WHEN SHOULD LABOR CONTRACTS BE NOMINAL?

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Abstract

We propose a theory to explain the choice between nominal and indexed labor contracts. We find that contracts should be indexed if prices are difficult to forecast and nominal otherwise. Our analysis is based on a principal-agent model developed by Jovanovic and Ueda (1997) in which renegotiation can take place once the nominal value of the agent's output is observed. Their model assumes that agents use pure strategy, with the strong result that only nominal contracts can be written without being renegotiated. But, in reality, we do observe indexed contracts. We resolve this weakness of their model by allowing agents to choose mixed strategies, and find that the optimal contract is indeed nominal for certain parameters. For other parameters, however, we show that the optimal contract is indexed. Our findings are consistent with two empirical regularities: that prices are more volatile with higher inflation, and that countries with high inflation tend to have indexed contracts.

JEL Classification: D8, E3, J4

Keywords: Nominal Contracts; Theory of Uncertainty and Information

1 Introduction

In this paper, we build on a model proposed by Jovanovic and Ueda (1997) to explain why we observe both nominal and indexed labor contracts. We show that this explanation is consistent with the empirical evidence that indexed contracts are more prevalent in high-inflation countries than in low-inflation ones.

Jovanovic and Ueda propose an explanation for why we observe nominal contracts. They use a principal-agent model with moral hazard, renegotiation and a *signal*, which is the nominal value of the agent's sales. Real sales are directly affected by the effort level, but the price level is not. Therefore the level of information on the exerted level of effort provided by the signal depends on the price variability. The more variable the price, the less information the signal conveys. This signal is observed after the agent makes his effort decision but before renegotiation takes place. Their model assumes that the agent follows a pure strategy in choosing his effort level and they show that although the contract *could* be indexed (that is, dependent on the real amount of the agent's sales), it will in fact be nominal (in other words, dependent only on the signal). This is because indexed contracts are not renegotiation-proof.

Assume, for instance, that the principal offers an indexed contract. Since the agent follows a pure strategy, the principal can infer, in equilibrium, the effort level that the agent has chosen. At the renegotiation stage, since the effort is sunk, the principal can offer a new contract that is independent of the real amount of sales. The agent, being risk-averse, prefers such a contract. So does the principal, because it reduces her wage bill. Nominal contracts, on the other hand, are renegotiation-proof because they depend only on the signal that is observed before renegotiation takes place.

We modify Jovanovic's and Ueda's model and, as in Fudenberg and Tirole (1990), allow the agent to choose his effort level according to a mixed strategy. Under this hypothesis, indexed contracts can also be renegotiation-proof. This is because the principal can no longer infer the agent's effort level. We consider which type of contract is best for different parameter values and show that the contract will be indexed if the signal does not provide much information and nominal otherwise.

To gain some intuition for this result, first note that if the principal offers a nominal contract, the agent will follow a pure strategy, whereas if the contract is indexed, the agent will choose a mixed strategy. Hence, using the argument above, if the agent chooses the optimal level of effort with probability one, then the contract is nominal. However, the principal must compensate the agent against the risk arising from the variability of the price level. Thus the cost of a nominal contract (as compared with the standard optimal contract in a world where there is no renegotiation) is the cost of compensating the agent against the price risk. Obviously, this cost varies inversely with the quality of the signal, approaching zero if the signal is near-perfect and tending to infinity as the signal becomes completely uninformative. The cost of an indexed contract, on the other hand, (again, compared to a world without renegotiation) comes from the fact that the agent does not choose the optimal level of effort with probability one. Since the indexed contract does not depend on the signal, this cost is independent of the quality of the nominal signal. Hence, the lower the volatility of the price level, the more likely it is that a nominal contract will be preferred. Conversely, the higher the volatility, the more likely it is that the principal will offer an indexed contract.

The model also suggests a channel through which inflation has real effects. Because higher inflation implies more uncertainty, indexed contracts will be chosen. But this choice has the unfortunate consequence that expected output is lower.

Holland (1986) provides empirical evidence that directly supports the results of the model. He considers US data for the period 1961-83 and finds that the prevalence of wage indexation is positively related to lagged values of a measure of inflation uncertainty. We can look for more, indirect, evidence by considering two related questions: are prices more volatile in an economy with high inflation, and are economies with high inflation more likely to have indexed contracts? There is considerable work devoted to the first question (see Cukierman (1984) and the references therein) that show that higher inflation is accompanied by more volatile prices. There is, however, surprisingly little work that considers whether high inflation will lead to more contracts being indexed, despite the fact that this seems to be a widely held belief. For example, Azariadis (1978) states that: "Wage escalation is even more widespread in countries which have recently experienced substantial inflationary episodes", but he does not provide any evidence. To our knowledge, Holland (1995) who shows that, in the postwar United-States, increases in inflation precede increases in wage indexation has done the only work on the question.[1] In light of these observations, our model suggests that high-inflation countries tend to have indexed contracts because prices in these countries are difficult to forecast.

[1]However, there has been a large literature on the effect of indexation on inflation. See for example Dornbusch and Simonsen (1983), Fisher (1986) and the references therein.

The body of literature related to our paper is fairly small. Jovanovic and Ueda indicate that two reasons have been suggested as to why contracts are nominal: "The price level cannot be observed in time (Lucas 1972) or is costly to incorporate into a contract (Dye 1981)." We are not aware of a paper that specifically considers the circumstances under which contracts should be nominal or indexed.

Azariadis-Cooper (1985a,b) and Cooper (1990) investigate economies for which there can be equilibria with nominal contracts as well as with indexed contracts. They incorporate ideas from the implicit contract literature (Azariadis, 1975 and Baily, 1974) into an overlapping generation model. Risk-averse consumers whose consumption is risky when they are old can obtain insurance from risk-neutral speculators who can sell them nominal claims. Azariadis-Cooper (1985a) show that the market for nominal claims will be open if consumers are sufficiently risk-averse, and that the resulting equilibrium is constrained-optimal. Azariadis-Cooper (1985b) and Cooper (1990) introduce a government that prints money to finance its consumption, and show that there are multiple equilibria. In particular, there is an equilibrium with nominal contracts if government policy is not " too variable." Intuitively, this result is similar to ours. Freeman and Tabellini (1998) present a model in which nominal contracts can be optimal under certain assumptions. However, they do not discuss when indexed or nominal contracts should be preferred. There is a literature concerned with the optimal degree of indexation of contracts (Gray, 1976 and Fischer, 1977), but the form of the contract in these models is given and not derived from first principles.

The rest of the paper is organized as follows. In Section 2 we describe the model. Then, in Section 3, we present the problem of the principal. We find it convenient to consider three separate cases: the case when agents work hard with probability one, the case when they work hard with probability zero, and the case when they follow a mixed strategy. By comparing the utility of the principal in each case, we can determine the contract that will be chosen. In Section 4 we present our main theorem, and in Section 5, we discuss the conditions under which the principal wants to provide incentives for the agents to work hard with strictly positive probability. Section 6 concludes.

2 The Model

2.1 Preferences

The presentation of the model closely follows Jovanovic and Ueda (1997). We consider the case of an individual (the principal) who faces the problem of giving another individual (the agent) the incentive to make some work effort. The principal cannot directly observe the agent's effort, but can only infer it from a signal s and then the output y. The level of effort influences the level of output through the probability of its occurrence, $F_e(y) \equiv prob(\text{output} \leq y|\text{effort} = e)$.

The agent has preferences over real consumption $c \in \mathbb{R}$ and effort $e \geq 0$ which can be represented by an additively separable utility function $U(c, e) = U(c) - \Psi(e)$. The function U is strictly increasing and strictly concave and $\lim_{c\to\infty} U(c) = \infty$.[2] We impose no limits on the amount the principal can promise the agent. The function $\Psi : \mathbb{R}_+ \to \mathbb{R}_+$ denotes the cost of exerting a particular effort. Finally, we let $\underline{c}$ denote the reservation level of consumption of the agent and define $u = U(\underline{c})$ the corresponding utility level.

The principal is risk neutral, and aims at maximizing the difference between her expected revenue and her expected wage bill. We assume that the utility of the principal is zero if she chooses not to hire the agent.

2.2 Signals

The *signal* is the nominal value of the agent's sales $s = Py$; P is the price level, which is distributed according to a given probability density function. The price level is eventually known by the principal before consumption takes place. This allows for the possibility of indexed contracts. The probability density function for the price level and the function $F_e(y)$ define an implicit *conditional* density function for the price level which is non-degenerate. This density function is conditional on the signal and the effort level and we denote it by $g(P|s, e)$. We assume that the support of $g(.|s, e)$ is identical to the support of $g(.|s', e')$ for $(s, e) \neq (s', e')$. Hence, upon observing a high signal, the principal does not know if sales were good and the price was low, or if sales were poor and the price was high.

[2] Fudenberg and Tirole (1990) assumes in addition that U is defined on $(-\infty, \infty)$ as well as $\lim_{c\to-\infty} U(c) = -\infty$. These assumptions are necessary for the uniqueness of the continuation equilibrium in their model. They are not needed for the analysis in the present paper.

2.3 Contracts

We now define a contract in this environment. Let $\mathcal{Q}$ denote the set of prices, $\mathcal{S}$ the set of signals and $\mathcal{E}$ the set of efforts.

Definition 2.1. *A contract is a function* $c : \mathcal{Q} \times \mathcal{S} \times \mathcal{E} \longrightarrow \mathbb{R}$.

Let $\mathcal{C}$ denote the contract space. A contract specifies the real income received by the agent for a given signal, a given state and an announced effort level. Hence if the signal is s, the price level is $P = s/y$ and the announced effort level is $\hat{e}$, then the real income received is $c\,(P, s, \hat{e})$. Because of strict concavity, each different level of income corresponds to a unique level of utility for the agent. We can thus think of a compensation scheme as specifying a level of utility, $U(c(P, s, \hat{e}))$ for each observed signal and price, and each announced level of effort. Abusing notation, we write the level of utility as $U(P, s, \hat{e})$.

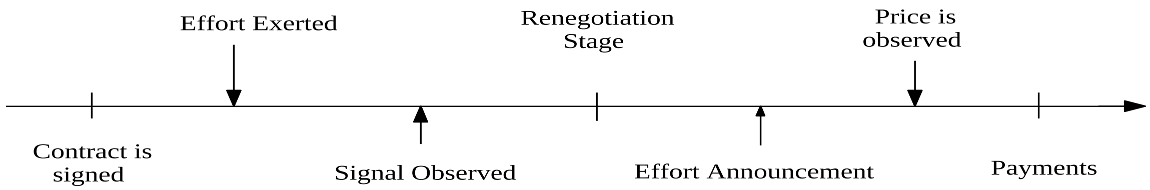
We let the principal offer contracts that depend on the announced effort level to allow her to screen the agent. We can restrict ourselves to the set $\mathcal{E}$ as the message space and assume that the agent will report his type truthfully by appealing to the revelation principle. Hence, we know that the reported effort is the actual effort, and so from now on we will write e instead of $\hat{e}$.

Definition 2.2. *We call a contract nominal if* $c(P, s, e) = c(s, e)$ *for all* P *(otherwise the contract is indexed).*

A contract is *nominal* if the compensation schemes depend only on the signal and the announced level of effort. If $c(P, s, e) \neq c(P', s, e)$, for some P and P', the compensation schemes depend also on the real outcome $y = s/P$. Given the signal, learning the level of prices is equivalent to learning the real outcome. Therefore, if $c(P, s, e) \neq c(P', s, e)$ for some P and P', we can say that the compensation scheme depends both on the signal and on the price level (i.e. on the real output level y) and we call it an *indexed* contract.

2.4 Game and Strategies

The extensive form of the game is as follows. First the principal offers the agent a contract. If it is refused, the agent receives reservation utility u and the principal cannot offer another contract; the game ends. Otherwise, the agent chooses an effort strategy and determines the effort level. The effort is made and the signal observed. The renegotiation stage starts



thereafter. At this stage, the principal can offer the agent a new contract. If the agent does not accept it, the original contract remains valid and the principal cannot offer another one. Otherwise, the new contract becomes valid. Following the renegotiation stage, the agent announces an effort level. Finally, the real outcome of the project is observed (i.e., the price level is realized) and payments are made. A strategy for the agent is a function μ with domain $\mathbb{R}_+ \times \mathcal{C}$ and range $[0, 1]$.

We restrict our analysis to renegotiation-proof contracts.

Definition 2.3. *A contract is renegotiation-proof if, given the realized signal, there is no other contract that Pareto dominates it at the renegotiation stage.*

The following result allows us to consider only renegotiation-proof contracts without any loss of generality.

Lemma 2.4. *Any equilibrium contract is payoff equivalent to a renegotiation-proof contract.*

Proof. On the contrary, suppose there is an equilibrium contract that is not payoff equivalent to a renegotiation-proof contract. By definition, there is a Pareto superior contract that can be offered by the principal to the agent at the renegotiation stage. This implies that the original contract's payoff is not an equilibrium payoff; a contradiction. □

3 The Principal's Problem

In the remainder of the paper we consider the case where there are two possible signals, H and L as well as two levels of real outcomes y_H and y_L and two levels of effort $\overline{e}$ and $\underline{e}$. We assume $H > L$, $y_H > y_L$ and $\overline{e} > \underline{e}$. The distribution functions become discrete probability measures, but we keep the same notation. Therefore, $F_e(s) \equiv prob(signal = s|e)$ for $s \in \{H, L\}$. For simplicity, we abuse notation and write $g(y_H|s, e) = g(s/y_H|s, e) \equiv prob(P = s/y_H|s, e)$. Therefore we use g interchangeably to denote the probability measure for the real output and for the price level.[3] Moreover we use $g(y|e) = \sum_s g(y|s,e)F_e(s)$ as the probability of being in state y if the agent chose a level of effort e. The cost of effort function Ψ is such that $\Psi(\underline{e}) = 0$ and $\Psi(\overline{e}) = \Psi > 0$.

To fix ideas, consider the following example. Assume the price level P can take three values, $P_1 < P_2 < P_3$ and the real amount y of sales is either y_H or y_L. Let $P_1 y_H = P_2 y_L = L$, $P_2 y_H = P_3 y_L = H$; assume also that $prob(P = P_3 \text{ and } y = y_H) = prob(P = P_1 \text{ and } y = y_L) = 0$. Hence, upon observing a high signal, the principal does not know if the amount of sales was good and the price was low or if the amount of sales was poor and the price medium.

Assumption 3.1. *$F_{\overline{e}}(H) > F_{\underline{e}}(H)$, $g(y_H|s,\overline{e}) > g(y_H|s,\underline{e})$ for $s = H, L$ and $g(y_H|\overline{e}) > g(y_H|\underline{e})$.*

The first inequality says that the high signal is more likely to occur if the agent has chosen the high effort. The second says that conditional on any signal, the high state is more likely when the agent has chosen the high effort. Finally, unconditional on the signal, the high state has a higher probability of occurrence when the agent has chosen the high effort.

We now describe the problem of the principal when she wants to induce effort with a strictly positive probability. Let μ denote the probability with which the agent chooses the high effort. We let $\mathcal{RP}$ be the set of renegotiation-proof contracts. These contracts cannot be Pareto-improved upon at the renegotiation stage. A point in $\mathcal{RP}$ is given by a probability $\mu \in [0,1]$ and of eight promised utility levels $\{U(y,s,e)\}$ for each $s \in \{H,L\}, y \in \{y_H, y_L\}$ and $e \in \{\overline{e}, \underline{e}\}$. Hence, we can write $\mathcal{RP}$ as:

$$\mathcal{RP} = \left\{ \begin{array}{c} \mu, \{U(y,s,e)\} \text{ for all } s,y,e \text{ such that:} \\ \\ \{U(y,s,e)\} \text{ for all } s,y,e, \text{ is renegotiation-proof} \end{array} \right\}.$$

We will characterize such contracts below.

The principal solves the following maximization problem (MP):

$$\underset{\mu,\{c(y,s,e)\}}{Max} \sum_{e=\overline{e},\underline{e}} \mu(e) \left\{ \sum_{s=H,L} F_e(s) \left[\sum_{y=y_H,y_L} g(y|s,e)(y - c(y,s,e)) \right] \right\}$$

[3] We can do this because given P and s we know the real output level $y = s/P$ and given y and s we also know the price level $P = s/y$.

subject to the following constraints[4]

(a) the *ex-ante* incentive compatibility constraint (IC),

(b) the individual rationality constraint (IR),

(c) the fact that μ must be a best response for the agent to $\{c(y,s,e)\}$ and

(d) the contract is renegotiation-proof.

As stated previously, the principal's objective function is the expected revenue minus the expected wage bill. The *ex-ante* incentive compatibility constraint says that the agent must get at least as much utility from choosing the high effort as he does from choosing the low effort and exactly the same utility if he randomizes. The other constraints are standard.

Instead of trying to solve this problem directly, it is useful to consider two different cases, $\mu = 1$ and $\mu \in (0,1)$. It is also useful to find the utility of the principal when she does not provide any incentives for the agent to work hard, so we will also look at the case where $\mu = 0$. As we show below, the contract takes a simple form in each case. Then, for a given set of parameters, we can compare the value the principal gets from the contract in each case and determine which one is best for her.

3.1 Case I: $\mu = 0$

Here, the principal does not provide the agent with any incentive to exert effort. Thus the wage offered is simply the reservation level of consumption $\underline{c}$, for all (y,s,e) .

The utility of the principal, denoted V_0, is given by

$$V_0 = y_L + g(y_H|\underline{e})\Delta - \underline{c},$$

where $\Delta \equiv y_H - y_L$.

This contract is renegotiation-proof.

[4]The constraints are written explicitly in the Appendix.

3.2 Case II: $\mu = 1$

Now the principal wants the agent to always exert high effort. This corresponds to the case presented in Jovanovic and Ueda.

First we derive the renegotiation-proof contract. From the renegotiation-proofness condition, it follows that $U(y, H, \overline{e}) = U(H)$ and $U(y, L, \overline{e}) = U(L)$ for all y, and $U(s, y, \underline{e}) = u$ for all s and y. The intuition is as in Jovanovic and Ueda: when $\mu = 1$, any contract that has $U(y_H, s, e) \neq U(y_L, s, e)$, for some s and e, will be renegotiated.

This contract is nominal since the utility levels offered are independent of the real amount of sales by the agent y. Nominal contracts are renegotiation-proof because the signal is observed before renegotiation takes place.

Combining the (IC) and (IR) constraints yields a unique solution to the principal's problem, given by

$$\begin{aligned} U(H) &= u + \Psi \left[\frac{1 - F_{\underline{e}}(H)}{F_{\overline{e}}(H) - F_{\underline{e}}(H)} \right], \\ U(L) &= u - \Psi \left[\frac{F_{\underline{e}}(H)}{F_{\overline{e}}(H) - F_{\underline{e}}(H)} \right]. \end{aligned}$$

Under this contract, the agent receives an amount that depends on the nominal value of his sales (the signal), but not on the real amount of those sales, and his expected utility is $u + \Psi$. Note that $U(H) - U(L)$ increases with Ψ and decreases with $F_{\overline{e}}(H) - F_{\underline{e}}(H)$.

Now we can find the principal's payoff from offering this contract. We denote it by V_1 and it is given by

$$V_1 = y_L + g(y_H | \overline{e})\Delta - W_1,$$

where $W_1 = F_{\overline{e}}(H)c(H) + (1 - F_{\overline{e}}(H))c(L)$ is the expected wage bill of the principal under this contract. $c(s)$ is such that $U(c(s)) = U(s)$ for $s \in \{H, L\}$.

The following technical lemma will be used in the proof of our main theorem. Loosely speaking, it states that, as the probability of observing the high signal conditioned on the high and low effort being exerted gets close, the wage bill of the principal under a nominal contract goes to infinity.

All proofs are in the Appendix.

Lemma 3.2. *Given $F_{\underline{e}}(H)$, $W_1 \longrightarrow \infty$ as $F_{\overline{e}}(H) \longrightarrow F_{\underline{e}}(H)$.*

3.3 Case III: $\mu \in (0,1)$

In this case, the contract offered by the principal is almost the one derived by Fudenberg and Tirole (1990), but is a little more complicated since we have a signal.[5]

We now derive the renegotiation-proof contract in this case. For a given signal, and a given μ, the form of the renegotiation-proof contract is given by Lemma 2.1 (p. 1284) in Fudenberg and Tirole, which is reproduced as Lemma 3.3 below:.

Lemma 3.3. *(FT, Lemma 2.1) Given a signal s, if $c = \{U(y_H, s, e), U(y_L, s, e)\}_{e \in \{\overline{e}, \underline{e}\}}$ is a renegotiation-proof contract for distribution μ, then*
either

$$g(y_H|\underline{e})U(y_H, s, \underline{e}) + g(y_L|\underline{e})U(y_L, s, \underline{e}) > g(y_H|\overline{e})U(y_H, s, \overline{e}) + g(y_L|\overline{e})U(y_L, s, \overline{e})$$

so that, at the renegotiation stage, the expected utility of an agent who chose low effort is strictly higher than the expected utility of an agent who chose high effort,
or
the following three conditions hold:

1. $U(y_H, s, \underline{e}) = U(y_L, s, \underline{e}) = u$,

2. $U(y_H, s, \overline{e}) \geq U(y_L, s, \overline{e})$,

3. $g(y_H|s, \underline{e})U(y_H, s, \underline{e}) + g(y_L|s, \underline{e})U(y_L, s, \underline{e}) = g(y_H|s, \underline{e})U(y_H, s, \overline{e}) + g(y_L|s, \underline{e})U(y_L, s, \overline{e})$.

so that, at the renegotiation stage, the incentive compatibility constraint is binding for agents who chose low effort.

Because an agent will never choose to work hard under the first type of contract, we consider only the second type of contract. Since the Conditions (1) to (3) must hold for

[5] Notice however that in the case where the signal in uncorrelated with the real outcome, the contract in our model is the same as in Fudenberg and Tirole (1990).

any signal s, we have in particular that $U(y, H, \underline{e}) = U(y, L, \underline{e}) = u$. To be consistent with Fudenberg and Tirole, we call condition (3) the *interim incentive compatibility* constraints. Notice that the last constraints do not include the cost of exerting high effort since it is sunk at the renegotiation stage.

When there is no signal, as in Fudenberg and Tirole, the constraint set yields a singleton, but this is not true here. For our case, four constraints must hold with equality: the interim incentive compatibility constraint, for each signal, the ex-ante incentive compatibility constraint and the individual rationality constraint. Using Lemma 3.3, we can see that the last two constraints are equivalent and we are left with:

$$
\begin{aligned}
u &= g(y_H|H, \underline{e})U(y_H, H, \overline{e}) + g(y_L|H, \underline{e})U(y_L, H, \overline{e}), \\
u &= g(y_H|L, \underline{e})U(y_H, L, \overline{e}) + g(y_L|L, \underline{e})U(y_L, L, \overline{e}), \\
\Psi + u &= F_{\overline{e}}(H)\left[g(y_H|H, \overline{e})U(y_H, H, \overline{e}) + g(y_L|H, \overline{e})U(y_L, H, \overline{e})\right] + \\
& \quad F_{\overline{e}}(L)\left[g(y_H|L, \overline{e})U(y_H, L, \overline{e}) + g(y_L|L, \overline{e})U(y_L, L, \overline{e})\right].
\end{aligned}
$$

These three equations in four unknowns give us a family of solutions that we can parameterize by one of the unknowns. We let $Z = U(y_H, L, \overline{e})$ and then express all the other utility levels in terms of Z. We get

$$
\begin{aligned}
U(y_L, L, \overline{e}) &= \frac{u}{g(y_L|L, \underline{e})} - \frac{g(y_L|L, \underline{e})}{g(y_L|L, \underline{e})}Z, \\
U(y_H, H, \overline{e}) &= \frac{1 - g(y_H|H, \underline{e})}{g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})}\chi(Z), \\
U(y_L, H, \overline{e}) &= \frac{u}{g(y_L|H, \underline{e})} - \frac{g(y_H|H, \underline{e})}{g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})}\chi(Z).
\end{aligned}
$$

The expression for $\chi(Z)$ is given in the Appendix, where we also show how to obtain μ as a function of Z. The utility levels offered in this contract depend both on the signal and on the real amount of sale. Given the nominal value of the sales, knowing the real amount is equivalent to knowing the price level. Thus we can think of this contract as depending on the signal and the level of prices. This is why we call it an indexed contract.

Again, given the form of this contract we can find the principal's payoff in this case. The problem of the principal now reduces to choosing the value of Z (denoted Z^*) that maximizes her profit.[6] Letting $V_\mu(Z^*)$ denote the maximum value of the objective function, we have

$$
\begin{aligned}
V_\mu(Z^*) =& y_L + \Delta[g(y_H|\underline{e}) + \mu(Z^*)(g(y_H|\overline{e}) - g(y_H|\underline{e}))] \\
& - \mu(Z^*)W_\mu(Z^*) - (1 - \mu(Z^*))\Phi(u),
\end{aligned}
$$

where W_μ is the expected wage bill of the principal if the agent chooses the high effort. It is given by

$$
\begin{aligned}
W_\mu &= F_{\overline{e}}(H)[g(y_H|H,\overline{e})c(y_H,H,\overline{e}) + g(y_L|H,\overline{e})c(y_L,H,\overline{e})] \\
&\quad + F_{\overline{e}}(L)[g(y_H|L,\overline{e})c(y_H,L,\overline{e}) + g(y_L|L,\overline{e})c(y_L,L,\overline{e})]
\end{aligned}
$$

where $c(y,s,e)$ is such that $U(c(y,s,e)) = U(y,s,e)$.

In this section, we have derived the contract that is offered by the principal and her payoff from that contract in three cases: $\mu = 0$, $\mu = 1$ and $\mu \in (0,1)$. Thus we can determine, for a given set of parameters, the case that gives the principal the highest utility. This in turn determines what type of contract she will offer.

It is interesting to note that the value of y_L has no effect on the type of contract the principal chooses to offer. Thus we can assume that y_L is high enough so that the principal always prefers to offer the agent a contract. Also, we will need two other lemmas for the proof of our main theorem.

Lemma 3.4. *For any $\varepsilon > 0$, there exists $\mu > 0$ small enough such that $V_\mu - V_0 < \varepsilon$.*

This lemma states that as the probability of exerting the high effort μ tends to zero, the value to the principal of offering an indexed contract tends to something equal to V_0 or lower.

Lemma 3.5. *There exists $\bar{\Delta} > 0$ big enough such that $V_1 > V_0$ for all $\Delta \geq \bar{\Delta}$.*

Δ is the gain from getting the high output over the low output. Therefore, for some high enough surplus, the principal always prefers to have the agent working hard.

[6] This Z exists as the principal maximizes a continuous objective function over a compact set.

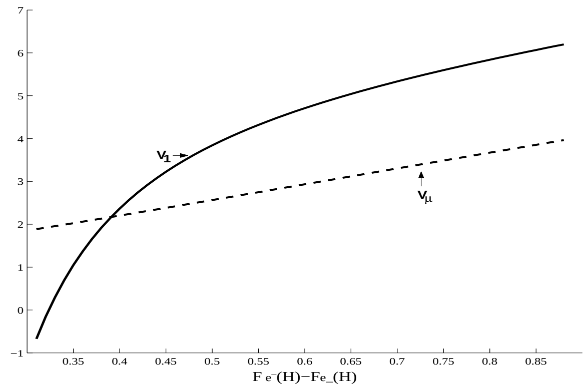


Figure 1: Value functions as $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ varies.

Notes. The parameters are: $y_H - y_L = 10$; $U(c) = log(c)$; $u = 0$; $g(y_L|L, \overline{e}) = 0.4$; $g(y_L|L, \underline{e}) = 0.9$; $\Psi = 1$; $g(y_H|H, \overline{e}) = 0.9$; $g(y_H|H, \underline{e}) = 0.4$; $F_{\underline{e}}(H)$ *is maintained fixed at 0.1 and* $F_{\overline{e}}(H)$ *is varied.*

4 Indexed versus Nominal Contracts

In this section, we show our main result: the principal will choose indexed contracts whenever the signal is not very informative about the agent's level of effort, and nominal contracts otherwise.

Let $\mathcal{P}$ denote a parameter vector $\{\underline{c}, \Psi, y_H, y_L\}$.

Theorem 4.1. *Given* $\mathcal{P}$ *and any* $\delta > 0$,

1. *Assume that* $F_{\underline{e}}(H), g(y|s, \overline{e})$, *all* y, s, $g(y_L|s, \underline{e})$, *all* s, *as well as the difference* $g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})$ *belong to* $[\delta, 1]$. *Then there is* $\epsilon(\mathcal{P}) > 0$ *such that:*
   If $F_{\overline{e}}(H) - F_{\underline{e}}(H) < \epsilon(\mathcal{P})$ *then* $V_\mu > V_1$.

2. *Assume that* $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ *belongs to* $[\delta, 1]$ *and* $V_1(\mathcal{P}) \geq V_0$. *Then there is* $\epsilon'(\mathcal{P}) > 0$ *such that:*
   If $g(y_H|H, \overline{e}) - g(y_H|H, \underline{e}) < \epsilon'(\mathcal{P})$ *then* $V_1 > V_\mu$.

Notice that under (1) above, the probabilities $F_{\overline{e}}(H)$ and $g(y|s, \overline{e})$, all y, s, $g(y_L|s, \underline{e})$, all s, and the difference $g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})$ are not necessarily fixed. As we change $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ they are allowed to change as long as they remain in the interval $[\delta, 1]$. A similar remark holds for (2). The first part of the theorem is illustrated in Figure 1.[7]

[7] The linearity of V_μ is implied by $u = 0$.

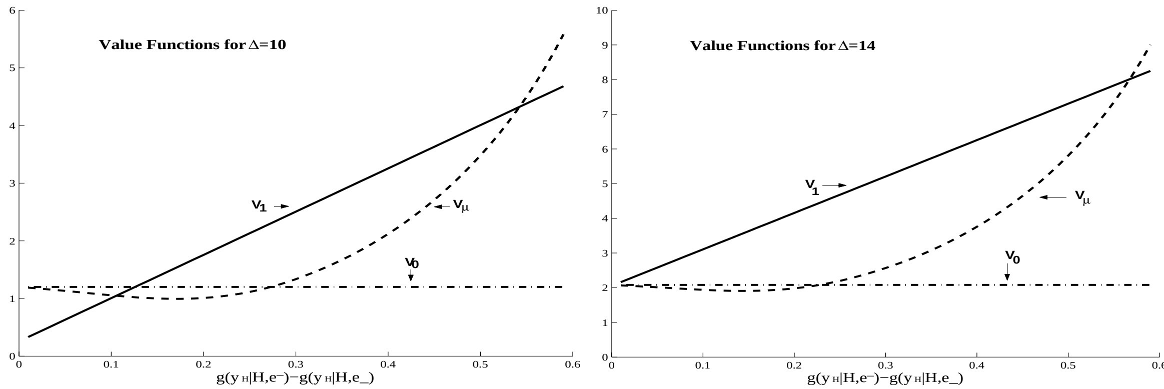


Figure 2: Value functions as $g(y_H|H,\overline{e}) - g(y_H|H,\underline{e})$ varies.

Notes. The parameters are: $y_H - y_L = \Delta$; $U(c) = log(c)$; $u = 0$; $g(y_L|L,\overline{e}) = 0.4$; $g(y_L|L,\underline{e}) = 0.9$; $\Psi = 1$; $F_{\overline{e}}(H) = 0.75$; $F\underline{e}(H) = 0.4$. $g(y_H|H,\underline{e})$ *is maintained fixed at 0.4 and* $g(y_H|H,\overline{e})$ *is varied.*

Also, for this second part of the theorem, note that we require that $V_1(\mathcal{P}) > V_0$ such that the principal would rather have the worker exerting high effort. If we did not have this requirement, the claim would not be true. The reason is that it is always possible to approximate arbitrarily closely the payoff of the no-effort contract using the indexed contract with a low probability of exerting effort. Since there always exists Δ sufficiently large such that the principal prefers the agent to work hard rather than not exert any effort, the condition $V_1(\mathcal{P}) \geq V_0$ is not a strong requirement. The discussion above as well as Theorem 4.1 are illustrated in Figure 2.

The theorem follows from a simple argument. As $F_{\overline{e}}(H) \longrightarrow F_{\underline{e}}(H)$, the cost to the principal of offering a nominal contract goes to infinity. The restrictions we impose on the other probability in our theorem assure that the cost of offering an indexed contract is bounded. As $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ gets smaller, the cost of offering the nominal contract increases, and the principal eventually prefers to choose an indexed contract. When the effort level has little effect on the realization of the signal and the agent's wage is tied to the signal only, it is costly to induce the agent to choose the right effort. To increase the incentives, the principal must increase the promised utility level for a good signal. This gets more expensive as the effect of effort on the signal gets smaller. The logic for the other case is very similar. As $g(y_H|H,\overline{e}) \longrightarrow g(y_H|H,\underline{e})$, the cost to the principal of offering an indexed contract approaches infinity, and thus the principal chooses μ smaller and smaller until eventually, the principal prefers to choose a nominal contract.

We want to show that this theorem supports the intuition that indexed contracts are used when the signal is not very informative and non-indexed contracts are used otherwise. Using Bayes rule, we write

$$F_{\overline{e}}(H) - F_{\underline{e}}(H) = \frac{F(H)}{\mu(1-\mu)}[g(\overline{e}|H) - \mu].$$

where $F(H) = \mu F_{\overline{e}}(H) + (1-\mu)F_{\underline{e}}(H)$. By definition, $g(\overline{e}|H) - \mu$ is the marginal information provided by the high signal. Hence, if this amount of information is small enough, $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ will be small and, by Theorem 4.1, indexed contracts are preferred.

Similarly, using Bayes rule we have

$$g(y_H|H,\overline{e}) - g(y_H|H,\underline{e}) = \frac{g(\underline{e},H)F(H)}{F_{\overline{e}}(H)(F(H) - g(\underline{e},H))}[g(\overline{e}|y_H,H) - g(\overline{e}|H)].$$

where $g(\underline{e},H)$ is the probability that the agent exerts the high level of effort *and* the high signal is observed. By definition, $g(\overline{e}|y_H,H) - g(\overline{e}|H)$ is the marginal information provided by the realization of the good real sales, given that the high signal was observed. If this information is very limited, the principal does not expect to learn much by observing the real output and therefore prefers to propose a nominal contract.

If prices can be forecasted perfectly, the nominal signal reveals all the relevant information. At the other extreme, the signal might be pure noise and reveal absolutely no information. We thus expect to see nominal contracts when prices are easy to forecast and indexed contracts when they are not. As mentioned earlier, the conclusions of our model are consistent with two empirical regularities: first higher inflation makes prices more difficult to forecast; second, countries experiencing high inflation tend to have more indexed contracts. Our model confirms that indexed contracts are more prevalent in high inflation countries because prices are hard to predict.

Our model also suggests a way in which inflation can have real effects. An increase in inflation may lead to a switch from nominal to indexed contracts. This in turn has an impact on the agent's effort. In our model, the expected output is lower under indexed contracts because the agent chooses high effort with a smaller probability than under nominal contracts.

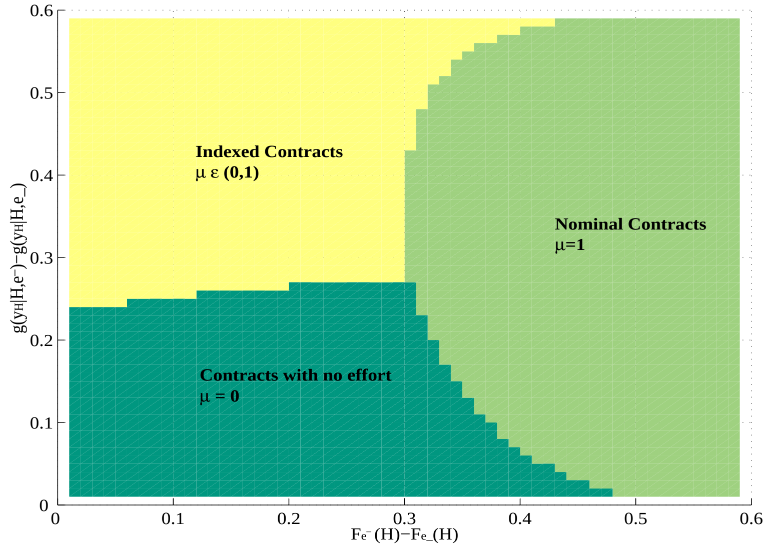


Figure 3: Choice of the Optimal Contract

Notes. The parameters are the same as in Figure 2 with $\Delta = 10$. $F_{\underline{e}}(H)$ *and* $g(y_H|H, \underline{e})$ *are maintained fixed at 0.4 and* $F_{\overline{e}}(H)$ *and* $g(y_H|H, \overline{e})$ *are varied.*

5 When Should the Agent Work Hard?

In this section, we determine parameter regions where it is optimal for the principal to induce high effort by the agent with strictly positive probability. We characterize situations where either $V_0 > max\{V_1, V_\mu\}$ or where $min\{V_1, V_\mu\} > V_0$. The most important parameters to analyze are the net gain from being in the good rather than the bad state Δ, and the cost of exerting high effort for the agent Ψ. Note here that once we know that $V_0 > max\{V_1, V_\mu\}$ or that $min\{V_1, V_\mu\} > V_0$, Theorem 1 applies so that we can find regions where $V_1 > V_\mu$ or where $V_1 < V_\mu$.

It is easy to see (the proofs are provided in the Appendix) that if Δ is very small, or if the cost Ψ of hard work is very high, the principal will not be willing to give the agent any incentive to work hard. Intuitively, if the cost of hard work is too high, it will be too expensive for the principal to elicit effort and she will choose not to do it. To summarize, we have $min\{V_1, V_\mu\} > V_0$ for Δ high enough or for Ψ small enough. Hence for these cases our theorem applies. Figure 3 illustrates our main theorem as well as the discussion above.

6 Conclusion

In this paper we explain the coexistence of indexed and nominal contracts. We modify a principal-agent model proposed by Jovanovic-Ueda (1997) with moral hazard, renegotiation and a signal (the nominal value of the agent's sales) that is observed before renegotiation takes place. Our model relaxes the Jovanovic-Ueda's assumption that agents choose pure strategies. We show that their explanation for the occurrence of nominal contracts is robust to the more general framework in the sense that for some parameters, the optimal contract is nominal in our model.

In order to keep the analysis tractable, we consider a simple model where the agent can choose from only two levels of effort; the signal takes two values, as does the real amount the agent's sales. However, we do not believe that our key results would be substantially altered by considering a more general setup.

Our findings imply that contracts should be indexed whenever the signal is not very informative and nominal otherwise. Since the signal in this model is the nominal value of the agent's sales, it will not be very informative if prices are difficult to forecast. Our result is consistent with the empirical regularities that higher inflation makes prices more difficult to forecast and that countries having high inflation tend to have indexed contracts. The theory thus suggests that because the signal will be less informative in countries with high inflation, these countries will choose to have indexed contracts. Conversely, in countries where inflation is low, the signal will carry enough information so that contracts will be nominal.

7 Appendix

In this Appendix, for convenience, we let $\Phi(U)$ be the inverse function corresponding to U, such that $\Phi(U(c)) = c$, all $c \in \mathbb{R}$.

7.1 Constraints for the Problem of the Principal

$$\begin{aligned}(IC)\quad & F_{\overline{e}}(H)[g(y_H|H,\overline{e})U(y_H,H,\overline{e}) + g(y_L|H,\overline{e})U(y_L,H,\overline{e})] \\ & + F_{\overline{e}}(L)[g(y_H|L,\overline{e})U(y_H,L,\overline{e}) + g(y_L|L,\overline{e})U(y_L,L,\overline{e})] - \Psi(\overline{e}) \\ & \geq F_{\underline{e}}(H)[g(y_H|H,\underline{e})U(y_H,H,\underline{e}) + g(y_L|H,\underline{e})U(y_L,H,\underline{e})] \\ & + F_{\underline{e}}(L)[g(y_H|L,\underline{e})U(y_H,L,\underline{e}) + g(y_L|L,\underline{e})U(y_L,L,\underline{e})]\end{aligned} \tag{7.1}$$

$$\begin{aligned}(IR)\quad & F_{\overline{e}}(H)[g(y_H|H,\overline{e})U(y_H,H,\overline{e}) + g(y_L|H,\overline{e})U(y_L,H,\overline{e})] \\ & + F_{\overline{e}}(L)[g(y_H|L,\overline{e})U(y_H,L,\overline{e}) + g(y_L|L,\overline{e})U(y_L,L,\overline{e})] - \Psi(\overline{e}) \geq u\end{aligned} \tag{7.2}$$

$$\begin{aligned}(BR)\quad & \mu(\overline{e}) \in argmax \\ & \mu\left\{F_{\overline{e}}(H)[g(y_H|H,\overline{e})U(y_H,H,\overline{e}) + g(y_L|H,\overline{e})U(y_L,H,\overline{e})]\right. \\ & \left.+F_{\overline{e}}(L)[g(y_H|L,\overline{e})U(y_H,L,\overline{e}) + g(y_L|L,\overline{e})U(y_L,L,\overline{e})] - \Psi(\overline{e})\right\} \\ & + (1-\mu)\left\{F_{\underline{e}}(H)[g(y_H|H,\underline{e})U(y_H,H,\underline{e}) + g(y_L|H,\underline{e})U(y_L,H,\underline{e})]\right. \\ & \left.+F_{\underline{e}}(L)[g(y_H|L,\underline{e})U(y_H,L,\underline{e}) + g(y_L|L,\underline{e})U(y_L,L,\underline{e})]\right\}\end{aligned} \tag{7.3}$$

$$(RP)\quad \left(\mu, \{U(y_H,s,e), U(y_L,s,e)\}_{s\in\{H,L\}}\right) \in \mathcal{RP}$$

7.2 Parametrization of the Indexed Contract

In the case where $\mu \in (0,1)$, we are left with three equations in four unknowns. These give us a family of solutions that we can parameterize by one of the unknowns. We let $Z = U(y_H, L, \overline{e})$

and then express all the other utility levels in terms of Z. We get

$$U(y_H, L, \overline{e}) = Z$$

$$U(y_L, L, \overline{e}) = \frac{u}{g(y_L|L, \underline{e})} - \frac{g(y_H|L, \underline{e})}{g(y_L|L, \underline{e})} Z$$

$$U(y_H, H, \overline{e}) = \frac{1 - g(y_H|H, \underline{e})}{g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})} \chi(Z)$$

$$U(y_L, H, \overline{e}) = \frac{u}{g(y_L|H, \underline{e})} - \frac{g(y_H|H, \underline{e})}{g(y_H|H, \overline{e}) - g(y_H|H, \underline{e})} \chi(Z)$$

$$\mu(Z) = min\{\mu_H(Z), \mu_L(Z)\}$$

$$\mu_s(Z) = \frac{\Phi'(u) \frac{g(y_H|s,\overline{e}) - g(y_H|s,\underline{e})}{g(y_H|s,\overline{e}) g(y_L|s,\overline{e})}}{\Phi'(U(y_H, s, \overline{e})(Z)) - \Phi'(U(y_L, s, \overline{e})(Z)) + \Phi'(u) \frac{g(y_H|s,\overline{e}) - g(y_H|s,\underline{e})}{g(y_H|s,\overline{e}) g(y_L|s,\overline{e})}} \text{ for } s \in \{H, L\}$$

$$\chi(Z) = cst - Z \frac{F_{\overline{e}}(L)}{F_{\overline{e}}(H)} \frac{g(y_H|L, \overline{e}) - g(y_H|L, \underline{e})}{g(y_L|L, \underline{e})}$$

$$cst = u \left[\frac{1}{F_{\overline{e}}(H)} - \frac{g(y_L|H, \overline{e})}{g(y_L|H, \underline{e})} \frac{F_{\overline{e}}(L) g(y_L|L, \overline{e})}{F_{\overline{e}}(H) g(y_L|L, \underline{e})} \right] + \frac{\Psi}{F_{\overline{e}}(H)}$$

The solution to the randomization behavior of the agent follows from a simple argument. At the renegotiation stage, once the signal has been realized, it must be the case that the principal does not want to renegotiate the contract. However, from Fudenberg and Tirole (1990) Lemma 2.2, we know that the contract will be renegotiated after signal s has been observed if the chosen randomization behavior μ is greater than μ_s - where μ_s is defined above. Therefore it must be the case that $\mu \leq min\{\mu_H, \mu_L\}$. Now, because the principal's payoff is linear in μ, the optimal solution for the principal is either at $\mu = 0$ or at $\mu = min\{\mu_H, \mu_L\}$. Since we consider the case where the principal prefers the agent to work (that is $y_H - y_L$ is sufficiently large), $\mu(Z) = min\{\mu_H(Z), \mu_L(Z)\}$ is the optimal solution for a given Z.

Therefore, the principal will just maximize his objective function with respect to Z.

Because the solution to the Fudenberg and Tirole problem is valid only if $U(y_H, s, \overline{e}) \geq U(y_L, s, \overline{e})$, we are able to obtain -using the above equalities - a lower and an upper bound on Z. Let $\overline{Z}$ and $\underline{Z}$ be respectively the upper and lower bound on Z. They are defined by the following equalities:

$$U(y_H, L, \overline{e})(\underline{Z}) = U(y_L, L, \overline{e})(\underline{Z})$$
$$U(y_H, H, \overline{e})(\overline{Z}) = U(y_L, H, \overline{e})(\overline{Z}),$$

and they have the following properties:

$$\underline{Z} = \underset{\underline{Z} \leq Z \leq \overline{Z}}{Argmax} U(y_H, H, \overline{e})(Z) - U(y_L, H, \overline{e})(Z) = \underset{\underline{Z} \leq Z \leq \overline{Z}}{Argmin} U(y_H, L, \overline{e})(Z) - U(y_L, L, \overline{e})(Z)$$

$$\overline{Z} = \underset{\underline{Z} \leq Z \leq \overline{Z}}{Argmax} U(y_H, L, \overline{e})(Z) - U(y_L, L, \overline{e})(Z) = \underset{\underline{Z} \leq Z \leq \overline{Z}}{Argmin} U(y_H, H, \overline{e})(Z) - U(y_L, H, \overline{e})(Z)$$

Furthermore, we have trivially that

$$\frac{\partial (U(y_H, H, \overline{e})(Z) - U(y_L, H, \overline{e})(Z))}{\partial Z} < 0$$

$$\frac{\partial (U(y_H, L, \overline{e})(Z) - U(y_L, L, \overline{e})(Z))}{\partial Z} > 0$$

Therefore, we have that $\mu(Z) < 1$, for all $Z \in [\underline{Z}, \overline{Z}]$. Hence, if the principal decides to set the mixing probability strictly between $(0, 1)$, the problem the principal faces is the following:

$$\underset{\underline{Z}\leq Z\leq \overline{Z}}{Max} \quad \mu(Z)\left\{g(y_H|\overline{e})y_H + (1-g(y_H|\overline{e}))y_L \right.$$

$$- F_{\overline{e}}(H)[g(y_H|H,\overline{e})\Phi(U(y_H,H,\overline{e})(Z)) + g(y_L|H,\overline{e})\Phi(U(y_L,H,\overline{e})(Z))]$$

$$\left. -F_{\overline{e}}(L)[g(y_H|L,\overline{e})\Phi(U(y_H,L,\overline{e})\,(Z)) + g(y_L|L,\overline{e})\Phi(U(y_L,L,\overline{e})(Z))]\right\}$$

$$+(1-\mu(Z))\left\{g(y_H|\underline{e})y_H + (1-g(y_H|\underline{e}))y_L - \Phi(u)]\right\}$$

subject to:

$$\mu(Z) = min\{\mu_H(Z), \mu_L(Z)\}$$

There exists a solution to this problem as we maximize a continuous function over a compact set.

7.3 Proofs

Proof of Lemma 3.2.
We can rewrite W_1 as

$$\begin{aligned} W_1 &= F_{\underline{e}}(H)\Phi(U(H)) + (1-F_{\underline{e}}(H))\,\Phi(U(L)) \\ &\quad + [F_{\overline{e}}(H) - F_{\underline{e}}(H)]\,[\Phi(U(H)) - \Phi(U(L))]\,. \end{aligned}$$

We know that $[F_{\overline{e}}(H) - F_{\underline{e}}(H)]\,[U(H) - U(L)]$ is non-negative and will show that $\Gamma \equiv F_{\underline{e}}(H)\Phi(U(H)) + (1-F_{\underline{e}}(H))\,\Phi(U(L))$ goes to infinity as $F_{\overline{e}}(H) \longrightarrow F_{\underline{e}}(H)$. Now,

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial F_{\overline{e}}(H)} &= F_{\underline{e}}(H)\Phi'(U(H))\left(-\Psi\frac{1-F_{\underline{e}}(H)}{\left[F_{\overline{e}}(H)-F_{\underline{e}}(H)\right]^2}\right) \\
&\quad + \left(1-F_{\underline{e}}(H)\right)\Phi'(U(L))\left(\Psi\frac{F_{\underline{e}}(H)}{\left[F_{\overline{e}}(H)-F_{\underline{e}}(H)\right]^2}\right) \\
&= \frac{\Psi}{\left[F_{\overline{e}}(H)-F_{\underline{e}}(H)\right]^2}\left[F_{\underline{e}}(H)\left(1-F_{\underline{e}}(H)\right)\right]\left[\Phi'(U(L))-\Phi'(U(H))\right] < 0.
\end{aligned}
$$

The last inequality holds by convexity of Φ and the fact that $U(H) > U(L)$. Also, by assumption, $F_{\overline{e}}(H) > F_{\underline{e}}(H)$, so as $F_{\overline{e}}(H)$ decreases, Γ increases at an increasing rate and tends to infinity.

□

Proof of Lemma 3.4.
From the expressions given above, we have that

$$V_\mu - V_0 = \mu\left[\underline{c} + \Delta\left(g(y_H|\overline{e}) - g(y_H|\underline{e})\right) - W_\mu\right].$$

The proof follows since $W_\mu \geq 0$ and $\underline{c} + \Delta\left(g(y_H|\overline{e}) - g(y_H|\underline{e})\right)$ is independent of μ.

□

Proof of Lemma 3.5.
We can write

$$V_1 - V_0 = \Delta\left[g(y_H|\overline{e}) - g(y_H|\underline{e})\right] + \underline{c} - W_1.$$

Notice that W_1 is independent of Δ. The lemma holds as $g(y_H|\overline{e}) > g(y_H|\underline{e})$.

□

Proof of Theorem 4.1.
(1) By Lemma 3.2, $W_1 \longrightarrow \infty$ as $F_{\overline{e}}(H) \longrightarrow F_{\underline{e}}(H)$. Our assumptions guarantee that W_μ is bounded so that for $F_{\overline{e}}(H) - F_{\underline{e}}(H)$ small enough, $V_\mu > V_1$.

(2) By Lemma 3.5 we can choose Δ big enough so that $V_1(\mathcal{P}) > V_0$ holds. As $g(y_H|H,\overline{e}) \rightarrow g(y_H|H,\underline{e})$, $U(y_H, H, \overline{e}) \rightarrow \infty$ and $U(y_L, H, \overline{e}) \rightarrow -\infty$.

Therefore, given $\Phi' > 0$ and $\Phi'' > 0$, we have that $\Phi'(U(y_H, H, \overline{e})) - \Phi'(U(y_L, H, \overline{e})) \rightarrow \infty$. Hence, $\mu_H(Z) \rightarrow 0$, for all Z, and since $\mu = min\{\mu_H(Z), \mu_L(Z)\}$, we have that $\mu(Z) \longrightarrow 0$ for all Z. Let $\varepsilon = (V_1 - V_0)/2$. From lemma 3.4 we know that as $\mu \longrightarrow 0$, $V_\mu - V_0 < \varepsilon$, so that $V_1 > V_\mu$.

□

7.4 Comparative Statics with Respect to Δ and Ψ

In this appendix, we derive some of the intuitive results formally.

We have already proved that if Δ is big enough, $V_1 > V_0$ and $V_\mu > V_0$. We also have:

Claim: There exists $\Delta > 0$ small enough such that $V_0 > V_1$ and $V_0 > V_\mu$.

Proof. Setting $\Delta = 0$, we see that the claim holds. By continuity, and because W_1 and W_μ are independent of Δ, the claim will hold for sufficiently small $\Delta > 0$. □

Claim:

There exists $\Psi > 0$ small enough such that $V_1 > V_0$ and $V_\mu > V_0$.

Proof. As $\Psi \longrightarrow 0$, we have that $U(H)$ and $U(L)$ tend to U, such that $W_1 \longrightarrow W_0$. Since $g(y_H|\overline{e}) > g(y_H|\underline{e})$, there is a Ψ low enough such that $V_1 > V_0$.

Similarly, for $V_\mu > V_0$. □

Note that with $\Psi = 0$, the principal will always choose $\mu = 1$. But even for small $\Psi > 0$, depending on the probabilities, it may be true that $V_\mu > V_1$.

Claim:

There exists $\Psi > 0$ large enough such that $V_0 > V_1$ and, for any $\varepsilon > 0$, $V_\mu - V_0 < \varepsilon$.

Proof. We have $V_1 - V_0 = \Delta\left[g(y_H|\overline{e}) - g(y_H|\underline{e})\right] + \Phi(u) - W_1$. The wage bill W_1 increases unboundedly with Ψ and thus for Ψ large enough, $V_0 > V_1$.

Also, $V_\mu - V_0 = \mu\left[\Phi(u) + \Delta\left(g(y_H|\overline{e}) - g(y_H|\underline{e})\right) - W_\mu\right]$. As Ψ increases, W_μ increases and μ decreases to zero. □

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