, we can have a maximum of n mergers. At $t = 1$, each pair of banks decide whether to merger or not. We solve the game by backward induction. Note that, in the integrated market, each bank faces the following deposit supply and loan demand schedule

$$R(D) = \frac{D}{a_1 + a_2} \equiv \frac{D}{2a} \quad \text{where } D = D_1 + D_2, \tag{30}$$

$$r(L) = \lambda - \frac{L}{b_1 + b_2} \equiv \lambda - \frac{L}{(1+\beta)b} \quad \text{where } L = L_1 + L_2. \tag{31}$$

Given that we have n banks of each type, mergers can occur between like banks (e.g. a merger between any two high-cost or any two low-cost banks) and unlike banks (merger between a high- and a low-cost bank). We assume that these two types of mergers are different. When banks with the same operating costs merge, the merged entity operates with the same cost as its predecessors and there are no efficiency gains. In contrast, a merger between a low- and a high-cost banks captures the entire efficiency gains in that the merged entity operates

[38] This simplification is easily modified if we assume that either (1) banks in market 2 have lower operating costs or (2) the loan market is smaller in region 2, $\beta \in (0, 1)$. In (2), loan rate and risk-taking in market 2 can be higher or lower than that in market 1 depending on the relative strength of the two opposing forces (i.e., region 1 has a larger loan market, but banks in market 2 are more inefficient). While this alternative complicates the analytical solution, it does not qualitatively alter the results in this extension.

as a low-cost bank.[39] We begin with the following useful result regarding the profitability of mergers between two banks with the same operating costs in any given market.

Lemma 5 *Consider any given market with deposit supply schedule, $R(D) = D/A$, and loan demand schedule, $r(L) = \lambda - L/B$, where $A, B > 0$ represent the size of the deposit and loan markets, respectively. Suppose further that a merger between two banks with the same operating costs yields no efficiency gains.*

(a) *If the market consists only of banks with the same operating costs, a merger between a pair of banks is profitable only if the pre-merger market structure is a duopoly (i.e., $n = 2$).*

(b) *If there are same number of low-cost and high-cost banks, then any merger between a pair of banks with same operating costs is never profitable.*

The first part of Lemma 5 asserts that mergers in any given market are profitable when there are only two banks. In other words, pairwise mergers must result in monopoly banking in order to be profitable.

Let $\pi_j(n_l, n_h)$ denote the expected profit of each type-j bank when there are n_l low-cost and n_h high-cost banks in any given market. If there only low-cost banks in the market, a profitable pairwise merger would require that $\pi_1(n-1, 0) > 2\pi_1(n, 0)$. Likewise, a profitable pairwise merger in any market comprising only high-cost banks requires that $\pi_2(0, n-1) > 2\pi_2(0, n)$. We find that these conditions hold only when the resulting (post-merger) market structure is a monopoly, that is, $\pi_1(1, 0) > 2\pi_1(2, 0)$ for market 1, and $\pi_2(0, 1) > 2\pi_2(0, 2)$ for market 2.[40] In what follows, we assume that $n > 2$ in each market so that pairwise mergers are ruled out in any segmented market.

Next, consider that there are n low-cost and n high-cost banks in any given market with $n \geq 2$. For merger between two low-cost banks to be profitable, we require $\pi_1(n-1, n) > 2\pi_1(n, n)$. On the other hand, for a merger between two high-cost banks to be profitable, we require that $\pi_2(n, n-1) > 2\pi_2(n, n)$. We establish that neither of the two inequalities hold when $n \geq 2$. Therefore, for mergers in the integrated market, we can restrict attention only to mergers between a pair of low- and high-cost banks.

Lemma 5 allows us to focus on two types of subgame perfect equilibria: (1) an IME without mergers which we term the "no-merger IME" and (2) an IME with mergers which we term the "merger IME". We describe them below.

The no-merger IME. A no-merger IME can prevail under two scenarios—either the planner does not permit mergers or banks do not find mergers profitable. In both cases, the market structure is an asymmetric Cournot market with n low-cost and n high-cost banks, each facing deposit supply and loan demand schedules given in (30) and (31), respectively. In equilibrium, we have $L = D$, and each type-j bank i solves

$$\pi_j^*(n, n) = \max_{L_{ij}} P(L) \left(r(L) - R(L) - c_j \right) L_{ij},$$

[39] Admittedly, this is an oversimplification. For our results, we require that the efficiency gains from mergers between dissimilar banks be greater than that those between similar banks. First, this assumption is motivated by the idea that synergies from mergers of dissimilar banks comes from the local knowledge that each bank possesses about its home market. Because by construction, similar banks in our setting share the same local market, the scope for synergies is naturally smaller. Second, one may view the assumption of zero efficiency gains as a normalization of the synergies from mergers of similar banks relative to that of dissimilar banks. Although this is not explicitly modeled, this assumption is intended to capture the notion that consolidation and competition within an individual market has exhausted all within-market efficiency gains prior to integration.

[40] That mergers may reduce the profits of the merged entity is a common feature of Cournot models. In particular, Salant, Switzer, and Reynolds (1983) have shown that for mergers to be profitable under Cournot competition, a critical fraction (about 80%) of the firms must merge.

where L_{ij} denotes the amount lent by a bank i of type j, and $P(L) = L/2\lambda(1+\beta)b$. The result of the maximization problem is summarized in the following lemma.

Lemma 6 *Let L_j^* denote the aggregate loans of the type-j banks in the no-merger IME. If $c < \bar{c} \equiv \frac{\lambda}{n+2}$, all banks make positive profits so that $L_j^* > 0$ for $j = 1, 2$, and aggregate loan volume is $L^* = L_1^* + L_2^*$ is*

$$L^* = \frac{ab(1+\beta)\left\{(3n+2)(2\lambda - c) + \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda - c)}\right\}}{4(2a + (1+\beta)b)(n+1)}. \tag{32}$$

Moreover, both high- and low-cost banks take the same level of risk, which is given by

$$\theta^* = \lambda - \frac{L^*}{2(1+\beta)b} = \lambda - \frac{a\left\{(3n+2)(2\lambda - c) + \sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda - c)}\right\}}{8(2a + (1+\beta)b)(n+1)}. \tag{33}$$

However, if $c \geq \bar{c}$, the difference in costs between the two bank types is sufficiently large so that high-cost banks cannot lend in the no-merger IME and $L_2^* = 0$. As a result, the no-merger IME comprises only low-cost banks. We will omit this relatively uninteresting case from further analysis.

The expression in (33) reveals that equilibrium loan rates and risk-taking in the no-merger IME depend on three factors. These include the bank-competitor effect, the bank-customer effect, and the operational efficiency effect which are captured by the effects on the loan rate and risk-shifting of n, β, and c, respectively. Among the three, the operational efficiency effect is the additional source of heterogeneity imposed on the baseline model. Loan rates and risk-taking in the no-merger IME depend on the relative strength of each of the three factors mentioned above. The following proposition compares the risk-shifting levels under the SME and the IME.

Proposition 7 *Let θ^* be the risk-shifting under the no-merger IME and θ_j^0 be the risk-shifting under the SME in market j, $j = 1, 2$. Assume $\beta \geq 1$ and $0 \leq c < \bar{c}$. The SME risk-taking in market 2 is always greater than that in the IME, (i.e., $\theta^* < \theta_2^0$). We obtain $\theta^* > (<)\, \theta_1^0$ if $c \geq (<)\, c^*(\beta)$, where $c^*(\beta) = \max\{0, \tilde{c}(\beta)\}$ with $\tilde{c}(1) = \bar{c}$ and $\tilde{c}'(\beta) < 0$.*

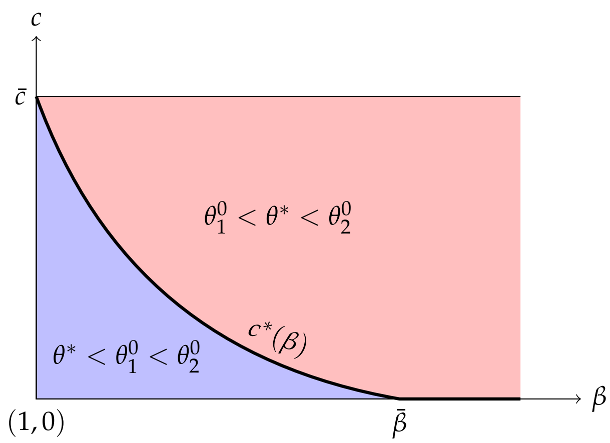

Figure 4: *Comparison of risk levels in the SME, and the no-merger IME.*

Figure 4 illustrates the result in Proposition 7. First, the negative bank-competitor effect from an increase in the number of banks in the no-merger IME tends to lower rates. Second, it follows from Proposition 4 and $\beta > 1$ that the bank-customer effect is positive (negative) with respect to the low-cost market or market 1 (high-cost market or market 2). Apart from the bank-competitor and bank-customer effects, there is a third effect that stems from differences in the operating costs of banks in the no-merger IME.

For high-cost banks, the bank-competitor and bank-customer effect are both negative, and this tends to lower loan rates and risk-taking. Additionally, competing with low-cost banks in the integrated markets also tends to lower loan rates for high-cost banks in the no-merger IME. When compared to the SME in market 2, all three effects tend to increase lending and lower loan rates and risk-taking by high-cost banks in the no-merger IME so that $\theta^* < \theta_2^0$.

For low-cost banks, the bank-competitor effect is negative but the bank-customer effect is positive. As shown above (under the assumption that banks are homogenous), there is a threshold value of β, exceeding which the bank-customer effect is sufficiently strong to outweigh the negative bank-competitor effect. It turns out that in the no-merger IME with heterogenous efficiency costs, the threshold value of β is declining in c according to the function $\tilde{c}(\beta)$ shown in Figure 4, where $\tilde{c}'(\beta) < 0$.

For $c = 0$, this threshold value is $\bar{\beta}$ and is defined by $\tilde{c}(\bar{\beta}) = 0$. With $c = 0$, the no-merger IME is the same as the IME described in the linear model above (where banks are homogenous). In this case, the differences in risk levels in the IME and SME in market 1 are explained entirely by the bank-competitor and bank-customer effects. It follows that $\theta^* > \theta_1^0$ if $\beta > \bar{\beta}$ when $c = 0$ as shown in Figure 4.

For $c \in (0, \bar{c})$, the threshold value of β is declining in c. The intuition is simple. Low-cost banks from market 1 can lend at higher rates (compared to their SME values) when they have a large cost advantage over their high-cost rivals in the integrated market (high c) or alternatively, as shown above, the bank-customer effect is sufficiently strong (high β). At high cost differences, a relatively moderate bank-customer effect is sufficient to allow the bank to lend at higher rates. The opposite happens when cost differences are low. Low-cost banks require a sufficiently strong bank-customer effect to lend at rates higher than their SME values. Of course, higher loan rates in the no-merger IME relative to the SME imply higher risk-shifting. And although we explain the intuition in terms of rates here, it is useful to recall that banks in the Cournot model optimally choose loan volumes.

The merger IME. For an IME with bank mergers, Lemma 5 allows us to focus only on mergers between unlike banks. We also assume that the merged entity captures the entire efficiency gains from the merger and, in terms of the linear model, operates with zero costs. Finally, because the integrated market has n banks of each type, symmetry ensures that a merger between any pair of low-cost and high-cost bank allows for the merger between all n pairs. Below, we show that in an IME with bank mergers, n pairwise mergers between each high-cost and each low-cost bank are profitable.

Mergers in the IME will yield n low-cost banks in the post-merger market. Each bank faces deposit supply and loan demand schedules given by (30) and (31) and solves

$$\pi_M(n) = \max_{L_i} P(L)\,(r(L) - R(L))\,L_i.$$

The solution to this maximization problem yields the following result.

Lemma 7 *When the planner allows pairwise bank mergers, we obtain*

$$\pi_M(n) > \pi_1^*(n, n) + \pi_2^*(n, n) \quad \text{for all } c < \bar{c}. \tag{34}$$

As a result, all pairwise mergers between low-cost banks and high-cost banks are profitable. Aggregate *loans in the merger IME are given by*

$$L_M = \frac{2\lambda ab(1+\beta)(n+1)}{(2a+b(1+\beta))(n+2)},$$

and the equilibrium risk-shifting is

$$\theta_M = \lambda - \frac{L_M}{2(1+\beta)b} = \lambda\left(1 - \frac{a(n+1)}{(2a+b(1+\beta))(n+2)}\right).$$

Inequality (34) shows that the sum of the profits of a low-cost bank and a high-cost bank in the no-merger IME is less than the profits of the merged entity formed by the two banks in the merger IME. Importantly, mergers are profitable under the same regularity conditions, $c < \bar{c}$, as that for the no-merger IME (Lemma 6). As a result, mergers of unlike banks are always profitable in the integrated market, and such mergers do not occur only if the planner disallows them. Risk-taking in the merger IME is summarized in the following proposition.

Proposition 8 *We obtain $\theta_1^0 \leq \theta_M < \theta_2^0$ for all $c < \bar{c}$, where θ_M and θ_j^0 are the risk-shifting in the merger IME and the SME in market j, $j = 1, 2$, respectively.*

The bank-competitor effect is zero because the number of banks in the merger IME is the same as that in the SME in either market. As a result, differences in loan rates and risk-taking between the merger IME and SME can be attributed to the remaining two factors, namely, the bank-customer effect and the efficiency gains. In terms of operational costs, mergers generate efficiency gains in that surviving banks are low-cost and this tends to lower loan rates and risk-taking relative to their high-cost predecessors. Meanwhile, the bank-customer effect is negative (positive) with respect to market 2 (market 1). The negative bank-customer effect together with efficiency gains helps lower the merger IME loan rate and risk-taking from their SME values in market 2 so that $\theta_M < \theta_2^0$. On the other hand, banks were already operating at low cost in market 1 so that the difference between θ_1^0 and θ_M stems entirely from the positive bank-customer effect, and we get $\theta_1^0 \leq \theta_M$.

The above analysis emphasizes the importance of the bank-customer effect even when banks can exit the integrated market through mergers. Proposition 8 shows that loan rates and risk-shifting are lower compared to their SME values in market 2 even though the total number of competitors did not increase in the merger IME. We demonstrate that competition under market integration can lower loan rates and risk-taking without a concomitant increase in the number of banks. Such a negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior or from efficiency gains from bank mergers or both.

Welfare effects of bank mergers. The planner permits mergers if the the expected aggregate welfare in the merger IME exceeds that in the no-merger IME. Let $W^*(c)$ and W_M denote aggregate welfare in the no-merger and merger IME, respectively. Using the expressions for aggregate welfare in the (29), we obtain

$$W^*(c) = \frac{(2a-(1+\beta)b)(L^*)^2}{4a(1+\beta)b} - \frac{(L^*)^3}{3\lambda(1+\beta)^2b^2} - \underbrace{\frac{cL^*L_2^*}{2\lambda(1+\beta)b}}_{P(L^*)cL_2^*},$$

$$W_M = \frac{(2a-(1+\beta)b)(L_M)^2}{4a(1+\beta)b} - \frac{(L_M)^3}{3\lambda(1+\beta)^2b^2}.$$

Mergers are permitted if $\Delta W(c) \equiv W^*(c) - W_M < 0$. We discuss the sign of $\Delta W(c)$ in the following numerical example which shows that for high efficiency gains, i.e., high values c, merger IME yields higher aggregate welfare than the non-merger IME. Thus, for high values of c, allowing two-bank mergers is the dominant choice for the planner.[41]

Example 1 Set $\lambda = 1.5$, $a = 4$, $b = 1$, $\beta = 1.2$ and $n = 3$. Figure 5 shows the function $\Delta W(c)$ for $c \in (0, 0.3)$, where $\bar{c} = 0.3$. The function, $\Delta W(c)$, intersects the horizontal axis at $c^* \approx 0.035$. Small differences in costs imply small efficiency gains from mergers. Accordingly, welfare gains from mergers are low for small differences in cost between high- and low-cost banks, $c \leq c^*$. However, if $c > c^*$, then efficiency gains are sufficiently large so that aggregate welfare in the merger IME exceeds aggregate welfare in the no-merger IME. ■

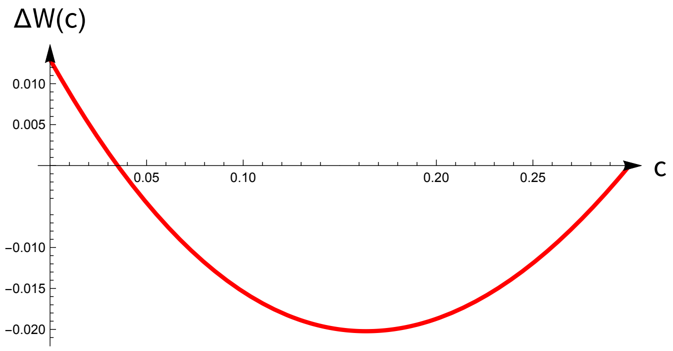

Figure 5: *Welfare effects of bank mergers.*

Example 1 conforms to the welfare analysis of horizontal mergers with efficiency gains in oligopoly markets (Williamson, 1968). Mergers increase bank market power and this tends to raise loan rates and lower deposit rates (by lowering loan volumes), thereby reducing expected customer (borrower and depositor) surplus. On the other hand, the opposite is true for efficiency gains from mergers because they tend to lower loan rates and raise deposit rates, thereby increasing expected bank surplus. As a result, net increase in aggregate welfare from mergers depends on the relative magnitudes of these two opposing forces. In sum, mergers are welfare enhancing (i.e., $\Delta W(c) < 0$) if efficiency gains are sufficiently large (i.e., $c > c^*$).

To summarize, at $t = 0$, the planner either allows or disallows bank mergers. This decision is made as described in Example 1. For $c \leq c^*$, the planner disallows bank mergers at $t = 0$ and we obtain the no merger IME described in Lemma 6 and Proposition 6. On the other hand, if $c > c^*$, the planner permits bank mergers at $t = 0$ and we obtain the merger IME described in Lemma 7 and Proposition 8. In our setting, there is no equilibrium wherein the planner permits mergers but banks do not find mergers profitable. Moreover, loan rates and risk-taking are higher in the merger IME because of the negative bank-competitor effect and the fact that there are fewer banks in the merger IME. The result is summarized in terms of the following proposition.

Proposition 9 *In the subgame perfect equilibrium of the merger game, a merger between a pair of high-cost bank and a low-cost bank occurs whenever efficiency gains are large (i.e., $c > c^*$). Moreover, $\theta_M > \theta^*$, that is, risk-taking in the merger IME is greater than risk-taking in the no-merger IME.*

This result has important implications. First, even when they are privately optimal (as described in Lemma 7), bank mergers may not be socially optimal. In essence, our model

[41] This result can be proved analytically, which we avoid because the expressions are cumbersome. The sketch of the proof is as follows. It is easy to show that $\Delta W(\bar{c}) = 0$, and $\lim_{c \to \bar{c}} d\Delta W(c)/dc > 0$. Therefore, because $\Delta W(c)$ is continuous on $[0, c]$, it must be the case that $\Delta W(c) < 0$ for large values of c (close to $\bar{c}$).

presents a rationale for merger reviews to determine that mergers do not reduce competition substantially. Second, higher aggregate welfare in the merger IME relative to the no-merger IME is also accompanied by higher risk-taking. This has financial stability implications. The planner can lower risk-taking by disallowing mergers. However, this comes at the cost of reducing aggregate welfare. With the inclusion of the *financial stability factor* in merger reviews under the Dodd Frank Act of 2010, welfare considerations are no longer the sole factor in approving bank mergers. Our framework presents scenarios in which the welfare effect and the risk-taking effect can provide conflicting recommendations for merger approvals.

Before concluding this section, it is important to review some of the results for this extension of the model. First, allowing for bank mergers does not materially change the results of the baseline model—the fact that a positive bank-customer effect tends to increase loan rate and risk-taking in the IME is still robust. Second, we find that market integration yields pro-competitive gains even without a concomitant increase in the number of banks (Proposition 8). As discussed above, the negative association between market integration and loan rates can originate from the bank-customer effect that alters price-taking behavior or from efficiency gains from bank mergers or both. While the empirical literature has focused on efficiency gains from mergers, our model shows that this can happen even in the absence of efficiency gains. Third, the model in this subsection is motivated by a large empirical literature that has highlighted the efficiency-enhancing role of mergers post integration (Berger et al., 1999; DeYoung et al., 2009). In particular, our model demonstrates how efficient banks merge with relatively inefficient, less profitable banks when heterogenous markets integrate (cf. footnote 10). The model also reflects the dominant pattern following market integration of across-market mergers between local and non-local banks as opposed to within-market mergers between local banks (Berger et al., 1999; Dick, 2006). Fourth, although our specification finds that mergers between like banks are rarely profitable, this is a common feature of Cournot models (Salant, Switzer, and Reynolds, 1983). It is possible that other modeling choices could account for the prevalence of mergers between similar banks. Lastly, we have focused mainly on the *unilateral effects* of mergers—that is, whether mergers yield welfare gains or losses. On the other hand, the *pro-collusive effects* of mergers can be no less important, especially in any post-merger IME where it is easier to sustain tacit collusion between banks that have similar cost structures and market shares (Bernheim and Whinston, 1990; Boyd and Graham, 1998). Moreover, the pro-collusive effects of mergers can potentially increase loan rates and risk-taking beyond that obtained in the merger IME.

6.2. Market integration and risk diversification

In this extension, we consider the possibility that market integration creates opportunities for diversification.[42] To this end, we return to the two-market linear model with homogenous costs in Section 5.3. We assume that borrower returns are perfectly correlated within each market where the probability of success is $p(\theta) = 1 - \theta/\lambda$. However, project returns are imperfectly correlated across the two markets, where $\sigma \in [0, 1]$ denotes the correlation coefficient between default risks in markets 1 and 2. Moreover, we assume that depositors are repaid by their banks only if projects are successful (and loans are repaid) for both group of borrowers.[43]

[42] Martinez-Miera and Repullo (2010) present a general model wherein projects are imperfectly correlated. In addition to the negative risk-shifting effect (the bank-competitor effect, in our case), they identify a countervailing margin effect—reduced bank revenues from performing loans due to lower loan rates create incentives for banks to take on more risk.

[43] The results obtained here are robust to the assumption that banks' debt obligation are met if at least one group succeeds. Nevertheless, the simplification here does not explicitly model idiosyncratic risk, thereby retaining the feature that entrepreneurial risk-taking is synonymous with risk-taking by banks (cf. footnote 22). As a result, the margin effect modeled in Martinez-Miera and Repullo (2010) is zero in this setting. If we had considered within market correlation, we would also have the same margin effect both in the segmented and the integrated markets.

The probability that projects in both markets succeed is

$$q(\theta) \equiv p(\theta)^2 + \sigma p(\theta)[1 - p(\theta)].$$

The linear inverse deposit supply and loan demand functions are as described in (23) with $n_1 = n_2 = n \geq 1$, $a_1 = a_2 = a > 0$, $b_1 = b > 0$ and $b_2 = \beta b$ with $\beta \geq 1$. Denote by D_j and L_j the aggregate deposits and loans in region j, respectively. We maintain the assumption that all loans are financed through deposits so that $L_j = D_j$ for $j = 1, 2$. Let θ_j^0 denote risk-shifting in the SME of market j. It follows that $\theta_1^0 < \theta_2^0$ because $\beta \geq 1$.

In the integrated market with $D = D_1 + D_2$ and $L = L_1 + L + 2$, the inverse deposit supply and loan demand functions are given by (30) and (31). The probability of success function of each borrower is given by $p(L) = p(\theta(r(L))) = L/2\lambda(1+\beta)b$, and

$$q(L) \equiv q(\theta(r(L))) = \left(\frac{L}{2\lambda(1+\beta)b}\right)^2 + \sigma\left(\frac{L}{2\lambda(1+\beta)b}\right)\left(1 - \frac{L}{2\lambda(1+\beta)b}\right).$$

We find that $q'(L) > 0$. Each bank i in the integrated market solves

$$\max_{L_i} q(L)\left(1 - \frac{L}{(1+\beta)b} - \frac{L}{2a}\right)L_i. \tag{35}$$

The following proposition analyzes the implications of risk-diversification benefits of market integration.

Proposition 10 *Let θ^* be the risk-shifting in the IME and θ_j^0 be the risk-shifting in the SME in market j, $j = 1, 2$. If $\beta \geq 1$, SME risk-taking in market 2 is always greater than that in the IME, (i.e., $\theta^* < \theta_2^0$). Moreover, there is a unique strictly decreasing function, $\tilde{\beta}(\sigma)$ with $\tilde{\beta}(1) > 1$ so that $\theta^* > (\leq)\, \theta_1^0$ according as $\beta > (\leq)\, \tilde{\beta}(\sigma)$.*

Increases in σ indicate higher correlation between returns and this reduces diversification benefits. Reducing diversification benefits reduces bank lending and increases loan rates. Put differently, a greater the diversification benefit (lower σ) tends to lower the loan rate.

Assuming that $\beta \geq 1$ implies, as shown above, that the bank-customer effect with respect to market 1 (market 2) is positive (negative). In market 2, the negative bank-customer effect reinforces the negative bank-competitor effect. Morever, increases in the diversification benefit also tends to lower the loan rate. As a result, we obtain $\theta^* < \theta_2^0$. In market 1, the positive bank-customer effect opposes the negative bank-competitor effect. We show that the threshold at which the positive bank-customer effect outweighs the negative bank-competitor effect depends on the diversification benefit, as measured by σ. The threshold function $\tilde{\beta}(\sigma)$ is decreasing in σ because the greater the diversification benefit (lower σ tending to lower loan rates), the larger is the threshold that the bank-customer effect needs to exceed for the IME loan rate to exceed the SME loan rate in market 1. To summarize, in a model incorporating imperfect correlations between default risks across markets, the relation between competition and risk-taking in the SME can still be reversed in the IME, albeit with a higher threshold.

6.3. Interbank lending

Next, we extend the linear model in Section 5 to include an interbank market. In particular, we assume that banks trade funds at the interbank rate, ρ, which is a policy variable chosen by the central bank (Freixas and Rochet, 2008, p. 79). The key assumption here is that all banks take ρ as given irrespective of whether markets are segmented or integrated. Under this assumption, we show below that banks' maximization problems for the SME and the IME are isomorphic.

We also show that the bank-customer effect of market integration is significant in determining the loan rate and risk-taking even when banks have this alternative funding source. Moreover, there are additional implications of interbank transactions for the bank-competitor effect of market integration as we describe below.

6.3.1. Interbank market and the SME

The model setup is similar to that in Section 5.1. Although individual banks operate in only one of m segmented markets, they face the same interbank rate, ρ, chosen by the central bank, where $\rho \in [0, \bar{\rho}]$ where $\bar{\rho} \leq \lambda$. Let the probability of success be given by $p(\theta) = 1 - \theta/\lambda$. Bank i in market j solves

$$\max_{\{L_{ij}, D_{ij}\}} P(L_j/b_j)[(r(L_j/b_j) - \rho)L_{ij} + (\rho - R(D_j/a_j)D_{ij}] \tag{36}$$

where $L_j = \sum_{i=1}^{n_j} L_{ij}$ and $D_j = \sum_{i=1}^{n_j} D_{ij}$. We show below that this problem is isomorphic to that for the IME. It follows that their solutions are similar as described below.

6.3.2. Interbank market and the IME

As in Section 4.2, we consider the integration of a subset of segmented markets, J_m, where $J_m = \{1, 2, \ldots, m\}$ and $|J_m| = m < |J|$. Banks are allowed to transact with each other at the interbank rate ρ set by the central bank. With (inverse) deposit supply and loan demand given by (18) and (20), respectively, bank i solves

$$\max_{\{L_i, D_i\}} P(L/b(m))[(r(L/b(m)) - \rho)L_i + (\rho - R(D/a(m))D_i], \tag{37}$$

where $L = \sum_{i=1}^{n(m)} L_i$ denotes aggregate loans and $D = \sum_{i=1}^{n(m)} D_i$ denotes aggregate deposits in the integrated market, J_m. The proposition below summarizes the effect of increased competition under the IME on loan rates and risk-shifting in the presence of interbank lending.

Proposition 11 *The following hold for the IME with an interbank market:*

(a) *If $n(m)$ is invariant to m (i.e., the bank-competitor effect is set to zero), the bank-customer effect is positive (negative) according as $\hat{b}(m) > (<)\, \hat{a}(m)$.*

(b) *If $a(m)$ and $b(m)$ are invariant to m (i.e., the bank-customer effect is set to zero), there is a unique threshold $\rho^* \in [0, \bar{\rho}]$ such that the bank-competitor effect is negative (positive) according as $\rho < (>)\, \rho^*$. Therefore, r and θ are decreasing (increasing) in m according as the interbank rate ρ is low (high).*

The bank-competitor and the bank-customer effects comprise the aggregate effect of market integration on the equilibrium loan rate and risk-shifting. It follows that the overall effect of market integration on the loan rate, and consequently, risk-taking is indeterminate.

Proposition 11 extends the results in Proposition 3 to the IME with an interbank market. Proposition 11(a) shows that a non-zero bank-customer effect exists even with interbank lending. Moreover, loan rate and risk-taking increase under market integration if this bank-customer effect is positive and sufficiently strong.

Proposition 11(b) shows that the bank-competitor effect is no longer unambiguously negative in the presence of interbank lending. To demonstrate this, we simplify by setting the bank-customer effect to zero so that any changes in m yield changes only in the number of banks, n. From the first-order conditions of the maximization problem in (37), we obtain the following relation between loan and deposit rates in the symmetric equilibrium (see Appendix for details):

$$R = \frac{\rho n}{n+1}, \tag{38}$$

$$b(n+2)(\lambda - r)^2 - b(\lambda - \rho)(n+1)(\lambda - r) = aR(\rho - R). \tag{39}$$

Condition (38) helps pin down the deposit rate as an increasing function of the number of banks, n. An increase in the number of banks, n implies a greater volume of aggregate deposits, and hence, a higher deposit rate in equilibrium (Allen and Gale, 2004). On the other hand, condition (39) reveals that the equilibrium loan rate depends on both the number of banks, n, and additionally, on the deposit rate, R. The dependence of the equilibrium loan rate on the deposit rate stems from the fact that the probability of default is endogenous in our framework (Dermine, 1986).[44] Differentiating (39), we obtain

$$\frac{dr}{dn} = \frac{\partial r}{\partial n} + \frac{\partial r}{\partial R} \cdot \frac{dR}{dn}. \tag{40}$$

From (39), it follows $\partial r/\partial n < 0$ and $\partial r/\partial R > 0$. Increasing the number of banks decreases individual banks' market power, which tends to raise deposit rates and lower loan rates. Ceteris paribus, increases in deposit costs pass-through to loan rates. The bank-competitor effect on loan rates comprises a negative direct effect—the first term on the right-hand side of (40), and a positive indirect effect that operates through deposit rates—the second term on the right-hand side of (40). Increasing the interbank rate dampens the negative direct effect but strengthens the positive indirect effect.[45] Thus, at low interbank rates, the negative direct effect dominates and the overall bank-competitor effect is negative. At high interbank rates, on the other hand, the positive indirect effect dominates and the overall bank-competitor effect is positive.

The full effect of market integration comprises the bank-competitor and bank-customer effects. Unlike the results in Sections 4.2 and 5, the bank-competitor effect is no longer unambiguously negative. With interbank lending, both bank-competitor and bank-customer effects can be positive or negative depending on the parameters of the model. It follows that the effect of market integration on loan rates and risk-taking can be positive or negative. Moreover, any existing relationship between competition and risk-taking in the SME can be reversed in the IME with a countervailing and sufficiently strong bank-customer effect. We illustrate this result in terms of the numerical example below. Example 2 confirms the robustness of our baseline findings in Propositions 5 and 6 in the presence of interbank lending.

Example 2 Let $a(m) = m^{\alpha}$, $b(m) = m^{\beta}$ and $n(m) = m^{\nu}$ with α, b, $\nu \in [0, 1]$. We allow the number of integrated markets, m, to vary between 2 and 10. We set $\alpha = \nu = 0.3$, $\lambda = 4$ and $\rho = 2.4$. Figure 6 depicts the equilibrium loan rate $r(m)$ as a function of the number of integrated markets. In the left panel, $r(m)$ is drawn for $\beta = 0.3$, thereby setting the bank-customer effect to zero. In this case, the equilibrium loan rate decreases under market integration because of the negative bank-competitor effect. The right panel depicts the reversal because of a positive and sufficiently strong bank-customer effect by setting $\beta = 0.6 > 0.3 = \alpha$. As a result, $r(m)$ is now increasing in m. ■

[44] Loan and deposit rates are independent of each other in the presence of strategic competition in both loan and deposit markets, and interbank lending as long as the probability of default is exogenous. Setting $p(\theta) = p^0$ in each bank's maximization problem yields the equilibrium loan rate, $r^* = (\lambda + \rho n)/(n+1)$, and the equilibrium deposit rate, $R^* = \rho n/(n+1)$. Notably, the loan rate is monotonically decreasing and the deposit rate is monotonically increasing in the number of banks, n.

[45] More formally, we obtain that $\partial((\partial r/\partial n))/\partial\rho < 0$ and $\partial((\partial r/\partial R)(dR/dn))/\partial\rho > 0$.

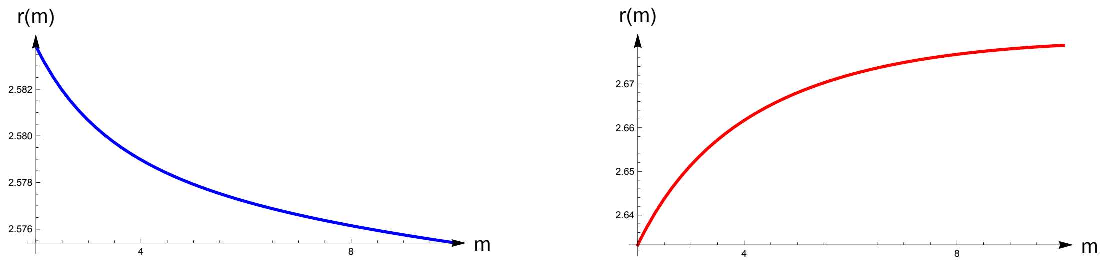

Figure 6: *In the presence of interbank lending, the negative association between loan rates and market integration in the left panel is driven by negative bank-competitor effect and a bank-customer effect set to zero. A strong and countervailing (positive) bank-customer effect reverses this association as shown in the right panel.*

7. Conclusion

The lack of consensus among studies examining the linkages between bank competition and risk-taking poses a significant challenge for policymakers and academics. Our model provides one possible reason behind the mixed empirical evidence on this question. In the context of integrating banking markets, we find that the effect of increased competition extends beyond that explained by a simple increase in the number of rival banks. Importantly, we point to a risk-incentive mechanism, namely the bank-customer effect of market integration, that can potentially reverse *any* observed association between bank competition and risk-taking under within-market competition. Moreover, while the predictions of existing theory are shown to prevail under specific conditions within our generalized framework, they are not the only outcomes of the model.

To the best of our knowledge, this paper is among the first to directly examine the effect of market integration on risk-taking. While future modeling efforts can include richer settings that improve our understanding about the linkages between competition and stability in banking, we have opted for a tractable approach. Even in this parsimonious setting with homogenous agents, the richer set of results demonstrate the need for a broader definition of increased competition that captures the evolution of competition. Most theory on bank competition tends to focus on changes in the number of competitor banks within a limited geographic area. Our results suggest the need for reexamining the idea that bank competition is confined to local individual markets.

Appendix A: Proofs

Proof of Proposition 1. We first characterize the maximization problem of the borrower. We omit the subscript j from the loan rate. Let $\phi(k) \equiv f(k)/k$ be the average product of investment. The first-order condition of (3) is given by

$$\underbrace{\left\{y(\theta) + y'(\theta) \cdot \frac{p(\theta)}{p'(\theta)}\right\}}_{h(\theta)} f(k) - rk = 0 \quad \Longleftrightarrow \quad h(\theta) = \frac{r}{\phi(k)}, \tag{41}$$

which defines $\theta = \theta(k; r)$. Note that the objective function of the maximization problem (3) is strictly concave on $\theta \in [0, \bar{\theta}]$ because $p(\theta) \geq 0$, $p'(\theta) < 0$, $p''(\theta) < 0$, $y(\theta) \geq 0$, $y'(\theta) > 0$ and $y''(\theta) < 0$, and hence, $\theta(k; r)$ is unique. Also, because $y''(\theta) \leq 0 < y'(\theta)$, $p'(\theta) < 0$ and

$p''(\theta) \leq 0$, we have

$$h'(\theta) = 2y'(\theta) + \frac{p(\theta)}{p'(\theta)} \left\{ y''(\theta) - \frac{y'(\theta)p''(\theta)}{p'(\theta)} \right\} \geq 2y'(\theta) > 0. \tag{42}$$

Differentiating (41) with respect to k and r, respectively, we obtain

$$\theta_k(k;\, r) = \frac{r\varepsilon_\phi(k)}{h'(\theta)k\phi(k)} = \frac{r[1-\varepsilon(k)]}{h'(\theta)k\phi(k)} > 0, \quad \text{and} \quad \theta_r(k;\, r) = \frac{1}{h'(\theta)\phi(k)} > 0, \tag{43}$$

where $\varepsilon_\phi(k) \equiv -k\phi'(k)/\phi(k)$ is elasticity of the average product, $\phi(k)$ with respect to k, and $\varepsilon(k) \equiv kf'(k)/f(k)$ is the output elasticity of investment. It is immediate to show that $\varepsilon_\phi(k) = 1 - \varepsilon(k) > 0$. The last inequality holds because $f(0) \geq 0$ and $f(k)$ is strictly concave.[46]

Now, let $U(k;\, r)$ be the value function of the maximization problem (3). Then, by the Envelope theorem, we have $U_k(k;\, r) = p(\theta)\{y(\theta)f'(k) - r\}$, and hence, the first-order condition of (5) is given by:

$$U_k(k;\, r) = 0 \quad \Longrightarrow \quad y(\theta)f'(k) = r. \tag{44}$$

The second-order necessary condition is given by

$$\begin{aligned} & p(\theta)\{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)\} + p'(\theta)\theta_k\{\underbrace{y(\theta)f'(k) - r}_{=0 \text{ by } (44)}\} \leq 0 \\ \Longrightarrow \quad & \underbrace{y'(\theta)\theta_k f'(k) + y(\theta)f''(k)}_{\Omega(k,r)} \leq 0. \end{aligned} \tag{45}$$

On the other hand, differentiating (44) with respect to r, and using the expression of θ_r from (43), we obtain

$$k'(r) = \frac{1 - (y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,\, r)}. \tag{46}$$

Observe that $h'(\theta) \geq 2y'(\theta)$ implies that $y'(\theta)/h'(\theta) \leq 1/2$. Therefore, the numerator of the last expression is strictly positive because $\varepsilon(k) < 1$. On the other hand, the denominator is negative by the second-order condition. Consequently, $k'(r) \leq 0$.

To show the last part of the proposition, note that

$$\frac{d\hat{\theta}}{dr} = \theta_r + \theta_k \cdot k'(r) = \frac{1}{h'(\theta)\phi(k)} + \frac{r\varepsilon_\phi(k)}{h'(\theta)k(r)\phi(k)} \cdot k'(r) = \frac{1}{h'(\theta)\phi(k)} \left\{ 1 + \varepsilon_\phi(k) \cdot \frac{rk'(r)}{k(r)} \right\}.$$

The above condition is an equilibrium condition where $k = k(r)$. Let $\eta(r) \equiv -rk'(r)/k(r)$ be the elasticity of the individual loan demand function. Because $h'(\theta),\, \phi(k) > 0$, from the last expression of $d\hat{\theta}/dr$, it follows that

$$\text{sign}[d\hat{\theta}/dr] = \text{sign}[1 - \eta(r)\varepsilon_\phi(k)].$$

[46] Note that, for any production function $f(k)$, strictly concave, the average product, $\phi(k)$ is strictly decreasing in k. Moreover, $\phi'(k) < 0$ is equivalent to $\varepsilon(k) < 1 \iff \phi(k) > f'(k)$. We show that $\varepsilon(k) < 1$ for a twice differentiable production function $f(k)$ that is strictly concave with $f(0) \geq 0$. Take any point $(k_0,\, f(k_0))$ on the graph of $f(k)$. Then, there is $\kappa \in (0,\, k_0)$ so that

$$\phi(k_0) \equiv \frac{f(k_0)}{k_0} \geq \frac{f(k_0) - f(0)}{k_0} = f'(\kappa) > f'(k_0).$$

The first (weak) inequality follows from the fact that $f(0) \geq 0$, the second equality follows from the Mean Value theorem, and the last (strict) inequality is implied by $f''(k) < 0$ and $\kappa < k_0$. This proves that $\varepsilon(k) < 1$ as k_0 has been chosen arbitrary.

The above condition is intuitive. Recall that the indirect effect works as follows. An increase in r decreases k because $k'(r) < 0$. This decrease in k increases $\phi(k)$ because $\phi'(k) < 0$, which, given (41), decreases $\hat{\theta}$. Clearly, the strength of the indirect effect depends on both the responsiveness of k with respect to r, i.e., $\eta(r)$, and that of $\phi(k)$ with respect to k, i.e., $\varepsilon_\phi(k)$. Thus, if $\eta(r)\varepsilon_\phi(k)$ is low (high), i.e., less (greater) than 1, the direct effect dominates (is dominated by) the indirect effect, and hence, risk-taking increases (decreases) with loan rate.

To prove that $d\hat{\theta}/dr > (<)\, 0$ according as $\varepsilon'(k) < (>)\, 0$, note that

$$\varepsilon'(k) = \frac{d}{dk}\left(\frac{kf'(k)}{f(k)}\right) = \frac{kf''(k) + f'(k)[1-\varepsilon(k)]}{f(k)} \tag{47}$$

Note that

$$\begin{aligned} \frac{d\hat{\theta}}{dr} &= \underbrace{\frac{r[1-\varepsilon(k)]}{h'(\theta)f(k)}}_{\theta_k} \cdot \underbrace{\frac{1-(y'(\theta)/h'(\theta))\varepsilon(k)}{\Omega(k,r)}}_{k'(r)} + \underbrace{\frac{k}{h'(\theta)f(k)}}_{\theta_r} \\ &= \frac{r[1-\varepsilon(k)][h'(\theta) - y'(\theta)\varepsilon(k)] + h'(\theta)k\Omega(k,r)}{[h'(\theta)]^2 f(k)\Omega(k,r)} \\ &\equiv \frac{Q}{[h'(\theta)]^2 f(k)\Omega(k,r)}. \end{aligned} \tag{48}$$

Using the expressions of θ_k, $\Omega(k,r)$, and the fact that $r = y(\theta)f'(k)$ form (44), we obtain

$$Q = h'(\theta)y(\theta)\left\{kf''(k) + f'(k)[1-\varepsilon(k)]\right\} = h'(\theta)y(\theta)f(k)\varepsilon'(k).$$

The last equality follows from (47) Therefore,

$$\frac{d\hat{\theta}}{dr} = \frac{y(\theta)\varepsilon'(k)}{h'(\theta)\Omega(k,r)},$$

which implies that

$$\text{sign}[d\hat{\theta}/dr] = \text{sign}[1-\eta(r)\varepsilon_\phi(k)] = -\,\text{sign}[\varepsilon'(k)]$$

because $y(\theta)$, $h'(\theta)$, $f'(k) > 0$ and $\Omega(k,r) \le 0$. This completes the proof of the proposition.

Examples in Table 1. For all the examples below, we assume that $p(\theta) = 1-\theta$, $y(\theta) = \theta$.

1. Consider $f(k) = k(1-k)$ defined on $[0, 1/2]$ so that $f'(k) > 0$. For this functional form, $\varepsilon(k)$ decreases with k. In this case, equilibrium risk-shifting is strictly increasing in r_j and is given by:
$$\theta^*(r_j) = \frac{1}{4}\left(1 + \sqrt{1+8r_j}\right).$$

2. Consider $f(k) = \sqrt{k_0 + k}$ with $k_0 > 0$ an $k \ge 0$. In this case, the elasticity of investment is increasing in k, i.e., $\varepsilon'(k) > 0$. The equilibrium risk-shifting is given by:
$$\theta^*(r_j) = \frac{1}{2} + \frac{\sqrt{2}\left(1 - 24k_0r_j^2 + \sqrt{1-12k_0r_j^2}\right)}{12\left(1 - 6k_0r_j^2 + \sqrt{1-12k_0r_j^2}\right)^{1/2}}.$$

 The above expression is decreasing in r_j.

3. Finally, let $f(k) = k^\delta$ with $\delta \in (0,1)$. In this case, $\varepsilon(k) = \delta$ for all k. The optimal risk-

taking is given by $\theta^*(r_j) = (2-\delta)^{-1}$, which is independent of the loan rate.

Proof of Lemma 1. The proof of this Lemma is identical to that of Lemma 2 below. Replace a, b, n and D_i in the proof of Lemma 2 respectively by a_j, b_j, n_j and D_{ij} to obtain the result.

Proof of Proposition 2. This proposition can be obtained as a special case of Proposition 3. Replace $a(m)$, $b(m)$, $n(m)$ and D_i respectively by a_j, b_j, n_j and D_{ij}, and set $\hat{a}(m) = \hat{b}(m) = 0$ in the proof of Proposition 2 to obtain the result.

Proof of Lemma 2. We suppress for the time being the argument m from $a(m)$, $b(m)$ and $n(m)$. Let $P(L/b) \equiv p(\theta^*(r(L/b)))$. With $L = \sum_{i=1}^n D_i = D$, the first-order condition of the maximization problem of bank i in the integrated market J_m is given by:

$$\begin{aligned}
&P(L/b)[r(L/b) - R(L/a)] + L_i[r(L/b) - R(L/a)]P'(L/b)\cdot\frac{1}{b}\\
&= P(L/b)L_i\left(R'(L/a)\cdot\frac{1}{a} - r'(L/b)\cdot\frac{1}{b}\right)\\
\iff L_i &= \frac{P(L/b)[r(L/b) - R(L/a)]}{P(L/b)\left(R'(L/a)\cdot\frac{1}{a} - r'(L/b)\cdot\frac{1}{b}\right) - [r(L/b) - R(L/a)]P'(L/b)\cdot\frac{1}{b}} \quad \text{for all } i.
\end{aligned}$$

Because the right-hand side of the above condition depends only on the aggregate loans, L, it immediately follows that $L_i = L_{i'}$ for any $i \neq i'$. Therefore, there are no asymmetric equilibria. In the (symmetric) IME, $L_i = L/n$ for all i. Thus, the above optimality condition boils down to:

$$\mu(L, a, b) - F(L, a, b, n) = 0, \tag{49}$$

where $\mu(L, a, b) \equiv r(L/b) - R(L/a)$ is the equilibrium intermediation margin, and

$$F(L, a, b, n) \equiv \frac{[bR'(L/a) - ar'(L/b)]\,P(L/b)L}{a\,[nbP(L/b) + P'(L/b)D]}.$$

The second order necessary condition is given by:

$$\mu_L(L, a, b) - F_L(L, a, b, n) < 0. \tag{50}$$

The equilibrium loan rate is given by $r(L/b)$, and the equilibrium risk-shifting is given by $\theta^*(r(L/b))$.

Proof of Proposition 3. The first-order condition (49) can be written as

$$\mu(L, a, b) = F(L, a, b, n) \equiv \frac{\zeta(L, a, b)}{\xi(L, b, n)},$$

where $\zeta(L, a, b) \equiv (b/a)R'(L/a) - r'(L/b) > 0$, and $\xi(L, b, n) \equiv \frac{bn}{L} + \eta(L/b)$ with $\eta(L/b) \equiv P'(L/b)/P(L/b)$. Because the optimal risk-shifting of the entrepreneurs may be increasing or decreasing in the loan rate, the sign of $P'(L/b)$ is indeterminate. If $P'(L/b) > 0$, then $\xi(L, b, n)$ is also positive. Therefore, we would assume that $\xi(L, b, n) \geq 0$ if $P'(L/b) < 0$. Note that

$$\mu_L = r'(L/b)\cdot\frac{1}{b} - R'(L/a)\cdot\frac{1}{a} < 0, \quad \mu_a = R'(L/a)\cdot\frac{L}{a^2} > 0, \quad \text{and} \quad \mu_b = -r'(L/a)\cdot\frac{L}{b^2} > 0.$$

On the other hand, from the expression of $F(L, a, b, n)$, we obtain

$$\begin{aligned}
F_L &= \frac{1}{\xi}\left[\frac{bR''(L/a)}{a^2} - \frac{r''(L/b)}{b} + F(L, a, b, n)\left(\frac{bn}{L^2} - \frac{\eta'(L/b)}{b}\right)\right], \\
F_a &= -\frac{b}{\xi a^2}\left(R''(L/a)(L/a) + R'(L/a)\right), \\
F_b &= \frac{1}{\xi}\left[\frac{R'(L/a)}{a} + \frac{Lr''(L/b)}{b^2} + F(L, a, b, n)\left(\frac{L\eta'(L/b)}{b^2} - \frac{n}{L}\right)\right], \\
F_n &= -\frac{\zeta b}{\xi^2 L} < 0.
\end{aligned}$$

Note that $F_L - \mu_L > 0$ by (50). Moreover, and $F_a < 0$ because $R''(L/a) \geq 0$, and hence, $\mu_a - F_a > 0$. Lastly, it is immediate to verify that

$$1 - \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)} = \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)}.$$

Totally differentiating the first-order condition (49), we obtain

$$\frac{dL}{L} = -\frac{nF_n}{L(F_L - \mu_L)} \cdot \frac{dn}{n} + \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} \cdot \frac{da}{a} + \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)} \cdot \frac{db}{b}.$$

Because $r = r(L/b)$, we have

$$\begin{aligned}
dr &= -r'(L/b)(L/b)\left(\frac{db}{b} - \frac{dL}{L}\right) \\
&= -r'(L/b)(L/b)\left\{\frac{nF_n}{L(F_L - \mu_L)} \cdot \frac{dn}{n} + \underbrace{\left(1 - \frac{b(\mu_b - F_b)}{L(F_L - \mu_L)}\right)}_{= \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)}}\frac{db}{b} - \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)} \cdot \frac{da}{a}\right\}. \qquad (51)
\end{aligned}$$

Because $\hat{n}(m) \equiv n'(m)/n(m)$, $\hat{a}(m) \equiv a'(m)/a(m)$ and $\hat{b}(m) \equiv b'(m)/b(m)$, it follows from (51) that

$$\frac{dr}{dm} = -r'(L/b)(L/b)\left[\frac{nF_n}{L(F_L - \mu_L)} \cdot \hat{n}(m) + \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)}\left(\hat{b}(m) - \hat{a}(m)\right)\right].$$

Define

$$Z_n \equiv -r'(L/b)(L/b) \cdot \frac{nF_n}{L(F_L - \mu_L)} \quad \text{and} \quad Z_c \equiv -r'(L/b)(L/b) \cdot \frac{a(\mu_a - F_a)}{L(F_L - \mu_L)}.$$

Because $r'(\cdot) < 0$, $F_n < 0$ and $F_L - \mu_L > 0$, we have $Z_n < 0$. On the other hand, $Z_c > 0$ as $r'(\cdot) < 0$, $F_L - \mu_L > 0$ and $\mu_a - F_a > 0$. This completes the proof of the Proposition.

Proof of Lemma 3. If $p(\theta) = 1 - \theta$ and $y(\theta) = \theta$, then $h(\theta) = 2\theta - 1$. The first-order condition (4) of the maximization problem (3) implies that

$$\theta(k; r) = \frac{1}{2}\left\{1 + \frac{rk}{f(k)}\right\}.$$

Then, (44) reduces to

$$\frac{1}{2}\left\{1+\frac{rk}{f(k)}\right\} f'(k) = r \quad \Longleftrightarrow \quad r = \frac{f(k)f'(k)}{2f(k)-kf'(k)}.$$

Note that $k = \lambda - r \iff r = \lambda - k$ is equivalent to

$$\underbrace{\frac{f(k)f'(k)}{2f(k)-kf'(k)}}_{r} = \lambda - k \quad \Longleftrightarrow \quad f'(k) = \frac{2f(k)(\lambda-k)}{f(k)+k(\lambda-k)} \equiv \Phi(k, f(k)). \tag{52}$$

Let $f(k_0) = z_0$ such that $(k_0, z_0) \in [0, \bar{k}] \times \mathbb{R}$. Observe that both $\Phi(k, z)$ and $\Phi_z(k, z)$ are continuous functions. The second assertion is true because

$$\Phi_z(k, z) = \frac{2z(\lambda-k)^2}{\{z+k(\lambda-k)\}^2}$$

whose denominator is different from zero if $z_0 > 0$ which implies that $\Phi_z(k, z)$ is a bounded function, and hence, $\Phi(k, z)$ is Lipschitz continuous in z. Therefore, there exists a unique $f(k)$ on $(0, \bar{k})$ which satisfies (52) (see Simmons, 2017, Theorem B, p. 634).

Proof of Proposition 6. With $R = D/a$ and $r = \lambda - (L/b)$, we have $\theta(L/b) = \lambda - (L/2b)$ and $P(L/b) \equiv 1 - \theta(L/b) = L/(2\lambda b)$. Therefore in the integrated market, bank i's objective function, using $L = D$, becomes

$$\frac{L}{2\lambda b}\left\{\lambda - \frac{L}{b} - \frac{L}{a}\right\} L_i.$$

The first-order condition of the above maximization problem with respect to L_i is given by:

$$\lambda L - \left(\frac{1}{a}+\frac{1}{b}\right) L^2 + L_i \left\{\lambda - 2\left(\frac{1}{a}+\frac{1}{b}\right) L\right\} = 0. \tag{53}$$

In the symmetric equilibrium, we have $L_i = L/n$ for all i. Substituting this into (53), and solving for $L > 0$ yields

$$L = \frac{\lambda ab(n+1)}{(a+b)(n+2)}.$$

Thus, the equilibrium risk-shifting of the integrated market is given by:

$$\theta(L/b) = \lambda - \frac{L}{2b} = \lambda\left\{1 - \frac{a(n+1)}{2(a+b)(n+2)}\right\}.$$

Substituting $n \equiv n(m) = \frac{1}{2}m(m+1)$, $a \equiv a(m) = m^{\alpha}$ and $b \equiv b(m) = m^{\beta}$ into the above expression, we obtain (22). Let $\gamma \equiv \beta - \alpha$. Because $0 \le \alpha, \beta \le 1$, we have $-1 \le \gamma \le 1$. Differentiating the expression in (22) with respect to m we obtain

$$\frac{d\theta}{dm} = \frac{\lambda m^{\gamma-1}(2+m+m^2)}{2(1+m^{\gamma})^2(4+m+m^2)} \underbrace{\left(\gamma - \frac{2m(1+2m)(1+m^{-\gamma})}{(2+m+m^2)(4+m+m^2)}\right)}_{h(m;\gamma)}. \tag{54}$$

Therefore, $\text{sign}[d\theta/dm] = \text{sign}[h(m;\gamma)]$. Note first that $\gamma \le 0$ or $\beta \le \alpha$ implies that $h(m;\gamma) < 0$ for all $m \ge 2$, and hence, $d\theta/dm < 0$. Next, consider $\gamma \in (0, 1]$. In this case, the bank-customer effect is positive. Note that

$$h_\gamma(m;\,\gamma) = 1 + \frac{2m^{1-\gamma}(1+2m)(\log m)}{(2+m+m^2)(4+m+m^2)} > 0 \quad \text{for all } \; m \geq 2,$$

$h(m;\,\gamma)$ is strictly increasing in θ for all $m \geq 2$. Write $h(m;\,\gamma) = \gamma - l(m)g(m;\,\gamma)$, where

$$l(m) \equiv \frac{2m(1+2m)}{(2+m+m^2)(4+m+m^2)} \quad \text{and} \quad g(m;\,\gamma) \equiv 1 + m^{-\gamma}.$$

Note that $l(m) > 0$ and $g(m;\,\gamma) > 0$ for all $m \geq 2$ and $\gamma > 0$. Clearly, $l(m)$ is strictly decreasing in m, that is, $l'(m) < 0$ for all $m \geq 2$, and $g(m;\,\gamma)$ is strictly decreasing in m for all $\gamma > 0$ as $g_m(m;\,\gamma) = -\gamma m^{-(1+\gamma)} < 0$. Therefore, $h_m(m;\,\gamma) = -(l'(m)g(m;\,\gamma) + l(m)g_m(m;\,\gamma)) > 0$. It can also be shown that $h(m;\,\gamma)$ is strictly concave in m. Next, note that

$$\lim_{m\to\infty} h(m;\,\gamma) = \gamma.$$

The above expression is equal to 0 for $\gamma = 0$, that is, $\beta = \alpha$, and is strictly positive for any $\gamma > 0$,that is, $\beta > \alpha$. Finally,

$$h(2;\,\gamma) = \gamma - \frac{1}{4}\left(1 + 2^{-\gamma}\right).$$

Because $h(2;\,\gamma)$ is strictly increasing in γ, $h(2;\,0) = -0.5$ and $h(2;\,0.5) \approx 0.073$, by the Intermediate Value theorem, there is a unique $x \in (0,\,0.5)$ such that $h(2;\,x) = 0$. It turns out that $x \approx 0.435$. Therefore, for any $\gamma \in (0,\,x)$, $h(m;\,\gamma) < 0$ if and only if $m < \widehat{m}(\gamma)$. Note that, for any $\gamma \in (0,\,x)$, we have $h(2;\,\gamma) < 0$ and $\lim_{m\to\infty} h(m;\,\gamma) = \gamma > 0$, and hence, the Intermediate Value theorem guarantees the existence of $\widehat{m}(\gamma)$. Because $h(m;\,\gamma)$ is strictly increasing in m for all $\gamma > 0$, $\widehat{m}(\gamma)$ is unique. Finally, $\widehat{m}'(\gamma) < 0$ as $h(m;\,\gamma)$ is strictly increasing in γ. Finally, for any $\gamma > x$, we have $h(m;\,\gamma) > 0$ for all $m \geq 2$. Figure 7 summarizes the above discussion.

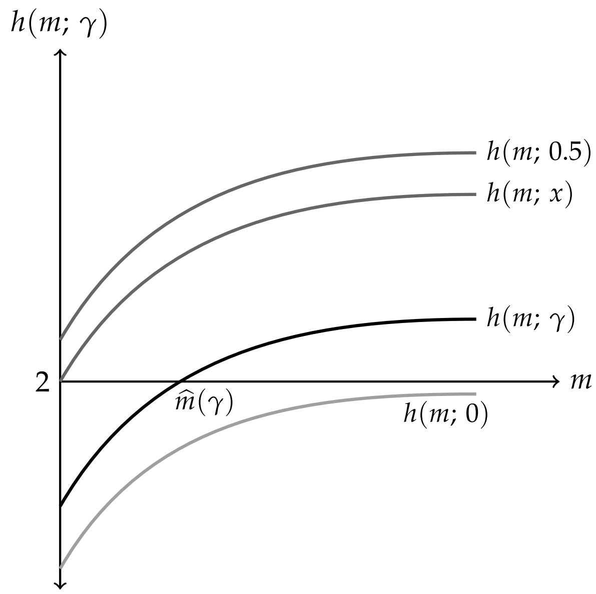

Figure 7: *For $\theta = 0$, $h(m;\,\gamma) < 0$, and hence, $d\theta/dm < 0$. For any $\gamma \in (0,\,x)$, $h(m;\,\gamma)$ intersects the horizontal axis at a unique point $\widehat{m}(\gamma)$, that is, $h(m;\,\gamma) < (>)\, 0$ if $m < (>)\, \widehat{m}(\gamma)$, and hence, $\theta(m)$ is U-shaped with respect to m. For any $\gamma \in (x,\,1]$, $h(m;\,\gamma) > 0$, and hence, $d\theta/dm > 0$.*

Proof of Lemma 4. For market j, we have $R_j(L_j) = L_j/a_j$ and $r_j(L_j) = \lambda - L_j/b_j$, and hence, $P_j(L_j) = L_j/2\lambda b_j$ where $L_j = \sum_{i=1}^{n_j} L_{ij}$ is the aggregate loans in market j. Each bank i in market j solves

$$\max_{L_{ij}} P_j(L_j)[r_j(L_j) - R_j(L_j) - c_j]L_{ij}.$$

In the symmetric equilibrium, we have $L_{ij} = L_j/n_j$. The first-order condition in the symmetric equilibrium is given by

$$\frac{L_j}{2\lambda b_j}\left(\lambda - \frac{L_j}{b_j} - \frac{L_j}{a_j} - c_j\right) + \frac{L_j}{n_j} \cdot \frac{(\lambda - c_j)a_j b_j - 2(a_j + b_j)L_j}{2\lambda a_j b_j^2} = 0.$$

Solving the above equation, we obtain

$$L_j^0 = \frac{(\lambda - c_j)a_j b_j(n_j+1)}{(a_j+b_j)(n_j+2)}. \tag{55}$$

The equilibrium risk-shifting is given by

$$\theta_j^0 = \lambda - \frac{L_j^0}{2\lambda b_j} = \lambda - \frac{(\lambda - c_j)a_j(n_j+1)}{2(a_j+b_j)(n_j+2)}. \tag{56}$$

Substituting $a_1 = a_2 = a$, $b_1 = b$, $b_2 = \beta b$, $n_1 = n_2 = n$, $c_1 = 0$ and $c_2 = c$ in the last expression, we obtain θ_1^0 and θ_2^0 in Lemma 4. Note that

$$\theta_2^0 - \theta_1^0 = \frac{a(n+1)[(a+b)c + \lambda b(\beta - 1)]}{2(a+b)(a+\beta b)(n+2)}.$$

The above expression is strictly positive for $\beta \geq 1$, and hence, $\theta_2^0 > \theta_1^0$.

Proof of Lemma 5. The detailed calculations for the proof are provided in Appendix B. We provide here only the sketch of the proof. We first prove part (a). Consider any market with parameters $\lambda > 1$, $A > 0$, $B > 0$, $n \geq 2$ and $z \in \{0, c\}$ (each bank's constant marginal cost). Using the expression in (55), the equilibrium aggregate loans are given by

$$L = \frac{(\lambda - z)AB(n+1)}{(A+B)(n+2)}$$

which yields the (common) expected profit for each bank:

$$\pi = \underbrace{\frac{L}{2\lambda B}}_{P(L)} \underbrace{\left(\lambda - \frac{L}{b} - \frac{L}{a} - z\right)}_{r(L)-R(L)-z} \cdot \frac{L}{n}.$$

The above is the common equilibrium profit level of each bank when there are n banks, each with marginal cost z. Let $\pi_j(n_l, n_h)$ be the equilibrium profit of each type $j = 1, 2$ bank when there are n_l low-cost and n_h high-cost banks in the market. So, in a market consisting of only low-cost bank, we have $z = 0$, and the common profit level is given by $\pi_1(n, 0)$. On the other hand, for a high-cost market, we have $z = c$, and the common profit level, $\pi_2(0, n)$. If any pair of banks merge, then the market consists of $n-1$ identical banks. So, for a two-bank merger to be profitable we require that $\pi_1(n-1, 0) > \pi_1(n, n) + \pi_1(n, n) = 2\pi_(n, n)$ (if market 1 is considered), or $\pi_2(0, n-1) > 2\pi_2(0, n)$ (if market 2 is considered) so that each bank in the merged entity obtains at least a profit strictly higher than $\pi_1(n, 0)$ (or $\pi_2(0, n)$), say $\frac{1}{2}[\pi_1(n-1, 0) - 2\pi_1(n, 0)]$ (or $\frac{1}{2}[\pi_2(0, n-1) - 2\pi_2(0, n)]$). We show that the above two strict inequalities does not hold if there are at least 3 banks in a given market, i.e., $n \geq 3$, and it holds only for $n = 2$, i.e., the merger must result in a monopoly banking.

The proof of part (b) is similar. If there are no mergers, then each type-j bank consumes $\pi_j(n, n)$. If two low-cost banks merge, each in the merged entity (as well as each of the $n-2$ same type rivals) obtains $\pi_1(n-1, n)$. On the other hand, if two high-cost banks merger, each in the merged entity (as well as each of the $n-2$ same type rivals) obtains $\pi_2(n, n-1)$. So, for the merger of two low-cost banks to be profitable, we require $\pi_1(n-1, n) > 2\pi_1(n, n)$, and for the merger of two high-cost banks to be profitable, we require $\pi_2(n, n-1) > 2\pi_2(n, n)$. We show that none of the last two inequalities holds if we have $n \geq 2$, i.e., there are at least two banks of each cost type (the requirement to form a merger between two banks of the same type).

Proof of Lemma 6. Note first that in the integrated market, we have $p(\theta) = 1 - \theta/\lambda$ and $r(L) = \lambda - L/(1+\beta)b$, which together imply that the probability of success as a function of the aggregate loan volume is given by $P(L) = L/2(1+\beta)b\lambda$. Moreover the equilibrium must respect $L = L_1 + L_2 = D$. The IME is characterized by within-group symmetry, i.e., $L_{ij} = L_j/n$ because there are n each type of banks. Therefore, the first-order condition of each type-j bank i yields

$$(L_1+L_2)\left(\lambda - \left(\frac{1}{2a} + \frac{1}{(1+\beta)b}\right)(L_1+L_2)\right) + \frac{L_1}{n}\left(\lambda - \left(\frac{1}{a} + \frac{2}{(1+\beta)b}\right)(L_1+L_2)\right) = 0,$$
$$(L_1+L_2)\left(\lambda - c - \left(\frac{1}{2a} + \frac{1}{(1+\beta)b}\right)(L_1+L_2)\right) + \frac{L_2}{n}\left(\lambda - c - \left(\frac{1}{a} + \frac{2}{(1+\beta)b}\right)(L_1+L_2)\right) = 0.$$

The above system of non-linear equations have three sets of solutions in (L_1, L_2). The first set is discarded because it has $L_1 = L_2 = 0$. The second set is also discarded because $L_1 < 0$ for $\lambda > 1$, a, $b > 0$, $\beta \geq 1$, $n \geq 2$ and $0 < c \leq \lambda$. The final expressions for L_1 and L_2 in the third set of the solutions are very cumbersome, so we omit them. The third set of solutions (L_1, L_2) yields

$$L^* = \frac{ab(1+\beta)\left\{(2\lambda - c)(3n+2) + \sqrt{c^2(3n+2)^2 + 4\lambda(\lambda - c)n^2}\right\}}{4(2a + (1+\beta)b)(n+1)}.$$

It turns out that $L_2^* > 0$ if $c < \lambda/(n+2) \equiv \bar{c}$. The expression of the equilibrium risk-shifting is obtained from

$$\theta^* = \lambda - \frac{L^*}{2(1+\beta)b}.$$

Proof of Proposition 7. The detailed calculations are in Appendix B. It is easy to show that $\theta_2^0 > \theta^*$ under $\lambda > 1$, a, $b > 0$, $\beta \geq 1$, $n \geq 2$ and $0 < c < \bar{c}$. The above holds because $L^* > L_2^0$. We next compare θ_1^0 and θ^*. Write θ^* as $\theta^*(\beta, c)$. It is immediate to show that $\theta^*(\beta, c)$ is strictly increasing in both the arguments. On the other hand, t_1^0 depends neither on β nor on c. Consider $\theta_1^0 - \theta^*(\beta, c) = 0$ which defines implicitly c as a function of β. Call this implicit function $\tilde{c}(\beta)$. Differentiating $\theta_1^0 - \theta^*(\beta, \tilde{c}(\beta)) = 0$ with respect to β we obtain

$$\tilde{c}'(\beta) = -\frac{\partial\theta^*/\partial\beta}{\partial\theta^*/\partial c}.$$

Because both the partial derivatives are positive, we have $\tilde{c}'(\beta) < 0$. In fact, $\tilde{c}(\beta)$ is given by

$$\tilde{c}(\beta) \equiv \frac{2\lambda[(a+\beta b)n + (\beta-1)b][(a+b)n - b(\beta-1)(n+1)^2}{(a+b)(n+2)[b(\beta-1)(5n+2) + 2a + b(3\beta-1)n^2]}.$$

It is immediate to see that $\tilde{c}(1) = \frac{\lambda}{n+2} \equiv \bar{c}$. On the other hand, $\lim_{\beta\to\infty} \tilde{c}(\beta) = -\infty$. However, c can take only non-negative values; so, we restrict attention to the function $c^*(\beta) = \max\{0, \tilde{c}(\beta)\}$. The threshold bank-customer effect is given by $\tilde{c}(\beta) = 0$ which yields

$$\bar{\beta} \equiv 1 + \frac{(a+b)n}{b(n+1)^2}.$$

This completes the proof of the proposition.

Proof of Lemma 7. Calculations for the first part is in Appendix B where we show that $\pi_M > \pi_1^* + \pi_2^*$. Note that, in a merger between a low-cost bank and a high-cost bank, the gain to the merged entity is $\pi_M - (\pi_1^* + \pi_2^*)$. Under the following surplus division rule

$$\tilde{\pi}_1 = \pi_1^* + \frac{1}{2}[\pi_M - (\pi_1^* + \pi_2^*)] \quad \text{and} \quad \tilde{\pi}_2 = \pi_2^* + \frac{1}{2}[\pi_M - (\pi_1^* + \pi_2^*)],$$

both banks gain because $\tilde{\pi}_1 > \pi_1^*$ and $\tilde{\pi}_2 > \pi_2^*$, and hence, such mergers are profitable. The expressions for the aggregate deposits and risk-shifting follow from (21) and (22) by substituting a by $2a$, and b by $(1+\beta)b$.

Proof of Proposition 8. Note that

$$\theta_1^0 - \theta_M = -\frac{\lambda ab(\beta-1)(n+1)}{2(a+b)(2a+(1+\beta)b)(n+2)} \leq 0 \quad \text{for } \beta \geq 1,$$
$$\theta_2^0 - \theta_M = \frac{a[(2a+(1+\beta)b)c + \lambda b(\beta-1)](n+1)}{2(a+\beta b)(2a+(1+\beta)b)(n+2)} > 0 \quad \text{for } \beta \geq 1,$$

which prove the proposition.

Proof of Proposition 9. Note that

$$\theta_M - \theta^* = \frac{a}{8(2a+(1+\beta)b)(n+1)}\left[\sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda-c)} - \left(c(3n+2) + \frac{2\lambda n^2}{n+2}\right)\right].$$

So,

$$\begin{aligned}
&\theta_M > \theta^* \\
\iff &\sqrt{c^2(3n+2)^2 + 4n^2\lambda(\lambda-c)} > c(3n+2) + \frac{2\lambda n^2}{n+2} \\
\iff &\lambda - c > \lambda\left(\frac{n}{n+2}\right)^2 + \frac{c(3n+2)}{n+2} \\
\iff &c < \frac{\lambda(n+2)}{4(n+1)}\left[1 - \left(\frac{n}{n+2}\right)^2\right] = \frac{\lambda}{n+2} \equiv \bar{c}.
\end{aligned}$$

Proof of Proposition 10. We provide the sketch of the proof. All the calculations are in Appendix B. The IME aggregate loans, $L^*(\beta, \sigma)$ is determined from the first-order condition of each bank's maximization problem evaluated at $L_i = L/2n$, which is given by

$$q(L)(r(L) - R(L)) + \frac{L}{2n}\cdot\frac{d}{dL}[q(L)(r(L) - R(L))] = 0.$$

The IME risk-shifting is given by $\theta^*(\beta,\sigma) = \lambda - \frac{L^*(\beta,\sigma)}{2(1+\beta)b}$. We next show that $\theta^*(\beta,\sigma)$ are strictly increasing in both arguments. We next compare $\theta_1^0 - \theta^*(\beta,\sigma)$. To prove the existence of a unique $\tilde{\beta}(\sigma)$ for each σ, we proceed as follows. We first show that $\theta^*(1,\sigma) > 0$ and $\lim_{\beta\to\infty}\theta^*(\beta,\sigma) < 0$. Hence, $\tilde{\beta}(\sigma)$ exists. Because θ_1^0 does not depend on β, $\theta_1^0 - \theta^*(\beta,\sigma)$ is strictly decreasing in β as $\theta^*(\beta,\sigma)$ is strictly increasing in β. Therefore, $\tilde{\beta}(\sigma)$ is unique for each σ, and is defined by

$$\theta_1^0 - \theta^*(\tilde{\beta}(\sigma),\sigma) = 0.$$

Differentiating the above with respect to σ we obtain

$$\tilde{\beta}'(\sigma) = -\frac{\partial\theta^*/\partial\sigma}{\partial\theta^*/\partial\beta} < 0$$

because both the numerator and the denominator are strictly positive. It is also easy to show that $\theta_2^0 > \theta^*(\beta,\sigma)$ for all (β,σ) (see Appendix B). This completes the proof of the proposition.

Proof of Proposition 11. To save on notations, write $a \equiv a(m)$, $b \equiv b(m)$ and $n \equiv n(m)$. Given that $R(D/a) = D/a$ and $r(L/b) = \lambda - (L/b)$, the first-order conditions associated with the maximization problem (37) with respect to L_i and D_i, under symmetry, i.e., $L_i = L/n$ and $D_i = D/n$ for all i, are given by

$$a(n+2)L^2 - ab(\lambda-\rho)(n+1)L = b(a\rho - D)D, \tag{57}$$
$$a\rho n = (n+1)D, \tag{58}$$

respectively. Solving the above system, we obtain the aggregate loans, L and deposits, D in equilibrium. Using $L = b(\lambda - r)$ and $D = aR$, we get (38) and (39). The equilibrium loan rate is thus given by:

$$r(a,b,n,\rho) = \lambda - \frac{L}{b} = \lambda - \frac{b(\lambda-\rho)(n+1)^2 + \sqrt{b^2(\lambda-\rho)^2(n+1)^4 + 4ab\rho^2 n(n+2)}}{2b(n+1)(n+2)}. \tag{59}$$

The equilibrium deposit rate, on the other hand, is given by:

$$R(n,\rho) = \frac{D}{a} = \frac{\rho n}{n+1}.$$

Note first that our assumption of $m \geq 2$ implies that $\min\{n(m)\} = n(2) \geq 2$. We require that $R(n,\rho) \leq \rho \leq r(a,b,n,\rho)$. The first inequality holds for any $n \geq 2$. The second inequality is written as

$$g(\rho) \equiv \frac{b(\lambda-\rho)^2}{a\rho^2} \geq \frac{n}{(n+1)^2}.$$

The last inequality is equivalent to

$$\rho \leq \frac{\lambda b(n+1)}{b(n+1)+\sqrt{an}} \equiv \bar{\rho} \leq \lambda.$$

Note that $\bar{\rho}$ is increasing in n and $\lim_{n\to\infty}\bar{\rho} = \lambda$.

We first prove part (a). Fix $n(m) = \bar{n}$ so that $n'(m) = 0$, i.e., the bank-competitor effect is zero. Note that

$$\frac{\partial r}{\partial a} = -\frac{n\rho^2}{(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2+4abn(n+2)\rho^2}} < 0,$$
$$\frac{\partial r}{\partial b} = \frac{an\rho^2}{b(n+1)\sqrt{b^2(n+1)^4(\lambda-\rho)^2+4abn(n+2)\rho^2}} = -\frac{a}{b}\cdot\frac{\partial r}{\partial a} > 0.$$

Then, differentiating r with respect to m, we obtain

$$\frac{dr}{dm} = \frac{\partial r}{\partial a}\cdot a'(m) + \frac{\partial r}{\partial b}\cdot b'(m) = \left\{ b'(m) - \frac{b(m)}{a(m)}\cdot a'(m) \right\} \frac{\partial r}{\partial b}$$

The above expression is positive (negative) according as $\hat{b}(m) \gtrless \hat{a}(m)$, i.e., the bank-customer effect is positive (negative).

We next prove part (b). By implicitly differentiating (38), we obtain

$$\frac{\partial r}{\partial R} = \frac{a(2R-\rho)}{b\{2(\lambda-r)(n+2)-(\lambda-\rho)(n+1)\}} > 0,$$
$$\frac{\partial r}{\partial n} = -\frac{(\lambda-r)(r-\rho)}{2(\lambda-r)(n+2)-(\lambda-\rho)(n+1)} < 0.$$

Using the expression of r in (59), it is easy to show that the denominator of each of the above two expressions is positive. On the other hand, from (39), it follows that

$$\frac{dR}{dn} = \frac{\rho}{(n+1)^2} = \frac{R}{n(n+1)} > 0.$$

Note that the sign of the bank-competitor effect is completely determined by that of dr/dn, which is given by:

$$\frac{dr}{dn} = \underbrace{\frac{\partial r}{\partial n}}_{\text{direct effect}} + \underbrace{\frac{\partial r}{\partial R}\cdot\frac{dR}{dn}}_{\text{indirect effect}}.$$

We first show that both the terms in the right-hand-side of the above expression is increasing in ρ. Substituting the equilibrium values of r and R into the expression of $\partial r/\partial n$, and differentiating with respect to ρ we obtain the following:[47]

$$\operatorname{sign}\left[\frac{\partial}{\partial\rho}\left(\frac{\partial r}{\partial n}\right)\right]$$
$$= \operatorname{sign}\left[1+\phi(a,b,n,\rho)\right],$$

where

$$\phi \equiv \frac{b^2(\lambda-\rho)^3(n+1)^7+8a^2\rho^3n^2(n+2)^2+2ab\rho(\lambda-\rho)n(n+1)^3(n+2)(\lambda(n+5)+(\lambda-\rho)(n-1))}{b^{\frac{1}{2}}(b(\lambda-\rho)^2(n+1)^4+4a\rho^2n(n+2))^{\frac{3}{2}}},$$

which is strictly positive. Clearly, $\partial(\partial r/\partial n)/\partial\rho < 0$, i.e., the negative direct effect dampens as ρ increases. As far as the indirect effect is concerned, using $R = \rho n/(n+1)$ in $\partial r/\partial R$ and dR/dn, and differentiating the indirect effect with respect to ρ, we obtain

$$\operatorname{sign}\left[\frac{\partial}{\partial\rho}\left(\frac{\partial r}{\partial R}\cdot\frac{dR}{dn}\right)\right] = \operatorname{sign}\left[b(\lambda-\rho)(2\lambda-\rho)(n+1)^4 - 2a\rho^2n(n+2)(n-3)\right],$$

[47] We omit the unmanageable algebraic expressions. The *Mathematica* codes are available upon request.

which is also positive for any $a > 0$, $b > 0$, $\lambda > 0$ and $0 < \rho \leq \lambda$. Therefore, dr/dn is strictly increasing in ρ. Let $f(\rho)$ denote dr/dn evaluated at ρ. Note that

$$f(0) = \frac{\lambda(b+1)\{b(n+1)-(n+3)\}}{4b(n+2)^2} \quad \text{and} \quad f(\lambda) = \frac{\lambda^4 a^2 bn\{n(n+1)+b(n^2+n-2)\}}{2(n+1)^2[\lambda^2 abn(n+2)]^{3/2}} > 0.$$

The sign of $f(0)$ ambiguous. There are the following possible cases. First, if $f(0) \geq 0$, i.e., $b \geq \frac{n+3}{n+1}$, then $f(\rho) \geq 0$ for all $\rho \geq 0$. Then, set $\rho^* = 0$. Thus, the equilibrium loan rate is increasing for all possible values of the interbank rate. Next, consider the case when $f(0) < 0$ i.e., $b < \frac{n+3}{n+1}$. Because $f(\lambda) > 0$ and $f'(\rho) > 0$, it follows from the Intermediate Value theorem, there is a unique $\rho^* \in (0, \lambda)$ such that $dr/dn < 0$ if and only if $\rho < \rho^*$. If $\rho^* \geq \bar{\rho}$, then set $\rho^* = \bar{\rho}$, i.e., $dr/dn < 0$ for all $\rho \in [0, \bar{\rho}]$. Otherwise, $\rho^* < \bar{\rho}$, and hence, the bank-competitor effect is positive if and only if $\rho > \rho^*$. This case is more likely for large $n(m)$ because $\bar{\rho} \to \lambda$ and $n \to \infty$ and $f(\lambda) > 0$. This completes the proof of part (b).

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