Dynamic Specifications in Optimizing Trend-Deviation Macro Models

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Abstract

As noted in surveys by Goodfriend and King (1997) and Walsh (1998) and exemplified by models analyzed in Taylor (1999), there is encouraging progress in developing optimizing trend-deviation macro models that provide useful insights into the transmission and design of monetary policy. Several controversial features of a minimalist trend-deviation model, with optimizing households, firms, and bond traders, are examined. Dynamic specifications are suggested to improve the data-based realism, while preserving the simplicity, of the minimalist model.

Keywords: New-Keynesian macro models; optimizing IS; Phillips curve; time-varying term premiums.

JEL: E3, E5

1 Introduction

In the last five years, there has been a remarkable flood of papers on a new variety of trend-deviation macro models used for analysis of monetary policy, both by academic researchers and by the staffs of several central banks. These models are a coherent blend of research in several disparate areas of macroeconomics and macrofinance. Essential elements include: intertemporal behavior of optimizing agents, drawn in large measure from work on equilibrium trend models; sticky pricing, based on New Keynesian models of producer pricing in monopolistic competitive markets; no-arbitrage models of financial asset valuation in bond, equity, and currency markets, drawn from both general equilibrium and time series models of pricing kernels; and interest rate feedback characterizations of monetary policy. This vintage of trend-deviation macro models is sufficiently distinctive that it has been dubbed the “new neoclassical synthesis” by Goodfriend and King (1997).[1]

One characteristic of this literature is that essential properties of even the larger-scale trend-deviation models can be captured by a small three-variable model, containing behavioral equations for the trend-deviation of aggregate output, $\tilde{y}$, the rate of inflation, π, and the one-period nominal interest rate that is controlled by the central bank, r.

This minimalist optimizing policy model consists of an “optimizing IS” equation, a “New Keynesian Phillips” curve, and a Taylor-style interest rate feedback rule (or, alternatively, a forward-looking inflation targeting rule, as in Batini and Nelson (2000)):

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} + aE_t\{(\rho_t - \bar{\rho})\} + \epsilon_{y,t}, \quad (1)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + b\tilde{y}_t + \epsilon_{\pi,t}, \quad (2)$$

$$r_t = c_0 + c_1\pi_t + c_2\tilde{y}_t + \epsilon_{r,t}, \quad (3)$$

where $E_t\{\rho_t - \bar{\rho}\}$ denotes equilibrium deviations of the ex ante real interest rate. In the case of a minimalist open-economy model, such as Ball (1999), output and consumer price inflation are altered by movements in the real exchange rate, and an additional behavioral equation reflecting real interest rate parity in global capital markets is added. Recent examples of minimalist three- or four-variable model descriptions of the optimizing behaviors of households, firms, and bond traders include Rotemberg and Woodford (1998), McCallum and Nelson (1999a), Clarida, Gali, and Gertler (1999), and Batini and Nelson (2000).

[1] Many researchers have contributed to the evolution of the new optimizing trend-deviation macro models. With apologies to those not cited, a few of the influential contributions include: Taylor (1993a, 1993b), Fuhrer and Moore (1995), Yun (1996), Rotemberg (1996), Jeanne (1997), Rotemberg and Woodford (1998), Kiley (1999), McCallum and Nelson (1999a), Ball (1999), Kim (2000) and the operational policy model, FRB/US, described in Brayton and Tinsley (1996) and Brayton et al. (1997).

The starkness of this model resembles the compactness of a textbook IS/LM model but the only valid similarity is that both models are useful in discussing issues in the transmission and design of monetary policy. In principle, all parameters in the minimalist trend-deviation model are based on specifications of the utility functions and resource constraints of households, firms, and bond traders in the economy (along with the constant-parameter description of the policy feedback or inflation targeting rule of the central bank). Further, because the reduced-form solution of the model is equivalent to a linear VAR with restricted coefficients, these restrictions (again, in principle) can be tested against the coefficients of an unrestricted VAR.

Despite the attractive communication and tractable computational properties of this model, several of its dynamic properties have been criticized in recent papers as markedly inconsistent with stylized empirical facts, such as those established by the atheoretic VAR analyses of Sims (1992), Bernanke and Blinder (1992), and Christiano, Eichenbaum, and Evans (1996).

Section 2 of this paper discusses behavioral assumptions and empirical shortcomings of the minimalist trend-deviation macro model, and suggests several dynamic specifications that are likely to improve the empirical realism of the model, without sacrificing the essential starkness of the model. Some new empirical evidence to support the suggested modifications is presented in section 3. Section 4 concludes.

2 Behavioral assumptions of the minimalist optimizing policy model

This section sketches the economy that underlies the minimalist policy model shown in equations (1), (2), and (3). Although a number of significant economic activities are glossed over in aggregation, such as business fixed investment and the valuation of foreign currencies, the economy is populated by four types of agents. Households consume, invest in financial assets, and supply labor inputs to firms. Financial asset traders forecast the future interest rate behavior of the central bank. Firms rent household labor, produce goods and services, and adjust product prices towards equilibrium markups over marginal costs of production. Finally, a central bank alters the short-term nominal interest rate in response to target deviations of inflation and trend deviations of output.

For brevity, the discussion focuses on the activities of households and firms and several implications for dynamic specifications of the optimizing IS equation, (1), and the New Keynesian Phillips curve, (2). Some consequences of alternative specifications of bond trader expectations and of collateral interest rate volatility consequences of central bank policy are discussed in section 3.

2.1 *Households and the optimizing IS equation*

The demand for aggregate output, equation (1), is based, principally, on household plans for intertemporal consumption. The discussion focuses on the potential policy transmission role of long-maturity bond rates and the simple first-order dynamics implied by a standard utility specification. A more general dynamic specification is obtained by allowing for endogenous habit aspirations, as in Fuhrer (2000), and a log linearization is shown to be equivalent to the higher-order Euler equation format suggested in Kozicki and Tinsley (1999) and Tinsley (forthcoming).

The representative household is infinitely-lived and maximizes its expected utility,

$$E_t\{\Upsilon_t\} \quad = \quad E_t\{\sum_{i=0}^{\infty} \beta^i \upsilon_{t+i}\}, \tag{4}$$

where β is a fractional discount factor.[2]

Although some differences in specifications will be noted, many current policy models are based on the assumption that utility is separable over time and over allocations to consumption, C, and hours worked by household members, H. A common specification is the CRRA utility function,

$$\upsilon(C_{t+i}, H_{t+i}) \quad = \quad [\frac{C_{t+i}^{1-\alpha}-1}{1-\alpha} - \frac{H_{t+i}^{1+\gamma}}{1+\gamma}], \tag{5}$$

[2]Household agents are heterogeneous in some optimizing models. For the purpose of discussing the minimalist model formulations, it is sufficient to assume all households are identical and interpret household equations as aggregate specifications.

with the coefficients, $\alpha > 0$ and $\gamma > 0$.

In each period, the representative household is subject to the budget constraint,

$$C_t + V_t + B_t \quad = \quad \frac{W_t}{P_t} H_t + Re^1_{a,t} V_{t-1} + Re^1_t B_{t-1}, \tag{6}$$

where uses of (real) funds are shown on the left of the equal sign and sources of funds on the right. Investments in the equity of firms, V_t, and in bonds, B_t, are deflated by the current aggregate price, P_t; W_t denotes the nominal wage rate for employed hours; and Re^1_a and Re^1 are the real, one-period gross returns to equity and bonds, respectively.

Several optimizing macro models, such as those developed by McCallum and Nelson (1999a) and Walsh (1998), include money balances in the household utility function and the budget constraint. While this (or a similar motivation for money demand such as a cash-in-advance constraint) can be essential for examining certain issues such as the financing of fiscal expenditures, we ignore this complication for two reasons: First, real outside money balances are a very small proportion of US household wealth, roughly around 1%. Even this is an overstatement because about 90% of base money is currency. A conservative estimate is that at least half of US currency is held abroad; see discussion and references in Doyle (2000). Second, household utility specifications are often separable in money balances. Although this provides an expedient, theory-based specification for money demand, money balances are functionally irrelevant in the benchmark model if the central bank pursues an interest rate policy aimed at stabilizing a nominal anchor, such as the inflation rate in equation (3).

The first-order conditions for optimal consumption, bond allocation, and hours supplied to firms are:

$$\frac{\partial \Upsilon}{\partial C_t} \quad = \quad E_t\{C_t^{-\alpha} - \Lambda_t\} = 0, \tag{7}$$

$$\frac{\partial \Upsilon}{\partial B_t} \quad = \quad E_t\{-\Lambda_t + \beta Re^1_{t+1} \Lambda_{t+1}\} = 0, \tag{8}$$

$$\frac{\partial \Upsilon}{\partial H_t} \quad = \quad E_t\{-H_t^{\gamma} + \Lambda_t \frac{W_t}{P_t}\} = 0, \tag{9}$$

where Λ_t is the Lagrangian multiplier on the t-period flow constraint in (6), $\Lambda \geq 0$.

By equation (7), Λ is the one-period marginal utility of consumption. Rearranging (8) gives the condition for no-arbitrage valuation,

$$E_t\{Re^1_{t+1} M_{t+1}\} \quad = \quad 1, \tag{10}$$

where M_{t+1} is the CCAPM pricing kernel for real returns,

$$M_{t+1} \quad = \quad \frac{\beta \Lambda_{t+1}}{\Lambda_t}.$$

Assuming real returns and the pricing kernel are joint lognormally distributed, equation (8) can be conveniently restated in the log linear format

$$\lambda_t \quad = \quad E_t\{\lambda_{t+1} + \rho_t + a_0 + \log(\beta)\}, \tag{11}$$

where lowercase indicates logs.[3] For the moment, we assume homoskedastic variances and covariances, which implies a_0 is constant,

$$a_0 \quad = \quad \frac{1}{2}\left[\text{var}(\rho) + \text{var}(m) + 2\text{cov}(\rho, m)\right], \tag{12}$$

where var(.) and cov(.) denote variances and covariances.

Using equation (7), equation (11) can be restated as the Euler equation for log consumption,

$$c_t \quad = \quad E_t\{c_{t+1} - \frac{1}{\alpha}\rho_t - \frac{\log(\beta) + a_0}{\alpha}\}. \tag{13}$$

Finally, assume $c_t \equiv y_t$ and restate equation (13), using equilibrium deviations, as the optimizing IS schedule shown earlier in (1),[4]

$$\tilde{y}_t \quad = \quad E_t\{\tilde{y}_{t+1}\} - \frac{1}{\alpha}E_t\{(\rho_t - \bar{\rho})\} + \epsilon_{y,t}. \tag{14}$$

An important implication of equation (14), discussed in Woodford (1999), is that the equilibrium deviation in output, $\tilde{y}$, is a function, not just of the one-period real interest rate, ρ, but of a real long-term interest rate, say ρ_n. To demonstrate this, successively eliminate expected future output terms in the first-order difference equation (14), showing that the current output gap is driven by expectations of future short-term rates,

$$\begin{aligned}
\tilde{y}_t \quad &= \quad -\frac{1}{\alpha}\sum_{i=0}^{\infty} E_t\{(\rho_{t+i} - \bar{\rho})\} + \epsilon_{y,t}, \\
&\simeq \quad -\frac{1}{\alpha}\sum_{i=0}^{n-1} E_t\{(\rho_{t+i} - \bar{\rho})\} + \epsilon_{y,t}, \\
&= \quad -\frac{n}{\alpha}E_t\{(\rho_{n,t} - \bar{\rho})\} + \epsilon_{y,t},
\end{aligned} \tag{15}$$

where the infinite sum in the first line of (15) is finite for stationary real rate deviations, and the last two lines link current output deviations to the current ex ante real rate on an n-period bond.

[3]The real return on a one-period discount bond is $Re^1_{t+1} = \frac{P^b_{0,t+1}}{P^b_{1,t}\Pi_{t+1}}$ where P^b denotes the price of the bond and Π is the gross inflation rate. The price of the bond is fixed at one at maturity, $P^b_{0,t+1} = 1$. Consequently, $\log Re^1_{t+1} = -p^b_t - \pi_{t+1} = r_t - \pi_{t+1} = \rho_t$.

[4]Alternatively, one can use first-order log approximations of a GDP identity, such as $y_t = s_c c_t + s_g g_t$, where s_c and s_g are the average GDP shares of interest-sensitive and autonomous GDP components, Rotemberg and Woodford (1998).

Because most optimizing policy models are calibrated or estimated with the output equation in the first-order format shown in (14), use of the real yield on a finite maturity bond, ρ_n, is only a suggestive approximation. Nevertheless, because we will return to the issue of policy-invariant term premiums, it is of interest to pin down a reasonable maturity choice for the long-term interest rate.[5] The small policy model estimated by Rudebusch and Svensson (1999) uses a four-quarter average of the one-period real interest rate in the output equation; Taylor (1993b) and Coenen and Wieland (2000) select a two-year rate as the relevant long-term interest rate; and Fuhrer and Moore (1995) use a 10-year real rate, noting that the average duration of the corporate BAA rate over their sample is 40 quarters.

Under a log form of the expectations hypothesis (EH), the ex ante n-period real interest rate is proportional to the average of one-period nominal interest rates less the average of one-period inflation rates expected over the n-period forecast horizon.

$$\begin{aligned} E_t\{\rho_{n,t}\} &\propto \frac{1}{n}E_t\{\sum_{i=0}^{n-1}(r_{t+i}-\pi_{t+i+1})\}, \\ &= r_{n,t} - E_t\pi_{n,t}. \end{aligned} \tag{16}$$

Under the EH hypothesis, the n-period nominal rate (on a zero coupon bond) is an average of bond traders' current forecasts of future one-period nominal rates. The expectations operator is dropped for $r_{n,t}$ under the assumption that current nominal interest rates of all maturities are contained in the t-period information set. The expectations operator is retained for the n-period inflation rate, π_n, because the inflation expectations of bond traders are not directly observed.[6]

Section 3 discusses some empirical evidence on the choice of the maturity, n, of the relevant interest rate in the IS equation (15); methods of approximating the inflation expectations of bond traders, and consequences of the common, but counterfactual, assumption that term and credit risk premiums are constant.

A potential empirical shortcoming of the benchmark optimizing IS equation, especially as formulated in equation (15), is that it implies immediate output responses to changes in the current bond rate. Evidence from empirical VAR models suggests delayed and smoothed responses by output, and components of output, to unexpected changes in interest rates. One specification that produces response lags is a fixed lag in expectations, such as used in Rotemberg and Woodford (1998), where current household expenditures are based on stale expectations, such as $E_{t-k}\rho_{n,t}$. Another approach that increases the persistence of the output

[5]As illustrated in Kozicki and Tinsley (2001), time-varying term premiums account for 50-80% of the historical variation in nominal 10-year bond yields under standard VAR characterizations of bond trader forecasts.

[6]Useful, but not certain, inferences may be drawn from surveys of investors or observed yields on indexed bonds.

gap, but does not address the issue of delayed expenditure responses, is to introduce serially correlated "taste" shocks to the utility function, vid. McCallum and Nelson (1999b).

Although order and installation lags are common in real business cycle models, they are not standard specifications in optimizing policy models because trend output is exogenous. Also uncommon are strictly convex costs of adjusting stocks or flows of durable goods or of collecting relevant information. Linearizations of FOCs associated with these adjustment costs imply higher-order Euler equations, such as

$$E_t\{a(L)a(\beta F)\tilde{y}_t\} \quad = \quad -a(1)a(\beta)\frac{n}{\alpha}E_t\{(\rho_{n,t}-\bar{\rho})\}+\epsilon_{y,t}, \tag{17}$$

where $a(.)$ is a scalar polynomial in the lag, L, and lead, F, operators. Kozicki and Tinsley (1999), for example, demonstrate that higher-order Euler equations have better empirical properties for a model of producers' durable equipment investment.

Fuhrer (2000) demonstrates that modifying the utility function to introduce endogenous "habit" produces aggregate consumption responses to policy shocks that better match the dynamic responses estimated by atheoretic time series models. In brief, the consumption component of the utility function is altered to

$$v(C_t,0) \quad = \quad \frac{(C_t\Gamma_t^{-\nu})^{1-\alpha}-1}{1-\alpha}, \tag{18}$$

where Γ is an accustomed aspiration level, whose log is approximated here by a weighted average of past log consumption,[7]

$$\log\Gamma_t \quad = \quad v(L)c_{t-1}, \qquad v(1)=1. \tag{19}$$

An additive, exogenous habit specification, where the distributed lag in (19) refers to the consumption levels of the household's neighbors, is sufficient to introduce lags into the output equation, and has been used by Campbell and Cochrane (1999) to interpret time-variation in equity risk premiums. By contrast, the contribution of Fuhrer (2000), and the earlier work of Carroll, Overland and Weil (2000), is to emphasize the intertemporal consequences of a multiplicative "endogenous" habit, where the lagged arguments of household habit refer to past consumption levels of the optimizing household. Under the endogenous habit specification, utility is no longer separable over time, and both lags and leads of consumption will appear in the FOC for consumption.

To illustrate, suppose habit is a two-period geometric average of past consumption, $v(L)=v_1+v_2L$, and, to reduce notation, absorb the habit parameter, ν, into the polynomial weights, $v(1)=\nu$. In this instance, the FOC in equation (7) becomes

$$\frac{\partial\Upsilon}{\partial C_t} \quad = \quad E_t\{C_t^{-\alpha}\Gamma_t^{-(1-\alpha)} - v_1\beta C_{t+1}^{1-\alpha}\Gamma_{t+1}^{-(1-\alpha)}C_t^{-1} - v_2\beta^2 C_{t+2}^{1-\alpha}\Gamma_{t+2}^{-(1-\alpha)}C_t^{-1} - \Lambda_t\} \;=\; 0. \tag{20}$$

[7] Fuhrer (2000) represents habit by a distributed lag of past levels of consumption, $\Gamma_t=(1-\rho)C_{t-1}+\rho\Gamma_{t-1}$.

Although recent policy models, such as McCallum and Nelson (1999b), have adopted endogenous habit specifications, the literature does not explicitly demonstrate that log linearization of an endogenous habit FOC, such as (20), yields the higher-order linear Euler specification suggested above in (17). To see this connection, rearrange the log-linearization of (20) to show the symmetry of coefficients on the leads and lags of consumption,

$$E_t\{(w_0 + w_1[L + \beta F] + w_2[L^2 + \beta^2 F^2])c_t - \lambda_t\} \quad = \quad 0, \tag{21}$$

where

$$\begin{aligned} w_0 &= \frac{-\alpha + v_1\beta[v_1(1-\alpha) - 1] + v_2\beta^2[v_2(1-\alpha) - 1]}{1 - v_1\beta - v_2\beta^2}, \\ w_1 &= \frac{-v_1(1-\alpha) + v_1\beta[v_2(1-\alpha)]}{1 - v_1\beta - v_2\beta^2}, \\ w_2 &= \frac{-v_2(1-\alpha)}{1 - v_1\beta - v_2\beta^2}. \end{aligned}$$

Polynomials with symmetric coefficients on lag and lead operators with like powers are self-reciprocating, and can be factored into the format shown in (17).[8]

The intuition for the equivalence of linearized FOCs from endogenous habit utility functions and of self-reciprocating, linear Euler equations is apparent by examining the utility function with a one-period habit specification,

$$v(C_t) \quad = \quad \frac{(C_t^{1-\nu}(\frac{C_t}{C_{t-1}})^{\nu})^{1-\alpha} - 1}{1-\alpha}, \tag{22}$$

Maximizing the concave utility function implies intertemporal smoothing of both the level and first-difference of consumption, as in Fuhrer (2000). The extension to higher-order endogenous habit is consistent with the generalized smoothing specifications explored in Kozicki and Tinsley (1999) and Tinsley (forthcoming), which includes smoothing of deviations from moving averages and of higher-order differences.

Finally, the log-linearized optimizing IS equation for a generalized endogenous habit specification is

$$E_t\{\frac{[(1-\alpha)w(L)w(\beta F) + w(\beta) - 2]}{w(\beta)}\tilde{y}_t - \sum_{i=0}^{\infty}(\rho_{t+i} - \bar{\rho})\} = 0, \tag{23}$$

where we use the condensed notation, $w(L) = 1 - v(L)$ and, as before, set $v(1) = \nu$. The infinite sum in expected real interest rate deviations is obtained by log linearizing equation (8) and solving forward for λ_t.

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[8] Self-reciprocating polynomials are discussed in Tinsley (1993)

2.2 *Firms and the New-Keynesian Phillips curve*

Turning to the business sector, in order for monetary policy to affect real output in the short-run, prices of differentiated goods in monopolistic product markets are sticky. The New Keynesian Phillips curve, shown in equation (2), is based on the assumption that it is costly to continuously adjust price levels. Implications of this specification are that the aggregate inflation rate is a jumping variable and inflation responds faster than output to policy shocks. The discussion shows that more general descriptions of pricing frictions lead to Phillips curves that contain additional lags and expected leads of inflation. These alternative specifications are more consistent with stylized empirical facts and do not require dropping the assumption of rational expectations.

The short-run production function of the ith firm is $Y_{i,t} = Z_t H_{i,t}$, where each firm has access to a common labor-augmenting productivity process, Z_t. Also, as noted earlier, household labor is paid the common nominal wage, W_t. Thus, each firm confronts an identical nominal marginal cost of production,

$$nmc_t = w_t - z_t. \tag{24}$$

There is a unit continuum of monopolistically competitive firms, each producing a differentiated output, $Y_{i,t}$. The outputs of all firms are purchased by a retail distributor and combined (using a Dixit-Stiglitz (1977) aggregator) into a composite product for sale to households, Y_t.

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}. \tag{25}$$

As in Kim (2000), maximizing retailer real profits, $\frac{P_i}{P}Y_i - \frac{WH_i}{P}$, implies the retailer's demand for the product of the ith firm is,

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t, \tag{26}$$

where θ is the common price elasticity of demand.

In addition, imposing zero profits on the retailer implies the aggregate price index,

$$P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{27}$$

Real profits of the ith firm are

$$\begin{aligned}\Xi_{i,t} &= \frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}H_{i,t}, \\ &= \left[\frac{P_{i,t}}{P_t} - \frac{W_t}{P_t Z_t}\right]\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t, \end{aligned} \tag{28}$$

where the second line is obtained using (26). In the absence of pricing frictions, the ith firm will choose its price, $P^*_{i,t}$, to maximize its real profits. The FOC condition,

$$\frac{\partial \Xi_{i,t}}{\partial P_{i,t}} \;=\; \frac{\theta-1}{\theta}\left(\frac{P_{i,t}}{P_t}\right)^{1-\theta} Y_t - \frac{W_t}{P_t Z_t}\left(\frac{P_{i,t}}{P_t}\right)^{-\theta} Y_t \;=\; 0, \tag{29}$$

can be simplified to the familiar condition,

$$p^*_t \;=\; m + nmc_t, \tag{30}$$

where p^* denotes the equilibrium log price; the log markup is $m = \log(\frac{1}{1-\frac{1}{\theta}})$; and the firm subscript can be dropped because the equilibrium price is the same for all firms.

However, as documented in Carlton (1986), the average lag in adjusting (US) producer prices is about a year. One popular characterization of time-dependent delays in price adjustment is the discrete-time variant of Calvo (1983) where each firm is subject to a geometric distribution of price adjustment delays.

In brief, the probability that a firm will be permitted to alter its price in any period is $(1-\lambda)$. This implies that the mean lag of adjustment is $\frac{1}{1-\lambda}$, and the probability of a price reset after $i-1$ periods without allowable resets is $(1-\lambda)\lambda^{i-1}$.

When a firm is allowed to reset its price, it must account for the prospect of future adjustment delays. Consequently, the firm selects a reset price level which is a weighted average of expected equilibrium prices over future periods, where the weights are the discounted survival probabilities of the current reset price. Accordingly, the log reset price of firms allowed to adjust prices in period t is

$$\begin{aligned} p_{r,t} &= E_t\{(1-\beta\lambda)\sum_{i=0}^{\infty}(\beta\lambda)^i p^*_{t+i}\}, \\ &= E_t\{\frac{(1-\beta\lambda)}{(1-\beta\lambda F)}p^*_t\}. \end{aligned} \tag{31}$$

The current aggregate log price is a geometric average of the current reset price of adjusting firms and the past prices of firms not yet able to adjust their prices,

$$\begin{aligned} p_t &= (1-\lambda)[p_{r,t} + \lambda p_{r,t-1} + \lambda^2 p_{r,t-2} + \ldots], \\ &= \frac{(1-\lambda)}{(1-\lambda L)}p_{r,t}. \end{aligned} \tag{32}$$

Combining (31) and (32), the dynamic behavior of the aggregate price is described by the linear difference equation,

$$E_t\{(1-\lambda L)(1-\beta\lambda F)p_t\} \;=\; (1-\lambda)(1-\beta\lambda)p^*_t. \tag{33}$$

Adding $\lambda(1+\beta)p_t$ to both sides of the equal sign, equation (33) can be restated as

$$\pi_t \;=\; E_t\beta\pi_{t+1} + \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}(p_t^* - p_t). \tag{34}$$

Note the only essential difference between the New Keynesian Phillips curve (2) and equation (34) is that the price gap in the latter, $p^* - p$, is represented by the output gap in the former, $\tilde{y}$. A demonstration of the connection between these equilibrium deviations requires two additional steps.

First, using the notation in (24), real marginal cost is $mc_t = nmc_t - p_t$, which implies that equilibrium real marginal cost is $mc_t^* = nmc_t - p_t^*$. Consequently, the equilibrium price deviation is the reverse of the equilibrium deviation in real marginal cost.

$$\begin{aligned} p_t^* - p_t &= (nmc_t - mc_t^*) - (nmc_t - mc_t), \\ &= mc_t - mc_t^*. \end{aligned} \tag{35}$$

Second, by (24), firms will demand labor at the log real wage

$$w_t - p_t \;=\; mc_t + z_t, \tag{36}$$

and, by equation (9), households will supply labor at the real wage

$$w_t - p_t \;=\; \alpha y_t + \gamma h_t. \tag{37}$$

These two equations imply that equilibrium deviations in real marginal cost are proportional to trend deviations in output,[9]

$$mc_t - mc_t^* \;=\; (\alpha+\gamma)\tilde{y}_t. \tag{38}$$

Substituting (35) and (38) into (34) gives the New Keynesian Phillips curve,

$$\pi_t \;=\; E_t\beta\pi_{t+1} + b\tilde{y}_t. \tag{39}$$

where $b = (\alpha+\gamma)\frac{(1-\lambda)(1-\beta\lambda)}{\lambda}$. Because models with Phillips curves are widely used by central banks and government agencies to simulate expected inflation consequences of alternative policies, it is very appealing to have a formulation that is based on a model of optimal pricing behavior.

Although the following equation neglects some standard bells and whistles, the core specification of Phillips curves used in operational policy models until the mid-1990s is

$$\pi_t \;=\; b_1(L)\pi_{t-1} + b_2\tilde{y}_t, \tag{40}$$

[9]Equating (36) and (37) gives $mc_t = (\alpha-1)y_t + (1+\gamma)h_t$. This implies (38) for $\tilde{y} = \tilde{h}$. Note that "trend" and "equilibrium" output are synonymous, and refer to the output associated with equilibrium pricing, $p = p^*$.

where a unit sum, $b_1(1) = 1$ is often imposed to avoid model simulations that suggest a long-run tradeoff between inflation and output.[10]

Of course, the two formulations are equivalent if the forward rational expectation is equal to the inflation autoregression, $E_t\beta\pi_{t+1} = b_1(L)\pi_{t-1}$. But this cannot be true for any anticipated monetary policy that significantly deviates from historical policy. Because the strength of the traditional specification in (40) is that it captures the strong autocorrelations of actual inflation rates, dynamic implications of the New Keynesian formulation in (39) have been the target of critical analysis, such as Fuhrer (1997), Gali and Gertler (1999), Mankiw (2000), and Ball (2000).

All criticisms are directed at the restriction that current inflation is a jumping variable in (39). For example, if the current output gap is negative, current inflation immediately jumps below inflation expected in the next period. Alternatively, if the central bank can directly reduce the inflation expectations of firms, disinflation is immediate and costless because the sacrifice ratio is zero. By contrast, atheoretic estimates in Ball (1994) suggest, if anything, historical sacrifice ratios have been higher in countries where the central bank has a stronger reputation for pursuing low-inflation policies. In addition, solving (39) forward indicates that current inflation responds solely to current and expected output gaps in the future,

$$\pi_t = E_t\{b\sum_{i=0}^{\infty}\beta^i\tilde{y}_{t+i}\}. \tag{41}$$

This suggests inflation responses to policy are faster than output responses, which is the reverse of policy response lags estimated by atheoretic VAR analyses.

Because historical inflation rates are significantly autocorrelated, there have been several efforts to justify a hybrid mixture of the forward-looking (39) and backwards-looking (40) Phillips curves, such as

$$\pi_t = b_1\pi_{t-1} + (1-b_1)E_t\beta\pi_{t+1} + b_2\tilde{y}_t, \tag{42}$$

where, in simple examples, $\beta = 1$ and $b_1 = .5$. This format implies that current inflation is a two-sided function of both expected and past output gaps. For $b_1 = .5$, the mean lag to anticipated changes in output is zero.

Without exception, all motivations of a two-sided formulation such as equation (42), rest on an assumption of non-rational behavior by a significant proportion, b_1, of agents: For example, a fraction of agents use adaptive expectations in Roberts (1977) and Ball (2000). In the interpretation offered by Gali and Gertler (1999), some firms use rule-of-thumb pricing rules. In the case of the real wage contracts model

[10] As noted by Sargent (1971), it is difficult to justify this restriction on empirical grounds unless the inflation rate is $I(1)$.

of Fuhrer and Moore (1995), the price base of the real wage comparison is not the average of prices expected over the life of the contract.

A more fundamental issue is that the dynamics of inflation and output are determined by the structure of the full model of the economy. Regardless of whether or not the sum of weights on inflation is unity, long-run inflation can be stationary if the central bank effectively targets a constant rate of inflation.

The lag dynamics of inflation are determined also by the effective forecast model of agents regarding expected inflation. One source of relatively slow historical inflation responses is that agents may have to learn about shifts in the policy target for inflation. Learning can significantly slow aggregate inflation adjustments, at least after major changes in policy, as illustrated in Kozicki and Tinsley (2001).

The analytical strength of the New Keynesian Phillips curve is that it is an implication of *any* linearized model of optimal pricing that is subject to smoothing restrictions, such as (33), which is restated as

$$E_t\{a(L)a(\beta F)p_t\} \quad = \quad a(1)a(\beta)p_t^*. \tag{43}$$

An alternative explanation of slow inflation responses, that does not require abandoning the assumption of rational expectations, may be that firms are subjected to more complicated frictions than those captured by the geometric polynomials, $a(L)^{-1} = \frac{1}{1-\lambda L}$, and $a(\beta F)^{-1} = \frac{1}{1-\lambda\beta F}$.

To illustrate, suppose the factor polynomial in equation (43) is second-order, $a(L) = 1 - \lambda_1 L - \lambda_2 L^2$. This implies the counterpart to equation (34) is

$$\begin{aligned} a(1)a(\beta)p_t^* \quad &= \quad E_t\{[a(1)a(\beta) + \lambda_1(1-L) - \lambda_1\lambda_2\beta(1-L) + \lambda_2(1-L^2) \\ &\qquad -\beta\lambda_1(F-1) + \lambda_1\lambda_2\beta^2(F-1) - \lambda_2\beta^2(F^2-1)]p_t\}, \\ &= \quad E_t\{a(1)a(\beta)p_t + d\pi_t + \lambda_2\pi_{t-1} - (d - \lambda_2(1-\beta))\beta\pi_{t+1} - \lambda_2\beta^2\pi_{t+2}\}, \end{aligned} \tag{44}$$

where the last line in (44) uses the definitions: $d \equiv (\lambda_1 + \lambda_2 - \lambda_1\lambda_2\beta)$; $(1 - L^2)p_t \equiv (\pi_t + \pi_{t-1})$; and $(F^2 - 1)p_t \equiv (\pi_{t+2} + \pi_{t+1})$.

Normalizing on the current inflation rate gives

$$\pi_t \quad = \quad \eta\pi_{t-1} + \beta(1 + \eta(1-\beta))E_t\pi_{t+1} - \eta\beta^2 E_t\pi_{t+2} + \gamma(p_t^* - p_t), \tag{45}$$

where $\eta = -\frac{\lambda_2}{d}$ and $\gamma = \frac{a(1)a(\beta)}{d}$.

A noteworthy property of this optimizing Phillips curve is that the addition of lagged inflation terms, such as π_{t-1}, also requires additional lead terms in expected inflation, such as $E_t\pi_{t+2}$, at least given the symmetry of the Euler equation format in (43).

Of course, if agents assume expected inflation in subsequent forecast periods is a random walk, $E_t\pi_{t+2} = E_t\pi_{t+1}$, then the effective optimizing equation resembles the format of the ad hoc hybrid equation (42) for $\beta = 1$,

$$\pi_t \;=\; \beta(1 - \eta\beta + \eta(1 - \beta))E_t\pi_{t+1} + \eta\pi_{t-1} + \gamma(p_t^* - p_t). \tag{46}$$

Because it is not always convenient to begin with the factor polynomial of the pricing Euler equation, such as $a(.)$ in (43), generalizations of New Keynesian Phillips curves can be directly generated also from a polynomial characterization of pricing frictions. The essential ingredient of optimal intertemporal planning is captured by assuming that firms minimize a quadratic expansion of expected profits about the optimal price path, p^*, subject to frictions on price adjustment,[11]

$$\min_p E_t\left\{\sum_{i=0}^{\infty}\beta^i[\frac{1}{2}(p_{t+i} - p^*_{t+i})^2 + \frac{1}{2}(v(L)p_{t+i})^2]\right\}, \tag{47}$$

where $v(L) = v_0 + v_1L + \ldots + v_mL^m$ is an m-order frictions polynomial. By assumption, frictions are only binding in disequilibrium, which implies $v(1) = 0$.

The FOC associated with minimizing the criterion (47) is

$$E_t\{v(L)v(\beta F)p_t\} \;=\; p_t^* - p_t. \tag{48}$$

This can be equivalently written in the price level format shown in equation (43).

Again, using the example of second-order friction polynomial, $v(L) = v_0 + v_1L + v_2L^2$, the left side of equation (48) is expanded to give

$$\begin{aligned} p_t^* - p_t &= E_t\{[v_0^2 + v_1^2\beta + v_2^2\beta^2 + (v_0v_1 + v_1v_2\beta)(L + \beta F) + v_0v_2(L^2 + \beta^2F^2)]p_t\}, \\ &= E_t\{[v(1)v(\beta) + f_1(1 - L) + f_2(1 - L^2) + f_1\beta(1 - F) + f_2\beta^2(1 - F^2)]p_t\}, \\ &= E_t\{(f_1 + f_2)\pi_t + f_2\pi_{t-1} - (f_1 + f_2\beta)\beta\pi_{t+1} - f_2\beta^2\pi_{t+2}\}, \end{aligned} \tag{49}$$

which, again, demonstrates the symmetry of coefficients on forward and lagged inflation rates. To reduce notation, the last two lines use the definitions: $f_1 \equiv -v_1(v_0 + v_2\beta)$ and $f_2 \equiv -v_0v_2$; also, the $v(1)v(\beta)$ term in the second line vanishes because $v(1) = 0$, as noted above.[12]

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[11] Alternatively, the original profit function in (28) can be included in the objective function, as in Rotemberg (1996), but this adds extra manipulations without changing essential results.

[12] In the case of a general m-order frictions polynomial, $v(L) = \sum_{i=0}^{m} v_iL^i$, the analogue to (49) is:

$$p_t^* - p_t \;=\; E_t\{\sum_{i=0}^{m-1}\pi_{t-i}(\sum_{j=i+1}^{m} f_j) - \sum_{i=1}^{m}\pi_{t+i}(\sum_{j=i}^{m}\beta^j f_j)\},$$

where the coefficients are $f_i \equiv -\sum_{j=0}^{m-i} v_jv_{j+i}\beta^j,\; i = 1, \ldots, m.$

Normalizing on the current inflation rate gives the New Keynesian Phillips curve format that is consistent with a second-order frictions specification,

$$\pi_t \quad = \quad g_0\pi_{t-1} + g_1\beta E_t\pi_{t+1} - g_0\beta^2 E_t\pi_{t+2} + g_2(p_t^* - p_t), \tag{50}$$

where $g_0 = -\frac{f_2}{f_1+f_2}$; $g_1 = \frac{f_1+f_2\beta}{f_1+f_2}$; and $g_2 = \frac{1}{f_1+f_2}$.[13] If $\beta \simeq 1$,then $g_1\beta = 1$, and a unit sum restriction on the coefficients of past and expected inflation rates is justified, even if inflation is stationary, $I(0)$.

3 Empirical evidence on alternative specifications

This section discusses some empirical tests of several assumptions and specifications used in representative optimizing models.

With regard to output equations, all optimizing policy models are simulated under the assumption that term and credit risk premiums are constant and, as noted earlier, models make various assumptions about the explicit or implicit maturity of the bond associated with the interest rate in the IS equation. By contrast, the interest rate in macro VAR models is usually the short-term nominal interest rate controlled by the central bank. Blinder and Bernanke (1993) suggest that the short-term nominal interest rate may capture also effects of imperfections in credit markets. The assumption of constant term and risk premiums is of particular interest because normative policy analyses with optimizing policy models, including those under "conservative" robust control assumptions, often suggest that monetary policy should be substantially more aggressive than historical policies in adjusting nominal interest rates to offset the effects of adverse shocks.[14]

In the case of New Keynesian Phillips curves, we have demonstrated that formulations under generalized pricing frictions will admit lagged inflation rates, without dispensing with the assumption of rational expectations. Empirical evidence reported in existing literature is very mixed on the relevance of lagged inflation rates in optimal pricing rules for aggregate price indexes, ranging from an important role in Fuhrer (1977) to a very modest influence in Gali and Gertler (1999), who suggest lagged inflation terms are "not quantitatively important." The key issue is whether or not movements of the aggregate price are adequately captured by geometric response weights. In this case, the aggregate price is sticky but the inflation rate is not, leading to the jumping inflation property of the conventional New Keynesian Phillips curve. Although previous work, such as Tinsley (forthcoming), indicates higher-order response weights and, therefore,

[13]Given that $v_0 + v_1 + v_2 = v(1) = 0$, the condition for $g_0 > 0$ is $(v_0 + v_2)(v_0 + v_2\beta) > v_0v_2$.

[14]For example, in Rotemberg and Woodford (1998), the simulated variance of the policy interest rate is larger than the historical variance by an order of magnitude.

lagged inflation rates are required to adequately explain producer price adjustments at the industry level, the explanatory role of lagged inflation rates could be diminished at higher levels of aggregation.

3.1 *Policy transmission through long-term interest rates*

As indicated earlier, one consequence of the optimizing IS equation is that the trend deviation of output responds to bond trader expectations of all future expected equilibrium deviations of real interest rates. This presumes that changes in long-term interest rates can be captured by policy model simulations of long-horizon forecasts of the short-term interest rate.

In fact, as demonstrated in Kozicki and Tinsley (2001, forthcoming), the expectations hypothesis (EH) is soundly rejected when bond trader forecasts are represented by conventional stationary or difference-stationary VAR models. This should be of some concern because linearized optimizing policy models are isomorphic to conventional VARs, albeit with restrictions on model coefficients. Because the characterization of bond trader forecasts makes a difference in the regression tests that follow, we consider three different VAR models to represent the bond trader forecast model. Details of these models are available in Kozicki and Tinsley (2001), so differences among the forecast models are only sketched here.

Each VAR contains six lags of monthly observations on: the demeaned log of US manufacturing utilization, $\tilde{y}$; the inflation rate of the PCE deflator, π; and the one-month nominal interest rate, r. The competing VAR models differ only in the specification of the equation intercepts. The VAR forecast model is cast into the companion form: $z_t = Hz_{t-1} + h$, where h is a vector that contains equation intercepts and zeros.

fixed endpoints model

If all variables in the VAR are stationary, the long-run forecast of the model is $\lim_{i\to\infty} \hat{z}_{t+i} = (I - H)^{-1}h = \bar{z}$. In long samples, the "fixed endpoints" of bond trader forecasts, $\bar{z}$, are simply the sample means of the components of z.

moving average endpoints model

If some of the variables in the VAR are difference-stationary, the long-run forecast of the model is $\lim_{i\to\infty} \hat{z}_{t+i} = \vec{H}z_{t-1} = \vec{z}_t$. Here, $\vec{H}$ is the limit of H^i as $i \to \infty$. The "moving average endpoints" of bond trader forecasts, $\vec{z}_t$, ordinarily change in each forecast period and are heavily influenced by recent observations.

shifting endpoints model

In the case of shifts in the policy target for inflation, simulated bond traders learn about these policy shifts by testing for changepoints in real-time regression models of inflation. Bond trader real-time estimates of the current policy target for inflation alter intercepts of the forecast model, h_t. Thus, for multiperiod forecasts formulated in period t, trader long-run forecasts approach $(I - H)^{-1} h_t = \bar{z}_t$. Kozicki and Tinsley (2001) show that the "shifting endpoints," $\bar{z}_t$, provide good approximations of agent long-run expectations, such as provided by surveys of 10-year forecasts.

In all three instances, finite-period forecasts of simulated bond traders are generated by $\hat{z}_{t+i} = \bar{z} + H^{i+1}(z_{t-1} - \bar{z})$, where $\bar{z}$ is replaced by $\vec{z}_t$ or $\bar{z}_t$ in the moving endpoints and shifting endpoints models, respectively.

Turning first to tests of the relevant maturity of real bond rates in an IS equation, the demeaned log of the US manufacturing utilization rate is again used to represent the trend deviation of output, $\tilde{y}$. The first three regressions reported in Table 1 compare use of a one-month real interest rate, ρ; a five-year real interest rate, ρ_{60}; and a ten-year real interest rate, ρ_{120}. In each instance, the ex ante real interest rate is measured by the historical nominal rate on the relevant constant-maturity, zero-coupon bond less forecasts of inflation rates over the maturity of the bond using the shifting endpoints VAR model.[15]

The p-values listed in the first boldface column indicate that the estimated equilibrium interest rate elasticities are strongly significant for both the 5-year and 10-year ex ante real interest rates. The elasticity of the 1-year rate is significant only for confidence levels at or below 94%. The equilibrium elasticity of the annualized 5-year rate is about .13. By equation (15), the implied estimate of the power coefficient of the utility function is $\hat{\alpha} = 2.6$.[16]

Equations 4 and 5 of Table 1 compare the contributions of the 5-year real rate against additional information in the 10-year real rate and the 1-month real rate, where the incremental information is measured by spreads from the 5-year rate. In both cases, any additional information in the 10-year or 1-month interest rates is statistically insignificant. Thus, we accept the ex ante 5-year rate as the best approximation of the long-term real rate in the output regression.

Also, it should be noted that the reverse regression format does not contradict the selection of the 5-year rate as the preferred approximation. That is, if either the 10-year or 1-month real rate is selected as the base rate and the spread of the 5-year rate over the base rate is included, the estimated equilibrium elasticity on the spread is statistically significant. Ordinarily, to condense presentation of remaining test comparisons, we will show only one comparison regression. Reverse regression formats will be shown only if inferences

[15] Yields on zero-coupon government bonds are a sample extension of those reported in McCulloch and Kwon (1993).

[16] This is an upper-bound estimate that does not adjust for the smaller share of interest-sensitive expenditures in GDP; see footnote 4 above.

are altered by the choice of the base rate regressor.

The last three equations in Table 1 compare the ex ante 5-year real rate (which uses 5-year inflation forecasts from the shifting endpoints VAR) with 5-year real rates constructed by 5-year inflation forecasts from the fixed endpoints (stationary) VAR and moving average (difference-stationary) VAR. With the exception of empirical term structure analyses in macrofinance, forecasts from stationary VARs appear to be the most common method used to generate expectations of simulated agents in macroeconomic models. Equation 6 indicates that the 5-year real rate generated by the stationary VAR is quite acceptable, although the equilibrium elasticity is about half that of the shifting endpoints VAR. However, equation 7 indicates that there is no additional information in the ex ante real rate constructed by the stationary VAR that is not already captured by the real rate constructed by the shifting endpoints VAR. The last equation in Table 1 contains the ex ante 5-year real rate generated by the moving average VAR. As indicated, the estimated equilibrium elasticity is not significant at any reasonable confidence level.

As noted earlier, standard assumptions underlying simulations of optimizing policy models are that term premiums are constant, and the interest rate effects of long-term bonds are adequately captured by long-horizon simulations. Table 2 provides several tests of the contributions of components of ex ante real rates, where ex ante real interest rates are decomposed into the averages of multiperiod forecasts of nominal rates and inflation rates, $\hat{\rho}_n = \hat{r}_n - \hat{\pi}_n$, and the residual term premium, $\phi_n = r_n - \hat{r}_n$, where r denotes historical nominal yields on constant maturity, zero coupon bonds. If the expectations of bond traders are well-replicated by the VAR forecast model, one would expect the estimated equilibrium elasticities of the two components to be statistically indistinguishable. By contrast, if the VAR simulations of short rate expectations do not closely resemble the forecasts of historical bond traders, the residual constructions of the "term premium" can contain sizeable errors due to poor replications of bond trader expectations.

The first two equations in Table 2 explore the expected real rate, $\hat{r}_{60}$, and term premium, ϕ_{60}, contributions of the 5-year ex ante real rate, again using the shifted VAR to generate 60-month forecasts of the nominal interest rate and the inflation rate. In equation 1, the equilibrium elasticities of both the 5-year expected rate and the 5-year term premium are significant, with a somewhat higher elasticity estimated for the term premium. However, the results in equation 2 demonstrate that the equilibrium elasticities are not statistically different, and we can continue to measure the ex ante real rate by adding the expected real rate and term premium components together.

The same regression tests are repeated for the expected rate and term premium components of the 5-year real rates generated by the fixed endpoints (stationary) VAR, $\rho_{60(f)} \equiv \hat{\rho}_{60(f)} + \phi_{60(f)}$. Here, the results are quite different. As shown in equation 3 of Table 2, both the expected 5-year real rate and the term premium components are significant, but the estimated equilibrium elasticity of the term premium is large and positive. The difference in equilibrium elasticities is also statistically significant at all levels of confidence.

This is an interesting result because, as demonstrated in Kozicki and Tinsley (2001), much of the movement in long-term bond rates is due to time-varying term premiums, if expected rates are generated by a stationary VAR. This is simply because long-horizon forecasts by stationary VARs approach sample means relatively quickly, so the lion's share of variation in historical bond rates is left to the residual term premium. The offsetting signs for the estimated elasticities of the expected rate and term premium components also helps explain why the estimated equilibrium elasticity of the fixed endpoints real rate, shown in equation 6 of Table 1, is lower than the estimated elasticities of real rates from the shifting endpoints VAR.

Next, there are a number of reasons, ranging from the absence of a fully-indexed tax structure to imperfect credit markets, to suspect that expenditure demands may be functions also of the level of *nominal* interest rates.[17] This possibility is explored in Table 3. Equation 1 indicates there is no additional equilibrium contribution from including also the 5-year nominal interest rate, along with the ex ante 5-year real interest rate.

A rather different story emerges in equations 2 and 3 of table 3 where the nominal 5-year nominal interest rate, r_{60}, is included with the 5-year real interest rate generated by the fixed endpoints (stationary) VAR, $\rho_{60(f)}$. As shown in equation 3, the nominal 5-year rate appears to eliminate any significant equilibrium contribution from the fixed endpoints 5-year real interest rate. Because the only difference between the two rates is the 5-year inflation prediction from the stationary VAR, this is another example of unsatisfactory properties of long-horizon predictions from stationary VARs.

The next set of equations pairs the 5-year real interest rate (generated by the shifted endpoints VAR) with the 3-year nominal interest rate, r_{36}, and the 1-month nominal rate, r_1. As with the nominal 5-year rate, there appears to be no additional equilibrium contribution from the nominal 3-year rate. However, not only is there additional information in the 1-month nominal rate (equation 5), the 1-month nominal rate also appears to dominate the 5-year real rate (equation 6).

This is a difficult result to explain on the basis of conventional theories. For example, if either of the two reasons cited above (partially-indexed taxation and imperfect credit markets) are operable, one would expect a long-maturity nominal interest component of the effective cost of borrowing to be statistically significant.

[17]One interpretation is that the household utility function is nonseparable in consumption and real money balances. This suggests real expenditures are a positive function of real money balances. Nelson (2000) finds that the trend deviation in output is significantly and positively related to *growth rates* of real base money balances. Assuming the demand for real base money is a negative function of the nominal interest rate, this suggests output trend deviations are negatively related to *changes* in the nominal interest rate.

One possible problem is that our constructions of real interest rates include term premiums on government bonds, but not private credit risk premiums. It may be possible that the 1-month nominal rate is a good proxy for cyclical movements in private credit risk spreads.[18] Equation 7 of Table 3 adds a private credit risk premium, ψ, to the ex ante 5-year real interest rate that is based on government bonds, ρ_{60}. The credit risk premium is measured by the difference between the corporate BAA rate and the yield on 5-year government coupon bonds. Equation 7 shows that the assumption that the two components share the same equilibrium elasticity is not rejected.

Finally, equation 8 of Table 3 indicates that the equilibrium response to the spread containing the 1-month nominal interest rate is not significant if the borrowing rate of the private sector is extended to include the credit risk premium, ψ. This supports the conjecture that the 1-month nominal interest rate is a proxy for cyclical movements in the private sector credit risk premium. Although ψ is the credit risk spread of investment-grade borrowers, its role complements the demonstration in Gertler and Lown (2000) that credit risk spreads for below-investment-grade borrowers are significant indicators of business cycles. In both instances, movements in private sector credit risk spreads reflect changes in lenders' estimates of the probability of future bankruptcy.

3.2 *Aggregate price adjustment and lagged inflation rates*

Perhaps the most controversial specification in the minimalist optimizing policy model is the absence of lagged inflation terms in the New Keynesian Phillips curve. This has nontrivial implications for a range of policy design issues, ranging from the lost output costs of price level targeting to the role of expectations in accelerating the effects of monetary policy. As discussed in section (2), the postulated absence of lagged inflation terms in the Phillips curve rests on the specification that the linearized FOC for optimal price behavior is a second-order difference equation.

This section provides a simple test of the assumption that lagged inflation terms do not appear in an optimizing Phillips curve. The test is based on estimation of the optimal pricing rule that is based on a linearized FOC for optimal pricing behavior, shown earlier in equation (43) and reproduced here,

$$E_t\{a(L)a(\beta F)p_t\} \quad = \quad a(1)a(\beta)p_t^*. \tag{51}$$

As demonstrated in section (2), a structural Phillips curve can be obtained from equation (51). If the factor polynomial, $a(.)$, is first-order, then no lagged inflation rate will appear in the structural Phillips curve.

[18] One reason for this conjecture is that previous work, Tinsley (1999), indicates government bond term premiums are increasing functions of the 1-month nominal rate.

If the factor polynomial is higher-order, the structural Phillips curve will contain *both* additional lags and leads of inflation. An advantage of this approach is that no additional ad hoc assumptions are required, such as abandoning assumptions of rational expectations or optimizing firms.

The construction of the equilibrium price, p^*, is similar to that in Gali and Gertler (1999), who specify that the desired price is a fixed markup over the unit cost of labor in the non-farm business sector. In our case, we use the equilibrium construction from the FRB/US quarterly model database for the price of private output, less energy, housing, and farm,[19]

$$p_t^* \quad = \quad .02pe_t + .98(w_t - \rho_{\tau,t}), \tag{52}$$

where pe is the log price of crude energy, w is the log of compensation per hour in nonfarm business, and ρ_τ is the trend of labor productivity in the nonfarm business sector.[20]

The parameters of the pricing equation in (51) are estimated following the method described in Tinsley (forthcoming) and Kozicki and Tinsley (1999). The optimal price FOC is restated in the stationary form,

$$\begin{aligned} \Delta\pi_t &= -a(1)p_{t-1} + a^*(L)\pi_{t-1} + E_t\{a(1)a(\beta)a(\beta F)^{-1}p_t^*\} \\ &= -a(1)(p_{t-1} - p_{t-1}^*) + a^*(L)\pi_{t-1} + E_t\{\sum_{i=0}^{\infty}\omega_{t+i}\Delta p_{t+i}^*\}. \end{aligned} \tag{53}$$

In the first line of (53), the m-order factor polynomial (where $m \geq 1$) is partitioned into level and difference terms, $a(L) \equiv a(1)L + (1 - a^*(L))(1 - L)$, where $a^*(.)$ is an $(m-2)$-order polynomial, $a^*(L) = a_0^* + a_1^*L + \ldots + a_{m-2}^*L^{m-2}$.[21] The second line partitions the forward path of the equilibrium price into an initial level and forward differences, π_{t+i}^*. Note that the forward weights, ω_{t+i}, are known functions of the parameters of the factor polynomial, $a(.)$. Finally, forecasts of forward changes in the equilibrium price are generated by a four-lag VAR in the two equilibrium price inputs, pe and $(w - \rho_\tau)$.

Thus, there are two sets of unknown coefficients. One contains the coefficients of the VAR used to generate forecasts of future movements in the equilibrium price. The other contains the parameters of the factor polynomial, $a(.)$. A two-step MLE is used, and sampling errors are adjusted for the "generated regressor" effect of incorporating forecasts generated by an estimated VAR.[22]

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[19]Equations in the FRB/US model are described in Brayton and Tinsley (1996) and Brayton, Mauskopf, Reifschneider, Tinsley, and Williams (1997). It should be noted that the aggregate price equations estimated for this paper are considerably simpler than the aggregate price equation in the FRB/US model, where the present aim is to focus on the lag structure of the price equation.

[20]Note that using the FRB/US estimate of trend productivity will understate the degree of smoothing required of actual prices, and will bias results against a finding of higher-order smoothing. In other words, a noisier equilibrium price requires more frictions on actual price movements to explain the observed disequilibrium price gap, $p_t - p_t^*$.

[21]The coefficients of $a^*(.)$ are moving sums of the coefficients of $a(.), a_i^* = -\sum_{k=i+2}^{m} a_k$.

[22]The estimator is discussed in Tinsley (forthcoming).

Table 4 presents estimates of the pricing equation under alternative assumptions about the degree of the factor polynomial, m. As noted in the table, the sample span is 1963Q1 - 1999Q4, and p-values are reported in parentheses.

The assumption of the purely forward-looking New Keynesian Phillips curve is that the degree of the factor polynomial, $a(.)$, in the pricing FOC is first-order, $m = 1$. This implies lagged inflation rates should not appear as regressors. As shown in the column headed by $m = 1$, the estimate of the single polynomial parameter, $a(1)$, is strongly significant with a zero p-value; the explained variation in the quarterly inflation rate is about 60%; and the mean response lag is about 6 quarters. However, there are two problems: First, as indicated by the zero p-value of the Breusch-Godfrey test, there is strong evidence of serial correlation in the estimated residuals. Second, the rational expectations overidentifying restrictions, consisting of nonlinear restrictions on the coefficients of the VAR forecast model and the parameters of the factor polynomial, are also strongly rejected, with a zero p-value.

The column headed by $m = 2$ in Table 4 lists results of the estimated pricing equation for a second-order factor polynomial. (There was no empirical support for $m > 2$.) Here, both the coefficients of the lagged price gap, $a(1)$, and the coefficient of the lagged inflation rate, a_0^*, are significant; the proportion of explained variation increases by over 40%; and the mean lag is about 7 quarters. Significantly, adding the lagged inflation rate eliminates both the residual serial correlation and the rejection of the rational expectations restrictions.

Thus, there appears to be strong empirical support for a hybrid format of the New Keynesian Phillips curve that contains a lagged inflation rate (and *two* lead inflation rates). In addition, contrary to other justifications for a lagged inflation rate, the assumption of rational expectations is maintained and not rejected by the data.

4 Concluding remarks

Whether it will retain the "new neo-classical synthesis" label or some other branding, the emergence of optimizing trend-deviation macro models that combine some of the better features of more specialized research, such as trend equilibrium models with optimizing agents, New Keynesian sticky price models, financial asset valuation models, and central bank policy feedback rules, is a welcome advance. Also, the rapid proliferation of these models to classrooms and central banks suggests the new models provide useful insights into the transmission of monetary policy.

But not always sensible policy advice. For example, in the minimalist optimizing model of section 1, the response of output to a change in the long-maturity real interest rate is immediate. Similarly, the inflation rate jumps in advance of expected future changes in output. In both cases, responses to anticipated monetary policy are powerful and swift.

Section 2 discussed three dynamic specifications that are likely to improve the realism of the minimalist three-variable model of output, inflation and the nominal rate (or four, if the real exchange rate is included, following Ball(1999)).

First, it is difficult to defend the tenacious hold of first-order difference equation specifications for output and inflation on empirical grounds. As indicated in section 2, recent empirical work suggests the empirical responses of output and inflation in the minimalist model are significantly improved by including endogenous habit in the optimizing IS equation and additional lags (and leads) of inflation rates in the New Keynesian Phillips curve. Both modifications are shown in this paper to be equivalent to use of linearized FOCs with higher-order, self-reciprocating polynomials in lag and lead operators. Evidence supporting a lag and an additional lead inflation rate in an aggregate price Phillips curve was presented in section 3.

Second, two sources of policy information are encapsulated in the Taylor-style interest rate feedback rule (or forward-looking equivalents). One source is the strength and persistence of the policy rate response to deviations of inflation from the policy target for inflation and to trend deviations of output. The persistence of the policy response is important if policy is to alter the average of expected short rates contained in the long-maturity real interest rate of the optimizing IS equation. The second source is bond trader estimates of the current policy target for inflation.

As demonstrated in section 3, the long-maturity real interest rate required by the output equation is sensitive to the forecast model used to proxy bond trader expectations. A VAR forecast model that captures learning lags in bond trader estimates of the policy target for inflation generates more plausible ex ante real interest rates than either a stationary or difference-stationary VAR in the three variables of the minimalist model.

In keeping with the linearized equations of the minimalist model, which maps to a reduced-form VAR model with restricted coefficients, a minimalist approximation of time-varying shifts in bond trader expectations of the policy target can be provided by a VAR model with shifting intercepts.

Third, the regression tests reviewed in section 3 also indicate statistically significant roles for time-varying term and credit risk premium components of the long-maturity real interest rate. We draw two conclusions from this. First, simulations of minimalist policy models that do not capture effects of the level and volatility of the policy interest rate on risk premiums are missing an important constraint on monetary policy. Only a few researchers, such as Rotemberg and Woodford (1998) and Williams (1999), suggest simulation procedures that partially compensate for this model deficiency, such as constraints on variation of simulated policy rates. Second, a more attractive long-run resolution is to incorporate theory-based estimates of endogenous risk premiums into policy models.

In principle, the addition of endogenous risk premiums need not alter the number of macro variables in the minimalist trend-deviation model. As discussed in Kozicki and Tinsley (2001), the interest rate risk premium is a function of expected conditional covariances of the policy interest rate and the pricing kernel. However, the construction of satisfactory pricing kernels based on macro variables remains unfinished business.[23]

Minimalist trend-deviation macro models are extremely useful in communicating significant issues in the design of monetary policy and in tests of empirical adequacy against properties of VARs. Moving away from the starkness of these models would diminish their usefulness in these capacities. As demonstrated in several papers by the authors, cited in sections 2 and 3, many of the modifications suggested in this paper can be formulated as variants of first-order companion matrix systems, which are tractable and maintain the desired restrictions on the number of macro variables and the linearity of model equations.

[23] Approximations of time-varying term premiums of long-maturity bond rates are illustrated in Tinsley (1999), where the log pricing kernel is a linear function of the policy interest rate.

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Table 1: Output Trend Deviations and the Maturity of the Real Bond Rate[a]

$$\tilde{y}_t = c_0 + c_1\tilde{y}_{t-1} + c_2(L)\Delta\tilde{y}_{t-1} + c_3\rho_{\text{base},t-1} + c_4(L)\Delta\rho_{\text{base},t-1}$$
$$+ \left[c_5(\rho_{\text{add},t-1} - \rho_{\text{base},t-1}) + c_6(L)\Delta(\rho_{\text{add},t-1} - \rho_{\text{base},t-1})\right] + a_t.$$

equation	interest rate[b]	c_1	c_2	c_3	c_4	c_5	c_6	R^2	SEE
1.	$\rho_{\text{base}} = \rho_1$	.964 (.00)	.569 (.00)	**-.038** **(.06)**	.052 (.59)			..98	.0082
2.	$\rho_{\text{base}} = \rho_{60}$	.956 (.00)	.510 (.00)	**-.129** **(.00)**	.296 (.23)			..98	.0078
3.	$\rho_{\text{base}} = \rho_{120}$	.962 (.00)	.520 (.00)	**-.152** **(.00)**	.256 (.40)			.98	.0079
4.	$\rho_{\text{base}} = \rho_{60}$ $\rho_{\text{add}} = \rho_{120}$	.955 (.00)	.510 (.00)	**-.109** **(.04)**	.200 (.53)	**- .062** **(.64)**	-.256 (.73)	.98	.0079
5.	$\rho_{\text{base}} = \rho_{60}$ $\rho_{\text{add}} = \rho_1$	.956 (.00)	.499 (.00)	**-.133** **(.00)**	.246 (.33)	**.029** **(.27)**	-.147 (.17)	.98	.0079
6.	$\rho_{\text{base}} = \rho_{60(f)}$[c]	.952 (.00)	.590 (.00)	**-.065** **(.00)**	.146 (.52)			.98	.0079
7.	$\rho_{\text{base}} = \rho_{60}$ $\rho_{\text{add}} = \rho_{60(f)}$	.955 (.00)	.523 (.00)	**-.127** **(.00)**	.308 (.22)	**- .009** **(.79)**	.263 (.64)	.98	.0079
8.	$\rho_{\text{base}} = \rho_{60(m)}$[d]	.969 (.00)	.563 (.00)	**-.007** **(.74)**	.052 (.92)			.98	.0081

[a]In each equation, the dependent variable is the demeaned log of the monthly capacity utilization rate of US manufacturing, $\tilde{y}$, and regressors include lags of interest rates and interest rate spreads. Consistent with a VAR(6), polynomials in the lag operator, L, indicated below are fourth-order. The sample span is 1967m1-1997m9, and parentheses contain p-values.

[b]Ex ante real bond rates are observed nominal interest rates less forecast inflation rates: 1-month real rate, $\rho_1 = r_1 - \hat{\pi}_1$; 5-year real rate, $\rho_{60} = r_{60} - \hat{\pi}_{60}$; and 10-year real rate, $\rho_{120} = r_{120} - \hat{\pi}_{120}$. Unless otherwise indicated, inflation rates are forecast by a shifting endpoints VAR in π, $\tilde{y}$, and r (see text).

[c]Five-year inflation rate forecast by fixed endpoints (stationary) VAR (see text).

[d]Five-year inflation rate forecast by moving average (difference-stationary) VAR (see text).

Table 2: Output Trend Deviations and the Term Premium[a]

$$\tilde{y}_t = c_0 + c_1\tilde{y}_{t-1} + c_2(L)\Delta\tilde{y}_{t-1} + c_3\rho_{\text{base},t-1} + c_4(L)\Delta\rho_{\text{base},t-1}$$
$$+ \left[c_5(\rho_{\text{add},t-1} - \rho_{\text{base},t-1}) + c_6(L)\Delta(\rho_{\text{add},t-1} - \rho_{\text{base},t-1})\right] + a_t.$$

equation	interest rate[b]	c_1	c_2	c_3	c_4	c_5	c_6	R^2	SEE
1.	$\rho_{\text{base}} = \hat{\rho}_{60}$	.955	.518	**-.104**	.180	**-.161**	.550	.98	.0079
	$\rho_{\text{add}} = \rho_{60}$	(.00)	(.00)	**(.05)**	(.62)	**(.02)**	(.18)		
	$\rho_{\text{add}} - \rho_{\text{base}} \equiv \phi_{60}$.								
2.	$\rho_{\text{base}} = \rho_{60}$	.955	.518	**-.104**	.180	**.057**	-.370	.98	.0079
	$\rho_{\text{add}} = \hat{\rho}_{60}$	(.00)	(.00)	**(.05)**	(.62)	**(.56)**	(.49)		
	$\rho_{\text{add}} - \rho_{\text{base}} \equiv -\phi_{60}$.								
3.	$\rho_{\text{base}} = \hat{\rho}_{60(f)}$ [c]	.969	.459	**-.477**	1.221	**.113**	.137	.98	.0078
	$\rho_{\text{add}} = \rho_{60}$	(.00)	(.00)	**(.00)**	(.13)	**(.05)**	(.63)		
	$\rho_{\text{add}} - \rho_{\text{base}} \equiv \phi_{60(f)}$.								
4.	$\rho_{\text{base}} = \rho_{60(f)}$	.969	.459	**-.477**	1,221	**-.590**	1.359	.98	.0078
	$\rho_{\text{add}} = \hat{\rho}_{60(f)}$	(.00)	(.00)	**(.00)**	(.13)	**(.00)**	(.16)		
	$\rho_{\text{add}} - \rho_{\text{base}} \equiv -\phi_{60(f)}$.								

[a] In each equation, the dependent variable is the demeaned log of the monthly capacity utilization rate of US manufacturing, $\tilde{y}$, and regressors include lags of interest rates and interest rate spreads. Consistent with a VAR(6), polynomials in the lag operator, L, indicated below are fourth-order. The sample span is 1967m1-1997m9, and parentheses contain p-values.

[b] The 5-year ex ante real bond rate is partitioned into three components: $\rho_{60} \equiv \hat{r}_{60} - \hat{\pi}_{60} + \phi_{60}$. The forecast of the 5-year, zero coupon nominal interest rate is the average of forecasts of the 1-month nominal interest rate over a 60-month horizon, $\hat{r}_{60} \equiv \frac{1}{60}\sum_{i=1}^{60}\hat{r}_{1,i}$. Similarly, the forecast 5-year inflation rate is an average of 1-month inflation forecasts over a 60-month horizon. The term premium is the residual difference between the observed 5-year nominal interest rate and the forecast nominal interest rate, $\phi_{60} \equiv r_{60} - \hat{r}_{60}$. Unless otherwise indicated, nominal interest rate and inflation rate forecasts are generated by a shifting endpoints VAR in π, $\tilde{y}$, and r (see text).

[c] Both the 5-year nominal interest rate and 5-year inflation rate are forecast by a fixed endpoints (stationary) VAR (see text).

Table 3: Output Trend Deviations and Nominal Interest Rates[a]

$$\tilde{y}_t = c_0 + c_1\tilde{y}_{t-1} + c_2(L)\Delta\tilde{y}_{t-1} + c_3\rho_{\text{base},t-1} + c_4(L)\Delta\rho_{\text{base},t-1}$$
$$+ \left[c_5(\rho_{\text{add},t-1} - \rho_{\text{base},t-1}) + c_6(L)\Delta(\rho_{\text{add},t-1} - \rho_{\text{base},t-1})\right] + a_t.$$

equation	interest rate[b]	c_1	c_2	c_3	c_4	c_5	c_6	R^2	SEE
1.	$\rho_{\text{base}} = \rho_{60}$	.952	.531	**-.124**	.329	**- .037**	.171	.98	.0078
	$\rho_{\text{add}} = r_{60}$	(.00)	(.00)	**(.00)**	(.22)	**(.21)**	(.72)		
2.	$\rho_{\text{base}} = \rho_{60(f)}$[c]	.957	.536	**-.071**	.306	**- .156**	.614	.98	.0079
	$\rho_{\text{add}} = r_{60}$	(.00)	(.00)	**(.00)**	(.25)	**(.04)**	(.36)		
3.	$\rho_{\text{base}} = r_{60}$	.957	.536	**-.071**	.306	**- .085**	.308	.98	.0079
	$\rho_{\text{add}} = \rho_{60(f)}$	(.00)	(.00)	**(.00)**	(.25)	**(.27)**	(.60)		
4.	$\rho_{\text{base}} = \rho_{60}$	.952	.546	**-.120**	.340	**.039**	.002	.98	.0078
	$\rho_{\text{add}} = r_{36}$	(.00)	(.00)	**(.00)**	(.18)	**(.17)**	(.99)		
5.	$\rho_{\text{base}} = \rho_{60}$	.954	.492	**-.115**	.496	**.052**	.209	.98	.0078
	$\rho_{\text{add}} = r_1$	(.00)	(.00)	**(.00)**	(.06)	**(.04)**	(.37)		
6.	$\rho_{\text{base}} = r_1$	.954	.492	**-.115**	.496	**.063**	.287	.98	.0078
	$\rho_{\text{add}} = \rho_{60}$	(.00)	(.00)	**(.00)**	(.06)	**(.18)**	(.37)		
7.	$\rho_{\text{base}} = \rho_{60} + \psi$	.941	.239	**-.117**	-.675	**.173**	2.979	.98	.0074
	$\rho_{\text{add}} = \rho_{60}$	(.00)	(.00)	**(.00)**	(.01)	**(.26)**	(.00)		
	$\rho_{\text{add}} - \rho_{\text{base}} \equiv -\psi.$								
8.	$\rho_{\text{base}} = \rho_{60} + \psi$	.948	.399	**-.122**	.056	**- .034**	.485	.98	.0078
	$\rho_{\text{add}} = r_1$	(.00)	(.00)	**(.00)**	(.86)	**(.19)**	(.04)		

[a]In each equation, the dependent variable is the demeaned log of the monthly capacity utilization rate of US manufacturing, $\tilde{y}$ and regressors include lags of interest rates and interest rate spreads. Consistent with a VAR(6), polynomials in the lag operator, L, indicated below are fourth-order. The sample span is 1967m1-1997m9, and parentheses contain p-values.

[b]The 5-year real bond rate, ρ_{60}, is the observed 5-year nominal interest rate, r_{60}, less the forecast 5-year inflation rate, $\hat{\pi}_{60}$. Additional nominal interest rates are: the 3-year rate, r_{36}; and the 1-month rate, r_1. The private credit risk premium, ψ, is measured by the spread between the corporate BAA interest rate and the nominal yield on 5-year Treasury coupon bonds.

[c]Five-year inflation rate forecast by fixed endpoints (stationary) VAR (see text).

Table 4: Aggregate Price Adjustment[a]

$$\Delta p_t = -a(1)[p_{t-1} - p^*_{t-1}] + a^*(L)\Delta p_{t-1} + \sum_{i=0}^{\infty} \omega_{t+i}\Delta\hat{p}^*_{t+i} + a_t.$$

response parameters[b]	m = 1	m = 2
$a(1)$	.14	.03
	(.00)	(.05)
a^*_0		.78
		(.00)
R^2	.59	.84
SEE	.0049	.0030
$BG(12)$ [c]	.00	.26
average responses		
mean lag (qtrs)	6.2	6.9
mean lead (qtrs)	5.4	6.8
RE restrictions[d]		
	.00	.40

[a]The dependent variable is the quarterly change of the log deflator of US final sales, p. The equilibrium log price, p^*, is a weighted average of the price of crude energy and unit labor costs. The sample span is 1963q1-1999q4, and parentheses contain p-values.

[b]m denotes the order of the polynomial, $a(L)^{-1}$, that generates the response weights to forward expectations, $\sum \omega_i$.

[c]Rejection probability (p-value) for the absence of residual serial correlations, Breusch-Godfrey test (12 lags).

[d]Rejection probability (p-value) of rational expectations overidentifying restrictions.