

Fiscal Sustainability: A Cross-Country Analysis

By Huixin Bi

Since the global financial crisis, public debt has risen rapidly in many advanced and emerging market economies. Every country faces a fiscal limit at which, for economic or political reasons, taxes and spending can no longer adjust to stabilize debt. The lingering debt crisis in Greece, which started in 2009, provides a sobering example of how approaching or going over the fiscal limit can grind the economy to a halt. But how much debt is too much?

Quantifying fiscal limits is challenging for two reasons. First, different countries have different capacities to service their debt. Two countries with similar debt levels may face drastically different default risks. For instance, Japan's general government debt reached 240 percent of its gross domestic product at the end of 2016, but its sovereign debt still has an A+ rating according to Standard and Poor's. In contrast, Italian government debt lost its A+ rating in 2011, when general government debt was below 120 percent of Italy's GDP. Second, the default risk associated with a given debt level may change depending on a country's economic and policy environment. Although the level of Italian government debt did not change materially from 2003 to 2007, Standard and Poor's downgraded its sovereign debt rating twice during the same period, citing weakening economic growth prospects.

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In this article, I propose a framework of fiscal limits to quantify the maximum level of debt a government can sustain given its economic and policy environment. Fiscal limits are intrinsically country-specific in this framework, as government policy, economic growth, and investors' preferences can vary greatly across countries. I find that countries with relatively low government expenditures, such as Japan and the United States, have significantly higher fiscal limits than countries with relatively high government expenditures. I also find that emerging market economies may have lower fiscal limits than advanced economies, as investors may discount their future fiscal surpluses more heavily.

Finally, my framework suggests sovereign default risks can rise rapidly in an economic downturn. Downturns reduce the maximum fiscal surplus that a government can collect in the near future, thereby lowering its capacity to repay debt. At the same time, downturns can induce a government to borrow more. A declining fiscal limit, together with rising debt, leads sovereign risks to surge rapidly. The framework highlights that debt levels viewed as safe in good times can quickly become unsustainable during an economic downturn.

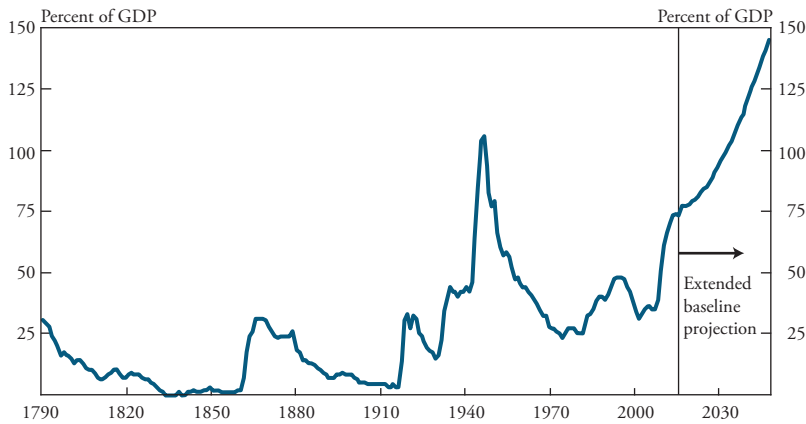
Section I introduces the framework of fiscal limits. Section II explains the properties of fiscal limits. Section III compares the distributions of fiscal limits for several advanced economies and emerging market economies. Section IV highlights the debt and default dynamics in an economic downturn.

I. A Sketch of the Fiscal Limit Framework

Since the global financial crisis, public debt has surged in many countries, including the United States. According to the Congressional Budget Office (CBO), federal government debt held by the public is now at its highest level since World War II and is projected to reach 150 percent of U.S. GDP by 2047 (Chart 1). The United States is not alone. Chart 2 shows that in recent years, public debt has picked up across many countries, particularly in advanced economies. The rapid increases in government debt reflect that economic woes weighed on government revenues during the recent financial crisis and prompted governments to adopt expansionary fiscal measures to stimulate their economies.

Chart 1

U.S. Federal Government Debt Held by the Public (1790–2040)



Source: Congressional Budget Office.

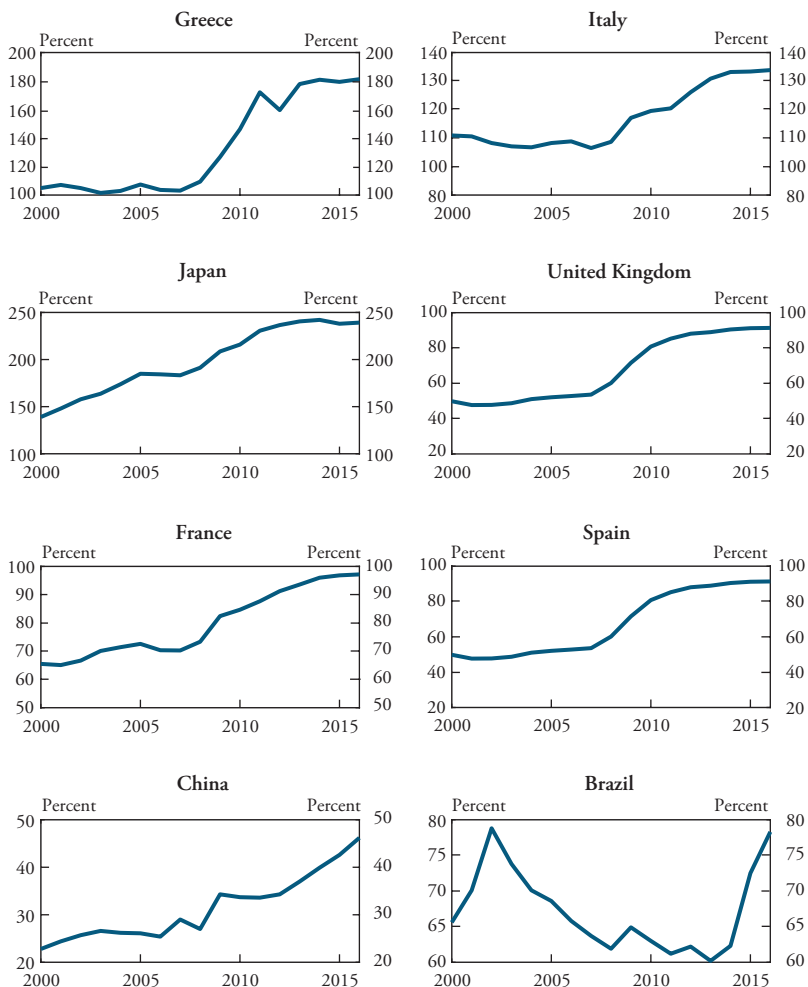
Every economy, however, faces a fiscal limit beyond which debts become unsustainable and sovereign default risks surge.¹ One way to identify a country's fiscal limit is through a standard asset pricing model, which expresses the value of an asset as the discounted sum of future dividend payments. The higher the dividends expected in the future, the higher the asset's value today. Similarly, fiscal limits, which measure the maximum level of debt a government can support given its economic and policy environment, are the discounted sum of the maximum fiscal surpluses that the government can possibly collect in the future. Ultimately, a government's ability to raise revenues is limited, as tax distortions discourage economic activity, thereby creating an upper bound on government revenues. On the spending side, economies require some minimum level of government expenditures to function, and most countries have adopted policies that effectively put a floor on spending that is well above this minimum. These considerations imply that fiscal surpluses, which are the difference between tax revenues and government expenditure, face some maximum ceiling.

Following Bi as well as Bi and Leeper, a government's fiscal limit can be expressed as:

$$b_t^* = \sum_{j=0}^{\infty} \underbrace{\beta_{t+j}}_{\text{discount factor}} \left(\underbrace{T_{t+j}^{\max}}_{\text{max tax revenue}} - \underbrace{g_{t+j}}_{\text{discretionary spending}} - \underbrace{Z_{t+j}}_{\text{transfers}} \right). \quad (1)$$

Chart 2

General Government Gross Debt as a Share of GDP across Countries (2000–16)



Source: International Monetary Fund (Haver Analytics).

The fiscal limit, b_t^* , is determined by the discounted stream of maximum future government surpluses. Given economic and policy conditions at time $t+j$, T_{t+j}^{\max} is the maximum tax revenue a government can collect. Government expenditures consist of discretionary spending that includes defense spending (g_{t+j}) and transfers that includes pensions and healthcare costs (z_{t+j}). Fiscal limits are the discounted sum of future

maximum fiscal surpluses. The further the discount factor, β_{t+j} , is below 1, the more investors discount future fiscal surpluses. Appendix A provides more technical details on the underlying model used to derive fiscal limits.²

Revenues

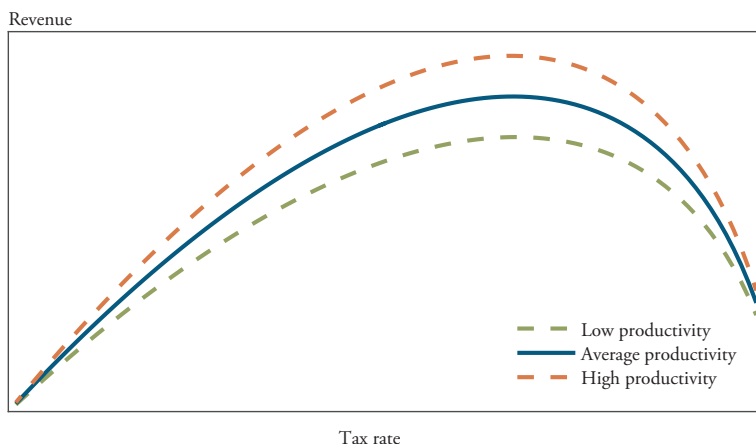
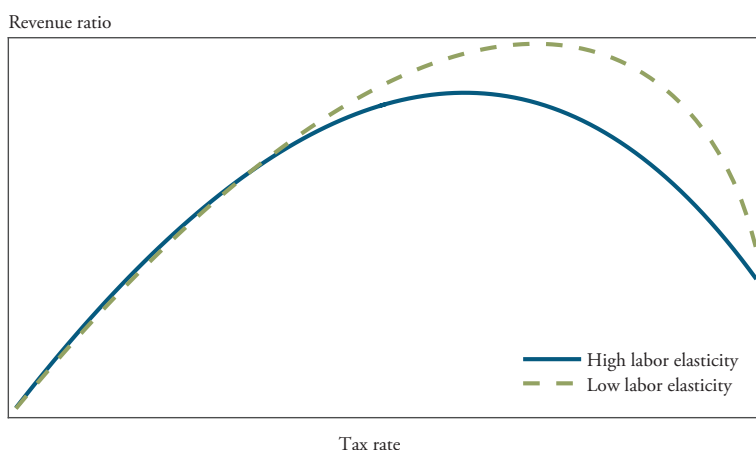
Governments face fiscal limits because they cannot indefinitely increase revenues by raising tax rates, as distortionary taxes create disincentives to work and invest. Consider, for example, an increase in the income tax. If households' work effort remains unchanged, the tax base also remains the same and tax revenues rise unambiguously. A higher income tax, however, reduces the after-tax return to working and, therefore, may induce households to work less. The resulting effect on revenue is ambiguous, but generally, increasing taxes raises revenue at low tax rates and reduces revenue at high tax rates.

Panel A of Figure 1 illustrates this inverted U-shape relationship between the tax rate and revenue, also known as the Laffer curve. Revenue climbs with the tax rate until it hits the peak of the Laffer curve, where revenue reaches its maximum level of T^{\max} . As the tax rate rises further, its distortionary effects reduce tax revenue. Notably, economic conditions can shift the Laffer curve up or down. Higher productivity, for example, boosts the tax base, raising the maximum revenue that government can collect. Likewise, lower productivity lowers the maximum tax revenue.

The Laffer curve can also shift due to changes in the elasticity of the labor supply to a tax change. A higher elasticity means that workers are more responsive to tax changes: all else equal, a hike in the tax rate discourages work effort more. Panel B of Figure 1 shows that a more elastic labor supply lowers the peak of the Laffer curve, reducing the maximum tax revenue that a government can collect. A more elastic labor supply may reflect society's intrinsic preferences for more leisure or poorer tax compliance as more people choose to work in the underground economy to evade taxes as the tax rate increases.

Expenditures

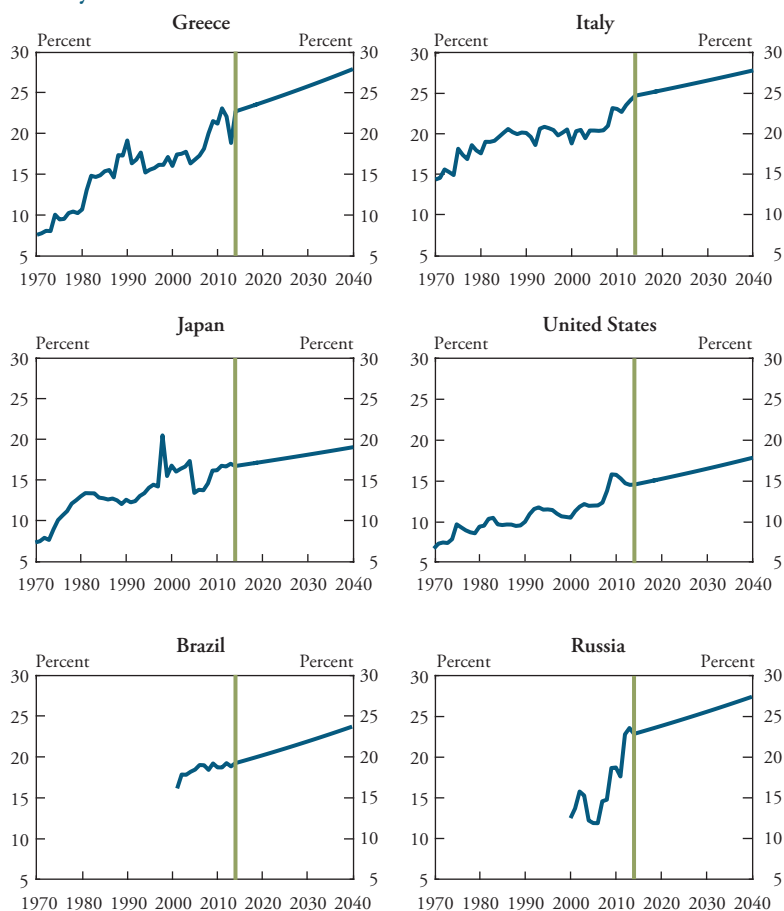
A certain level of government spending is necessary to keep institutions and society functioning properly, and most countries have

*Figure 1***Examples of Laffer Curves****Panel A: Laffer Curves across Different Productivity Levels****Panel B: Laffer Curves with Different Labor Elasticities**

adopted policies that effectively put a floor on spending that is well above the minimum. Over the last few decades, government spending has been growing as a share of output. Discretionary spending, however, has stayed fairly stable or even declined in some countries, leaving limited, if any, room for further cuts. Discretionary spending in the United States, for example, has declined by 6.3 percent of GDP since 1967 and is projected to edge down 1 percent further in the next 10 years (CBO).

Chart 3

Government Transfers Are Projected to Grow as a Share of GDP in Many Countries

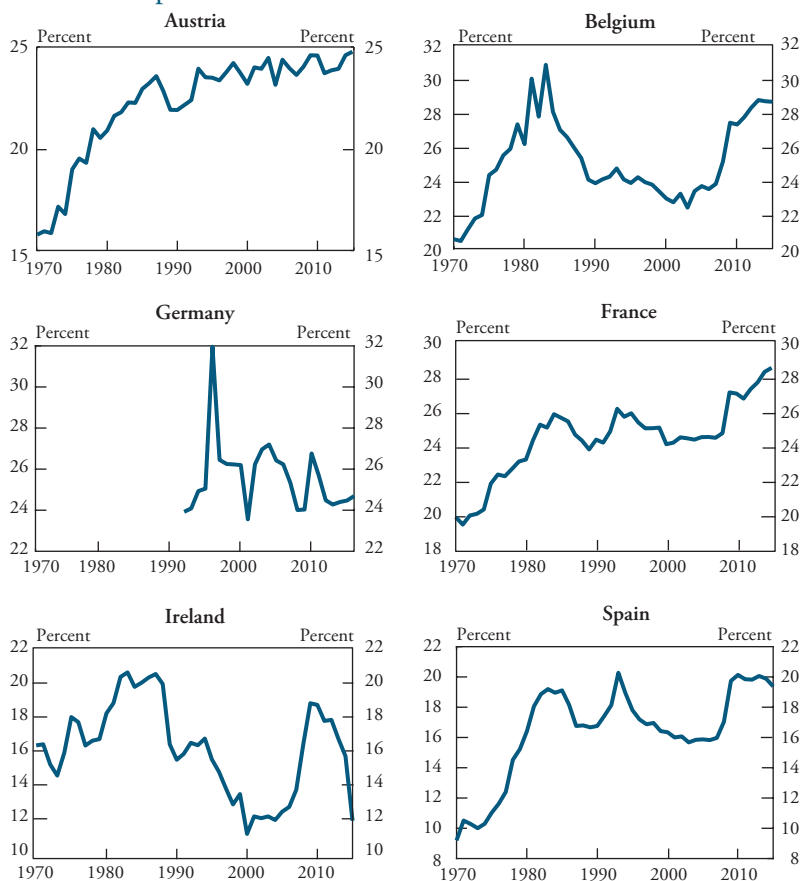


Sources: Organisation for Economic Co-operation and Development (OECD) and IMF (Haver Analytics).

Instead, growth in transfers has largely driven the growth in government expenditures. Drazen put it well: the growth in transfers “has been one of the most dramatic changes in government expenditures in the last sixty years.” Without fundamental reforms, the upward trajectory in transfer spending will continue as aging populations require higher spending on pensions and healthcare. Chart 3 shows that the share of transfer spending in GDP has been growing steadily in Greece, Italy, Japan, and the United States since 1970. The trajectory is particularly steep in Greece: transfers have grown steadily from 8 percent

Chart 4

Government Transfers Fluctuated as a Share of GDP in some European Countries



Source: OECD.

of GDP in 1970 to over 22 percent in 2016 and are expected to rise further to 27 percent by 2040. In emerging market economies, the historical trend is less obvious due to data limitations; going forward, however, transfer expenditures are projected to grow at a similar pace to advanced economies. Chart 3 shows that the shares of output spent on transfers in Brazil and Russia are projected to exceed those in the United States and Japan by 2040.

Fundamental reforms and faster-than-expected economic growth can—and historically, have been able to—reverse this trend (Chart 4).

Belgium undertook fiscal consolidations in the 1980s and 1990s, successfully stabilizing the growth of transfers until the recent financial crisis. And when the economies of Ireland and Spain boomed during the 1990s and early 2000s, their shares of output spent on transfers declined and stabilized at lower levels. Mounting challenges related to entitlement and healthcare reform in some countries, however, have created substantial uncertainty about when governments will undertake such reforms.

II. Properties of Fiscal Limits

Understanding why some countries have a greater capacity to service their debts than others—and why sovereign default risks can rise rapidly in an economic downturn—requires an understanding of both the distribution of fiscal limits and their dependence on economic conditions and policy.

Distribution of fiscal limits

The fiscal limit is not a static point but rather a random variable whose value depends on uncertain future economic and policy conditions. In particular, fiscal limits depend on both the stream of maximum fiscal surpluses that governments can collect in the future and the discount factors that investors apply to government debt. These future surpluses and discount factors, in turn, depend on future economic and policy conditions. For instance, a future recession would reduce the maximum tax revenues a government could collect, while a boom would raise them. Future economic conditions may also affect investors' preferences for holding government bonds and, therefore, the discount factor they apply to the bonds. In addition, policy changes by future governments could raise or lower the level of future government expenditures. Due to these uncertainties, the fiscal limit today is a random variable characterized by a probability distribution. While there are many possible sources of uncertainty, I focus on uncertainty about the productivity level and government expenditures.

The economy's productivity level fluctuates over the business cycle. Following previous research (see, for example, King, Plosser, and Rebelo), I allow the productivity level to follow a persistent and stationary process: a shock immediately alters the productivity level today, which

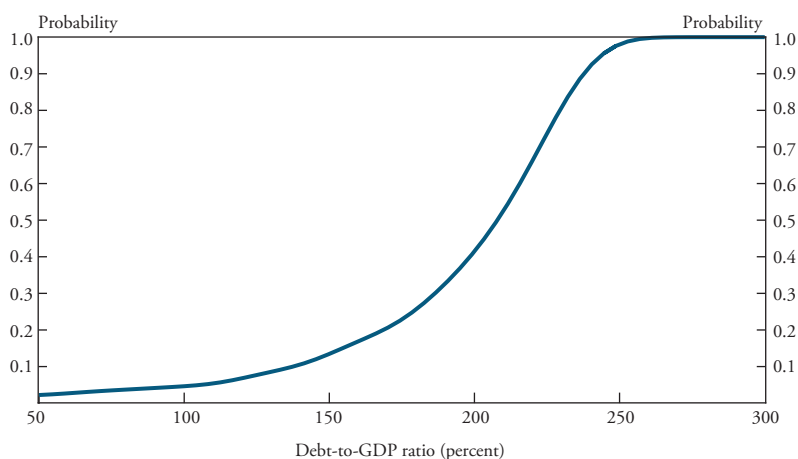
returns to its long-run stable level over time in the absence of further shocks. How soon it returns to its long-run stable level depends on how persistent the productivity process is. The more persistent the process is, the longer it will take a shock to run its course and the larger the effect it will have on the economy overall.

Fiscal policy itself can be another source of uncertainty. Today's government might allow expenditures on transfers to grow, reflecting either demographic shifts that lead to a greater demand for retirement benefits or rapid growth in the government's share of the economy. Future governments, on the other hand, may adopt fiscal reforms that stabilize expenditures—though these reforms may not be permanent. Chart 4 shows that the share of output spent on transfers grows at some times and stabilizes at others. In the framework of fiscal limits, then, I allow the share of output spent on transfers to switch between two regimes: it either fluctuates around its long-run stable level in a “stable” regime, or it grows over time in an “unstable” regime. Neither regime is permanent; instead, governments can switch between the two regimes with probabilities that reflect the likelihood and the persistence of reforms. Unlike transfer expenditures, government discretionary spending has stayed fairly stable. Accordingly, I allow it to follow a persistent and stationary process.

Uncertainty about future economic and policy conditions, captured by the random nature of the productivity level and government expenditures, gives rise to a probability distribution of the fiscal limit. Absent this uncertainty, the fiscal limit today would be a static point, as the government would have a known and fixed capacity to service its debt. As long as debt remained below this point, the government would be able to service its debts; an increase of its debt to this point or beyond would force the government to default on its debt. Economic and policy conditions do change over time in an unpredictable way, however, so the fiscal limit becomes a random variable described by a probability distribution (see appendix B for further details on how the distribution is computed).

Figure 2 shows a distribution of fiscal limits—in other words, the chart shows the probability that a certain debt level can be supported given the current level of, and possible future changes to, economic and policy conditions. The debt levels in the upper tail of the distribution can

Figure 2
A Hypothetical Distribution of Fiscal Limits



Source: Author's calculations.

be maintained only if the economy receives a run of good shocks. Given the low probability of receiving such good shocks, governments have a high risk of default if their debt reaches the upper tail. Similarly, the debt levels in the lower tail can be maintained even if the economy receives a run of bad shocks; as a result, default is less likely at these levels.

According to the hypothetical distribution in Figure 2, a government's default probability is 0.4 when its outstanding debt level reaches 200 percent of its GDP. That is, due to uncertainty about the underlying economy's future fundamentals, there is a 40 percent chance that a debt-to-GDP ratio of 200 percent will be unsustainable. At a debt level at the far left tail of the distribution of fiscal limits (for instance, a debt-to-GDP ratio of 50 percent), the default probability is essentially nil, and an increase in outstanding debt may have little effect on sovereign default risk. As the debt level moves to the right in the distribution of the fiscal limit, however, the risk of default can surge rapidly.

State-dependence of the fiscal limit distribution

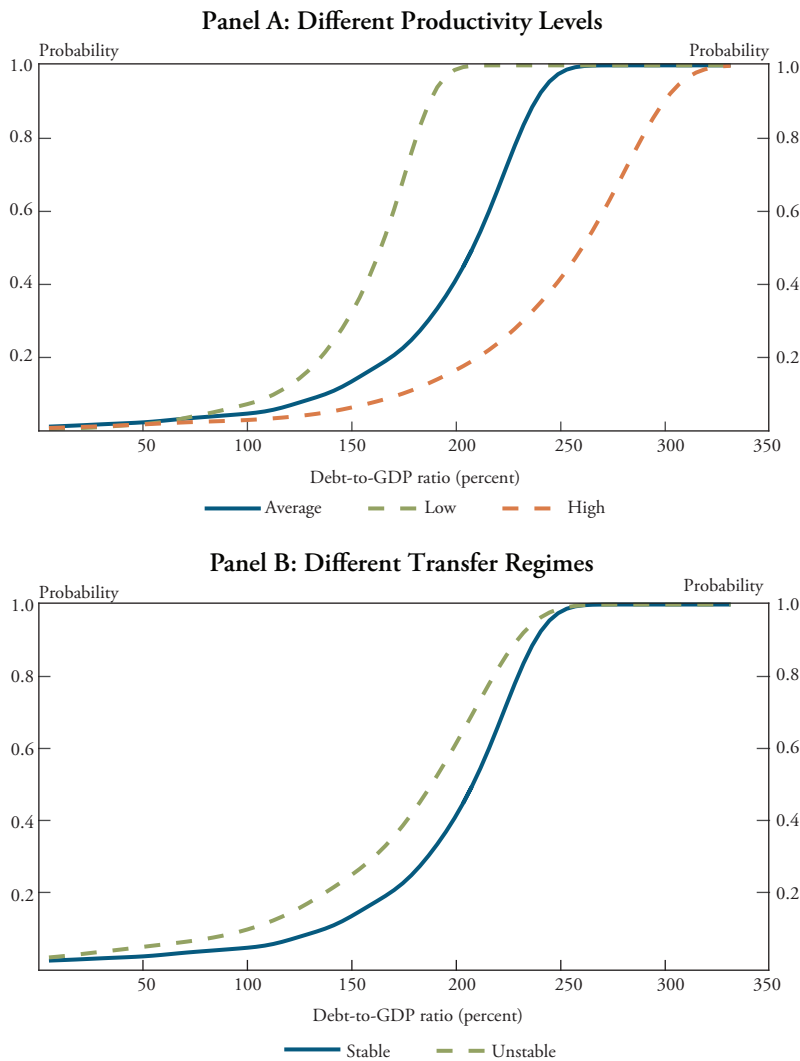
Economic and policy conditions can shift the distribution of fiscal limits. The probability of whether a given debt level is sustainable at a given time depends on the state of the economy at that time—for instance, whether the productivity level is high or low, or whether

government transfers are stable or growing.³ Thus, the distribution of the fiscal limit is state-dependent.

Panel A of Figure 3 shows that a lower productivity level shifts the distribution of the fiscal limit to the left, raising the probability a government will default on its debt. The solid blue line shows the distribution for a hypothetical economy at its long-run stable level without any shocks, the green dashed line shows the distribution for the same economy with a low productivity level, and the orange dashed line shows the distribution with a high productivity level. The panel highlights that a country's productivity level can play a significant role in determining its government's capacity to service its debt. In this example, when the outstanding debt level reaches 200 percent of GDP, the probability of default is less than 0.2 at a high productivity level and 0.4 if the economy is at its long-run stable level. At a low productivity level, default is unavoidable. A higher productivity level, if persistent, raises the maximum tax revenues the government can collect—not only in the current period, but also in the near future. The resulting higher maximum primary surpluses shift up the distribution of fiscal limits, reducing the probability of default at any given level of debt. In contrast, lower productivity levels shift down the distribution of fiscal limits, raising the probability of default at all debt levels.

Panel B of Figure 3 plots the distributions for the stable and unstable transfer regimes. When transfers are in the stable regime and expected to remain there for some time, the distribution permits the economy to support a relatively high level of debt as shown by the blue line. When transfers are in the unstable regime and expected to remain there for some time, the resulting reduction in future surpluses shifts the distribution of fiscal limits to the left, as shown by the green dashed line. In this example, when transfers switch from the stable to the unstable regime, the probability of default at a debt level of 200 percent of GDP rises from 0.4 to 0.6. The more persistent the unstable regime is, the larger the effect such a switch would have on the default probability.

Figure 3
Hypothetical Distributions of State-Dependent Fiscal Limits



Source: Author's calculations.

III. Country-Specific Distributions of Fiscal Limits

Distributions of fiscal limits are intrinsically country-specific, as government policy, economic growth, and investors' preferences can vary greatly across countries. In some countries, relatively high debt levels may be associated with minimal default risk if distributions of fiscal limits imply the economy can easily support more debt. In other countries, even relatively low debt levels may carry a substantial risk of default if the economy cannot generate sufficiently large future surpluses.

In this section, I use the proposed framework to compare the distributions of fiscal limits across four advanced economies: Greece, Italy, Japan, and the United States. Except in the United States, each of these countries has amassed debt well above 100 percent of GDP. Despite this, the riskiness of their bonds differs in global financial markets. In addition, I apply the framework to four emerging market economies—Brazil, China, India, and Russia—and examine how their fiscal limits differ from the advanced economies.

I use two measures to quantify cross-country differences in the distributions of fiscal limits: the median fiscal limit and the dispersion within state-dependent distributions. The median fiscal limit is the debt level associated with a probability of default of 0.5. All else equal, a country with a higher median fiscal limit has more space to support its debt and is therefore less likely to run into its fiscal limit. The dispersion within state-dependent distributions measures the effect of economic and policy conditions on fiscal limits. A wider dispersion within a country's distribution implies that economic and policy conditions have a larger effect in shifting distributions. In other words, when the country receives negative shocks, its risk of default may rise more rapidly.

Advanced economies

I calibrate the framework of fiscal limits to Greece, Italy, Japan, and the United States to compare the distributions of their fiscal limits. Table 1 summarizes some statistics across those countries—specifically, the average of the ratio of discretionary government spending to GDP from 1971 to 2015; the average of the ratio of transfers to GDP for the same period; and the standard deviations of the productivity level, which is measured by real GDP per worker, from 1951 to 2014 (see Appendix A for details on how the other parameters are calibrated).

Table 1
Selected Statistics across Advanced Economies

Variables	Greece (percent)	Italy (percent)	Japan (percent)	United States (percent)
Ratio of discretionary spending to GDP	21.0	20.0	14.0	19.0
Ratio of transfer spending to GDP	15.5	19.7	13.4	14.2
Ratio of total government expenditures to GDP	36.5	39.7	27.4	33.2
Standard deviation of productivity level	0.045	0.027	0.025	0.016

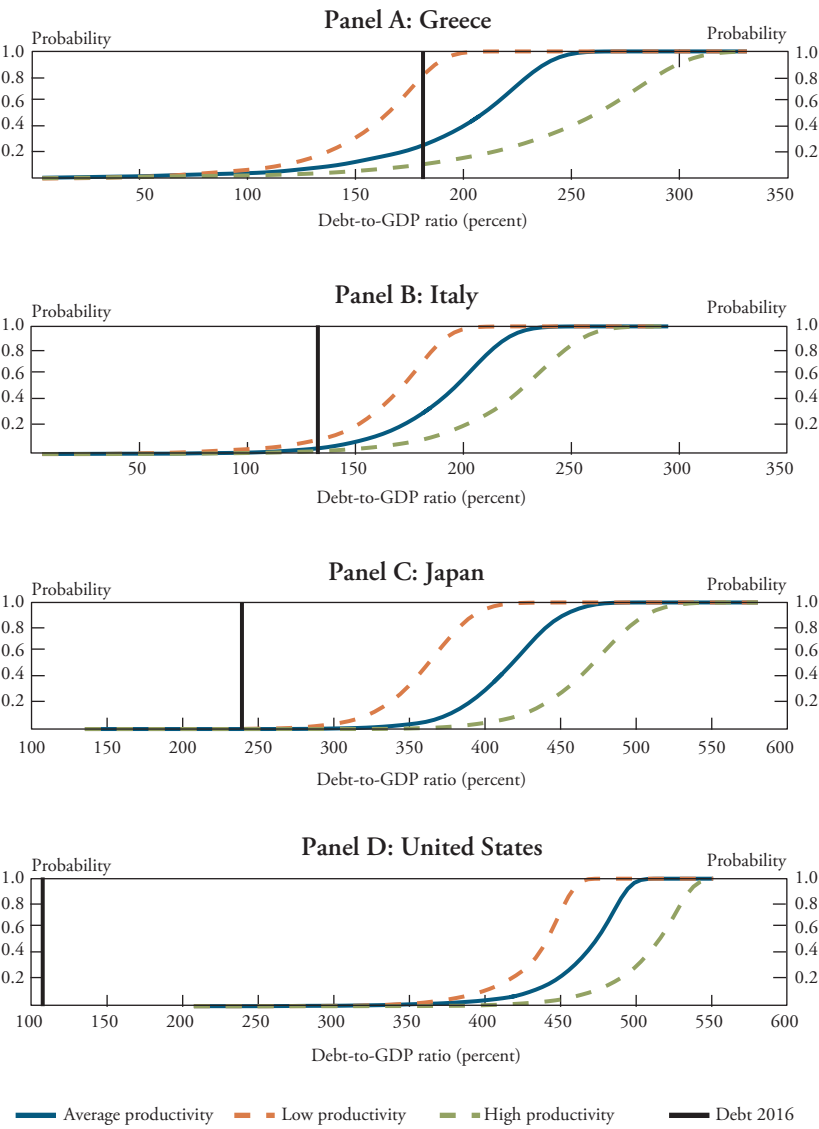
Sources: IMF and OECD.

The table shows that Japan and the United States have lower transfer and discretionary government spending ratios, on average, than Greece and Italy. For instance, total government spending, which is the sum of discretionary and transfer spending in the first two rows, accounts for 36.5 percent and 39.7 percent of GDP in Greece and Italy, respectively, compared with only 27.4 percent in Japan and 33.2 percent in the United States. In addition, Greece has the most volatile productivity: its productivity is almost three times as volatile as in the United States as shown by the comparison of standard deviations of productivity levels in Table 1.

Chart 5 compares the distributions of fiscal limits across the four countries. In each panel, the blue line represents the distribution of the fiscal limit when the economy is at its long-run stable level; the dashed orange line represents the distribution when the productivity level is four standard deviations lower than its long-run level, as might occur during an unusually deep recession; and the dashed green line represents the distribution when the productivity level is four standard deviations higher than its long-run level, as might occur in an exceptionally good economic environment. The black vertical line shows the government debt-to-GDP ratio in 2016 for each country.

The top two panels compare the distributions of fiscal limits in Greece and Italy. Panel A shows that in Greece, a country with a debt level of over 180 percent of GDP in 2016, the probability of default is 0.25 when productivity is at its long-run level and 0.12 when productivity is four standard deviations higher than its long-run level. When productivity is four standard deviations lower than its long-run level, however, the probability of default jumps to 0.8. The Italian distribution, shown in Panel B, is comparable to the Greek with one notable

Chart 5
Comparing Distributions of Fiscal Limits across Advanced Economies



Source: Author's calculations.

distinction: the dispersion of the state-dependent distributions is narrower due to smaller fluctuations in Italian productivity levels (Table 1). This seemingly small difference in the dispersion can have a significant effect on sovereign default risk. The narrower dispersion for Italy implies

that a negative shock—for example, a decline in productivity—leads to a smaller leftward shift in the distribution than for Greece. The smaller shift in the distribution means that the probability of default does not increase as rapidly as in Greece, dampening the rise in debt interest payments and, ultimately, the outstanding debt level in Italy. Overall, the distribution of Italian fiscal limits shifts less to the left and the outstanding debt level rises less, both of which dampen the increase in the Italian government's risk of default.

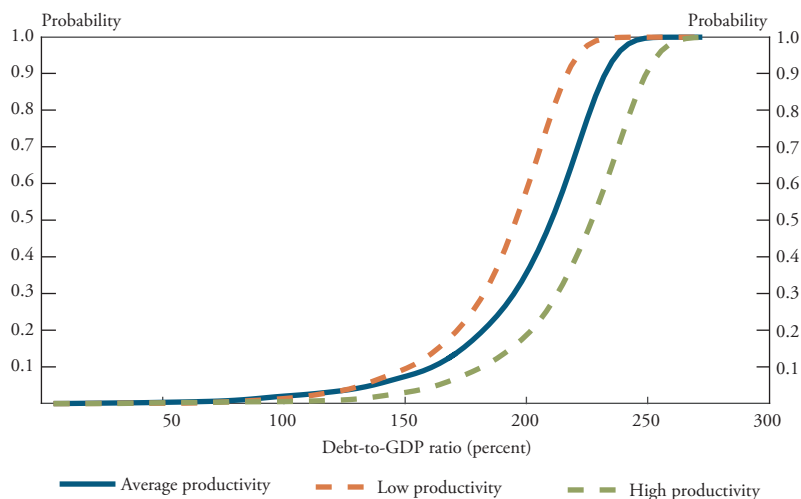
The bottom two panels of Chart 5 show that the fiscal limits for Japan (Panel C) and the United States (Panel D) are much higher. For instance, the probability of default associated with a debt-to-GDP ratio of 300 percent is near zero for both countries regardless of the productivity level. This is consistent with the countries' lower government spending. Since fiscal limits are the discounted sum of maximum fiscal surpluses in the future, even a relatively small difference in government expenditures each period can have a large cumulative effect on fiscal limits. The difference in government expenditures between Japan and the United States and between Greece and Italy leads to dramatic differences in their distributions of fiscal limits.⁴

Comparing the panels for Japan and the United States shows that the United States has a lower probability of default. The median debt threshold is lower in Japan than in the United States regardless of the productivity level in each country. As a result, given the same economic and policy conditions, the United States can support more debt than Japan.⁵ In addition, the dispersion of the state-dependent distributions is wider for Japan. As discussed above, the wider dispersion indicates that Japan's risk of default would likely increase more rapidly should the outstanding debt level approach the fiscal limit.

One important caveat in bringing this fiscal limit measure to policy debates is that in calculating fiscal limits, I assume that the government could raise tax rates to the peak of Laffer curves. Depending on the model specifications, Laffer curves often peak when taxes reach the range of 50 to 70 percent. This range of tax rates would be politically challenging, if possible at all, to reach in the United States, considering that the average tax rate was 25.3 percent from 1970 to 2015.⁶ As a result, the fiscal limits for the United States in Chart 5 likely overestimate the room the country actually has to increase its debt.

Chart 6

Alternative Distributions of Fiscal Limits for the United States: Keeping the Maximum Tax Rate at 40 Percent



Source: Author's calculations.

Chart 6 shows the distributions of fiscal limits under an alternative scenario: I assume the government can no longer tax at the peak of the Laffer curve; instead the maximum tax rate is fixed at 40 percent. Lowering the maximum tax rate to 40 percent shifts the distributions to the left by a substantial margin. For instance, the probability of default associated with a debt-to-GDP ratio of 200 percent is above 0.3 when the productivity is at its long-run level and 0.6 when the productivity is four standard deviations lower than its long-run level, compared with a probability of zero regardless of productivity level if the government could tax at the peak of the Laffer curve.

Emerging market economies

Applying the framework of fiscal limits to China, India, Brazil, and Russia highlights key differences between the sustainable levels of government debt in advanced economies and major emerging market economies.

The emerging market economies differ from the advanced economies in their shares of government spending in GDP, in the volatility of their productivity levels, and in their discount factors. Table 2 shows

Table 2

Selected Data Statistics across Emerging Market Economies

Variables	Brazil (percent)	China (percent)	India (percent)	Russia (percent)
Ratio of discretionary spending to GDP	10.3	7.7	7.6	15.0
Ratio of transfer spending to GDP	18.5	14.2	12.6	17.0
Ratio of total government expenditures to GDP	28.8	21.9	20.2	32.0
Standard deviation of productivity level	0.048	0.060	0.037	0.090

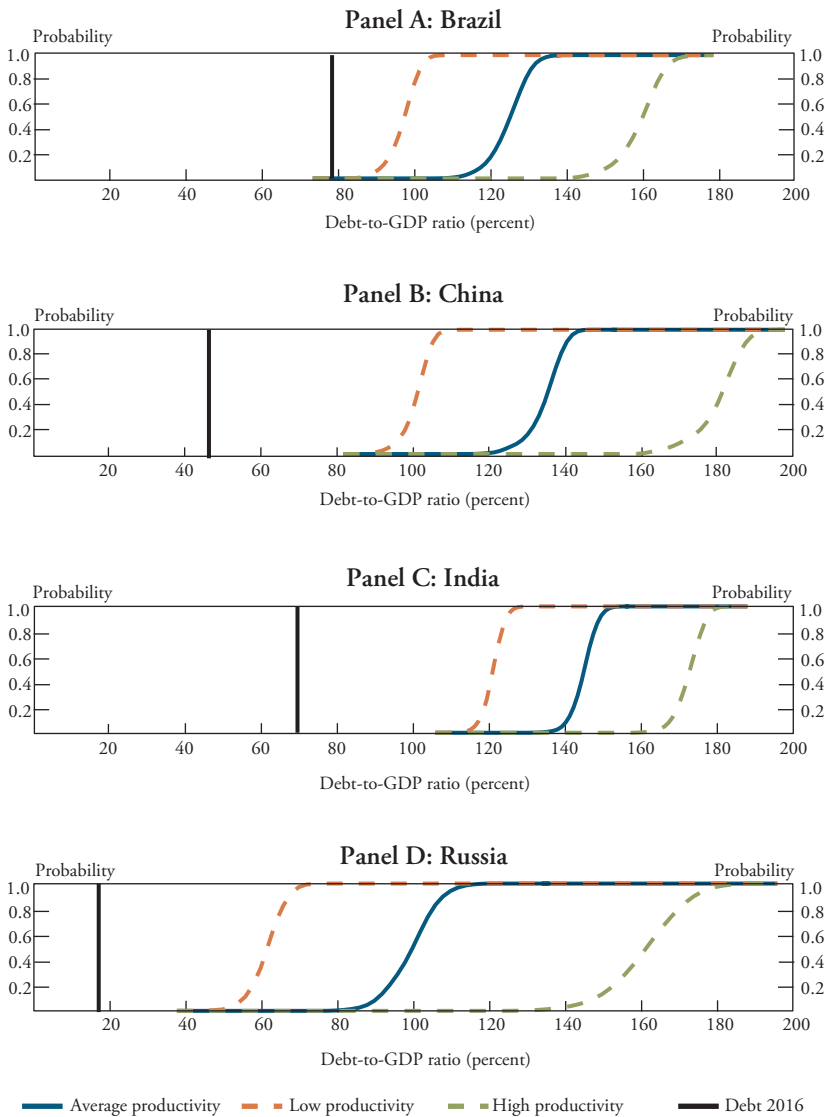
Sources: IMF and OECD.

that total government expenditures in China and India are low compared with the advanced economies at 21.9 percent and 20.2 percent of GDP, respectively. In contrast, government expenditures in Brazil (at 28.8 percent) and Russia (at 32.0 percent) are closer to those in Japan and the United States. Productivity levels are, in general, more volatile in the emerging market economies than in the advanced economies. For instance, productivity is four times as volatile in Russia as it is in the United States.

More importantly, investors are less willing to hold debt issued by emerging market economies unless they are compensated by extra returns on those bonds. The JP Morgan Emerging Market Bond Index tracks total returns for external debt in emerging markets and shows that governments in these countries on average pay a substantial premium over United States Treasury bond yields. In the framework of fiscal limits, this difference in investors' preferences is captured by a lower long-run discount factor in emerging market economies than in advanced economies. The lower the discount factor, the more heavily investors discount future fiscal surpluses and the lower the distribution of fiscal limits becomes. I calibrate the long-run level of the discount factor to be 0.9 for emerging market economies compared with 0.98 for advanced economies.

Chart 7 compares the distributions of fiscal limits across the four emerging market economies. Brazil and Russia, which have larger shares of government expenditures, have lower fiscal limits. Assuming that productivity in each country is at its long-run level, the median debt level is 100 percent of GDP for Russia and 125 percent for Brazil compared with 135 percent for China and 145 percent for India. As

Chart 7
Comparing Distributions of Fiscal Limits across Emerging Market Economies



Source: Author's calculations.

Russia's productivity level is substantially more volatile, its state-dependent distributions are quite dispersed: as the productivity level increases from a low to a high level, the median debt level rises from 60 percent of GDP to over 140 percent of GDP.

Fiscal limits are substantially lower in emerging market economies than in advanced economies due to the lower discount factor. Regardless of productivity level, the probability of default for all four emerging market economies would be 1 at a debt level of 190 percent of GDP; in Japan and the United States, the probability of default would be essentially zero at the same debt level.

As all four emerging market economies had government debt levels lower than their fiscal limits in 2016, their default probabilities are essentially zero. However, sovereign default risks can rise rapidly when economic and policy conditions deteriorate. In the next section, I apply the framework of fiscal limits to the Greek debt crisis to highlight the debt and default dynamics in an economic downturn.

IV. Debt and Default Dynamics in an Economic Downturn

The state-dependent nature of fiscal limit distributions implies that a debt level that is safe in good times can quickly become unsustainable when the economy receives a negative shock. Such a shock lowers the maximum fiscal surplus a government can collect, shifting the distribution of its fiscal limit to the left. At the same time, the shock lowers the actual fiscal surplus that the government collects—and as fiscal surpluses turn to deficits, government debt begins to accumulate. The leftward shift in the distribution of the fiscal limit, in conjunction with rising debt, increases the risk of default. Ballooning interest payments on debt may force the government to borrow even more, driving its debt level closer to the fiscal limit. This vicious cycle pushes the outstanding government debt and the distribution of the fiscal limit toward each other, consistent with the observation in debt crises that once default risks start to rise, they do so rapidly.

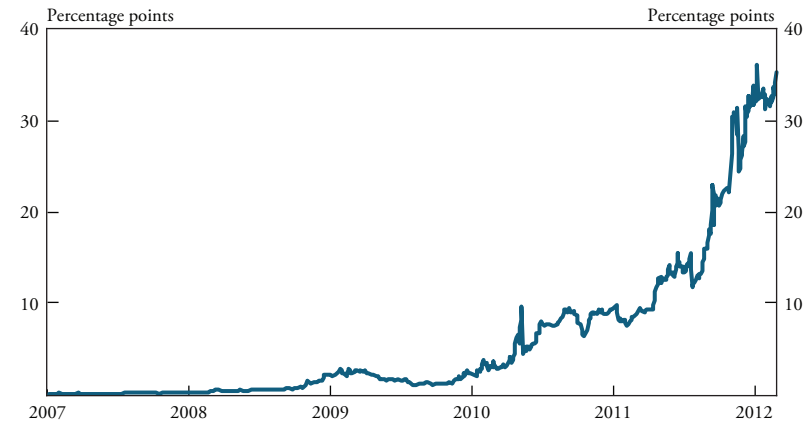
The recent Greek debt crisis provides a salient example. During a period of robust economic growth from 2001 to 2007, the Greek government borrowed at roughly the same low cost as Germany. Investors were confident about the Greek government's creditworthiness even

though its debt stood at about 100 percent of GDP while the German government's debt stood below 70 percent of GDP. But this confidence deteriorated rapidly after 2009, when the Greek economy plunged into a recession and investors began to doubt that the amount of debt the Greek government had accrued was sustainable.⁷ Chart 8 shows that the interest rate spread between Greek and German bonds rose to over 30 percentage points by early 2012, leading the Greek government to renegotiate the terms of its debt with investors.

Such a staggering increase in spreads stimulated a debate over whether financial markets had mispriced default risks in Greece before or after the start of the crisis. Aizenman, Hutchison, and Jinjark; and De Grauwe and Ji show that linear empirical models cannot predict the magnitude of sovereign risk premiums during the crisis, suggesting financial markets had indeed underpriced the sovereign risk before the crisis or overpriced it afterward. The framework of fiscal limits, on the other hand, provides an alternative and nonlinear approach to shed light on the debate. Shocks to economic and policy conditions today can change both the distribution of fiscal limits and the outstanding debt liability. Such shifts may have little effect on debt sustainability when the prevailing level of debt is far away from the fiscal limit, but they may have a significant effect on default risk when the level of debt is already close to the limit.

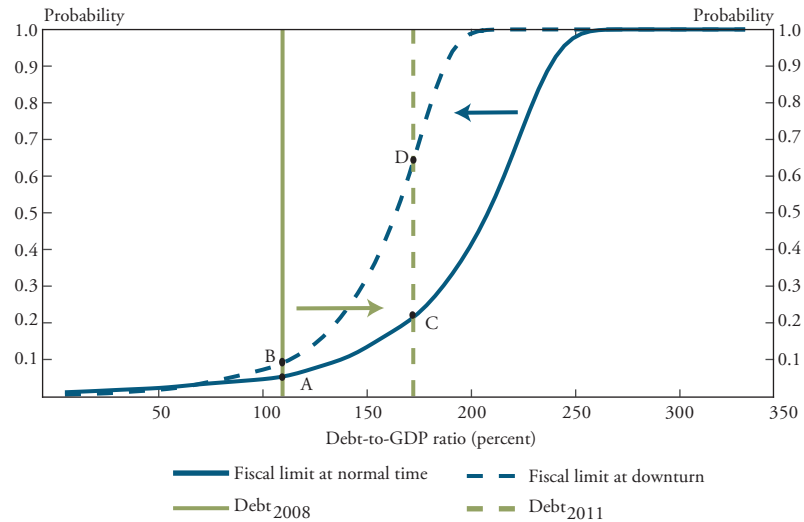
Chart 9 illustrates these dynamics for Greece. In 2008, when Greece's debt stood at 110 percent of GDP, the probability that the Greek government would run into its fiscal limit was about 0.05 (point A), assuming productivity was at its long-run stable level. Even if the economy received a large negative shock while productivity was four standard deviations lower than its long-run level, the probability of default in 2008 would be only marginally higher at 0.09 (point B). This estimate, however, relies on an unrealistic assumption—specifically, that a negative shock to the economy will not raise the government's outstanding debt liability. In fact, Greek government debt ballooned from 110 percent of GDP in 2008 to over 170 percent of GDP in 2011. Had productivity stayed at its long-run level in 2011, the probability of default would have risen modestly to 0.2 (point C); with productivity four standard deviations lower than its long-run level, however, the probability of default jumped above 0.6 (point D). Together, the rise

Chart 8
Interest Rate Spread between 10-Year Greek and German Government Bonds (2007–12)



Source: Reuters (Haver Analytics).

Chart 9
Outstanding Debt Levels and Distributions of Fiscal Limits in Greece



Sources: IMF and author's calculations.

in government debt and the shift in the distribution of the fiscal limit explain the surge in the interest rate spread between Greek and German government bonds after 2009 in terms of a jump in the risk of default.

V. Conclusion

By mapping policy and economic fundamentals to fiscal limit distributions, this article provides a quantitative framework to examine fiscal sustainability. The state-dependent property of fiscal limits is not just of theoretical interest; instead, understanding state dependence can be important for policymakers, as it explains why debt levels that are viewed as safe in good times can quickly become unsustainable in an economic downturn.

In addition, the distributions of fiscal limits are intrinsically country-specific. There are no one-size-fits-all fiscal limits—the same debt level can be associated with drastically different probabilities of default for different countries, as each country has distinct economic and policy fundamentals. For the European periphery countries, credible fiscal reforms may not only help stabilize the outstanding government debt levels, but may also shift up their fiscal limits, thereby making their debt less risky.

Appendix A

Model Specification

I employ a simple theoretical model that draws on Bi. The model consists of three sectors: household, firm, and government. Each period, a representative firm produces goods, y_t , using a technology that is linear in hours worked, $1-L_t$, where L_t denotes the proportion of time spent in leisure. Its productivity level, A_t , is determined by an exogenous AR(1) process.

$$y_t = A_t(1-L_t) \quad (A1)$$

$$\ln \frac{A_t}{A} = \rho^A \ln \frac{A_{t-1}}{A} + \varepsilon_t^A \quad \varepsilon_t^A \sim N(0, \sigma_A^2) \quad (A2)$$

A representative household receives wage income from the firm and lump-sum transfers from the government, z_t , and pays taxes on its wage income, $w_t(1-L_t)$, at the rate τ_t . The household also receives payoffs from the asset it purchased in the previous period, a_{t-1} , and decides its asset holding in the current period, a_t . For each unit of asset purchased at price q_t at time t , the household expects to receive one unit of payoff in the next period. Given the budget constraint, the household decides how much to work and consume, c_t , each period to maximize welfare.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \quad (A3)$$

$$s.t. \quad w_t(1-\tau_t)(1-L_t) + z_t - c_t = a_t q_t - a_{t-1} \quad (A4)$$

From this optimization problem, I derive the first-order conditions as follows. Equation (A5) shows that the marginal rate of substitution between consumption and leisure equals the after-tax wage, while equation (A6) reflects that the household can use assets to smooth its consumption path.

$$\frac{u_L(t)}{u_c(t)} = w_t(1-\tau_t) \quad (A5)$$

$$q_t = \beta E_t \left[\frac{u_c(t+1)}{u_c(t)} \right] \quad (A6)$$

The government collects tax revenues and issues bonds to finance its discretionary spending, g_t , and transfers. Unlike the household, which solves an explicit optimization problem, the government obeys simple rules in setting its discretionary spending and transfers. The discretionary spending obeys an AR(1) process, shown in equation (A7). The lump-sum transfers follow a regime-switching process. In one regime, transfers are stationary; in the other regime, transfers grow exponentially, and the growth is captured by the parameter μ^z . The transfers can move from the stationary regime to the explosive regime, dictated by a regime-switching index, rs_t^z . Depending on the transition matrix, transfers can rise steadily over prolonged periods. In both regimes, transfers can respond to the change in productivity with respect to its steady state, capturing automatic stabilizers built in transfers. The processes for government expenditures are as follows:

$$\ln \frac{g_t}{g} = \rho^g \ln \frac{g_{t-1}}{g} + \varepsilon_t^g \quad \varepsilon_t^g \sim N(0, \sigma_g^2) \quad (\text{A7})$$

$$z_t \equiv z(rs_t^z, A_t) = \begin{cases} z + \zeta^z(A_t - A) & \text{if } rs_t^z = 1 \\ \mu^z z_{t-1} + \zeta^z(A_t - A) & \text{if } rs_t^z = 2 \end{cases} \quad (\text{A8})$$

with $\mu^z > 1$ and rs_t^z following a transition matrix of $\begin{pmatrix} p_1^z & 1-p_1^z \\ 1-p_2^z & p_2^z \end{pmatrix}$. To compute the fiscal limit, the discount factor, β_{t+j} , depends on investors' preferences for holding assets to smooth consumption:

$$\beta_{t+j} = q_t q_{t+1} \cdots q_{t+j}. \quad (\text{A9})$$

The model is calibrated at annual frequency. The discount rate is 0.98 for advanced economies and 0.9 for emerging market economies. The utility function is assumed to be $u(c_t, L_t) = \log c_t + \phi \log L_t$. The total amount of time and the productivity level at the steady state are normalized to 1. The household spends 25 percent of its time working and, given the logarithm utility function, the Frisch elasticity of labor supply is 3. The leisure preference parameter, ϕ , is calibrated to match the consumption-over-GDP ratio ($1-g/y$) and tax rate for different countries (Table A-1), in addition to the labor supply. As a result, the leisure preference parameter can vary for different countries. The

process parameters for discretionary government spending and transfers are calibrated to country-specific fiscal data from the IMF and OECD during the period of 1971–2015.⁸ The process parameters for productivity levels are calibrated to country-specific data on real GDP per worker from the Penn World Table during the period of 1951–2014. Table A-1 summarizes the calibrations across countries. Bi provides more details on the model and solution.

Table A-1
Model Calibration

Variables	Greece	Italy	Japan	United States	China	India	Brazil	Russia
g/y	0.21	0.2	0.14	0.19	0.077	0.076	0.103	0.15
z/y	0.155	0.197	0.134	0.11	0.142	0.126	0.185	0.17
τ	0.276	0.353	0.169	0.253	0.215	0.193	0.344	0.344
μ^z	1.0079	1.0046	1.005	1.0078	1.012	1.0067	1.008	1.007
β	0.98	0.98	0.98	0.98	0.9	0.9	0.9	0.9
p_1^z	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
p_2^z	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
ρ^A	0.18	0.51	0.74	0.11	0.25	0.53	0.35	0.28
σ_g	0.045	0.0265	0.0254	0.0157	0.06	0.037	0.048	0.09
ρ^g	0.53	0.92	0.97	0.87	0.75	0.75	0.74	0.82
σ_g	0.02	0.013	0.04	0.014	0.05	0.05	0.06	0.044

Appendix B

Computing Distributions of Fiscal Limits

I use the Markov Chain Monte Carlo method to simulate the distributions of fiscal limits. It works as follows: assuming the economy starts at a certain state (zr_t, A_t, g_t) , I simulate a path for the productivity level and the transfer regime going forward for a long period of time, which follow their stochastic processes. Given this path, I compute the path of discount factors, which depend on people's optimization conditions as detailed in appendix A, and the path of maximum fiscal surpluses, which depend on the peak of the Laffer curve. With those in hand, I can compute the discounted sum of future maximum surpluses, b_t^* , that is associated with this specific path of future shocks. Next, I do another simulation with a different path of future shocks and compute another discounted sum of future maximum surpluses. I repeat this process many times. A sufficiently large number of simulations provides a good approximation of all possible fiscal limits conditional on the current state (zr_t, A_t, g_t) , from which I can compute its distribution, $f(b_t^*)$.

Endnotes

¹Other than outright default, governments can (and often do) resort to other options, including high inflation, to deflate their debt, as discussed in Reinhart and Rogoff. In this article, however, I focus on real government debt and exclude the possibility of deflating debt burdens through inflation.

²By focusing on real government debt, this article abstracts from the discount factor channel, through which central banks can affect fiscal limits by influencing real interest rates and thus the discount factor.

³Compared with the productivity level or government transfers, government discretionary spending has a much smaller effect on the distribution of fiscal limits in this framework (Bi).

⁴Table A-1 shows that Japan and the United States have lower tax rates on average than Greece and Italy. To match the data, the parameter that governs households' preferences is calibrated slightly higher in Japan and the United States, which reduces the maximum tax revenues. Given the same labor supply elasticity across all countries, however, the differences in the maximum revenues are negligibly small and, as a result, government expenditures largely determine the differences in maximum fiscal surpluses.

⁵This is because historically, Japan has lower tax rates than the United States on average, which, in my model, is captured by Japanese households' lower tolerance for being taxed.

⁶The average tax rate is defined as total tax revenue, including tax revenue from federal, state, and local governments, as a share of GDP.

⁷Bi and Leeper offer more background on how the "fiscal data revisions" announced by the Greek Ministry of Finance in 2009 and 2010 affected risk premiums on Greek government debt.

⁸The data sample is shorter for emerging market economies due to data limitations.

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