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Optimal Monetary Policy Regime Switches*

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Abstract

Given regime switches in the economy's growth rate, optimal monetary policy rules may respond by switching policy parameters. These optimized parameters differ across regimes and from the optimal choice under fixed regimes, particularly in the inflation target and interest rate inertia. Optimal switching rules produce welfare gains relative to constant rules, with switches in the implicit real interest rate used for policy and the degree of interest rate inertia producing the largest gains. However, gains from switching rules decrease if the monetary authority trades-off the probability of low rates, or if it may misidentify the regime.

Keywords: growth rate; optimal policy; regime switching; Taylor rule; inflation target

JEL Codes: C63, E31, E52

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1 Introduction

The slow growth after the financial crisis prompted a renewed debate about objectives and conduct of monetary policy. In particular, slow growth and a number of additional factors have contributed to lowering the equilibrium real rate of interest, leading to questions about how, if at all, the systematic conduct of monetary policy should respond to this shift. This debate appeared in public speeches by policymakers, and is highlighted in minutes of the Federal Open Market Committee in April, 2015:

> "One participant suggested that, in part because of the evidence that the equilibrium real interest rate was low by historical standards, the Committee should discuss the possibility of increasing its longer-run inflation objective...[Another participant] noted, in particular, that a decision to raise the Committee's longer-run inflation objective might work against the achievement of maximum employment and price stability because such a change could undermine the Committee's credibility and, in addition, lead to adverse changes in inflation dynamics that could pose significant challenges for policy-makers."
> – Board of Governors of the Federal Reserve System (2015)

The quotation highlights opposing views on whether monetary policy objectives should change given a lower real interest rate, and some of the counter-arguments for doing so. Even more recently, Federal Reserve officials have framed structural change in explicit terms of regime shifts, and have thought about appropriate policy responses (Bullard (2016)).

More broadly, after the crisis some economists argued that the Federal Reserve should alter its implicit rule to allow either a higher inflation target, a less aggressive response to inflation deviations, or a more aggressive response to the output gap. For example, Rogoff (2008), Blanchard et al. (2010), and Ball (2013) called for an explicit move to an inflation target from 2 percent to the range of 4-6 percent. Others, such as Taylor (2012), saw changes in the implicit policy rule and called for a return to rules used at an earlier time.

This paper studies optimal simple rules for monetary policy when the structural economy experiences regime shifts. Using a standard New Keynesian model modified so that growth rate switches between high and low growth regimes, it finds optimal simple rules that are fully regime dependent, and compares their performance against optimized rules that constrain some or all of the policy parameters to be identical across regimes. The switching in the growth rate process tends to generate a low real interest rate in one regime, which causes the monetary authority to need to adjust its policy rule.

For the optimal rules, the policy parameters differ from those that would be chosen if each regime occurred in isolation, indicating that modifying the rule due to the presence of regime switching is important to the conduct of optimal policy. Further, even conditional on starting from an environment with a fixed policy rule that does not adjust with regime changes, changing to rules with switching parameters increases welfare. A rule where all policy parameters switch is welfare-preferred, but rules that only allow a subset of policy parameters to change increase welfare as well. In particular, a rule that keeps all policy parameters fixed except the real rate used in the policy rule in each regime generates over half the welfare gains of a fully switching environment; on top of this change, allowing changes in the degree of interest rate inertia nearly generates the entire welfare gains of a fully switching rule. These results suggest that allowing flexibility for monetary policy in the face of shifts in the structural economy is important for conducting monetary policy, and constraining policy to follow a single rule regardless of the regime can generate inferior economic outcomes.

Switches between high and low growth rate regimes drive persistent fluctuations in the real interest rate of the economy, but are not the only way of generating such fluctuations. An alternative setup, under which regimes in preferences between intertemporal utility drive changes in the real interest rate, generates similar optimal policy outcomes. This result highlights that it is not changes in the growth rate that matter directly, but rather growth rate switches matter because they affect the real interest rate.

As highlighted in the above quotation, however, there are possibly reasons to doubt the advantages of allowing flexibility in the policy rule. In particular, the baseline optimal policy results assume the monetary authority wishes to keep the nominal interest rate positive across regimes at the same rate. As a result, in a low growth regime, the real interest rate being low warrants a large shift in the policy rule relative to the high growth regime. If instead, the monetary authority is willing to make a trade off in the frequency of a positive nominal rate, the policy rule needs to shift less and the gains to having a switching rule decline.

A second reason why a fully switching monetary policy rule may be less optimal than the baseline results suggest is if the monetary authority possibly misperceives the regime. Having a flexible rule can generate benefits if the low growth rule is put in place only when the economy is actually in the low growth regime, but it could generate unnecessary distortions such as through a higher inflation target if the economy is actually in the high growth regime. As a result, the gains to having a fully flexible rule relative to a constant rule will decrease as the chance of misidentifying the regime increase.

Much of the literature on optimal monetary policy with simple rules assumes constant rules over time. For example, Schmitt-Grohe and Uribe (2007) characterize optimal simple rules in an economy without instability in the structural economy, and show these rules nearly replicate welfare achieved by a Ramsey planner. In the context of regime switches in the structural economy, this paper shows simple rules that switch alongside the structural economy welfare dominate fixed rules. More recently, Billi (2011), Coibion et al. (2012), and Blanco (2015) consider the optimal inflation target in the presence of the zero lower bound on the nominal interest rate. In these cases, the monetary authority sets a constant inflation target that weighs the costs of higher inflation against the chance of hitting the zero lower bound. The optimal simple rules considered in this paper allow this trade-off to be regime-dependent, which enables the authority to set a higher inflation target in regimes where hitting the zero bound is relatively more likely. Further, this paper shows that the optimal choice of switching parameters depends heavily on the optimal choice of fixed parameters. For example, the optimal choice of inflation targets across regimes also depends on the choice of inertia common to both regimes.

Papers studying switching in monetary policy rules often have non-optimal switches that are independent of any underlying changes in the economy. For example, Davig and Leeper (2007) and Bianchi (2013) consider switches in the coefficients dictating how the monetary authority responds to deviations from its targets, Schorfheide (2005) and Liu et al. (2011) allow for switches in the inflation target, and Foerster (2016) considers both types. However, in each of these frameworks, changes in the monetary policy rule occur randomly and without regard to the state of the private economy. In contrast, this paper motivates regime switching in the policy rule as an optimal response to switches in the private economy.

In the case where papers consider optimal policy with regime switching in the private economy, the monetary authority may face a reduced-form representation of the structural economy as in Blake and Zampolli (2011). On the other hand, Debortoli and Nunes (2014) interpret regime switching in monetary policy as coming from explicit changes in the authority's loss function. Davig (2016) shows how regime switches in price-setting behavior in the structural economy map into switches in the loss function when the authority operates with discretion. In contrast to these frameworks, this paper considers optimal simple rules, and how changes in the structural economy affect the optimal choice of policy parameters.

The remainder of the paper proceeds as follows: Section 2 presents the model, Section 3 discusses computation of the optimal policy rules, Section 4 contains the main results on optimal policy rules with growth rate switches, Section 5 shows the main results hold in an alternative environment with preference regimes, Section 6 considers two modified models that push optimal rules towards less switching, and Section 7 concludes.

2 Model

This section describes a prototypical New Keynesian model. The six key features of the model are: (i) nominal rigidities without indexation of prices that create a role for inflation stabilization near price stability, (ii) a markup shock that generates a trade-off between output and inflation stabilization, (iii) a preference shock that affects the inter-temporal decisions of the household, (iv) regime switching that affects the growth rate of the economy, and (v) government spending financed solely by lump-sum taxes, and (vi) a monetary authority with only access to a Taylor rule for nominal interest rates.

The following presents the model's several parts: households, final and intermediate goods firms, fiscal policy, the monetary authority, and how regimes switch.

2.1 Households

Households maximize lifetime expected discounted utility of the form

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left(\log C_t - H_t\right), \tag{1}$$

where $\mathbb{E}_0$ is the expectations operator conditional on information at time 0, $\beta \in (0,1)$ is the discount factor, d_t is an inter-temporal preference shifter, C_t is consumption, and H_t is hours worked. Households face the budget constraint

$$C_t + \frac{B_t}{P_t} + T_t = W_t H_t + R_{t-1}\frac{B_{t-1}}{P_t} + D_t, \tag{2}$$

where B_t denotes bonds purchased at time t that pay out a gross nominal interest rate R_t at $t+1$, T_t is real lump-sum taxes paid to the government, W_t is the real wage rate, and D_t is real dividend payments from firms. The inter-temporal preference shifter follows

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}. \tag{3}$$

Standard optimality conditions for the household produce an Euler equation of the form

$$\beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}}\right)\left(\frac{d_{t+1}}{d_t}\right)\frac{R_t}{\Pi_{t+1}} = 1, \tag{4}$$

which highlights how, all else equal, shocks or regime shifts that decrease the growth rate of consumption will increase the term C_t/C_{t+1} and hence will tend to lower the nominal interest rate. Likewise, given the autoregressive process in equation (3), negative realizations of $\varepsilon_{d,t}$ will tend to increase d_{t+1}/d_t, which, all else equal, will then tend to lower R_t.

2.2 Firms

There are two types of firms: intermediate goods firms that produce with labor, and final good firms that bundle intermediate goods into a final output to be consumed by households and the government.

2.2.1 Final Good Firms

A competitive final good producer combines a continuum of intermediate goods $Y_{j,t}$, $j \in [0,1]$, by constant elasticity of substitution technology to produce a final good

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{1}{1+m_t}} dj \right)^{1+m_t}, \tag{5}$$

where m_t denotes the time-varying net markup. This specification implies the demand for a good $Y_{j,t}$ depends on its relative price, the markup, and aggregate demand by

$$Y_{j,t}^d = \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1+m_t}{m_t}} Y_t. \tag{6}$$

The net markup m_t follows an autoregressive process

$$\log m_t = (1-\rho_m) \log m_{ss} + \rho_m \log m_{t-1} + \sigma_m \varepsilon_{m,t}. \tag{7}$$

Positive markup shocks $\varepsilon_{m,t}$ produce opposite movements in inflation and output, generating a trade-off for their stabilization.

2.2.2 Intermediate Goods Firms

Intermediate goods producers are indexed by j and have production functions

$$Y_{j,t}^s = A_t H_{j,t}, \tag{8}$$

where total factor productivity nests stationary and unit root components:

$$\log A_t = \log \omega(s_t) + \log A_{t-1} + a_t, \tag{9}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}. \tag{10}$$

In this case, the mean growth rate of TFP, $\omega(s_t)$, switches with the regime variable s_t.

Intermediate goods firms adjust prices according to Rotemberg pricing without indexation of prices to inflation. Consequently, the firm's maximization problem is to choose $H_{j,t}$ and $P_{j,t}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0 P_t} \left(\left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1}{m_t}} Y_t - W_t H_{j,t} - \frac{\phi_p}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right) \tag{11}$$

where λ_t denotes the marginal utility of consumption for the household. Firms are also subject to the constraint that supply (8) must meet demand (6) at the posted price. The price adjustment generates a cost of inflation in output terms, as deviations from price stability generate progressively higher losses in output.

2.3 Fiscal Policy

The government purchases a fraction ζ_t of aggregate output Y_t, so

$$G_t = \zeta_t Y_t. \tag{12}$$

The fraction of goods purchased satisfies

$$g_t = \frac{1}{1-\zeta_t} \tag{13}$$

where g_t follows an autoregressive process

$$\log g_t = \left(1-\rho_g\right)\log g_{ss} + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t}. \tag{14}$$

The government collects lump-sum taxes to cover spending, and nominal bonds are in zero net supply. Hence the aggregate resource constraint is given by

$$Y_t = C_t + G_t + \frac{\phi_p}{2}\left(\Pi_t - 1\right)^2 Y_t. \tag{15}$$

This resource constraint again highlights the cost of inflation, as deviations from price stability produce losses in output that cannot go to consumption or the government. Importantly, fiscal policy does not have access to a production subsidy to firms that eliminates the distortions associated with monopolistic competition; the inefficiency from imperfect competition must be taken into account by the monetary authority when setting policy (Woodford (2003)).

2.4 Monetary Policy

The monetary authority follows a Taylor rule of the form

$$\frac{R_t}{R^*\left(s_t\right)} = \left(\frac{R_{t-1}}{R^*\left(s_t\right)}\right)^{\rho_r(s_t)} \left(\left(\frac{\Pi_t}{\Pi^*\left(s_t\right)}\right)^{\psi_\pi(s_t)} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{ss}}\right)^{\psi_y(s_t)}\right)^{1-\rho_r(s_t)}, \tag{16}$$

which includes

$$R^*(s_t) = \Pi^*(s_t)\, r(s_t), \text{ where } r(s_t) = \frac{\omega(s_t)}{\beta}. \tag{17}$$

This form allows for regime switches in the real interest rate $r(s_t)$, the inflation target $\Pi^*(s_t)$, the degree of interest rate inertia $\rho_r(s_t)$, and the responsiveness to the inflation and output gaps, $\psi_\pi(s_t)$ and $\psi_y(s_t)$, respectively. The output gap is in terms of de-trended output $\tilde{Y}_t = Y_t/A_t$. Note that the neutral nominal rate $R^*(s_t)$ switches, and is made up of the inflation target, $\Pi^*(s_t)$, and the steady state real rate, $r(s_t)$, that would prevail if each regime occurred in isolation.

2.5 Regime Switching

To summarize the regime switching in the model, the structural economy experiences switches in the growth rate $\omega(s_t)$. At the same time, the monetary policy rule switches parameters ($r(s_t)$, $\rho_r(s_t)$, $\psi_\pi(s_t)$, $\psi_y(s_t)$, $\Pi^*(s_t)$). The regime follows a Markov process governed by a transition matrix with elements $P_{i,j} = \Pr(s_t = j|s_{t-1} = i)$. Assuming two regimes with symmetric probabilities given by $p = \Pr(s_t = s_{t-1})$, the transition matrix is given by

$$P = \begin{bmatrix} P_{1,1} & P_{1,2} \\ P_{2,1} & P_{2,2} \end{bmatrix} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}. \tag{18}$$

3 Computation and Welfare

Given the model previously described, this section turns to the calibration and solution method, how the monetary authority sets optimal implementable rules, and the welfare calculations.

3.1 Calibration and Solution

The set of parameters shown in Table 1 describe preferences and production assuming the unit of time is a quarter. These parameters largely follow the estimates in a similar model by Schorfheide (2005). The economy switches between two regimes of high and low growth with probability $p = 0.95$. In particular, in the first, high growth regime, the economy grows at 2.5%

Table 1: Fixed Parameters

Description	Parameter Value
Discount Factor	$\beta = 0.9987$
Steady State Fraction of Government Purchases	$\zeta_{ss} = 0.2$
Steady State Net Markup	$m_{ss} = 0.10$
Cost of Price Adjustment	$\phi_p = 160$
Technology Shock Persistence	$\rho_a = 0.8$
Government Spending Shock Persistence	$\rho_g = 0.9$
Inter-temporal Preference Shock Persistence	$\rho_d = 0.9$
Markup Shock Persistence	$\rho_m = 0.9$
Technology Shock Std Dev	$\sigma_a = 0.003$
Government Spending Shock Std Dev	$\sigma_g = 0.012$
Inter-temporal Preference Shock Std Dev	$\sigma_d = 0.01$
Markup Shock Std Dev	$\sigma_m = 0.01$
Growth Rate, High Growth Regime	$\omega(1) = 1.025^{1/4}$
Growth Rate, Low Growth Regime	$\omega(2) = 1.005^{1/4}$
Transition Probability	$p = 0.95$

annually, while in the second, low growth regime, the economy grows at 0.5% annually. These two regimes capture concerns about switches to slower growth regimes (for example, Gordon (2012)), as well as a lower real interest rate (Summers (2014)). The presence of growth rate switches affects the household's Euler equation (4) and the intermediate goods firms' problem via technology (8). In addition, lower growth, by affecting the real interest rate, also changes the neutral nominal rate in the monetary policy rule (17). In the low growth regime, the real rate tends to be low, which corresponds to a lower–and hence a greater chance of negative–nominal rate.

Given the switching in the structural economy and the monetary policy rule, as well as a need to perform welfare calculations, the results in Section 4-6 use the perturbation method for Markov-switching DSGE models from Foerster et al. (2013). This methodology allows for second-order approximations to the decision rules, which enable accurate welfare calculations and capture the certainty non-equivalence generated by Markov-switching, especially in the monetary policy rule (see Foerster (2016)). In addition, perturbation allows for checking determinacy generated by the monetary policy rule, which is key for policy to be implementable.

3.2 Optimal Implementable Rules

Given the economy described by the parameterizations in Table 1, the monetary authority sets the policy parameters for each regime $\left\{r\left(s_t\right), \rho_r\left(s_t\right), \psi_\pi\left(s_t\right), \psi_y\left(s_t\right), \Pi^*\left(s_t\right)\right\}$ in the simple rule given by equation (16) to be optimal within the class of implementable rules. The following two definitions detail these terms.

Definition 1 (Implementable) *For a policy rule to be implementable, it must meet the following three conditions:*

1. *The policy parameters generate a unique equilibrium when considering mean square stability (MSS) of minimum state variable (MSV) solutions.*

2. *The stochastic steady state in each regime admit non-negative dynamics for the net nominal interest rate. If $\boldsymbol{\mu}_R\left(s_t\right)$ and $\boldsymbol{\sigma}_R\left(s_t\right)$ denote the mean and standard deviations for the log of the gross interest rate conditional on regime s_t, respectively, then this condition requires*

$$\boldsymbol{\mu}_R\left(s_t\right) - 2\boldsymbol{\sigma}_R\left(s_t\right) > 0 \text{ for each } s_t. \tag{19}$$

3. *The policy coefficients are in the following intervals for each s_t: $\rho_r\left(s_t\right) \in [0,1)$, $\psi_\pi\left(s_t\right) \in [0,5]$, and $\psi_y\left(s_t\right) \in [0,5]$.*

Condition (1) requires that the policy parameters across regimes produce a unique equilibrium when considered as a whole. The MSS concept allows temporarily explosive regimes as long as the entire system has finite first and second moments in expectation. Under these circumstances, satisfying determinacy regime-by-regime is neither necessary nor sufficient for achieving determinacy overall, and determinacy regions can be complex functions of all the policy parameters (see Davig and Leeper (2007) and Foerster (2016)).

Condition (2) imposes that hitting a negative net nominal interest rate should be at least a two standard deviation event in each regime, which ensures a sufficiently low probability of that occurring. Note that the perturbation solution approach allows these negative dynamics, and this condition requires a low volatility relative to the average value in each regime.[1]

Lastly, condition (3) restricts the policy parameters to intervals that are the correct sign and within reasonable bounds. In particular, the monetary authority increases the nominal rate with inflation and the output gap, but there is a limit on how strongly they can respond, and the nominal rate can have positive inertia.[2]

As Section 4 shows, two key features of the implementability definition play an important role in the results: the two standard deviation part of condition (2) and the upper bound of 5 on $\psi_\pi(s_t)$ in condition (3). The results highlight how these assumptions impact the results, and the Appendix shows robustness to alternatives–namely that making condition (2) be a one or three standard deviation constraint, and raising or lowering the upper bound on $\psi_\pi(s_t)$–generate similar qualitative results.

[1] From a technical standpoint, perturbation does not easily handle the occasionally binding zero lower bound constraint; an alternative would be to solve the model globally (for example, Coibion et al. (2012)), but this procedure would be too slow given the number of policy parameters to optimize over. In addition, global solutions may not handle indeterminacy well (see Richter et al. (2014)).

From an economic standpoint, recent experience in Europe and Japan shows that policy rates can indeed go below zero, and there may be an effective lower bound that is negative. Perturbation has the benefit of not explicitly taking a stand on a hard bound for the nominal rate, by simply seeking to limit the probability that the nominal rate goes below zero.

[2] In general, high values of $\Pi^*(s_t)$ near indeterminacy regions generate numerical inaccuracies that distort the welfare calculation. As a result, the planner does not have bounds on $\Pi^*(s_t)$, but instead searches for maxima around a net inflation rate of zero. An alternative, following Schmitt-Grohe and Uribe (2004), would be to restrict the policy parameters to be sufficiently far from an indeterminacy border. Given the number of policy parameters, along with the complexity of finding indeterminacy regions with regime switching, this restriction would be prohibitively difficult to check.

Among the set of policy parameters that generate an implementable rule, the monetary authority chooses those that are optimal in the sense that they maximize the household's expected lifetime utility given an initial condition.

Definition 2 (Optimal) *For an implementable policy to be optimal, it must maximize the household's value function*

$$V_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left(\log C_t - H_t\right) \tag{20}$$

given initial conditions.

By using the household's preferences, the monetary authority is making optimal policy from the household's view, rather than holding its own objective function.

3.3 Measuring Welfare

In order to compute and compare welfare, the monetary authority maximizes the expected lifetime utility of the household conditional on an initial condition. The benchmark case for each model is one in which the economy experiences regime switches, but the authority sets a constant rule–that is, a rule with policy parameters that do not vary with the regime, and the real rate used in the policy rule is fixed $r(s_t) = \bar{\omega}/\beta$, where $\bar{\omega}$ denotes the ergodic mean of $\omega(s_t)$. The monetary authority then considers different switches in the policy parameters $r(s_t)$, $\rho_r(s_t)$, $\psi_\pi(s_t)$, $\psi_y(s_t)$, $\Pi^*(s_t)$ shown in Table 2.

Under the real rate only rule, the monetary authority must fix the policy parameters, but adjusts $r^*(s_t) = \omega(s_t)/\beta$. The additional alternative cases allow various portions of the policy parameters to switch with the regimes. If a certain policy parameter cannot switch, then the authority picks a constant value across regimes; for example, in all the rules when the inflation target cannot switch, then $\Pi^*(s_t) = \Pi^*$ for all s_t. Note that, because the constant rule uses a fixed real rate, it is not nested in the real rate only or full switching rules, but is nested in the target, inertia, inflation response, and output gap response only rules.

For the initial condition used to compute welfare, the monetary authority uses the steady state of the switching economy with a constant rule. Since the regime-independent inflation

Table 2: Different Rules and the Parameters that Switch

	$r(s_t)$	$\Pi^*(s_t)$	$\rho_r(s_t)$	$\psi_\pi(s_t)$	$\psi_y(s_t)$
Constant Rule	$\bar{\omega}/\beta$				
Real Rate Only	$\omega(s_t)/\beta$				
Target Only	$\bar{\omega}/\beta$	x			
Inertia Only	$\bar{\omega}/\beta$		x		
Inflation Response Only	$\bar{\omega}/\beta$			x	
Out Gap Response Only	$\bar{\omega}/\beta$				x
Real Rate + Target	$\omega(s_t)/\beta$	x			
Real Rate + Inertia	$\omega(s_t)/\beta$		x		
Full Switching	$\omega(s_t)/\beta$	x	x	x	x

target is a policy choice and in steady state the inflation rate equals the inflation target, the authority implicitly chooses this initial state when selecting the inflation target for the constant rule. This assumption puts the constant rule on the best possible relative welfare terms, as the authority in alternative cases will face transition dynamics to a different steady state implied by alternative policy choices. For the initial regime, the monetary authority chooses coefficients before realizing the regime, and forms expectations according to the ergodic distribution of regimes.[3]

As a result, the monetary authority in the constant rule case picks policy coefficients to maximize the conditional expectation

$$V^{cr}\left(x_{ss}^{cr}\right) = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t d_t \left(\log C_t^{cr} - H_t^{cr}\right) \middle| x_{-1} = x_{ss}^{cr}\right], \tag{21}$$

where cr denotes constant rule, x denotes the vector of state variables, and C_t^{cr} and H_t^{cr} respectively denote the optimal consumption and hours choices under the optimized constant rule. Note that the state $x_{-1} = x_{ss}^{cr}$ highlights that the initial state is the steady state that in turn

[3] An alternative would be to condition the starting welfare on each of the two growth regimes, and solve optimal policy rules in each case. These values tend to be numerically different–which highlights time-inconsistencies in choosing optimal rules–but gives qualitatively similar results given the setup and parameterization of Table 1.

depends on policy parameters in the constant rule. Denoting the ergodic distribution across regimes by $\xi = [\xi_1, \xi_2]$, this expectation can be expressed as

$$V^{cr}\left(x_{ss}^{cr}\right) = \sum_s \xi_s \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t d_t \left(\log C_t^{cr} - H_t^{cr}\right)\middle| x_{-1} = x_{ss}^{cr}, s_0 = s\right] \tag{22}$$

In the alternative policy rule specifications, the monetary authority maximizes

$$V^{a}\left(x_{ss}^{cr}\right) = \sum_s \xi_s \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t d_t \left(\log C_t^{a} - H_t^{a}\right)\middle| x_{-1} = x_{ss}^{cr}, s_0 = s\right] \tag{23}$$

where a denotes the alternative specification, while C_t^a and H_t^a denote the optimal consumption and hours choices under this rule, respectively. Again, the dependence on the constant rule steady state x_{ss}^{cr} highlights that the constant rule steady state is the point of comparison.

Given the above definitions, the welfare cost associated with a move from policy specification cr to a is implicitly given by Υ^a, where Υ^a satisfies

$$V^{a}\left(x_{ss}^{cr}\right) = \sum_s \xi_s \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t d_t \left(\log\left(C_t^{cr}\left(1 + \Upsilon^a\right)\right) - H_t^{cr}\right)\middle| x_{-1} = x_{ss}^{cr}, s_0 = s\right]. \tag{24}$$

Under this definition, positive values of Υ^a indicate the alternative policy welfare-dominates the constant rule policy, while negative values indicate a preference for the constant rule. Since the absolute values of these welfare numbers tend to be small, the results show figures relative to the absolute value of the full switching (fs) case $\left(\Upsilon^a / \left|\Upsilon^{fs}\right|\right)$. The full switching case gives the monetary authority the most flexibility in terms of setting its rule, making it a useful benchmark.

4 Optimal Policy Rules with Growth Rate Switches

This section presents the main set of results on optimal rules in the presence of regime switches for the economy described by the parameters in Table 1. The monetary authority sets an optimal implementable rule, where the rule varies in which parameters can switch at the same time as the growth rate.

Given the framework discussed in Section 2, the monetary authority faces a number of trade-offs when setting policy. First, the presence of nominal rigidities without price indexation implies a need to keep inflation low and stable; in particular, exact price stability minimizes the resource cost from inflation in equation (15). However, given that the policy instruments are only lump-sum taxes and a nominal interest rate rule and not a production subsidy to eliminate the distortion from monopolistic competition, the first-best inflation rate will not necessarily be zero (Schmitt-Grohe and Uribe (2007)). In the case of non-zero inflation targets, the model does not contain features such as quality improvements or heterogeneity among individuals that might warrant higher inflation targets, so these inflation targets will tend to be low in absolute terms (Schmitt-Grohe and Uribe (2010)). Second, the presence of a markup shock generates opposing co-movement between inflation and the flexible-price output gap, which in turn creates a trade-off for stabilization policy. Third, the constraint for non-negative dynamics for the net nominal interest rate imposed by the implementability condition creates incentives to increase the nominal rate and limit how much it fluctuates; the authority will tend to want to raise the inflation target, make the nominal rate more inertial, or limit the responsiveness to movements in the inflation rate or output gap. These incentives work in contrast to the ability to keep inflation stable at a level around price stability.

A final trade-off more unique to the current setup is that the presence of regime switches–in both the private economy and in monetary policy–generate determinacy considerations and expectational effects (Davig and Leeper (2007), Liu et al. (2009), and Foerster (2016)) that the monetary authority must internalize when setting optimal policy. The interaction between regimes can imply that the parameters in one regime could possibly imply indeterminacy if that regime occurred in isolation, but switches between regimes mean the overall equilibrium satisfies determinacy. Low values of the inflation response parameter $\psi_\pi(s_t)$ typically produce indeterminacy in economies without regime switching; the current setup implies that it is possible for low values of this parameter in one regime provided that policy in the other regime ensures determinacy. In addition, expectational effects of regime switching alter the equilibrium outcomes based on a given policy, and the monetary authority must take these effects into account. For example, in the case when the inflation target may switch between regimes, firms and households will internalize this switching, leading to differences in behavior and hence realized inflation relative to the case when regime changes do not exist. Under these circumstances, realized inflation in each regime may differ from that regime's inflation target, and the authority sets policy knowing this result will occur.

To summarize, in the high growth regime, TFP grows at an annual rate of 2.5%, while in the low growth regime it grows at 0.5%. These growth regimes matter for both households and firms: growth affects the inter-temporal consumption decision made by households as seen in the Euler equation (4), as well as firms' production function (8) and hence the forward-looking nature of price setting. Experiencing high and low growth therefore affects output, inflation, and interest rates directly. In particular, expectations for low growth lower the real rate; by extension, the nominal rate will tend to be lower, which increases the chance of a negative nominal rate. In addition, expectations of regime switches also affect behavior by households and firms, as they take into account the fact that future regimes may differ from the current one. As a result, regime switches affect the optimal implementable monetary policy rules.

4.1 Optimal Monetary Policy Rules

Table 3 shows the optimal monetary policy parameters under several policy rules. In the no switching cases, the economy experiences either the high or low growth regimes exclusively rather than experiencing regime switching. In the high-growth regime only case, the optimized rule is similar to the results in Schmitt-Grohe and Uribe (2007): the nominal rate responds as strongly as possible to inflation, very little to the output gap, and exhibits a moderate degree of inertia. In addition, the optimal inflation target is nearly zero, which in steady state almost eliminates any output loss generated by positive inflation.

In the low growth only case, households face lower consumption growth, which lowers the real interest rate. Since the optimal implementable rule must limit the probability of hitting a negative net nominal rate, the optimized coefficients produce a strong degree of inertia and an above-zero inflation target. The higher inflation target attempts to counteract the decrease in the real rate, while the higher inertia component makes the nominal rate less volatile and hence less likely to become negative. If the monetary authority had to maintain a moderate level of

Table 3: Optimal Policy Rules

	High Growth Rule				Low Growth Rule				Welfare
	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*	$\Upsilon^a / \lvert\Upsilon^{fs}\rvert$
No Switching Cases									
High Growth Only	0.63	5	0.05	0.02	–	–	–	–	–
Low Growth Only	–	–	–	–	0.96	5	0.09	0.27	–
Regime Switching Cases									
Non-Optimized	0.63	5	0.05	0.02	0.96	5	0.09	0.27	$*$
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Real Rate Only	0.93	5	0.08	0.17	0.93	5	0.08	0.17	0.583
Target Only	0.93	5	0.08	0	0.93	5	0.08	0.45	0.529
Inertia Only	0.81	5	0.08	0.33	0.95	5	0.08	0.33	0.329
Inflation Resp Only	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Out Gap Resp Only	0.94	5	0.10	0.18	0.94	5	0.08	0.18	0.005
Real Rate+Target	0.93	5	0.07	0.02	0.93	5	0.07	0.28	0.641
Real Rate+Inertia	0.73	5	0.08	0.24	0.95	5	0.08	0.24	0.938
Full Switching	0.65	5	0.07	0.10	0.95	5	0.08	0.31	1

Note: $*$ denotes the Non-Optimized rule under regime switching is not implementable. Inflation target expressed in percentage points at an annualized rate.

inertia seen in the high growth only case, it would generate a higher probability of a negative nominal rate, and hence the inflation target would need to be raised further to maintain an implementable policy. In other words, by jointly altering the inertia parameter and the inflation target relative to their values in the high growth only case, the monetary authority can move each by a smaller amount while producing better welfare outcomes than if they could only adjust one.[4]

[4]The Appendix shows robustness results that highlight changing two key aspects of the implementability condition (the non-negative dynamics constraint and the upper bound on $\psi_\pi(s_t)$) do not generate qualitative differences in the results.

Turning to the economy with regime switching, the non-optimized rule simply uses the optimal rules from the no switching cases when the economy does switch between high and low growth regimes. In this case, expectational effects cause excess nominal rate volatility, which in turn violates the non-negative rate dynamics constraint in the implementability definition. The following subsection shows this violation occurs in the high growth regime; expectations of a switch to a lower growth regime overwhelm the low inflation target and low interest rate inertia, generating a nominal rate that is negative too often. As a result, using the rules from the no switching cases–even if they are optimal in that context–produces sub-optimal outcomes in an environment with structural change.

In the case of the constant rule, the economy experiences regime switches, but the monetary authority does not respond with switches in policy and keeps the policy parameters fixed. The optimal choice of parameters then has to balance the outcomes in each regime. Since the low growth regime tends to have a lower real rate and is hence more likely to produce a negative nominal rate, the constant rule has a high degree of inertia and a positive inflation target. In these circumstance the monetary authority is effectively weighing the probability of a negative nominal rate during the low growth regime versus a loss of output in the high growth regime.

The real rate only case has optimized coefficients that are similar to the constant rule, but generate different economic outcomes because the nominal rate tends to be higher in the high growth regime and lower in the low regime than under the constant rule. In the constant rule, the neutral nominal interest rate is between the regime-specific values when the real rate can change.

When only one parameter can switch, the other parts of the policy rule must stay constant across the regimes, and the optimal values of these parameters differ from the constant rule. In addition, the parameter that is allowed to switch has to compensate for the other parameters being fixed. For example, when only the inflation target can switch, it moves to a larger extent than the non-optimized rules because the other parameters are fixed, and the constant inertia is high to help avoid negative nominal rates. When the inertia can switch, the monetary authority sets a higher inflation target than in the constant rule, since lowering the inertia makes the nominal rate more volatile. Finally, in the inflation response only case, there is no change from the constant rule, as the monetary authority refuses to adjust this parameter even when allowed to do so. In the case with regime switching, it would be possible to have a passive rule with a lower inflation response in one regime–perhaps as a way of muting the fluctuations in the nominal rate in response to shocks–and still satisfy determinacy, but the inflation response only rule does not have this feature.

The full switching rule allows all parameters to change and has many of the features of both the high and low growth only cases, as well as the cases when only one parameter can switch. During the high growth regime, the optimal rule has a moderate degree of interest rate inertia and a relatively low but positive value of the inflation target. During the low growth regime, the optimal rule increases the persistence of the nominal rate and raises the inflation target as it attempts to minimize the probability of hitting a negative nominal rate. In both cases, the regime switching does have expectational effects, which lead to slightly different policy parameters than in the no switching cases. For example, the full switching model has a higher inflation target in both regimes than in each of the no switching cases; in the high growth regime, the higher inflation target is needed to offset a lower real rate generated by expectations of a switch to the low growth regime, and in the low growth regime, the higher inflation target is needed to offset realized inflation below the inflation target caused by expectations of a switch to a lower inflation target regime.

The final column of Table 3 shows the welfare cost measure in the regime switching cases. These welfare costs start from the constant rule specification, meaning the constant rule has welfare cost of zero; in addition, they are normalized so the full switching rule has an absolute value of one. All switching rules have non-negative welfare numbers, showing they make households at least as well off even conditional on starting from the constant rule's steady state. The full switching case has the largest welfare gains, as giving the authority flexibility to change all parameters produces the most benefit. In addition, among rules with only one source of switching, changing the real rate produces the largest welfare gains, followed by the inflation target and the interest rate smoothing. Interestingly, the two cases that combine the real rate with either the inflation target or the inertia show that the welfare gains from changing multiple components are not necessarily additive: changing the real rate and the inflation target serve as substitutes, with a total welfare gain that is less than the sum of its parts, while switching in the real rate and the inertia are complements since the total welfare gain that is more than the sum of its parts. The real rate and inertia switching produces around 94% of the increase from the constant rule to the full switching rule. Higher interest rate inertia helps keep the nominal rate positive with relatively low inflation targets, giving the monetary authority the best possible trade-off between avoiding a negative nominal rate and avoiding the output loss from high inflation targets.

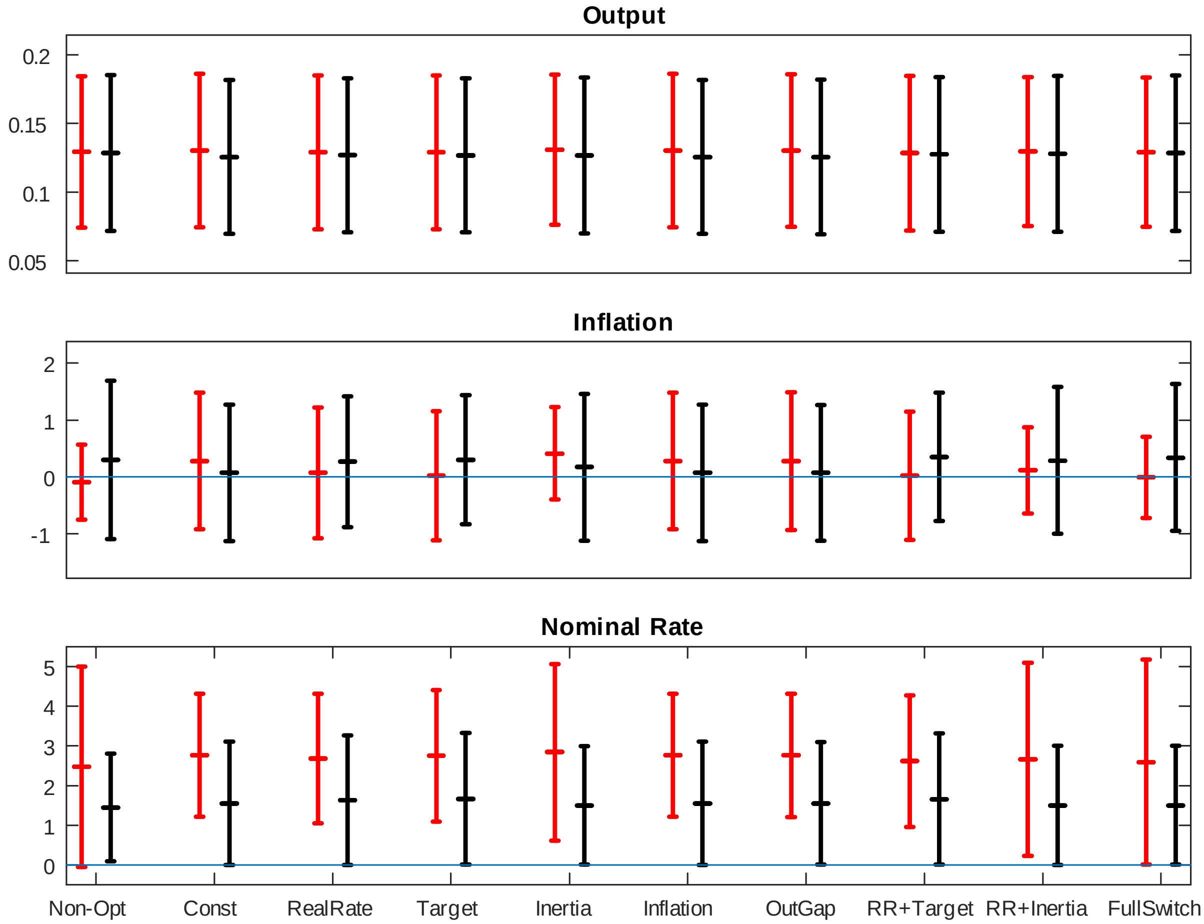
4.2 Economic Performance under Optimal Monetary Policy Rules

In order to shed light on the effects of the optimal monetary policy rules, Figure 1 displays how output, inflation, and the nominal rate respond under the different rules. The plots show the mean and two standard deviation intervals for each regime across rules. In the case of the non-optimized rule, the figure shows how in the high growth regime, the combination of a low inflation target and moderate inertia combined with more volatility due to expectational effects generates a non-implementable rule, as a negative rate occurs within two-standard deviations of the mean.

Comparing the economic performance of the various optimal rules gives a good indication as to why ones that are more flexible perform better in welfare terms. While the differences in output are minor, the levels and volatility of inflation and the nominal rate vary more across rules. When the policy rules allow interest rate inertia switches–the inertia only, real rate plus inertia, and full switching rules–the high growth regimes have relatively lower inertia components, which translates to a more volatile nominal rate and less volatile inflation. The low growth regime, by contrast, has a higher degree of inertia and often more volatile inflation as a consequence of smaller fluctuations in the nominal rate.

The importance of a non-negative rate for implementable policy is striking in that, for each rule, the optimal policy sets parameters such that in the low growth regime the non-negative

Figure 1: Economic Outcomes under Optimal Rules



Note: Mean values and two-standard deviation intervals shown for the high growth regime in red, and the low growth regime in black. Inflation and the nominal rate expressed in percentage points at an annualized rate.

dynamics constraint nearly exactly binds. In the cases where portions of the rule are constrained to not switch, the high growth regime has nominal rate intervals that are much farther away from the non-negative rate restriction. The full switching rule, and to a lesser extent the inertia and real rate plus inertia switching rules, are the rules that achieve the highest welfare gains and have the non-negativity constraint for both regimes much closer to binding. Therefore, a main reason why these rules perform the best is that the monetary authority can maximize the trade-off between keeping the nominal rate positive and not producing too much distortion through inflation fluctuations.

Table 4: Parameters for the Preference Switching Model

Description	Parameter Value
Constant Growth Rate	$\omega = 1.015^{1/4}$
High Preference Regime	$\delta(1) = 1.023$
Low Preference Regime	$\delta(2) = 0.977$

5 Optimal Policy with Preference Switches

This section considers an alternative setup, where instead of switching in the growth rate of the economy, there is switching in the process governing intertemporal preferences. This case allows for regime shifts in the real rate without affecting the rate of growth. In particular, this alternative economy modifies the intertemporal preference shifter (3) to follow

$$\log d_t = (1 - \rho_d)\,\delta(s_t) + \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}, \tag{3'}$$

while the TFP process (9) no longer has changes in the trend:

$$\log A_t = \log \omega + \log A_{t-1} + a_t. \tag{9'}$$

Table 4 shows the alternative calibration. All else equal, the values of for $\delta(s_t)$ generate a real rate similar to those in the growth rate regime switching economy, but without the differences in trend growth. These changes in the preference shock process, through the household's Euler equation (4), affect the trade-off between consumption in the current and future periods. In the low preference regime, marginal utility is low given a fixed consumption level, which encourages savings and pushes the market clearing interest rate down. This lower rate means the nominal rate is more likely to become negative absent any policy changes. Importantly, in contrast to the case with growth regimes, the high and low preference regimes by themselves do not have an impact on behavior since they simply scale marginal utility; instead, it is the presence of switches between these two regimes that matter.

Table 5: Optimal Policy Rules, Economy with Preference Switches

	High Pref Rule				Low Pref Rule				Welfare
	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*	$\Upsilon^a / \lvert\Upsilon^{fs}\rvert$
No Switching Cases									
High Pref Only	0.88	5	0.07	0.07	–	–	–	–	–
Low Pref Only	–	–	–	–	0.88	5	0.07	0.07	–
Regime Switching Cases									
Non-Optimized	0.88	5	0.08	0.07	0.88	5	0.07	0.07	$*$
Constant Rule	0.92	5	0.08	0.15	0.92	5	0.08	0.15	0
Real Rate Only	–	–	–	–	–	–	–	–	–
Target Only	0.91	5	0.07	0.12	0.91	5	0.07	0.26	0.032
Inertia Only	0.75	5	0.06	0.13	0.92	5	0.06	0.13	0.995
Inflation Resp Only	0.92	5	0.08	0.15	0.92	5	0.08	0.15	0
Out Gap Resp Only	0.92	5	0.10	0.12	0.92	5	0.05	0.12	0.008
Real Rate+Target	–	–	–	–	–	–	–	–	–
Real Rate+Inertia	–	–	–	–	–	–	–	–	–
Full Switching	0.76	5	0.07	0.09	0.92	5	0.07	0.15	1

Note: $*$ denotes the Non-Optimized rule under regime switching is not implementable. Inflation target expressed in percentage points at an annualized rate.

5.1 Optimal Monetary Policy Rules with Preference Switches

Table 5 displays the optimal parameters for the different monetary policy rules. In the no switching cases, both the high and low preference regimes have the same optimal rules. Since the level of the preference shock simply scales the household's utility, being in the high or low preference regime forever does not affect their decisions. Both cases then have high inertia, a strong inflation response and weak output gap response, and a small but nonzero inflation target.

In the cases when there is regime switching, the changes between high and low preferences generate fluctuations in the real interest rate that require a response by optimal monetary policy. The non-optimized rule, which is constant and optimal for both high and low preferences in isolation, is no longer implementable, as it generates interest rate fluctuations that produce a negative rate too frequently. The next subsection shows that, because in the low preference regime the real rate is depressed, the nominal rate hits negative territory within two standard deviations. Again, this result highlights the fact that in the presence of regime switching, optimal rules from the case without switching will not necessarily be optimal. In fact, the preference switching application is notable because both regimes in isolation have an identical optimal rule, but this rule is not implementable when regime switches occur.

The optimal constant rule with regime switches, on the other hand, takes into account the fluctuations in the real interest rate generated by the high and low preference regimes. As noted, the non-optimized rule violates implementability by generating a negative nominal rate too frequently in the low preference regime, so the optimal constant rule responds to this fact by having a higher inflation target and a higher inertia component. As with the growth rate switches, these raise the mean of the nominal rate and decrease its standard deviation.

In the cases when only one of the parameters can switch, the results mimic those seen for the growth rate switches case.[5] However, while growth rate switches directly affected the household's Euler equation and the price setting equation for firms, the preference switches only directly alter the Euler equation, and hence the general equilibrium effects differ. Consequently, the numerical differences tend to be smaller. For example, when interest rate inertia is the only parameter that can switch, the response to the output gap and the inflation target decline, but only slightly.

The full switching rule combines many of the features from the cases where only one parameter can switch. In the high preference regime, the inflation target is lower and the interest rate inertia is lower than in the low preference regime. These changes help keep the nominal rate higher and less volatile when the real rate is low so that the policy rule is implementable.

The final column of Table 5 again shows the welfare cost measure from the regime switching

[5] Note that $r(s_t)$ does not change in the presence of preference switches.

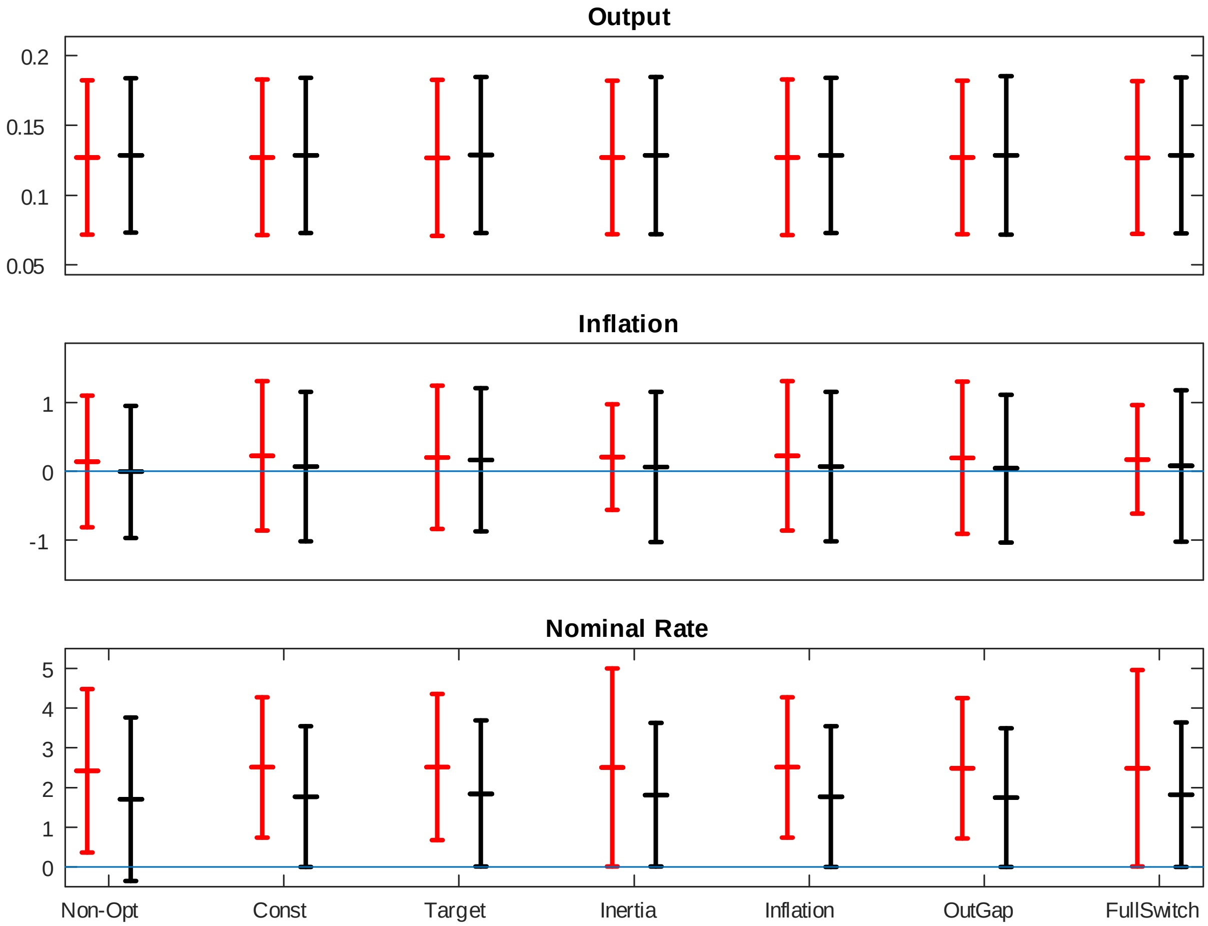
cases. All of the welfare costs are non-negative despite starting from the constant rule specification, implying that instituting a switching rule either raises welfare or keeps it the same. The ability to switch inflation responses, as noted, generates no differences in the optimal rule, so the welfare gain is zero. For the other switching rule specifications the welfare change is positive, and most of the gains come from switches in the interest rate inertia. Despite the fact that optimal policy primarily affects the inflation target and the inertia component, simply having the inertia component switch generates 99.5% of the gains from a fully switching rule. These results all suggest that more flexibility in the monetary policy rule produces better welfare outcomes, and that most of this gain is due to changes in how strongly policy responds to current conditions versus moving only gradually.

5.2 Economic Performance under Optimal Monetary Policy Rules with Preference Switches

Figure 2 shows how output, inflation, and the nominal rate react to the different optimized policy rules in the two preference regimes. For the non-optimized rule, as discussed, the low preference regime has a lower real rate that in turn produces a low nominal rate. The nominal rate falls below zero too frequently, which implies the rule is not implementable.

Similar to the results for growth rate switching, the behavior of the nominal rate and inflation give a good indication as to what drives optimal policy. In the low preference regime, the real rate is lower, which tends to lower the nominal rate; the monetary authority attempts to offset this effect, primarily through a higher inflation target and a larger degree of interest rate inertia. For each rule, the optimal policy implies that in the low preference regime the implementability restriction on non-negative rate dynamics just binds. For the rules that achieve the highest welfare–the inertia only and full switching–this constraint also binds for the high preference regime. For these rules, in high preference regime the monetary authority lets the nominal rate fluctuate more, which in turn produces lower volatility of inflation and hence higher welfare gains.

Figure 2: Economic Outcomes under Optimal Rules, Economy with Preference Switches



Note: Mean values and two-standard deviation intervals shown for the high preference regime in red, and the low preference regime in black. Inflation and the nominal rate expressed in percentage points at an annualized rate.

6 Conditions Diminishing the Gains of Switching Rules

The results of Sections 4 and 5 show that allowing the monetary policy rule to switch along with the private economy can produce welfare gains. This Section returns to the original setup of switching in the trend growth rate and considers two modifications of the framework described in Sections 2 and 3 that decrease the differences in the optimal full switching rule across regimes, which in turn makes the optimal full switching rule look more like the optimal constant rule. In particular, by modifying the non-negative nominal rate constraint in the implementability definition, and by relaxing the assumption of perfect synchronization between the private economy and the monetary authority, the full switching rule approaches a constant rule.

6.1 Trade-off for a Low Nominal Rate

In the definition of implementability, condition (2) required that the nominal rate must exhibit non-negative dynamics, which was defined as the mean minus two standard deviations needing to be positive for each regime. That is, the original implementability condition required

$$\boldsymbol{\mu}_R(s_t) - 2\boldsymbol{\sigma}_R(s_t) > 0 \text{ for each } s_t, \tag{25}$$

where $\boldsymbol{\mu}_R(s_t)$ and $\boldsymbol{\sigma}_R(s_t)$ denote the mean and standard deviations for the log of the gross interest rate in regime s_t. The fact that this condition needed to hold regime-by-regime drove many of the results seen in Section 4, since optimal switching rules adjusted their parameters such that this constraint nearly bound, as this outcome tended to minimize the distortions.

The original formulation for the non-negativity of the nominal rate implied a constant bound on the probabilities of realizing a negative rate. Since the distribution of shocks across the two regimes does not change, the probability of the nominal rate being below zero is roughly the same in each regime. While this formulation captures many intuitive features of an occasionally binding constraint on nominal rates, since it implies a desire to avoid a negative rate equally across regimes, an alternative restriction could instead bound the total chance of hitting a negative rate, and allow the monetary authority to consider a trade-off in the regime-dependent probabilities.

As a result, an alternative condition (2) in the definition of implementability would be

$$\sum_{s_t} \xi_{s_t} \left[\boldsymbol{\mu}_R(s_t) - 2\boldsymbol{\sigma}_R(s_t)\right] > 0 \tag{26}$$

where again $\xi = [\xi_1, \xi_2]$ is the ergodic distribution across regimes, which equals $\xi_1 = \xi_2 = 1/2$ given the symmetry in the regime probabilities. This formulation bounds the total probability of hitting a negative rate across regimes, and allows for higher probabilities in one regime if the other regime has a lower probability. Note that any policy satisfying the original non-negativity constraint also satisfies this alternative definition, but the reverse is not necessarily true.

Table 6: Optimal Policy under Different Non-Negativity Restrictions

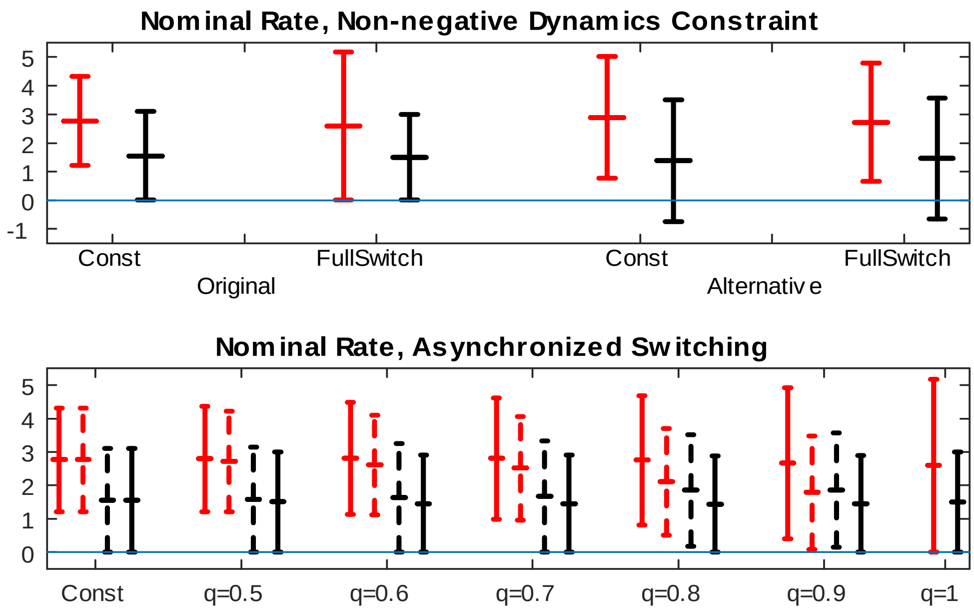
	High Growth Rule				Low Growth Rule				Welfare
Non-Negativity	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*	$\Upsilon^a_{nn} / \lvert \Upsilon^{fs}_{orig} \rvert$
Original									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.65	5	0.07	0.10	0.95	5	0.08	0.31	1
Alternative									
Constant Rule	0.87	5	0.08	0.15	0.87	5	0.08	0.15	0
Full Switching	0.88	5	0.07	0.10	0.87	5	0.06	0.10	0.279

Note: Inflation target expressed in percentage points at an annualized rate.

Table 6 shows the results for the original non-negativity constraint and the alternative, focusing on only the optimal constant and full switching rules. Under the alternative non-negativity constraint, both the optimal constant and full switching rules differ than under the original constraint. The alternative constant rule has a lower inflation target and a lower degree of inertia than the original constant rule. When the monetary authority can effectively trade-off a higher probability in the low growth regime for a lower probability in the high growth regime, it does not need to have such a high target and degree of inertia, as these both create distortions from higher and more variable inflation. Similarly, the alternative full switching rule has a lower inflation target, especially in the low growth regime.

The fact that the alternative full switching rule is much closer to the alternative constant rule highlights that, the original non-negativity constraint played a large role in dictating differences between the rules across regimes. When nominal rates can be negative with a higher probability in one regime, the monetary authority with access to a full switching rule nevertheless sets a rule that is similar to the constant rule. The last column of Table 6 shows the welfare measure normalized by the welfare gains of the full switching rule in the original constraint. In the alternative setup the gains to full switching are only 27.9% of their magnitude under the original constraint. This result again highlights that, if the monetary authority is able to have a higher

Figure 3: Economic Outcomes under Optimal Rules, Variants on the Model



Note: Mean values and two-standard deviation intervals shown for the high growth rate regime in red, and the low growth rate regime in black. In lower panel, solid lines indicate the correct rule is in place, dashed lines indicate the incorrect rule is in place. Nominal rate expressed in percentage points at an annualized rate.

probability of negative rates in the low growth regime, the optimal full switching rule becomes more like the optimal constant rule, and the benefits to having a switching rule decrease.

The top panel in Figure 3 shows the economic outcomes for the nominal rate across the original and alternative non-negativity constraints. In the original constraint, the constant rule sought to have two standard deviations of the nominal rate away from its mean be as close to zero as possible, and the full switching rule had the same effects for both regimes. In contrast, in the alternative setup both the constant and full switching rule have a negative nominal rate more frequently than a two standard deviation event in the low growth regime, and offset this by having a lower probability in the high growth regime.

6.2 Optimal Rules without Synchronized Switching

The baseline results required perfect synchronization of the monetary policy rule with switches in the private economy. In this formulation, when the economy switches from high to low growth, or vice versa, the monetary authority immediately recognizes the switch, and implements the correct rule. Given uncertainty about the growth regime, this assumption may be too strict, and optimal rules would need to be robust to error in identifying the regime.

Specifically, consider a modification of the Markov switching framework where the private economy experiences growth rate switches with probability $1-p$ as before, but each period the monetary authority correctly assesses the growth regime with probability q. When $q=1$, this modified assumption collapses back to the original model, since the monetary authority always correctly identifies the regime. When $q=0.5$, they correctly assess the regime half the time, and when $q=0$ they always misidentify the regime.[6] In this case, there are four regimes: high growth rate with a high growth rule ($s_t=1$), high growth rate with a low growth rule ($s_t=2$), low growth rate with a high growth rule ($s_t=3$), and low growth rate with a low growth rule ($s_t=4$). Under these circumstances, regimes $s_t=1,4$ indicate the monetary authority has put the correct rule in place, and $s_t=2,3$ indicate the wrong rule is in place. The transition probabilities in this case become

$$P=\begin{bmatrix} pq & p\left(1-q\right) & \left(1-p\right)\left(1-q\right) & \left(1-p\right)q \\ pq & p\left(1-q\right) & \left(1-p\right)\left(1-q\right) & \left(1-p\right)q \\ \left(1-p\right)q & \left(1-p\right)\left(1-q\right) & p\left(1-q\right) & pq \\ \left(1-p\right)q & \left(1-p\right)\left(1-q\right) & p\left(1-q\right) & pq \end{bmatrix}. \tag{27}$$

When setting the optimal rule, households, firms, and the monetary authority all understand that the monetary authority will implement the correct rule in each period with probability q. Consequently, when setting an optimal rule, the monetary authority adapts the coefficients to

[6] The probabilty q can be thought of as a shorthand for a learning process, but allows for incorrectly realized rules in a framework close to the original specification that doesn't resort to requiring a filtering problem by the monetary authority. The results are symmetric for $q<0.5$, since a monetary authority that misidentified the regime more than half the time would simply flip the labels on its rules.

this probability.

Table 7 shows the results for the constant and full switching rules as q decreases from 1.0 to 0.5. The constant rule is identical for all values of q, since if the monetary authority has to set a rule that does not vary across regimes, then the probability of misidentifying the regime becomes irrelevant. By contrast, the full switching rule differs as q changes, but the changes in the optimal parameters are not necessarily monotonic in q. This result occurs because of the expectational effects of policy and growth rate switches. For example, in setting the high growth rule, when q decreases from 1.0 to 0.9, the monetary authority must raise the inflation target, since with a 10% probability it will incorrectly put the high growth rule in place during the low growth regime, which will tend to lead to negative nominal interest rates. As q decreases, therefore, the high growth rule's inflation target also increases. However, the Table shows that for $q = 0.9$, the low growth rule also has a higher inflation target, which is not needed to keep a positive nominal rate when the regime is incorrectly identified, but because having a higher target in the low growth regime helps raise inflation in the high growth regime as well. Put another way, the monetary authority is using expectational effects instead of raising the inflation target in the high growth rule by an even greater extent. As q decreases from 1.0 to 0.5, the magnitude of the expectational effects changes as well.

The last column of Table 7 shows the welfare gain in the full switching rule over the constant rule, normalized by the gain from the full switching rule when $q = 1.0$. As q decreases, the welfare gain decreases in a monotonic fashion, until there is a loss when $q = 0.5$, meaning the constant rule welfare dominates the full switching rule. This result implies that as the monetary authority becomes increasingly prone to putting the incorrect rule in place, then eventually–due to the costs associated with the wrong real rate used in the policy rule–the optimal policy should be a constant rule. The constant rule therefore has the advantage in this case of setting a real rate component that is fixed; this feature ends up contributing to welfare losses when $q = 1.0$, but becomes positive for welfare when $q = 0.5$.

The bottom panel in Figure 3 shows the economic outcomes for the nominal rate as q varies. Similar to the baseline results, the full switching rules set the mean nominal rate less two-

Table 7: Optimal Policy with Asynchronized Regimes

	High Growth Rule				Low Growth Rule				Welfare
	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*	$\Upsilon_q^a / \left\lvert \Upsilon_{q=1}^{fs} \right\rvert$
$q = 1.0$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.65	5	0.07	0.10	0.95	5	0.08	0.31	1
$q = 0.9$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.78	5	0.08	0.16	0.96	5	0.08	0.46	0.780
$q = 0.8$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.87	5	0.10	0.19	0.96	5	0.05	0.48	0.544
$q = 0.7$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.89	5	0.10	0.33	0.96	5	0.05	0.24	0.300
$q = 0.6$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.91	5	0.10	0.32	0.96	5	0.08	0.15	0.098
$q = 0.5$									
Constant Rule	0.94	5	0.09	0.18	0.94	5	0.09	0.18	0
Full Switching	0.93	5	0.09	0.36	0.95	5	0.08	0	-0.019

Note: Inflation target expressed in percentage points at an annualized rate.

standard deviations as close to zero as possible to minimize the distortions. When q is less than 1.0, the authority faces a constraint that is binding for the regimes when the incorrect rule is in place (the dashed lines). As q decreases, this constraint becomes more problematic when the high growth rule is in place during the low growth regime (the black dashed line). These cases lead to a nominal rate that is often far away from zero in the high growth regime, leading to distortions and hence welfare losses.

7 Conclusion

Motivated by recent arguments for changing the conduct of monetary policy in the presence of regime shifts in the growth rate of the economy, this paper has studied optimal monetary policy rules that switch parameters. These optimal rules differ from the optimal choice under fixed regimes, with the real interest rate used in the policy rule, the inflation target, and degree of inertia all playing important roles. At the same time, deviations from the benchmark environment can erode the gains from switching rules, particularly if the monetary authority may misidentify the regime.

The results therefore point to the benefits of having a flexible rule that can switch when there are structural shifts in the economy rather than a purely constant rule, but at the same time provide caution as to when the added flexibility may not be quite as beneficial. Additional considerations beyond the scope of the simple New Keynesian model considered here, such as more frictions or the central bank facing communication difficulties, remain interesting extensions.

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8 Appendix: Robustness of Implementability Definition

The results for optimal policy hinge on the definition of implementable policy, which requires putting a limit on negative nominal rate dynamics and bounds on the coefficients. This Appendix shows that the optimal rules in the non-switching cases produce similar qualitative results under modified definitions of implementable policy.

First, all of the results for optimal policy require an inflation response of $\psi_{\pi}(s_t) = 5$ regardless of the regime, rule, or switching behavior. For both the high and low growth only cases, Panel A of Table 8 shows that changing condition (3) in the implementability from $\psi_{\pi}(s_t) \in [0, 5]$ to either $\psi_{\pi}(s_t) \in [0, 3]$ or $\psi_{\pi}(s_t) \in [0, 8]$ produces qualitatively similar results. Regardless of the assumptions on the upper bound for $\psi_{\pi}(s_t)$, the inflation response hits the upper bound, and moving from the high to the low growth regime, optimal policy responds with a combination of increasing the inertia to a high value, slightly increasing the output gap response, and increasing the inflation target. Given the qualitative similarities of these cases to the results originally included in Table 3, changing the upper bound on the inflation response would not have a qualitative impact on the results with regime switching.

Second, many of the results are driven by the condition limiting a negative nominal rate to be a two-standard deviation event in each regime. Again, for both the high and low growth cases, Panel B of Table 8 shows that changing condition (2) in the implementability definition to be either one or three standard deviations produces qualitatively similar results. When the constraint on a non-negative nominal rate is strengthened by increasing from one to three standard deviations–which in turn lowers the probability of a negative nominal rate–the optimal inflation target and the degree of inertia both increase. Since these results are qualitatively similar to the results in Table 3, the effects with regime switching would also be qualitatively similar.

Table 8: Optimal Policy with Varying Implementability Definitions

Panel A:	High Growth Regime				Low Growth Regime			
Coefficient Bounds	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*
$\psi_\pi(s_t) \in [0,5]$								
High Growth Only	0.63	5	0.05	0.02	–	–	–	–
Low Growth Only	–	–	–	–	0.96	5	0.09	0.27
$\psi_\pi(s_t) \in [0,3]$								
High Growth Only	0.68	3	0.04	0	–	–	–	–
Low Growth Only	–	–	–	–	0.95	3	0.07	0.20
$\psi_\pi(s_t) \in [0,8]$								
High Growth Only	0.66	8	0.07	0.01	–	–	–	–
Low Growth Only	–	–	–	–	0.97	8	0.13	0.32

Panel B:	High Growth Regime				Low Growth Regime			
Non-Negativity Constraint	ρ_r	ψ_π	ψ_y	Π^*	ρ_r	ψ_π	ψ_y	Π^*
$\boldsymbol{\mu}_R(s_t) - 2\boldsymbol{\sigma}_R(s_t) > 0$								
High Growth Only	0.63	5	0.05	0.02	–	–	–	–
Low Growth Only	–	–	–	–	0.96	5	0.09	0.27
$\boldsymbol{\mu}_R(s_t) - \boldsymbol{\sigma}_R(s_t) > 0$								
High Growth Only	0.50	5	0.07	0	–	–	–	–
Low Growth Only	–	–	–	–	0.84	5	0.06	0.14
$\boldsymbol{\mu}_R(s_t) - 3\boldsymbol{\sigma}_R(s_t) > 0$								
High Growth Only	0.88	5	0.06	0.07	–	–	–	–
Low Growth Only	–	–	–	–	0.98	5	0.10	0.28

Note: Inflation target expressed in percentage points at an annualized rate.