

A Bottleneck Capital Model of Development

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Abstract

A simple augmentation of the Ramsey-Cass-Koopmans growth model allows it to match observed transitions by initially poor economies. A high-convexity installation cost directly dampens investment demand for a first capital input. The resulting scarcity acts as a bottleneck, strongly dampening demand for investment in a complementary capital input as well. The match to observed transitions holds both for narrow and broad interpretations of capital. In either case, the bottleneck capital's share of factor income need not be large.

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1 Introduction

To what extent can neoclassical growth theory help us to understand development? The Ramsey-Cass-Koopmans framework of capital accumulation by intertemporally optimizing agents is one of a few benchmark models underlying modern macroeconomics (Ramsey, 1926; Cass, 1965; Koopmans, 1965). But a number of authors have argued that the neoclassical model implies strongly counterfactual transition paths for output, savings, and real interest rates (Christiano, 1989; Hayashi, 1989; King and Rebelo, 1993). King and Rebelo, in particular, show that this failure holds across numerous permutations of the basic model including those allowing for a broad capital factor share, investment installation costs, minimum threshold consumption levels, alternative production functions, and vintage capital effects.

In contrast, Gilchrist and Williams (2001) argue that a neoclassical model in which installed capital embodies vintage-specific technology can approximately match transition dynamics. But their result holds only for an economy that is already relatively close to its steady state. Moreover, they rely in part on an assumed gradual phase-in of improved technology.

The present paper argues that a much simpler augmentation of the neoclassical model enables it to approximately match observed transitions for an economy with initial output that is far below its steady state. The key to doing so is to allow for two complementary capital inputs, one of which is subject to an increasing marginal installation cost that is highly convex. This installation cost chokes off investment demand for the capital input to which it applies. Such low investment acts as a bottleneck, greatly diminishing investment demand for the complementary capital input as well. As a result, a moderate real interest rate suffices to equate investment and saving.

The paper proceeds as follows: Section 2 describes the observed transitions by five initially-poor high-growth economies. Section 3 lays out a neoclassical growth model augmented to include an installation cost to investment for each of two complementary capital inputs. Section 4 shows that increasing the convexity of the marginal installation cost for one of the two capital inputs enables the model to match observed transitions. A last section

briefly concludes.

2 Observed Transitions

The neoclassical growth model implies transition dynamics for several important economic variables including per worker output growth, savings, the expected real interest rate, and the marginal value of installed capital. The present section describes observed transitions of these.

Figure 1 shows per worker output growth (left-hand-side panels) and savings rates (right-hand-side panels) for five initially-poor, high-growth economies. The horizontal axes give the ratio of per worker GDP to contemporary U.S. per worker GDP. This is meant to serve as a proxy for the ratio of current to steady-state output. The dots represent the years 1960, 1970, 1980, and 1990.¹

The observed growth rates are distinguished by three key characteristics. First is that growth generally decreases as economies develop. Such decreasing growth is commonly referred to as “convergence.”² Second is that for some economies, growth is characterized by a “hump-shaped” pattern of first increasing and then decreasing growth. Third is that even for these rapidly developing economies, annual growth remains below 8%.

The observed savings rates are distinguished by two key characteristics. First is that savings generally increases as economies develop. Second is that for some economies, savings peaks at an intermediate output level and thus displays a hump-shaped pattern.

Neither capital’s expected real rate of return nor its marginal value is directly observable. Even so, time series data suggest that rates of return during development may be as high as 20%, but probably not much above this and perhaps much lower. For example, the highest

¹Underlying data for West Germany is from the Penn World Table Version 5.6 (Summers, Heston, Aten, and Nuxoll, 1994). Underlying data for remaining countries is from the Penn World Table Version 6.1 (Heston, Summers, and Aten, 2002). The cyclical component of each data series has been removed using the Hodrick-Prescott (1997) filter with a smoothing parameter of 100.

²In contrast, the “speed of convergence”—the rate at which per capita output closes the log gap to the proxy of its steady state—increases during the transitions of all five economies. For example, Taiwan’s speed of convergence increases from 3.2% in 1970, the year in which its growth is highest, to 8.1% in 1998.

smoothed return to holding a broad basket of Japanese stocks for any calendar year over the period 1914 to 2000 was 19.7%. The highest smoothed return to holding such a basket for any consecutive ten calendar years over the same period was 13%. Similarly, the highest smoothed returns to holding a broad basket of U.S. stocks for any single calendar year and for any consecutive 10 calendar years over the period 1870 to 2000 were 15.7% and 14.1%. Both the Japanese and late 19th century U.S. smoothed time series are also characterized by periods during which rates of return were rising.³

Lastly, observed average values of firms' capital suggest that plausible marginal values extend at least up to 2 and possibly somewhat higher. Average capital values are typically calculated as the ratio of a firm's equity plus liabilities to the book value of its tangible assets. Under a restrictive set of conditions, this ratio will equal the marginal value of a firm's capital (Hayashi, 1982).⁴ For a large panel of U.S. manufacturing firms over the period 1977 to 1986, Barnett and Sakellaris report median, mean, and 75th-percentile average capital values of 1.23, 1.79, and 1.95. Adjusted for tax considerations, these rise to 1.79, 2.89, and 3.21. In a rapidly developing economy with no stochastic risk, average capital values should be even higher.

The augmented neoclassical model that follows can approximately match the growth and savings transition paths shown in Figure 1. Furthermore, it implies real interest rates and marginal values of physical capital that are well within plausible bounds.

3 An Augmented Cass-Koopmans Growth Model

The theoretical framework is simply the standard Ramsey-Cass-Koopmans closed-economy setup along with an installation cost to capital investment. A first novel feature is that production requires two capital inputs. As investment in both consists of only the output good, the model retains its one-sector simplicity. A second novel feature is a generalization of the typical quadratic specification of installation costs.

³Underlying stock and inflation indices are from www.globalfinddata.com and have been smoothed using the Hodrick-Prescott (1997) filter with a smoothing parameter of 100.

⁴With multiple capital inputs, as in the model herein, these conditions are not met (Wildasin, 1984; Hayashi and Inoue, 1991).

3.1 Firms

The economy is made up of a large number of identical firms, each using constant-returns-to-scale (CRS) Cobb-Douglas technology to combine two capital inputs, K_1 and K_2 , and labor. Aggregate production and the evolution of the capital inputs are given by

$$Y(t) = AK_1(t)^{\alpha_1} K_2(t)^{\alpha_2} (L(t)e^{xt})^{1-\alpha_1-\alpha_2} \quad (1)$$

$$\dot{K}_i(t) = I_i(t) - \delta_i K_i(t) \quad i = 1, 2 \quad (2)$$

The parameters A , x , and δ_i respectively measure total factor productivity, the rate of labor-augmenting technological progress, and the rate of capital depreciation. The Cobb-Douglas exponent on each of the factors measures its share of factor income. These shares are assumed to lie between 0 and 1.

Firms maximize the net present value of future cash flows. Let $z_i \equiv I_i/K_i$ be the rate of investment in capital input i . Then

$$V(t) = \int_t^\infty \left(Y(s) - w(s)L(s) - I_1(s) \left(1 + \psi_1(z_1(s)) \right) - I_2(s) \left(1 + \psi_2(z_2(s)) \right) \right) e^{-\int_t^s r(v)dv} ds \quad (3)$$

The term $\psi_i(z_i) \equiv (b_i/(1 + \phi_i))z_i^{\phi_i}$ represents the *average* cost of installing I_i units of capital. It implies a *marginal* installation cost per unit of $b_i z_i^{\phi_i}$. The parameter ϕ_i determines the convexity of this marginal cost schedule. The parameter b_i shifts it up and down. Setting ϕ_i equal to 1 captures the case in which the marginal installation cost increases linearly with z_i . A linear increasing marginal installation cost is commonly labeled as “quadratic” in the sense that the *total* installation cost with respect to I_i is indeed so. More generally, a positive ϕ_i corresponds to a convex total installation cost or, equivalently, to an increasing marginal installation cost. Henceforth, discussion of the convexity of installation costs will refer to marginal costs. Also, I_i will be referred to as “realized investment.” Realized investment plus installation costs will be referred to as “gross investment.”⁵

⁵Introducing investment installation costs via the value function by assuming that each unit of realized investment requires more than one unit of output follows Abel (1982) and Abel and Blanchard (1983). Alternatively, installation costs can be introduced via the capital accumulation equation by assuming that

Writing firms' dynamic optimization problem in current-value Hamiltonian form and solving gives firms' desired levels of labor and realized investment. In particular, realized investment can be written as an increasing function of each capital input's marginal installed value, q_i :

$$I_i(t) = b_i^{-\frac{1}{\phi_i}} (q_i(t) - 1)^{\frac{1}{\phi_i}} K_i(t) \quad i = 1, 2 \quad (4)$$

Note that ϕ also corresponds to the reciprocal of the elasticity of realized investment with respect to the amount by which the marginal value of capital exceeds one.

The evolution of capital per normalized worker, $\hat{k}_i(t) \equiv K_i/(Le^{xt})$, and the marginal value of installed capital can then be solved by substituting (4) back into the Hamiltonian first-order conditions:

$$\dot{\hat{k}}_i(t) = \left(b_i^{-\frac{1}{\phi_i}} (q_i(t) - 1)^{\frac{1}{\phi_i}} - \delta_i - x - n \right) \hat{k}_i(t) \quad i = 1, 2 \quad (5)$$

$$\begin{aligned} \dot{q}_i(t) &= (r(t) + \delta_i) q_i(t) - \alpha_i A \hat{k}_i(t)^{\alpha_i-1} \hat{k}_j(t)^{\alpha_j} - \frac{\phi_i (q_i(t) - 1)^{\frac{\phi_i+1}{\phi_i}}}{b_i^{\frac{1}{\phi_i}} (\phi_i + 1)} \\ (i, j) &= (1, 2), (2, 1) \end{aligned} \quad (6)$$

3.2 Individuals

Individual utility and asset accumulation are given by

$$U(t) = \int_t^\infty \frac{c(s)^{1-\theta} - 1}{1-\theta} e^{-(\rho-n)(s-t)} ds \quad (7)$$

$$\dot{\text{assets}}(t) = (r(t) - n) \text{assets}(t) + w(t) - c(t) \quad (8)$$

The parameters θ , ρ , and n respectively measure the reciprocal of the elasticity of intertemporal substitution, individuals' rate of time preference, and the rate of population growth.

each unit of gross investment results in less than one unit of installed capital (Uzawa, 1969). The two approaches yield qualitatively similar results (Hayashi, 1982). Under both specifications, there are several possible accounting conventions. One is to treat installation costs as output goods that are used up during installation. Another is to treat them as the price differential between consumption and investment goods. With installation costs introduced via the value function, they can also be treated as foregone output (Treadway, 1970). Except for a few cases noted in the text, all numerical results are robust to the various accounting treatments.

Individuals' normalized consumption, $\hat{c}_i(t) \equiv C_i/(Le^{xt})$, evolves according to the standard Euler equation,

$$\dot{\hat{c}}(t) = \frac{1}{\theta} (r(t) - \rho - x\theta) \hat{c}(t) \quad (9)$$

3.3 Closed-Economy Constraints

The first key characteristic defining a closed economy is the aggregate resource constraint that absorption (consumption plus realized investment plus installation costs) not exceed output,

$$C(t) + I_1(t) (1 + \psi(z_1(t))) + I_2(t) (1 + \psi(z_2(t))) \leq Y(t) \quad (10)$$

The instantaneous real interest rate, $r(t)$, adjusts to make this hold. The second key characteristic is that individuals' asset wealth must always equal the installed value of capital,

$$\widehat{\text{assets}}(t) = q_1(t)\hat{k}_1(t) + q_2(t)\hat{k}_2(t) \quad (11)$$

3.4 Steady State

An asymptotic limit on the rate of debt accumulation implies that steady-state per capita consumption and capital stock must each grow at the rate of exogenous technological progress, x . Setting each of (9) and (5) equal to zero implies a constant steady-state interest rate, $r^* = \rho + x\theta$, and a constant steady-state marginal value of capital, $q_i^* = 1 + b_i (\delta_i + n + x)^{\phi_i}$.

These and (6) then determine steady-state normalized capital intensity,

$$\hat{k}_i^* = A^{\frac{1}{1-\alpha_i-\alpha_j}} \left(\frac{\alpha_i (1 + \phi_i)}{\tilde{b}_i} \right)^{\frac{1-\alpha_j}{1-\alpha_i-\alpha_j}} \left(\frac{\alpha_j (1 + \phi_j)}{\tilde{b}_j} \right)^{\frac{\alpha_j}{1-\alpha_i-\alpha_j}} \quad (12)$$

where j indexes the capital input complementary to k_i and \tilde{b}_i is a strictly increasing function of the installation level parameter, b_i . Note that an increase in b_i decreases the steady-state level both of the capital input to which it directly applies as well as of the complementary capital input.

4 Transition Dynamics

With a high-convexity marginal installation cost on one of the two capital inputs, the augmented neoclassical model can approximately match observed transitions. The ability to do so applies to economies with initial relative output much lower than that used in the King and Rebelo critique. It also holds for a wide range of parameterizations both of the combined share of factor income of the two capital inputs as well as of the factor share of the capital input to which the high-convexity installation cost applies.

The following subsection discusses the parameterization of the model. Section 4.2 shows the effects of applying a high-convexity marginal installation cost when there is only a single capital input. Section 4.3 applies a high-convexity marginal installation cost to one of the two capital inputs when their combined factor share is relatively narrow. Section 4.4 does the same when capital's combined share is relatively broad.

4.1 Parameterization

The parameter choices most affecting transition dynamics are the combined capital share of factor income and the convexity of marginal installation costs. Additionally, the *initial* ratio between the two capital inputs shapes dynamics during the early phase of transitions.

Remaining parameters are set at benchmark values. The depreciation rate of each capital input is set to 6%. Technological progress and population growth are each set to 2%. The reciprocal of the elasticity of intertemporal substitution is set to 1.5. And the rate of time preference is set to 3%. Together these imply a steady-state real interest rate of 6%.

Appropriate factor income shares closely depend on how the capital inputs are interpreted. For example, capital can be interpreted narrowly to include only privately-owned physical inputs to production. If so, a one-third combined share for the two inputs is consistent with national income data across a large number of developed and developing nations (Gollin, 2002). A first problem with a narrow interpretation is that a large portion of measured labor income represents the return to human capital. A benchmark estimate is that human capital's share of factor income is one-third (Mankiw, Romer, Weil, 1992). A second problem is that a narrow interpretation of capital misses some critical inputs to aggregate

production. Many of these inputs are subject to accumulation and so are akin to capital goods.

Among such capital-like inputs, an especially tangible one is public infrastructure. Since the services of public capital are rarely priced, its literal share of factor income is close to zero. But with Cobb-Douglas production, the exponent on an input also corresponds to the elasticity of output with respect to it. Estimates of U.S. tangible wealth combined with the assumption that labor's factor share is twice that of private capital suggest, via (12), an exponent on private capital of 0.31 and one on public capital of 0.08. More formal empirical research suggests that the elasticity of output with respect to public capital may be much higher (Aschauer, 1989; Kocherlakota and Yi, 1996; Fernald, 1999).

Many other capital-like inputs to production are less tangible. Human capital acquired through formal education is one of these. Other theoretical examples include learning-by-doing capital (Arrow, 1962), organization capital (Prescott and Visscher, 1980), general knowledge capital (Romer, 1986), innovation technology capital (Grossman and Helpman, 1991), and imitation technology capital (Parente and Prescott, 1994). National institutions can also function as capital-like inputs. Especially for developing economies, credibly establishing secure property rights, transparent rule-of-law, and well-functioning capital markets may require explicit and ongoing investments of resources (Hall and Jones, 1999).

The present model can approximately match observed transitions both for narrow and broad interpretations of capital. The first exercise below models just a single capital input in order to isolate the effect of allowing the marginal installation cost to be convex. The capital input is assumed to have a four-ninths share of factor income. It might be interpreted as private physical capital plus some portion of public or intangible capital. The second exercise explicitly splits this four-ninths share into α_1 equal to one-third and α_2 equal to one-ninth. The third exercise assumes that both α_1 and α_2 are equal to one-third. In this latter case, a human capital interpretation of k_2 seems especially appropriate.

No benchmark exists for parameterizing the level of the capital installation costs. To keep such costs low near the steady state, q_1^* and q_2^* are each assumed to equal 1.02. Doing so residually determines b_1 and b_2 . Qualitative results are extremely robust to alternative assumed levels. For illustrative purposes, the first exercise below also shows a transition

with q^* equal to 3.

Transition dynamics are very sensitive to the convexity of installation costs. Estimates using firm-level data on physical capital investment suggest that ϕ lies between 2 and 4 (Abel, 1980; Abel and Eberly, 2001). Also using firm-level data, Barnett and Sakellaris (1998) find that ϕ is higher at high rates of investment than at low ones. And using aggregate data, Jermann's (1998) calibration of a real business cycle model to match a set of observed moments implies ϕ to be slightly above 4. On the other hand, relaxing the assumption of perfect competition, Cooper and Ejarque (2001) estimate ϕ to be close to 1.

The convexity of installation costs for intangible investment would intuitively seem to be higher than for tangible investment. Consider human capital formation: simply devoting more resources to it (for example, expanded student enrollment, more time devoted to learning, more and fatter textbooks) beyond some point probably does not result in much additional human capital.

For the exercise below in which there is only a single capital input, transition dynamics are shown for ϕ equal to 1 and to 6. Note that the latter is higher than can be justified empirically. In the second and third exercises, ϕ equal to 6 will apply only to investment in the second capital good. To give a sense of magnitude, by assumption a marginal installed capital value of 1.02 elicits a 10% rate of realized investment regardless of ϕ . Eliciting a 20% rate requires respective marginal values of 1.04 and 2.28 for ϕ equal to 1 and to 6.

A final parameterization choice concerns the initial ratio of k_2 to k_1 . The steady-state ratio between the two is determined by (12). But for any initial relative output, there is a continuum of relative capital intensities, $\tilde{k} \equiv (k_2/k_1)/(k_2^*/k_1^*)$. Numerical solutions show that for any assumed initial value, \tilde{k} quickly converges to a "neutral" transition path. With identical installation costs for the two capital inputs, \tilde{k} equals 1 everywhere along its neutral path. In this case, transitions along the neutral path are equivalent to transitions when there is just a single capital input. But for the scenarios below in which $\phi_2 > \phi_1$, the neutral \tilde{k} path lies below 1. Because it has a higher marginal installation cost, k_2 tends to be relatively more scarce during development. For these scenarios, transitions are shown for initial \tilde{k} both equal to and below its neutral value.⁶

⁶The paper's supplemental materials show transitions for unitary initial capital ratios.

4.2 Single Capital Input

Numerical results from specializing production to a single capital input isolate the effect of the installation-cost convexity. Figure 2 shows dynamics as an economy transitions from initial output that is 20% its steady-state level. Two of the sets of transition paths maintain the standard convexity assumption of ϕ equal to 1 while assuming alternative installation cost levels corresponding to q^* equal to 1.02 and q^* equal to 3. The third set of transition paths is characterized by the lower of these installation-cost levels along with an installation-cost convexity of ϕ equal to 6.

There are three main problems with the standard implementation of the neoclassical model. First is that it implies implausibly high transitional growth. With q^* equal to 1.02 and ϕ equal to 1, initial per capita output is 41% (Panel A). Second is that it counterfactually implies a downward-sloping transition path for savings (Panel B). Third is that it implies an implausibly high return to capital during the transition. The real interest rate is 70% initially and remains above 20% until relative output has attained more than half its steady-state value (Panel C).

As shown by King and Rebelo, increasing the level of the installation cost does not solve these problems. Assuming q^* to equal 3 rather than 1.02 corresponds to a hundred-fold increase in the level of installation costs (as measured by the parameter b). Nevertheless, the respective initial growth and interest rates of 14.3% and 25.4% remain implausibly high, and the savings transition path remains downward sloping. Additionally, the initial capital marginal value of 8.5 seems unlikely.

The underlying ineffective dampening of investment demand arises because as the cost parameter b increases, so too does the marginal benefit of investment. Installation costs are proportional to I/K . Higher current investment increases the future denominator and so lowers future installation costs. Such lower future costs at least partly offset higher current ones. In some cases, they can completely do so. Under the present parameterization, initial growth asymptotes down only to 11% as b goes to infinity.

In contrast, increasing the convexity of the installation cost strongly dampens investment demand. Savings and investment can then be equated at a moderate real interest rate. With q^* equal to 1.02 and ϕ equal to 6, initial per capita output growth of 9.1% is only

slightly high relative to observed transitions. The savings transition path is now upward sloping, and the initial interest rate of 15.9% is well within the range of observed values. The underlying strong dampening of investment demand arises because realized marginal installation costs decline steeply as the transition proceeds (Panel D). Hence there is a big incentive to delay investment. Increasing a single parameter, ϕ , thus allows the neoclassical model to approximately match observed transitions for three key macroeconomic variables.

Nevertheless, there are two main drawbacks to the high-convexity parameterization. First is that the 7.3 initial marginal value of installed capital is probably too high.⁷ Second is that the degree of convexity needed to obtain the approximate match is itself higher than can be justified by empirical estimates. A production function that allows for two independent capital inputs, only one of which is subject to a high-convexity installation cost, resolves both of these concerns.

4.3 Narrow Combined Capital Share of Income

With two independent capital inputs, a high-convexity marginal installation cost on one acts like a bottleneck, slowing investment in both. This is true even when the high-convexity cost applies to capital receiving a relatively small share of factor income.

Figure 3 shows transition dynamics for an economy in which a first capital input receives one-third of factor income and a second capital input receives just one-ninth of it. For expositional purposes, K_1 will be referred to as “tangible capital” and K_2 , as “intangible capital.” Such labels are not meant to preclude alternative interpretations. Under the assumed factor income shares, K_2 might correspond to a physical input such as government infrastructure or to some sort of organization capital.

Transition paths are shown for three different assumptions regarding the convexity of installation costs and the *initial* ratio of intangible-to-tangible capital, $\tilde{k}(0)$. Under all three sets of assumptions, the convexity of tangible capital’s installation cost, ϕ_1 , is held fixed at 1. In the first scenario, ϕ_2 is also assumed to equal 1 as is $\tilde{k}(0)$. The scenario thus collapses to the case of a single capital input with a four-ninths income share (which is one of the

⁷But at the 38% initial relative output assumed by King and Rebelo, the marginal value of capital is only 2.9.

low-convexity parameterizations in the previous subsection). Under the second and third scenarios, ϕ_2 is assumed to equal 6. For scenario two, $\tilde{k}(0)$ is assumed to be neutral. As discussed above, this lies along the transition path to which all initial capital ratios converge. At 20% relative output, the neutral \tilde{k} turns out to equal 0.007. In scenario three, intangible capital is assumed to be especially scarce: $\tilde{k}(0)$ is set equal to 0.001. As shown in Panel A, \tilde{k} converges to its neutral path by the time relative output has risen to 40%. The dynamics implied by scenarios two and three thus differ only during the initial phase of the transition.

A high-convexity marginal installation cost that applies only to intangible capital slows growth nearly as effectively as one that applies to all capital. The 10.6% initial growth rate under scenario two is only slightly higher than the 9.2% initial growth rate under the high-convexity parameterization in the previous subsection. By directly choking off intangible capital investment, a high ϕ_2 indirectly dampens tangible capital investment. Scenario two also achieves a slightly upward-sloping savings transition profile (Panel C), a plausible initial real interest rate (Panel D), and a low initial marginal value of tangible capital (Panel E).

A potential drawback of a high ϕ_2 is that it implies an extremely high initial marginal value of intangible capital, though this drops sharply as the economy develops. With the neutral assumed \tilde{k} of scenario two, initial q_2 exceeds 300. It then falls to 26 at 40-percent relative output and further to just below 5 at 60-percent relative output (Panel F).

There are several reasons for thinking that some capital inputs may, indeed, have extremely high marginal values in a rapidly developing economy. If k_2 is interpreted as infrastructure and other public capital, then its price will not be directly observable. Numerous examples from developing countries of grid-locked traffic, telecommunications and utility breakdowns, corruption, and poorly functioning capital markets suggest that the marginal value of public capital may exceed its materials cost by an order of magnitude. Alternatively, if k_2 is interpreted as organization capital, then its price may be reflected only in a market value that is averaged together with k_1 . Under scenario two, the initial average value of

capital— q_1 and q_2 respectively weighted by their current levels, k_1 and k_2 —is just 1.79.⁸

A different reason for thinking that an extremely high initial q_2 may be plausible follows from interpreting installation costs as arising from a concave transformation between investment goods and consumption goods (Kim 2001). Under this interpretation, the ratio q_2/q_2^* measures the normalized relative *price* of intangible capital goods in terms of consumption goods. In a closed economy, numerous capital-like goods are likely to have extremely high prices during the earliest phases of development. For example, many developing countries lack a domestic supply of tertiary education on specialty subjects. More generally, capital goods that become available only as an economy develops have infinite initial price.

Two additional problems characterize the transition paths under scenario two. First is that the initial growth rate of 10.6% remains high relative to observed growth rates. Second is that the upward sloping savings profile is fragile to how savings is measured. Under many interpretations of intangible capital, investment in it is unlikely to be accounted as such in the national income accounts. The savings required to finance intangible investment may be labelled as consumption or take the form of foregone output. Proxied by gross investment in just tangible capital, the savings profile implied by scenario two is flat.

Allowing for some extra initial scarcity of intangible capital, as under scenario three, solves both of these latter problems. Doing so puts even more downward pressure on initial investment demand thereby lowering the initial growth, savings, and interest rates. The initial relative scarcity of intangible capital corresponds to an initial relative abundance of tangible capital. As a result, q_1 begins the transition from below its steady state. The associated low initial demand for tangible capital investment more than offsets the high initial demand for intangible capital investment. Initial growth is just 7.6%, and an initial interest rate of 10.4% suffices to equate savings and investment.⁹ As the high rate of intangible

⁸This weighted average value of capital measures the ratio of the net present value of future cash flows relative to depreciated realized investment. If a firm owns all of its productive inputs and book value is based on realized investment, it is equivalent to the sum of the firm's equity and liabilities relative to its book value.

⁹The relationship between initial \tilde{k} and initial growth is nonmonotonic. Numerical results suggest that for a given relative output level, there is a unique \tilde{k} at which growth is minimized. The supplemental figures show an example in which $\tilde{k}(0)$ is assumed to be below this growth-minimizing level.

capital investment makes tangible capital less relatively abundant, overall investment demand increases thereby requiring a rise in real interest rates. A rising interest rate, in turn, robustly underpins an upward-sloping savings transition profile. Unsurprisingly, increasing the initial scarcity of intangible capital also increases its marginal installed value. Under scenario three, initial q_2 exceeds 1500. As discussed immediately above, the plausibility of such a high value depends on the interpretation of intangible capital.

Overall, a high-convexity marginal installation cost that applies to only a small portion of capital can approximately match observed transitions of growth, savings, and the real interest rate while maintaining a low marginal value for the remaining larger portion of capital. But this combination also implies an extremely high initial marginal value of the bottleneck capital. The next subsection shows that with a broader combined factor share for the two capital inputs, the initial marginal value of the bottleneck capital can itself remain moderate.

4.4 Broad Combined Capital Share of Income

A number of authors have argued that increasing capital's share of factor income suffices to allow the the neoclassical model to match observed transitions (e.g., Mankiw, Romer, Weil, 1992; Barro and Sala-i-Martin, 1995). While doing so does ameliorate the model's calibration problems, nevertheless a high-convexity marginal installation cost on at least a small portion of capital is still required to completely solve them. With two capital inputs, the benefit of a broader combined factor share is that it substantially lowers the initial marginal value of the bottleneck capital.

Figure 4 shows transition dynamics for an economy in which a “tangible” and an “intangible” capital input each account for one-third of factor income. The tangible capital input, K_1 , is assumed to have a linear marginal installation cost. Transition paths are shown for the same three scenarios of the previous subsection: ϕ_2 and $\tilde{k}(0)$ equal to 1, ϕ_2 equal to 6 and a neutral $\tilde{k}(0)$, and ϕ_2 equal to 6 and initially scarce intangible capital. For the second scenario, the neutral $\tilde{k}(0)$ turns out to equal 0.55 (Panel A). For the third scenario, $\tilde{k}(0)$ is assumed to equal 0.11.

The first scenario collapses to the case of production with a single capital input that

accounts for two-thirds of factor income. Two problems characterize its transition dynamics. First is that the initial per capita output growth of 11.1% exceeds the highest observed growth rate by more than 3 percentage points (Panel B). Second is that the savings transition profile is downward sloping (Panel C). Additionally, the initial real interest rate of 20% is at the upper bound of plausibility (Panel D).¹⁰

Increasing ϕ_2 to 6, along with a neutral $\tilde{k}(0)$, considerably dampens demand for investment in both types of capital. The initial per capita output growth and real interest rates respectively fall to 7.8% and 14%. The initial tangible capital marginal value is only 1.03 (Panel E). The initial intangible capital marginal value of 2.5 is easily plausible (Panel F). Lastly, the savings profile becomes upward sloping, though this lattermost result is fragile to alternatively measuring savings by gross investment in only tangible capital.

Allowing for some initial scarcity of intangible capital robustly aligns modeled dynamics with observed ones. A hump-shaped transition growth profile increases from 5.2% to 6% before falling off.¹¹ The savings profile slopes upward, even when measured by only tangible gross investment. Intangible capital's initial marginal value of 8.8 is probably plausible. In addition, a hump-shaped interest rate profile first increases from 3.3% to 10.4% before falling off to its steady-state value of 6%.

Virtually identical growth, savings, and interest rate transition paths to those in scenarios two and three are implied by maintaining the same two-thirds combined capital share and letting α_2 equal one-ninth. Quite similar ones are implied by maintaining the combined capital share and letting α_2 equal only 1/27. Conversely, applying the high-convexity installation cost to the entire two-thirds capital share implies transition paths for these variables that are identical to those of scenario two.¹² Such results illustrate that it is primarily the

¹⁰With a linear installation cost on a single capital input along with a two-thirds capital factor share, the minimum steady-state capital marginal value for which growth is below 8% is 1.56. With a steady-state marginal capital value of 1.02, the minimum capital factor share for which initial per capita output growth is below 8% is 74%. In both cases, the savings profile is essentially flat.

¹¹The initially increasing growth rate no longer holds when installation costs are assumed to take the form of forgone output. During the initial phase of the transition, tangible investment rapidly increases as do the associated installation costs. The latter increasingly depress measured output, and so measured growth does not accelerate.

¹²Illustrations are included in the paper's supplemental materials.

highest-convexity installation cost, rather than some average of all installation costs, that determines transition dynamics.

The trade-off from decreasing the factor share of the bottleneck capital is an increase in its initial marginal value. With a neutral initial \tilde{k} and alternatively assuming that α_2 equals one ninth and $1/27$, initial q_2 respectively equals 7.6 and 34.2. With sufficient initial k_2 scarcity to imply dynamics similar to those of scenario three, initial q_2 respectively equals 63 and 976. Once again, the plausibility of such high marginal values is not clear. What does seem clear is that for a given marginal installed value, plausibility increases as the bottleneck capital's factor share becomes smaller.

5 Conclusions

A simple augmentation of the Ramsey-Cass-Koopmans growth model allows it to approximate transition dynamics for economies that are initially far below their steady-state output. A high-convexity installation cost directly dampens investment demand for a first capital input. The resulting scarcity acts as a bottleneck, strongly dampening demand for investment in a complementary capital input as well. Growth is thus slowed, and savings and investment can be equated at a moderate real interest rate. As the economy develops, the savings rate increases and the marginal installed value of the complementary capital input remains low.

The strong dampening of investment demand holds even when the bottleneck capital's share of factor income is quite low. Similarly, it holds for a wide range of parameterizations of the combined factor share of the two capital inputs. For some parameterizations, the bottleneck capital's initial marginal value can be extremely high. The plausibility of such high values is unclear since they may not be directly observable. Under some interpretations, bottleneck capital inputs may be unpriced. Under other interpretations, their value may be observed only when averaged with the value of complementary capital inputs. Under still other interpretations, bottleneck capital inputs may not be available at any price during the early phases of development.

Identifying bottleneck capital inputs stands out as an important priority for future re-

search. An especially tangible class of potential bottlenecks is publicly financed or regulated infrastructure. Examples include transportation facilities; electrical generation and distribution; water, sewer, and solid waste disposal systems; and telecommunications and postal systems. Some less tangible potential bottlenecks are institutions from which intermediate-input services flow. Examples of such services include the enforcement of property rights, the allocation of credit, and the provision of insurance. Within firms, bottlenecks may arise from many forms of organization capital. For example, increasingly capital intensive production may require significant on-the-job learning and a continual rematching of workers to tasks. For individuals, human capital formation surely exhibits rapidly diminishing returns to investment above some level. Even the brightest, hardest-working students can learn only so fast.

Lastly, the bottleneck model underscores the blurred distinction between capital accumulation and productivity growth. Total factor productivity growth is typically measured as the residual of output growth after accounting for the accumulation of physical and human capital. But many residual sources of growth can themselves be thought of as subject to accumulation. Thus an additional interpretation of the bottleneck capital input is that it represents “TFP capital.” Indeed, several of the scenarios discussed above are characterized by dynamics that would seem to indicate rapidly increasing TFP. For instance, output growth accelerates, and both the real interest rate and marginal installed value of tangible capital rise. Although interpreting bottleneck capital as TFP capital gives only limited insight into the sources of total factor productivity, nevertheless it illustrates the flexibility of the neoclassical growth framework to incorporate even poorly understood aspects of development.

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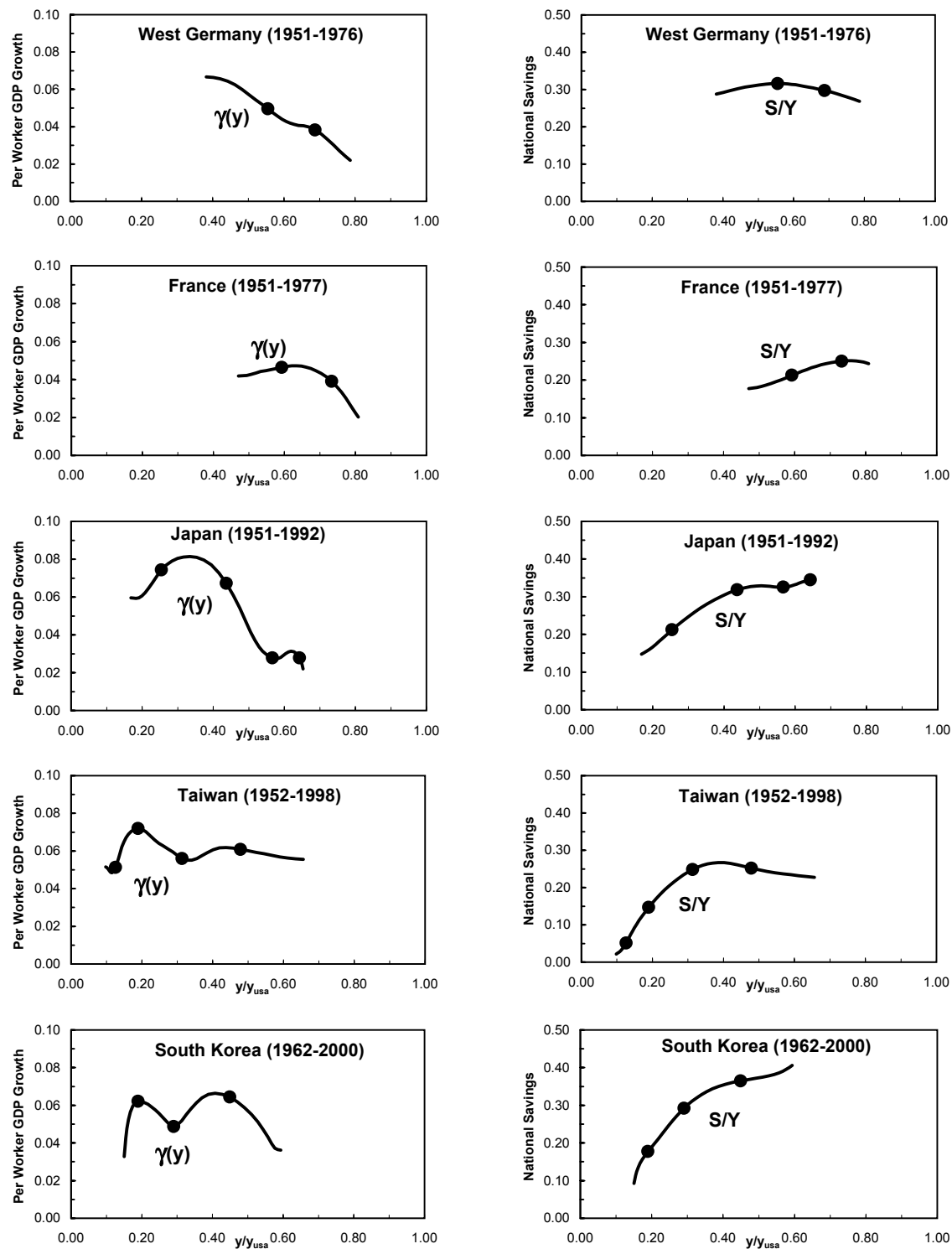
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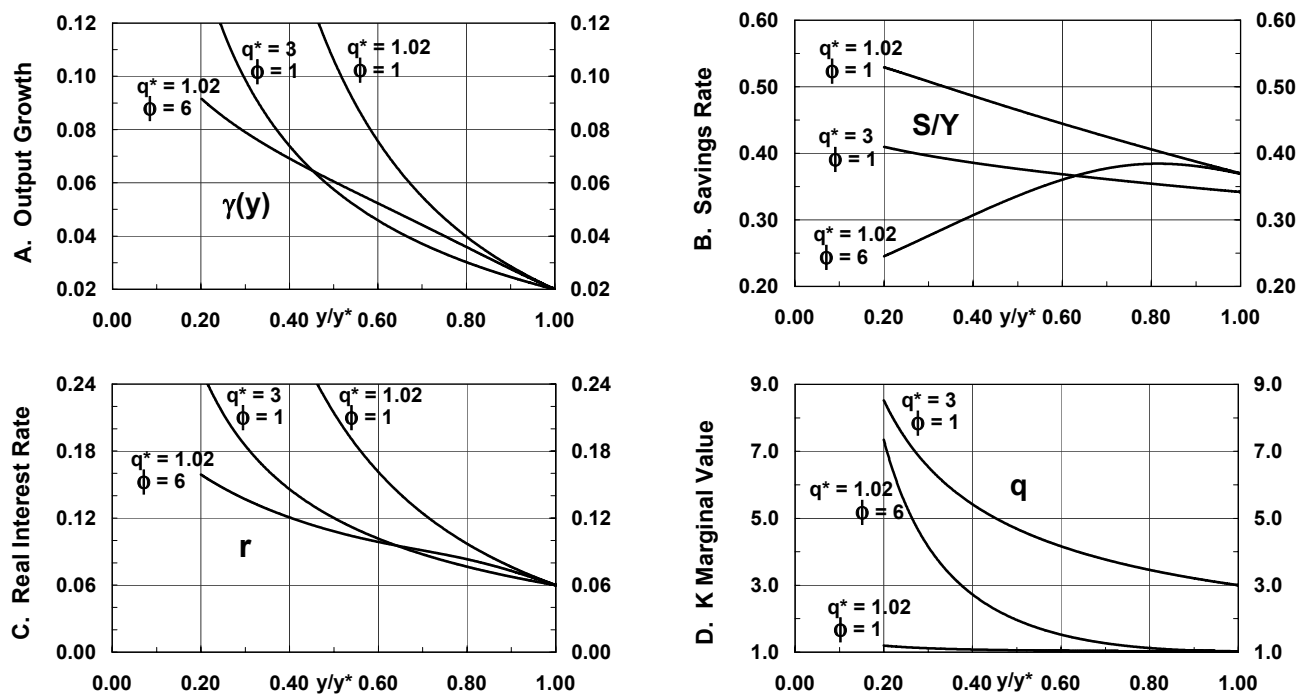
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Figure 1: Transition Dynamics for Selected Countries



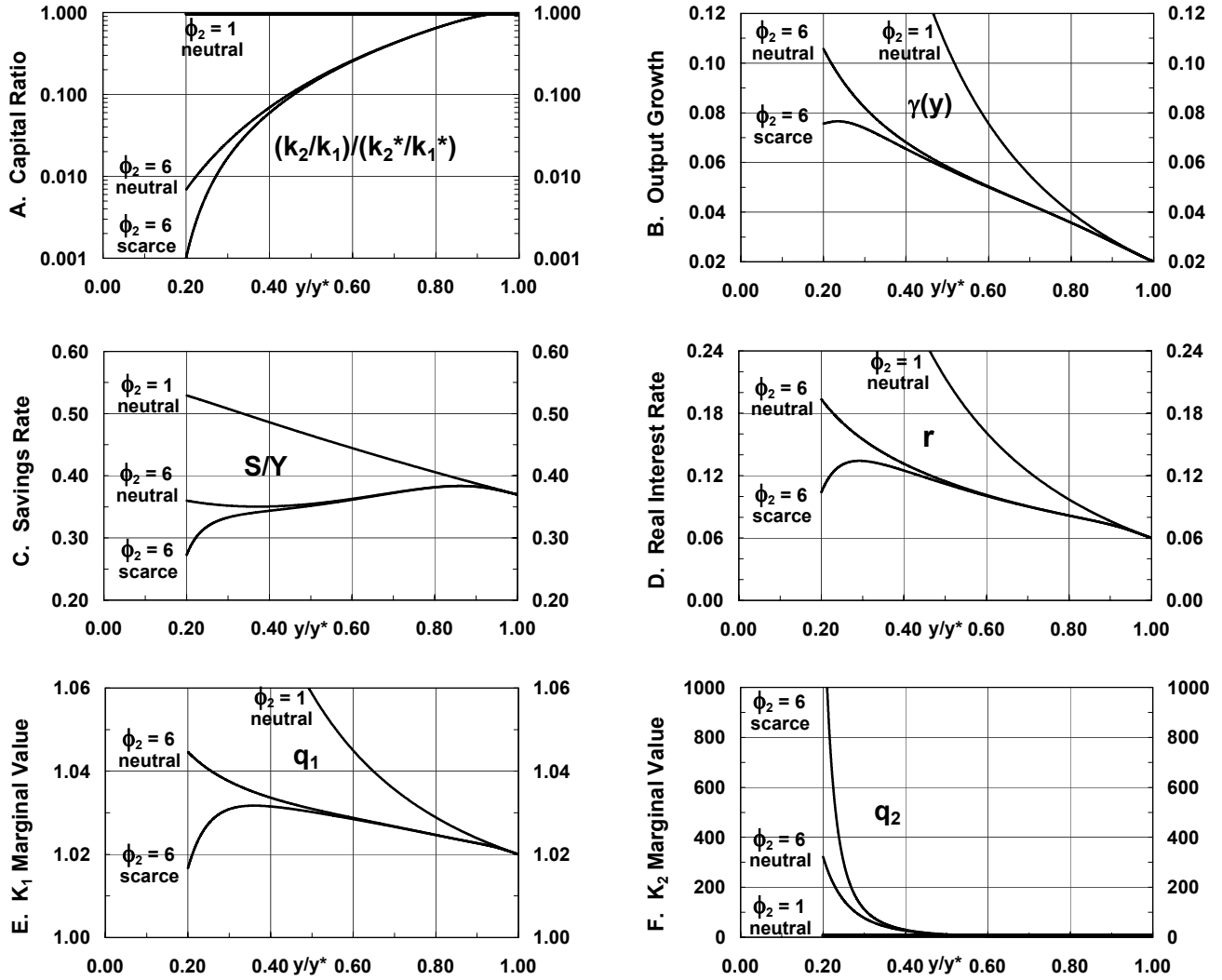
Dots represent the years 1960, 1970, 1980, and 1990. Underlying data is from the Penn World Table Version 5.6 (for West Germany) and Version 6.1 (remaining countries) (Summers, Heston, Aten, and Nuxoll, 1994; Heston, Summers, and Aten, 2002). The cyclical component of each data series has been removed using the Hodrick-Prescott filter (Hodrick and Prescott, 1997) with the smoothing parameter set equal to 100.

Figure 2: Single Capital Input



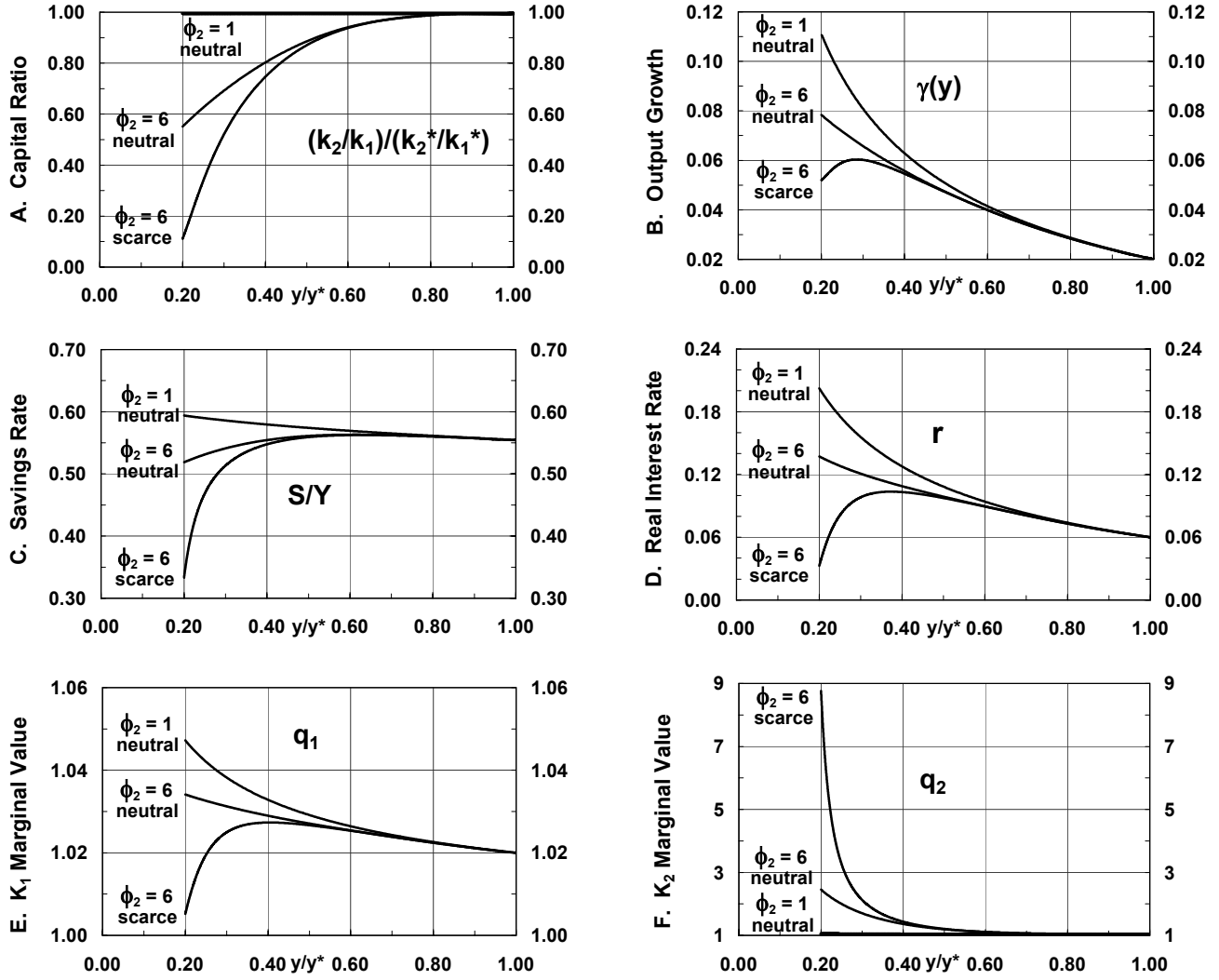
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Figure 3: Narrow Combined Capital Share



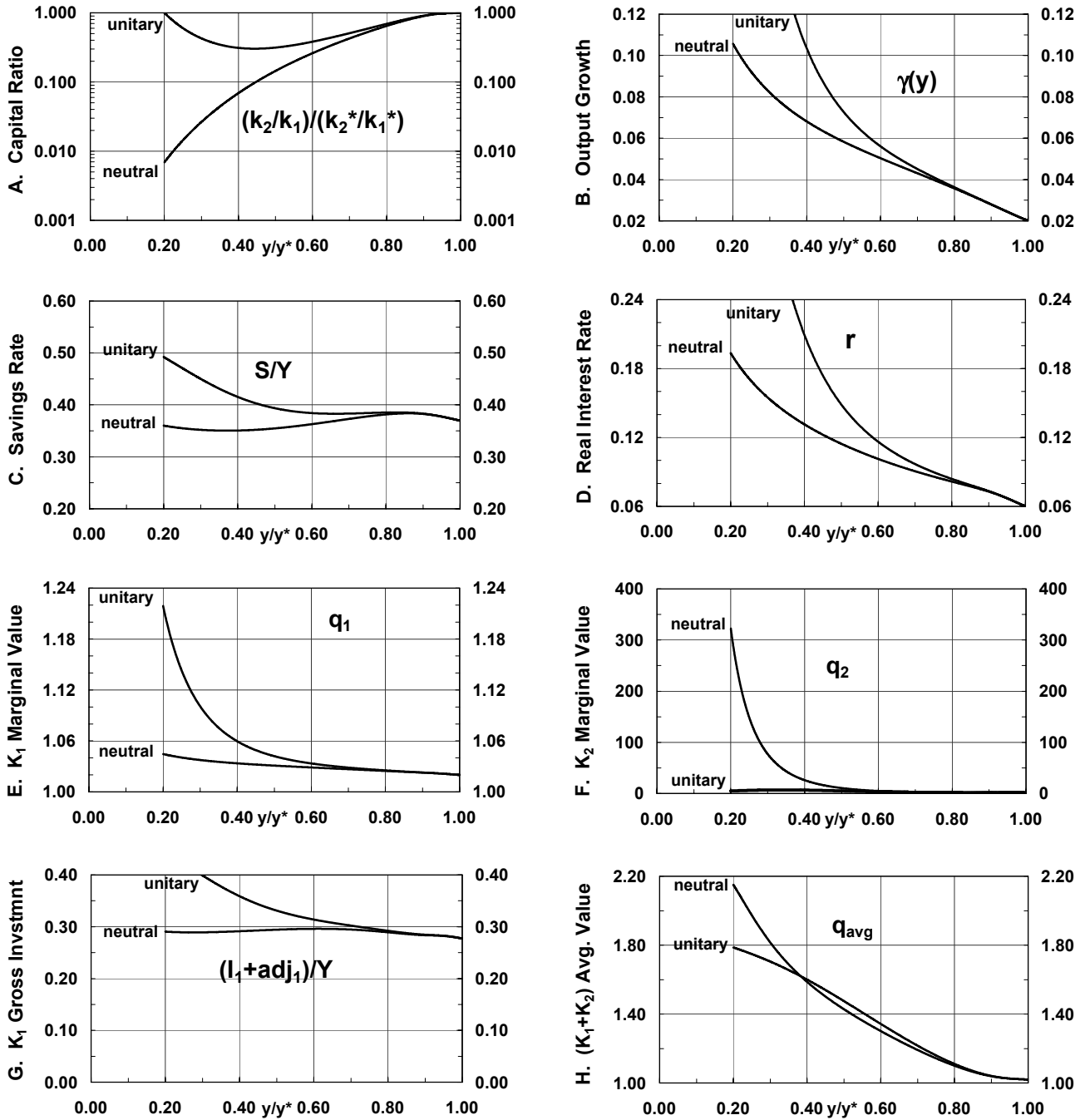
Parameters: Capital Shares: $\alpha_1 = 0.33$; $\alpha_2 = 0.11$. Steady-State Marginal Values: $q_1^*, q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1$; $\phi_2 = 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.

Figure 4: Broad Combined Capital Share



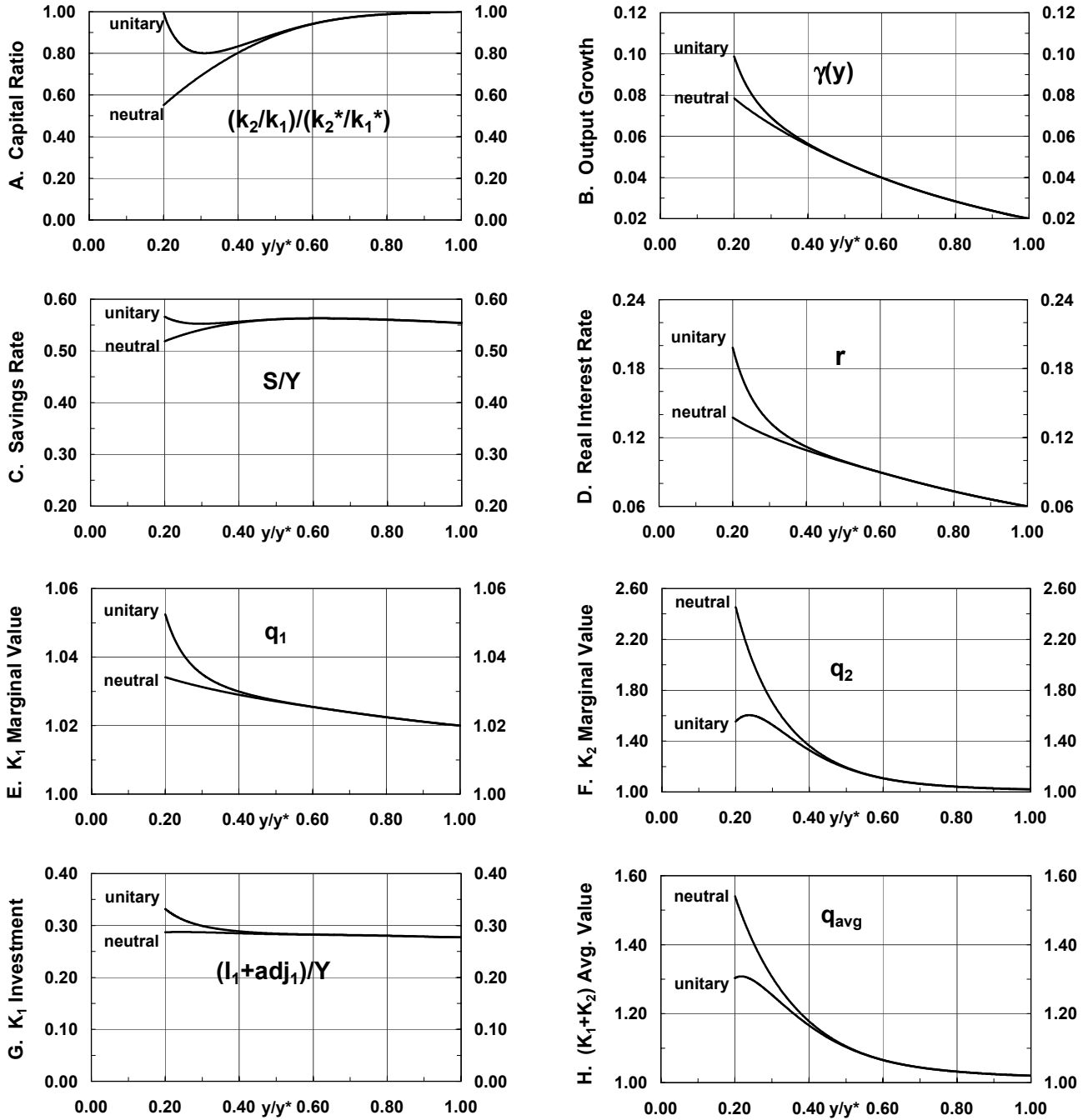
Parameters: Capital Shares: $\alpha_1 = 0.33$; $\alpha_2 = 0.33$. Steady-State Marginal Values: q_1^* , $q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1$; $\phi_2 = 1, 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.

Supplemental Figure 1: Neutral vs. Unitary Initial Capital Ratio, Narrow Combined Capital Share



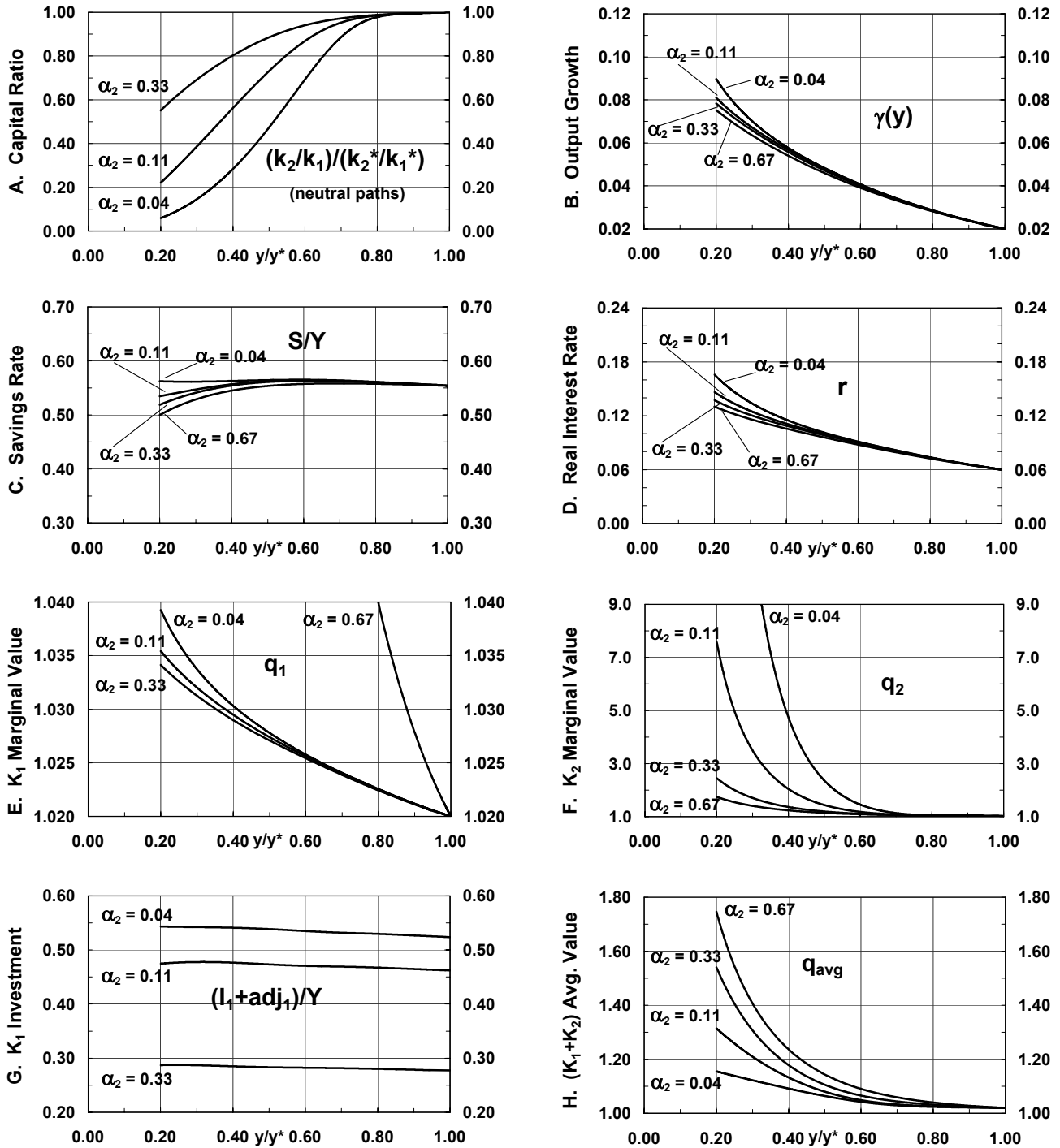
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Supplemental Figure 2: Neutral vs. Unitary Initial Capital Ratio, Broad Combined Capital Share



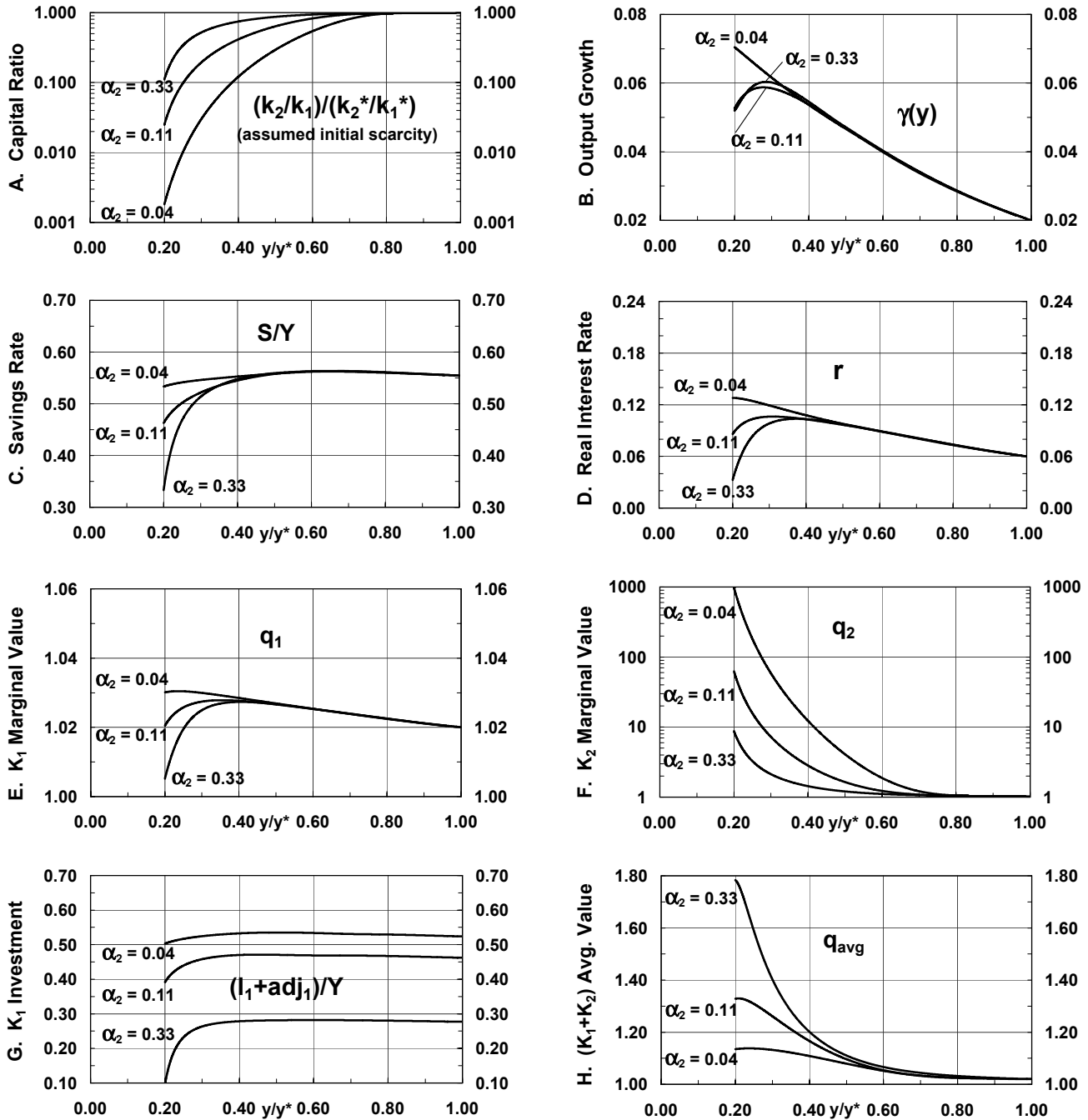
Parameters: Capital Shares: $\alpha_1 = 0.33$; $\alpha_2 = 0.33$. Steady-State Marginal Values: q_1^* , $q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1$; $\phi_2 = 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.

Supplemental Figure 3: Alternative K_2 Shares, Broad Combined Capital Share, Neutral Paths



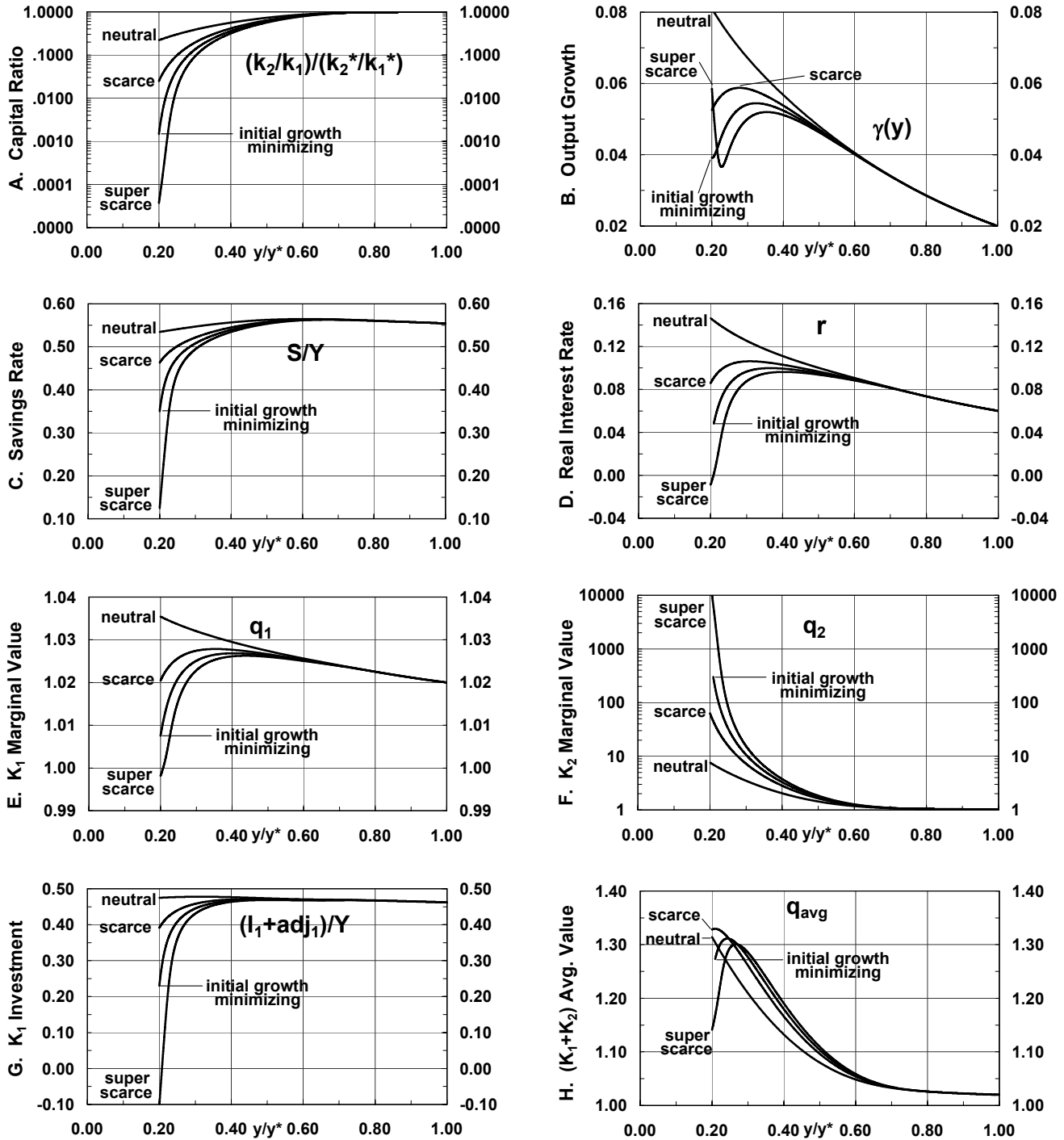
Parameters: Capital Shares: $\alpha_1 + \alpha_2 = 0.67$; $\alpha_2 = 0.67, 0.33, 0.11, 0.037$. Steady-State Marginal Values: $q_1^*, q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1, \phi_2 = 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.

Supplemental Figure 4: Alternative K_2 Shares, Broad Combined Capital Share, Initial K_2 Scarcity



Parameters: Capital Shares: $\alpha_1 + \alpha_2 = 0.67$; $\alpha_2 = 0.33, 0.11, 0.037$. Steady-State Marginal Values: q_1^* , $q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1$, $\phi_2 = 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.

Supplemental Figure 5: Varying Initial Scarcity, Broad Combined Capital Share, Narrow Bottleneck,



Parameters: Capital Shares: $\alpha_1 = 0.56$; $\alpha_2 = 0.11$. Steady-State Marginal Values: $q_1^*, q_2^* = 1.02$. Installation Cost Convexity: $\phi_1 = 1$, $\phi_2 = 6$. Capital Depreciation: $\delta_1, \delta_2 = 0.06$. Population Growth: $n = 0.02$. Technological Progress: $x = 0.02$. Intertemporal Elasticity (Reciprocal): $\theta = 1.5$. Time Preference: $\rho = 0.03$.