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Yulei Luo, Jun Nie, and Eric R. Young

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Yulei Luo†
University of Hong Kong

Jun Nie‡
Federal Reserve Bank of Kansas City

Eric R. Young§
University of Virginia

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Abstract

In this paper we examine the effects of model misspecification (robustness or RB) on international consumption correlations in an otherwise standard small open economy model with endogenous capital accumulation. We show that in the presence of capital mobility in financial markets, RB lowers the international consumption correlations by generating heterogeneous responses of consumption to productivity shocks across countries facing different macroeconomic uncertainty. In addition, we show that RB can also improve the model's predictions

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†School of Economics and Finance, Faculty of Business and Economics, The University of Hong Kong, Hong Kong. E-mail: yluo@econ.hku.hk.

‡Economic Research Department, Federal Reserve Bank of Kansas City. E-mail: jun.nie@kc.frb.org.

§Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: ey2d@virginia.edu.

in other three moments of consumption dynamics: the relative volatility of consumption to income, the persistence of consumption, and the correlation between consumption and output. After calibrating the RB parameter using the detection error probabilities, we show that the model can explain the observed international consumption correlations as well as the other consumption moments quantitatively. Finally, we show that the main conclusions of our benchmark model do not change in an extension in which the agent cannot observe the state perfectly due to finite information-processing capacity.

Keywords: Robustness, Imperfect State Observation, International Consumption Correlations, Consumption Dynamics.

JEL Classification Numbers: D83, E21, F41, G15.

1 Introduction

A common assumption in international business cycle models is that world financial markets are complete in the sense that individuals in different countries are able to fully insure country-specific income risks using international financial markets. Under this assumption, the models predict that consumption (or consumption growth) is highly correlated across countries, and in some cases the international consumption correlation is equal to 1 regardless of income or output correlations. The intuition is that since consumers are risk averse they will choose to smooth consumption over time by trading in international financial markets. However, in the data cross-country consumption correlations are very low and are generally lower than corresponding income correlations in many countries.[1] For example, Backus, Kehoe, and Kydland (1992) solve a two-country real business cycle model and argue that the puzzle that empirical consumption correlations are lower than output correlations is the most striking discrepancy between theory and data.[2] In the literature, the empirical low international consumption correlations have been interpreted as indicating international financial markets imperfections – for examples, see Kollman (1996), Baxter and Crucini (1995), Lewis (1996), and Kehoe and Perri (2002).

Other extensions have been proposed to make the models better fit the data. For example, Devereux, Gregory, and Smith (1992) show that in the perfect risk-sharing model nonseparability between consumption and leisure has the potential to reduce the cross-country consumption correlation. Stockman and Tesar (1995) show that the presence of nontraded goods in the complete-market model can also improve the models' predictions. Kollman (1996) shows that asset market incompleteness can generate significantly lower cross-country consumption correlations. Wen (2007) shows that adding country-specific demand shocks can also help explain the cross-country business cycle comovements within a complete-market framework. In addition, Fuhrer and Klein (2006) show that habit formation has important implications for international consumption correlations. In particular, they show that with a shock to the interest rate habit formation *by itself* can generate positive consumption correlations across countries even in the absence of international risk sharing and common income shocks. They then argue that if habit is a good characterization of consumers' behavior, the absence of international risk sharing is *even more striking* than standard tests suggest; that is, existing studies may overstate the extent to which common consumption movements across countries reflect international risk sharing because

[1] Table 1 reports the cross-country consumption and income correlations using the G-7 data.

[2] Pakko (1996) shows that, in the presence of complete asset markets, consumption should be more highly correlated with total world income than with domestic income, while the data shows the opposite. This result provides an alternative standard for evaluating models of international business cycles.

some of them are due to habit.[3]

All of the above papers have assumed that agents in the economy fully trust the probability model they use to make decisions. However, in reality, agents may not be able to know exactly the model generating the data, and they are concerned about whether their model is somehow misspecified. In this paper, we examine how introducing the preference for robustness (RB, a concern for model misspecification) into an otherwise standard small open economy (SOE) model with capital accumulation can significantly improve the models' predictions on the international consumption correlations puzzle we discussed above.[4]

Our benchmark model is based on Hansen, Sargent, and Tallarini (1999, henceforth, HST) and Hansen and Sargent (2007a) in which we assume that consumers can observe the state perfectly. Hansen and Sargent (1995, 2007) first introduced robustness into economic models. In robust control problems, agents are concerned about the possibility that their model is misspecified in a manner that is difficult to detect statistically; consequently, they make their decisions as if the subjective distribution over shocks was chosen by a malevolent nature in order to minimize their expected utility (the solution to a robust decision-maker's problem is the equilibrium of a max-min game between the decision-maker and nature).[5] Robustness models produce precautionary savings but remain within the class of LQ-Gaussian models, which leads to analytical simplicity.[6] We find that RB can help improve the model's consistency with the empirical evidence on international consumption correlations.[7] Specifically, we show that in the presence of capital mobility in international financial markets, RB lowers international consumption correlations by introducing heterogenous responses of consumption to productivity shocks across countries which are facing different levels of macroeconomic uncertainty. That is, we have uncovered a novel

[3]Baxter and Jermann (1997) also argue that the international diversification puzzle is *"worse than you think"* due to nontraded labor income being correlated with the return to domestic assets.

[4]We adopt a small-open economy setting with quadratic utility and linear state transition equation rather than two-country general equilibrium setting with CRRA utility and stochastic interest rates in this paper for two reasons. First, as argued in Hansen and Sargent (2007), if the objective function is not quadratic or the state transition equation is not linear, worst possible distributions due to RB are generally non-Gaussian, which significantly complicates the computational task. Second, there does not exist a two-country general equilibrium in which the equilibrium interest rate is constant. See Section 3.1 for a detailed discussion.

[5]We interpret fear of model misspecification as an information imperfection because it implies that the true data-generating process is unknown.

[6]A second class of models that produces precautionary savings but remains within the class of LQ-Gaussian models is the risk-sensitive model of Hansen, Sargent, and Tallarini (1999). See Hansen and Sargent (2007a) and Luo and Young (2010) for detailed comparisons of the two models.

[7]Recently, some papers have incorporated model uncertainty into small open economy models and examined the effects of robustness on business cycles and monetary policy. See Cook (2002), Leitemo and Söderström (2008), Dennis, Leitemo, and Söderström (2009), and Justiniano and Preston (2010),.

channel through which the fundamental economic shocks in different countries can interact with agents' concerns about model misspecification (i.e., model uncertainty), which, in turn, reduces consumption correlations across these countries.

After calibrating the RB parameter using plausible detection error probabilities in our benchmark model, we find that it can better match the data on international consumption correlations quantitatively. In addition, we also compare the implications for the key stochastic properties of consumption in individual countries: (i) the relative volatility of consumption to income, (ii) the first-order autocorrelation of consumption, and (iii) the contemporaneous consumption-income correlation in individual countries between the FI-RE and RB models after calibrating plausible RB parameters. We find that RB can also significantly improve the performance of the model in terms of these consumption moments: each model economy displays more realistic consumption dynamics.

In an extension, in addition to the concern for model misspecification, we assume that consumers face state uncertainty (SU) due to imperfect state observations. This assumption can be rationalized using the rational inattention (RI) hypothesis proposed in Sims (2003).[8] The key idea of RI is that agents have imperfect information about the state of the world due to limited information-processing capacity and learn slowly.[9] Specifically, we assume that consumers only observe noisy signals about the true state when making optimal decisions and thus need to extract the true signals using imperfect observations. Here we assume that the noisy signal is the sum of the true state and an iid noise, which is standard in the signal extraction literature.[10] In this extension, we can see that incorporating state uncertainty can lead to additional model uncertainty. As we will show, the state uncertainty that agents face can significantly amplify the effects of RB on reducing international consumption correlations across countries.

Specifically, we find that the interactions between RB and SU generates two competing forces on international consumption correlations. First, the model with SU generates gradual responses of consumption growth to income shocks due to imperfect state observation. Just like the habit formation or sticky expectations hypotheses (infrequent updating as in Bacchetta and van Wincoop 2010), this channel increases cross-country consumption correlations. Second, the noise due to imperfect observation reduces consumption correlations across countries because it increases

[8]Luo and Young (2013) showed that within the univariate linear-quadratic framework, if the signal-to-noise ratio is given in the traditional signal extraction problem with state uncertainty, signal extraction and rational inattention are observationally equivalent in the sense that they lead to the same decision rules. Therefore, in this paper we use state uncertainty to make the model more general and help better focus on the key intuitions.

[9]In contrast, the full-information rational expectations (RE) hypothesis assumes that ordinary households can observe all available information without errors.

[10]For example, Muth (1960), Lucas (1972), Morris and Shin (2002), and Angeletos and La'O (2009).

consumption volatility but has no effect on the covariance of consumption across countries; the noise shocks are independent across countries. These effects both disappear if there is no model uncertainty due to RB. The intuition is that the two forces have the same effect on the consumption adjustment processes across countries in the absence of heterogenous degrees of fundamental uncertainty and the preference for RB, which thus does not affect international consumption correlations. Using the same calibration procedure, we find that the interaction between RB and SU further improves the model's quantitative predictions on the observed consumption correlations, and that we require only a small departure from perfect state observation to fit the data well. In addition, we also find that the effects of SU on the other stochastic properties of consumption are not significant and mixed for individual countries in G-7 we studied.

The remainder of the paper is organized as follows. Section 2 reviews the standard full-information rational expectations small open economy (SOE) model and discuss the puzzling implications for international consumption correlations in the model. Section 3 presents the RB model, calibrates the model misspecification parameter, and show to what extent RB can improve the model's performance. Section 4 introduces RB into the SOE model with SU and shows that SU can further improve the model's key predictions. Section 5 concludes.

2 A Full-information Rational Expectations Small Open Economy Model

2.1 Model Setup

In this section we present a full-information rational expectations (RE) version of a small open economy (SOE) model and will discuss how to incorporate robustness (RB) into this stylized model in the next sections. Following the literature, we assume that the model economy is populated by a continuum of identical infinitely-lived consumers, and the only asset that is traded internationally is a risk-free bond. Following Glick and Rogoff (1995), we formulate the full-information RE-SOE model as

$$\max_{\{c_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right] \tag{1}$$

subject to the flow budget constraint

$$b_{t+1} = Rb_t - c_t + y_t, \tag{2}$$

where $u(c_t) = -\frac{1}{2} (\overline{c} - c_t)^2$ is the utility function, c_t is consumption, $\overline{c}$ is the bliss point, $R \geq 1$ is the exogenous and constant gross world interest rate, b_t is the amount of the risk-free world bond held at the beginning of period t, and y_t is net income in period t and is defined as output less than investment and government spending. Here we assume that the household sector takes y_t as given and later we will model how y_t is determined endogenously in the firm sector. The model assumes perfect capital mobility in that domestic consumers have access to the bond offered by the rest of the world and that the real return on this bond is the same across countries. In other words, the world risk-free bond provides a mechanism for domestic households to smooth consumption using the international capital market. Finally we assume that the no-Ponzi-scheme condition is satisfied.

A similar problem can be formulated for the rest of the world (ROW). We use an asterisk ("$*$") to represent the rest of world variables. For example, we assume that y_t^* is the aggregate income of the rest of the world (G-7, OECD, or EU). Furthermore, we assume that domestic endowment and the ROW endowment are correlated. We will specify the structure of the income processes later after we discuss how to determine y_t endogenously.

Let $\beta R = 1$; optimal consumption is then determined by permanent income:

$$c_t = (R-1)\, s_t \tag{3}$$

where

$$s_t = b_t + \frac{1}{R}\sum_{j=0}^{\infty} R^{-j} E_t\left[y_{t+j}\right]$$

is the expected present value of lifetime resources, consisting of financial wealth (the risk free foreign bond) plus human wealth. In order to facilitate the introduction of robustness and rational inattention we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model with a general income process to a univariate model with iid innovations to permanent income s_t that can be solved in analytically.[11] Letting s_t be defined as a new state variable, we can reformulate the PIH model as

$$v(s_0) = \max_{\{c_t, s_{t+1}\}_{t=0}^{\infty}} \left\{ E_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t)\right]\right\} \tag{4}$$

subject to

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1}, \tag{5}$$

where the time $(t+1)$ innovation to permanent income can be written as

$$\zeta_{t+1} = \frac{1}{R}\sum_{j=t+1}^{\infty}\left(\frac{1}{R}\right)^{j-(t+1)} \left(E_{t+1} - E_t\right)\left[y_j\right]; \tag{6}$$

[11] See Luo (2008) for a formal proof of this reduction. Multivariate versions of the RI model are numerically, but not analytically, tractable, as the variance-covariance matrix of the states cannot generally be obtained in closed form.

$v(s_0)$ is the consumer's value function under RE. Under the RE hypothesis, this model with quadratic utility leads to the well-known random walk result of Hall (1978),

$$\Delta c_t = \frac{R-1}{R}\left(E_t - E_{t-1}\right)\left[\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j\left(y_{t+j}\right)\right] \tag{7}$$
$$= (R-1)\,\zeta_t,$$

which relates the innovations to consumption to income shocks.[12] In this case, the change in consumption depends neither on the past history of labor income nor on anticipated changes in labor income. In addition, certainty equivalence holds, and thus uncertainty has no impact on optimal consumption.

2.2 Capital Accumulation and Endogenous Net Output

In this subsection, we follow Glick and Rogoff (1995) and Gruber (2002), and explicitly model capital accumulation. Consequently, net output is determined endogenously. Specifically, we assume that the production function is given by

$$y_t = a_t k_t^{\alpha} - \frac{g}{2}\frac{i_t^2}{k_t},$$

where k_t is the capital stock, i_t is gross investment, $\frac{g}{2}\frac{i_t^2}{k_t}$ measures the loss of output due to adjustment costs, and a_t is a multiplicative country-specific productivity shock that follows

$$a_{t+1} = (1-\rho)\,\overline{a} + \rho a_t + \epsilon_{t+1}, \tag{8}$$

where ϵ_{t+1} is a Gaussian innovation with mean 0 and variance ω^2. Linearizing this production function around the steady state, we have

$$y_t \cong \alpha_a a_t + \alpha_k k_t + \alpha_i i_t, \tag{9}$$

where $\alpha_a = \overline{k}^{\alpha} > 0$, $\alpha_k = \alpha\overline{a}\overline{k}^{\alpha-1} + g\delta^2/2 > 0$, $\alpha_i = -g\delta < 0$, and the variables with a bar are the corresponding steady state values. Net output, $\widetilde{y}_t = y_t - i_t$, can be thus written as

$$\widetilde{y}_t = \alpha_a a_t + \alpha_k k_t + (\alpha_i - 1)\, i_t. \tag{10}$$

The firm's optimization problem is to maximize output minus total investment plus the corresponding adjustment costs

$$\max \sum_{t=j}^{\infty}\left(\frac{1}{R}\right)^{t-j}\left[a_t k_t^{\alpha} - \frac{g}{2}\frac{i_t^2}{k_t} - i_t\right]$$

[12] Under RE the expression of the change in individual consumption is the same as that of the change in aggregate consumption.

subject to the capital accumulation equation

$$k_{t+1} = (1-\delta) k_t + i_t,$$

for $t \geq j$. Here we assume that the household sector owns the firm. Following the same procedure used in Glick and Rogoff (1995), we can solve for optimal capital accumulation and investment rules as follows:

$$k_t = \lambda_1 k_{t-1} + \frac{\alpha \overline{k}^{\alpha}}{g\lambda_2} \sum_{j=t}^{\infty} \left(\frac{1}{\lambda_2}\right)^{j-t} E_{t-1}[a_j] + \Omega, \tag{11}$$

$$i_t = \lambda_1 i_{t-1} + \lambda_i \Delta a_t, \tag{12}$$

where $\Omega = \frac{\alpha \overline{a} \overline{k}^{\alpha}}{g(1-\lambda_2 \cdot L)}$ is an irrelevant constant term, $\lambda_i = \frac{\rho \alpha \overline{k}^{\alpha}}{g\lambda_2(1-\rho)}$, and $\lambda_1 \in (0,1)$ and $\lambda_2 > 1$ satisfy $\lambda_1 + \lambda_2 = 1 + R - (\alpha - 1)\alpha \overline{a} \overline{k}^{\alpha-1}/g$ and $\lambda_1 \lambda_2 = R$. Using (6), (10), (11), and (12), we can express the innovation to perceived income as:

$$\zeta_{t+1} \equiv \frac{1}{R} \sum_{j=t+1}^{\infty} \left(\frac{1}{R}\right)^{j-(t+1)} (E_{t+1} - E_t)[y_j]$$
$$= \Xi \epsilon_{t+1},$$

where

$$\Xi = \frac{1}{R-\rho}\left[1 + \frac{\alpha\rho(R+\delta)}{g(R-\lambda_1)(\lambda_2-\rho)}\right]. \tag{13}$$

This expression means that the innovation to permanent income is a linear function of the innovation to aggregate productivity.

For the rest of world, we have a similar expression:

$$\zeta_{t+1}^* \equiv \Xi^* \epsilon_{t+1}^*,$$

where $\Xi^* = \frac{1}{R-\rho^*}\left[1 + \frac{\alpha\rho^*(R+\delta^*)}{g^*(R-\lambda_1^*)(\lambda_2^*-\rho^*)}\right]$ and

$$a_{t+1}^* = (1-\rho)\overline{a}^* + \rho^* a_t^* + \epsilon_{t+1}^*, \tag{14}$$

where ϵ_{t+1}^* is a Gaussian innovation with mean 0 and variance ω^{*2}. It is worth noting that in the SOE-RBC setting, given that labor is inelastic, consumption-saving and investment decisions can first be modeled independently and then be combined together because the first-order conditions of the consumer and the firm are considered separately and then combined.[13]

[13]See Glick and Rogoff (1995) and Gruber (2002) for detailed discussions on this specification.

To model the observed income correlations across countries, we assume that the correlation between ε_{t+1} and ε_{t+1}^* is given:

$$\text{corr}\left(\varepsilon_{t+1}, \varepsilon_{t+1}^*\right) = \phi.$$

Given the productivity processes, (8) and (14), the productivity correlation is

$$\text{corr}\left(a_{t+1}, a_{t+1}^*\right) = \Pi_a \phi, \tag{15}$$

where $\Pi_a = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$. Note that $\Pi_a = 1$ when $\rho = \rho^*$ and $\Pi_a < 1$ when $\rho \neq \rho^*$.

2.3 Implications for Consumption Correlations

In the full-information RE model proposed in Section 2.1, consumption growth can be written as

$$\Delta c_t = (R-1)\,\Xi \epsilon_t,$$

which means that consumption growth is white noise and the impulse response of consumption to the income shock is *flat* with an immediate upward jump in the initial period that persists indefinitely (see the solid line in Figure 1). However, as has been well documented in the consumption literature, the impulse response of aggregate consumption to aggregate income takes a *hump-shaped* form, which means that aggregate consumption growth reacts to income shocks gradually. Similarly, for the rest of the world,

$$\Delta c_t^* = (R-1)\,\Xi^* \epsilon_t^*.$$

The international consumption correlation can thus be written as

$$\text{corr}(\Delta c_t, \Delta c_t^*) = \text{corr}(\epsilon_t, \epsilon_t^*) = \frac{1}{\Pi_a} \text{corr}\left(a_t, a_t^*\right). \tag{16}$$

Note that when aggregate productivity processes in both countries are unit roots, i.e., $\rho = \rho^* = 1$, the correlation reduces to $\text{corr}(\Delta c_t, \Delta c_t^*) = \text{corr}(\Delta a_t, \Delta a_t^*) = \phi$. It is clear that if the estimated productivity persistence parameters, ρ and ρ^*, are different and less than 1, $\Pi_a < 1$ and $\text{corr}\left(c_t, c_t^*\right) > \text{corr}\left(a_t, a_t^*\right)$. This prediction would not be consistent with the empirical evidence if the output process can be written as a linear function of productivity, as international consumption correlations are lower than output correlations for most pairs of countries.

3 The Effects of RB on Consumption Correlations

3.1 The RB Version of the SOE Model

Robust control emerged in the engineering literature in the 1970s and was introduced into economics and further developed by Hansen, Sargent, and others. A simple version of robust optimal control considers the question of how to make decisions when the agent does not know the probability model that generates the data. Specifically, an agent with a preference for robustness considers a range of models surrounding the given approximating model, (5):

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \tag{17}$$

where ν_t distorts the mean of the innovation, and makes decisions that maximize lifetime expected utility given this worst possible model (i.e., the distorted model).[14] To make that model (5) is a good approximation when (17) generates the data, we constrain the approximation errors by an upper bound η_0:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^{t+1} \nu_t^2 \right] \leq \eta_0, \tag{18}$$

where $E_0 [\cdot]$ denotes conditional expectations evaluated with model uncertainty, and the left side of this inequality is a statistical measure of the discrepancy between the distorted and approximating models. Note that the standard full-information RE case corresponds to $\eta_0 = 0$. In the general case in which $\eta_0 > 0$, the evil agent is given an intertemporal entropy budget $\eta_0 > 0$ which defines the set of models that the agent is considering. Following Hansen and Sargent (2007), we compute robust decision rules by solving the following two-player zero-sum game: a minimizing decision maker chooses optimal consumption path $\{c_t\}$ and a maximizing evil agent chooses model distortions $\{\nu_t\}$.

Following Hansen and Sargent (2007a), a simple robustness version of the SOE model proposed in Section 2.1 can be written as

$$v(s_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2} (\bar{c} - c_t)^2 + \beta \left[\vartheta \nu_t^2 + E_t \left[v(s_{t+1}) \right] \right] \right\} \tag{19}$$

subject to the distorted transition equation (i.e., the worst-case model), (17), where $\vartheta > 0$ is the Lagrange multiplier on the constraint specified in (18) and controls how bad the error can be.[15] As shown in Hansen, Sargent, and Tallarini (1999) and Hansen and Sargent (2007a), this class of models produces precautionary behavior while maintaining tractability within the LQ-Gaussian

[14] In this paper we consider model uncertainty due to robustness in the household sector and assume that the firm sector has no doubt about their model. This assumption is not implausible because in reality many firms have professional managers and economists to help them gather and analyze information about the structure of the economy and some important macroeconomic conditions.

[15] Formally, this setup is a game between the decision-maker and a malevolent nature that chooses the distortion process ν_t. $\vartheta \geq 0$ is a penalty parameter that restricts attention to a limited class of distortion processes; it can be mapped into an entropy condition that implies agents choose rules that are robust against processes which are close to the trusted one. In a later section we will apply an error detection approach to calibrate ϑ in small-open economies.

framework. When domestic aggregate productivity follows an AR(1) process, (8), solving this robust control problem yields the following proposition:

Proposition 1 *Under RB, the consumption function is*

$$c_t = \frac{R-1}{1-\Sigma} s_t - \frac{\Sigma \bar{c}}{1-\Sigma}, \tag{20}$$

the mean of the worst-case shock is

$$\nu_t \omega_\zeta = \frac{(R-1)\Sigma}{1-\Sigma} s_t - \frac{\Sigma}{1-\Sigma} \bar{c},$$

and $s_t \left(= b_t + \frac{y_t}{R-\rho}\right)$ *is governed by*

$$s_{t+1} = \rho_s s_t + \frac{\Sigma}{1-\Sigma} \bar{c} + \zeta_{t+1}, \tag{21}$$

where $\zeta_{t+1} = \Xi \epsilon_{t+1}$, $\Sigma = R\omega_\zeta^2 / (2\vartheta) > 0$ *measures the effect of robustness on consumption, and* $\rho_s = \frac{1-R\Sigma}{1-\Sigma} \in (0,1)$.

Proof. See Appendix 6.1. ■

Our univariate RB model with unique state variable s_t leads to the same consumption function as the corresponding multivariate RB model in which the state variables are b_t and y_t. The key difference between these two models is that in our univariate RB model the evil agent distorts the transition equation of permanent income s_t, whereas in the multivariate RB model the evil agent distorts the income process y_t. Theoretically, the preference of robustness, ϑ, affects both the coefficients attached to b_t and y_t in the consumption function of the multivariate model. That is, in the multivariate model RB may affect the relative importance of the two state variables in the consumption function, whereas in the univariate model the relative importance of the two effects are *fixed* by reducing the state space. However, after solving the two-state model numerically using the standard procedure proposed in Hansen and Sargent (2007a), we can see that the two models lead to the same decision rule; see Luo, Nie, and Young (2012) for a detailed proof. The key reason is that in our univariate model the evil agent is not permitted to distort the law of motion for b_t as it is an accounting equation and has been used to obtain the transition equation of s_t, whereas in the multivariate RB model we also only need to consider the distortion to y_t as there is no innovation to b_t in the resource constraint.

The effect of the preference for robustness, Σ, is jointly determined by the RB parameter, ϑ, and the volatility of the permanent income, ω_ζ. This interaction provides a novel channel that the income shock can affect optimal consumption adjustments for different countries. That is, when there is a preference for robustness (i.e., $\vartheta < \infty$), the different volatilities for the income processes in two countries will imply different consumption adjustments. This effect will disappear (i.e., $\Sigma = 0$) if there is no preference for robustness (i.e., $\vartheta \to \infty$).

The consumption function under RB, (20), shows that the preference for robustness, ϑ, affects the precautionary savings increment, $-\frac{\Sigma}{1-\Sigma}\overline{c}$. The smaller the value of ϑ the larger the precautionary saving increment, provided $\Sigma < 1$.

Proposition 2 $\Sigma < 1$.

Proof. The second-order condition for a minimization by nature is

$$A = \frac{1}{2}\frac{R(R-1)}{1 - R\omega_{\zeta}^{2}/(2\vartheta)} > 0,$$

which can be rearranged into

$$\vartheta > \frac{1}{2}R\omega_{\zeta}^{2}.$$

Using the definition of Σ we obtain

$$\Sigma < 1.$$

■

The consumption function also implies that the stronger the preference for robustness, the larger the marginal propensity to consume out of permanent income and therefore the more consumption responds initially to changes in permanent income; that is, under RB consumption is more sensitive to unanticipated shocks to permanent income. This response is referred to as "making hay while the sun shines" in van der Ploeg (1993), and reflects the precautionary aspect of these preferences. Note that for the ROW, we have analogous results: we can just replace s_t and ζ_t with s_t^* and ζ_t^*, respectively. It is worth noting that it is straightforward to show that the robust consumption function, (20), can also be obtained by solving the following risk-sensitive SOE model:

$$v(s_t) = \max_{c_t}\left\{-\frac{1}{2}(\overline{c} - c_t)^2 + \beta\mathcal{R}_t[v(s_{t+1})]\right\} \tag{22}$$

subject to (21), and the distorted expectation operator $\mathcal{R}_t$ is defined by

$$\mathcal{R}_t[v(s_{t+1})] = -\frac{1}{\alpha}\log E_t[\exp(-\alpha v(s_{t+1}))],$$

where $\alpha > 0$ measures higher risk aversion of the agent *vis a vis* the von Neumann-Morgenstern specification.[16] Risk-sensitivity (RS) was first introduced into the LQG framework by Jacobson

[16]The detailed proof is available from the authors by request. The observational equivalence between the risk-sensitive and robust LQG models has been well established in the literature. See Hansen, Sargent, and Tallarni (1999), Backus, Routledge, and Zin (2004), and Luo and Young (2010).

(1973) and extended by Whittle (1981).[17] In the risk-sensitive SOE specified in (22), the agents are prudent in the sense that they minimizes the expected value of an exponential transformation of a quadratic welfare loss function and adjust optimal consumption more aggressively to changes in income.

As mentioned before, we adopt the small-open economy model with the constant interest rate and quadratic utility rather than a two-country general equilibrium model with CRRA utility (e.g., Kollman 1996 and Daniel 1997) for two reasons.[18] First, most existing RB models assume that the objective functions are quadratic and the state transition equations are linear, consequently, worst-case distributions are Gaussian. However, if the objective functions are not quadratic or the transition equations are not linear, worst-case distributions are generally non-Gaussian. As argued in Hansen and Sargent (2007a), the most difficult part in solving such non-LQ RB models is computational: representing the worst-case distribution parsimoniously enough that the model is tractable.[19] Second, there does not exist a two-country general equilibrium in which the general equilibrium interest rate is constant. Specifically, consider a simple RE full-information two-country general equilibrium model in which the home country's budget constraint and consumption decision are characterized by (2) and (3), respectively, and the agents in the foreign country solve the same problem in which its variables are denoted with an asterisk.[20] In general equilibrium, the bond market-clearing condition is

$$b_t + b_t^* = 0 \text{ for all } t. \tag{23}$$

By Walras' law, (23) implies that the global resource constraint should also hold for all t:

$$c_t + c_t^* = y_t + y_t^* \equiv y_t^w, \tag{24}$$

where y_t^w denotes exogenously given current world output. Using the expected resource constraint,

$$E_{t-1}\left[y_t^w\right] = E_{t-1}\left[c_t\right] + E_{t-1}\left[c_t^*\right] = \frac{1}{R}\left(\frac{1}{\beta}c_{t-1} + \frac{1}{\beta^*}c_{t-1}^*\right),$$

[17]Hansen and Sargent (1995) introduced discounting into the RS specification and showed that the resulting decision rules are time-invariant; van der Ploeg (1993) applied this preference to examine its implications for precautionary savings; Hansen, Sargent, and Tallarini (1999) also explored its implications for precautionary savings and asset prices; and Luo and Young (2010) examined its implications for consumption and precautionary savings when consumers only have finite capacity.

[18]Benigno and Nistico (2012) revisited the international home-bias puzzle in both Hansen-Sargent's robust model and Epstein-Zin's recursive utility model. Colacito and Croce (2012) show that RB can endogenously generate international disagreement about endowment growth in a complete-market two-country model.

[19]Solving the model actually poses no problems; see Young (2012) for an example. In this paper we are interested in estimating ϑ, which requires us to sample from the worst-case distribution; when the worst-case distribution is not of known form this sampling becomes difficult.

[20]Note that here we relax the assumption that $\beta R = 1$ such that β could be different in the two countries.

we can easily obtain the expression for the general equilibrium interest rate:

$$R = \frac{1}{E_{t-1}\left[y_t^w\right]}\left(\frac{1}{\beta}c_{t-1} + \frac{1}{\beta^*}c_{t-1}^*\right). \tag{25}$$

However, given that $c_{t-1} = (R-1)\,s_{t-1}$ and $c_{t-1}^* = (R-1)\,s_{t-1}^*$, the right-hand side of there (25) is a time-$(t-1)$ random variable, i.e., there does not exist a constant R such that (25) holds. It is obvious that this argument also holds for the RB model as both c_{t-1} and c_{t-1}^* are random variables at $t-1$.

3.2 Implications of RB for International Consumption Correlations

Combining (20) with (21), consumption dynamics under RB in the domestic and average country are

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\,\Sigma\overline{c}}{1-\Sigma} + \frac{R-1}{1-\Sigma}\zeta_t, \tag{26}$$

and

$$c_t^* = \rho_s^* c_{t-1}^* + \frac{(R-1)\,\Sigma^*\overline{c}}{1-\Sigma^*} + \frac{R-1}{1-\Sigma^*}\zeta_t^*, \tag{27}$$

respectively. Figure 1 also illustrates the response of aggregate consumption to an income shock ε_{t+1} in the domestic country; comparing the solid line (full-information RE) with the dash-dotted line (RB), it is clear that RB raises the sensitivity of consumption to unanticipated changes in income. Given these two expressions, we have the following proposition about the cross-country consumption correlation:

Proposition 3 *Under RB, the consumption correlation between the home country and the ROW can be written as*

$$\operatorname{corr}\left(c_t, c_t^*\right) = \frac{\Pi_s}{\Pi_a}\operatorname{corr}\left(a_t, a_t^*\right); \tag{28}$$

where

$$\Pi_s = \frac{\sqrt{\left(1-\rho_s^2\right)\left(1-\rho_s^{*2}\right)}}{1-\rho_s\rho_s^*}, \tag{29}$$

$\Pi_a = \frac{\sqrt{(1-\rho^2)(1-\rho^{*2})}}{1-\rho\rho^*}$, $\rho_s = \frac{1-R\Sigma}{1-\Sigma}$, $\rho_s^* = \frac{1-R\Sigma^*}{1-\Sigma^*}$, *and we use the facts that* $\operatorname{corr}\left(\zeta_t, \zeta_t^*\right) = \operatorname{corr}\left(\epsilon_t, \epsilon_t^*\right) = \phi$ *and* $\operatorname{corr}\left(a_t, a_t^*\right) = \Pi_a\phi$.

Proof. See Appendix 6.4. ■

Expression (28) clearly shows that the degrees of preference for robustness (RB), ρ_s and ρ_s^* (Σ and Σ^*) , affect the consumption correlation across countries. The value of Π_s defined in (29) measures to what extent RB changes consumption correlations across countries. It is straightforward to show that when the effects of RB, Σ, are the same in the two economies,

($\rho_s = \rho_s^*$ or $\Sigma = \Sigma^*$), $\Pi_s = 1$. In this case, RB has *no* impact on the consumption correlation: $\text{corr}(c_t, c_t^*) = \text{corr}(a_t, a_t^*)/\Pi_a$, which is just the correlation obtained in the standard RE-SOE model, (16). Note that Σ can be written as $\Sigma^* + \Delta_\Sigma$ where Δ_Σ is defined as the difference between the domestic country and the ROW. Therefore, both the degree of RB in the ROW and the difference between the degrees of RB in the two economies affects the consumption correlation across countries.[21]

Figure 2 shows that the impact of RB on consumption correlations, Π_s, is increasing with Σ^*, the degree of RB in the ROW, for different values of Δ_Σ. (Here we set $R = 1.04$.) Note that holding Δ_Σ constant, Π_s is also increasing with Σ. Consumption is more sensitive to income shocks when Σ is larger, which by itself increases consumption correlations across countries when the difference in RB is fixed. Figure 2 also shows that Π_s is decreasing in the difference between Σ and Σ^*, Δ_Σ, for given values of Σ^*. For example, when $\Sigma^* = 0.05$, $\Pi_s = 0.84$ if $\Sigma - \Sigma^* = 0.1$, and $\Pi_s = 0.69$ if $\Sigma - \Sigma^* = 0.2$. After calibrating the model we will examine the net effect of RB on the consumption correlations.

3.3 Implications for Other Stochastic Properties of Consumption under RB

Given (9) and (26), we can obtain other key stochastic properties of consumption. The following proposition summarizes the implications of RB for the relative volatility, persistence, and correlation with output of consumption in the home country:

Proposition 4 *Under RB, the relative volatility of consumption to income is:*

$$\mu_{cy} \equiv \frac{\text{sd}(c_t)}{\text{sd}(y_t)} = \frac{(R-1)\,\Xi}{\Gamma_y}\sqrt{\frac{1}{(1-\Sigma)^2\left(1-\rho_s^2\right)}}, \tag{30}$$

the first-order autocorrelation of consumption is:

$$\rho_c \equiv \text{corr}(c_t, c_{t-1}) = \frac{\text{cov}(c_t, c_{t-1})}{\sqrt{\text{var}(c_t)\,\text{var}(c_{t-1})}} = \rho_s, \tag{31}$$

and the contemporaneous correlation between consumption and output is

$$\rho_{cy} \equiv \text{corr}(c_t, y_t) = \frac{\text{cov}(c_t, y_t)}{\sqrt{\text{var}(c_t)}\sqrt{\text{var}(y_t)}} = \frac{\sqrt{1-\rho_s^2}}{\Gamma_y\,(1-\rho_s\rho)}, \tag{32}$$

where Ξ is defined in (13), $\rho_s = \frac{1-R\Sigma}{1-\Sigma}$, and $\Gamma_y = \sqrt{\frac{1}{1-\rho^2} + \frac{\alpha_k^2(1+\rho\lambda_1)}{(1-\rho\lambda_1)\left[(1+\rho\lambda_1)^2-(\rho+\lambda_1)^2\right]} + \frac{2\rho\alpha_k}{(1-\rho\lambda_1)(1-\rho^2)}}$.

[21] In the next section, we will calibrate the RB parameter Σ and Σ^* using detection error probabilities and show that the values of Σ and Σ^* are different.

Proof. See Appendix 6.4. ■

It is clear from (31) that the first-order autocorrelation of consumption (ρ_c) is decreasing with the amount of model uncertainty (Σ) since $\rho_c = \rho_s$ and $\frac{\partial \rho_s}{\partial \Sigma} < 0$. Using these explicit expressions, Figure 3 shows that the impact of RB on these three key stochastic properties of consumption. We can see from the figure that the first-order autocorrelation of consumption (ρ_c) is decreasing with the amount of model uncertainty (Σ), while the contemporaneous correlation between consumption and output (ρ_{cy}) is increasing with Σ. The relative volatility of consumption to income (μ_{cy}) is not a monotonic function of Σ. It is decreasing with Σ when Σ is low and is increasing with Σ when Σ is high enough. The intuition behind this result is as follows. From (26) that an increase in Σ has two opposite effects on consumption volatility: (i) the increase in Σ raises the MPC of consumption out of permanent income, which makes consumption more sensitive to the productivity shock and more volatile and (ii) the increase in Σ reduces the persistence of consumption measured by ρ_s, which leads to less volatile consumption. In the next section, after calibrating the RB parameter using the data, we show that in the presence of RB, the model generates more realistic relative volatility of consumption to output in the individual countries in the G-7.

3.4 Calibrating the RB Parameter

In this subsection we follow Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a) to calibrate the RB model. Specifically, we calibrate the model by using the model detection error probability that is based on a statistical theory of model selection (the approach will be precisely defined below). We can then infer what values of the RB parameter ϑ imply reasonable fears of model misspecification for empirically-plausible approximating models. In other words, the model detection error probability is a measure of how far the distorted model can deviate from the approximating model without being discarded; standard significance levels for testing are then used to determine what reasonable fears entail. It is important to be clear that the detection error probability (which will be defined below) is considered to be the "deep parameter, not ϑ; ϑ is a reduced form parameter that depends on the underlying detection error probability as well as the variance of the disturbances.

3.4.1 The Definition of the Model Detection Error Probability

Let model **A** denote the approximating model and model **B** be the distorted model. Define $p_{\mathbf{A}}$ as

$$p_{\mathbf{A}} = \text{Prob}\left(\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) < 0 \middle| \mathbf{A} \right), \tag{33}$$

where $\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right)$ is the log-likelihood ratio. When model **A** generates the data, p_A measures the probability that a likelihood ratio test selects model **B**. In this case, we call $p_{\mathbf{A}}$ the probability of the model detection error. Similarly, when model B generates the data, we can define $p_{\mathbf{B}}$ as

$$p_{\mathbf{B}} = \text{Prob}\left(\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) > 0 \middle| \mathbf{B}\right). \tag{34}$$

Following Hansen, Sargent, and Wang (2002) and Hansen and Sargent (2007a), the detection error probability, p, is defined as the average of $p\mathbf{A}$ and $p_{\mathbf{B}}$:

$$p(\vartheta) = \frac{1}{2}(p_{\mathbf{A}} + p_{\mathbf{B}}), \tag{35}$$

where ϑ is the robustness parameter used to generate model **B**. Given this definition, we can see that $1-p$ measures the probability that econometricians can distinguish the approximating model from the distorted model. Now we show how to compute the model detection error probability in the RB model.

3.4.2 Calibrating the RB Parameter in the SOE Model

The general idea of the calibration exercise is to find a value of ϑ (or Σ) such that the detection error probability $p(\vartheta)$ equals a given value (say, 10 percent). We use the domestic country to illustrate the procedure. Under RB, assuming that the approximating model generates the data, the state, s_t, evolves according to the transition law

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1}, \\ &= \frac{1-R\Sigma}{1-\Sigma}s_t + \frac{\Sigma}{1-\Sigma}\bar{c} + \zeta_{t+1}. \end{aligned} \tag{36}$$

In contrast, assuming that the distorted model generates the data, s_t evolves according to

$$\begin{aligned} s_{t+1} &= Rs_t - c_t + \zeta_{t+1} + \omega_\zeta \nu_t, \\ &= s_t + \zeta_{t+1}. \end{aligned} \tag{37}$$

The following procedure explains how to compute $p_{\mathbf{A}}$ and $p_{\mathbf{B}}$:

1. Simulate $\{s_t\}_{t=0}^T$ using (36) and (37) a large number of times. The number of periods used in the simulation, T, is set to be the actual length of the data for each individual country.[22]
2. Count the number of times that $\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) < 0 \middle| \mathbf{A}$ and $\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) > 0 \middle| \mathbf{B}$ are each satisfied.

[22] As our data covers the 1950-2010 period, T is set to be 61 accordingly.

3. Determine $p_{\mathbf{A}}$ and $p_{\mathbf{B}}$ as the fractions of realizations for which $\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) < 0 \Big| \mathbf{A}$ and $\log\left(\frac{L_{\mathbf{A}}}{L_{\mathbf{B}}}\right) > 0 \Big| \mathbf{B}$, respectively.

In practice, given Σ, to simulate the $\{s_t\}_{t=0}^{T}$ we need to know the standard deviation of the shock to the permanent income, ζ_t, in (36) and (37) and the value of $\bar{c}$. The standard deviation of ζ_t equals $\Xi\sigma(\epsilon_t)$ where is Ξ is given by (13). To determine the value of Ξ, we first use the G-7 data to estimate the productivity process, (8); see Table 1 for the estimated ρ and ρ^* for the individual countries in the G-7 group. Next, we follow the existing literature (Glick and Rogoff 1995, Gruber 2002, and Marquez 2004) and use the following parameter values: $R = 1.04$, $\alpha_a = 1$, and the share of capital, α, for Canada, Italy, UK, France, Germany, Japan, United States are 0.37, 0.52, 0.32, 0.36, 0.46 and 0.34 respectively. The depreciation rate δ is set to be 0.05.[23] The remaining parameter values can be recovered after estimating λ_1 and λ_i in the investment equation, (12). Glick and Rogoff (1995) estimated $\lambda_1 = 0.9$ and $\lambda_i = 0.36$ by running a pooled time-series regression for G-7 countries. In contrast, Marquez (2004) estimated the same investment equation for individual countries in G-7 by using a updated sample: a sample for $1960-1977$ from Glick and Rogoff (1995) with revisions over $1978-1990$ and extensions through 1998, and found that the estimated λ_i, the response of investment to the productivity shock, is much lower in individual countries. In this paper, we adopt his updated estimates when we conduct quantitative analysis. We use the local coefficient for relative risk aversion $\gamma = -\frac{u''(c)c}{u'(c)} = \frac{c}{\bar{c}-c}$ to recover $\bar{c} = \left(1+\frac{1}{\gamma}\right)c$ where c is mean consumption. Here we set $\gamma = 2$ as the benchmark case. For the ROW, we follow the same procedure to pin down the parameter values in the calibration.

3.5 Data

To implement the calibration procedure we describe in the previous section, we need to first define our empirical counterpart of the domestic country and the ROW. We defines ROW as the GDP-weighted average of the G-7 excluding the country we choose as the domestic country. For instance, when we choose Canada as the domestic country, we use the other six countries in G-7 to define ROW.[24] In addition, we choose one of the relatively small countries in G-7 as the domestic country. In practice, we have applied our analysis to 4 different countries, Canada, Italy, UK, and

[23] This is line with the estimates in the literature. For example, Nadiri and Prucha (1993) estimated the annual depreciation rate of physical capital to be 0.059 in the manufacturing sector. Our quantitative results are not sensitive to the value of this parameter.

[24] In this paper we follow Crucini (1999) and focus on G-7 countries. Pakko (1998) used 15 OECD countries to study cross-country correlations.

France and report the results in the next section.[25]

The annual data we use come from the Penn World Table 7.1 which covers the period from 1950 to 2010. All the variables are measured in the US currency of year 2005. We apply the HP filter (with a smoothing parameter of 100) to the time series before computing the statistics. Table 1 summarizes the key statistics. The numbers in the parentheses are the GMM-corrected standard errors of the statistics across countries.

3.6 Main Findings

Table 2 reports the calibration results and some key data statistics (the stochastic properties of aggregate productivity and consumption and output correlations across countries) for the four countries we set as the domestic country in turn. In the calibration exercise, following Hansen and Sargent (2007), we set the detection error probability, p, to be a plausible value, 10 percent (i.e., with 90 percent probability consumers can distinguish the approximating model from the distorted model).[26]

Given these estimation and calibration results, Table 3 compares the implications for the international consumption correlations between the FI-RE and RB models when $p = 0.1$. Our key result here is that RB improves the performance of the model in terms of the cross-correlations of consumption; at the estimated Σ each country displays a lower correlation with the foreign aggregate. Furthermore, in most countries the cross-correlations of consumption are below the income correlations under RB, whereas the opposite holds for FI-RE. Quantitatively, we can see from Table 3 that the improvements are significant for most countries. For example, in Canada, the correlation is reduced from 0.80 to 0.55 which is much closer to the empirical counterpart (0.49). In U.K., the correlation is reduced to 0.69, which matches the empirical value (0.69) perfectly. There are two clear patterns apparent in Tables 2 and 3. The reduction in the consumption correlation is decreasing in Π_s and increasing in Σ. The large improvement in the Canada correlation is therefore due to the relatively low value of Π_s and relatively high value of Σ; we will elaborate more on this point in the next paragraph. These findings are consistent with our theoretical results obtained in Section 3.2.

[25] We do not model the three largest countries in G-7, US, Germany and Japan, as the domestic country because they are too large to be modeled as small open economies. There are also data issues for Germany relating to the unification.

[26] As shown in Section 2.3, consumption in full-information (FI) RE models follows a random walk process which does not allow us to explicitly write down the expression for $\text{corr}(c, c^*)$ in that case. Thus, we approximate the value of $\text{corr}(c, c^*)$ by choosing a value of Σ extremely close to zero to approximate the predictions of the FI-RE model.

[Insert Tables 2-3 Here]

To examine whether the findings are robust, we do sensitivity analysis by using a different value of the detection error probabilities (p). Specifically, when p is set to 5%, Tables 4 and 5 clearly show that our main finding that the presence of RB reduces consumption correlations across countries is robust to different value of the detection error probability. In addition, it is also clear from the tables that as p varies from 10 percent to 5 percent, RB has almost no impact on corr (c_t, c_t^*). For example, when p falls from 0.1 to 0.05, corr (c_t, c_t^*) decreases from 0.55 to 0.54 in Canada; and remains almost unchanged in Italy, the UK, and France. The intuition for this conclusion is as follows. As p decreases, it generally leads to a lower calibrated ϑ (and ϑ^*) and a higher calibrated Σ (and Σ^*); this combination generates two competing effects on consumption correlations. First, the increase of Σ^* will increase Π_s (as we can see from Figure 2), so consumption correlations will increase. Second, reducing p not only increases Σ^* but also increases the difference $\Sigma - \Sigma^*$ (which means the increase of Σ is more than that of Σ^* as shown in Table 2). This increase of $\Sigma - \Sigma^*$ decreases the consumption correlation, as we can see from Figure 2 as well. Hence, these two offsetting effects imply that consumption correlations do not change much as p varies.

[Insert Tables 4-5 Here]

Table 6 compares the implications for the stochastic properties of consumption: the relative volatility of consumption to income, the first-order autocorrelation of consumption, and the contemporaneous consumption-income correlation in individual countries between the FI-RE and RB models when $p = 0.1$. Our key result here is that RB significantly improves the performance of the model in terms of these consumption moments; at the estimated Σ each model economy has more realistic consumption dynamics. Quantitatively, we can see from the first four columns in Table 6 that the improvements are significant for all countries we studied. For example, in Canada, the relative consumption volatility falls from infinity in the FI-RE case to 1.25 in the RB case. The autocorrelation declines from 1 to 0.89, which is closer to the empirical counterpart, 0.72. The consumption-income correlation increases from 0 to 0.66, which is much closer to its empirical counterpart, 0.74. These findings are consistent with our theoretical results obtained in Section 3.2.

To examine whether the findings are robust, we do sensitivity analysis by using a different value of the detection error probabilities (p). Specifically, when $p = 0.05$, Table 7 clearly show that our main finding that the presence of RB can significantly improve the model's predictions on these three consumption moments is robust to a different value of the detection error probability.

[Insert Tables 6-7 Here]

4 Extension: The RB Model with Imperfect State Observation

In this section, we consider an important extension by considering another important informational friction, imperfect state observation due to rational inattention, in the RB model, and examine whether this extension can further improve the model's predictions on the international output-consumption correlations and other important consumption moments.

4.1 State Uncertainty due to Imperfect Observations

We assume that consumers in the model economy cannot observe the true state s_t perfectly and only observes the noisy signal

$$s_t^* = s_t + \xi_t, \tag{38}$$

when making decisions, where ξ_t is the iid Gaussian noise due to imperfect observations. The specification in (38) is standard in the signal extraction literature and captures the situation where agents happen or choose to have imperfect knowledge of the underlying shocks.[27] Since imperfect observations on the state lead to welfare losses, agents use the processed information to estimate the true state.[28] Specifically, we assume that households use the Kalman filter to update the perceived state $\widehat{s}_t = E_t[s_t]$ after observing new signals in the steady state in which the conditional variance of s_t, $\Upsilon_t = \text{var}_t(s_t)$, has converged to a constant Σ:

$$\widehat{s}_{t+1} = (1-\theta)(R\widehat{s}_t - c_t) + \theta(s_{t+1} + \xi_{t+1}), \tag{39}$$

where θ is the Kalman gain (i.e., the observation weight).[29] Note that in the signal extraction problem, the Kalman gain can be written as

$$\theta = \Upsilon\Lambda^{-1}, \tag{40}$$

where Υ is the steady state value of the conditional variance of a_{t+1}, $\text{var}_{t+1}(a_{t+1})$, and $\Lambda = \text{var}_t(\xi_{t+1})$ is the variance of the noise. Υ and Λ are linked by the following updating equation for

[27] For example, Muth (1960), Lucas (1972), Morris and Shin (2002), and Angeletos and La'O (2009). This assumption is also consistent with the rational inattention idea that ordinary people only devote finite information-processing capacity to processing financial information and thus cannot observe the states perfectly, as shown in Luo and Young (2013).

[28] See Luo (2008) for details about the welfare losses due to information imperfections within the partial equilibrium permanent income hypothesis framework.

[29] θ measures how much uncertainty about the state can be removed upon receiving the new signals about the state.

the conditional variance in the steady state:

$$\Lambda^{-1} = \Upsilon^{-1} - \Psi^{-1}, \tag{41}$$

where Ψ is the steady state value of the *ex ante* conditional variance of s_{t+1}, $\Psi_t = \text{var}_t(s_{t+1})$. Multiplying ω_ζ^2 on both sides of (41) and using the fact that $\Psi = R^2\Upsilon + \omega_\zeta^2$, we have

$$\omega_\zeta^2 \Lambda^{-1} = \omega_\zeta^2 \Upsilon^{-1} - \left[R^2 \left(\omega_\zeta^2 \Upsilon^{-1}\right)^{-1} + 1\right]^{-1}, \tag{42}$$

where $\omega_\zeta^2 \Upsilon^{-1} = \left(\omega_\zeta^2 \Lambda^{-1}\right)\left(\Lambda \Upsilon^{-1}\right)$. Define the signal-to-noise ratio (SNR) as $\pi = \omega_\zeta^2 \Lambda^{-1}$. We obtain the following equality linking SNR (π) and the Kalman gain (θ):

$$\pi = \theta \left(\frac{1}{1-\theta} - R^2\right). \tag{43}$$

Solving for θ yields

$$\theta = \frac{-\left(1+\pi-R^2\right) + \sqrt{\left(1+\pi-R^2\right)^2 + 4R^2\pi}}{2R^2}, \tag{44}$$

where we omit the negative values of θ because both Σ and Λ must be positive. Note that given π, we can pin down Λ using $\pi = \omega_\zeta^2 \Lambda^{-1}$ and Υ using (40) and (44). Luo and Young (2013) showed that within this univariate framework, if the signal-to-noise ratio (π) is given, the traditional signal extraction problem with state uncertainty and the rational inattention problem in which the Kalman gain (θ) is an increasing function of channel capacity (κ) (i.e., $\theta = 1 - \exp(-2\kappa)$) are observationally equivalent when $\pi = (1 - \exp(-2\kappa))\left(\exp(2\kappa) - R^2\right)$.

Combining (5) with (39), we obtain the following equation governing the perceived state $\widehat{s}_t$:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \eta_{t+1}, \tag{45}$$

where

$$\eta_{t+1} = \theta R \left(s_t - \widehat{s}_t\right) + \theta \left(\zeta_{t+1} + \xi_{t+1}\right) \tag{46}$$

is the innovation to the mean of the distribution of perceived permanent income,

$$s_t - \widehat{s}_t = \frac{(1-\theta)\zeta_t}{1-(1-\theta)R\cdot L} - \frac{\theta\xi_t}{1-(1-\theta)R\cdot L} \tag{47}$$

is the estimation error where L is the lag operator, and $E_t[\eta_{t+1}] = 0$. Note that η_{t+1} can be rewritten as

$$\eta_{t+1} = \theta\left[\left(\frac{\zeta_{t+1}}{1-(1-\theta)R\cdot L}\right) + \left(\xi_{t+1} - \frac{\theta R \xi_t}{1-(1-\theta)R\cdot L}\right)\right], \tag{48}$$

where $\omega_\xi^2 = \text{var}(\xi_{t+1}) = \frac{1}{\theta}\frac{1}{1/(1-\theta)-R^2}\omega_\zeta^2$. Expression (48) clearly shows that the estimation error reacts to the fundamental shock positively, while it reacts to the noise shock negatively. In addition, the importance of the estimation error is decreasing with θ. More specifically, as θ increases, the first term in (48) becomes less important because $(1-\theta)\zeta_t$ in the numerator decreases, and the second term also becomes less important because the importance of ξ_t decreases as θ increases.[30]

4.2 The RB-SU Version of the SOE Model

To introduce robustness into this model, we assume that the agent thinks that (45) is the approximating model. Following Hansen and Sargent (2007a), we surround (45) with a set of alternative models to represent the preference for robustness:

$$\widehat{s}_{t+1} = R\widehat{s}_t - c_t + \omega_\eta \nu_t + \eta_{t+1}. \tag{49}$$

Under SU the innovation η_{t+1} that the agent distrusts is composed of two MA(∞) processes and includes the entire history of the exogenous income shock and the endogenous noise, $\{\zeta_{t+1}, \zeta_t, \cdots, \zeta_0; \xi_{t+1}, \xi_t, \cdots, \xi_0\}$.[31] Following Hansen and Sargent (2007a) and Luo and Young (2010), the robust PIH problem with imperfect state observation can be written as

$$\widehat{v}(\widehat{s}_t) = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}(c_t - \overline{c})^2 + \beta E_t \left[\vartheta \nu_t^2 + \widehat{v}(\widehat{s}_{t+1}) \right] \right\}, \tag{50}$$

subject to (49) and (48), and $s_0 \sim N\left(\widehat{s}_0, \sigma^2\right)$ is fixed. (50) is a standard dynamic programming problem. The following proposition summarizes the solution to the RB model with imperfect state observation.

Proposition 5 *Given ϑ and θ, the consumption function under RB and SU is*

$$c_t = \frac{R-1}{1-\widetilde{\widetilde{\Sigma}}}\widehat{s}_t - \frac{\widetilde{\widetilde{\Sigma}}\overline{c}}{1-\widetilde{\widetilde{\Sigma}}}, \tag{51}$$

the mean of the worst-case shock is

$$\omega_\eta \nu_t = \frac{(R-1)\widetilde{\widetilde{\Sigma}}}{1-\widetilde{\widetilde{\Sigma}}}\widehat{s}_t - \frac{\widetilde{\widetilde{\Sigma}}}{1-\widetilde{\widetilde{\Sigma}}}\overline{c}, \tag{52}$$

and $\widehat{s}_t$ is governed by

$$\widehat{s}_{t+1} = \rho_s \widehat{s}_t + \eta_{t+1}. \tag{53}$$

[30]Note that when $\theta = 1$, $\text{var}(\xi_{t+1}) = 0$.

[31]The RB-SU model proposed in this paper encompasses the hidden state model discussed in Hansen, Sargent, and Wang (2002), Hansen and Sargent (2007b), and Hansen, Mayer, and Sargent (2010); the main difference is that none of the states in the RB-SU model are perfectly observable (or controllable).

where $\rho_s = \frac{1-R\widetilde{\Sigma}}{1-\widetilde{\Sigma}} \in (0,1)$,

$$\widetilde{\Sigma} = R\omega_\eta^2 / (2\vartheta) = \frac{\theta}{1-(1-\theta)R^2}\Sigma > \Sigma, \tag{54}$$

$$\omega_\eta^2 = \mathrm{var}(\eta_{t+1}) = \frac{\theta}{1-(1-\theta)R^2}\omega_\zeta^2 > \omega_\zeta^2, \textit{ for } \theta < 1. \tag{55}$$

Proof. See Appendix 6.4.1. ■

It is clear from (51)-(55) that RB and SU affect the consumption function via two channels in the model: (1) the marginal propensity to consume (MPC) out of the perceived state $\left(\frac{R-1}{1-\widetilde{\Sigma}}\right)$ and (2) the dynamics of the perceived state ($\widehat{s}_t$). Given $\widehat{s}_t$, stronger degrees of SU and RB increase the value of $\widetilde{\Sigma}$, which increases the MPC. Furthermore, from (54) and (55), we can see that imperfect state observation can amplify the importance of model uncertainty measured by $\widetilde{\Sigma}$ in determining consumption and precautionary savings.

In the representative agent case, the individual dynamics are identical to aggregate dynamics. Combining (45) with (51) yields the change in aggregate consumption in the RB-SU economy:

$$\Delta c_t = \frac{(1-R)\widetilde{\Sigma}}{1-\widetilde{\Sigma}}(c_{t-1}-\overline{c}) + \frac{R-1}{1-\widetilde{\Sigma}}\left(\frac{\theta\zeta_t}{1-(1-\theta)R\cdot L} + \theta\left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right), \tag{56}$$

where L is the lag operator and we assume that $(1-\theta)R < 1$.[32] This expression shows that consumption growth is a weighted average of all past permanent income and noise shocks. Note that (56) can be written in the following AR(1) process:

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}} + \frac{R-1}{1-\widetilde{\Sigma}}\eta_t, \tag{57}$$

where

$$\eta_t = \theta\left[\frac{\zeta_t}{1-(1-\theta)R\cdot L} + \left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right] \tag{58}$$

is an iid innovation to aggregate consumption with mean 0 and variance

$$\omega_\eta^2 \equiv \mathrm{var}(\eta_t) = \frac{\theta}{1-(1-\theta)R^2}\omega_\zeta^2 > \omega_\zeta^2 \tag{59}$$

for $\theta < 1$. Figure 1 also shows how SU can help generate the smooth and hump-shaped impulse response of consumption to the income shock, which, as argued in Sims (2003) and Reis (2006), fits the VAR evidence better. In the rest of the world, we have a similar dynamic equation for c_t^*:

$$c_t^* = \rho_s^* c_{t-1}^* + \frac{(R-1)\widetilde{\Sigma}^*\overline{c}^*}{1-\widetilde{\Sigma}^*} + \frac{R-1}{1-\widetilde{\Sigma}^*}\eta_t^*, \tag{60}$$

[32] This assumption is innocuous, since it is weaker than the condition needed for convergence of the filter (it requires that $\kappa > \frac{1}{2}\log(R) \approx \frac{R-1}{2}$). The condition implies that consumption is responsive enough to the state to 'zero out' the effect of the explosive root in the Euler equation; see Sims (2003).

where

$$\eta_t^* = \theta^* \left[\frac{\zeta_t^*}{1-(1-\theta^*)R\cdot L} + \left(\xi_t^* - \frac{\theta^* R \xi_{t-1}^*}{1-(1-\theta^*)R\cdot L} \right) \right].$$

4.3 Robust Consumption Correlations under SU

Given (57) and (60), we have the following proposition about the cross-country consumption correlation under RB and SU:

Proposition 6 *The consumption correlation between the two economies under RB and SU is*

$$\operatorname{corr}(c_t, c_t^*) = \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}} = \frac{\Pi}{\Pi_a} \operatorname{corr}(a_t, a_t^*), \tag{61}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty} \left\{ \left[\sum_{j=0, j\le k}^{k} \left(\rho_\theta^j \rho_s^{k-j} \right) \right] \left[\sum_{j=0, j\le k}^{k} \left(\rho_\theta^{*j} \rho_s^{*k-j} \right) \right] \right\}}{\sqrt{\Xi_1 \Xi_2}}, \tag{62}$$

$$\Xi_1 = \frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2 - (\rho_s+\rho_\theta)^2 \right]} +$$
$$\frac{1}{\theta\left(1/(1-\theta) - R^2\right)} \left\{ \frac{1}{1-\rho_s^2} + \frac{(\theta R)^2 (1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2 - (\rho_s+\rho_\theta)^2 \right]} \right\},$$

$$\Xi_2 = \frac{(1+\rho_s^*\rho_\theta^*)}{(1-\rho_s^*\rho_\theta^*)\left[(1+\rho_s^*\rho_\theta^*)^2 - (\rho_s^*+\rho_\theta^*)^2 \right]} +$$
$$\frac{1}{\theta^*\left(1/(1-\theta^*) - R^2\right)} \left\{ \frac{1}{1-\rho_s^{*2}} + \frac{(\theta^* R)^2 (1+\rho_s^*\rho_\theta^*)}{(1-\rho_s^*\rho_\theta^*)\left[(1+\rho_s^*\rho_\theta^*)^2 - (\rho_s^*+\rho_\theta^*)^2 \right]} \right\}.$$

Proof. See Appendix 6.5. ■

We assume that the SU parameters are the same in the domestic country and the rest of the world: $\theta^* = \theta$ and $\rho_\theta^* = \rho_\theta$.[33] Π will converge to Π_s, (29) defined in Section 3.2, as θ converges to 1. The presence of endogenous noises, ξ_{t-j} and ξ_{t-j}^* ($j \ge 0$), in the expressions for the dynamics of aggregate consumption does not affect the covariance under RB and SU, $\operatorname{cov}(c_t, c_t^*)$, as all noises are iid and are also independent of the exogenous income shocks ($\zeta_{t-j},\ j \ge 0$). Therefore, the presence of the common noise shocks will further reduce the consumption correlations across countries as they increase the variances of both c_t and c_t^*.

[33] It is straightforward to show that allowing for the heterogeneity in θ will further reduce the cross-country correlations by a factor, $\Xi = \sqrt{\frac{[1-((1-\theta^*)R)^2][1-((1-\theta)R)^2]}{[1-(1-\theta)(1-\theta^*)R^2]^2}}$. This case may be of interest, however, since Luo and Young (2013) show that it can imply infrequent updating as in Bacchetta and van Wincoop (2010). Since it lies beyond our purposes here and poses calibration challenges, we leave it for future work.

From (61) and (62), it is clear that the aggregate endogenous noise due to finite capacity plays an important role in determining the consumption correlations. Some recent papers have shown the importance of noise shocks for aggregate fluctuations. For example, Angeletos and La'O (2009) show how dispersed information about the underlying aggregate productivity shock contributes to significant noise in the business cycle and helps explain cyclical variations in observed Solow residuals and labor wedges in the RBC setting. Lorenzoni (2009) examines how demand shocks (noisy news about future aggregate productivity) contribute to business cycles fluctuations in a new Keynesian model. Here we will show that an aggregate noise component can improve the model's predictions on consumption correlation across countries.

Figure 4 illustrates how the consumption correlation is increasing with Σ^*. As in the previous section, Figure 5 illustrates how the consumption correlation is decreasing with $\Sigma - \Sigma^*$. Note that given the SU parameter (θ), the effects of Σ^* and $\Sigma - \Sigma^*$ on the correlation are the same as in the RB model. From these two figures, it is also clear that given the Σ^* or $\Sigma - \Sigma^*$, the correlation is increasing with the degree of observation imperfection, i.e., the Kalman gain (θ). In other words, as the Kalman gain decreases, the noise channel dominates the slow adjustment channel, which leads to lower consumption correlations.

To distinguish the noise channel from the slow adjustment channel, we conduct the following experiment. If we shut down the noise channel, i.e., $\xi_{t-j} = 0$, where $j \geq 0$, (61) can thus be reduced to

$$\operatorname{corr}(c_t, c_t^*) = \frac{\Pi}{\Pi_y} \operatorname{corr}(y_t, y_t^*), \tag{63}$$

where

$$\Pi = \frac{\sum_{k=0}^{\infty} \left\{ \left[\sum_{j=0, j\leq k}^{k} \left(\rho_\theta^j \rho_s^{k-j} \right) \right] \left[\sum_{j=0, j\leq k}^{k} \left(\rho_\theta^j \rho_s^{*k-j} \right) \right] \right\}}{\sqrt{\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2 - (\rho_s+\rho_\theta)^2\right]} \frac{(1+\rho_s^*\rho_\theta)}{(1-\rho_s^*\rho_\theta)\left[(1+\rho_s^*\rho_\theta)^2 - (\rho_s^*+\rho_\theta)^2\right]}}}.$$

Note that here we have assumed that $\theta^* = \theta$ and $\rho_\theta^* = \rho_\theta$. Figures 6 and 7 illustrate how the interaction between RB and SU affects consumption correlations across countries when $\Sigma = 0.7$ and $R = 1.04$. Note that here we use Π to measure to what extent RB and SU can affect the correlation because Π converges to 1 as Σ^* and Σ reduces to 0 and θ increases to 1. They show that given the level of finite capacity measured by θ, the consumption correlation is increasing with the degree of Σ^* (RB in the ROW) and is decreasing with the difference of RB in the two economies, $\Sigma - \Sigma^*$. These results are the same as those obtained the RB model in which $\theta = 1$ (channel capacity, κ, is infinite). In addition, it is also clear that given the Σ^* or $\Sigma - \Sigma^*$, the correlation is decreasing with the degree of observation imperfection. That is, the gradual response of consumption to income shocks due to imperfect observations by itself increases the consumption correlation. Comparing Figure 4 (Figure 5) with Figure 6 (Figure 7), it is clear that the effect of θ on the consumption correlation differs in sign for the RB-SU case and the $\xi = 0$ case. The intuition is that in the special case the common noise shock has zero variance that is a missing effect. In the RB-SU case, the effect of reducing θ increases the variance of c_t and c_t^* and thus reduces the consumption correlation. In other words, this effect dominates the positive effect of slow adjustment identified above and the correlation is increasing in θ.

The effect of SU on cross-country consumption correlations in the case with $\xi = 0$ is similar to that of habit formation, because habit formation also leads to slow adjustments in consumption.[34] As shown in Fuhrer and Klein (2006), the presence of habit formation increases the correlation of consumption across countries and the empirical evidence of high consumption correlations might reflect habit persistence rather than common income risks or risk sharing. In addition, this special case can also be compared to the sticky expectations (SE) model. The idea of SE is to relax the assumption that all consumers' expectations are completely updated at every period and assume that only a fraction of the population update their expectations on permanent income and re-optimize in any given period.[35] As shown in Carroll and Slacalek (2006), SE also generates the same predictions for aggregate consumption dynamics as habit formation. Consequently, it is straightforward to show that SE generates the same predictions on international consumption correlations as habit formation and the special case of $\xi = 0$ do.

4.4 Implications for Other Stochastic Properties of Consumption under RB and SU

Given (9) and (57), we can obtain other key stochastic properties of consumption in the robust model with imperfect state observations. The following proposition summarizes the implications of RB and SU for the relative volatility, persistence, and correlation with output of consumption in the home country:

Proposition 7 *Under RB and SU, the relative volatility of consumption to income is:*

$$\begin{aligned}\mu_{cy} &\equiv \frac{\mathrm{sd}\,(c_t)}{\mathrm{sd}\,(y_t)} \\ &= \frac{\theta\,(R-1)\,\Xi}{\Gamma_y}\sqrt{\frac{(1+\rho_s\rho_\theta)}{\left(1-\widetilde{\Sigma}\right)^2(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\frac{1+\rho_\theta^2}{\theta\,(1-\rho_\theta R)}},\end{aligned} \tag{64}$$

the first-order autocorrelation of consumption is:

$$\rho_c \equiv \mathrm{corr}\,(c_t, c_{t-1}) = \rho_s. \tag{65}$$

[34]See Luo (2008) for a detailed proof for the observational equivalence between this special SU case and habit formation.

[35]Reis (2006) uses the term "inattentiveness" to characterize the infrequent adjustment behavior of consumers.

and the contemporaneous covariance between consumption and output is

$$\rho_{cy} \equiv \operatorname{corr}(c_t, y_t) = \frac{\sqrt{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2 - (\rho_s+\rho_\theta)^2\right]}}{\Gamma_y (1-\rho\rho_s)(1-\rho\rho_\theta)\sqrt{1+\rho_s\rho_\theta}\sqrt{1+(1-\theta+\rho_\theta R)/[\theta(1-\rho_\theta R)]}}, \qquad (66)$$

where $\Gamma_y = \sqrt{\frac{1}{1-\rho^2} + \frac{\alpha_k^2(1+\rho\lambda_1)}{(1-\rho\lambda_1)\left[(1+\rho\lambda_1)^2-(\rho+\lambda_1)^2\right]} + \frac{2\rho\alpha_k}{(1-\rho\lambda_1)(1-\rho^2)}}$.

Proof. See Appendix 6.6. ■

It is clear from (65) that the first-order autocorrelation of consumption (ρ_c) is decreasing with the amount of model uncertainty (Σ) since $\rho_c = \frac{1-R\widetilde{\Sigma}}{1-\widetilde{\Sigma}}$, $\widetilde{\Sigma} = \frac{\theta}{1-(1-\theta)R^2}\Sigma$, and $\frac{\partial\rho_s}{\partial\Sigma} < 0$. In addition, the autocorrelation is increasing with the Kalman gain (θ) since $\frac{\partial\rho_c}{\partial\theta} > 0$. It is worth noting that the explicit expression for the autocorrelation under RB and SU, (65), is the same as that under RB. The intuition behind this result is as follows. In the RB-SU model, both the slow adjustment channel and the noise channel affect the first-order autocovariance of consumption. Specifically, the slow adjustment of consumption to the productivity shock increases ρ_c if we shut down the noise channel, while the quick impulse response of consumption to the noise creates a negative correlation of consumption over time. The two opposite mechanisms are just cancelled out in the representative agent setting. Figure 8 clearly shows that the impact of SU on the consumption autocorrelation is very small for plausibly calibrated Σ when the deviation from the FI-RI case is not large.

Using these explicit expressions, Figure 8 also shows that the impacts of RB on these three key stochastic properties of consumption are similar to that in the RB model. That is, the contemporaneous correlation between consumption and output (ρ_{cy}) is increasing with Σ, and the relative volatility of consumption to income (μ_{cy}) is not a monotonic function of Σ. It is decreasing with Σ when Σ is not sufficiently high and is increasing with Σ when Σ is high enough. The intuition for these result is the same as that we discussed in the RB case: There are two opposite effects of RB on consumption volatility; one increases the MPC and one decreases the consumption persistence. In addition, we can see from the figure that the relative consumption volatility and the consumption-output correlation are increasing with and decreasing with the degree of imperfect-state-observation (i.e., decreasing with the Kalman gain), respectively.

4.5 Main Findings

To illustrate the quantitative implications of the RB-SU model on the consumption correlation and other consumption moments, we fix the the RB parameter at the same levels we obtain in

Section 3.6 and vary the SU parameter, θ, to slightly deviate from the FI-RE case in which $\theta = 1$.[36] As in Section 3.6, we set the detection error probability, p, to be a plausible value, 10%. Table 6 reports the implied consumption correlations (between the domestic country and ROW) between the FI-RE, RB, and RB-SU models. There are several interesting observations in the table. First, $\text{corr}(c_t, c_t^*)$ is decreasing with the degree of SU (i.e., increasing with θ). The intuition is that in the presence of the noise, the effect of the noise dominates the effect of gradual consumption adjustments on cross-country consumption correlations. This contrasts with the results in the case in which $\xi = 0$. Note that in that case $\text{corr}(c_t, c_t^*)$ is decreasing with the degree of SU (θ), and the effect of SU on $\text{corr}(c_t, c_t^*)$ is similar to that of habit formation or sticky expectations.

As we can see from Table 6, for all the countries we consider here, introducing SU into the RB model enables the model to better fit the data on consumption correlations for three countries (Canada, Italy, and France) and slightly worsens the model's prediction for the UK. For example, for Canada, when $\theta = 0.95$ (i.e., 95 percent of the uncertainty is removed upon receiving a new signal about the innovation to permanent income), the RB-SU model predicts that $\text{corr}(c_t, c_t^*) = 0.49$, which exactly matches the empirical counterpart.[37] For Italy, when $\theta = 0.95$, the RB-SU model predicts that $\text{corr}(c_t, c_t^*) = 0.44$, which is close to the empirical counterpart. For the other consumption moments, we can see the table that SU has almost no impact on the consumption autocorrelation and the model's predictions are almost the same as that in the RB case. The impacts of SU on the consumption volatility and consumption-output correlation are mixed. In some countries, the RB-SU model performs better than the RB model and in some countries it performs slightly worse than the RB model. In summary, there are no significant differences in the predictions of the two models on these three consumption moments. Note that a less-than-one value of θ can be rationalized by examining the welfare effects of finite channel capacity.[38]

To examine whether the findings are robust, we do sensitivity analysis using different values of the detection error probabilities (p) in the calibration. As in the last section, here we also set

[36] We also recalibrated the value of Σ in the RB-SU model after setting $\theta = 0.9$ and 0.95 and found that the recalibration leads to similar values of Σ. The results from these calibration exercises are available from the authors.

[37] In this paper we only consider small deviations of the SU model from the standard FI-RE model and set θ to be close to 1. In the RI literature, to explain the observed aggregate fluctuations and the effects of monetary policy on the macroeconomy, the calibrated values of θ are lower and deviate more from the FI-RE case. For example, Adam (2005) found $\theta = 0.4$ based on the response of aggregate output to monetary policy shocks. Luo (2008) found that if $\theta = 0.5$, the otherwise standard permanent income model generates realistic relative volatility of consumption to labor income.

[38] See Luo and Young (2010) for details about the welfare losses due to imperfect observations in the RB model; they are uniformly small.

$p = 0.05$. Table 7 shows that our main findings in the benchmark RB-SU model are very robust; varying the value of p does not have significant effects on the consumption correlations and other consumption moments.

5 Conclusion

In this paper we provide further evidence that movements in consumption across countries can be understood easily when viewed through the lens of the SOE model with capital accumulation that incorporates robust decision-making; combined with the results in Luo, Nie, and Young (2012) on the model's ability to capture the dynamics of the current account, we can safely say that the interaction of robustness and imperfect state observation has a role in future open-economy macro studies. The model used here has many virtues – it is analytically tractable (leaving nothing hidden behind numerical computations), it displays precautionary savings, and it resolves the classic *excess sensitivity* and *excess smoothness* puzzles in aggregate consumption. However, it does have some shortcomings, such as reliance on a constant return to savings, linear-quadratic functional forms, and a univariate source of risk. The absence of shocks to the interest rate may be of particular importance, given the results in Neumeyer and Perri (2005) regarding the importance of such disturbances. We are working to relax these limitations currently in order to confront the model with more aspects of small open economy behavior.

6 Online Appendix (Not for Publication)

6.1 Solving the Robust Model

To solve the Bellman equation (19), we conjecture that

$$v(s_t) = -As_t^2 - Bs_t - C,$$

where A, B, and C are undetermined coefficients. Substituting this guessed value function into the Bellman equation gives

$$-As_t^2 - Bs_t - C = \max_{c_t} \min_{\nu_t} \left\{ -\frac{1}{2}(\bar{c} - c_t)^2 + \beta E_t \left[\vartheta \nu_t^2 - As_{t+1}^2 - Bs_{t+1} - C \right] \right\}. \tag{67}$$

We can do the min and max operations in any order, so we choose to do the minimization first. The first-order condition for ν_t is

$$2\vartheta\nu_t - 2AE_t\left[\omega_\zeta \nu_t + Rs_t - c_t\right]\omega_\zeta - B\omega_\zeta = 0,$$

which means that

$$\nu_t = \frac{B + 2A\left(Rs_t - c_t\right)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta. \tag{68}$$

Substituting (68) back into (67) gives

$$-As_t^2 - Bs_t - C = \max_{c_t}\left\{-\frac{1}{2}\left(\bar{c} - c_t\right)^2 + \beta E_t\left[\vartheta\left[\frac{B + 2A\left(Rs_t - c_t\right)}{2\left(\vartheta - A\omega_\zeta^2\right)}\omega_\zeta\right]^2 - As_{t+1}^2 - Bs_{t+1} - C\right]\right\},$$

where

$$s_{t+1} = Rs_t - c_t + \zeta_{t+1} + \omega_\zeta\nu_t.$$

The first-order condition for c_t is

$$\left(\bar{c} - c_t\right) - 2\beta\vartheta\frac{A\omega_\zeta}{\vartheta - A\omega_\zeta^2}\nu_t + 2\beta A\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right)\left(Rs_t - c_t + \omega_\zeta\nu_t\right) + \beta B\left(1 + \frac{A\omega_\zeta^2}{\vartheta - A\omega_\zeta^2}\right) = 0.$$

Using the solution for ν_t the solution for consumption is

$$c_t = \frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + \frac{\bar{c}\left(1 - A\omega_\zeta^2/\vartheta\right) + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}. \tag{69}$$

Substituting the above expressions into the Bellman equation gives

$$\begin{aligned}
&-As_t^2 - Bs_t - C \\
&= -\frac{1}{2}\left(\frac{2A\beta R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + \frac{-2\beta A\bar{c} + \beta B}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right)^2 \\
&+ \frac{\beta\vartheta\omega_\zeta^2}{\left(2\left(\vartheta - A\omega_\zeta^2\right)\right)^2}\left[\frac{2AR\left(1 - A\omega_\zeta^2/\vartheta\right)}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t + B - \frac{2\bar{c}\left(1 - A\omega_\zeta^2/\vartheta\right)A + 2\beta AB}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}\right]^2 \\
&- \beta A\left[\left(\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right)^2 + \omega_\zeta^2\right] \\
&- \beta B\left[\frac{R}{1 - A\omega_\zeta^2/\vartheta + 2\beta A}s_t - \frac{-B\omega_\zeta^2/\vartheta + 2c + 2B\beta}{2\left(1 - A\omega_\zeta^2/\vartheta + 2\beta A\right)}\right] - \beta C.
\end{aligned}$$

Given $\beta R = 1$, collecting and matching terms, the constant coefficients turn out to be

$$A = \frac{R\left(R - 1\right)}{2 - R\omega_\zeta^2/\vartheta}, \tag{70}$$

$$B = -\frac{R\bar{c}}{1 - R\omega_\zeta^2/\left(2\vartheta\right)}, \tag{71}$$

$$C = \frac{R\omega_\zeta^2}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)} + \frac{R\bar{c}^2}{2\left(1 - R\omega_\zeta^2/2\vartheta\right)\left(R - 1\right)}. \tag{72}$$

Substituting (70) and (71) into (69) yields the consumption function (20) in the text.

We impose parameter restrictions so that $A > 0$, implying the value function is concave; these restrictions amount to requiring that ϑ not be too small and are shown in the text to imply $\Sigma < 1$.

6.2 Solving the Robust SU Model

To solve the Bellman equation (50) subject to 49, we conjecture that

$$v\left(\widehat{s}_t\right) = -C - B\widehat{s}_t - A\widehat{s}_t^2,$$

where A, B, and C are undetermined coefficients. The detailed procedure is similar to that in Appendix 6.1. Here we only need to replace ω_ζ^2 with ω_η^2 in the constant terms obtained in (70), (71), and (72):

$$\begin{aligned}
A &= \frac{R\left(R-1\right)}{2 - R\omega_\eta^2/\vartheta}, \\
B &= -\frac{R\overline{c}}{1 - R\omega_\eta^2/\left(2\vartheta\right)}, \\
C &= \frac{R\omega_\eta^2}{2\left(1 - R\omega_\eta^2/2\vartheta\right)} + \frac{R\overline{c}^2}{2\left(1 - R\omega_\eta^2/2\vartheta\right)\left(R-1\right)}.
\end{aligned}$$

Using these coefficients, we can obtained the consumption function (51) and the worst possible rule (52) in the text.

6.3 Deriving International Consumption Correlations under RB

Given the AR(1) expressions for c_t and c_t^*, (26) and (27), the consumption correlation between the home country and the rest of the world (ROW) can be written as:

$$\begin{aligned}
&\operatorname{corr}\left(c_t, c_t^*\right) \\
&= \frac{\operatorname{cov}\left(c_t, c_t^*\right)}{\sqrt{\operatorname{var}\left(c_t\right)\operatorname{var}\left(c_t^*\right)}}, \\
&= \frac{\sqrt{\left(1-\rho_s^2\right)\left(1-\rho_s^{*2}\right)}}{1-\rho_s\rho_s^*}\frac{\operatorname{cov}\left(\zeta_t, \zeta_t^*\right)}{\omega_\zeta\omega_{\zeta^*}},
\end{aligned}$$

which is just (28) in the main text. Note that here we use the following facts:

$$\begin{aligned}
\operatorname{var}(c_t) &= \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^2}, \\
\operatorname{var}(c_t^*) &= \left(\frac{R-1}{1-\Sigma^*}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^{*2}}, \\
\operatorname{cov}(c_t, c_t^*) &= \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\zeta_t}{1-\rho_s \cdot L}, \frac{R-1}{1-\Sigma^*}\frac{\zeta_t^*}{1-\rho_s^* \cdot L}\right) \\
&= \frac{R-1}{1-\Sigma}\frac{R-1}{1-\Sigma^*}\frac{1}{1-\rho_s\rho_s^*}\operatorname{cov}(\zeta_t, \zeta_t^*).
\end{aligned}$$

6.4 Deriving Other Stochastic Properties of Consumption under RB

6.4.1 Relative Volatility of Consumption to Output under RB

We first compute the variance of output. Substituting the parameter values into the capital accumulation equation, $k_t = \lambda_1 k_{t-1} + \frac{\alpha \bar{k}^\alpha \rho}{g(\lambda_2 - \rho)} a_{t-1}$, yields an AR(2) process:

$$k_t = \lambda_1 k_{t-1} + a_{t-1} = \frac{a_{t-1}}{1-\lambda_1 \cdot L} = \frac{\epsilon_{t-1}}{(1-\rho \cdot L)(1-\lambda_1 \cdot L)}.$$

Taking unconditional variance on both sides of k_t yields:

$$\operatorname{var}(k_t) = \operatorname{var}\left(\frac{\epsilon_{t-1}}{(1-\rho \cdot L)(1-\lambda_1 \cdot L)}\right) = \frac{1+\rho\lambda_1}{(1-\rho\lambda_1)\left[(1+\rho\lambda_1)^2 - (\rho+\lambda_1)^2\right]}\omega^2.$$

The covariance between k and a is

$$\begin{aligned}
\operatorname{cov}(a_t, k_t) &= \operatorname{cov}\left(\frac{\epsilon_t}{1-\rho \cdot L}, \frac{\epsilon_{t-1}}{(1-\rho \cdot L)(1-\lambda_1 \cdot L)}\right) \\
&= \rho \operatorname{cov}\left(\epsilon_{t-1} + \rho\epsilon_{t-2} + \rho^2\epsilon_{t-3} + \cdots, \epsilon_{t-1} + (\rho+\lambda_1)\epsilon_{t-2} + \left(\rho^2 + \rho\lambda_1 + \lambda_1^2\right)\epsilon_{t-3} + \cdots\right) \\
&= \frac{\rho}{(1-\rho\lambda_1)(1-\rho^2)}\omega^2
\end{aligned}$$

Given that output is

$$\begin{aligned}
y_t &\cong a_t + \alpha_k k_t + \alpha_i i_t \\
&= a_t + \frac{\alpha_k a_{t-1}}{1-\lambda_1 \cdot L},
\end{aligned}$$

taking unconditional variance on both sides of y_t gives:

$$\begin{aligned}
\operatorname{var}(y_t) &= \operatorname{var}(a_t) + \alpha_k^2 \operatorname{var}(k_t) + 2\operatorname{cov}\left(a_t, \frac{\alpha_k a_{t-1}}{1-\lambda_1 \cdot L}\right) \\
&= \Gamma_y^2 \omega^2
\end{aligned}$$

where $\Gamma_y = \sqrt{\frac{1}{1-\rho^2} + \frac{\alpha_k^2(1+\rho\lambda_1)}{(1-\rho\lambda_1)\left[(1+\rho\lambda_1)^2-(\rho+\lambda_1)^2\right]} + \frac{2\alpha_k\rho}{(1-\rho\lambda_1)(1-\rho^2)}}$.

Given the AR(1) expression for c_t, (26), the relative volatility of consumption to income can be written as:

$$\mu = \frac{\mathrm{sd}(c_t)}{\mathrm{sd}(y_t)} = \frac{(R-1)\Xi}{\Gamma_y}\sqrt{\frac{1}{(1-\Sigma)^2(1-\rho_s^2)}},$$

which is just (30) in the main text. Note that here we use the following fact that $\mathrm{var}(c_t) = \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^2}$.

6.4.2 Consumption Persistence under RB

Given the AR(1) expressions for c_t, (26), it is straightforward to show that the first-order autocorrelation of consumption is:

$$\rho_c \equiv \mathrm{corr}(c_t, c_{t-1}) = \frac{\mathrm{cov}(c_t, c_{t-1})}{\sqrt{\mathrm{var}(c_t)\,\mathrm{var}(c_{t-1})}} = \frac{\mathrm{cov}\left(\rho_s c_{t-1} + \frac{R-1}{1-\Sigma}\zeta_t, c_{t-1}\right)}{\sqrt{\mathrm{var}(c_t)\,\mathrm{var}(c_{t-1})}} = \rho_s.$$

6.4.3 Consumption-Output Correlations under RB

Given the AR(1) expressions for c_t, (26), and the output process, $y_t = a_t + \frac{\alpha_k \epsilon_{t-1}}{(1-\rho\cdot L)(1-\lambda_1\cdot L)}$, the contemporaneous covariance between consumption and output can be written as

$$\begin{aligned}
\mathrm{cov}(c_t, y_t) &= \mathrm{cov}\left(\frac{R-1}{1-\Sigma}\frac{\Xi\epsilon_t}{1-\rho_s\cdot L}, a_t + \frac{\alpha_k\epsilon_{t-1}}{(1-\rho\cdot L)(1-\lambda_1\cdot L)}\right) \\
&= \Xi\frac{R-1}{1-\Sigma}\mathrm{cov}\left(\frac{\epsilon_t}{1-\rho_s\cdot L}, \frac{\epsilon_t}{1-\rho\cdot L} + \frac{\alpha_k\epsilon_{t-1}}{(1-\rho\cdot L)(1-\lambda_1\cdot L)}\right) \\
&= \Xi\frac{R-1}{1-\Sigma}\left[\mathrm{cov}\left(\frac{\epsilon_t}{1-\rho_s\cdot L}, \frac{\epsilon_t}{1-\rho\cdot L}\right) + \alpha_k\rho_s\,\mathrm{cov}\left(\frac{\epsilon_{t-1}}{1-\rho_s\cdot L}, \frac{\epsilon_{t-1}}{(1-\rho\cdot L)(1-\lambda_1\cdot L)}\right)\right] \\
&= \Xi\frac{R-1}{1-\Sigma}\left[\mathrm{cov}\left(\frac{\epsilon_t}{1-\rho_s\cdot L}, \frac{\epsilon_t}{1-\rho\cdot L}\right) + \alpha_k\rho_s\,\mathrm{cov}\left(\frac{\epsilon_{t-1}}{1-\rho_s\cdot L}, \frac{\epsilon_{t-1}}{(1-\rho\cdot L)(1-\lambda_1\cdot L)}\right)\right] \\
&= \Xi\frac{R-1}{1-\Sigma}\left[\frac{1}{1-\rho\rho_s} + \frac{\alpha_k\rho_s}{(1-\rho\rho_s)(1-\lambda_1\rho_s)}\right]\omega^2 \\
&= \frac{R-1}{1-\Sigma}\Xi\frac{1}{1-\rho\rho_s}\omega^2
\end{aligned}$$

Given that $\mathrm{var}(c_t) = \left(\frac{R-1}{1-\Sigma}\right)^2 \frac{\omega_\zeta^2}{1-\rho_s^2}$ and $\mathrm{var}(y_t) = 5.8\omega^2$, the contemporaneous covariance between consumption and output is then

$$\mathrm{corr}(c_t, y_t) \equiv \frac{\mathrm{cov}(c_t, y_t)}{\sqrt{\mathrm{var}(c_t)}\sqrt{\mathrm{var}(y_t)}} = \frac{\Xi\frac{R-1}{1-\Sigma}\frac{1}{1-\rho\rho_s}}{\sqrt{\Xi^2\left(\frac{R-1}{1-\Sigma}\right)^2\frac{1}{1-\rho_s^2}}\sqrt{5.8}} = \frac{\sqrt{1-\rho_s^2}}{\sqrt{5.8}(1-\rho_s\rho)},$$

which is just (32) in the main text.

6.5 Deriving International Consumption Correlations under RB and SU

Given the AR(2) expressions for c_t and c_t^*, (57) and (60), the consumption correlation between the home country and the rest of the world can be written as:

$$
\begin{aligned}
&\operatorname{corr}(c_t, c_t^*) \\
&\equiv \frac{\operatorname{cov}(c_t, c_t^*)}{\sqrt{\operatorname{var}(c_t)\operatorname{var}(c_t^*)}} \\
&= \frac{1+(\rho_\theta+\rho_s)(\rho_\theta^*+\rho_s^*)+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)+\cdots}{\sqrt{\left\{\begin{array}{c}\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\lambda^2}{\theta(1/(1-\theta)-R^2)}\left[\frac{1}{1-\rho_s^2}+\frac{(\theta R)^2(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\end{array}\right\}\times\left\{\begin{array}{c}\frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^{*2}}{\theta^*(1/(1-\theta^*)-R^2)}\left[\frac{1}{1-\rho_s^{*2}}+\frac{(\theta^* R)^2\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right]\end{array}\right\}}}\frac{\operatorname{cov}(\zeta_t, \zeta_t^*)}{\omega_\zeta\omega_{\zeta^*}} \\
&= \frac{\sum_{k=0}^{\infty}\left\{\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^j\rho_s^{k-j}\right)\right]\left[\sum_{j=0,j\le k}^{k}\left(\rho_\theta^{*j}\rho_s^{*k-j}\right)\right]\right\}}{\sqrt{\Xi_1\Xi_2}}\phi,
\end{aligned}
$$

which is just (61) in the text. Note that here we use the following facts:

$$
\begin{aligned}
\operatorname{var}(c_t) &= \left(\frac{R-1}{1-\Sigma}\right)^2\theta^2\left\{\begin{array}{c}\frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\frac{\operatorname{var}(\bar{\xi}_t)}{1-\rho_s^2}+\frac{(\theta R)^2(1+\rho_s\rho_\theta)\operatorname{var}(\bar{\xi}_t)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\end{array}\right\} \\
&= \left(\frac{R-1}{1-\Sigma}\right)^2\theta^2\left\{\begin{array}{c}\frac{(1+\rho_s\rho_\theta)\omega_\zeta^2}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\left[\frac{1}{1-\rho_s^2}+\frac{(\theta R)^2(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\operatorname{var}(\bar{\xi}_t)\end{array}\right\} \\
&= \left(\frac{R-1}{1-\Sigma}\right)^2\theta^2\left\{\begin{array}{c}\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]} \\ +\left[\frac{1}{1-\rho_s^2}+\frac{(\theta R)^2(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\right]\frac{\lambda^2}{(1/(1-\theta)-R^2)\theta}\end{array}\right\}\omega_\zeta^2,
\end{aligned}
$$

$$
\operatorname{var}(c_t^*) = \left(\frac{R-1}{1-\Sigma^*}\right)^2\theta^{*2}\left\{\begin{array}{c}\frac{\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]} \\ +\frac{\lambda^2}{(1/(1-\theta^*)-R^2)\theta^*}\left[\frac{1}{1-\rho_s^{*2}}+\frac{(\theta^* R)^2\left(1+\rho_s^*\rho_\theta^*\right)}{\left(1-\rho_s^*\rho_\theta^*\right)\left[\left(1+\rho_s^*\rho_\theta^*\right)^2-\left(\rho_s^*+\rho_\theta^*\right)^2\right]}\right]\end{array}\right\}\omega_{\zeta^*}^2,
$$

and

$$\begin{aligned}
\operatorname{cov}\left(c_t, c_t^*\right) &= \operatorname{cov}\left(\frac{R-1}{1-\Sigma}\frac{\theta\left(\zeta_t+\xi_t-R\xi_{t-1}\right)}{\left(1-\rho_s\cdot L\right)\left(1-(1-\theta)R\cdot L\right)}, \frac{R-1}{1-\Sigma^*}\frac{\theta^*\left(\zeta_t^*+\xi_t^*-R\xi_t^*\right)}{\left(1-\rho_s^*\cdot L\right)\left(1-(1-\theta^*)R\cdot L\right)}\right) \\
&= \frac{\theta\theta^*\left(R-1\right)^2}{\left(1-\Sigma\right)\left(1-\Sigma^*\right)}\operatorname{cov}\left(\frac{\zeta_t}{\left(1-\rho_s\cdot L\right)\left(1-(1-\theta)R\cdot L\right)}, \frac{\zeta_t^*}{\left(1-\rho_s^*\cdot L\right)\left(1-(1-\theta^*)R\cdot L\right)}\right) \\
&= \frac{\theta\theta^*\left(R-1\right)^2}{\left(1-\Sigma\right)\left(1-\Sigma^*\right)}\operatorname{cov}\left(\begin{array}{c}\zeta_t+\left(\rho_\theta+\rho_s\right)\zeta_{t-1}+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\zeta_{t-2} \\ +\left(\rho_\theta^3+\rho_s\rho_\theta^2+\rho_s^2\rho_\theta+\rho_s^3\right)\zeta_{t-3}+\cdots, \\ \zeta_t^*+\left(\rho_\theta^*+\rho_s^*\right)\zeta_{t-1}^*+\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)\zeta_{t-2}^* \\ +\left(\rho_\theta^{*3}+\rho_s^*\rho_\theta^{*2}+\rho_s^{*2}\rho_\theta^*+\rho_s^{*3}\right)\zeta_{t-3}^*+\cdots\end{array}\right) \\
&= \frac{\theta\theta^*\left(R-1\right)^2}{\left(1-\Sigma\right)\left(1-\Sigma^*\right)}\left[1+\left(\rho_\theta+\rho_s\right)\left(\rho_\theta^*+\rho_s^*\right)+\left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\left(\rho_\theta^{*2}+\rho_s^*\rho_\theta^*+\rho_s^{*2}\right)+\cdots\right]\operatorname{cov}\left(\zeta_t,\zeta_t^*\right).
\end{aligned}$$

6.6 Deriving Other Stochastic Properties of Consumption under RB and SU

6.6.1 Relative Volatility of Consumption to Output under RB and SU

Under RB and SU, the consumption process, (57),

$$c_t = \rho_s c_{t-1} + \frac{\left(R-1\right)\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}} + \frac{R-1}{1-\widetilde{\Sigma}}\overline{\eta}_t,$$

where

$$\overline{\eta}_t = \theta\left[\frac{\zeta_t}{1-(1-\theta)R\cdot L} + \left(\xi_t - \frac{\theta R\xi_{t-1}}{1-(1-\theta)R\cdot L}\right)\right]$$

can be written as the following ARMA$(2,1)$ process:

$$\left(1-\phi_1\cdot L-\phi_2\cdot L^2\right)c_t = \theta\frac{R-1}{1-\widetilde{\Sigma}}\left(\zeta_t+\xi_t-R\xi_{t-1}\right),$$

where $\phi_1=\rho_s+\rho_\theta=\rho_s+(1-\theta)R$, $\phi_2=-\rho_s\rho_\theta=-\rho_s(1-\theta)R$, and

$$\operatorname{var}\left(\zeta_t+\xi_t-R\xi_{t-1}\right) = \left[1+\frac{\left(1-\theta\right)\left(1+R^2\right)}{\theta\left[1-\left(1-\theta\right)R^2\right]}\right]\omega_\zeta^2$$

Therefore,

$$\operatorname{var}\left(c_t\right) = \frac{\left(1+\rho_s\rho_\theta\right)}{\left(1-\rho_s\rho_\theta\right)\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]}\left[1+\frac{\lambda\left(1-\theta\right)\left(1+R^2\right)}{\theta\left[1-\left(1-\theta\right)R^2\right]}\right]\left(\theta\frac{R-1}{1-\widetilde{\Sigma}}\right)^2\omega_\zeta^2$$

and the relative volatility of consumption to output is

$$\begin{aligned}
\mu &\equiv \frac{\operatorname{sd}\left(c_t\right)}{\operatorname{sd}\left(y_t\right)} \\
&= \Xi\theta\left(\frac{R-1}{1-\widetilde{\Sigma}}\right)\sqrt{\frac{\left(1+\rho_s\rho_\theta\right)}{\left(1-\rho_s\rho_\theta\right)\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]}\left[1+\frac{\left(1-\theta+\rho_\theta R\right)}{\theta\left(1-\rho_\theta R\right)}\right]}/\Gamma_y
\end{aligned}$$

where we use the fact that $\operatorname{sd}\left(y_t\right)=\Gamma_y$, which is just (64) in the main text.

6.6.2 Consumption Persistence under RB and SU

Given that

$$c_t = \rho_s c_{t-1} + \frac{(R-1)\,\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}} + \frac{R-1}{1-\widetilde{\Sigma}}\eta_t,$$

where

$$\eta_t = \theta\left[\frac{\zeta_t}{1-\rho_\theta \cdot L} + \left(\xi_t - \frac{\theta R \xi_{t-1}}{1-\rho_\theta \cdot L}\right)\right],$$

the first-order covariance of consumption under RB and SU is

$$\begin{aligned}
\operatorname{cov}\left(c_t, c_{t-1}\right) &= \operatorname{cov}\left(\rho_s c_{t-1} + \frac{R-1}{1-\widetilde{\Sigma}}\eta_t, c_{t-1}\right) \\
&= \operatorname{cov}\left(\rho_s c_{t-1}, c_{t-1}\right) + \frac{R-1}{1-\widetilde{\Sigma}}\operatorname{cov}\left(\eta_t, c_{t-1}\right) \\
&= \rho_s \operatorname{var}\left(c_{t-1}\right) + \left(\frac{R-1}{1-\widetilde{\Sigma}}\right)^2 \operatorname{cov}\left(\eta_t, \frac{\eta_{t-1}}{1-\rho_s \cdot L}\right) \\
&= \rho_s \operatorname{var}\left(c_{t-1}\right)
\end{aligned}$$

because

$$\begin{aligned}
\operatorname{cov}\left(\eta_t, \frac{\eta_{t-1}}{1-\rho_s \cdot L}\right) &= \operatorname{cov}\left(\theta\left[\frac{\zeta_t}{1-\rho_\theta \cdot L} + \left(\xi_t - \frac{\theta R \xi_{t-1}}{1-\rho_\theta \cdot L}\right)\right], \frac{\theta\left[\frac{\zeta_{t-1}}{1-\rho_\theta \cdot L} + \left(\xi_{t-1} - \frac{\theta R \xi_{t-2}}{1-\rho_\theta \cdot L}\right)\right]}{1-\rho_s \cdot L}\right) \\
&= \left[\frac{\rho_\theta \theta^2}{\left(1-\rho_\theta^2\right)\left(1-\rho_\theta \rho_s\right)} - \frac{\theta^2\left(\theta R\right)\left(1-\rho_\theta R\right)}{\left(1-\rho_\theta^2\right)\left(1-\rho_\theta \rho_s\right)}\frac{\left(1-\theta\right)}{\theta\left(1-\rho_\theta R\right)}\right]\omega_\zeta^2 \\
&= 0.
\end{aligned}$$

Here we use the facts that

$$\begin{aligned}
\operatorname{cov}\left(\theta\frac{\zeta_t}{1-\rho_\theta \cdot L}, \theta\frac{\zeta_{t-1}}{\left(1-\rho_\theta \cdot L\right)\left(1-\rho_s \cdot L\right)}\right) &= \rho_\theta \theta^2 \operatorname{cov}\left(\frac{\zeta_{t-1}}{1-\rho_\theta \cdot L}, \frac{\zeta_{t-1}}{\left(1-\rho_\theta \cdot L\right)\left(1-\rho_s \cdot L\right)}\right) \\
&= \frac{\rho_\theta \theta^2}{\left(1-\rho_\theta^2\right)\left(1-\rho_\theta \rho_s\right)}\omega_\zeta^2
\end{aligned}$$

and

$$\operatorname{cov}\left(\theta\left(-\frac{\theta R\xi_{t-1}}{1-\rho_\theta\cdot L}\right),\frac{\theta\left(\xi_{t-1}-\frac{\theta R\xi_{t-2}}{1-\rho_\theta\cdot L}\right)}{1-\rho_s\cdot L}\right)$$

$$=\operatorname{cov}\left(\theta\left(-\frac{\theta R\xi_{t-1}}{1-\rho_\theta\cdot L}\right),\frac{\theta\left[\xi_{t-1}\left(1-\rho_\theta\cdot L\right)-\theta R\xi_{t-2}\right]}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)$$

$$=\theta^2\operatorname{cov}\left(-\frac{\theta R\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{\xi_{t-1}-R\xi_{t-2}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)$$

$$=-\theta^2\left(\theta R\right)\operatorname{cov}\left(\frac{\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{\xi_{t-1}-R\xi_{t-2}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)$$

$$=-\theta^2\left(\theta R\right)\left[\operatorname{cov}\left(\frac{\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{\xi_{t-1}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)-\operatorname{cov}\left(\frac{\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{R\xi_{t-2}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)\right]$$

$$=-\theta^2\left(\theta R\right)\left[\operatorname{cov}\left(\frac{\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{\xi_{t-1}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)-\rho_\theta R\operatorname{cov}\left(\frac{\xi_{t-2}}{1-\rho_\theta\cdot L},\frac{\xi_{t-2}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)\right]$$

$$=-\theta^2\left(\theta R\right)\left(1-\rho_\theta R\right)\operatorname{cov}\left(\frac{\xi_{t-1}}{1-\rho_\theta\cdot L},\frac{\xi_{t-1}}{\left(1-\rho_s\cdot L\right)\left(1-\rho_\theta\cdot L\right)}\right)$$

$$=-\frac{\theta^2\left(\theta R\right)\left(1-\rho_\theta R\right)}{\left(1-\rho_\theta^2\right)\left(1-\rho_\theta\rho_s\right)}\operatorname{var}\left(\xi_t\right),$$

where $\operatorname{var}\left(\xi_t\right)=\frac{\lambda(1-\theta)}{\theta(1-\rho_\theta R)}\omega_\zeta^2$. The first-order correlation of consumption under RB and SU is

$$\begin{aligned}\rho_c&\equiv\operatorname{corr}\left(c_t,c_{t-1}\right)\\&=\frac{\rho_s\operatorname{var}\left(c_{t-1}\right)+\left(\frac{R-1}{1-\widetilde{\Sigma}}\right)^2\frac{\rho_\theta\theta^2}{\left(1-\rho_\theta^2\right)\left(1-\rho_\theta\rho_s\right)}\left(1-\lambda\right)\omega_\zeta^2}{\sqrt{\operatorname{var}\left(c_t\right)\operatorname{var}\left(c_{t-1}\right)}}\\&=\rho_s+\frac{\frac{\rho_\theta}{1-\rho_\theta^2}\left(1-\lambda\right)}{\frac{\left(1+\rho_s\rho_\theta\right)}{\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2}\left[1+\frac{\lambda(1-\theta+\rho_\theta R)}{\theta(1-\rho_\theta R)}\right]}\end{aligned}$$

where $\operatorname{var}\left(c_t\right)=\frac{\left(1+\rho_s\rho_\theta\right)}{\left(1-\rho_s\rho_\theta\right)\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]}\left[1+\frac{\lambda(1-\theta+\rho_\theta R)}{\theta(1-\rho_\theta R)}\right]\left(\theta\frac{R-1}{1-\widetilde{\Sigma}}\right)^2\omega_\zeta^2$, which is just (65) in the main text. Note that when $\lambda=1$, $\rho_c=\rho_s$. When $\lambda=0$,

$$\rho_c=\rho_s+\frac{\rho_\theta\left[\left(1+\rho_s\rho_\theta\right)^2-\left(\rho_s+\rho_\theta\right)^2\right]}{\left(1+\rho_s\rho_\theta\right)\left(1-\rho_\theta^2\right)}.$$

6.6.3 Consumption-Output Correlations under RB and SU

Given that

$$c_t=\rho_s c_{t-1}+\frac{\left(R-1\right)\widetilde{\Sigma}\overline{c}}{1-\widetilde{\Sigma}}+\frac{R-1}{1-\widetilde{\Sigma}}\overline{\eta}_t,\tag{73}$$

where

$$\eta_t=\theta\left[\frac{\zeta_t}{1-\rho_\theta\cdot L}+\left(\xi_t-\frac{\theta R\xi_{t-1}}{1-\rho_\theta\cdot L}\right)\right],$$

the contemporaneous covariance between consumption and output can be written as

$$\begin{aligned}
\operatorname{cov}(c_t, y_t) &= \operatorname{cov}\left(\begin{array}{c} \frac{R-1}{1-\Sigma}\left(\frac{\theta\zeta_t}{(1-\rho_\theta\cdot L)(1-\rho_s\cdot L)} + \theta\left(\frac{\xi_t}{1-\rho_s\cdot L} - \frac{\theta R\xi_{t-1}}{(1-\rho_\theta\cdot L)((1-\rho_s\cdot L))}\right)\right), \\ a_t + \frac{\alpha_k\epsilon_{t-1}}{(1-\lambda_1\cdot L)(1-\rho\cdot L)} \end{array}\right) \\
&= \theta\frac{R-1}{1-\Sigma}\operatorname{cov}\left(\frac{\zeta_t}{(1-\rho_\theta\cdot L)(1-\rho_s\cdot L)}, \frac{\epsilon_t}{1-\rho\cdot L} + \frac{\alpha_k\epsilon_{t-1}}{(1-\lambda_1\cdot L)(1-\rho\cdot L)}\right) \\
&= \theta\frac{R-1}{1-\Sigma}\Xi\operatorname{cov}\left(\frac{\epsilon_t}{(1-\rho_\theta\cdot L)(1-\rho_s\cdot L)}, \frac{\epsilon_t}{1-\rho\cdot L} + \frac{\alpha_k\epsilon_{t-1}}{(1-\lambda_1\cdot L)(1-\rho\cdot L)}\right) \\
&\cong \theta\frac{R-1}{1-\Sigma}\Xi\operatorname{cov}\left(\begin{array}{c} \epsilon_t + (\rho_\theta+\rho_s)\epsilon_{t-1} + \left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\epsilon_{t-2} + \left(\rho_\theta^3+\rho_s\rho_\theta^2+\rho_s^2\rho_\theta+\rho_s^3\right)\epsilon_{t-3} + \cdots, \\ \epsilon_t + \rho\epsilon_{t-1} + \rho^2\epsilon_{t-2} + \cdots \end{array}\right) \\
&= \theta\frac{R-1}{1-\Sigma}\Xi\left(1 + (\rho_\theta+\rho_s)\rho + \left(\rho_\theta^2+\rho_s\rho_\theta+\rho_s^2\right)\rho^2 + \left(\rho_\theta^3+\rho_s\rho_\theta^2+\rho_s^2\rho_\theta+\rho_s^3\right)\rho^3 + \cdots\right)\omega^2 \\
&= \theta\frac{R-1}{1-\Sigma}\Xi\frac{1}{(1-\rho\rho_s)(1-\rho\rho_\theta)}\omega^2,
\end{aligned}$$

and

$$\begin{aligned}
\rho_{cy} \equiv \operatorname{corr}(c_t, y_t) &= \frac{\theta\frac{R-1}{1-\Sigma}\Xi\frac{1}{(1-\rho\rho_s)(1-\rho\rho_\theta)}}{\Gamma_y\sqrt{\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\left[1+\frac{\lambda(1-\theta+\rho_\theta R)}{\theta(1-\rho_\theta R)}\right]\left(\theta\frac{R-1}{1-\Sigma}\right)^2\Xi^2}} \\
&= \frac{1}{\Gamma_y(1-\rho\rho_s)(1-\rho\rho_\theta)\sqrt{\frac{(1+\rho_s\rho_\theta)}{(1-\rho_s\rho_\theta)\left[(1+\rho_s\rho_\theta)^2-(\rho_s+\rho_\theta)^2\right]}\left[1+\frac{\lambda(1-\theta+\rho_\theta R)}{\theta(1-\rho_\theta R)}\right]}}
\end{aligned}$$

which is just (66) in the main text.

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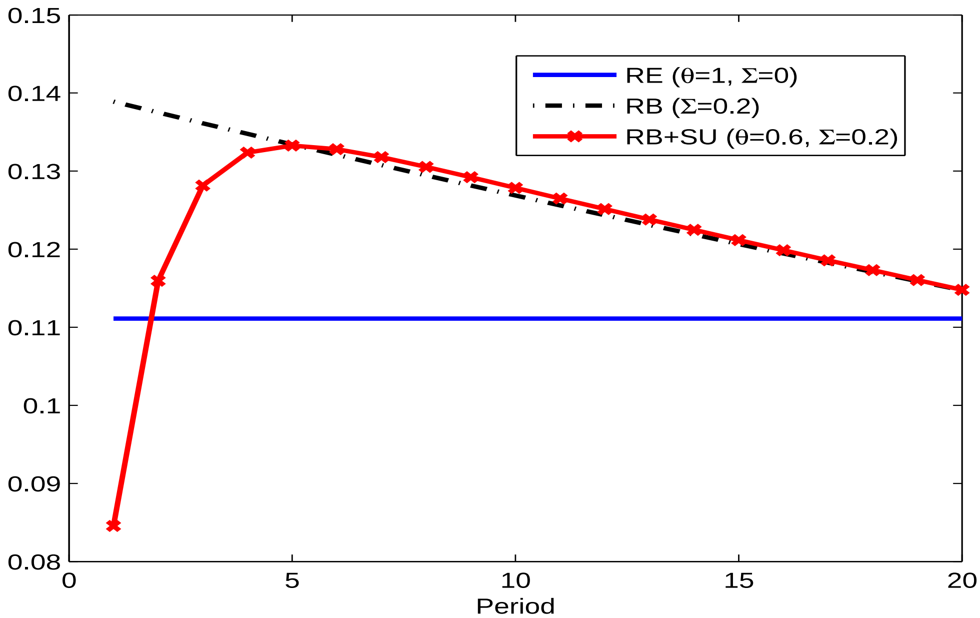

Figure 1: Impulse Responses of Consumption to Income Shocks

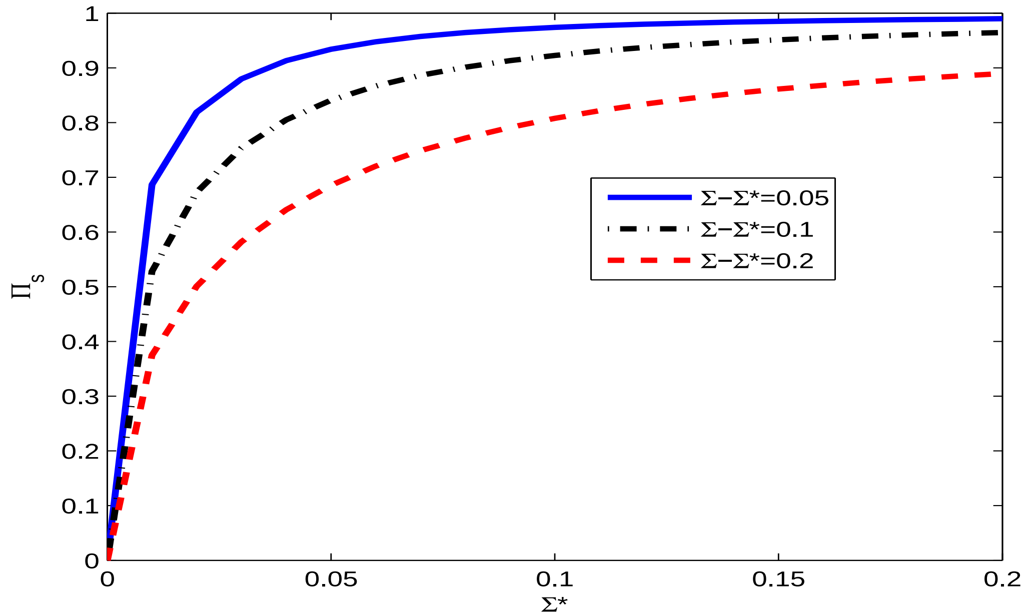

Figure 2: International Consumption Correlation under RB

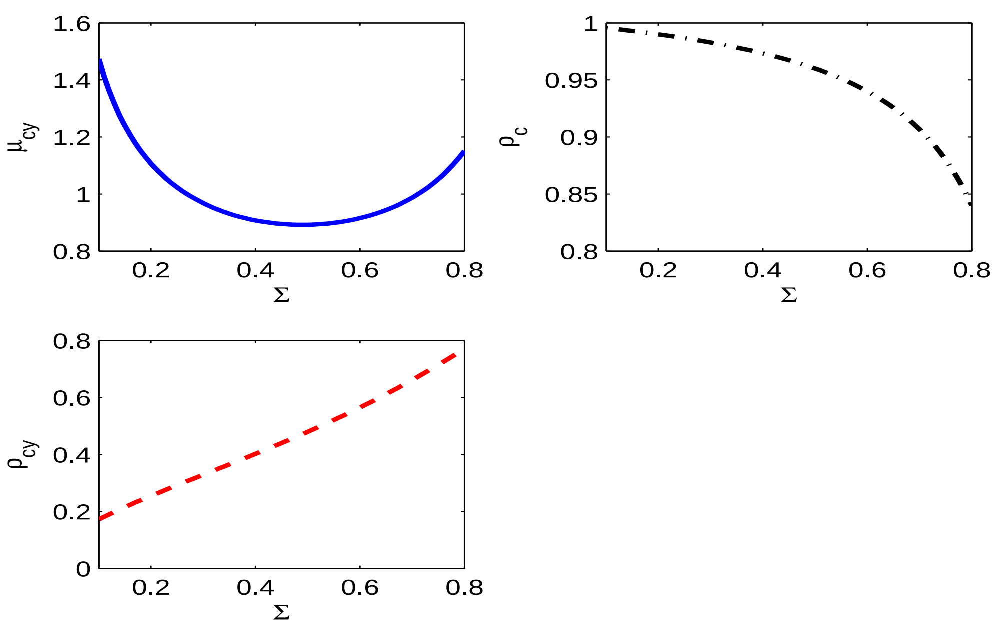

Figure 3: Stochastic Properties of Consumption under RB

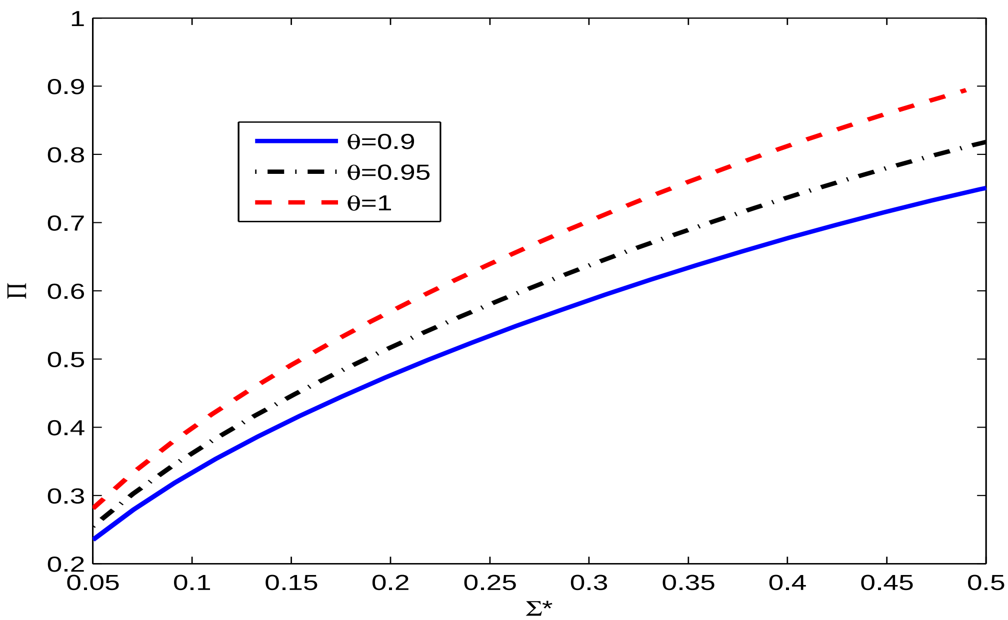

Figure 4: International Consumption Correlation under RB and SU.

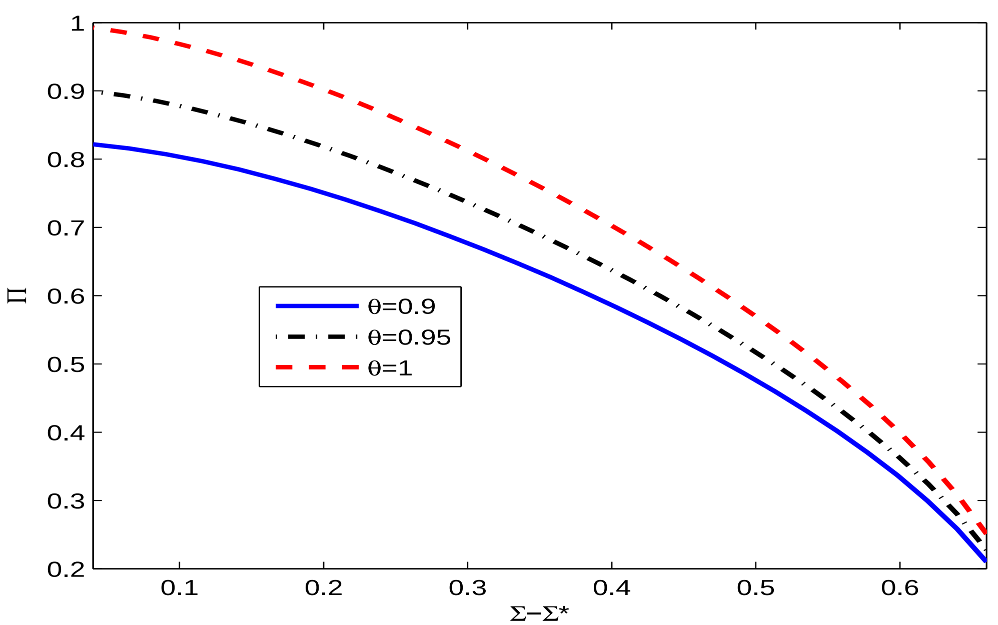

Figure 5: International Consumption Correlation under RB and SU.

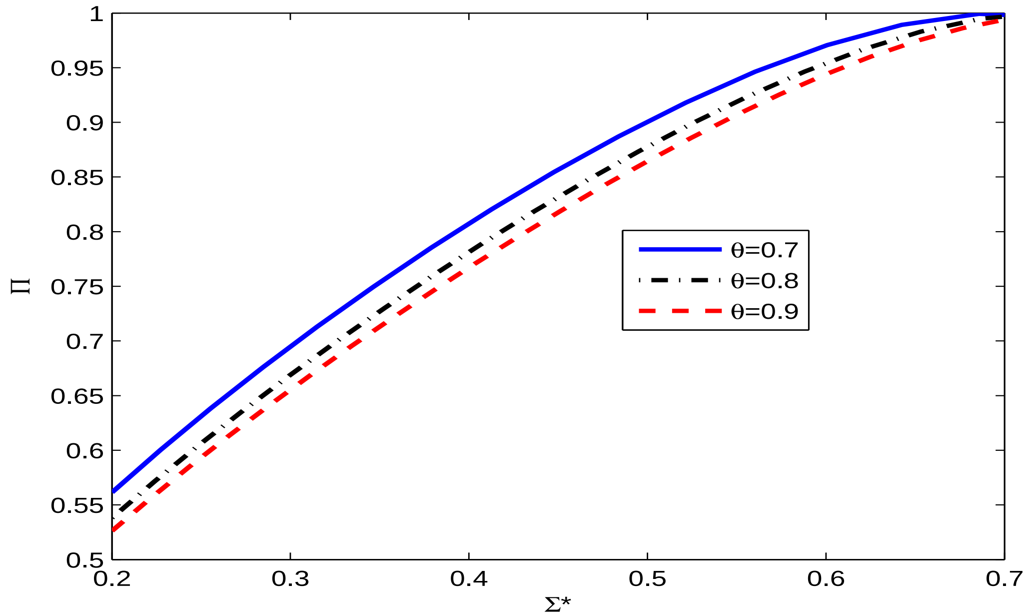

Figure 6: International Consumption Correlation under RB and SU when $\xi = 0$.

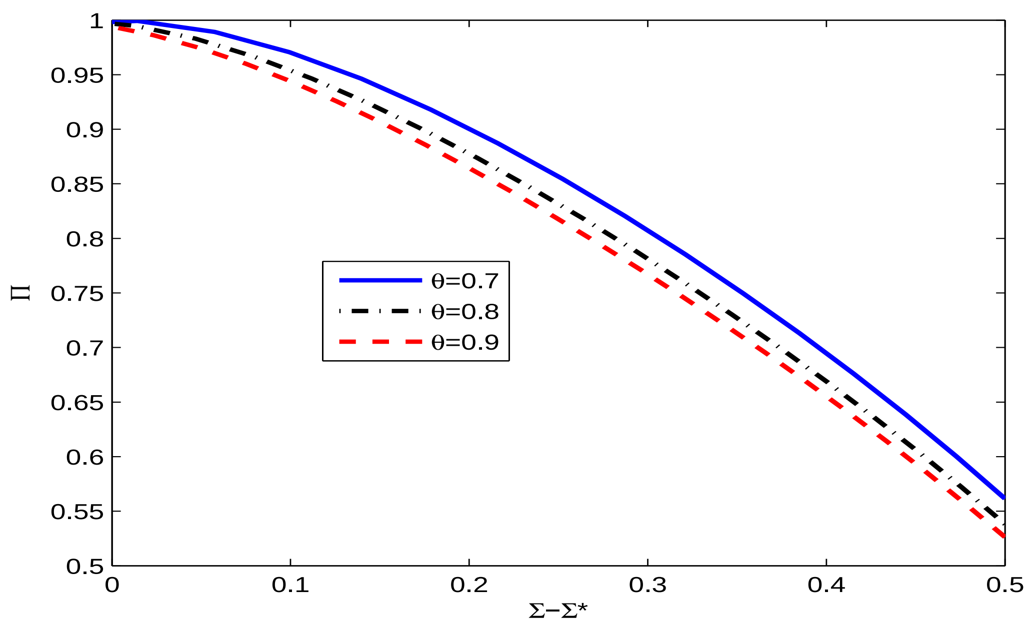

Figure 7: International Consumption Correlation under RB and SU when $\xi = 0$.

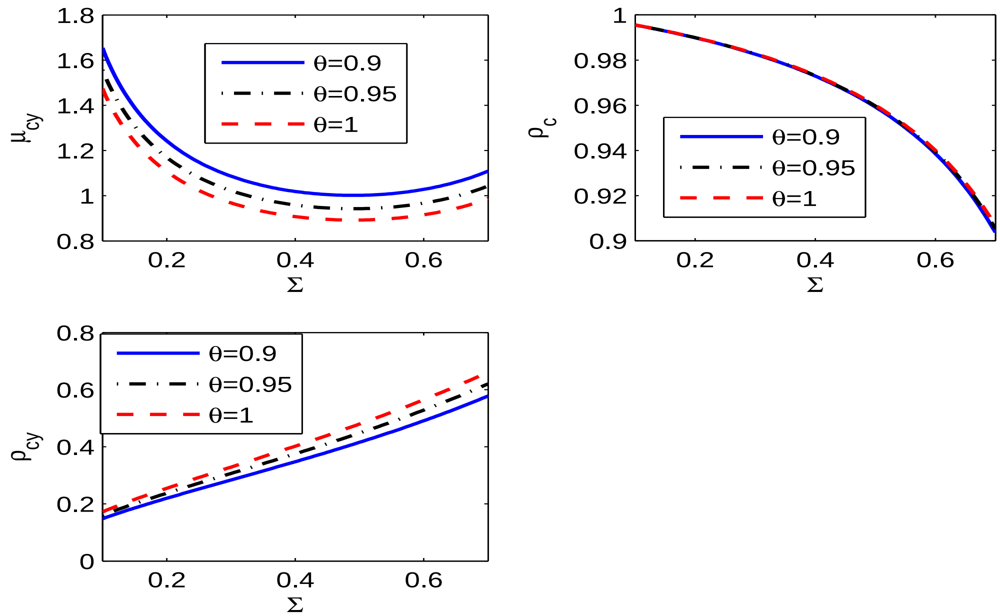

Figure 8: Stochastic Properties of Consumption.

Table 1: Summary of Statistics

	ρ_a	ρ_a^*	ρ_c	corr (a, a^*)	corr (y, y^*)	corr (c, c^*)	corr (c, y)	sd$(c)/$ sd(y)
Canada	0.62(0.06)	0.50(0.08)	0.72(0.05)	0.78(0.05)	0.82(0.06)	0.49(0.07)	0.74(0.08)	0.98(0.14)
Italy	0.42(0.11)	0.51(0.07)	0.70(0.05)	0.55(0.15)	0.55(0.17)	0.34(0.17)	0.76(0.04)	1.18(0.23)
UK	0.65(0.09)	0.49(0.07)	0.75(0.06)	0.70(0.09)	0.75(0.10)	0.69(0.11)	0.95(0.01)	1.55(0.09)
France	0.58(0.08)	0.50(0.07)	0.72(0.04)	0.63(0.11)	0.53(0.17)	0.41(0.16)	0.87(0.03)	0.93(0.10)

Table 2: Estimation and Calibration Results for Different Countries ($p = 0.1$)

	ρ	ρ^*	corr_a	$\text{corr}^D(y, y^*)$	$\text{corr}^M(y, y^*)$	$\text{corr}^D(c, c^*)$	Π_a	Π_s	Σ	Σ^*
Canada	0.62	0.50	0.78	0.82	0.78	0.49	0.98	0.69	0.73	0.31
Italy	0.42	0.51	0.55	0.55	0.55	0.34	0.99	0.88	0.78	0.57
UK	0.65	0.49	0.70	0.75	0.70	0.69	0.97	0.96	0.40	0.27
France	0.58	0.50	0.63	0.53	0.63	0.41	0.99	0.88	0.60	0.35

Table 3: Comparing Consumption Correlations ($p = 0.1$)

	Data (corr (y, y^*))	Data(corr (c, c^*))	RE(corr (c, c^*))	RB(corr (c, c^*))
Canada	0.82	0.49	0.80	0.55
Italy	0.55	0.34	0.56	0.49
UK	0.75	0.69	0.72	0.69
France	0.53	0.41	0.63	0.55

Table 4: Estimation and Calibration Results for Different Countries ($p = 0.05$)

	ρ	ρ^*	corr_a	$\text{corr}^D(y, y^*)$	$\text{corr}^M(y, y^*)$	$\text{corr}^D(c, c^*)$	Π_a	Π_s	Σ	Σ^*
Canada	0.62	0.50	0.78	0.82	0.78	0.49	0.98	0.68	0.80	0.40
Italy	0.42	0.51	0.55	0.55	0.55	0.34	0.99	0.89	0.83	0.66
UK	0.65	0.49	0.70	0.75	0.70	0.69	0.97	0.95	0.49	0.35
France	0.58	0.50	0.63	0.53	0.63	0.41	0.99	0.87	0.69	0.43

Table 5: Comparing Consumption Correlations ($p = 0.05$)

	Data (corr (y, y^*))	Data(corr (c, c^*))	RE(corr (c, c^*))	RB(corr (c, c^*))
Canada	0.82	0.49	0.79	0.54
Italy	0.55	0.34	0.56	0.49
UK	0.75	0.69	0.72	0.69
France	0.53	0.41	0.63	0.55

Table 6: Comparing Consumption Moments from Different Models ($p = 0.1$)

	Data	FI-RE	RB ($p = 0.1$)	RB+SU ($\theta = 0.95$)	RB+SU ($\theta = 0.9$)	RB+SU ($\theta = 0.85$)
Canada						
corr (c, c^*)	0.49	0.80	0.55	0.49	0.45	0.41
$\text{sd}(c)/\,\text{sd}(y)$	0.98	$+\infty$	1.25	1.32	1.41	1.51
ρ_c	0.72	1.00	0.89	0.89	0.89	0.89
ρ_{cy}	0.74	0.00	0.66	0.62	0.58	0.54
Italy						
corr (c, c^*)	0.34	0.56	0.49	0.44	0.40	0.37
$\text{sd}(c)/\,\text{sd}(y)$	1.18	$+\infty$	1.15	1.21	1.29	1.38
ρ_c	0.70	1.00	0.86	0.85	0.85	0.85
ρ_{cy}	0.76	0.00	0.70	0.65	0.60	0.55
UK						
corr (c, c^*)	0.69	0.72	0.69	0.63	0.57	0.53
$\text{sd}(c)/\,\text{sd}(y)$	1.55	$+\infty$	1.42	1.50	1.59	1.70
ρ_c	0.75	1.00	0.97	0.97	0.97	0.97
ρ_{cy}	0.95	0.00	0.41	0.38	0.36	0.33
France						
corr (c, c^*)	0.41	0.63	0.55	0.50	0.46	0.42
$\text{sd}(c)/\,\text{sd}(y)$	0.93	$+\infty$	1.71	1.81	1.92	2.05
ρ_c	0.72	1.00	0.94	0.94	0.94	0.94
ρ_{cy}	0.87	0.00	0.58	0.54	0.50	0.46

Table 7: Comparing Consumption Moments from Different Models ($p = 0.05$)

	Data	FI-RE	RB ($p = 0.05$)	RB+SU ($\theta = 0.95$)	RB+SU ($\theta = 0.9$)	RB+SU ($\theta = 0.85$)
Canada						
corr (c, c^*)	0.49	0.79	0.54	0.49	0.44	0.40
$\mathrm{sd}(c)/\mathrm{sd}(y)$	0.98	$+\infty$	1.42	1.50	1.59	1.71
ρ_c	0.72	1.00	0.84	0.84	0.83	0.83
ρ_{cy}	0.74	0.00	0.74	0.70	0.65	0.61
Italy						
corr (c, c^*)	0.34	0.56	0.49	0.44	0.40	0.37
$\mathrm{sd}(c)/\mathrm{sd}(y)$	1.18	$+\infty$	1.28	1.36	1.44	1.55
ρ_c	0.70	1.00	0.80	0.80	0.79	0.79
ρ_{cy}	0.76	0.00	0.78	0.73	0.68	0.63
UK						
corr (c, c^*)	0.69	0.72	0.69	0.62	0.57	0.52
$\mathrm{sd}(c)/\mathrm{sd}(y)$	1.55	$+\infty$	1.39	1.47	1.56	1.67
ρ_c	0.75	1.00	0.96	0.96	0.96	0.96
ρ_{cy}	0.95	0.00	0.49	0.45	0.42	0.39
France						
corr (c, c^*)	0.41	0.63	0.55	0.50	0.45	0.41
$\mathrm{sd}(c)/\mathrm{sd}(y)$	0.93	$+\infty$	1.82	1.92	2.05	2.19
ρ_c	0.72	1.00	0.91	0.91	0.91	0.91
ρ_{cy}	0.87	0.00	0.67	0.63	0.58	0.54