Variable Elasticity Demand and Inflation Persistence

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October 2016; updated April 2018
RWP 16-09
https://dx.doi.org/10.18651/RWP2016-09
This paper supersedes the old version:
"Price Dispersion and Inflation Persistence"
Variable Elasticity Demand and Inflation Persistence*

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April 2018

Abstract

We propose a novel explanation for the well-known persistent inflation response to monetary policy shocks by introducing variable elasticity demand curves in a staggered price model with trend inflation. The demand curves induce strategic complementarity in price setting and thus generate inflation persistence under positive trend inflation through the effect on inflation dynamics of a measure of price dispersion, which differs from relative price distortion. We also show that credible disinflation leads to a gradual decline in inflation and a fall in output and that lower trend inflation reduces inflation persistence, as observed around the time of the Volcker disinflation.

JEL Classification: E31, E52

Keywords: Inflation persistence; Variable elasticity demand curve; Trend inflation; Price dispersion; Credible disinflation

*The authors are grateful to Susanto Basu, Brent Bundick, Olivier Coibion, Ferre De Graeve, Michael Dotsey, Mark Gertler, Andrew Foerster, Jeffrey Fuhrer, Michal Marencak, Juan Rubio-Ramírez, Argia Sbordone (discussant), Lee Smith, Raf Wouters, and participants at 2017 North American Summer Meeting of the Econometric Society, 25th Annual Symposium of the Society for Nonlinear Dynamics and Econometrics, Spring 2017 Midwest Macroeconomics Meeting, Federal Reserve System Committee Meeting on Macroeconomics, Summer Workshop on Economic Theory 2017, and seminars at the Federal Reserve Bank of Kansas City, the National Bank of Belgium, and the University of Kansas for comments and discussions. The views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan, the Federal Reserve Bank of Kansas City, or the Federal Reserve System.

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1 Introduction

An extensive empirical literature has documented the persistent response of inflation to monetary policy shocks (e.g., Christiano, Eichenbaum, and Evans, 1999, 2005; Boivin, Kiley, and Mishkin, 2011). Figure 1 displays impulse responses of the federal funds rate, inflation, output, and labor productivity to an expansionary policy shock in an estimated structural vector autoregression (VAR). The figure illustrates that inflation rises during some quarters following the shock to a peak level and then returns gradually to the pre-shock level. In addition, both output and labor productivity rise after the shock.

To account for the empirical evidence, most of the previous studies have assumed intrinsic inertia of inflation in dynamic stochastic general equilibrium (DSGE) models. Popular sources of the inertia are price indexation to past inflation (Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007) and backward-looking price setters (Galí and Gertler, 1999). However, the assumption of intrinsic inflation inertia remains controversial for at least three reasons. First of all, it is an ad hoc assumption. Second, the price indexation implies that all prices change in every period, which contradicts the micro evidence that many individual prices remain unchanged for several months, as argued by Woodford (2007). Third, Benati (2008) demonstrates that the degree of inflation persistence varies with monetary policy regimes, and thus concludes that inflation inertia may not be intrinsic.

There have been a few exceptions that use DSGE models to explain inflation persistence without assuming inflation is intrinsically inertial. Mankiw and Reis (2002) develop a sticky information model to account for the persistent response of inflation to monetary policy shocks. Dupor, Kitamura, and Tsuruga (2010) introduce sticky information in a staggered price model of Calvo (1983) and find that lagged inflation emerges endogenously in the model-induced Phillips curve. A similar finding is also obtained by Sheedy (2010), who instead

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1 Woodford (2007) and Fuhrer (2011) review different theories of intrinsic inertia in inflation.

2 Galí and Gertler (1999) suggest that “it is worth searching for explanations of inflation inertia beyond the traditional ones that rely heavily on arbitrary lags” (p. 219).

3 Fuhrer (2011) discusses the distinction between “intrinsic” versus “inherited” persistence in inflation.

4 Mankiw and Reis (2002) indicate that “[t]he key empirical fact that is hard to match, however, is not the high autocorrelations of inflation, but the delayed response of inflation to monetary policy shocks” (p. 1311).
Notes: The solid lines are impulse responses obtained by estimating a structural VAR during the period 1955:Q1–2008:Q4 using the federal funds rate, the log difference of the GDP deflator, and the logs of real GDP, output per hour in the business sector, and the KR-CRB spot commodity price index (all commodities). The interest and inflation rates are annualized. The lag length of the VAR is six quarters as determined by the AIC. A history of monetary policy shocks is recovered from the error terms under the identifying assumption that no variables of the model, except for the federal funds rate and the commodity price index, respond contemporaneously to such a shock. The dashed lines are 90 percent confidence intervals obtained from 1,000 bootstrap replications of the VAR.
incorporates an upward-sloping hazard function in the Calvo model so that prices that have remained fixed for longer are more likely to be changed. Cogley and Sbordone (2008) embed drifting trend inflation in a Calvo model with price indexation only to past inflation under the assumption of subjective expectations based on the anticipated utility model of Kreps (1998). They empirically show no role of the price indexation and thus conclude that the intrinsic inflation inertia is not needed for the model to explain U.S. inflation dynamics in the presence of the drifting trend inflation.

Our paper proposes a novel explanation for inflation persistence, using a Calvo staggered price model with constant trend inflation. This model potentially generates a persistent response of inflation to a monetary policy shock, because the model-implied inflation dynamics can be affected by relative price distortion, which is intrinsically inertial under the staggered price setting, and thus inflation possibly inherits persistence from the distortion.\(^5\) A plausibly calibrated version of the model, however, shows that relative price distortion induces little persistence in the response of inflation to policy shocks. The model also incorporates a fixed cost of production so that the production technology exhibits increasing returns to scale. Nevertheless, labor productivity presents a counterfactual decline after an expansionary policy shock, since output rises less than labor input due to an increase in relative price distortion following the shock.

In the model with trend inflation, our paper introduces variable elasticity demand curves for goods along the lines of Kimball (1995), Dotsey and King (2005), Levin et al. (2008), Shirota (2015), and Kurozumi and Van Zandweghe (2016).\(^6\) The variable elasticity demand curves induce strategic complementarity in price setting and cause the model-implied in-

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\(^5\)This is the case if labor input is homogeneous; if it is firm-specific, there is no effect of relative price distortion on the model-implied inflation dynamics, as pointed out by Kurozumi and Van Zandweghe (2017). Damjanovic and Nolan (2010) show that a decreasing returns to scale production technology with only homogeneous labor as an input, along with a long average duration of price change of two years, amplifies relative price distortion and makes it more persistent, thus generating a persistent response of inflation to a monetary policy shock. At the same time, however, their model generates a counterfactual decline in output after an expansionary policy shock. Therefore, they indicate that “further work is required to understand this and reconcile it with how one typically thinks the economy responds to such a shock” (p. 1096).

\(^6\)In a state-dependent pricing model, Dotsey and King (2005) examine implications of variable elasticity demand curves for inflation and output persistence and find that the demand curves raise persistence.
flation dynamics to be influenced by a measure of price dispersion under positive trend inflation. This dispersion is intrinsically inertial under the staggered price setting and differs from relative price distortion, which coincides with demand dispersion in the model.

In our model, inflation shows a persistent response to an expansionary monetary policy shock, with a hump shape and a gradual decline, as documented by the empirical literature. This response is inherited from the aforementioned price dispersion, which exhibits a hump-shaped response to the shock and rises with inflation, consistent with the empirical finding of Sheremirov (2015) that the correlation between inflation and the dispersion of regular prices is positive. An economic intuition for the inflation response is as follows. In response to the expansionary policy shock under positive trend inflation, firms increase their products’ prices for profit maximization if they can change them. The variable elasticity demand curves induce larger elasticity of demand for goods with higher relative prices and thereby dampen the price increases. This strategic complementarity in price setting under positive trend inflation causes inflation to display the persistent response with the hump shape and the gradual decline.

In the presence of the variable elasticity demand curves, relative price distortion shows a muted response to the expansionary policy shock (under positive trend inflation). This is because the demand curves lead to smaller elasticity of demand for goods with lower relative prices as well as larger elasticity of demand for goods with higher relative prices, and thereby keep demand dispersion—which coincides with the relative price distortion—from increasing in response to the shock. This finding is consistent with that of Nakamura et al. (2017), who indicate little sensitivity of relative price distortion—“inefficient price dispersion” in their terms—to changes in inflation, using the BLS micro-data on consumer prices. Owing to the muted response of relative price distortion and the increasing returns to scale production technology arising from the fixed cost, both output and labor productivity rise after the expansionary policy shock in the model, as in the VAR illustrated in Figure 1.

This paper also contributes to the literature on credible disinflation. As Fuhrer (2011) points out, intrinsic inertia of inflation plays a key role in canonical New Keynesian (NK) models, where a credible permanent reduction in trend inflation induces a gradual adjustment

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7See, e.g., Ball (1994), Fuhrer and Moore (1995), and Mankiw and Reis (2002).
of inflation to its new trend rate and a decline in output. These responses align closely with historical experiences; for instance, they are reminiscent of the U.S. economy’s evolution around the time of the Volcker disinflation. Without the intrinsic inertia, inflation jumps to its new trend rate, while output never deviates from its steady-state value. However, in our model, credible disinflation leads to a gradual decline in inflation and a fall in output even in the absence of intrinsic inertia in inflation.

The paper further shows that lower trend inflation reduces inflation persistence, by extending the model so that staggered wage setting and variable elasticity demand curves for labor are also incorporated. A number of empirical studies indicate that inflation persistence has decreased in the U.S. since the early 1980s, around the time of the Volcker disinflation. The leading explanation for the measured decrease in inflation persistence since the 1980s involves a more active monetary policy response to inflation. Thus our extended model provides a novel explanation: the fall in trend inflation caused inflation persistence to decrease. Under the staggered wage setting, the real wage is intrinsically inertial and thus becomes an additional source of inflation persistence. Then, under positive trend inflation, the variable elasticity demand curves for labor cause the model-implied wage dynamics to be influenced by a measure of wage dispersion, which is also intrinsically inertial and increases with inflation. Owing to these effects, the extended model better accounts for the empirical evidence on U.S. inflation dynamics.

Our results suggest that introducing the variable elasticity demand curves improves Calvo staggered price models with trend inflation, in line with the conclusions of previous studies. The canonical case of constant elasticity demand curves has several drawbacks. Higher trend

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8Empirical studies that find a decrease in inflation persistence in the early to mid 1980s include Cogley and Sargent (2002), Stock and Watson (2007), Cogley, Primiceri, and Sargent (2010), and Fuhrer (2011). Owing to differences in methodology and measures of inflation, some studies find no change in inflation persistence in the post-World War II period (Pivetta and Reis, 2007; Benati, 2008).

9Previous studies examining the source of the change in inflation persistence include Benati and Surico (2008), Carlstrom, Fuerst, and Paustian (2009), Cogley, Primiceri, and Sargent (2010), and Davig and Doh (2014).

10Variable elasticity demand curves have been widely used as a source of strategic complementarity or, more generally, real rigidity in DSGE models; see, e.g., Eichenbaum and Fisher (2007) and Smets and Wouters (2007).
inflation causes not only a larger loss in steady-state output relative to its natural rate—in violation of the natural rate hypothesis—but also a counterfactual weaker relationship between output and inflation, as pointed out by Ascari (2004) and Levin and Yun (2007). Moreover, it leads to higher likelihood of indeterminacy of equilibrium, as indicated by Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011). Once the variable elasticity demand curves are incorporated, the violation of the natural rate hypothesis becomes minor and the indeterminacy is largely prevented, as shown by Kurozumi and Van Zandwegrhe (2016), while lower trend inflation weakens the relationship between output and inflation, as demonstrated by Shirota (2015).

The remainder of the paper proceeds as follows. Section 2 presents a Calvo staggered price model with constant trend inflation, variable elasticity demand curves for goods, and a fixed cost of production. Section 3 shows that a plausibly calibrated version of the model can explain the empirical evidence presented by the VAR, using impulse responses to monetary policy shocks. Section 4 applies the model to an analysis on credible disinflation. Section 5 extends the model by introducing staggered wage setting and variable elasticity demand curves for labor, and demonstrates that lower trend inflation reduces inflation persistence. Section 6 concludes.

2 Model

To account for the empirical evidence presented by the VAR, this paper uses a Calvo staggered price model with constant trend inflation, variable elasticity demand curves for goods, and a fixed cost of production. The model consists of households, a monetary authority, composite-good producers, and firms. A key feature of the model is that each period a fraction of individual goods’ prices remains unchanged even under nonzero trend inflation in

\footnote{For a micro-foundation of variable elasticity demand curves, see Benabou (1988), Heidhues and Koszegi (2008), and Gourio and Rudanko (2014) among others. Benabou develops a model of customer search, where a search cost gives rise to a reservation price above which a customer continues to search for a seller. Heidhues and Koszegi consider customers’ loss aversion, which increases the price responsiveness of demand at higher relative to lower market prices. Gourio and Rudanko construct a model of customer capital, where firms have a long-term relationship with customers whose demand is unresponsive to a low price.}
line with the micro evidence on prices, while the remaining fraction is set by firms that face variable elasticity demand curves. In what follows, the behavior of each economic agent is described in turn.

2.1 Households

There is a representative household that consumes a composite good $C_t$, supplies (homogeneous) labor $N_t$, and purchases one-period riskless bonds $B_t$ so as to maximize the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log C_t \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right)$$

subject to the budget constraint

$$P_tC_t + B_t = P_tw_tN_t + i_{t-1}B_{t-1} + T_t,$$

where $E_t$ denotes the expectation operator conditional on information available in period $t$, $\beta \in (0, 1)$ is the subjective discount factor, $\sigma_n \geq 0$ is the inverse of the elasticity of labor supply, $P_t$ is the price of the composite good, $w_t$ is the real wage, $i_t$ is the gross interest rate on the bonds and is assumed to be equal to the monetary policy rate, and $T_t$ consists of lump-sum taxes and transfers and firm profits received.

Combining the first-order conditions for utility maximization with respect to consumption, labor supply, and bond holdings yields

$$w_t = C_tN_t^{\sigma_n}, \quad (1)$$

$$1 = E_t \left( \frac{\beta C_t}{C_{t+1}} \frac{i_t}{\pi_{t+1}} \right), \quad (2)$$

where $\pi_t = P_t/P_{t-1}$ is the gross inflation rate of the composite good’s price.

2.2 Monetary authority

The monetary authority conducts policy according to a rule of the sort proposed by Taylor (1993). This rule adjusts the interest rate in response to deviations of the inflation rate from its trend rate and allows for policy inertia:

$$\log i_t = \rho \log i_{t-1} + (1-\rho) \left[ \log i + \phi_\pi (\log \pi_t - \log \pi) \right] + \varepsilon_{it}, \quad (3)$$
where $i$ is the gross steady-state interest rate, $\pi$ is the gross trend (or steady-state) inflation rate, $\rho \in [0, 1]$ and $\phi_\pi \geq 0$ represent the degrees of policy inertia and the policy response to inflation, and $\varepsilon_{it}$ is an i.i.d. shock to monetary policy.

### 2.3 Composite-good producers

There are a representative composite-good producer and a continuum of firms $f \in [0, 1]$, each of which produces an individual differentiated good $Y_t(f)$. As in Kimball (1995), the composite good $Y_t$ is produced by aggregating individual goods $\{Y_t(f)\}$ with

$$
\int_0^1 F_p \left( \frac{Y_t(f)}{Y_t} \right) df = 1. \tag{4}
$$

Following Dotsey and King (2005) and Levin et al. (2008), the function $F_p(\cdot)$ is assumed to be of the form

$$
F_p \left( \frac{Y_t(f)}{Y_t} \right) = \frac{\gamma_p}{(1 + \varepsilon_p)(\gamma_p - 1)} \left[ (1 + \varepsilon_p) \frac{Y_t(f)}{Y_t} - \varepsilon_p \right]^{2\nu-1} + 1 - \frac{\gamma_p}{(1 + \varepsilon_p)(\gamma_p - 1)},
$$

where $\gamma_p \equiv \theta_p(1 + \varepsilon_p)$. The parameter $\varepsilon_p$ governs the curvature of the demand curve for each individual good, which is given by $-\varepsilon_p \theta_p$. In the special case of $\varepsilon_p = 0$, the aggregator (4) is reduced to the CES one $Y_t = \left[ \int_0^1 (Y_t(f))^{(\theta_p-1)/\theta_p} df \right]^{\theta_p/(\theta_p-1)}$, where $\theta_p > 1$ represents the elasticity of substitution between individual goods. The general case of $\varepsilon_p < 0$ is of particular interest in this paper. It induces strategic complementarity in price setting through variable elasticity demand curves, as explained below.

The composite-good producer maximizes profit

$$
P_t Y_t - \int_0^1 P_t(f)Y_t(f)df
$$

subject to the aggregator (4), given individual goods’ prices $\{P_t(f)\}$. Combining the first-order conditions for profit maximization and the aggregator (4) leads to

$$
\frac{Y_t(f)}{Y_t} = \frac{1}{1 + \varepsilon_p} \left[ \left( \frac{P_t(f)}{P_t d_{p,t}} \right)^{-\gamma_p} + \varepsilon_p \right], \tag{5}
$$

$$
d_{p,t} = \left[ \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{1-\gamma_p} df \right]^{1-\gamma_p}, \tag{6}
$$

$$
1 = \frac{1}{1 + \varepsilon_p} d_{p,t} + \frac{\varepsilon_p}{1 + \varepsilon_p} e_{p,t}, \tag{7}
$$

9
where \( d_{p,t} \) is the Lagrange multiplier on the aggregator (4), which represents the real marginal cost of producing the composite good and is a measure of price dispersion as shown in (6), and

\[
e_{p,t} \equiv \int_0^1 \frac{P_t(f)}{P^t} df.
\]  

(8)

In the special case of \( \epsilon_p = 0 \), where the aggregator (4) becomes the CES one as noted above, eqs. (5)-(7) can be reduced to \( Y_t(f) = Y_t(P_t(f)/P^t)^{-\theta_p}, P_t = [\int_0^1 (P_t(f))^{1-\theta_p} df]^{1/(1-\theta_p)} \), and \( d_{p,t} = 1 \), respectively.

Figure 2: Variable and constant elasticity demand curves.

Notes: The case of \( \epsilon_p = 0 \), that is, a constant elasticity demand curve is displayed by the dotted line. The case of \( \epsilon_p = -9 \), that is, a variable elasticity demand curve is illustrated by the dashed and the solid lines, which respectively assume a trend inflation rate \( \bar{\pi} \) of zero and 2.5 percent annually. The values of other model parameters used here are reported in Table 1.

Eq. (5) is the demand curve for each individual good \( Y_t(f) \). The price elasticity of demand for the good is then given by

\[
\eta_{p,t} = \theta \left[ 1 + \epsilon_p - \epsilon_p \left( \frac{Y_t(f)}{Y_t} \right)^{-1} \right].
\]
Figure 2 illustrates the demand curve (5) using two values of the curvature parameter, $\epsilon_p = 0$ and $\epsilon_p = -9$. In the case of $\epsilon_p = 0$ (the dotted line), $\eta_{p,t} = \theta_p$, that is, the demand curve is a constant elasticity one. In the case of $\epsilon_p = -9$, the elasticity $\eta_{p,t}$ varies inversely with relative demand $Y_t(f)/Y_t$. That is, relative demand for an individual good becomes more price-elastic for an increase in the relative price of the good, whereas the demand becomes less price-elastic for a decrease in the price. The figure shows the variable elasticity demand curve under a trend inflation rate $\bar{\pi} (\equiv 4\log \pi)$ of zero (the dashed line) and 2.5 percent annually (the solid line), respectively. A rise in trend inflation shifts up the demand curve as it increases steady-state price dispersion $d_p$.

The output of the composite good is equal to the household’s consumption:

$$Y_t = C_t. \quad (9)$$

### 2.4 Firms

Each firm $f$ produces one kind of differentiated good $Y_t(f)$ using the production technology that is given by

$$Y_t(f) = N_t(f) - \phi \quad (10)$$

if $N_t(f) > \phi$; otherwise, $Y_t(f) = 0$, where $\phi > 0$ denotes a fixed cost of production. In the presence of the fixed cost, the production technology exhibits increasing returns to scale. It is assumed throughout the paper that $N_t(f) > \phi$ for each firm $f$ in every period $t$. Then, each firm $f$ minimizes cost $w_t N_t(f)$ subject to the production technology (10), given the real wage $w_t$. The first-order condition for cost minimization shows that each firm’s real marginal cost is identical and equal to the real wage:

$$mc_t = w_t. \quad (11)$$

The labor market clearing condition is given by $N_t = \int_0^1 N_t(f) \, df$. Combining this condition with the demand curve (5) and the production technology (10) yields

$$Y_t = \frac{N_t - \phi}{\Delta_t}, \quad (12)$$

where

$$\Delta_t \equiv \frac{s_t + \epsilon_p}{1 + \epsilon_p} \quad (13)$$
represents relative price distortion and

\[ s_t \equiv \int_0^1 \left( \frac{P_t(f)}{P_t d_{p,t}} \right)^{-\gamma_p} df. \]  

Then, combining eqs. (5), (13), and (14) shows that the relative price distortion coincides with a measure of demand dispersion,

\[ \Delta_t = \int_0^1 \frac{Y_t(f)}{Y_t} df. \]  

Here it is worth noting that the relative price distortion \( \Delta_t \) measures the inefficiency of aggregate production under staggered price setting. Because all firms share the same production technology \( (10) \), if all prices are flexible then all firms produce the same amount, and thus equation (15) demonstrates no relative price distortion \( \Delta_t = 1 \) and the aggregate production equation \( (12) \) implies no inefficiency in producing aggregate output \( Y_t \) using aggregate labor input \( N_t \). On the other hand, staggered price setting generates price dispersion and hence demand dispersion, which increases the inefficiency of aggregate production, that is, the relative price distortion \( \Delta_t \). The relationship between price and demand dispersion then weakens in the presence of the variable elasticity demand curves, as shown later.

In the face of the demand curve \( (5) \) and the real marginal cost \( mc_t \), firms set their products’ prices on a staggered basis as in Calvo \( (1983) \). In each period, a fraction \( \alpha_p \in (0, 1) \) of firms keeps prices unchanged, while the remaining fraction \( 1 - \alpha_p \) of firms sets the price \( P_t(f) \) so as to maximize relevant profit

\[
E_t \sum_{j=0}^{\infty} \alpha_p^j q_{t,t+j} \left( \frac{P_t(f)}{P_{t+j}} - mc_{t+j} \right) \frac{Y_{t+j}}{1 + \epsilon_p} \left[ \left( \frac{P_t(f)}{P_{t+j} d_{p,t+j}} \right)^{-\gamma_p} + \epsilon_p \right],
\]

where \( q_{t,t+j} \) is the stochastic discount factor between period \( t \) and period \( t + j \), which meets the equilibrium condition \( q_{t,t+j} = \beta^j C_t/C_{t+j} \) for the log utility of consumption. Using the composite-good market clearing condition \( (9) \), the first-order condition for profit maximization can be rewritten as

\[
E_t \sum_{j=0}^{\infty} (\alpha_p \beta)^j \left[ \left( \frac{P_t^*}{d_{p,t+j}} \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} \right)^{-\gamma_p} \left( P_t^* \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} - \frac{\gamma_p}{\gamma_p - 1} mc_{t+j} \right) - \frac{\epsilon_p}{\gamma_p - 1} p_t^* \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} \right] = 0,
\]

where \( p_t^* \) is the relative price set by firms that can change prices in period \( t \). Moreover, under
the staggered price setting, eqs. (6), (8), and (14) can be reduced to, respectively,

\[(d_{p,t})^{1-\gamma_p} = \alpha_p \left( \frac{d_{p,t-1}}{\pi_t} \right)^{1-\gamma_p} + (1 - \alpha_p)(p_t^*)^{1-\gamma_p}, \tag{17}\]

\[e_{p,t} = \alpha_p \left( \frac{e_{p,t-1}}{\pi_t} \right) + (1 - \alpha_p)p_t^*, \tag{18}\]

\[(d_{p,t})^{-\gamma_p} s_t = \alpha_p \left( \frac{d_{p,t-1}}{\pi_t} \right)^{-\gamma_p} s_{t-1} + (1 - \alpha_p)(p_t^*)^{-\gamma_p}. \tag{19}\]

The equilibrium conditions consist of (1)–(3), (7), (9), (11)–(13), and (16)–(19). For the steady state to be well defined, it is assumed throughout the paper that the following condition is satisfied:

\[\alpha_p \max(\pi^{\gamma_p}, \pi^{-1}) < 1. \tag{20}\]

This condition is rewritten as \(\alpha_p \max(\pi^{\gamma_p}, \pi^{-1}) < 1\) in the special case of constant elasticity demand curves (i.e., \(\epsilon_p = 0\)), while the condition is always met in the special case of zero trend inflation (i.e., \(\pi = 1\)).

### 2.5 Log-linearized equilibrium conditions

Log-linearizing the equilibrium conditions around the steady state with trend inflation \(\pi\) under the assumption (20) and rearranging the resulting equations yields

\[
\hat{Y}_t = E_t \hat{Y}_{t+1} - (\hat{\iota}_t - E_t \hat{\pi}_{t+1}),
\]

\[\hat{\iota}_t = \rho \hat{\iota}_{t-1} + (1 - \rho) \phi_s \hat{\pi}_t + \epsilon_{\iota t}, \tag{21}\]

\[\hat{m}_{c_t} = \hat{w}_t, \tag{22}\]

\[\hat{w}_t = \hat{Y}_t + \sigma_n \hat{\pi}_t, \tag{23}\]

\[\hat{Y}_t = \left( 1 + \frac{\phi}{Y} \frac{1 + \epsilon_p}{s + \epsilon_p} \right) \hat{\pi}_t - \hat{\Delta}_t, \tag{24}\]

\[\hat{\Delta}_t = \alpha_p \pi^{\gamma_p} \hat{\Delta}_{t-1} + \frac{s}{s + \epsilon_p} \frac{\gamma_p \alpha_p \pi^{\gamma_p-1}(\pi - 1)}{1 - \alpha_p \pi^{\gamma_p-1}} \left( \hat{\pi}_t + \hat{d}_{p,t} - \hat{d}_{p,t-1} \right), \tag{25}\]

\[\hat{\pi}_t = \beta \pi E_t \hat{\pi}_{t+1} + \kappa \hat{m}_{c_t} - \left( \alpha_p \beta \pi^{\gamma_p} + \frac{1}{\alpha_p \pi^{\gamma_p-1}} - \kappa_d \right) \hat{d}_{p,t} + \hat{d}_{p,t-1} + \beta \pi E_t \hat{d}_{p,t+1} + \varphi_t + \psi_t, \tag{26}\]

\[\hat{d}_{p,t} = \frac{\alpha_p \pi^{-1}(1 + \epsilon_p \pi^{\gamma_p})}{1 + \epsilon_p} \hat{d}_{p,t-1} - \frac{\epsilon_p \alpha_p \pi^{-1}(\pi^{\gamma_p} - 1)}{(1 + \epsilon_p)(1 - \alpha_p \pi^{-1})} \hat{\pi}_t, \tag{27}\]

\[\hat{\pi}_t = \beta \pi E_t \hat{\pi}_{t+1} + \kappa \hat{m}_{c_t} - \left( \alpha_p \beta \pi^{\gamma_p} + \frac{1}{\alpha_p \pi^{\gamma_p-1}} - \kappa_d \right) \hat{d}_{p,t} + \hat{d}_{p,t-1} + \beta \pi E_t \hat{d}_{p,t+1} + \varphi_t + \psi_t, \tag{28}\]
\( \varphi_t = \alpha_p \beta_1 \pi^{\gamma_{p}-1} E_t \varphi_{t+1} + \kappa_f \left( E_t \dot{Y}_{t+1} - \dot{Y}_t + \gamma_p E_t \dot{\pi}_{t+1} + \gamma_p (1 - \alpha_p \beta_1 \pi^{\gamma_{p}-1}) E_t \dot{d}_{p,t+1} - \dot{i}_t \right), \) (29)

\( \psi_t = \alpha_p \beta_1 \pi^{\gamma_{p}-1} E_t \psi_{t+1} - \frac{\epsilon_{p2}\beta(\pi^{\gamma_{p}+1}-1)(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})}{\pi^{\gamma_{p}} \left[ \gamma_p - 1 - \epsilon_{p2}(1+\gamma_p) \right]} \left( E_t \dot{Y}_{t+1} - \dot{Y}_t - \dot{i}_t \right), \) (30)

where hatted variables denote log-deviations from steady-state values, \( s \) is the steady state value of \( s_t \) that is given by \( s = (1-\alpha_p)/(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})/(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1}), \)

\( \kappa \equiv \frac{(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})(1-\alpha_p \beta_1 \pi^{\gamma_{p}})}{\alpha_p \pi^{\gamma_{p}-1}[1-\epsilon_{p2}\gamma_p/(\gamma_p - 1 - \epsilon_{p2})]}, \epsilon_{p2} \equiv \epsilon_{p1} \frac{1-\alpha_p \beta_1 \pi^{\gamma_{p}-1}}{1-\alpha_p \beta_1 \pi^{\gamma_{p}-1}}, \epsilon_{p1} \equiv \epsilon_p \left( 1-\alpha_p \beta_1 \pi^{\gamma_{p}-1} \right) \gamma_p/(\gamma_p-1), \)

\( \kappa_d \equiv \frac{\gamma_p(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})}{\alpha_p \pi^{\gamma_{p}-1}[1-\epsilon_{p2}(1+\gamma_p)/(\gamma_p - 1)]} \left( \frac{\gamma_p - 1 - \epsilon_{p2} (1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})}{\gamma_p - 1 - 1-\alpha_p \beta_1 \pi^{\gamma_{p}-1}} - 1 \right), \kappa_f \equiv \frac{\beta(\pi - 1)(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1})}{1-\epsilon_{p2}(1+\gamma_p)/(\gamma_p - 1)}, \)

and \( \varphi_t \) and \( \psi_t \) are auxiliary variables that drive inflation in response to expected changes in future demand and expected discount rates on future profit under nonzero trend inflation.

In particular, \( \psi_t \) is relevant to the variable elasticity demand curves. Eq. (21) is the spending Euler equation, (22) is a Taylor-type monetary policy rule, (23) is the marginal cost equation, (24) is the labor supply equation, (25) is the aggregate production equation, (26) is the law of motion of the relative price distortion \( \hat{\Delta}_t, \) (27) is a generalized NK Phillips curve (GNKPC), (28) is the law of motion of the price dispersion \( \hat{d}_{p,t}, \) and (29) and (30) are forward-looking equations for the auxiliary variables \( \varphi_t \) and \( \psi_t. \)

In the special case of constant elasticity demand curves (i.e., \( \epsilon_{p} = 0 \)), the price dispersion’s law of motion (28) implies that \( \hat{d}_{p,t} = 0, \) and thus there is no role for the dispersion \( \hat{d}_{p,t}. \)

In addition, the auxiliary variable \( \psi_t \)’s equation (30) shows that \( \psi_t = 0. \) Consequently, the aggregate production equation (25), the relative price distortion’s law of motion (26), the GNKPC (27), and the auxiliary variable \( \varphi_t \)’s equation (29) are reduced to, respectively,

\( \dot{Y}_t = \left( 1 + \frac{\phi}{\gamma s} \right) \dot{N}_t - \hat{\Delta}_t, \) (31)

\( \hat{\Delta}_t = \alpha_p \beta_1 \hat{\Delta}_{t-1} \left( \frac{\theta_p \alpha_p \pi^{\theta_{p}-1}(\pi - 1)}{1-\alpha_p \pi^{\theta_{p}-1}} \right) \hat{\pi}_t, \) (32)

\( \hat{\pi}_t = \beta_1 \pi E_t \hat{\pi}_{t+1} \left( \frac{1-\alpha_p \pi^{\theta_{p}-1} (1-\alpha_p \beta_1 \pi^{\gamma_{p}})}{\alpha_p \pi^{\theta_{p}-1}} \right) \hat{m}_t + \varphi_t, \) (33)

\( \varphi_t = \alpha_p \beta_1 \pi^{\gamma_{p}-1} E_t \varphi_{t+1} + \beta(\pi - 1)(1-\alpha_p \beta_1 \pi^{\gamma_{p}-1}) \left( E_t \dot{Y}_{t+1} - \dot{Y}_t + \theta_p E_t \dot{\pi}_{t+1} - \dot{i}_t \right). \) (34)

Therefore, the log-linearized equilibrium conditions are composed of (21)–(24) and (31)–(34).

Moreover, in the special case of zero trend inflation (i.e., \( \pi = 1 \)), the laws of motion of the relative price distortion (26) and the price dispersion (28) imply that \( \hat{\Delta}_t = 0 \) and \( \hat{d}_{p,t} = 0, \)
respectively, and thus both the distortion $\Delta_t$ and the dispersion $\hat{d}_{p,t}$ play no role. Besides, the auxiliary variables’ equations (29) and (30) show that $\varphi_t = 0$ and $\psi_t = 0$, respectively. Therefore, the aggregate production equation (25) is reduced to

$$\dot{Y}_t = \left(1 + \frac{\phi}{\nu}\right) \dot{N}_t,$$

and the GNKPC (27) is reduced to the canonical NK Phillips curve

$$\dot{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha_p)(1 - \alpha_p\beta)}{\alpha_p[1 - \epsilon_p\theta_p/(\theta_p - 1)]} \hat{m}_{ct}.$$

Hence, the log-linearized equilibrium conditions consist of (21)–(24), (35), and (36).

Regarding inflation dynamics in the log-linearized equilibrium conditions (21)–(30), two points are particularly worth noting. First, the relative price distortion $\Delta_t$ has an influence on inflation dynamics under nonzero trend inflation through the aggregate production equation (25), the labor supply equation (24), the marginal cost equation (23), and the GNKPC (27). Thus inflation potentially inherits persistence from the distortion, which is intrinsically inertial under the staggered price setting as shown in the distortion’s law of motion (26).

Second, and more importantly, $\epsilon_p < 0$—which induces strategic complementarity in price setting through the variable elasticity demand curves—causes the price dispersion $\hat{d}_{p,t}$ to have an influence on inflation dynamics under nonzero trend inflation directly through the GNKPC (27) and indirectly through the relative price distortion’s law of motion (26) and the auxiliary variable $\varphi_t$’s equation (29). Hence, inflation possibly inherits persistence from the price dispersion, which is intrinsically inertial under the staggered price setting as shown in the dispersion’s law of motion (28). Therefore, the relative price distortion $\Delta_t$ and the price dispersion $\hat{d}_{p,t}$ are potential sources of persistence in the response of inflation to monetary policy shocks.

### 3 Impulse Response Analysis

This section examines impulse responses to monetary policy shocks in the model presented in the preceding section, and demonstrates that a plausibly calibrated version of the model can account for the empirical evidence the VAR has shown above.
3.1 Calibration of model parameters

The calibration of parameters in the quarterly model is summarized in Table 1. As is common in the literature, we set the subjective discount factor at $\beta = 0.99$; the inverse of the elasticity of labor supply at $\sigma_n = 1$; the probability of no price change at $\alpha_p = 0.75$, which implies that the average frequency of price change is four quarters; and the parameter governing the elasticity of substitution between individual goods at $\theta_p = 10$, which implies a desired price markup of 11 percent. As for the parameter governing the curvature of demand curves, the paper considers two values: $\epsilon_p = -9$ (variable elasticity demand curves) and $\epsilon_p = 0$ (constant elasticity demand curves). The former value then implies a curvature of the demand curves of $-\epsilon_p\theta_p = 90$, which is on the high side but within a wide range found in the literature surveyed by Dossche et al. (2010); we will also consider a smaller curvature in Section 5. The fixed production cost $\phi$ is chosen so that firm profit is zero in steady state. The trend inflation rate is set at 2.5 percent annually, which is the average inflation rate of the personal consumption expenditure (PCE) price index over the period 1985:Q1–2008:Q4.\footnote{To meet the assumption (20) under the calibration, the trend inflation rate needs to be greater than $-1.4$ percent annually when $\epsilon_p = -9$. In the special case of constant elasticity demand curves (i.e., $\epsilon_p = 0$), the rate needs to be less than $+12.2$ percent annually.} The degrees of policy inertia and the policy response to inflation are chosen at $\rho = 0.9$ and $\phi_x = 1.5$, respectively.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$\beta$</td>
<td>Subjective discount factor</td>
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</tr>
<tr>
<td>$\sigma_n$</td>
<td>Inverse of the elasticity of labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>Probability of no price change</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Parameter governing the elasticity of substitution between individual goods</td>
<td>10</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Parameter governing the curvature of demand curves</td>
<td>$-9$ or $0$</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Gross trend inflation rate</td>
<td>$1.025^{1/4}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Degree of policy inertia</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>Degree of the policy response to inflation</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 1: Calibration of parameters in the quarterly model
3.2 Impulse responses to monetary policy shocks

The VAR in Figure 1 demonstrates two empirical facts: (i) inflation shows a persistent response to an expansionary monetary policy shock, with a hump shape and a gradual decline; and (ii) both output and labor productivity rise after the shock. This subsection shows that our model can account for the two empirical facts, using the calibration of parameters presented in Table 1.

We begin by considering the special case of constant elasticity demand curves (i.e., $\epsilon_p = 0$). In this case, the two empirical facts are hard to explain by the model, where the relative price distortion $\hat{\Delta}_t$ has an effect on inflation dynamics (under nonzero trend inflation) but the price dispersion $\hat{d}_{p,t}$ does not, as shown by the log-linearized equilibrium conditions (21)–(24) and (31)–(34). The dashed lines in Figure 3 display the impulse responses to a negative 70 basis points monetary policy shock under a trend inflation rate $\bar{\pi}$ of 2.5 percent annually.\footnote{The size of the monetary policy shock is set equal to one standard deviation of the shock to the federal funds rate in the VAR illustrated in Figure 1.} Regarding the empirical fact (i), inflation jumps about 1.25 percentage points in annualized terms on impact and its response dies out within about two years, as displayed in the top right panel of the figure. While the relative price distortion exhibits a hump-shaped response to the shock as can be seen in the bottom left panel, the gradual decline in the inflation response is caused mainly by the interest rate inertia in the monetary policy rule. This is evident from noting that the inflation response under the positive trend inflation rate is similar to that under zero trend inflation—which is illustrated by the dotted line in the top right panel—and the distortion no longer affects inflation dynamics under zero trend inflation. Therefore, the relative price distortion induces little persistence in the response of inflation to the policy shock (under the positive trend inflation rate).

As for the empirical fact (ii), the starkest implication of the constant elasticity demand curves (i.e., $\epsilon_p = 0$) is that labor productivity falls after the expansionary monetary policy shock as shown in the middle right panel of Figure 3. Such a fall is at odds with the rise in labor productivity in the VAR illustrated in Figure 1. The fall in labor productivity results from an increase in the relative price distortion. The aggregate production equation (31) is
Notes: The figure presents the impulse responses to a negative 70 basis points monetary policy shock under the calibration of model parameters reported in Table 1. The interest and inflation rates are displayed in annualized terms. The solid lines represent the case of $\epsilon_p = -9$, that is, variable elasticity demand curves. The dashed lines show the case of $\epsilon_p = 0$, that is, constant elasticity demand curves, where the price dispersion exhibits no (first-order) response. The latter case is also illustrated by the dotted lines, which instead assume zero trend inflation and thus both the price dispersion and the relative price distortion have no responses.
rewritten as

\[ \hat{Y}_t - \hat{N}_t = \frac{\phi}{Y} \frac{1}{s} \hat{N}_t - \hat{\Delta}_t, \]

which shows that labor productivity \( \hat{Y}_t - \hat{N}_t \) rises with aggregate labor input \( \hat{N}_t \) in the presence of the increasing returns to scale production technology arising from the fixed cost \( \phi > 0 \), while it declines with the relative price distortion \( \hat{\Delta}_t \).\(^{14} \) The effect of the distortion dominates after the initial impact of the shock, as can be seen in the middle right panel of Figure 3. That is, the distortion lowers labor productivity, because it increases the inefficiency of aggregate production and thus aggregate output rises less than aggregate labor input. Therefore, the response of labor productivity (inversely) reflects that of the relative price distortion displayed in the bottom left panel.

Once the variable elasticity demand curves are taken into consideration (i.e., \( \epsilon_p = -9 \)), the model can account for the two empirical facts, as shown by the solid lines in Figure 3. The middle two panels of the figure demonstrate that both output and labor productivity rise after the shock, in line with the empirical fact (ii).\(^{15} \) At the same time, the top right panel illustrates that inflation exhibits a persistent response to the expansionary policy shock, with a hump shape and a gradual decline, as is consistent with the empirical fact (i). While the inflation response reaches a peak after just one quarter in the model, Section 5 will show that extending the model by incorporating staggered wage setting and variable elasticity demand curves for labor further increases inflation persistence.

The difference between the cases of variable elasticity demand curves (the solid lines) and constant elasticity demand curves (the dashed lines) is caused mainly by the presence of the price dispersion \( \hat{d}_{p,t} \), as can be seen in the difference between the log-linearized equilibrium conditions (21)–(30) in the former case and (21)–(24) and (31)–(34) in the latter. The dispersion exhibits a hump-shaped response to the shock, as displayed in the bottom right panel of Figure 3, and has a significant influence on inflation dynamics mainly through the

\(^{14}\) Basu and Fernald (2001) evaluate different explanations for the procyclicality of labor productivity.

\(^{15}\) While output and labor productivity display hump-shaped responses in the VAR illustrated in Figure 1, they do not in our model. Adding habit formation in consumption preferences to the model would generate hump-shaped responses of output and labor productivity and would provide an additional source of inflation persistence. As this is well understood, our paper omits habit formation to clarify its contribution to related literature.
GNKPC (27), where the past, present, and expected future values of the price dispersion drive inflation. Therefore, inflation inherits the response to the policy shock mostly from the price dispersion. An economic intuition for the inflation response is as follows. In response to the expansionary policy shock under positive trend inflation, firms increase their products’ prices for profit maximization if they can change them. The variable elasticity demand curves induce larger elasticity of demand for goods with higher relative prices and thereby reduce the price increases. This strategic complementarity in price setting under positive trend inflation causes inflation to show the persistent response with the hump shape and the gradual decline.

In the presence of the variable elasticity demand curves, the relative price distortion $\Delta_t$ displays a muted response to the expansionary policy shock, as illustrated in the bottom left panel of Figure 3. An economic intuition for the distortion’s response is that the demand curves lead to smaller elasticity of demand for goods with lower relative prices as well as larger elasticity of demand for goods with higher relative prices, and thereby keeps demand dispersion (that is, the relative price distortion) from increasing in response to the shock.¹⁶ Owing to the muted response of relative price distortion and the increasing returns to scale production technology arising from the fixed cost, both output and labor productivity rise after the expansionary policy shock.

4 Credible Disinflation

An alternative approach to evaluate inflation persistence is to examine the effects of disinflation. This section studies the transition from one steady state to another one with lower positive trend inflation in our model, and shows that a credible disinflation leads to a gradual decline in inflation and a fall in output in the model.

Around the time of the Volcker disinflation in the early 1980s, the U.S. economy underwent a gradual decline in inflation and a recession. To account for this evolution, existing

¹⁶Goods produced by price-adjusting firms have a higher relative price, and the larger elasticity lessens a decrease in demand for the goods because of the induced strategic complementarity in price setting. On the other hand, goods produced by non-adjusting firms have lower relative prices, and the smaller elasticity lessens an increase in demand for them.
literature has stressed that intrinsic inertia of inflation plays a key role in canonical NK models. As Fuhrer (2011) points out, when intrinsic inertia of inflation is absent in an NK model, a credible permanent reduction in trend inflation causes inflation to jump to its new trend rate and output to have no deviation from its steady-state value. Once the intrinsic inertia is embedded in the model, the credible disinflation generates a gradual adjustment of inflation to its new trend rate and a temporary decline in output.

The U.S. economy’s evolution around the time of the Volcker disinflation can be explained by our model, where inflation has no intrinsic inertia. To show this, the following experiment is carried out. In period 0, the economy is in steady state with a trend inflation rate of three percent annually. At the start of period 1, trend inflation is reduced suddenly and credibly to two percent annually. For simplicity, it is assumed that there is no policy inertia, i.e., $\rho = 0$. Denote the vector of endogenous state variables in the log-linearized model by $\hat{k}_t = \log k_t - \log k(\pi)$; for instance, $k_t = [\Delta_t, d_{p,t}]'$ in the case of variable elasticity demand curves and $k_t = \Delta_t$ in the case of constant elasticity demand curves. Here $k(\pi)$ denotes the vector of steady-state values of $k_t$, which stresses that these values are functions of trend inflation $\pi$. Because in period 0 all variables are in steady state, in period 1 the lagged endogenous state variables under the new trend inflation rate are given by $\log k(\pi^0) - \log k(\pi^1)$, where $\pi^0 = 1.03^{1/4}$ and $\pi^1 = 1.02^{1/4}$. Then, the solution of the log-linearized model under the trend inflation rate $\pi^1$ is used to compute inflation and output in period $t = 1, 2, 3, \ldots$

Figure 4 displays the responses of inflation and output to the sudden and credible reduction in trend inflation from three to two percent annually, using the calibration of other model parameters reported in Table 1, except for $\rho = 0$. In this figure the dotted lines represent the responses in a canonical NK model with intrinsic inertia of inflation (and constant elasticity demand curves, i.e., $\epsilon_p = 0$), which can be derived by altering our model so that firms which keep prices unchanged in the model instead update prices by fully indexing to recent past inflation as in Christiano, Eichenbaum, and Evans (2005). In this model, inflation declines gradually toward its new trend rate, while output falls temporarily and then rebounds gradually to the initial steady-state value, in line with the responses indicated by

17The disinflation is sudden in that agents did not anticipate the possibility of a change in trend inflation before period 1. The disinflation is credible in that agents believe that the new rate of trend inflation is permanent.
Figure 4: Credible disinflation.

Notes: The figure displays the responses of inflation and output to a sudden and credible reduction in trend inflation from three to two percent annually, using the calibration of other model parameters reported in Table 1, except for $\rho = 0$. The dotted lines represent the responses in an NK model with intrinsic inertia of inflation (and constant elasticity demand curves, i.e., $\epsilon_p = 0$). The solid lines show the responses in our model ($\epsilon_p = -9$), while the dashed lines illustrate those in the special case of constant elasticity demand curves ($\epsilon_p = 0$).

Fuhrer (2011). Similar responses are obtained in our model, as illustrated by the solid lines in the figure. One difference between the NK model and ours is that output in our model rebounds to its new steady-state value associated with the new rate of trend inflation, which is lower than the initial value of steady-state output.$^{18}$

$^{18}$Kurozumi and Van Zandweghe (2016) show that the variable elasticity demand curves can cause steady-state output to become an increasing function of trend inflation, in contrast with the case of constant elasticity demand curves. This is because the variable elasticity demand curves alter the effects of trend inflation on the two components of steady-state output, the steady-state average markup and the steady-state relative price distortion. In the case of constant elasticity demand curves, the responses of inflation and output to the sudden and credible reduction in trend inflation are displayed by the dashed lines. In this case, inflation drops rapidly to the new rate of trend inflation, while output rises immediately to its new steady-state value, which exceeds the initial one because of the decrease in the steady-state relative price distortion associated with the reduction in trend inflation.
5 Extension with Staggered Wage Setting

This section extends the model by introducing not only staggered wage setting as in Erceg, Henderson, and Levin (2000) but also variable elasticity demand curves for labor, and examines their implications for inflation persistence and the effect of a decline in trend inflation on inflation persistence.

5.1 Extended model

In the extended model, the representative household has a continuum of members \( h \in \{0, 1\} \), each of which supplies an individual differentiated labor service \( N_t(h) \). The household maximizes the utility function

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ \log C_t - \int_0^1 \frac{(N_t(h))^{1+\sigma_n}}{1 + \sigma_n} dh \right]
\]

subject to the budget constraint

\[
P_tC_t + B_t = \int_0^1 W_t(h)N_t(h)dh + i_{t-1}B_{t-1} + T_t.
\]

Assuming additive separability in preferences and complete contingent claims markets for consumption implies that all members make a joint consumption-saving decision. Thus, the utility maximization problem leads to the same consumption Euler equation as (2).

There is a representative labor packer that provides labor input \( N_t \) for firms by aggregating individual labor services \( \{N_t(h)\} \) with

\[
\int_0^1 F_w \left( \frac{N_t(h)}{N_t} \right) dh = 1,
\]

where the function \( F_w(\cdot) \) takes the same form as \( F_p(\cdot) \) in Section 2, but with parameters \( \epsilon_w, \theta_w, \) and \( \gamma_w \) instead of \( \epsilon_p, \theta_p, \) and \( \gamma_p \). Note that \( \theta_w > 1 \) and \( \gamma_w \equiv \theta_w(1+\epsilon_w) \) and that the case of \( \epsilon_w \leq 0 \) is considered in the following subsections. Combining the first-order conditions for the labor packer’s problem and the aggregator (37) yields

\[
\frac{N_t(h)}{N_t} = \frac{1}{1 + \epsilon_w} \left[ \left( \frac{W_t(h)}{W_t d_{w,t}} \right)^{-\gamma_w} + \epsilon_w \right],
\]

\[
d_{w,t} = \left[ \int_0^1 \left( \frac{W_t(h)}{W_t} \right)^{1-\gamma_w} dh \right]^{1-\gamma_w},
\]

\[
1 = \frac{1}{1 + \epsilon_w} d_{w,t} + \frac{\epsilon_w}{1 + \epsilon_w} e_{w,t}.
\]
where \( W_t(h) \) is the nominal wage of the labor service \( N_t(h) \), \( W_t \) is the labor input price for firms, \( d_{w,t} \) is the Lagrange multiplier on the aggregator (37) and is a measure of wage dispersion as shown in (39), and

\[
e_{w,t} \equiv \int_0^1 \frac{W_t(h)}{W_t} \, dh. \tag{41}
\]

In the face of the demand curve (38), wages are set on a staggered basis as in Calvo (1983).\(^{19}\) In each period, a fraction \( \alpha_w \in (0,1) \) of wages remains unchanged, while the remaining fraction \( 1-\alpha_w \) is set so as to maximize the relevant utility function

\[
E_t \sum_{j=0}^{\infty} (\alpha_w)^j \left[ \frac{\left( N_{t+j}(h) \right)^{1+\sigma_n}}{1+\sigma_n} + \Lambda_{t+j} \frac{W_t(h)}{P_{t+j}} N_{t+j}(h) \right]
\]

subject to the demand curve

\[
N_{t+j}(h) = \frac{N_{t+j}}{1+\epsilon_w} \left[ \left( \frac{W_t(h)}{W_{t+j}d_{w,t+j}} \right)^{-\gamma_w} + \epsilon_w \right],
\]

where \( \Lambda_t \) is the real value of the Lagrange multiplier on the household’s budget constraint and meets the condition \( \Lambda_t = 1/C_t \) for the log utility of consumption. The first-order condition for the staggered wage setting can be rewritten as

\[
E_t \sum_{j=0}^{\infty} (\alpha_w)^j \frac{N_{t+j}}{C_{t+j}} \left[ \left( \frac{w_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}}} {w_t \prod_{k=1}^j \frac{1}{\pi_{t+k}}} \right)^{-\gamma_w} - \frac{\epsilon_w}{\gamma_w - 1} w_t^* \prod_{k=1}^j \frac{1}{\pi_{t+k}} \right] = 0,
\]

where \( w_t^* \) is the real wage that can be set in period \( t \). Moreover, under the staggered wage setting, eqs. (39) and (41) can be reduced to, respectively,

\[
(w_tD_{w,t})^{1-\gamma_w} = \alpha_w \left( \frac{w_{t-1}D_{w,t-1}}{\pi_t} \right)^{1-\gamma_w} + (1-\alpha_w)(w_t^*)^{1-\gamma_w}, \tag{43}
\]

\[
w_te_{w,t} = \alpha_w \frac{w_{t-1}e_{w,t-1}}{\pi_t} + (1-\alpha_w)w_t^*. \tag{44}
\]

The equilibrium conditions consist of (40) and (42)–(44) in addition to (2), (3), (7), (9), (11)–(13), and (16)–(19). Hence, incorporating staggered wage setting and variable elasticity demand curves for labor—which introduces the three additional variables \( w_t^*, d_{w,t}, \) and \( e_{w,t} \)—replaces the labor supply equation (1) with the four equations (40) and (42)–(44).

\(^{19}\)For the micro evidence on wages, see, e.g., Barattieri, Basu, and Gottschalk (2014).
5.2 Implications for inflation persistence

This subsection shows that introducing staggered wage setting and variable elasticity demand curves for labor better accounts for the empirical response of inflation to monetary policy shocks. To this end, in addition to the calibration of parameters reported in Table 1 (except for $\epsilon_p$), the probability of no wage change is set at $\alpha_w = 0.75$, which implies that the average frequency of wage change is four quarters, and the parameter governing the elasticity of substitution between individual labor services is chosen at $\theta_w = 10$, which implies a desired wage markup of 11 percent. As for the parameters governing the curvature of demand curves for goods and labor, we consider two cases: $(\epsilon_p, \epsilon_w) = (3, 3)$ (variable elasticity demand curves for goods and labor) and $(\epsilon_p, \epsilon_w) = (-9, 0)$ (variable elasticity demand curves for goods and constant elasticity demand curves for labor). Using these calibrations, the equilibrium conditions are log-linearized around the steady state.

Figure 5 presents impulse responses to an expansionary monetary policy shock in the extended model. The dashed lines represent the case of $(\epsilon_p, \epsilon_w) = (-9, 0)$, where inflation rises for four quarters following the shock to a peak level and then declines gradually, as displayed in the top right panel of the figure. Under the staggered wage setting, the real wage, which coincides with the real marginal cost in the model, becomes intrinsically inertial and thereby the response of inflation becomes more hump-shaped and more persistent than that in the baseline model illustrated in Figure 3. This feature becomes more pronounced in the presence of the variable elasticity demand curves for labor, so that a larger value of the goods demand curve curvature parameter $\epsilon_p$, along with a negative value of the labor demand curve curvature parameter $\epsilon_w$, generates a similar response of inflation to that in the case of $(\epsilon_p, \epsilon_w) = (-9, 0)$, as shown by the solid lines that represent the case of $(\epsilon_p, \epsilon_w) = (-3, -3)$. Therefore, the extended model better accounts for the empirical response of inflation to monetary policy shocks than the baseline model.

A summary statistic of the persistence in impulse responses to a shock is the half-life, defined as the number of quarters until the size of the response falls to half of its size on impact of the shock. While the half-life of the inflation response is 4 quarters in the baseline model (illustrated in Figure 3), the half-life in Figure 5 is 15 quarters in the cases
Figure 5: Impulse responses to an expansionary monetary shock in the extended model.

Notes: The figure presents the impulse responses to a negative 70 basis points monetary policy shock under the calibration of model parameters reported in Table 1 (except for $\epsilon_p$) along with $a_w = 0.75$ and $\theta_w = 10$. The interest and inflation rates are displayed in annualized terms. The dashed lines represent the case of $(\epsilon_p, \epsilon_w) = (-3, -3)$, that is, variable elasticity demand curves for goods and labor. The solid lines show the case of $(\epsilon_p, \epsilon_w) = (-9, 0)$, that is, variable elasticity demand curves for goods and constant elasticity demand curves for labor.
of \((\epsilon_p, \epsilon_w) = (-9, 0)\) and \((-3, -3)\).^{20}

### 5.3 New explanation for the decline in U.S. inflation persistence

A number of empirical studies indicate that inflation persistence has decreased in the U.S. since the early 1980s. Cogley and Sargent (2002) employ spectral analysis to estimate inflation persistence and find that the persistence displays a similar pattern to the level of inflation: both the level and the persistence of inflation increased in the 1970s and decreased gradually from the early 1980s onward. Cogley, Primiceri, and Sargent (2010) use predictability as a measure of persistence, as shocks that are more persistent make time series more predictable. They show that the persistence of the inflation gap (i.e., the gap between actual and trend inflation) rose in the 1970s and fell during and after the Volcker disinflation in the early 1980s. Stock and Watson (2007) characterize inflation as consisting of a transitory and a permanent component, and show empirically that the variance of the permanent component increased in the 1970s before declining in the mid 1980s. Fuhrer (2011) examines the persistence in various measures of inflation using different methods, and finds that inflation persistence has decreased for headline inflation but less so for core inflation (which excludes food and energy prices).^{21}

As our model relates inflation dynamics to trend inflation, it may shed light on the measured decrease in inflation persistence from its high levels in the 1970s to lower levels beginning in the 1980s, around the time of the Volcker disinflation. Most previous studies attribute the decrease in inflation persistence to a more active monetary policy response to inflation, sometimes in combination with changes in the volatility of shocks to the U.S. econ-

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20 We found that introducing a cost channel of monetary policy transmission further increases inflation persistence. Assuming that firms must borrow working capital to pay workers at the beginning of each period leads the real marginal cost to depend on the nominal interest rate in addition to the real wage. Then, using the same calibration as for Figure 5 with \((\epsilon_p, \epsilon_w) = (-3, -3)\), the inflation response peaks six quarters after the shock and the half-life lengthens to 22 quarters.

21 Because there are multiple ways of measuring persistence, and because various inflation measures have different properties, the evidence on changes in inflation persistence is not as clear-cut as the observation that trend inflation has declined. Notably, Pivetta and Reis (2007) and Benati (2008) find no evidence of a significant change in inflation persistence in the post-World War II period.
Thus, our model allows us to consider an alternative explanation: the decline in trend inflation caused inflation persistence to decrease.

Figure 6: Impulse responses at high and low trend inflation rates.

Notes: The figure presents impulse responses to a negative 70 basis points monetary policy shock under the calibration of model parameters reported in Table 1, except for $\epsilon_p = -3$, along with $\alpha_w = 0.75$, $\theta_w = 10$, and $\epsilon_w = -3$. The interest and inflation rates are displayed in annualized terms. The solid and the dashed lines assume, respectively, a trend inflation rate $\bar{\pi}$ of 6.5 percent and 2.0 percent annually.

The extended model shows that lower trend inflation reduced inflation persistence. Figure 6 illustrates impulse responses to an expansionary monetary policy shock at a trend inflation rate of 6.5 percent.

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22 Cogley, Primiceri, and Sargent (2010) attribute the decrease in inflation gap persistence primarily to a decline in the volatility of shocks to drifting trend inflation, with a secondary role for the monetary policy response to inflation. A shock to drifting trend inflation in their estimated model is reminiscent of the credible disinflation examined in Section 4, although in their model a decline in trend inflation leads inflation to undershoot the new trend rate initially.
inflation rate $\pi$ of 6.5 percent annually (the solid lines) and 2.0 percent annually (the dashed lines). The former value is the average inflation rate of the PCE price index over the period 1970:Q1–1984:Q4, while the latter is the Federal Reserve’s target for the PCE inflation rate since 2012. Using the same calibration as for Figure 5 with $\epsilon_p = \epsilon_w = -3$ (except for the rate of trend inflation), the top right panel of Figure 6 shows that the response of inflation to the policy shock is more persistent under the higher trend inflation rate than under the lower one. Indeed, the half-life of the inflation response is 22 quarters at the trend inflation rate of 6.5 percent annually, and it declines to 14 quarters at the rate of 2.0 percent annually. Under positive trend inflation, the variable elasticity demand curves for goods and labor cause the price dispersion $\hat{d}_{p,t}$ to influence inflation dynamics and the wage dispersion $\hat{d}_{w,t}$ to influence the dynamics of wages and hence inflation. Price and wage dispersion increase with inflation, as displayed in the figure, and therefore lower trend inflation leads to lower persistence of inflation. Thus, our model provides a new explanation for the evidence that inflation persistence decreased around the time of the Volcker disinflation. In this explanation, the decreases in trend inflation and inflation persistence are no coincidence; the decline in trend inflation reduced inflation persistence.

6 Conclusion

This paper has proposed a novel explanation for two well-known empirical facts: (i) inflation exhibits a persistent response to an expansionary monetary policy shock, with a hump shape and a gradual decline; and (ii) both output and labor productivity rise after the shock. Specifically, the paper has shown that introducing variable elasticity demand curves in a Calvo staggered price model with positive trend inflation and a fixed cost of production can account for the two empirical facts. The demand curves induce strategic complementarity in price setting and thus generate a persistent response of inflation to an expansionary policy shock, with a hump shape and a gradual decline, through the effect of a measure of price dispersion on inflation dynamics. The price dispersion is intrinsically inertial under the staggered price setting and has a significant influence on the model-implied inflation dynamics, where the past, present, and expected future values of the dispersion drive inflation. Therefore, inflation inherits its persistent response to the policy shock mostly from the price dispersion.
The variable elasticity demand curves also give rise to a muted response of relative price distortion—which differs from the price dispersion—to the shock and thus lead output and labor productivity to rise following the shock, owing to the increasing returns to scale production technology arising from the fixed cost. The paper has also demonstrated that in the model a credible permanent reduction in trend inflation leads to a gradual decline in inflation and a fall in output as observed around the time of the Volcker disinflation. Moreover, the paper has extended the model by introducing staggered wage setting and variable elasticity demand curves for labor, and has shown that the extended model better accounts for the empirical response of inflation to monetary policy shocks than the baseline model. The extended model has also demonstrated that lower trend inflation reduced inflation persistence, providing a new explanation for the measured decrease in inflation persistence around the time of the Volcker disinflation.

Our analysis poses new questions for further research. Previous studies with DSGE models have suggested other sources of inflation persistence, such as sticky information and an upward-sloping hazard function, which raises the question: what is the most empirically relevant among the competing sources? An empirical investigation of this question by estimating DSGE models with each of the sources is a fruitful avenue for future research. Moreover, our model’s implication that a decline in trend inflation may have led to lower persistence of inflation provides an alternative view to the leading explanation, which holds that lower inflation persistence resulted from a more active monetary policy response to inflation. By estimating our extended model, future research could examine the relative importance of the two views. Conversely, the model predicts that a rise in trend inflation would increase inflation persistence, and thus could lead to longer-lasting deviations of inflation from a central bank’s target level. Therefore, the effect of trend inflation on inflation persistence is an additional factor that could be considered in research about the optimal inflation rate and in the debate on whether central banks should adopt a higher inflation target.

Coibion (2010) and Dupor, Kitamura, and Tsuruga (2010) show that an NK model with intrinsic inertia of inflation empirically outperforms a sticky information model. The latter authors also demonstrate that introducing sticky information in an NK model exhibits a similar empirical performance to incorporating intrinsic inflation inertia in the model.
References


