FORECAST-BASED MONETARY POLICY

Jeffery D. Amato
Thomas Laubach

OCTOBER 1999

RWP 99-10

Research Division
Federal Reserve Bank of Kansas City

Jeffery D. Amato is an economist at the Bank for International Settlements. Thomas Laubach is an economist at the Federal Reserve Bank of Kansas City. The authors gratefully acknowledge many helpful discussions with Michael Woodford, as well as comments from Mark Gertler, Peter Ireland, Julio Rotemberg, Argia Sbordone, and John Taylor. All remaining errors are those of the authors. The views expressed herein are those of the authors and do not necessarily reflect those of the Bank for International Settlements, the Federal Reserve Bank of Kansas City, or the Federal Reserve System.

Amato e-mail: jeffery.amato@bis.org
Laubach e-mail: tlaubach@frbkc.org

Abstract

A number of central banks use (published or unpublished) forecasts of goal variables as key ingredients in their decisions for instrument settings. This use of forecasts is modelled as a particular form of objective with the minimization of which the central bank is charged. We use an estimated optimization-based model with staggered price and wage setting to analyze the welfare properties of such objectives and their implications for the form of instrument rules. We find that stabilizing expected price inflation at a horizon of two years around target dominates policies of stabilizing inflation at shorter or longer horizons. However, stabilizing all fluctuations, not just forecastable ones, in both wage and price inflation leads to the closest approximation to the welfare-optimal rule.

1 Introduction

Forecasts of goal variables such as output and inflation play an important role in the policy process of many central banks. A common rationale for the use of forecasts in decisions about interest rates is that monetary policy affects those goal variables only with substantial lags. Absent some intermediate target variable which would largely capture the effect of the current stance of monetary policy on the future values of those goal variables, it might seem optimal for the central bank to set its instrument such that the forecast of the goal variables conditional on current information and the current interest rate coincides with one's targets for the goal variables.

A second rationale for the use of forecasts in monetary policy decisions has been suggested by the experience of countries that use inflation targets as monetary policy strategy. In inflation-targeting countries, the central bank is charged with achieving and maintaining a specific level of inflation in some price index. There is no equivalent numerical goal for output that monetary policy should achieve. It has been argued that, if the central bank would attempt to maintain inflation period by period at exactly the target, the instrument movements necessary for doing so would induce a large amount of undesirable volatility in output. Goodhart (1998) suggests that, by adjusting the instrument such as to stabilize the *forecast* of inflation at some appropriate horizon around the target level, the central bank can largely succeed in stabilizing actual inflation, while avoiding destabilizing effects on output.

Recently, the concept of "forecast targeting" has received considerable attention both in theoretical and empirical work (Svensson 1997, 1999a, 1999b, Batini and Haldane 1999, Batini and Nelson 1999). These authors model forecast targeting as a regime in which the central bank is charged with stabilizing its own forecast of some variable, typically inflation, at some horizon around a target value.[1] The implication of such a procedure is that the central bank does not attempt to offset either current fluctuations in its goal variable, or fluctuations that are expected to occur prior to its chosen forecast horizon. In models that are not formulated in terms of individual optimizing behaviour, Svensson shows that optimal policy implies forecast targeting, while Batini and Haldane and Batini and Nelson

[1] Our analysis is concerned with the role that the central bank's own forecasts play in setting monetary policy. Accordingly we are not analyzing any problems associated with the central bank's use of outside forecasts, as considered in Bernanke and Woodford (1997).

show that such use of forecasts has desirable properties according to some ad hoc criterion.

However, this view of the appropriate use of forecasts in setting monetary policy, while intuitively appealing, may be misleading. As Lucas (1976) pointed out, due to the forward-looking nature of private sector behaviour, changes in the rule under which monetary policy operates may cause changes in private sector responses. Hence, the lags with which monetary policy affects the goal variables may depend on private sector expectations about how monetary policy operates, and in this sense be largely endogenous. In this paper, we study the consequences of using forecasts as a guide for monetary policy within a framework which places great emphasis on the forward-looking behaviour of private agents. Similar to the literature cited above, forecast-based monetary policy is modelled as a regime in which the central bank is charged with setting policy such that the deviation of the forecast of a specific variable at a specific horizon from some target value is minimized. We are particularly interested in the question whether certain features of the economy imply welfare benefits resulting from the use of forecasts in the decision-making process of the central bank.

Because our model is based on optimizing behaviour of households, the representative household's welfare provides a natural benchmark for the evaluation of alternative objectives delegated to the central bank. In contrast to the literature on optimal delegation, beginning with Rogoff (1985), the reason for delegating an objective different than the representative household's welfare to the central bank is not an assumed inability of the central bank to act under commitment. We assume that the central bank is able to commit itself to setting policy such as to achieve the objective delegated to it. Instead, our analysis is motivated by the observation that in practice the objectives delegated to central banks are not necessarily reflecting some judgment about the preferences of society. For example, as mentioned earlier, in inflation targeting countries there exists no target for output equivalent to the target for inflation delegated to the central bank. Yet, this does not imply that in these countries monetary policy is oblivious to the output consequences of its decisions. A possible rationale for such delegation might be that delegation of a more complex objective, for example one involving some definition of the output gap, leads to problems in holding the central bank accountable for its performance, if only because the output gap is measured with great uncertainty. Delegating an objective defined in terms of inflation only, but in such a way as to prevent undesirable output variability, may then be attractive.

To analyze the welfare properties of various objectives for monetary policy, we use a model in which households maximize their utility by choosing consumption and setting wages in a staggered fashion, and firms engage in staggered price setting for their products. In previous work (Amato and Laubach 1999a), we analyzed the welfare properties of forecast-based monetary policy in a model with imperfect competition and staggered price setting in goods markets, but perfectly competitive labour markets. Extending the analysis to a model with imperfectly competitive goods and labour markets is interesting for several reasons. First, evidence on staggered *wage* setting is certainly at least as persuasive as evidence on staggered *price* setting. Furthermore, as demonstrated in Erceg (1997), staggered wage setting generates a flat marginal cost schedule at the individual firm level, and hence persistent output effects of monetary shocks, without assumptions on the elasticity of labour supply that are in conflict with evidence from micro data.

More directly related to the questions raised above, Erceg et al. (1999) show that the model with staggered price and wage setting generates a tradeoff between the variability of price inflation, wage inflation, and the output gap. If there is more than one source of nominal rigidity in the economy, stabilizing the price level does not imply stabilizing output around the Pareto-optimal level that would obtain in the flexible price and wage case. This is in contrast to the results of Rotemberg and Woodford (1997), who show that in a model with only one nominal rigidity, complete output gap and inflation stabilization is feasible, and hence no output-inflation variability tradeoff exists. A model with staggered price and wage setting, therefore, provides a framework in which the validity of Goodhart's suggestion mentioned earlier can be assessed.

Because we wish to evaluate economic performance under alternative policy rules, not only is it important to spell out the model in terms of individual optimizing behaviour for the reasons pointed out by Lucas (1976), but the model should also perform well in explaining the historical data. To this end, in Amato and Laubach (1999b) we estimate the model with sticky prices and wages using methods developed in Rotemberg and Woodford (1997). Estimation permits a rigorous evaluation of the model's empirical performance. The estimated model is used for performing the simulations reported in this study. The estimation method and results are contained in the companion paper, and accordingly the discussion of estimation issues below is brief.

Our findings are twofold. First, a policy that aims solely at stabilizing price inflation comes fairly close in terms of welfare to the welfare-optimal policy. In this case, monetary policy should aim at stabilizing only fluctuations in inflation that are forecastable two years ahead, as opposed to stabilizing all fluctuations. A policy that stabilizes the conditional expectation of inflation two years ahead around target reduces welfare losses compared to a policy that aims at stabilizing current inflation, or inflation four years ahead. The higher unconditional variance of inflation under a policy that stabilizes expected inflation two years ahead, compared to a policy of current inflation stabilization, is more than offset by a reduction in the variance of wage inflation. Second, however, a policy aimed at stabilizing all fluctuations, not just forecastable ones, in both price and wage inflation dominates a policy of stabilizing just price inflation. Stabilizing merely fluctuations in price and wage inflation that are forecastable at some horizon results in sizeable welfare losses. Hence, if an objective in terms of more than one variable is to be delegated to the central bank, stabilizing forecasts is inferior to stabilizing all fluctuations of those variables. This second result is reminiscent of Friedman's (1975) insight, in that monetary policy is best conducted under an objective which resembles the household's welfare as closely as possible.

The remainder of the paper is structured as follows. In section 2 we present the model and briefly discuss estimation. In section 3 we present the objective that characterizes individual welfare, and introduce the forecast-based objectives. Finally, we compare economic performance under the different interest-rate policies that are optimal for the various objectives. Section 4 concludes. The welfare objective is derived in an Appendix.

2 An Estimated Model for Policy Evaluation

In this section we introduce a structural model of price inflation, wage inflation and output determination similar to the model developed in Erceg et al. (1999). As mentioned earlier, real effects of monetary policy in this model are due to imperfect competition and staggered price and wage setting in goods and labour markets. Following the description of the model, we briefly discuss estimation of the structural parameters of this model.

2.1 The Structural Model

The economy consists of a continuum of households and firms, and there is a continuum of differentiated, perishable goods and differentiated kinds of labour services. Each household is the monopolistic supplier of one kind of labour service, and consumes a CES aggregate of all the differentiated goods. The household sets a nominal wage for its labour services, and supplies as many hours as are demanded at its chosen wage. Each firm is the monopolistic producer for one good, and uses a CES aggregate of households' labour services in the production process. The firm sets a price for its good, and satisfies demand at this price. Because the analysis focusses on the effects of monetary policy at the business cycle horizon, capital accumulation is not modelled.

Household i's utility is defined over the index C_t^i, where

$$C_t^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

z denotes a specific good, and $\theta > 1$ parameterizes the elasticity of substitution in the household's preferences between the various goods. As θ gets large, goods become ever closer substitutes, whereas if θ approaches 1 from above, goods are less and less substitutable. Hence θ also measures the market power of each of the firms located on the interval [0,1], with market power decreasing in θ.

The "consumption-based price index" is defined as

$$P_t \equiv \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{2}$$

The price index P_t denotes the minimum amount the household has to spend to obtain one unit of the composite good C_t defined as in (1). Maximizing the index (1) for a given level of consumption expenditure, the household allocates consumption across individual products according to

$$c_t^i(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} C_t^i. \tag{3}$$

Household i is the sole supplier of labour services h^i, and its objective is to maximize

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t (u(C_t^i; \xi_t) - v(h_t^i; \zeta_t)) \right] \tag{4}$$

subject to a demand schedule for its labour services and the budget constraint

$$E_t[\delta_{t,t+1} A_{t+1}^i] \leq A_t^i + W_t^i h_t^i + \Pi_t - P_t C_t^i \tag{5}$$

Within each period, the household derives utility $u(\cdot; \xi_t)$ from consumption C_t^i as defined in (1), while supplying hours h_t^i reduces utility, as indicated by the function $v(\cdot; \zeta_t)$. In the budget constraint, P_t denotes the price index defined in (2), and A_t denotes the nominal value of the household's holdings of financial assets at the beginning of period t. W_t^i is the hourly wage that household i charges, and Π_t the household's share in firms' profits, which we assume are distributed lump-sum to households. $\delta_{t,\tau}$ is a stochastic discount factor, pricing in period t assets whose payoffs are in period τ. Financial markets are assumed to be complete, and in particular there exists a riskless one-period nominal bond, the gross return on which is given by $R_t \equiv (E_t \delta_{t,t+1})^{-1}$. The stochastic disturbance ξ_t is interpreted as preference or "demand" shock, while ζ_t is a disturbance to labour supply. The household's choice variables are consumption and hours or, given the demand function for its labour services, its wage.

Firm z is the monopolistic supplier of good z, which it produces according to the production function

$$y_t(z) = e^{\eta_t} \bar{K}^a H_t(z)^{1-a} \tag{6}$$

where η_t denotes a stochastic technology disturbance, the capital stock employed by each firm is fixed at $\bar{K}$, and the firm's labour input is a CES aggregate of different households' labour services

$$H_t(z) = \left[\int_0^1 h_t^i(z)^{\frac{\phi-1}{\phi}} di \right]^{\frac{\phi}{\phi-1}} \tag{7}$$

The parameter $\phi > 1$ characterizes the elasticity of substitution between the various types of labour services. The wage index W_t is defined as

$$W_t \equiv \left[\int_0^1 (W_t^i)^{1-\phi} di \right]^{\frac{1}{1-\phi}} \tag{8}$$

Maximizing the index (7) for a given level of wage payments, firm z allocates demand for individual labour services according to

$$h_t^i(z) = \left[\frac{W_t^i}{W_t} \right]^{-\phi} H_t(z). \tag{9}$$

Aggregate demand for output is defined as $Y_t = C_t + G_t$, where $C_t \equiv \int_0^1 C_t^i di$, and G_t is an exogenously given component of demand for output, which is assumed to be determined one period ahead. Assuming that G_t is allocated across the different goods by maximizing an index defined analogous to the consumption index (1), the demand faced by firm z is given by

$$y_t(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} Y_t. \tag{10}$$

Analogously, by integrating (9) across firms, the demand for its labour services faced by household i is

$$h_t^i = \left[\frac{W_t^i}{W_t}\right]^{-\phi} H_t \tag{11}$$

where $H_t \equiv \int_0^1 H_t(z)dz$.

We now characterize households' utility-maximizing consumption and wage decisions, and firms' profit-maximizing price choices. Because we wish to use solution methods for linear rational expectations models, the equilibrium conditions we use are log-linear approximations to the exact, nonlinear first order conditions of households and firms. For reasons discussed in Woodford (1999a) the welfare analysis later on is facilitated by log-linearizing around the efficient steady state, i.e. the steady state corresponding to a situation without market power and nominal rigidities in goods and labour markets. The efficient steady state level of output is determined by the condition that households' marginal rate of substitution between labour and consumption equal marginal product of labour, i.e.

$$\frac{v_h(H(\bar{Y});0)}{u_c(\bar{Y}-\bar{G};0)} = (1-a)(\bar{Y}/\bar{K})^{-\frac{a}{1-a}} \tag{12}$$

where $\bar{Y}$ and $\bar{G}$ denote the steady state values of output and exogenous demand respectively. The presence of market power of households and firms implies that, absent some offsetting policy, the steady state output level is below this efficient level of output. To justify log-linearizing the exact equilibrium conditions around the efficient steady state, below we will have to assume that tax policies are in place which offset the inefficiencies caused by imperfect competition in goods and labour markets. Furthermore, we log-linearize around a steady state in which there is zero price and wage inflation.

Households are assumed to choose their consumption purchases two periods ahead, i.e. C_t^i is chosen in $t-2$.[2] The decision lag for consumption implies that the household's Euler equation takes the form

$$E_t u_c(C_{t+2}^i;\xi_{t+2}) = E_t\lambda_{t+2}^i P_{t+2} \tag{13}$$

[2] Although this choice of decision lag is somewhat arbitrary, it is no more arbitrary than choosing to specify our model at a quarterly frequency - or, for that matter, *any* frequency - in the absence of compelling evidence to the contrary. As in Rotemberg and Woodford (1997), we choose a two quarter lag to match the timing of the maximum impact of a monetary policy shock on output in our model to that in the VAR. Instead, we could introduce and estimate a free parameter that captures the average decision lag of households due to, e.g., time-to-build constraints.

where λ_t^i denotes household i's marginal utility of income at date t. Since households are free to take investment decisions each period with immediate effect, λ_t has to satisfy

$$\lambda_t = \beta E_t[R_t \lambda_{t+1}] \tag{14}$$

Dropping the superscript i implicitly assumes that, because of complete markets, households insure themselves against all idiosyncratic risk, and therefore the path of consumption is identical across households. Let $\hat{\lambda}_t$ denote the percentage deviation of $\lambda_t P_t$ from its steady state value. Then the log-linear approximation of (14) is

$$\begin{aligned} \hat{\lambda}_t &= E_t[\hat{R}_t - \pi_{t+1} + \hat{\lambda}_{t+1}] && (15) \\ &= \sum_{T=t}^{\infty} E_t[\hat{R}_T - \pi_{T+1}] && (16) \end{aligned}$$

where $\hat{R}_t$ is the percentage deviation of the interest rate from its steady state value consistent with zero inflation. The log-linear approximation of the Euler equation (13) is therefore

$$-\tilde{\sigma} E_t[\hat{C}_{t+2} - \tilde{\xi}_{t+2}] = \sum_{T=t+2}^{\infty} E_t[\hat{R}_T - \pi_{T+1}] \tag{17}$$

where $\hat{C}_t \equiv (C_t - \bar{C})/\bar{C}$ denotes the percentage deviation of consumption from its steady state value $\bar{C}$, $\tilde{\sigma} \equiv -u_{cc}(\bar{C})\bar{C}/u_c(\bar{C})$, and $\tilde{\xi}_t \equiv -(u_{c\xi}(\bar{C})/u_{cc}(\bar{C})\bar{C})\xi_t$ is the disturbance to the marginal utility of consumption.

Log-linearizing aggregate demand around the steady state yields

$$\hat{Y}_t = s_c \hat{C}_t + \tilde{G}_t \tag{18}$$

where $\hat{Y}_t \equiv (Y_t - \bar{Y})/\bar{Y}$, $\tilde{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, and $s_c \equiv \bar{C}/\bar{Y}$. By substituting from the log-linearized aggregate demand equation for C_t, the Euler equation can be written as

$$\hat{Y}_t = -\sigma^{-1} E_{t-2} \sum_{T=t}^{\infty} [\hat{R}_T - \pi_{T+1}] + \hat{G}_t \tag{19}$$

where $\sigma \equiv \tilde{\sigma}/s_c \equiv -u_{cc}(\bar{C})\bar{Y}/u_c(\bar{C})$, and $\hat{G}_t \equiv \tilde{G}_t + s_c E_{t-2}\tilde{\xi}_t$. Equation (19) is the model's "IS equation".

The assumption for wage and price adjustment we use is Rotemberg and Woodford's (1997) variant of Calvo's (1983) staggered price setting. Each period a fraction $1 - \lambda$ of households is chosen at random and independent of their individual histories, and is being offered the opportunity to set a new wage. Hence, from the perspective of an individual household, the wage set in period t applies with probability 1 in period t, with probability λ it applies in period $t+1$, with probability λ^2 in period $t+2$ and so forth. Rotemberg and Woodford assume furthermore that at the end of period $t-1$, a fraction γ^w of those households who choose a new wage can apply this wage beginning at date t, the remaining fraction $1-\gamma^w$ applies this wage beginning at date $t+1$. Let W_t^1 denote the wage chosen in $t-1$ by those households whose wage comes into effect in period t, and let W_t^2 denote the wage chosen in $t-2$ by those households whose wage comes into effect in t. The aggregate wage level is then given by

$$W_t = [\lambda W_{t-1}^{1-\phi} + (1-\lambda)\gamma^w (W_t^1)^{1-\phi} + (1-\lambda)(1-\gamma^w)(W_t^2)^{1-\phi}]^{\frac{1}{1-\phi}} \tag{20}$$

The wage W_t^1 is chosen to maximize

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left[\lambda_T(1+\tau_w)W_t^1\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T - v\left(\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T;\zeta_T\right)\right] \tag{21}$$

Since the wage chosen at the end of period $t-1$ will apply at time t with probability 1, at time $t+1$ with probability λ and so forth, the household discounts utility in future periods *conditional* on W_t^1 still applying by $(\lambda\beta)^{T-t}$. Marginal utility of income at any point in time is the same across households. Therefore, the household's utility from charging wage W_t^1 in period T is given by the product of marginal utility of income and earnings (the first term in brackets) less the disutility from supplying $(W_t^1/W_T)^{-\phi}H_T$, the number of hours demanded at wage W_t^1 and aggregate wages and hours W_T and H_T (the second term in brackets). τ_w denotes a subsidy for employment. By choosing $\tau_w = (\phi-1)^{-1}$, the effect of imperfect competition in labour markets on the steady state output level can be offset.

The first-order condition for W_t^1 can be expressed as

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T\left[v_h\left(\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T;\zeta_T\right) - \frac{\phi-1}{\phi}\lambda_T P_T(1+\tau_w)\frac{W_t^1}{P_T}\right] = 0. \tag{22}$$

Households choose their nominal wage in period $t-1$ such that the discounted sum of expected future real wages $(1+\tau_w)W_t^1/P_T$ equals the discounted sum of expected future marginal rates of substitution between consumption and leisure $v_h(h_{t,T}^1;\zeta_T)/(\lambda_T P_T)$ times a markup $\frac{\phi}{\phi-1}$, where we used $h_{t,T}^1$ as shorthand for the number of hours supplied in period T at wage W_t^1.

In the Appendix we derive a log-linear approximation to this first-order condition. Using this log-linear approximation as well as the corresponding relation for W_t^2 and the log-linear approximation of the wage index (20), we obtain the following law of motion for the rate of wage inflation $\pi_t^w \equiv \log(W_t/W_{t-1})$:

$$\pi_t^w = (1-\psi^w)E_{t-2}\pi_t^w + \psi^w \left[\kappa^w(\hat{Y}_t - \hat{Y}_t^w) - \frac{\kappa^w(1-a)}{\omega+\sigma(1-a)}(\hat{w}_t + \nu_{t-1}) + \beta E_{t-1}\pi_{t+1}^w\right]. \quad (23)$$

The parameter $\omega \equiv v_{hh}(\bar{H};0)\bar{H}/v_h(\bar{H};0)$ measures the elasticity of the disutility of labour supply at the steady state level of hours $\bar{H}$. The coefficient

$$\kappa^w \equiv \frac{(1-\lambda)(1-\lambda\beta)}{\lambda} \; \frac{\omega+\sigma(1-a)}{(1+\phi\omega)(1-a)}$$

describes the elasticity of wage inflation with respect to the gap between actual output $\hat{Y}_t$ and

$$\hat{Y}_t^w \equiv \frac{1-a}{\omega+\sigma(1-a)}E_{t-1}\left[\frac{\omega}{1-a}\eta_t + \omega\tilde{\zeta}_t + \sigma\hat{G}_t\right], \quad (24)$$

the level of output consistent with stable wage inflation. The coefficient $\psi^w \equiv \gamma^w\lambda/(1-\gamma^w(1-\lambda))$ equals 1 for $\gamma^w = 1$, the case in which all wage adjustments are effective the following period. The term $\hat{w}_t \equiv \log(W_t/P_t)$ denotes the percentage deviation of the real wage from its steady state. Positive deviations of the real wage from steady state reduce wage inflation. Finally,

$$\nu_{t-1} \equiv E_{t-1}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) - E_{t-2}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1})$$

is the revision from $t-2$ to $t-1$ in expectations of the long-term real interest rate in period t. Such revisions reduce wage inflation because they raise the returns households expect from their future earnings.

Price adjustment by firms is modelled analogous to wage adjustment by households. Each period a fraction $1-\alpha$ of firms is chosen at random and independent of their individual histories, and is being offered the opportunity to adjust their price. At the end of period $t-1$, a fraction γ^p of those who choose a new price can apply this price beginning at date t, the remaining fraction $1-\gamma^p$ applies this price beginning at date $t+1$. Let p_t^1 denote the price chosen in $t-1$ by those firms whose price comes into effect in period t, and let p_t^2 denote the price chosen in $t-2$ by those firms whose price comes into effect in t. The aggregate price level is then given by

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)\gamma^p(p_t^1)^{1-\theta} + (1-\alpha)(1-\gamma^p)(p_t^2)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (25)$$

The price p_t^1 is chosen to maximize

$$E_{t-1}\sum_{T=t}^{\infty}\alpha^{T-t}\delta_{t,T}\left[(1+\tau_p)p_t^1\left(\frac{p_t^1}{P_T}\right)^{-\theta}Y_T - W_T\left(\left(\frac{p_t^1}{P_T}\right)^{-\theta}\frac{Y_T}{e^{\eta_T}}\right)^{\frac{1}{1-a}}\right] \tag{26}$$

Since the price chosen at the end of period $t-1$ will apply at time t with probability 1, at time $t+1$ with probability α and so forth, the firm discounts future profits *conditional* on p_t^1 still applying by $\alpha^{T-t}\delta_{t,T}$, where $\delta_{t,T}$ is the stochastic discount factor introduced in (5). The first term in brackets denotes revenues in period T at price p_t^1, the second term the firm's labour cost implied by the level of output that is demanded in period T at price p_t^1. τ_p denotes a subsidy for producing output. By choosing $\tau_p = (\theta-1)^{-1}$, the effect of imperfect competition in goods markets on the steady state output level can be offset.

The first-order condition with respect to p_t^1 can be written as

$$\begin{aligned} &E_{t-1}\sum_{T=t}^{\infty}\alpha^{T-t}\delta_{t,T}\left(\frac{p_t^1}{P_T}\right)^{-\theta}Y_T \\ &\cdot\left[(1+\tau_p)p_t^1 - \frac{\theta}{\theta-1}(1-a)^{-1}e^{\frac{-\eta_T}{1-a}}W_T\left(\left(\frac{p_t^1}{P_T}\right)^{-\theta}Y_T\right)^{\frac{a}{1-a}}\right] = 0. \end{aligned} \tag{27}$$

Firms set the price in period $t-1$ such that the price, adjusted for the subsidy, equals a weighted average of expected future marginal cost at the level of output demanded at price p_t^1, times a markup $\frac{\theta}{\theta-1}$.

A log-linear approximation to this first-order condition is derived in the Appendix. Using this log-linear approximation as well as the corresponding relation for p_t^2 and the log-linear approximation of the price index (25), the law of motion for the rate of price inflation $\pi_t \equiv \log(P_t/P_{t-1})$ is given by

$$\pi_t = (1-\psi^p)E_{t-2}\pi_t + \psi^p\left[\kappa^p(\hat{Y}_t - \hat{Y}_t^p) + \frac{\kappa^p(1-a)}{a}\hat{w}_t + \beta E_{t-1}\pi_{t+1}\right]. \tag{28}$$

The coefficient

$$\kappa^p \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\,\frac{a}{1-a+\theta a}$$

denotes the elasticity of price inflation with respect to the gap between actual output $\hat{Y}_t$ and

$$\hat{Y}_t^p \equiv a^{-1}E_{t-1}\eta_t, \tag{29}$$

the level of output consistent with stable price inflation. The coefficient $\psi^p \equiv \gamma^p\alpha/(1-\gamma^p(1-\alpha))$ equals 1 for $\gamma^p = 1$, the case in which all price adjustments are effective the following period. Unlike in the wage inflation equation, positive deviations of the real wage from steady state increase price inflation.

In addition to the IS and wage and price inflation equations, a fourth structural equation is necessary to determine the paths of the four endogenous variables $\{\hat{Y}_t, \pi_t, \pi_t^w, \hat{R}_t\}$. For the estimation of this model, monetary policy is assumed to be described by a feedback rule for the one-period nominal interest rate of the form

$$\hat{R}_t = \sum_{k=1}^{3} \mu_k \hat{R}_{t-k} + \sum_{k=0}^{2} \psi_k \hat{w}_{t-k} + \sum_{k=0}^{2} \phi_k \pi_{t-k} + \sum_{k=0}^{2} \theta_k \hat{Y}_{t-k} + \epsilon_t \tag{30}$$

To summarize, the model consists of the IS equation (19), the wage inflation equation (23), the price inflation equation (28), and the feedback rule for the interest rate. Except for stochastic disturbances, wage and price inflation in this model are predetermined one period ahead, output two periods. The structural disturbances of the model are $\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p$, and ϵ_t. The first three of these shocks are themselves predetermined one period ahead, and so are wage and price inflation and output. The model parameters are $\beta, \sigma, \omega, a, \alpha, \theta, \gamma^p, \lambda, \phi, \gamma^w$, and the parameters of the feedback rule.

2.2 Model Solution and Estimation

The simulations of alternative policy rules that we perform in the next section require us to specify stochastic processes for the shocks and provide values of the model parameters. To obtain estimates of the shocks and model parameters, we adopt the approach taken by Rotemberg and Woodford (1997), which is extended to the model considered here in Amato and Laubach (1999b). We briefly describe this approach to estimation; see Amato and Laubach (1999b) for further details.

Our data set is for the U.S., and is comprised of quarterly observations on real (chain-weighted) GDP, the GDP deflator, compensation per hour in the nonfarm business sector, and the federal funds rate, from 1980:1 to 1997:3.[3,4] We are presenting our empirical results

[3] Quarterly values of the federal funds rate are computed as within-quarter averages of (effective) daily rates.

[4] Because we wish to identify the historical interest rate rule from the VAR, it is important that the VAR be estimated over a sample period in which policy can be characterized by an interest rate rule with constant coefficients. Several empirical studies of U.S. monetary policy have identified a change in policy behaviour around the beginning of the Volcker chairmanship in 1979 (e.g. Clarida et al., 1998). By contrast, policy since the disinflation of the early 1980s has displayed a high degree of stability in the sense of being well described by a rule like (30).

in terms of the real wage instead of wage inflation because we find impulse responses of the real wage more convenient to interpret, and because in other work the effects of monetary policy on wages is measured as effects on real wages, not wage inflation. Given our definition of variables in the previous subsection, the two are linked by $\pi_t^w = \hat{w}_t - \hat{w}_{t-1} + \pi_t$. To express the data conformable with the theoretical series $\{\hat{Y}_t, \pi_t, \hat{w}_t, \hat{R}_t\}$ of the model, the data on real GDP data are logarithmized and a linear trend is removed, inflation is computed as log first differences of the GDP deflator, the real wage is computed as the logarithm of compensation per hour deflated by the GDP deflator and a linear trend is removed, and the federal funds rate is expressed at quarterly rate. Let $\{y_t, \pi_t, w_t, r_t\}$ denote these series, which are conformable with their theoretical counterparts up to a constant.

The estimation process has three steps. The first step is to construct and estimate a vector autoregression (VAR) for the model's four endogenous variables. The theoretical model implies that, because they are predetermined, output, inflation, and the real wage are not contemporaneously affected by an interest rate innovation, while the form of the interest rate rule (30) allows for contemporaneous feedback from output, inflation, and the real wage to the interest rate. This is sufficient to identify the parameters of the historical interest rate rule and the series of interest rate innovations $\{\epsilon_t\}$. Let $Z_t = (r_t, w_{t+1}, \pi_{t+1}, y_{t+1})'$, and let $\bar{Z}_t = (Z_t', Z_{t-1}', Z_{t-2}')'$. The reason for defining Z_t in this manner is that the elements of Z_t all belong to the period t information set, since output, inflation, and the real wage are predetermined. The structural form of a VAR(3) in Z_t can then be written as

$$T\bar{Z}_t = m + A\bar{Z}_{t-1} + \bar{e}_t \tag{31}$$

where T is an identity matrix with a lower triangular 4 by 4 submatrix in the upper left corner, the first four rows of A contain coefficients, and the last eight rows of the VAR are identities. Accordingly, the last eight elements of $\bar{e}_t$ are zeros. The first four elements are mutually orthogonal, so that the first four diagonal elements of the covariance matrix V of $\bar{e}_t$ are distinct from zero, and all remaining elements of V are zero. Under our identifying assumption, the first row of A contains the coefficients of the historical interest rate rule (30), and the first element of $\bar{e}_t$ is ϵ_t. [5]

The second step is to choose the model parameters $(\beta, \sigma, \omega, a, \alpha, \theta, \gamma^p, \lambda, \phi, \gamma^w)$ based on

[5] As pointed out above, the series $\{y_t, \pi_t, w_t, r_t\}$ are conformable with their theoretical counterparts up to constants. By including the constant m in the VAR, the coefficients in the first row of A can be interpreted as the coefficients in (30).

Table 1: Structural Parameters

σ	.26	α	.66	λ	.66
ω	.20	γ^p	.56	γ^w	.56
β	.99	κ^p	.019	κ^w	.035
a	.25	$\frac{\theta}{\theta-1}$	1.19	$\frac{\phi}{\phi-1}$	1.13

the first and second moments of our data series as captured by (31). The parameters β and a can be recovered from the first moments of the data. Since β^{-1} is the steady-state gross real rate of return in our model, and the average *ex post* real interest rate in our sample is one percent (on a quarterly basis), we set β equal to 0.99. Our choice of a, equal to 0.25, combined with our estimate for the average markup in goods markets reported below, is compatible with an average labour share of 0.63.

The remaining structural parameters are chosen so that the responses of the endogenous variables in the model to an exogenous monetary policy shock, ϵ_t, match as closely as possible the responses estimated from the VAR. Unfortunately, inspection of the model equations (19), (23), and (28), reveals that not all of the parameters $\sigma, \omega, \alpha, \theta, \gamma^p, \lambda, \phi$, and γ^w are separately identified. The three parameters α, θ, and γ^p appear in the model only through κ^p and ψ^p in the price inflation equation (28); therefore, at most two of these parameters can be estimated. Likewise, we can estimate only two of the three parameters λ, ϕ, and γ^w, since they appear in the model only through κ^w and ψ^w in the wage inflation equation (23). Based on several survey studies, we follow Rotemberg and Woodford by setting $\alpha \equiv 0.66$, which implies that prices remain unchanged on average for three quarters. Similarly, we impose $\lambda \equiv 0.66$. Finally, as discussed in Amato and Laubach (1999b), although γ^p, γ^w, and ω are each identified (given values for α and λ), the ratio γ^w/γ^p and ω are not separately well-determined from the data. We therefore set $\gamma^w \equiv \gamma^p$, which has the interpretation of imposing equal measures of exogenous rigidity in prices and wages (under the assumption $\alpha \equiv \lambda$). Given these values, the remaining parameters $\sigma, \omega, \psi^p, \kappa^p$, and κ^w are estimated by minimizing the squared differences between the model's and the VAR's responses during quarters 1 to 5 following a monetary policy shock in quarter 0.

The estimates, or implied values, for the parameters $\sigma, \omega, \kappa^p, \theta, \gamma^p, \kappa^w, \phi$, and γ^w are displayed in Table 1. The estimate of σ implies an elasticity of intertemporal substitution of consumption of 3.9. This is larger than what has been found in the non-durable consumption literature and what is typically assumed in the real-business cycle literature (e.g. values between one and two), but it is smaller than Rotemberg and Woodford's estimate of 6.25. However, since the variable C in our model - as in Rotemberg and Woodford's - proxies for all interest-rate sensitive components of output, and not just non-durable consumption, a value higher than two appears justified. The estimate of ω implies an elasticity of labour supply of 5.0, which is about half the size of Rotemberg and Woodford's estimate.[6] The plausibility of our estimate is difficult to determine from the micro panel data literature, since the functional forms used in that literature are based on first-order conditions derived in a setting with flexible wages. Nonetheless, using panel data results as a guide, an estimate of five is larger than what is typically found, but not implausibly so. The estimate of κ^p implies a steady-state markup of prices over marginal cost of 19%, which is quite similar to Rotemberg and Woodford's value of 15%. Finally, the estimate of κ^w implies a steady-state markup of the real wage over the marginal rate of substitution of 13%, which, as with our estimate of the steady-state price markup, is neither so low nor so high to be regarded as implausible.

Figure 1 presents the impulse responses of the four endogenous variables to a monetary policy shock. The solid lines are the model's predictions given our parameter estimates, and the dashed lines are the responses estimated from the VAR. Overall, over the first five quarters after the shock, the responses of the model closely match those of the VAR. The main discrepancies are in the inflation and real wage responses, primarily from the fact that the model cannot replicate, for any parameter values, the hump in inflation three to four quarters after the shock and the hump in the real wage two quarters after the shock.

Given the estimates of the VAR and the structural parameters, the third estimation step is to choose the shock processes so that the model responses of output, inflation, and the real wage to innovations in the structural shocks match exactly the responses of those variables in the VAR to the three remaining VAR disturbances.

[6] The elasticity of labour supply is not separately identified in Rotemberg and Woodford's model, even though a similar quantity implicitly appears in their parametrization. Instead, they derive an estimate of this elasticity based on their estimate of σ and calibrated values for a and the elasticity of the average real wage with respect to variations in output that are orthogonal to preference and technology shocks.

3 Alternative Objectives for Monetary Policy

The goal of this section is to characterize the behaviour of the economy under various objectives for monetary policy, and to evaluate the desirability of these objectives in terms of their welfare properties. The previous section emphasized the fact that deriving a structural model from individual optimizing behaviour has the advantage that the coefficients in the resulting model equations have a structural interpretation and, if the model is correctly specified, should remain invariant under alternative policies. A second advantage of an optimization-based model, which is exploited in this section, is the ability to perform welfare comparisons between alternative policy rules, in that the representative household's lifetime utility provides a model-consistent evaluation criterion.

The section starts by providing an approximation to the lifetime utility of the representative household, expressed in terms of a weighted sum of the variances of the endogenous variables. This approximation facilitates the evaluation of the welfare consequences of alternative policies. The second subsection presents what we call "forecast-based objectives" for monetary policy. The purpose is to formalize the notion that the central bank chooses its instrument at any point in time such that the forecast s periods ahead of some variable is equal to some target for this variable. As argued in the Introduction, such objectives provide a plausible description of the way in which many central banks decide about instrument settings. It appears therefore of interest to evaluate the welfare properties of such objectives, and in particular the importance of the "policy horizon" s.

As mentioned in the previous section, given an objective for monetary policy, there are two alternative formulations of the policy-makers' problem. First, one can solve for the plan, i.e. the path for the endogenous variables, that is optimal under the specified objective by maximizing the objective subject to the constraint that the structural equations (the IS, wage inflation, and price inflation equations) hold at all points in time. This approach does not involve specifying a particular functional form for the interest rate rule. Second, one can posit some specific functional form of an interest rate rule, and then choose the parameters of this rule to maximize the objective. This approach is particularly interesting in light of the results of Rotemberg and Woodford (1999) who show that, in the context of their model, the optimal rule within quite simple classes of interest rate rules comes close to achieving the welfare level obtained under the optimal plan. It has been argued that such simple rules are attractive because they enhance the transparency of monetary policy. Our results using this approach are presented in the third subsection.[7]

3.1 An Expression for the Representative Household's Welfare

The criterion for evaluating alternative policies is the representative household's welfare, which can be expressed as

$$W = E\left[u(C_t;\xi_t) - \int_0^1 v(h_t^i;\zeta_t)di\right] \tag{32}$$

This objective is a simple transformation of the unconditional expectation of the household's lifetime utility (4), where the expectation is taken over all possible histories prior to date zero. Due to the assumption of perfect insurance among households, consumption is identical across households, and hence the first term inside brackets in (32) does not have a household index attached. The second term in brackets is understood as an average over possible histories of households' opportunity to change their wages.

In the Appendix we derive a second-order Taylor approximation of (32) around the same steady state considered in the log-linear approximations in section 2. This second-order approximation has the advantage that it can be evaluated in terms of the log-linear approximations to the model's exact equilibrium conditions derived in section 2. Specifically, the approximation can be expressed as

$$\begin{aligned} W &= -\Omega\Big[var(\pi_t) + (\psi^{p^{-1}}-1)var(\pi_t - E_{t-2}\pi_t) + (E\pi_t)^2 + c_1 var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e]) \\ &+ c_2\{var(\pi_t^w) + (\psi^{w^{-1}}-1)var(\pi_t^w - E_{t-2}\pi_t^w) + (E\pi_t^w)^2\}\Big] &(33)\\ &= -\Omega[L + (1+c_2)\bar{\pi}^2] &(34)\end{aligned}$$

where Ω, c_1 and c_2 are combinations of the model's parameters, and

$$\begin{aligned} L &= var(\pi_t) + (\psi^{p^{-1}}-1)var(\pi_t - E_{t-2}\pi_t) + c_1 var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e]) \\ &+ c_2\Big[var(\pi_t^w) + (\psi^{w^{-1}}-1)var(\pi_t^w - E_{t-2}\pi_t^w)\Big] &(35)\end{aligned}$$

is the welfare loss associated with variability of the output gap and price and wage inflation. The measure of potential output

$$\hat{Y}_t^e = \frac{a}{\omega + a + \sigma(1-a)}\,\hat{Y}_t^p + \left(1 - \frac{a}{\omega + a + \sigma(1-a)}\right)\hat{Y}_t^w \tag{36}$$

[7] All simulations are performed under the assumption that the central bank is able to commit itself to a certain interest-rate policy, i.e. policies are not constrained to being time-consistent.

is the Pareto-efficient level of output, expressed as percent deviation from $\bar{Y}$, that would obtain under completely flexible prices and wages. In transforming (33) to (34) we made use of the fact that, because the real wage is assumed stationary, $E(\pi_t^w)$ has to equal $\bar{\pi} \equiv E(\pi_t)$.[8]

The form of this loss function is similar to those assumed *ad hoc* in many studies of monetary policy design, and similar to the concern for output and inflation variability expressed e.g. in Taylor (1979). The coefficients c_1 and c_2 express the weights of output gap and wage inflation variability relative to price inflation variability in (35). For our parameter estimates, $c_1 = .007$ and $c_2 = .89$. The small value of c_1 implies that avoiding output variability caused by fluctuations in the efficient level of output is mostly undesirable, because it entails comparatively large welfare losses due to dispersion of relative prices and wages.

The presence of the first moment $\bar{\pi}^2$ in (34) is due to the fact that even a constant, perfectly anticipated rate of inflation different from zero forces households and firms to adjust their wages and prices whenever they have the opportunity to do so. The implied dispersion of relative prices is welfare reducing because at any point in time the condition that the real wage equal the marginal rate of substitution is violated for most households, and likewise the condition that price equal marginal cost is violated for most firms. The first moment term is important once it is taken into account that nominal interest rates cannot fall below zero in an economy where non-interest-bearing money is held. Suppose a given interest rate policy implies an unconditional standard deviation $\sigma(R)$ for the nominal interest rate, and that under such a policy all realizations of the interest rate are confined to an interval of size $k\sigma(R)$ on each side of the steady state value $\bar{R}$. For the zero lower bound on nominal interest rates to hold at all times, $\bar{R} \geq k\sigma(R)$ has to hold. Since $\bar{R} = \bar{\pi} + \rho$, i.e. the steady state nominal interest rate equals the steady state inflation rate plus the steady state real interest rate, we have that $\bar{\pi} \geq k\sigma(R) - \rho$. This last inequality shows that a more volatile interest rate policy can only be implemented at the cost of a higher steady state inflation rate, which reduces welfare. In the results reported below, we take this constraint into account by minimizing the objective

$$W = -\Omega[L + (1 + c_2)(\max\{k\sigma(R) - \rho, 0\})^2] \tag{37}$$

The values of k and ρ are set to 2.46 and 3.04% respectively, which have been obtained

[8] Because the second-order approximation (34) is taken around a steady state of zero wage and price inflation, the term $\bar{\pi}^2$ has to be small for the approximation to remain valid.

from the estimated VAR.

3.2 Forecast-Based Objectives

As discussed in the Introduction, the rationale for having policy decisions depend explicitly on forecasts may be sought either in the dynamic response of goal variables to interest rate changes, or in the argument that by stabilizing forecasts for a *subset* of goal variables, the outcome may resemble the one in which policy aims at stabilizing the realized values of *all* goal variables. To formalize the use of forecasts in policy decisions, we assume that the objective delegated to the central bank can be described by

$$\min_{\{R_t\}} E\left[(E_t\pi_{t+s_1,t+s_2} - \bar{\pi})^2 + \chi_1(E_t[\hat{Y}_{t+s_1,t+s_2} - \hat{Y}^e_{t+s_1,t+s_2}])^2 + \chi_2(E_t\pi^w_{t+s_1,t+s_2} - \bar{\pi})^2\right] \tag{38}$$

where $x_{t+s_1,t+s_2} \equiv \frac{1}{s_2-s_1+1}\sum_{s=s_1}^{s_2} x_{t+s}$ denotes the average of variable x between periods $t+s_1$ and $t+s_2$. This loss function penalizes deviations of forecasts, i.e. *conditional* expectations, of goal variables from target. The outer (*unconditional*) expectation makes the optimal path of interest rates (e.g. the interest rate rule) independent of the state of the economy when it is chosen. It is instructive to rewrite the objective (38) as

$$\min_{\{R_t\}} var(E_t\pi_{t+s_1,t+s_2}) + \chi_1 var(E_t[\hat{Y}_{t+s_1,t+s_2} - \hat{Y}^e_{t+s_1,t+s_2}]) + \chi_2 var(E_t\pi^w_{t+s_1,t+s_2}). \tag{39}$$

For the special case of (39) with $s_1 = s_2 = 0$, the central bank is charged with minimizing some combination of the unconditional variances of the goal variables. It is important to note, however, that the case with the additional assumption $\chi_2 = 0$ does *not* correspond to the case most commonly considered in the literature because the measure of the output gap in (39) has a different interpretation from what most authors use. This point is emphasized by both Rotemberg and Woodford (1997) and Erceg et al. (1999).[9] By increasing s_1 and s_2, the central bank is instructed to stabilize only the component of fluctuations in the goal variables that is forecastable at a particular horizon. In respect of the zero lower bound for nominal interest rates, we append (39) to penalize excessive variation in R_t (as in the

[9]In fact, what most authors use as an output gap in their central bank objective, e.g. Taylor (1993), is exactly our variable $\hat{Y}$, the deviation of log output from its steady state. In our empirical work, we measure the steady state output level using a linear trend estimated over our sample. See Amato and Laubach (1999b) for further discussion of this issue.

previous subsection). The problem is thus to minimize

$$\begin{aligned} FBO(s_1,s_2) &= var(E_t\pi_{t+s_1,t+s_2}) + \chi_1 var(E_t[\hat{Y}_{t+s_1,t+s_2} - \hat{Y}^e_{t+s_1,t+s_2}]) \\ &+ \chi_2 var(E_t\pi^w_{t+s_1,t+s_2}) + (1+\chi_2)(\max\{k\sigma(R)-\rho,0\})^2 \end{aligned} \tag{40}$$

Inspection of the welfare loss (35) associated with variability of the output gap and wage and price inflation suggests that stabilizing merely some forecastable component of fluctuations in the goal variables is not welfare improving. Not only are *all* fluctuations in all three endogenous variables welfare-reducing, fluctuations in wage and price inflation that are *unforecastable* two periods ahead are particularly undesirable, as they cause additional distortions due to the particular specification of price setting considered in the model. Hence, the rationale that, due to lags in the transmission mechanism, monetary policy should aim at stabilizing forecasts instead of actual values of goal variables is certainly not an implication of our model. This holds despite the fact that in our model monetary policy does have lagged effects on all the endogenous variables, as Figure 1 demonstrates.

Suppose, however, that monetary policy is directed at stabilizing only a subset of the variables entering the welfare objective (37), i.e. that in (40) either χ_1 or χ_2 or both equal 0. In that particular case, stabilizing some forecastable component of the variable(s) entering (40) may lead to welfare improvements, as measured by the objective (37), compared to stabilizing all fluctuations in that subset of variables. The case of $\chi_1 = \chi_2 = 0$, combined with an appropriate horizon, may for instance be viewed as a reasonable description of inflation targeting, as suggested by Goodhart (1998). Delegation of such a restricted objective may be sensible since the delegation of a less restricted objective involves specifying values for χ_1 and χ_2. It is now widely accepted that inflation stabilization should be a goal of monetary policy, whereas, apart from the guidance provided by the welfare objective (37), it is less clear how values for χ_1 and χ_2 would be determined in the political process. In the next subsection, we report results for the cases in which either χ_1, or χ_2, or both are set to 0.

Before doing so, it may be instructive to contrast our interpretation of forecast-based monetary policy with other representations in the literature. For instance, Batini and Haldane (1999) propose an interpretation of forward-looking monetary policy based on a specific form of interest-rate rule:

$$\hat{R}_t = a(E_t\pi_{t+s} - \pi^T) + c\hat{R}_{t-1} \tag{41}$$

where π^T denotes the target value for inflation. They consider the implications of such a rule for various forecast horizons, indexed by s, in a calibrated model of a small open economy. Rules of this form are optimal in the class of unrestricted rules only if the s-period-ahead forecast of quarterly inflation is the best linear combination of the state variables, which is generally unlikely to be the case.[10] A more serious problem with such a prescription for policy is that, without assuming a substantial degree of backward-looking dynamics in the structural equations (such as in Batini and Haldane's model), such a rule leads to indeterminacy of rational expectations equilibrium for large ranges of coefficients in (41).

3.3 Results for Simple Rules

Interest-rate rules that implement the optimal plan for some given objective are generally very complicated. Rotemberg and Woodford (1999) show that, for their model, rules confined to a few terms closely approximate the welfare achieved by unrestricted optimal plans. Also, because simple rules are more transparent, they are more likely to be inferred by private agents, thereby increasing the chance that a committed policy will reap its benefits. The form of simple rule we use is a generalization of Taylor's (1993) rule that includes feedback from real wages and lagged interest rates:

$$\hat{R}_t = a\hat{w}_t + b\pi_t + c\hat{Y}_t + d\hat{R}_{t-1} \tag{42}$$

Tables 2 and 3 present results from simulating the model under various interest-rate rules, which have been obtained by minimizing the objective (40) over the coefficients a, b, c, and d in (42) for the horizons listed at the top of the table. In Table 2 we report results for the case in which $\chi_1 = \chi_2 = 0$, while in Table 3 we consider results when either χ_1 or χ_2 is different from 0. For comparison, the final two columns of Table 2 present results under the rule that minimizes the welfare criterion (37), and under the historical rule estimated in the VAR. For each different objective, the table first presents the resulting coefficients for the interest-rate rule, followed by the unconditional variances of the model's endogenous variables, the level of steady-state inflation necessary to avoid the zero lower bound for nominal interest rates to be binding, and finally the value of the term inside brackets of the welfare criterion (37). The variances of wage and price inflation and the interest rate are

[10] If the policymaker holds rational expectations, then one may substitute the reduced form for the forecast. This implies the resulting rule is operationally more cumbersome than it appears. Nonetheless, as a communication strategy, the use of forecast-based rules may be a convenient means for allowing policy to respond to a richer information set.

Table 2: Minimization of Forecast-Based Objectives: $\chi_1 = \chi_2 = 0$

$Horizon$	1	1-4	5-8	9-12	13-16	W	VAR
a	-.0374	.0250	.0509	.0137	-.1652	.2619	
b	.7567	.6953	.6749	.6405	.6101	.7578	
c	.0258	.0019	-.0061	.0022	-.0027	.0151	
d	1.071	1.028	1.017	1.011	.966	1.122	
$var(R)$	1.570	1.554	1.549	1.546	1.544	1.557	6.14
$var(\pi^w)$	2.29	2.16	2.11	2.15	2.48	2.06	3.94
$var(\pi)$	.358	.377	.395	.383	.446	.453	2.00
$var(\hat{Y})$	10.94	12.45	13.19	12.54	13.08	12.32	4.12
$var(\text{gap})$	9.29	9.65	9.92	9.73	9.94	10.10	10.76
$\bar{\pi}$	.041	.026	.021	.018	.016	.029	3.05
W	5.436	5.306	5.283	5.329	5.972	5.127	26.89

expressed in annualized percentage points, while the variance of output and the output gap are measured in percentage deviations from trend. To facilitate comparison, all variances, including those under the historical rule, have been computed under the assumption that no monetary policy shocks are present, i.e. $\epsilon_t = 0$ at all times.

The size of the coefficients in the various interest-rate rules, in particular that of a and c, is difficult to interpret, as neither the real wage nor output enters any of the objectives directly, but wage inflation and the output gap instead. The response to current inflation is strong, that to current output weak and in some cases negative, under all the rules. Most striking, however, is the size of the coefficient on the lagged interest rate. Woodford (1999b) provides a rationale for such inertial behaviour of monetary policy by arguing that in the presence of the zero lower bound on nominal interest rates and welfare costs of inflation, a commitment to highly persistent interest-rate changes is optimal.

All simulations are characterized by very low (compared to historical standards) interest rate variability. The low variability is again attributable to the high degree of interest rate inertia under all the simulated rules, and the fact that in our rational expectations model this degree of inertia is both anticipated by agents and credible. Furthermore, comparison of the values presented in the second to last line of Table 2 shows that the steady-state inflation rate $\bar{\pi}$ induced by the interest-rate variability (as discussed in section 3.1) is very small, indicating that the welfare gains from further stabilization that could be achieved by a more variable interest-rate policy are too small to warrant the concomitant increase in $\bar{\pi}$.

Significant differences in economic performance under the various interest-rate rules appear in the comparisons of the variances of inflation, output, and the output gap. First, the variance of inflation under all the simulated rules is only a small fraction of its historical value. This reflects the large weight given to inflation stabilization under all the objectives, which is furthermore perfectly understood by agents. The variance of output is much larger under any of the simulated rules than under the historical one, while the same is not true for the output gap. Whereas in the model the only rationale for output stabilization is stabilization of output around its efficient level, such behaviour does not seem to characterize historical policy. This raises the question whether the highly variable process of estimated disturbances to the efficient level of output $\hat{Y}^e$ resembles those disturbances that in the mind of policymakers were not to be accomodated by policy.

Among the various simulated rules reported in Table 2, the one minimizing expected inflation 5 to 8 quarters ahead comes closest in terms of welfare to the one that is optimal under the objective (37). Minimizing the variance of expected inflation 1 to 4 or 9 to 12 quarters ahead generates only marginally higher welfare losses. By contrast, minimizing the unconditional variance or, more severely, the variance of expected inflation at the horizon 13 to 16 quarters ahead leads to considerably higher welfare losses. The main reason for these results is the interaction between the variances of wage and price inflation. While the unconditional variances of price inflation as well as the output gap increase almost monotonically with the horizon, by moving from horizon 1 to horizon 5 to 8 the unconditional variance of wage inflation is reduced sufficiently to more than offset the welfare loss from the increase in price and output gap variance. The results thus suggest a tradeoff between the variability of wage inflation and price inflation. The variances obtained under the rule that minimizes (37) also suggest that a slight increase in the variance of price inflation is necessary to achieve a substantial reduction in the variability of wage inflation.

The first four columns in Table 3 report results for the case in which the coefficient χ_1 is set to c_1, the weight on the output gap term in the welfare objective (37), and $\chi_2 = 0$. Compared to the results presented in Table 2, the variance of wage inflation is higher for the shortest horizon, the case in which just the unconditional variances enter the objective (40), but lower for the remaining three horizons. Exactly the opposite is true for price inflation and, interestingly, the output gap, the variances of which are lower at horizon 1

Table 3: Minimization of Forecast-Based Objectives: $\chi_1, \chi_2 \neq 0$

	$\chi_1 = c_1, \chi_2 = 0$				$\chi_1 = 0, \chi_2 = c_2$			
Horizon	1	1-4	5-8	9-12	1	1-4	5-8	9-12
a	-.0306	.0559	.0610	.0870	.1281	.0509	.1517	.0427
b	.7816	.6405	.6479	.6359	.5760	.5467	.5061	.5352
c	.0364	.0024	-.0187	-.0293	.0028	-.0050	-.0663	-.0478
d	1.090	1.020	.994	.981	1.019	.985	.926	.925
$var(R)$	1.595	1.546	1.545	1.547	1.535	1.536	1.546	1.540
$var(\pi^w)$	2.31	2.10	2.08	2.06	2.01	2.06	2.32	2.20
$var(\pi)$	.358	.392	.430	.477	.442	.430	1.022	.676
$var(\hat{Y})$	10.49	12.64	14.53	15.96	13.11	13.62	27.87	20.45
var(gap)	9.23	9.84	10.43	11.08	10.36	10.32	18.21	13.22
$\bar{\pi}$	.065	.018	.016	.019	.007	.008	.018	.012
W	5.443	5.257	5.336	5.399	5.197	5.326	6.894	6.060

when the output gap term is included in (40), but higher at the remaining horizons. Hence, charging the central bank with output gap stabilization seems to have the desired effect only if the central bank is charged with minimizing the variability of all fluctuations, and not just forecastable ones, in the goal variables. In terms of welfare, the conflicting changes in the variances of the various goal variables as the horizon changes lead to the lowest welfare loss for horizon 1 to 4, but affords only a slight reduction in welfare losses compared to the case without output gap stabilization entering (40). Increasing χ_1 to values larger than c_1 leads initially to marginal improvements in welfare, but already at a value $\chi_1 = 0.1$ welfare is reduced compared to $\chi_1 = 0$.

The last four columns in Table 3 consider the case in which $\chi_1 = 0$ and χ_2 is set equal to the coefficient c_2 in (37). In this case, stabilization of *all* fluctuations in wage and price inflation leads to a sizeable reduction in the variance of wage inflation, as one might expect, compared to the case of $\chi_2 = 0$, while the variances of price inflation and the output gap increase moderately. Of all the rules considered in Tables 2 and 3, this one is the closest approximation to the welfare-optimal rule displayed in Table 2. By contrast, stabilizing only those fluctuations in price and wage inflation which are forecastable more than four quarters ahead leads to much higher variances of the output gap and price and wage inflation compared to the case of $\chi_2 = 0$. Somewhat paradoxically, while stabilizing only expected price inflation at a horizon of 5 to 8 quarters helped reduce the unconditional variance of wage inflation, stabilizing both expected price and wage inflation at the same horizon leads to a sharp increase in the unconditional variances of both.

4 Conclusions

The goal of this study is to establish which structural features of the economy might suggest the use of forecasts in the process of setting interest rates. To evaluate the welfare consequences of delegating certain forecast-based objectives to the central bank, we specify a small structural model with staggered wage and price setting, and use the representative household's welfare as the measure for evaluation. Our two findings are, first, if an objective defined only in terms of price inflation variability is delegated to the central bank, then indeed stipulating that only the fluctuations forecastable 5 to 8 quarters ahead should be stabilized is welfare-improving. Second, however, if the objective delegated to the central bank may be specified over the variability of more than one goal variable, the best alternative to delegating the welfare objective (37) directly is to delegate the minimization of the unconditional variances of both wage and price inflation. In this case, stabilizing merely forecastable fluctuations in these two variables leads to significant welfare losses.

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A Log-linear Approximations

A.1 Wage and Price Inflation

In this Appendix we derive equations (23) and (28). The first step is to compute a log-linear approximation to equation (22)). Let $\hat{v}_t^1 \equiv \log(W_t^1/W_t)$. The ratio W_t^1/W_T can then be approximated as $\hat{v}_t^1 - \sum_{k=1}^{T-t} \pi_{t+k}^w$. Similarly, the ratio

$$\frac{W_t^1}{P_T} = \frac{W_t^1}{W_t}\frac{W_t}{P_t}\frac{P_t}{P_T}$$

is approximated by $\hat{v}_t^1 + \hat{w}_t - \sum_{k=1}^{T-t} \pi_{t+k}$. Finally, using the production function (6), the deviation of hours from steady state can be expressed as $\hat{H}_t = \frac{1}{1-a}(\hat{Y}_t - \eta_t)$.

With this notation, the log-linear approximation of (22) can be written as

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left\{\omega\left[\frac{\hat{Y}_T - \eta_T}{1-a} - \tilde{\zeta}_T - \phi(\hat{v}_t^1 - \sum_{k=1}^{T-t}\pi_{t+k}^w)\right] - \hat{\lambda}_T - (\hat{v}_t^1 + \hat{w}_t - \sum_{k=1}^{T-t}\pi_{t+k})\right\} = 0, \quad (43)$$

where $\tilde{\zeta}_t \equiv -(v_{h\zeta}(\bar{H};0)/v_{hh}(\bar{H};0)\bar{H})\zeta_t$ is the disturbance to the marginal disutility of labour supply. Combining (16) and (19) yields

$$E_{t-1}\hat{\lambda}_T = -\sigma E_{t-1}[\hat{Y}_T - \hat{G}_T] \;\forall T \geq t+1 \quad (44)$$

while taking expectations as of $t-1$ of (15) yields

$$\begin{aligned} E_{t-1}\hat{\lambda}_t &= E_{t-1}[\hat{R}_t - \pi_{t+1} + \hat{\lambda}_{t+1}] \\ &= E_{t-1}[\hat{R}_t - \pi_{t+1} - \sigma(\hat{Y}_{t+1} - \hat{G}_{t+1})] \\ &= -\sigma E_{t-1}[\hat{Y}_t - \hat{G}_t] + \nu_{t-1} \end{aligned} \tag{45}$$

where

$$\begin{aligned} \nu_{t-1} &\equiv E_{t-1}[R_t - \pi_{t+1} - \sigma(\hat{Y}_{t+1} - \hat{Y}_t - \hat{G}_{t+1} + \hat{G}_t)] \\ &= E_{t-1}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) - E_{t-2}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) \end{aligned}$$

Substituting these expressions for $E_{t-1}\hat{\lambda}_T$ into (43) and collecting terms, (43) can be written as

$$\begin{aligned} E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\Bigg\{\left(\frac{\omega}{1-a}+\sigma\right)\hat{Y}_T - \frac{\omega}{1-a}\eta_T - \omega\tilde{\zeta}_T - \sigma\hat{G}_T \\ -(1+\omega\phi)\hat{v}_t^1 + \omega\phi\sum_{k=1}^{T-t}\pi_{t+k}^w - \hat{w}_t + \sum_{k=1}^{T-t}\pi_{t+k}\Bigg\} - \nu_{t-1} = 0. \end{aligned} \tag{46}$$

Furthermore, we transform the double summation

$$\begin{aligned} \sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\sum_{k=1}^{T-t}\pi_{t+k} &= \sum_{T=t+1}^{\infty}(\lambda\beta)^{T-t}\sum_{k=0}^{\infty}(\lambda\beta)^k\pi_T \\ &= (1-\lambda\beta)^{-1}\left(\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\pi_T - \pi_t\right) \end{aligned}$$

The double sum involving π_t^w is transformed analogously.

We next wish to obtain an expression for $\hat{v}_t^1$ in terms of π_t^w. Dividing both sides of (20) by W_t and taking the logarithm yields

$$0 \simeq (1-\lambda)\gamma^w\hat{v}_t^1 + (1-\lambda)(1-\gamma^w)\hat{v}_t^2 - \lambda\pi_t^w \tag{47}$$

Since $W_t^2 = E_{t-2}W_t^1$,

$$\hat{v}_t^2 = E_{t-2}\hat{v}_t^1 - (\pi_t^w - E_{t-2}\pi_t^w) \tag{48}$$

Substituting this expression into (47) we obtain

$$\pi_t^w = \frac{1-\lambda}{\lambda}\left[\gamma^w\hat{v}_t^1 + (1-\gamma^w)(E_{t-2}\hat{v}_t^1 - (\pi_t^w - E_{t-2}\pi_t^w))\right] \tag{49}$$

Taking expectations as of $t-2$ on both sides, $E_{t-2}\pi_t^w = \frac{1-\lambda}{\lambda}E_{t-2}\hat{v}_t^1$ and hence

$$\frac{1-\lambda}{\lambda}\hat{v}_t^1 = \frac{1}{\psi^w}\pi_t^w - \frac{1-\psi^w}{\psi_w}E_{t-2}\pi_t^w \tag{50}$$

where $\psi^w \equiv \gamma^w\lambda/(1-\gamma^w(1-\lambda))$ is defined as in (23). Substituting (50) for $\hat{v}_t^1$ in (46) and using the transformation for the double sums and the fact that $E_{t-1}\nu_{t+j} = 0\ \forall j \geq 0$ we obtain (23).

The derivation of (28) involves the same steps as above. Let $\hat{p}_t^1 \equiv \log(p_t^1/P_t)$. Then (27) can be approximated as

$$E_{t-1}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left\{\frac{1-a+\theta a}{1-a}\hat{p}_t^1 - \frac{1}{1-a}(a\hat{Y}_T - \eta_T) - \hat{w}_t - \sum_{k=1}^{T-t}\pi_{t+k}^w - \frac{\theta a}{1-a}\sum_{k=1}^{T-t}\pi_{t+k}\right\} = 0. \quad (51)$$

The double sums in (51) are being transformed as before. Furthermore, dividing (25) by P_t and taking the logarithm, and using the fact that $p_t^2 = E_{t-2}p_t^1$, we can derive an expression for $\hat{p}_t^1$ in terms of π_t analogous to (50),

$$\frac{1-\alpha}{\alpha}\hat{p}_t^1 = \frac{1}{\psi^p}\pi_t - \frac{1-\psi^p}{\psi_p}E_{t-2}\pi_t \quad (52)$$

where $\psi^p \equiv \gamma^p\alpha/(1-\gamma^p(1-\alpha))$ is defined as in (28). Substituting (52) for $\hat{p}_t^1$ in (51) and using the transformation for the double sums we obtain (28).

A.2 The Representative Household's Welfare

In this Appendix we derive the second-order approximation (33) to the representative household's welfare (32), using some results of Rotemberg and Woodford's (1997) Appendix 3. Specifically, we form a second-order Taylor series expansion of (32) around the steady state characterized by the efficient output level $\bar{Y}$ defined in (12) and zero wage and price inflation. Hence, we form the approximation around the same steady state around which the model's exact equilibrium conditions have been log-linearized.

Since the demand side of our model is identical to Rotemberg and Woodford's, the second-order approximation of $u(C_t;\xi_t)$ is identical to their equation (9.10) as well, which we reproduce here:

$$u(C_t;\xi_t) = u_c\bar{Y}\hat{Y}_t + \frac{1}{2}(u_c\bar{Y} + u_{cc}\bar{Y}^2)\hat{Y}_t^2 - u_{cc}\bar{Y}^2\hat{G}_t\hat{Y}_t + unf + tip + \mathcal{O}(||\xi||^3) \quad (53)$$

where unf stands for terms that are unforecastable two periods ahead (since in our model monetary policy affects output only with a lag of two periods), and tip denotes terms that are independent of monetary policy. $||\xi||$ is a bound on the amplitude of fluctuations in the exogenous disturbances, which we take to be the same for ξ, ζ, and η. The term $\mathcal{O}(||\xi||^3)$ indicates that terms of third or higher order in the deviations of the various variables from their steady-state values are being neglected.

Similarly, a second-order approximation of household i's disutility of labour supply is given by

$$v(h_t^i;\zeta_t) = v_h\bar{H}\hat{h}_t^i + \frac{1}{2}(v_h\bar{H} + v_{hh}\bar{H}^2)\hat{h}_t^{i2} - v_{hh}\bar{H}^2\tilde{\zeta}_t\hat{h}_t^i + tip + \mathcal{O}(||\xi||^3). \quad (54)$$

Integrating this expression over i yields

$$\int_0^1 v(h_t^i;\zeta_t)di = v_h\bar{H}E_i[\hat{h}_t^i] + \frac{1}{2}(v_h\bar{H} + v_{hh}\bar{H}^2)\left(E_i[\hat{h}_t^i]^2 + var_i(\hat{h}_t^i)\right) - v_{hh}\bar{H}^2\tilde{\zeta}_tE_i[\hat{h}_t^i] + tip + \mathcal{O}(||\xi||^3). \quad (55)$$

By integrating (7) over z, we obtain

$$H_t = \left[\int_0^1 (h_t^i)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}. \tag{56}$$

Using this expression and the fact that for a random variable X, $\log E[X] = E[\log X] + \frac{1}{2} var(\log X)$, we obtain that

$$\hat{H}_t \equiv \log(H_t/\bar{H}) = E_i[\hat{h}_t^i] + \frac{\phi-1}{2\phi} var_i(\hat{h}_t^i). \tag{57}$$

Solving (57) for $E_i[\hat{h}_t^i]$ and substituting in (55) yields

$$\int_0^1 v(h_t^i;\zeta_t)di = v_h \bar{H}\hat{H}_t + \frac{v_h\bar{H}}{2}(1+\omega)\hat{H}_t^2$$
$$+\frac{v_h\bar{H}}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i) - v_{hh}\bar{H}^2\tilde{\zeta}_t\hat{H}_t + tip + \mathcal{O}(\|\xi\|^3) \tag{58}$$

where ω is defined as in (23).

We next wish to substitute for $\hat{H}_t$ in (58) in terms of output. To do so, note first that the definition of $H_t = \int_0^1 H_t(z)dz$ implies that

$$\hat{H}_t = E_z[\hat{H}_t(z)] + \frac{1}{2} var_z(\hat{H}_t(z)). \tag{59}$$

Firms' production function in turn implies that

$$E_z[\hat{H}_t(z)] = (1-a)^{-1}(E_z[\hat{y}_t(z)] - \eta_t), \;\; var_z(\hat{H}_t(z)) = (1-a)^{-2} var_z(\hat{y}_t(z)) \tag{60}$$

and therefore

$$\hat{H}_t = (1-a)^{-1}(E_z[\hat{y}_t(z)] - \eta_t) + \frac{1}{2(1-a)^2} var_z(\hat{y}_t(z)). \tag{61}$$

Finally, deriving an expression for $\hat{Y}_t$ analogous to (59), substituting from this expression for $E_z[\hat{y}_t(z)]$ in (61), and substituting the resulting expression for $\hat{H}_t$ into (58) yields

$$\int_0^1 v(h_t^i;\zeta_t)di = \frac{v_h\bar{H}}{1-a}\left[\hat{Y}_t + \frac{1+\omega}{2(1-a)}\hat{Y}_t^2\right] - \frac{v_h\bar{H}}{1-a}\left[\omega\tilde{\zeta}_t\hat{Y}_t + \frac{1+\omega}{1-a}\eta_t\hat{Y}_t\right]$$
$$+\frac{v_h\bar{H}}{1-a}\left[\frac{1}{2}\left(\frac{1}{1-a} - \frac{\theta-1}{\theta}\right) var_z(\hat{y}_t(z)) + \frac{1-a}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i)\right] + tip + \mathcal{O}(\|\xi\|^3). \tag{62}$$

Because the efficient steady-state level of output is characterized by (12), it follows that

$$\frac{v_h\bar{H}}{1-a} = u_c\bar{Y}.$$

Hence,

$$u(C_t;\xi_t) - \int_0^1 v(h_t^i;\zeta_t)di = u_c\bar{Y}\left[\frac{\omega+a+\sigma(1-a)}{1-a}\left(\hat{Y}_t\hat{Y}_t^e - \frac{1}{2}\hat{Y}_t^2\right)\right.$$
$$\left.-\frac{1}{2}\left(\frac{1}{1-a} - \frac{\theta-1}{\theta}\right) var_z(\hat{y}_t(z)) - \frac{1-a}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i)\right] + tip + \mathcal{O}(\|\xi\|^3) \tag{63}$$

where $\hat{Y}_t^e$ is the efficient level of output defined in (36). Taking the unconditional expectation of (63) then leads to an expression for (32) of the form

$$W = -\frac{u_c \bar{Y}}{2}\left[\frac{\omega + a + \sigma(1-a)}{1-a}\left(E[\hat{Y}_t^2] - 2E[\hat{Y}_t \hat{Y}_t^e]\right)\right.$$
$$\left. + \left(\frac{1}{1-a} - \frac{\theta - 1}{\theta}\right) E[var_z(\hat{y}_t(z))] + (1-a)(\phi^{-1} + \omega) E[var_i(\hat{h}_t^i)]\right] + tip + \mathcal{O}(\|\xi\|^3). \quad (64)$$

We now wish to substitute for each of the three terms involving unconditional expectations in (64). First, rearranging the definition of $var(\hat{Y}_t - \hat{Y}_t^e)$ yields

$$E[\hat{Y}_t^2] - 2E[\hat{Y}_t \hat{Y}_t^e] = var(\hat{Y}_t - \hat{Y}_t^e) + E[\hat{Y}_t]^2 - E[\hat{Y}_t^{e2}] + E[\hat{Y}_t^e]^2 - 2E[\hat{Y}_t]E[\hat{Y}_t^e] \quad (65)$$

The second and last terms on the right-hand side of (65) are zero because the unconditional expectation of output from its long-run trend is zero by definition. The third and fourth terms equal $-var(\hat{Y}_t^e)$, a term that is independent of policy. Hence, in (64) we can substitute $var(\hat{Y}_t - \hat{Y}_t^e)$ for the left-hand side of (65). Taking account of the fact that interest rates affect output only with two periods lag, we instead susbtitute $var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e])$ in (64).

Second, from the demand functions for households' labour services (9) and producers' goods (10) it follows that

$$E[var_i(\hat{h}_t^i)] = \phi^2 E[var_i(\log W_t^i)] \quad (66)$$

and

$$E[var_z(\hat{y}_t(z))] = \theta^2 E[var_z(\log p_t(z))]. \quad (67)$$

Following the argument in Rotemberg and Woodford's Appendix 3, these equations can be rewritten as

$$E[var_i(\hat{h}_t^i)] = \phi^2 \frac{\lambda}{(1-\lambda)^2}\left[var(\pi_t^w) + (\psi^{w^{-1}} - 1) var(\pi_t^w - E_{t-2}\pi_t^w) + (E\pi_t^w)^2\right] \quad (68)$$

and

$$E[var_z(\hat{y}_t(z))] = \theta^2 \frac{\alpha}{(1-\alpha)^2}\left[var(\pi_t) + (\psi^{p^{-1}} - 1) var(\pi_t - E_{t-2}\pi_t) + (E\pi_t)^2\right] \quad (69)$$

where ψ^w and ψ^p are defined as in (23) and (28) respectively. Substituting (65), (68), and (69) into (64) and noting that

$$(1-a)(\phi^{-1} + \omega)\phi^2 \frac{\lambda}{(1-\lambda)^2} = \frac{1 - \lambda\beta}{(1-\lambda)\kappa^w}\phi((1-a)\sigma + \omega)$$

and

$$\left(\frac{1}{1-a} - \frac{\theta - 1}{\theta}\right)\theta^2 \frac{\alpha}{(1-\alpha)^2} = \frac{1 - \alpha\beta}{(1-\alpha)\kappa^p}\frac{\theta a}{1-a}$$

we obtain (33), where

$$\Omega \equiv \frac{u_c \bar{Y}}{2}\frac{1 - \alpha\beta}{(1-\alpha)\kappa^p}\frac{\theta a}{1-a}$$

$$c_1 \equiv \left[\frac{1 - \alpha\beta}{(1-\alpha)\kappa^p}\frac{\theta a}{1-a}\right]^{-1}\frac{\omega + a + \sigma(1-a)}{1-a}$$

and

$$c_2 \equiv \left[\frac{1-\alpha\beta}{(1-\alpha)\kappa^p}\frac{\theta a}{1-a}\right]^{-1} \frac{1-\lambda\beta}{(1-\lambda)\kappa^w}\phi((1-a)\sigma+\omega).$$

Figure 1: Impulse Responses to a Monetary Shock