PORTFOLIO CHOICE IN TAX-DEFERRED AND ROTH-TYPE SAVINGS ACCOUNTS

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Abstract

This paper uses numerical methods to compare optimal portfolios in tax-deferred and Roth-type savings accounts. Income and payroll taxes affect optimal portfolios in tax-deferred and Roth-type plans differently. For workers with assets in only one type of plan, the optimal equity share in a tax-deferred account could be higher or lower than in a Roth, depending on initial wealth. The differences in optimal portfolios between plans are large at short investment horizons but smaller at longer horizons. This paper also studies the 'asset location' decision of workers with assets in plans of both types.

JEL classification: N920, O510, R110, R120, R230

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1. Introduction

Since 1998, Roth IRAs have been available to U.S. workers as an alternative or additional saving vehicle to traditional tax-deferred accounts such as IRAs and 401ks. Both Roth and traditional accounts are tax-sheltered accounts (TSAs), but their tax treatments differ. Workers may contribute pre-tax earnings to traditional TSAs. Withdrawals are then taxed as ordinary income. Workers can only contribute post-tax earnings to Roth-type accounts, but withdrawals are tax free. In both types of TSA, asset returns inside the accounts are free from dividend and capital gains taxes. This article focuses on the difference between taxation of contributions to and withdrawals from TSAs, ignoring several other differences between traditional and Roth-type accounts.

Much previous analysis considers whether workers would prefer to save in traditional or Roth-type accounts. Other interesting questions include how the rules of both types of account affect workers' incentives to save and hold risk, and the effect of these incentives on the distribution of wealth and tax revenues. President Bush's proposal in 2003 to replace IRAs and 401ks by Retirement Savings Accounts and Lifetime Savings Accounts, which would both be Roth-type plans, increased interest in these questions. While this proposal is not yet law, the Economic Growth and Tax Relief Reconciliation Act of 2001 permitted firms to offer Roth 401k plans starting in 2006.[1]

This paper models the environment surrounding workers' decisions in more detail than previous literature. This literature simplifies the tax system on withdrawals from traditional TSAs by assuming it is proportional. Much previous literature also assumes that asset returns are riskless. This paper models the true non-proportionality of the combined income and payroll tax system, and assumes that asset returns are stochastic.

[1] The EGTRRA also specified that the Roth 401k program would expire or 'sunset' in 2011.

The analysis is entirely in partial equilibrium, and assumes there is no possibility that the tax system will change over time. This paper shows that, under these assumptions, optimal portfolios within traditional and Roth-type TSAs differ.

The intuition for this result is as follows. Given non-proportional taxation of withdrawals from traditional TSAs, the future average tax rate on withdrawals has a probability distribution that depends on the distribution of wealth, and thus on the portfolio mix of the account. Therefore optimal portfolios within traditional TSAs are a function of the tax system. Even though the same tax system applies to contributions to Roth-type accounts, it affects the optimal portfolio in these accounts differently, and not at all in special cases. This is because, since withdrawals from Roths are not taxed, the portfolio in a Roth-type account does not affect future tax rates.

Numerical analysis shows that if workers hold only traditional TSAs or only Roths, the optimal share of equity in a traditional TSA one year prior to withdrawal of funds ranges from 2.5 percentage points lower to 15 percentage points higher than that in a Roth, depending on the worker's wealth. It is assumed that workers have constant-relative-risk-aversion utility with a risk-aversion parameter of 3, expect no Social Security benefits, and hold no financial assets outside their TSAs. If the workers expect Social Security benefits, the differences between optimal equity shares can be even larger. The differences in optimal equity shares can either grow or decline as the investment horizon lengthens, depending on the level of wealth in the accounts.

In this environment, workers' choice between traditional and Roth TSAs differs from that in previous literature. In some cases workers would prefer not to convert a traditional TSA to a Roth even thought the expected tax rate on withdrawals from the traditional TSA exceeds the tax rate payable on conversion. Previous literature finds that workers would wish to convert traditional TSAs to Roths in this case. With risky asset returns and non-proportional taxes, the tax rate on withdrawals covaries positively with portfolio returns, and thus provides insurance against poor asset returns. Workers would be willing to accept a higher mean tax rate in the traditional TSA than was on offer in a Roth TSA to gain this insurance.

This paper also explores the 'asset location' decision of households with assets in both traditional and Roth TSAs. This decision is only interesting if taxes are non-proportional, since this creates a difference between traditional and Roth-type TSAs. Many U.S. households now hold both types of account. Numerical analysis shows that low-wealth U.S. workers would wish to concentrate their equity holdings in traditional TSAs whilst medium-wealth workers would wish to concentrate their wealth holdings in Roth IRAs. At high wealth levels households would again optimally concentrate their equity holdings in their traditional TSAs. Thus conclusions about the optimal portfolio in any account appear sensitive to the details of the choices facing workers.

This paper is organized as follows. Section two reviews prior literature. Section three provides summary statistics on holdings of tax-sheltered savings accounts. Section four describes theoretically why optimal portfolios within traditional and Roth-type accounts differ. Section five uses numerical methods to explore the optimal portfolios assuming workers hold only one type of account, and preferences between the two types of account. Section six considers the asset-location problem of workers who hold both traditional and Roth-type accounts. Section seven concludes.

2. Literature Review

Most existing literature on Roth and traditional IRAs compares them in an environment of certainty, where there is only one, safe asset. Kutner et. al. (2001) and Burman, Gale and Weiner (2001) consider which types of workers would prefer to contribute to traditional and which to Roth IRAs in such an environment. The typical result is that workers would prefer to contribute to a Roth IRA if their tax rate while working is lower than that when retired. By contrast, in the current paper workers must compare a known current tax rate to a distribution of future tax rates.

Dickson (2003) considers a choice between Roth and traditional IRAs in an environment where the tax system on withdrawals is proportional, but at a stochastic rate because of uncertainty about future tax policy. He shows that in this case, if the expected future tax rate equals the tax rate today, risk-averse workers would prefer to save in Roth IRAs. His model has one, riskless asset and thus no implications for portfolio choice.

Much public finance literature considers the effect of taxes and welfare benefits on incentives to save in models with certain asset returns. For example, Gokhale and Kotlikoff (2001) examine workers' incentives to save in traditional and Roth TSAs in such a model. They show that since withdrawals from traditional TSAs increase taxes on Social Security benefits but withdrawals from Roth IRAs do not, some poorer workers have incentives to save in Roth IRAS but not in traditional TSAs.

Other public finance literature examines the cost of guarantees of minimum values of defined-contribution (DC) pensions. Smetters (2002) shows that such guarantees increase workers' incentives to hold equity in their pension accounts, which would increase the cost of any such guarantees. While progressive taxation of withdrawals from traditional IRAs does not guarantee any minimum level of withdrawals, it could also increase workers' incentives to hold equity.

Theoretical and empirical research also explores the portfolio choice of workers who hold assets both within and outside TSAs. These workers face both an 'asset allocation' decision of how much equity to hold and an 'asset location' decision of where to hold their equity. Clemens and Sialm (1999) and Poterba, Shoven and Sialm (2000) consider the optimal portfolios of such workers given stochastic asset returns. Both papers assume the tax rate on withdrawals from TSAs is an exogenous constant. A natural extension to the current paper would be to study the asset location decision between taxable accounts and TSAs in an environment of risky asset returns and non-proportional taxation of withdrawals from traditional TSAs.

In more purely finance work, Campbell, Cocco, Gomes and Maenhout (2001) study the evolution of workers' optimal portfolios over their lifetimes, where labor incomes are uncertain and declining at high ages. Workers are assumed able to rebalance their portfolios each year, so dynamic programming methods are needed to solve for the path of the optimal equity share. Their paper assumes all taxes are proportional, however, and so says little about the effects of taxes on optimal portfolios.

2. Holdings of Tax-Sheltered Savings Accounts in 2001

The Survey of Consumer Finances is a triennial survey of U.S. households' balance sheets, pensions and incomes. Table 1 provides some statistics on households' ownership of TSAs from the 2001 Survey, which was the first to ask about ownership of

Roth IRAs. Although the SCF over-samples richer households, Table 1 presents estimates for the country overall, using population weights supplied by the SCF. The SCF data and weights imply that U.S. households held $2.22 trillion in 401k or other employer-sponsored TSAs and $3.25 trillion in IRAs in 2001.[2]

Column 1 of Table 1 shows that an estimated 54 million or 51 percent of U.S. households held a TSA of some type in 2001. Of these households, the mean wealth in TSAs was just over $100,000 in TSAs, well below the mean financial wealth of $318,000. This suggests that many households held significant financial assets outside their TSAs. The remaining columns show that many households hold several types of TSA. An estimated 32.2 percent of households held a 401k or other employer-sponsored TSA, and 31.4 percent an individual IRA of some type.[3] An estimated 8.6 million or 8.1 percent of households held a Roth IRA in 2001. Column 5 shows that only an estimated 2.3 million of the 8.6 million households with Roth IRAs held no traditional TSA. Thus an estimated 6.3 million households face a decision of how to allocate their portfolios across their Roth and traditional IRAs. Section six below analyzes this decision.

While the SCF includes variables on the percent of TSA wealth held in equity, the permissible survey answers restrict the detail of the information collected. A holder of a 401k or IRA can state that it is held 'mostly in stock', 'mostly in bonds', or 'split between the two', but cannot express this split more precisely.[4] The SCF codes answers of 'mostly' to mean entirely in the given asset and 'split' to mean split equally between types of asset. The bottom two rows of Table 1 show that, given these codings, around

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[2] Federal Reserve (2003) states that U.S. households held $2.54 trillion in IRAs of all types in 2001.

[3] SIMPLE IRAs, SEP IRAs and SAR-SEP IRAs, which are all employer-sponsored accounts, are included in the category '401k/thrift' and not 'Any IRA'.

[4] See questions X3631 and X4234 in the 2001 SCF Codebook, at www.federalreserve.gov/pubs/oss/oss2/2001/codebk2001.txt.

70 percent of assets in TSAs are held in equity. The current paper focuses on describing optimal portfolios in theory and leaves the empirical question of whether workers choose optimal portfolios to future research.

Another source of data on holdings of TSAs in the U.S. is the Investment Company Institute's annual survey of IRA ownership. This has a smaller sample than the SCF, but attempts to be representative rather than over-sampling the rich. ICI (2002) reports that Roth IRAs were held by 11.9 million households in 2001 and 12.9 million households in 2002.

3. A Simple Model of Portfolio choice in Roth and Traditional TSAs

This section uses a simple model to show that in general optimal portfolios in Roth and traditional IRAs differ. This result is particularly clear if there is no labor or transfer income after the portfolio is chosen, since in this case personal taxation will affect the optimal portfolio in the Roth only through a wealth effect on the tolerance for risk. If labor or transfer income is expected after the portfolio is chosen, personal taxation will affect the optimal portfolio inside a Roth IRA more generally.

Consider a worker who lives for two periods, and whose labor earnings are exogenously fixed at W in the first period and zero in the second. There is one riskless and one risky asset. The worker must choose first-period consumption, C_1, and the share of his portfolio α that he will hold in the risky asset between periods 1 and 2, assuming his portfolio will be continuously rebalanced between these periods. Absent taxation, the worker's problem is

$$\max_{C_1, C_2, \alpha} \quad U(C_1) + \beta E[U(C_2)] \quad \text{subject to} \tag{1}$$

$$C_2 \leq [W - C_1] \exp[r_f + \alpha(r_e - r_f) + \alpha(1-\alpha)\sigma^2/2] \quad ,$$

where r_f is the log of the return on the riskless asset, and $r_e \sim N(r_f + \mu, \sigma^2)$ is the log of the return on the risky asset. The term $\alpha(1-\alpha)\sigma^2/2$ appears in the worker's portfolio return since the log of an average is greater than the average of logs.[5] This paper treats C_1 as given so as to focus on the worker's choice of α. If the worker has the constant relative risk aversion (CRRA) utility function $U(C) = C^{1-\theta}/(1-\theta)$, the optimal α can be shown to be

$$\alpha^* = \frac{\mu + \sigma^2/2}{\theta\sigma^2} . \tag{2}$$

Now suppose labour earnings are taxed at an average rate $\tau(W)$ and that the worker saves in a Roth IRA. The budget constraint becomes

$$C_2 \leq [W(1-\tau(W)) - C_1] \exp[r_f + \alpha(r_e - r_f) + \alpha(1-\alpha)\sigma^2/2] \ . \tag{1'}$$

Thus from the perspective of portfolio choice, introducing taxation is equivalent to reducing labor earnings W. Thus the worker's optimal α will change only due to a wealth

[5] See Campbell and Viceira (2001), p.28.

effect on the taste for risk. Given CRRA utility, there is no such wealth effect, so the optimal share of the risky asset is unchanged. Merton (1969) and Samuelson (1969) show that if the second period is thought of as occurring T periods after the first, and assuming asset returns are temporally IID and utility is CRRA, the optimal equity share (2) is the same for any horizon T.

Now suppose all savings are devoted to a traditional IRA or other tax-deferred savings account. The budget constraint in the above problem becomes

$$
\begin{aligned}
&C_2 \le \left[W - C_1(1-\tau(W))^{-1}\right]\exp\left[r_f + \alpha(r_e - r_f) + \alpha(1-\alpha)\sigma^2/2\right]\times \\
&\left[1-\tau\left(\left[W - C_1(1-\tau(W))^{-1}\right]\exp\left[r_f + \alpha(r_e - r_f) + \alpha(1-\alpha)\sigma^2/2\right]\right)\right]
\end{aligned}
\quad (3)
$$

Given a non-proportional tax system $\tau(.)$, the average tax rate on withdrawals from the traditional IRA in the second period depends on the size of these withdrawals (since this is a two-period model, all funds in the IRA are withdrawn in the second period). Thus the probability distribution of future average tax rates depends on the level of savings and α. Thus we may expect that α^* will be a function of the schedule $\tau(.)$, as is confirmed by the simulations below.

As discussed above, prior literature such as Shoven and Sialm (1999) assume the average tax rates on labor income and withdrawals from traditional IRAs to be the constants τ_W and τ_R respectively, regardless of wealth or portfolio choice. In this case the budget constraint (3) simplifies to

$$C_2 \leq \left[W\left(1-\tau_W\right)-C_1\right]\exp\left[r_f+\alpha\left(r_e-r_f\right)+\alpha\left(1-\alpha\right)\sigma^2/2\right]\frac{1-\tau_R}{1-\tau_W} \tag{3'}$$

Here again, given CRRA utility, the tax rates τ_W and τ_R do not affect the optimal portfolio.

The analysis above also notes that the same equity share over time would be optimal for workers holding Roth IRAs who had no labor or transfer income. Bodie, Merton and Samuelson (1992) show that workers who continued to earn labor income after choosing their portfolios would wish to adjust α continuously. The simulations below show that workers holding traditional IRAs would also wish continuously to adjust α even if they expected no future labor or transfer income. Numerical analysis cannot model workers as re-optimizing α continuously. Campbell et. al. (2001) model workers as reoptimizing once per year. Shoven and Sialm (1999) do not permit workers to reoptimize or rebalance at all. Most of the numerical analysis below assumes workers can re-optimize over α every quarter. In order to simplify the numerical analysis, however, workers with wealth in both traditional and Roth IRAs are assumed constrained to choose a single (α_R,α_{TR}) pair that remains constant over time.

5. Optimal Equity Shares for Workers with Either Traditional or Roth TSAs

This section compares the optimal equity shares within traditional and Roth TSAs, denoted α_{TR}^* and α_R^* respectively, assuming workers have account of only one type and hold no assets outside these accounts. Asset returns are assumed to be stochastic and taxation of withdrawals from the traditional IRA to be non-proportional. It also analyzes workers' choice of whether to convert a traditional IRA to a Roth in this environment.

Basic Setup

The basic setup is similar to problem (1) above, but assumes the worker chooses between two risky assets, equities and bonds. Both assets are assumed to have temporally IID log-normal distributions, though stock and bond returns in any period are correlated. Thus the worker solves

$$\max_{\{\alpha_1,\ldots,\alpha_{T-1}\}} \quad E_0\left[U\left(C_T\right)\right]$$

$$\text{s.t.} \qquad C_T = \left(SSB_T + A_T\right)\left[1-\tau\left(A_T, SSB_T\right)\right],$$

$$A_T = A_0 \prod_{t=1}^{T-1} \exp\left(\begin{array}{l}\left(\alpha_t\mu_e + \left(1-\alpha_t\right)\mu_b + \frac{1}{2}\alpha_t\left(1-\alpha_t\right)\left(\sigma_e^2+\sigma_b^2-2\sigma_{eb}\right)\right)\\ +\sqrt{\left(\alpha_t^{\,2}\sigma_e^2 + \left(1-\alpha_t\right)^2\sigma_b^2 + 2\alpha_{ti}\left(1-\alpha_{ti}\right)\sigma_{eb}\right)}s_t\end{array}\right) \qquad (5)$$

where SSB_T are Social Security benefits in the retirement period, quarterly log returns on equity and bonds are $r_e \sim N(\mu_e, \sigma^2_e)$ and $r_b \sim N(\mu_b, \sigma^2_b)$ respectively, $\text{Cov}(r_e, r_b) = \sigma_{eb}$, and the $s_t \sim$ i.i.d. $N(0,1)$. The horizon T is measured in quarters, so workers are assumed able to reoptimize over α every quarter. Given these assumptions, the return on their portfolios each quarter is drawn from a log-normal distribution, whose mean and variance depend on α_t. Campbell and Viceira (2002: 29) discuss why (5) is the portfolio return when both assets are risky. The worker is assumed to start with assets A_0 but to have no labor income. We can think of the worker as knowing he will die at the end of period T. More plausibly, we can think of the worker as being uncertain of his length of life but knowing that at date T he will use his entire pre-tax IRA wealth to buy a riskless annuity, thereafter withdrawing the annuity payments each year, and consuming the post-tax value of these payments. The latter interpretation is consistent with the tables below if 'IRA Wealth' is understood as referring to a pre-tax annuity payment on the worker's IRA wealth. Workers are assumed to have CRRA utility with parameter $\theta = 3$; changing this parameter changed the magnitudes but not the flavour of the results. While model (5) is clearly a stylized model of a near-retiree's optimization problem, it is useful in examining the effects of taxes on IRA withdrawals on optimal portfolio choices.

A numerical algorithm is used to find α_{TR}^*, or more correctly the schedule of values $\alpha_{TR,t}^*(A)$, since α_{TR}^* depends on the initial level of wealth and the date. For any candidate α_{TR}, this algorithm samples 100,000 times from the lognormal distribution of portfolio returns.[6] α_{TR}^* is defined as the equity share which give the highest utility on average over these 100,000 returns. Dynamic programming methods are necessary to solve backwards for $\alpha_{TR,T-1}^*(A)$, $\alpha_{TR,T-2}^*(A)$ and so on. For example, $\alpha_{TR,T-1}^*(A)$ implies a value function $V_{TR,T-1}(A)$ of wealth that period. This value function is then used to find $\alpha_{TR,T-2}^*(A)$. $V_{TR,T-1}(A)$ is evaluated at a series of discrete wealth levels, and estimated between these levels using linear interpolation. The algorithms are guaranteed only to find an α_{TR} that maximizes utility locally. Plots of expected utilities against α_{TR} showed

[6] It is necessary to sample from this distribution a large number of times since small probabilities of disastrous asset returns affect the portfolio choices of highly risk-averse workers.

that expected utility is a smooth and single-peaked function α_{TR}, however, so local maxima are also global maxima. α_R^* can be found analytically.[7]

Asset Returns

The parameters of the distributions of the log of total real returns to bond and equity are constructed from the real returns of long-term U.S. government bonds and U.S. large-company stocks during 1925-2000 quoted in *Ibbotson's Yearbook* for 2001. It is assumed that these log returns are normally distributed, covary in any period, and are IID over time. Table 1 shows the annual means, variances and covariance of these log returns. There is a considerable equity premium, so optimal equity shares tend to be high even given the high risk aversion embodied in the assumption that $\theta = 3$.

Taxation of IRA Withdrawals

Since withdrawals from Roth IRAs are not taxed and retirees without other income would pay no tax on their Social Security benefits, in the Roth case, $\tau(.)=0$ for all A_T in problem (5). Workers with traditional IRAs are assumed to pay federal income and payroll taxes at the rates for 2002, but no state income taxes, on withdrawals from traditional IRAs. All workers are assumed to pay tax at the rate for a married couple filing jointly and to claim deductions from taxable income of $5,000 per year. Table 2 describes the payroll taxes and the federal income tax system for married couples in

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[7] Campbell and Viceira (2001: 30) show that

$$\alpha_R^* = \frac{\mu_e - \mu_b + \frac{1}{2}\left(\sigma_e^2 + \sigma_b^2 - 2\sigma_{eb}\right) + (1-\theta)\left(\sigma_{eb} - \sigma_b^2\right)}{\theta\left(\sigma_e^2 + \sigma_b^2 - 2\sigma_{eb}\right)}$$

2002. Workers are assumed to be ineligible for the Earned Income Tax Credit, a wage subsidy for poor households with children, Supplemental Security Income, a welfare system for poor old people, and the Alternative Minimum Tax, which applies to high earners with large deductions from taxable income.

The simulations include Social Security benefits in taxable income according to the rules of the Federal tax code, as follows. Provisional income is non-Social Security income plus tax-exempt interest and half of Social Security benefits. If provisional income for a married couple exceeds $32,000 but not $44,000, half of the excess over $32,000 or half of Social Security benefits, whichever is smaller, is included in taxable income. If provisional income exceeds $44,000, the amount included in taxable income is the minimum of (i) 50 percent of benefits or $6,000, whichever is smaller, plus 85 percent of (provisional income - $44,000) and (ii) 85 percent of benefits.

Optimal Portfolios when No Social Security Benefits are Expected

Figure 1 shows $\alpha_R^*(A)$ and $\alpha_{TR}^*(A)$ assuming $SSB_T = 0$ in all cases, at one to four quarters before the withdrawal date T. $\alpha_R^* = 0.711$ for all wealth levels in all periods. Three main results about α_{TR}^* are apparent. First, α_{TR}^* differs greatly from α_R^* for some wealth levels. For example, $\alpha_{TR,T-4}^*(\$76,250) = 0.879$, 16.2 percentage points above α_R^*. Second, α_{TR}^* can be either higher or lower than α_R^*. Third, α_{TR}^* varies less by initial wealth as the investment horizon increases.

To give some intuition as to why non-proportional taxes can make α_{TR}^* both higher and lower than α_R^*, Figure 2 shows α_{TR}^* and α_R^* for horizons of one and two quarters, assuming the tax system comprises only the Social Security payroll tax. This tax system is proportional up to the ceiling (in 2002) of $84,900, but regressive at higher incomes. We may also describe $84,900 as the single tax-rate 'node', or income level at which the marginal tax rate changes. α_R^* is again 0.711 for all wealth levels. For low A_0, $\alpha_{TR}^* \approx \alpha_R^*$ since Pr(A_T>$84,900) is low and thus the tax system is nearly proportional. For A_0 approaching $84,900, $\alpha_{TR}^* > \alpha_R^*$ because both a higher mean and variance of A_T increase Pr(A_T>$84,900) and thus the probability that the average tax rate on withdrawals falls below 12.4 percent. For A_0 just over $84,900, α_{TR}^* falls towards α_R^*, because while a higher mean return increases Pr(A_T>$84,900), a higher variance of returns reduces this probability. For A_0>$95,000, the latter effect dominates and $\alpha_{TR}^* < \alpha_R^*$. For very large A_0, Pr(A_T<$84,900) becomes small, and the tax system approximates a proportional system with a zero rate. Thus Figure 2 shows that a single tax-rate node can a α_{TR}^* either higher or lower than α_R^*.

A comparison of the schedules $\alpha_{TR,t}^*(A)$ at investment horizons of one and two quarters in Figure 2 shows two effects of a longer horizon. First, the spikes upward and downward around the tax-rate node of $84,900 are shallower at the longer horizon. Second, further away from the node, the difference between α_{TR}^* and α_R^* is larger at the longer horizon. This reflects that the marginal effect of α on the probability of wealth moving to the other side of the node decreases with the horizon close to the node but rises with the horizon far from the node. Thus it is not simply the case that α_{TR}^* and α_R^* differ less at longer horizons.

Choice Between Traditional and Roth IRAs

Previous analyses of the choice between traditional and Roth IRAs have assumed that asset returns are riskless and that taxes are proportional. The following is a simple model of a worker's choice of whether to convert an existing traditional IRA to a Roth in an environment of risky asset returns and non-proportional taxes on withdrawals from traditional IRAs.

Assume that conversion of a traditional IRA to a Roth incurs tax in the present at the average rate τ_C, and that this tax will be paid out of the assets in the traditional IRA. For simplicity, assume that IRA conversion can only occur at date T-4. The numerical methods described above define the value function $V_{TR,T-4}(A)$ of wealth that period. Given CRRA utility, which implies α_R^* is constant over time, the expected utility from holding wealth *A* in a Roth IRA at T-h is given by

$$EU_{R,T-h}(A) = (1-\theta)^{-1} A^{1-\theta} \exp\left[(1-\theta)h\left(\mu_p + \frac{1}{2}(1-\theta)\sigma_p{}^2\right)\right], \qquad (6)$$

where μ_P and $\sigma_P{}^2$ are respectively the quarterly mean and variance of a portfolio with constant equity share α_R^*.[8] Defining the function $\hat{\tau}(A)$ by

$$EU_{R,T-4}\left[A(1-\hat{\tau}(A))\right] = V_{TR,T-4}(A) \qquad (7)$$

[8] Given $EU(A_T) = E\left[A_T^{1-\theta}/(1-\theta)\right]$, (6) follows from the fact that $\ln(A_T) \sim N\left(\ln(A_0) + \mu_P h, \sigma_P^2 h\right)$.

implies the worker should convert his traditional IRA to a Roth if and only if $\hat{\tau}(A) > \tau_C$. Equations (6) and (7) imply that $\hat{\tau}(A)$ is given by

$$\hat{\tau}(A) = 1 - A^{-1}\left[(1-\theta)\exp\left((\theta-1)h\left(\mu_P + \frac{1}{2}(1-\theta)\sigma_P^2\right)\right)V_{TR,T-h}(A)\right]^{\frac{1}{1-\theta}} \qquad (8)$$

The threshold tax rate $\hat{\tau}(A)$ does not appear in previous literature. Dickson (2003) finds that workers should hold Roth IRAs whenever $\tau_C \leq \bar{\tau}$, where $\bar{\tau}$ is the expected average tax rate on withdrawals from the traditional IRA. Thus in the current setup, $\hat{\tau}(A) < \bar{\tau}$ for any A would give a different result, since it would be optimal not to convert a traditional IRA to a Roth if $\hat{\tau}(A) < \tau_C < \bar{\tau}$. We should note, however, that for Dickson $\bar{\tau}$ depends only on the rules of the tax system. In the current setup, $\bar{\tau}$ depends on the tax system, wealth A, the date, the preference parameter θ, and the parameters of the distributions of asset returns. $\bar{\tau}$ can be calculated from the numerical simulations above in a similar manner to the value function.

Figure 3 compares $\hat{\tau}$ and $\bar{\tau}$ at date T-4 at wealth levels from \$30,000 to \$150,000. At all these wealth levels, $\hat{\tau}$ is lower between 1.5 and 1.8 percentage points lower than $\bar{\tau}$. Therefore, allowing for non-proportional taxes on withdrawals from traditional IRAs, there is a range of tax rates on converting a traditional IRA to a Roth, τ_C, such that $\tau_C < \bar{\tau}$ and yet the worker would not wish to convert. The intuition for this result is that under progressive taxation, the future tax rate on withdrawals from traditional IRAs will generally be positively correlated with asset returns. Therefore the prospect of progressive taxation creates insurance against poor asset returns, in exchange for which workers would be willing to accept a higher mean tax rate. There would be no such covariance if taxes on withdrawals were proportional.

It should be noted that the partial equilibrium analysis of this paper does not explain how the government would raise sufficient tax revenue in the event of very poor asset returns. As Smetters (2002) points out, younger taxpayers would be greatly averse to a tax system that, in the event of poor asset returns, taxed them more heavily to make up for taxing older workers more lightly.

Optimal Portfolios when Social Security Benefits Are Expected

The calculations above assume workers expect no Social Security benefits. Figure 4 shows optimal equity shares in traditional and Roth IRAs assuming all workers expect riskless Social Security benefits of $10,000 in period T. The addition of riskless Social Security benefits generally encourages workers to hold more risk in their IRA accounts. Indeed, in some cases $\alpha_{TR}^{*}(A)$ exceeds 100 percent. Workers could only hold more than 100 percent of their accounts in equity by selling bonds short, which may be difficult in practice. In Figure 4, $\alpha_{TR}^{*}(A)$ again varies widely, with peaks and troughs that become smaller as the investment horizon lengthens. The $\alpha_{TR}^{*}(A)$ schedules in Figure 4 have more peaks and troughs than those in Figure 1, since the rules for the taxation of Social Security benefits add more nodes to the tax system than were present in Figure 1.

In this simulation α_{R}^{*} does not depend on the investment horizon, as in the simulation without Social Security benefits. In contrast to the previous simulation, however, α_{R}^{*} does depend on *A*, since larger *A* implies riskless Social Security benefits are a smaller proportion of overall wealth. This dependence of α_R^* on the worker's accumulated post-tax savings in his Roth IRA implies α_R^* also depends on his past average tax rates on labor income. This means it is impossible to compare α_{TR}^* and α_R^* for identical workers in this simulation, since we cannot state how much wealth a worker with wealth *A* in a traditional IRA would have had in a Roth IRA had he saved in such an account. However, taxation clearly affects α_{TR}^* and α_R^* differently in the presence of Social Security benefits. The prospect of non-proportional taxation on withdrawals from traditional IRAs creates the large peaks and troughs in α_{TR}^* shown in Figure 4, while different rates of past taxation of labor income merely move α_R^* along its much flatter schedule.

6. Asset Location Between Roth and Traditional TSAs

This section considers the optimal portfolio of a worker who for exogenous reasons holds assets in both traditional and Roth IRAs and thus faces an 'asset location decision' between the two. This problem is interesting since, as stated above, a considerable number of households hold both traditional TSAs and Roth IRAs. Further, there is only an incentive to hold different portfolios in the two accounts if taxes on withdrawals are modeled realistically, as in (3) above. The simplified tax system (3') used in previous literature produces no such incentive.

The worker's problem in this context is to choose an optimal pair of equity shares, (α_R^*, α_T^*). Several simplifying assumptions are made to reduce the complexity of this problem. In particular, workers are models as choosing one (α_{TR}, α_R) pair for all time, rather than reoptimizing over (α_{TR}, α_R) every quarter. Although dynamic-programming methods similar to those employed above could be employed to model workers as regularly reoptimizing their equity shares, they require considerable computer time. With two state variables, A_{TR} and A_R, this problem would require calculating the value function at a two-dimensional grid of values, rather than only at a one-dimensional range, as above. Further, at each point in the (A_{TR}, A_R) grid it would be necessary to solve a two-variable maximization problem, which would consume more time than a single-variable problems. The results of the single-account problems modeled in the previous section were qualitatively very similar whether workers were assumed to optimize over one equity share for all time or were permitted to reoptimize over this share. Thus the results of this section may differ little from those of a dynamic-programming model in which workers were allowed to reoptimize. Analysis of such a dynamic-programming model remains an interesting task, however.[9]

It is assumed further that the equity shares in workers' traditional and Roth IRAs are fixed to ($\alpha^*_{TR}, \alpha^*_R$) at the start of each year, but that the portfolios in each account are not rebalanced during the course of each year. This assumption permits the correlation between the returns inside the traditional and Roth IRAs to be modeled in a simple manner. Thus workers maximize

[9] Later versions of this paper may include such a dynamic-programming solution to this problem.

$$\max_{a_{TR}, a_R} \quad EU(C_T),$$

$$\text{s.t.} \qquad C_2 = A_{R,T} + A_{TR,T}\left(1 - \tau\left(A_{TR,T}\right)\right),$$

$$A_{R,T} = \frac{A_0}{2} \prod_{t=1}^{T-1} \left[\alpha_R . \exp\left(r_{et}\right) + \left(1 - \alpha_R\right) \exp\left(r_{b_t}\right)\right], \tag{9}$$

$$A_{TR,T} = \frac{A_0}{2} \prod_{t=1}^{T-1} \left[\alpha_{TR} . \exp\left(r_{et}\right) + \left(1 - \alpha_{TR}\right) \exp\left(r_{b_t}\right)\right],$$

where annual log returns to equity (r_e) and bonds (r_b) have the distributions shown in Table 2. It is assumed that workers initially hold equal wealth in their traditional and Roth IRAs. This implies that in a post-tax sense, over half their wealth is in their Roth IRAs.

It is instructive to consider the case that $\tau(.) = k$, so that the tax system is proportional. The worker can then be thought of as holding two Roth IRAs, one containing wealth $A_0/2$ and the other $A_0(1-k)/2$. This is equivalent to holding one Roth, so the worker would be indifferent as to how his overall holdings of equity were split between the two accounts. This shows that the level of taxation on withdrawals from the traditional IRA is irrelevant to optimal asset location. It also suggests that the optimal equity share in the two accounts could be indeterminate if the tax system expected to apply to withdrawals from the traditional IRA was approximately proportional.

There were multiple numerical solutions to (9) in some cases. The algorithm used to find (α_R^*, α_T^*) requires a starting pair ($\alpha_{R,0}, \alpha_{TR,0}$) for its search, and stops at a local utility maximum. For some levels of A_0, this algorithm returned different local utility maximizers for different starting values. In cases where the utility maxima associated with these maximizers could be ranked, the figures below show only the (α_R,α_{TR}) pair that produced the highest utility. In one case several (α_R,α_{TR}) pairs produced the same highest level of utility. In this case the figures below show no (α_R,α_{TR}) pair as maximizing utility.

Figure 5 shows optimal equity shares for an investment horizon of one year. We see that, for A_0 <\$130,000, the worker wishes to concentrate equity holdings in his Roth account. The optimal share of equity in this account exceeds 100 percent. No optimal equity shares are shown for A_0=\$60,000, since in this case multiple (α_R,α_{TR}) pairs produced the same maximal level of utility. For \$130,000 ≤ A_0 ≤ \$230,000, the worker prefers to concentrate equity in the traditional account. In some cases the optimal share of equity in this account exceeds 200 percent and the optimal share in the Roth IRA is negative. For A_0>\$230,000 the worker reverts to wishing to hold most of his equity in his Roth account. As mentioned above, holding more than 100 percent of an account in equity requires selling bonds short, which may be difficult in practice. Figure 5 also shows the equity share of the two accounts combined. This is much more stable than the optimal equity shares in either individual account, and has a maximum of 1.04. Further work could repeat this exercise with the constraint that equity shares in either account or in both accounts only could not exceed 100 percent.

Figure 6 repeats the same exercise for an investment horizon of three years. Comparing figures 5 and 6, we see that the differences between α_{TR}^* and α_R^* are smaller at the longer horizon. This is similar to the result in the previous section that spikes upward and downward in α_{TR}^* are smaller at longer horizons. At this horizon the optimal equity shares within the individual accounts exceed 100 percent for some levels of A_0, but the optimal share in both accounts combines is below 100 percent for all levels of initial wealth.

Intuition for the results about the relationship between ($\alpha^*_{TR}, \alpha^*_R$) and A_0 can again be gained by repeating this maximization exercise assuming simple hypothetical tax systems. Figures 7 and 8 show optimal equity shares at a three-year investment horizon. Figure 7 assumes withdrawals from the traditional IRA are subject only to Social Security payroll taxes. Under this regressive tax system, workers prefer to concentrate their equity holdings in their traditional IRAs for all levels of A_0. Figure 8 assumes traditional IRA withdrawals are taxed at marginal rates of 15 percent below $46,700 and 27 percent above this amount. Given this progressive tax system, workers prefer to hold concentrate their equity holdings in their Roth IRAs. Thus it appears that workers wish to concentrate their equity holdings in their Roth IRAs in ranges where the tax system is progressive. Since the overall payroll and income tax system is progressive for low and high incomes and regressive for middling incomes, this would explain the double switch in preferred equity location shown in Figures 5 and 6.

7. Conclusion

This paper examines the optimal portfolios of workers who hold all their assets in either traditional or Roth-type tax-sheltered savings accounts or in a mixture of the two. This paper's innovation is to model the future tax on withdrawals from tax-deferred accounts accurately, rather than assuming it is proportional. In this environment, optimal portfolios within tax-deferred and Roth accounts differ, and workers' calculus of whether to save in a Roth or tax-deferred account differs from that in prior literature. Workers with both Roth-type and tax-deferred accounts wish to locate equity almost entirely in one account or the other, depending on their wealth. The results of the analysis of workers with assets in both Roth-type and tax-deferred accounts suggests that conclusions about workers' optimal portfolios in any particular account are sensitive to what other accounts workers hold, how much wealth they hold in these accounts, and tax laws relating to these accounts. Therefore future research may wish to test whether the qualitative results in this paper are robust to a richer description of workers' choices.

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Table 1: Summary Statistics of Ownership of Tax-Sheltered Savings Accounts (TSAs), 2001 Survey of Consumer Finances					
	1	**2**	**3**	**4**	**5**
Households Owning:	Any TSA	401(k) or Thrift	Any IRA	Roth IRA	Roth IRA Only
Percent of Households	51.1	32.2	31.4	8.1	2.2
Number of Households (thousands)	54,447	34,338	33,400	8,623	2,331
Mean Age of Household Head	47.6	42.6	51.8	45.9	47.6
Mean Assets in TSAs	100,375	94,834	139,337	101,095	24,280
Mean Household Financial Assets	318,411	219,788	466,742	317,113	174,648
Percent 401k/Thrift Assets in Equity	71.2	71.2	74.8	78.5	
Percent IRA Assets in Equity	59.4	67.7	59.4	73.5	62.8

Note: the SCF contains data for 4,422 households; statistics for the U.S. population are extrapolated using population weights contained in the data set. The SCF asks about holdings of equity in thrift-type plans and IRAs but not specifically in Roth IRAs.

Table 2: Parameters of Distributions of U.S. Real Annual Log Stock and Bond Returns, 1925-2000			
	Mean	Variance	Covariance (stocks with bonds)
Long-Term Government Bonds	0.0215	0.01	
Large-Company Stocks	0.0745	0.038	0.00476

Source: *Ibbotson's Yearbook* 2001, Table 5.2.

Table 3: Federal Payroll Taxes and Income Taxes for a Married Couple filing Jointly, 2002.		
Federal Income Tax		
Brackets of Taxable Income, Dollars		Marginal Income Tax Rate (Percent)
Lower Bound	Upper Bound	
0	12,000	10
12,000	46,700	15
46,700	112,850	27
112,850	171,950	30
171,950	307,050	35
307,050	………..	38.6
Brackets of Ordinary Income, Dollars		
Social Security Payroll Tax		
0	84,900	12.4
Medicare Payroll Tax		
0	……….	2.9

Note: data from IRS. It is assumed that neither the Earned Income Tax Credit or Alternative Minimum Tax apply. Text describes the rules for the inclusion of Social Security benefits in taxable income.

Figure 1: Optimal Equity Share in Traditional and Roth IRAs One to Four Quarters Before Withdrawal, Assuming No Social Security Benefit.

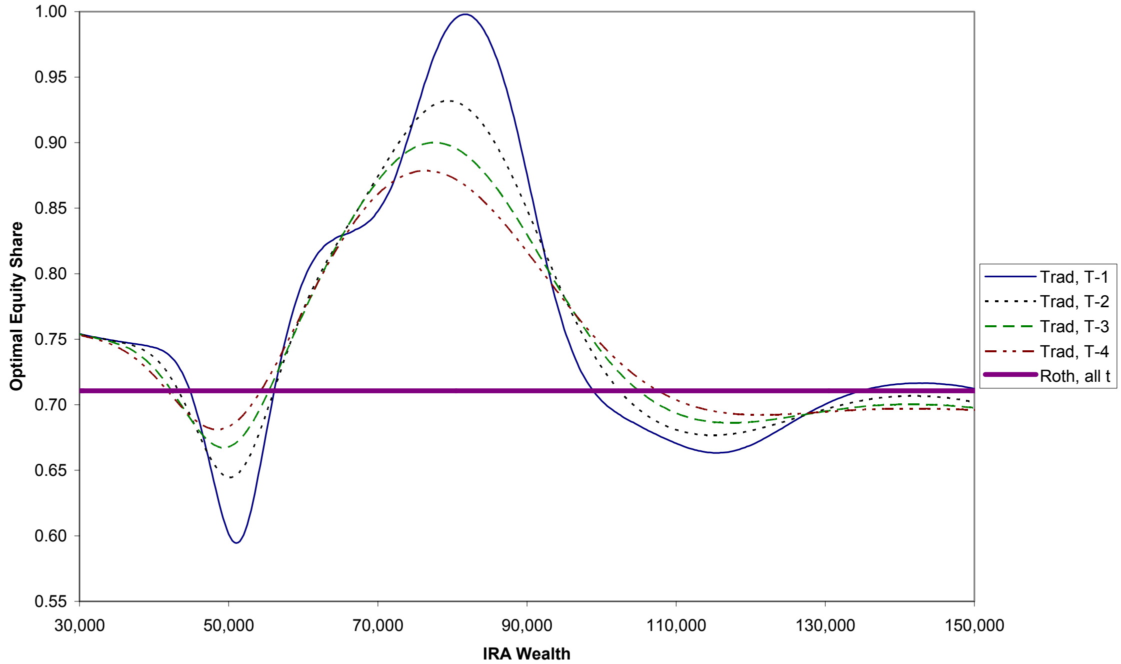

Figure 2: Optimal Equity Shares in Traditional and Roth IRAs for Tax System Comprising only Social Security Payroll Tax, One and Two Quarters Before Withdrawal.

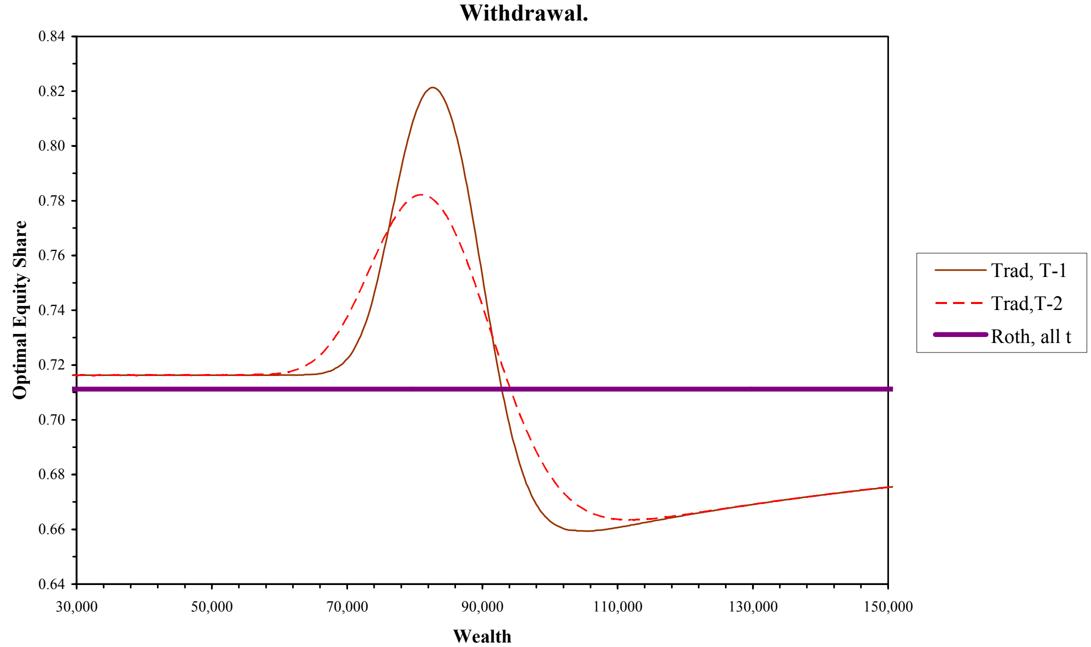

Figure 3: Tax-Rate Threshold for Trad-to-Roth IRA Conversion (t-hat) and Expected Tax Rate in Traditional IRA (t-bar), One Year Before Withdrawal.

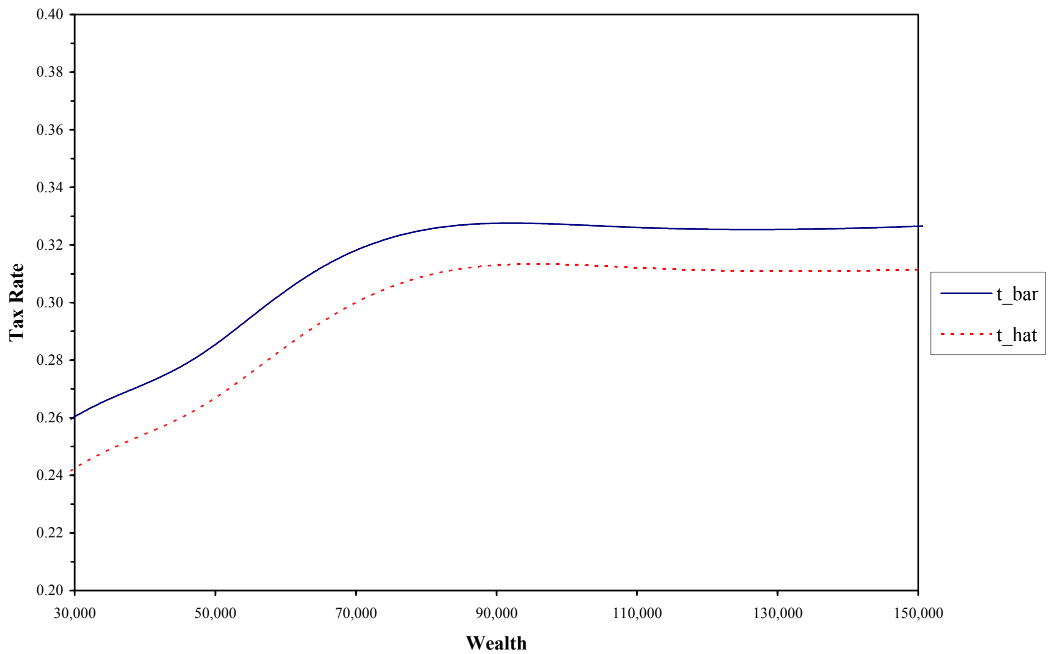

Figure 4: Optimal Equity Shares in Traditional and Roth IRAs One to Four Quarters Before Withdrawal, Social Security Benefit of $10,000 Expected at Withdrawal Date.

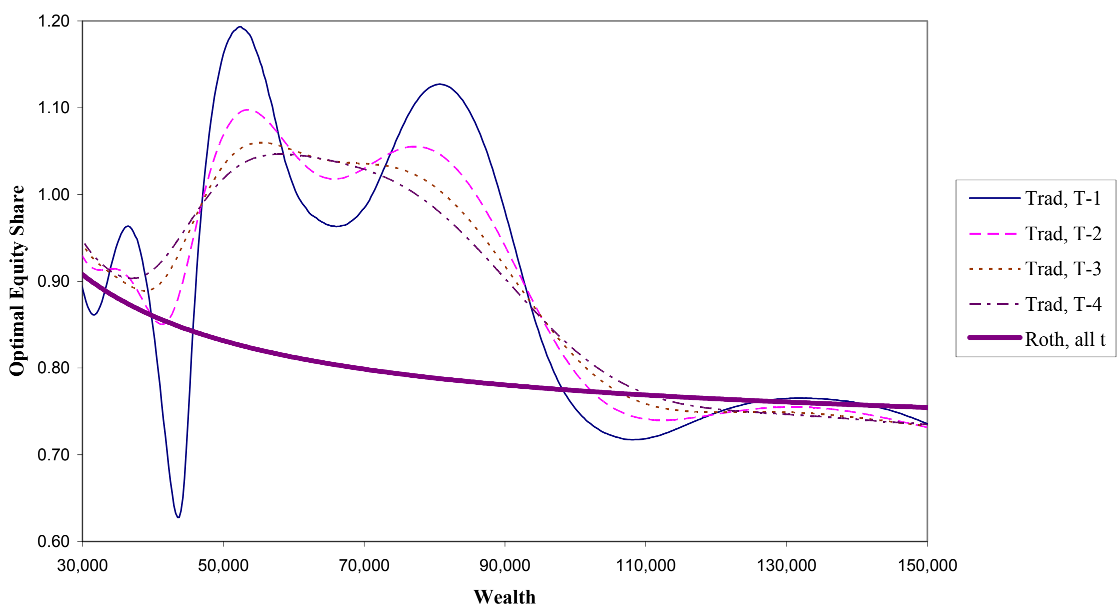

Figure 5: Optimal Equity Shares, Traditional and Roth IRA, Saver with Half Wealth in Each, One Year Prior to Liquidation

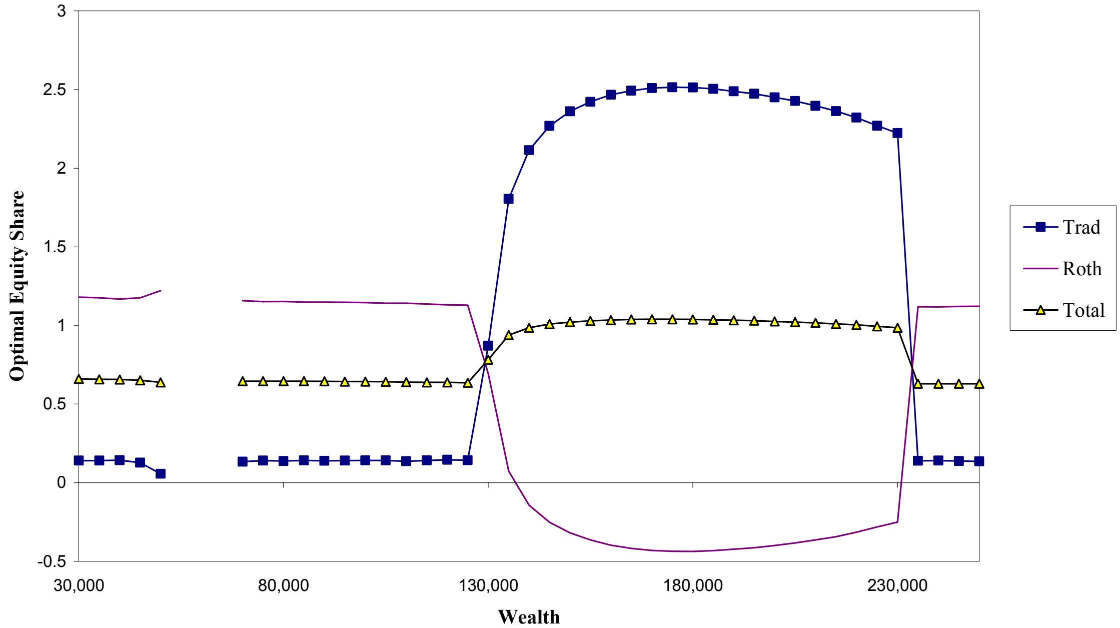

Figure 6: Optimal Equity Shares in Traditional and Roth IRA, Saver with Half Wealth in Each, Three Years Prior to Liquidation

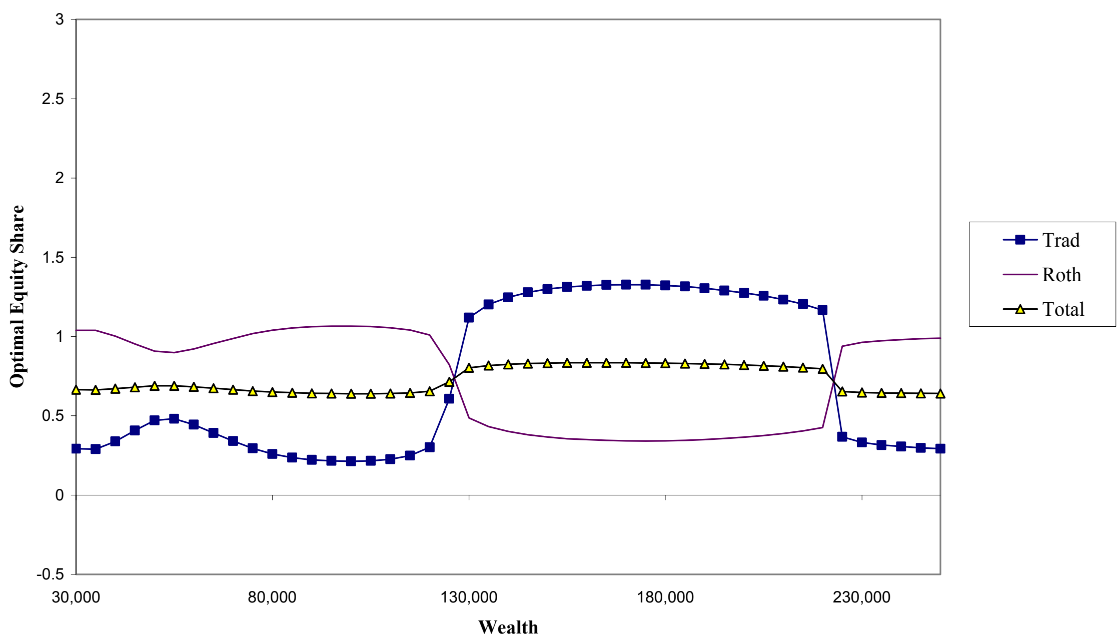

Figure 7: Optimal Equity Shares in Traditional and Roth IRAs, Three Years Prior to Withdrawal, Withdrawals from Traditional IRA Subject to Social Security Tax Only.

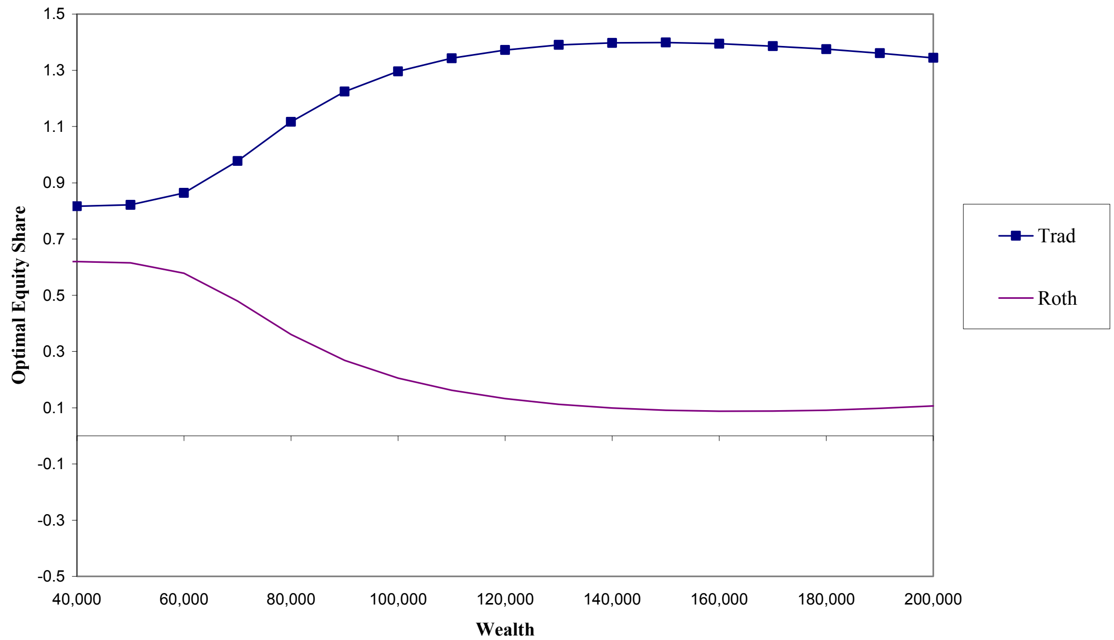

Figure 8: Optimal Equity Shares in Traditional and Roth IRA, Three Years Prior to Withdrawal, Withdrawals from Traditional IRA Subject to Simplified Progressive Tax System.