INFLATION TO TARGET:
WHAT INFLATION TO TARGET?

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Abstract

This paper derives a central bank's objective function and optimal policy rule for an economy with both CPI and PPI inflation rates. It implements constrained-optimal policy rules with minimal information requirement, and evaluates the robustness of these simple rules when the central bank may not know the exact sources of shocks or nominal rigidities. One of the main findings is that monetary policy that ignores PPI inflation rate or PPI sector shocks can result in significant welfare loss.

1 Introduction

Stabilizing the variability in inflation and output gap has been an important goal for many central banks around the world. It has also been viewed as the objective of a central bank in most studies of optimal monetary policy rules. In both policy practice and academic research, the inflation target, either explicit or implicit, is almost uniformly measured by the cost of living index, the CPI, even though the cost of production index, the PPI, is also readily observable and the cyclical behaviors of the two measures of inflation are quite different. Table 1 presents evidence that most countries that have adopted an explicit inflation-targeting policy have been targeting CPI inflation or its variants (see also the comprehensive survey in Bernanke, et al. (1999)). Table 2 presents the different cyclical behaviors of the two measures of inflation in the U.S. economy. It reveals that CPI inflation is in general less volatile and more persistent than PPI inflation (see also Clark (1999)).[1]

In the context of the "New Keynesian Synthesis," many authors have argued that, by stabilizing fluctuations in CPI inflation, the central bank could effectively stabilize the variability of the output gap that measures the deviation of actual output from its natural rate level. If the natural rate is close to being optimal, the argument goes, such a policy would then be welfare improving and hence desirable.[2] The reasoning behind such arguments is typically based upon a dynamic general equilibrium model with some sources of nominal rigidity, and is thus built on microeconomic foundations. The basic model is flexible enough to allow for several sources of nominal rigidities in the form of sticky prices in multiple sectors [e.g., Mankiw and Reis (2002)] or in multiple countries [e.g., Benigno (forthcoming) and Clarida, et al. (2002)], or in the form of sticky prices and sticky nominal wages [e.g., Erceg, et al. (2000) and Amato and Laubach (2003)]. An important insight from these studies is that, in the presence of multiple sources of nominal rigidities, complete stability of CPI inflation does not always lead to stability of output gap, because of a tradeoff between stabilizing output gap and relative price gaps. In a recent survey of this literature, Woodford (2003a, Chapter 6) notes that "the question of which price index it is most desirable to stabilize remains an important topic for further study."

In the spirit of this strand of literature, our paper analyzes the design and implementation of optimal monetary policy in a DSGE model with multiple sources of nominal rigidities and

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[1]One possible reason why CPI is less volatile than PPI is that the central bank has been targeting CPI. But this pattern holds even for the period in the mid 1930's [see Means (1935)], casting doubt on the role of monetary policy in shaping the cyclical behaviors of CPI and PPI.

[2]See, for example, Clarida, et al. (1999), Goodfriend and King (2001), and Woodford (2003a), among others.

therefore multiple price indices for the monetary authority to consider stabilizing. As a key point of departure from the literature, however, our model features an input-output linkage between sectors that is supported by empirical evidence yet remarkably overlooked in the literature.[3]

In the model, final consumption goods are produced through two stages of processing. At each stage of processing, there is a continuum of firms producing differentiated goods. The prices of both intermediate production inputs and final consumption goods are determined by staggered nominal contracts. The price index of the intermediate goods corresponds broadly to the PPI, while that of the finished goods corresponds to the CPI. We derive the objective function of a benevolent central bank from the first principle, and, under this objective, we characterize optimal monetary policy and compare the welfare implications of several simple interest-rate rules.

Our analytical results reveal that, along with variations in CPI inflation and output gap, the central bank should also care about variations in PPI inflation and the gap of the real marginal cost in the production of intermediate goods. Variation in the real marginal cost gap enters the benevolent CB's loss function as a separate term, which cannot possibly be rewritten as a combination of the other three terms in the loss function, because fluctuations in the relative price of intermediate goods to final goods have an allocative role. This stands in contrast to the two-sector model of Aoki (2001) featuring a single source of nominal rigidity, so that fluctuations in the sectoral relative price have no allocative role, and the first-best allocation can be achieved. Here, to achieve Pareto optimal allocations would require not only complete stabilization of the output gap and CPI inflation rate, but also complete stabilization of the PPI inflation rate and the marginal-cost gap, and thus of the relative-price gap. We show that it is impossible for monetary policy to attain the Pareto optimum allocation except in the special cases where the two sectors are buffeted by identical productivity shocks, or the prices of intermediate goods or finished goods are flexible, or the processing of finished goods does not require the use of primary factors. In the latter two cases, fluctuations in the sectoral relative price have no allocative role. In general, the central bank faces tradeoffs in stabilizing the four components in its optimally derived objective function: the output gap, CPI inflation, PPI inflation, and the marginal-cost gap.

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[3]For a DSGE model with multiple stages of processing and the implications of the input-output connections on monetary policy transmission, see Huang and Liu (2001) and the references therein.

Since the first-best allocation is in general not attainable, the optimality of a monetary policy depends on the relative weights assigned to the four components that the central bank should care about in its objective function. In contrast, in a standard one-sector model with staggered price setting, the weights assigned to the components in policymakers' objective functions are irrelevant for the determination of optimal monetary policy. In such a one-sector model, the central bank faces no tradeoffs in stabilizing the output gap and CPI inflation, since keeping constant the CPI inflation rate would also eliminate variations in the output gap. In fact, the first-best welfare levels are obtainable in this class of models under a remarkably simple policy of extreme CPI inflation targeting (e.g., Goodfriend and King (2001)).[4]

The weights assigned to the output gap and CPI inflation in a policymaker's objective function in our current model are similar to those in a standard one-sector model. The weights assigned to PPI inflation and the marginal-cost gap depend on the share of intermediate goods used in the processing of finished goods. Denote the share by ϕ. The weight on PPI inflation is increasing in ϕ, while the weight on the marginal-cost gap is a concave function of ϕ and achieves its maximum at $\phi = 0.5$. Therefore, a greater value of ϕ leads the central bank to care more about the variability in PPI inflation, while a moderate value of ϕ results in its most concern about the variation in the marginal cost.

This role played by ϕ in shaping the policymaker's objective function is a unique feature of our model with a vertical input-output structure. If ϕ is smaller than 0.5, intermediate production inputs become less important in the processing of final consumption goods, and thus the central bank should be less concerned with variations in both PPI inflation and the marginal cost faced by intermediate good producers. If ϕ is greater than 0.5, intermediate inputs become more important, and the central bank should be more concerned with variations in PPI inflation. However, as PPI inflation receives more direct attention, less attention needs to be paid to the marginal cost gap, since variations in the former are attributable in part to variations in the latter. This implication of the input-output structure on the

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[4]Woodford (2003a) presents a few examples where optimal monetary policy faces a tradeoff between stabilizing the inflation rate and the output gap, even if there is a single source of nominal rigidity. The examples include introducing exogenous cost-push shocks or imposing conditions that prevent nominal interest rate from hitting the zero lower bound. As we will make clear below, in our model, a "cost-push" term arises endogenously from the input-output connections, a unique feature of our model.

central bank's objective function has significant consequences on the determination of optimal monetary policy.[5]

The optimal monetary policy using short nominal interest rate as an instrument is characterized by a very complicated rule. To implement the optimal monetary policy using such a rule requires the central bank to possess perfect information about the leads and lags of the inflation rates and the gaps. A daunting task! Nevertheless, the welfare level under the optimal monetary policy provides a natural benchmark that can be used to evaluate the performance of alternative interest-rate rules that are easily implementable using the information set of the policymaker. We examine various such simple rules. We find that a hybrid inflation-targeting rule that sets the short nominal interest rate to respond to variations in both CPI inflation and PPI inflation induces a welfare level that is very close to the second best (i.e., to that under the optimal monetary policy). The incorporation of output gap as an additional targeting variable does not produce marked changes in the level of welfare. In contrast, an optimal Taylor rule, which targets variations in CPI inflation and the output gap, can result in substantial welfare losses compared to the second best. In general, a policy rule that ignores PPI inflation tends to generate greater welfare losses than a hybrid rule that targets an "inflation index" that includes both CPI and PPI inflation. In the policy reaction function under the hybrid inflation-targeting rule, the relative weights assigned to the CPI and PPI inflation rates depend on the intermediate-input share ϕ, along with other structural parameter values. Under plausible parameter values, the relative weight assigned to PPI inflation in the optimal inflation index lies between 0.4 and 0.5.

By constructing the model with two stages of processing, we are also able to analyze the sensitivity of the welfare losses when the central bank's perceived sources of shocks or of nominal rigidities may differ from the actual sources. To do this, we maintain that both CPI and PPI are sticky in the baseline model. We find that, when the central bank does not know

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[5]In a model with sticky prices and sticky wages (call it SP-SW), such as the one in Erceg, et al. (2000) and Amato and Laubach (2003), optimal monetary policy also feature a tradeoff between stabilizing the output gap, the price inflation, and the wage inflation. Such a model and its implications on monetary policy, however, do not correspond to any special cases of our model. In the general case with $0 < \phi < 1$, the SP-SW model has no counterpart of the marginal cost gap that appears in the central bank's objective function in our model. Even in the special case with $\phi = 1$, when the marginal cost gap drops out of the objective function, the implications on monetary policy still differ in the two types of models: as we show below in Section 3, the policy tradeoff disappears and the first-best allocation is attainable if $\phi = 1$, but there is always a tradeoff in the model with sticky prices and sticky wages.

the actual source of shocks, one way to avoid big welfare losses is to assume that the shocks hit both sectors or just the PPI sector, and to formulate an optimal interest rate rule that target the inflation index under such assumption. In other words, the loss would be small even if such an assumption was wrong. In contrast, the potential welfare loss would be large if the central bank formulate its policy based on the belief that the shock hits just the CPI sector, and the belief turns out to be wrong. We also find that, if the monetary policy is formulated based on the assumption that only one sector has sticky prices (either the CPI sector or the PPI sector) while the truth is that stickiness lies in both sectors, the potential welfare losses are large regardless of the source of shocks.

The paper is organized as follows. Section 2 presents the model and Section 3 describes equilibrium dynamics. Section 4 characterizes optimal monetary policy and derives a utility-based objective function for a benevolent central bank. Section 5 discusses implementation of the optimal policy and Section 6 examines the potential welfare losses when the central bank's perceived sources of shocks or of nominal rigidities may differ from the actual sources. Section 7 considers alternative policy objectives. Section 8 presents evidence that supports the baseline model based on an inflation accounting exercise. Section 9 concludes.

2 The Model

In the model economy, there is a large number of identical and infinitely lived households. The representative household is endowed with one unit of time and derives utility from consumption and leisure. The production of consumption goods goes through two stages of processing, from intermediate goods to finished goods. At each processing stage, there is a large number of firms producing differentiated products. The production of intermediate goods requires labor as the only input, while the production of finished goods requires both labor and a composite of intermediate goods as inputs. The final consumption good is a composite of differentiated finished goods.

2.1 The Household

The representative household has a utility function given by

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t[U(C_t) - V(N_t)], \tag{1}$$

where E is an expectations operator, $\beta \in (0,1)$ is a subjective discount factor, and C_t and N_t are consumption and labor hours, respectively. In each period t, the household faces a budget

constraint

$$\bar{P}_{ft}C_t + \mathrm{E}_t D_{t,t+1} B_{t+1} \leq W_t N_t + \Pi_t + B_t - T_t, \tag{2}$$

where B_{t+1} denotes the holdings of a one-period state-contingent nominal bond that matures in period $t+1$ with a payoff of one unit of currency in the appropriate event, $D_{t,t+1}$ is the period-t cost of such bonds, W_t is the nominal wage rate, Π_t is a claim to all firms' profits, and T_t is a lump-sum tax. The consumption good is a composite of differentiated finished goods. In particular,

$$C_t = \left[\int_0^1 Y_{ft}(j)^{\frac{\theta_f - 1}{\theta_f}} dj \right]^{\frac{\theta_f}{\theta_f - 1}}, \tag{3}$$

where $Y_{ft}(j)$ denotes the output of finished good j and $\theta_f > 1$ is the elasticity of substitution between the differentiated finished goods. Solving the household's expenditure-minimization problem results in the demand schedule for finish good j given by

$$Y_{ft}^d(j) = \left[\frac{P_{ft}(j)}{\bar{P}_{ft}} \right]^{-\theta_f} C_t, \tag{4}$$

where $P_{ft}(j)$ denotes the price of good j and the consumer price index $\bar{P}_{ft}$ is given by

$$\bar{P}_{ft} = \left[\int_0^1 P_{ft}(j)^{1-\theta_f} dj \right]^{\frac{1}{1-\theta_f}}. \tag{5}$$

Solving the household's utility-maximization problem results in a labor supply equation and an intertemporal Euler equation, given respectively by

$$\frac{W_t}{\bar{P}_{ft}} = \frac{-V_{nt}}{U_{ct}}, \tag{6}$$

and

$$D_{t,\tau} = \beta^{\tau - t} \frac{U_{c\tau}}{U_{ct}} \frac{\bar{P}_{ft}}{\bar{P}_{f\tau}}, \tag{7}$$

where U_{ct} is the marginal utility of consumption, and V_{nt} is the marginal disutility of working. Let $R_t = [\mathrm{E}_t D_{t,t+1}]^{-1}$ denote the nominal return on a risk-free bond (i.e., the nominal interest rate). It follows from (7) that

$$U_{ct} = \beta \mathrm{E}_t U_{c,t+1} R_t \frac{\bar{P}_{ft}}{\bar{P}_{f,t+1}}. \tag{8}$$

2.2 Firms and Optimal Price-Setting

To produce a type j finished good requires inputs of labor and a composite of intermediate goods, with a constant returns to scale (CRS) technology given by

$$Y_{ft}(j) = \bar{Y}_{mt}(j)^{\phi} (A_{ft} N_{ft}(j))^{1-\phi}, \tag{9}$$

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where $\bar{Y}_m(j) = [\int_0^1 Y_m(j,i)^{\frac{\theta_m-1}{\theta_m}} di]^{\frac{\theta_m}{\theta_m-1}}$ denotes the input of composite intermediate goods used by j, $N_f(j)$ is the input of homogeneous labor, $\theta_m > 1$ is the elasticity of substitution between differentiated intermediate goods, and A_{ft} is a productivity shock to the finished good sector.

To produce a type i intermediate good requires labor as the only input, with a CRS technology

$$Y_{mt}(i) = A_{mt} N_{mt}(i), \tag{10}$$

where $N_{mt}(i)$ is the input of homogeneous labor and A_{mt} is a productivity shock to the intermediate good sector.

The productivity shocks each follows a log-difference stationary process. In particular, we assume that

$$\ln(A_{k,t+1}/A_{k,t}) = \rho_k \ln(A_{kt}/A_{k,t-1}) + \varepsilon_{k,t+1}, \quad k \in \{f,m\}, \tag{11}$$

where ε_{ft} and ε_{mt} are mean-zero, iid normal processes that are mutually independent, with finite variances given by σ_f^2 and σ_m^2, respectively.

Firms are price-takers in the input markets and monopolistic competitors in the product markets. Within each processing stage, firms set prices in a staggered fashion in the spirit of Calvo (1983). In particular, in period t, all firms receive an iid random signal that determines whether or not they can set a new price. The probabilities that firms in the finished good sector and the intermediate good sector can adjust prices are $1 - \alpha_f$ and $1 - \alpha_m$, respectively. Thus, by the law of large numbers, a fraction $1 - \alpha_k$ of firms in sector $k \in \{f,m\}$ can adjust prices while the rest of the firms have to stay put.

If a finished good producer j can set a new price in period t, it chooses the new price $P_{ft}(j)$ to maximize the expected present value of its profits

$$E_t \sum_{\tau=t}^{\infty} \alpha_f^{\tau-t} D_{t,\tau} [P_{ft}(j)(1 + \tau_f) - V_{f\tau}] Y_{f\tau}^d(j), \tag{12}$$

where τ_f denotes a subsidy to finished good producers, $V_{f\tau}$ is the unit production cost, and $Y_{f\tau}^d(j)$ is the demand schedule for $j's$ output given by (4). A firm has to solve a cost-minimization problem, taking the input prices as given, regardless of whether it can adjust its price. The solution yields the factor demand functions

$$Y_{mt}^d(i) = \phi \frac{V_{ft}}{\bar{P}_{mt}} \left[\frac{P_{mt}(i)}{\bar{P}_{mt}}\right]^{-\theta_m} \int_0^1 Y_{ft}^d(j) dj, \tag{13}$$

$$N_{ft}^d = (1 - \phi) \frac{V_{ft}}{W_t} \int_0^1 Y_{ft}^d(j) dj, \tag{14}$$

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where $\bar{P}_{mt} = [\int_0^1 P_{mt}(i)^{1-\theta_m} di]^{\frac{1}{1-\theta_m}}$ is the price index of intermediate goods, that is, the producer price index (PPI), and the unit cost function is given by

$$V_{ft} = \bar{\phi} \bar{P}_{mt}^{\phi} \left(\frac{W_t}{A_{ft}} \right)^{1-\phi}, \tag{15}$$

where $\bar{\phi}$ is a constant determined by ϕ. Solving (12) gives the optimal pricing decision rule

$$P_{ft}(j) = \frac{\mu_f}{(1+\tau_f)} \frac{\mathrm{E}_t \sum_{\tau=t}^{\infty} \alpha_f^{\tau-t} D_{t,\tau} V_{f\tau} Y_{f\tau}^d(j)}{\mathrm{E}_t \sum_{\tau=t}^{\infty} \alpha_f^{\tau-t} D_{t,\tau} Y_{f\tau}^d(j)}, \tag{16}$$

where $\mu_f = \theta_f/(\theta_f - 1)$ measures the markup. The optimal price is thus an effective markup (adjusted for subsidy) over a weighted average of the marginal costs in the future periods during which the price is expected to remain in effect. Similarly, the optimal pricing decision for an intermediate good producer i who can adjust its price in period t is given by

$$P_{mt}(i) = \frac{\mu_m}{(1+\tau_m)} \frac{\mathrm{E}_t \sum_{\tau=t}^{\infty} \alpha_m^{\tau-t} D_{t,\tau} V_{m\tau} Y_{m\tau}^d(i)}{\mathrm{E}_t \sum_{\tau=t}^{\infty} \alpha_m^{\tau-t} D_{t,\tau} Y_{m\tau}^d(i)}, \tag{17}$$

where $\mu_m = \theta_m/(\theta_m - 1)$ is a markup, τ_m denotes a subsidy to intermediate good producers, the demand function $Y_{m\tau}^d(i)$ is given by (13), and the unit cost function is obtained from cost-minimization and is given by

$$V_{m\tau} = \frac{W_\tau}{A_{m\tau}}. \tag{18}$$

The solution to the cost-minimization problem also yields the firm's demand for labor. By aggregating the labor demand across firms in the intermediate good sector, we get

$$N_{mt}^d = \frac{1}{A_{mt}} \int_0^1 Y_{mt}^d(i) di. \tag{19}$$

Given the demand for labor in the two sectors in (14) and (19), labor market clearing implies that, in each period t,

$$N_t = N_{ft}^d + N_{mt}^d. \tag{20}$$

The bond market clearing implies that $B_t = 0$ for all t. The markets for the composite goods also clear in an equilibrium. Finally, the production subsidies are financed by lump-sum taxes so that $T_t = \tau_f \bar{P}_{ft} C_t + \tau_m \bar{P}_{mt} \bar{Y}_{mt}$, where $\bar{Y}_m = [\int_0^1 Y_m^d(i)^{\frac{\theta_m-1}{\theta_m}} di]^{\frac{\theta_m}{\theta_m-1}}$ is the composite of all intermediate goods.

Since our objective is to find an optimal monetary policy in this economy and to compare the welfare implications of alternative monetary policy rules, we do not specify a particular policy here. Under any given monetary policy, we can define an equilibrium in this economy. An *equilibrium* consists of allocations C_t, N_t, B_{t+1} for the representative household; allocations

$Y_{ft}(j)$, $\bar{Y}_{mt}(j)$, and $N_{ft}(j)$, and price $P_{ft}(j)$ for finished good producer $j \in [0,1]$; allocations $Y_{mt}(i)$ and $N_{mt}(i)$, and price $P_{mt}(i)$ for intermediate good producer $i \in [0,1]$; together with prices $D_{t,t+1}$, $\bar{P}_{ft}$, $\bar{P}_{mt}$, and wage W_t, that satisfy the following conditions: (i) taking the prices and the wage as given, the household's allocations solve its utility maximizing problem; (ii) taking the wage and all prices but its own as given, each finished good producer's allocations and price solve its profit maximizing problem; (iii) taking the wage and all prices but its own as given, each intermediate good producer's allocations and price solve its profit maximizing problem; and (iv) markets for bonds, labor, and the composite goods produced at each processing stage clear.

3 Equilibrium dynamics

We focus on a symmetric equilibrium in which a firm in each sector is identified by the time at which it can set a new price. Thus, we can drop the individual indexes of firms and denote P_{ft} the price of finished goods, P_{mt} the price of intermediate goods, etc..

3.1 The steady state

Since there is no trend-growth in productivity, a steady state in this economy obtains if $A_m = A_f = 1$. In the steady state, the optimal pricing rules (16) and (17) reduce to

$$P_f = \frac{\mu_f}{1 + \tau_f} V_f, \quad P_m = \frac{\mu_m}{1 + \tau_m} V_m. \tag{21}$$

In addition, symmetry implies that the pricing decision in each sector coincides with the sector's price index. Using the expressions for the unit cost functions (15) and (18), we can obtain a solution for the steady state real wage, which, along with the household's optimal labor supply decision (6), lead to

$$\frac{-V_n(N)}{U_c(C)} = (1 - \Phi_f)(1 - \Phi_m)^\phi \tilde{\phi}^{-1} \equiv (1 - \Phi)\tilde{\phi}^{-1}, \tag{22}$$

where $1 - \Phi_k \equiv (1 + \tau_k)/\mu_k$ for $k \in \{f, m\}$, and it measures an inefficiency wedge caused by monopolistic competition and distortionary subsidies.

To solve for aggregate employment, we first use the labor demand equations (14) and (19), and the steady state relations $Y_f(j) = Y_f = \bar{C}$, to obtain

$$N_m = \phi\tilde{\phi}(1 - \Phi_m)^{1-\phi} C, \quad N_f = (1 - \phi)\tilde{\phi}(1 - \Phi_m)^{-\phi} C. \tag{23}$$

Aggregate employment is then obtained by summing up the labor demand of each sector:

$$N = N_m + N_f = \eta C, \tag{24}$$

where $\eta \equiv \tilde{\phi}(1 - \phi\Phi_m)/(1 - \Phi_m)^{\phi}$.

Finally, we obtain solutions for steady state consumption and employment using (22) and (24).

3.2 The flexible-price equilibrium and the natural rate

With flexible prices, the pricing decisions are synchronized across firms so that we can follow a similar procedure as in solving the steady state equilibrium to obtain solutions for the flexible-price equilibrium. In particular, the optimal pricing rules (16) and (17) imply that the real marginal cost in each sector is a constant. This observation, along with the expressions for the unit cost functions (15) and (18), and the labor supply decision (6), leads to

$$\frac{-V_n(N_t^*)}{U_c(C_t^*)} = (1 - \Phi)\tilde{\phi}^{-1}A_{mt}^{\phi}A_{ft}^{1-\phi}, \tag{25}$$

where N_t^* and C_t^* denote the aggregate employment and consumption in the flexible-price equilibrium, and the right-hand-side is simply the solution for the real consumption wage.

The sectoral labor demand functions (14) and (19) imply that

$$N_{mt}^* = \phi \frac{V_{ft}^*}{P_{mt}^* A_{mt}} C_t^*, \quad N_{ft} = \frac{1 - \phi}{1 - \Phi_m} \frac{V_{ft}^*}{P_{mt}^* A_{mt}} C_t^*. \tag{26}$$

The aggregate employment is then given by the sum of the sectoral demand:

$$N_t^* = N_{mt}^* + N_{ft}^* = \frac{1 - \phi\Phi_m}{1 - \Phi_m} \frac{V_{ft}^*}{P_{mt}^* A_{mt}} C_t^*. \tag{27}$$

Thus, the sectoral employments are proportional to aggregate employment.

Next, we combine the optimal pricing equation and the unit cost function in the finished good sector to obtain a solution to the relative price of intermediate goods

$$Q_t^* = \frac{1 - \Phi_f}{\tilde{\phi}(1 - \Phi_m)^{1-\phi}} \left(\frac{A_{ft}}{A_{mt}}\right)^{1-\phi}, \tag{28}$$

where $Q_t^* \equiv P_{mt}^*/P_{ft}^*$ is the relative price of intermediate goods in units of consumption good.

Given the solution for the relative price Q_t^*, along with the fact that the real marginal cost V_{ft}^*/P_{ft}^* is a constant (equal to $1 - \Phi_f$), we can obtain a solution for the term V_{ft}^*/P_{mt}^* in (27) and thus express N_t^* as a function of C_t^* only:

$$N_t^* = \eta \frac{C_t^*}{A_{mt}^{\phi} A_{ft}^{1-\phi}}. \tag{29}$$

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Finally, combining (25) and (29) gives the solutions for N_t^* and C_t^*.

A more explicit closed-form solution can be obtained when we log-linearized these equilibrium conditions around the steady state. In particular, the log-linearized version of (25) is given by

$$\omega n_t^* + \sigma c_t^* = \phi a_{mt} + (1 - \phi)a_{ft}, \tag{30}$$

where $\omega = \frac{V_{nn}(N)N}{V_n(N)}$ and $\sigma = \frac{-U_{cc}(C)C}{U_c(C)}$ denote the relative risk aversion with respect to labor hours and consumption (evaluated at the steady state), and a lowercase variable denotes the log-deviation of the corresponding level from its steady state value. The log-linear version of (29) is given by

$$n_t^* = c_t^* - [\phi a_{mt} + (1 - \phi)a_{ft}]. \tag{31}$$

This equation, along with (30), results in the solution for aggregate consumption or real GDP given by

$$c_t^* = \frac{1 + \omega}{\omega + \sigma}[\phi a_{mt} + (1 - \phi)a_{ft}]. \tag{32}$$

In what follows, we refer to c_t^* as the "natural rate" of output, since it is the equilibrium real GDP without sticky-price distortions. Once the solution for consumption is obtained, we can use a log-linearized version of the household's intertemporal Euler equation (8) to solve for the equilibrium real interest rate. In particular, the linearized intertemporal Euler equation is given by

$$c_t^* = \mathrm{E}_t c_{t+1}^* - \sigma^{-1}(r_t^* - \mathrm{E}_t \pi_{f,t+1}^*), \tag{33}$$

where r_t^* and π_{ft}^* denote the nominal interest rate and the CPI inflation rate, respectively. Let $rr_t^* = r_t^* - \mathrm{E}_t \pi_{f,t+1}^*$ denote the (ex-ante) real interest rate. It follows from (32), (33), and the shock processes (11) that

$$rr_t^* = \phi \rho_m \Delta a_{mt} + (1 - \phi)\rho_f \Delta a_{ft}, \tag{34}$$

where $\Delta a_{kt} = a_{kt} - a_{k,t-1}$ is the productivity growth rate in sector $k \in \{f, m\}$.

3.3   The sticky-price equilibrium

We now characterize the sticky-price equilibrium. We begin with defining some notations. Let $\tilde{v}_{kt} = \ln(V_{kt}/P_{kt}) - \ln(V_k/P_k)$ denote the log-deviation of sector k's real marginal cost from steady state, for $k \in \{f, m\}$, $\tilde{q}_t = \ln(Q_t/Q) - q_t^*$ denote the "relative-price gap," and $\tilde{c}_t = \ln(C_t/C) - c_t^*$ denote the "output gap." The real marginal costs involve both the real consumption wage and the relative price of the basket of intermediate goods. The real wage is

related to the output gap and aggregate employment through the labor supply equation. Without loss of generality, we assume that $\omega = 0$ (corresponding to linear preferences in labor hours, which can be justified by labor indivisibility). Under this assumption, it is straightforward to show that the real marginal costs are related to the gaps by

$$\tilde{v}_{ft} = \phi\tilde{q}_t + (1 - \phi)\sigma\tilde{c}_t, \quad \tilde{v}_{mt} = \sigma\tilde{c}_t - \tilde{q}_t. \tag{35}$$

Next, we log-linearize the optimal pricing decision rules (16) and (17) around a zero-inflation steady state and make use of the log-linearized relations between the price indices and pricing decisions in both sectors to get

$$\pi_{ft} = \beta E_t \pi_{f,t+1} + \kappa_f(\phi\tilde{q}_t + (1 - \phi)\sigma\tilde{c}_t), \tag{36}$$

$$\pi_{mt} = \beta E_t \pi_{m,t+1} + \kappa_m(\sigma\tilde{c}_t - \tilde{q}_t), \tag{37}$$

where $\kappa_k = (1 - \beta\alpha_k)(1 - \alpha_k)/\alpha_k$ for $k \in \{f, m\}$.

Then, by log-linearizing the intertemporal Euler equation (8) around steady state and subtract the flexible-price counterpart (33) from the resulting equation, we can obtain an Euler equation in terms of the gaps:

$$\tilde{c}_t = E_t\tilde{c}_{t+1} - \sigma^{-1}(r_t - E_t\pi_{f,t+1} - rr_t^*), \tag{38}$$

where r_t and π_{ft} are the log-deviations of the nominal interest rate and the CPI inflation rate from steady state and rr_t^* is the real interest rate in the flexible-price equilibrium given by (34).

Finally, the law of motion of the relative price gap is given by

$$\tilde{q}_t = \tilde{q}_{t-1} + \pi_{mt} - \pi_{ft} - \Delta q_t^*, \tag{39}$$

where $\Delta q_t^* = q_t^* - q_{t-1}^*$ and, by log-linearizing (28) around steady state, it is given by

$$\Delta q_t^* = (1 - \phi)(\Delta a_{ft} - \Delta a_{mt}). \tag{40}$$

Clearly, if the shocks are identical across the two sectors (i.e., $\Delta a_{ft} = \Delta a_{mt}$), or if intermediate goods are the only input for the finished good sector (i.e., $\phi = 1$), then the relative price in the flexible-price equilibrium does not respond to the shocks.

For any given monetary policy, equations (36)-(39) fully characterize the equilibrium dynamics under sticky prices.

4 Optimal monetary policy

We now turn to examining the issue of optimal monetary policy. In the model, there are two sources of inefficiencies. One comes from monopolistic competition, under which there is a steady state markup distortion; and the other comes from staggered price-setting, with which dynamic equilibrium fluctuations are possibly inefficient. Our purpose is to analyze the stabilizing properties of monetary policy rules in the dynamic equilibrium. Thus, without loss of generality, we assume that the production subsidies exactly offset the steady-state markup distortions, so that the only possible source of inefficiency would be staggered price setting and that, if prices were allowed to adjust instantaneously, the equilibrium allocation would be Pareto optimal. Under this assumption, an equilibrium allocation is Pareto optimal only if the relative price gap and output gap are both zero. A natural question then arises:

4.1 Can monetary policy attain the first-best allocation?

The answer to this question is negative for general parameter values and shock processes. The following proposition formally establishes this result.

Proposition 1. *In the baseline model with sticky prices in both sectors and with labor being used in the production of both intermediate goods and finished goods (i.e., with $0 < \phi < 1$), there exists no monetary policy that can attain the Pareto optimal allocation unless the two sectors are buffeted by identical shocks.*

Proof: Suppose there were a monetary policy under which the equilibrium allocation under sticky prices would be Pareto optimal. Then, in such an equilibrium, the gaps would be completely closed for every period. That is, $\tilde{c}_t = \tilde{q}_t = 0$ for all t. It follows from (36) and (37) that $\pi_{ft} = \pi_{mt} = 0$ for all t. Since we also have $\Delta\tilde{q}_t = 0$, (39) and (40) imply that $\pi_{mt} - \pi_{ft} = (1 - \phi)(\Delta a_{ft} - \Delta a_{mt})$, contradicting that $\pi_{ft} = \pi_{mt} = 0$ unless $\Delta a_{ft} = \Delta a_{mt}$ for all t.

Q.E.D.

Corollary 1. *If $\phi = 1$, that is, when no labor is required in producing final goods, there is no policy tradeoff and the Pareto optimal allocation is attainable regardless of the source of shocks.*

The Corollary provides a counter-example to the conventional view that, in a two-sector models, the first-best allocation cannot be achieved if the prices in both sectors are sticky [e.g., Erceg, et al. (2000) and Woodford (2003a)]. A key difference of our model from the standard two-sector models in the literature is the input-output connections between the sectors. As the

share of intermediate input ϕ takes an extreme value of one, the relative price in the efficient equilibrium would not respond to the shocks.

But in general, if $\phi \in (0, 1)$, the first-best allocation cannot be attained.[6] The main reason is that, in the efficient equilibrium, both output and the relative price of intermediate goods to final goods fluctuate in response to productivity shocks unless the two shocks are identical, in which case, only output would fluctuate. The fluctuations in output and in the relative price in the efficient equilibrium create a trade-off facing the monetary authority: it can stabilize either the output gap or the relative price gap, but not both. Since fluctuations in the relative price in the sticky-price equilibrium have an allocative role, Pareto optimal allocation is not attainable.

This result stands in contrast to that obtained in a standard one-sector model, which predicts that the Pareto optimal allocation can be attained by complete stabilization of CPI inflation, since it also leads to complete stabilization of the output gap [e.g., Clarida, et al. (1999), Goodfriend and King (2001), and Woodford (2003a)]. In the one-sector models, however, a tradeoff between stabilizing output gap and inflation can arise if an ad-hoc "cost-push shock" is introduced in the Phillips-curve relation [e.g., Clarida, et al., 1999]. In our model, there is no ad-hoc cost-push shocks. Yet, a "cost-push" term arises endogenously in the finished good sector's Phillips-curve relation (36). To see this, we rewrite the real marginal cost in the finished good sector in (35) to obtain $\tilde{v}_{ft} = \sigma \tilde{c}_t - \phi \tilde{v}_{mt}$, where the first term is the output gap, just as in the one-sector model, and the second term corresponds to a cost-push "shock," which is here determined by the real marginal cost in the intermediate good sector. As we will show below, the real marginal cost in the intermediate good sector plays an important role in the objective function that a benevolent central bank tries to minimize.

4.2 A utility-based objective function for optimal monetary policy

Given that the Pareto optimal allocation is in general not attainable, a natural question arises: What is a second-best monetary policy? To answer this question requires to have a well-defined welfare criterion, or an objective function for the central bank. We now formally derive such an objective function based on the representative household's utility function.

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[6]In the extreme case with $\phi = 0$, one sector would be shut off, and the model would reduce to a standard one-sector model.

By replacing the event argument with a time subscript, we can rewrite the household's utility function (1) as

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t U_t, \quad \text{where} \quad U_t = U(C_t) - V(N_t). \tag{41}$$

We begin with a second-order approximation of the period utility $U(C_t)$ around steady state:

$$U(C_t) = U_c(C)C\left(c_t + \frac{1-\sigma}{2}c_t^2\right) + t.i.p. + \mathcal{O}(\|a\|^3), \tag{42}$$

where c_t denotes the log-deviation of consumption (in the sticky-price equilibrium) from steady state, $t.i.p.$ refers to the terms independent of policy, and $\mathcal{O}(\|a\|^3)$ summarizes all terms of the third or higher orders. Since the output gap is defined as $\tilde{c}_t = c_t - c_t^*$, the period utility function can be expressed in terms of the output gap

$$U(C_t) = U_c(C)C\left(\tilde{c}_t + \frac{1-\sigma}{2}\tilde{c}_t^2 + (1-\sigma)c_t^*\tilde{c}_t\right) + t.i.p. + O(\|a\|^3), \tag{43}$$

where c_t^* denotes the log-deviation of consumption in the flexible-price equilibrium from steady state.

We next approximate the period disutility function of working, which, after imposing the labor market clearing conditions (20), is given by $V(N_t) = V(N_{ft} + N_{mt})$. Taking a second-order approximation around steady state leads to

$$V(N_t) = V_n(N)N\left\{(1-\phi)n_{ft} + \phi n_{mt} + \frac{1}{2}[(1-\phi)n_{ft}^2 + \phi n_{mt}^2]\right\} + t.i.p. + \mathcal{O}(\|a\|^3), \tag{44}$$

where we have used the steady state ratios $N_f/N = 1-\phi$ and $N_m/N = \phi$, and we have also set the relative risk aversion parameter $\omega = V_{nn}N/V_n$ to zero to simplify expressions. To express $V(N_t)$ in terms of the gaps, we use the definitions $\tilde{n}_{ft} = n_{ft} - n_{ft}^*$ and $\tilde{n}_{mt} = n_{mt} - n_{mt}^*$, along with the flexible-price equilibrium condition $n_{ft}^* = n_{mt}^* = n_t^*$, to obtain

$$V(N_t) = V_n(N)N\left\{(1-\phi)\tilde{n}_{ft} + \phi\tilde{n}_{mt} + \tfrac{1}{2}[(1-\phi)\tilde{n}_{ft}^2 + \phi\tilde{n}_{mt}^2] + n_t^*[(1-\phi)\tilde{n}_{ft} + \phi\tilde{n}_{mt}]\right\}$$
$$+ t.i.p. + \mathcal{O}(\|a\|^3). \tag{45}$$

We now use the labor demand equations (14) and (19) and their flexible-price counterparts to express the sectoral employment gaps in terms of the output gap and the relative price gap, and obtain

$$\tilde{n}_{ft} = \phi\tilde{q}_t + (1-\phi\sigma)\tilde{c}_t + \ln(G_{ft}), \tag{46}$$

$$\tilde{n}_{mt} = -(1-\phi)\tilde{q}_t + (1+(1-\phi)\sigma)\tilde{c}_t + \ln(G_{ft}) + \ln(G_{mt}), \tag{47}$$

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where we have imposed the unit cost functions in (15) and (18), and the variable $G_{kt} \equiv \int_0^1 (P_{kt}(i)/P_{kt})^{-\theta_k} di$ measures the price-dispersions caused by staggered price setting in sector $k \in \{f, m\}$. A second-order approximation of $\ln(G_{kt})$ around steady state yields

$$\ln(G_{kt}) = \frac{\theta_k}{2} \int_0^1 [\ln(P_{kt}(i)) - \ln(\bar{P}_{kt})]^2 di + \mathcal{O}(\|a\|^3) \equiv \frac{\theta_k}{2} \sigma_{kt}^2 + \mathcal{O}(\|a\|^3), \quad k \in \{f, m\}. \quad (48)$$

Substituting equations (46), (47), and (48) into (45) and using the steady state relation $U_c(C)C = V_n(N)N$, we get

$$V(N_t) = U_c(C)C \left\{ \tilde{c}_t + \frac{\theta_f}{2} \sigma_{ft}^2 + \frac{\phi\theta_m}{2} \sigma_{mt}^2 + \frac{1+\sigma^2\phi(1-\phi)}{2} \tilde{c}_t^2 + \frac{\phi(1-\phi)}{2} \tilde{q}_t^2 - \sigma\phi(1-\phi)\tilde{c}_t\tilde{q}_t + n_t^*\tilde{c}_t \right\}$$
$$+ t.i.p. + \mathcal{O}(\|a\|^3). \quad (49)$$

Finally, since (30) and (31) imply that $(1 - \sigma)c_t^* = n_t^*$, by subtracting $V(N_t)$ in (49) from $U(C_t)$ in (43), we obtain

$$U_t = -\frac{U_c(C)C}{2} \left\{ \sigma\tilde{c}_t^2 + \phi(1-\phi)(\sigma\tilde{c}_t - \tilde{q}_t)^2 + \theta_f\sigma_{ft}^2 + \phi\theta_m\sigma_{mt}^2 \right\} + t.i.p. + \mathcal{O}(\|a\|^3). \quad (50)$$

Note that the second term on the right hand side of the equation involves the gap of the real marginal cost in the intermediate good sector. Thus, given that $U_c(C) > 0$, the fluctuations in the output gap and in the marginal-cost gap, as well as the dispersion of prices, tend to lower welfare. Following a similar procedure described in Woodford (2003a), we can relate the price-dispersion terms to the variability of the inflation rates:

$$\sum_{t=0}^{\infty} \beta^t \sigma_{kt}^2 = \kappa_k^{-1} \sum_{t=0}^{\infty} \beta^t \pi_{kt}^2 + t.i.p. + \mathcal{O}(\|a\|^3), \quad k \in \{f, m\}, \quad (51)$$

where $\kappa_k = (1 - \beta\alpha_k)(1 - \alpha_k)/\alpha_k$ measures the responsiveness of the inflation rate in sector k to changes in the real marginal cost, as shown in the Phillips-curve relations (36) and (37). Substitution of (51) into (50) leads to

$$W \equiv \mathrm{E} \sum_{t=0}^{\infty} \beta^t U_t = -\frac{U_c(C)C}{2} \mathrm{E} \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + \mathcal{O}(\|a\|^3), \quad (52)$$

where W measures the welfare and the quadratic loss function is given by

$$L_t = \sigma\tilde{c}_t^2 + \phi(1-\phi)(\sigma\tilde{c}_t - \tilde{q}_t)^2 + \theta_f\kappa_f^{-1}\pi_{ft}^2 + \phi\theta_m\kappa_m^{-1}\pi_{mt}^2. \quad (53)$$

The loss function (53) reveals that the benevolent central bank should care about not only fluctuations in the output gap and CPI inflation, as what a one-sector model would suggest, but also the variability of the marginal-cost gap in the intermediate good sector and PPI

inflation. In a standard one-sector model, however, the central bank's loss function consists of only the variances of CPI inflation and the output gap, and the relative weights assigned to the two components are irrelevant for the determination of optimal monetary policy. In such a model, the monetary authority faces no tradeoff in stabilizing output gap and CPI inflation, since keeping constant the CPI inflation rate would also minimize the variability in the output gap. In fact, the first-best welfare levels can be obtained in this class of models by following a remarkably simple policy of extreme CPI inflation-targeting.

In our model, the benevolent central bank's loss function also involves the two additional terms, variations in PPI inflation and the gap of the real marginal cost in the production of intermediate goods. The variability of the real marginal cost gap is present as a separate term in the loss function, and cannot possibly be rewritten as a combination of the other three terms in the loss function, because fluctuations in the sectoral relative price have an allocative role. This stands in contrast to the two-sector model such as that in Aoki (2001), where there is a single source of nominal rigidity, so that fluctuations in the relative price have no allocative role, and the first-best allocation is attainable. Here, to achieve Pareto optimal allocation would require not only complete stabilization of the output gap and CPI inflation rate, but also complete stabilization of the PPI inflation rate and the marginal-cost gap, and thus the relative-price gap. Hence, as we have already shown in Proposition 1 and Corollary 1, it is impossible for monetary policy to attain the Pareto optimum allocation except in the special cases where the two sectors are buffeted by identical productivity shocks, or the prices of intermediate goods or finished goods are flexible, or the processing of the finished goods does not require the use of primary factors. In the latter two cases, fluctuations in the relative price play no allocative role. In general, the central bank faces tradeoffs in stabilizing the four components in its optimally derived objective function: the output gap, CPI inflation, PPI inflation, and the marginal-cost gap. Since the first-best allocation is in general not attainable, the optimality of a monetary policy depends on the relative weights assigned to the four components that the central bank should care about in its objective function.

In light of (53), the weights assigned to the output gap and CPI inflation in the central bank's objective function in our model are similar to those obtained in a standard one-sector model where the policymaker's loss function features only these two components. The optimal weights assigned to PPI inflation and the marginal-cost gap are uniquely derived from our model, and they depend on the share of intermediate input in the production of finished goods (i.e., the parameter ϕ). In the loss function L_t in (53), the weight on PPI inflation increases

with ϕ while the weight on the marginal-cost gap is a concave function of ϕ and achieves its maximum at $\phi = 0.5$. Therefore, a greater value of ϕ leads the central bank to care more about the variability in PPI inflation, while a more moderate value of ϕ (close to 0.5) would justify a greater concern about variations in the marginal-cost gap.

The role played by ϕ in shaping the policymaker's objective function is a unique feature of our model with a chain-like input-output structure. If ϕ falls below 0.5, the intermediate input becomes a less important factor in the production of the final consumption goods, and the central bank becomes less concerned about variations in both PPI inflation and the marginal cost in the intermediate good sector. If ϕ rises above 0.5, the intermediate inputs become more important, and the central bank becomes more concerned about variations in the PPI inflation. However, as PPI inflation receives more direct attention, less attention needs to be paid to the marginal cost, since variations in the former are attributable in part to variations in the latter. This implication of the input-output structure on the central bank's objective function has significant consequences on the determination of the optimal monetary policy rule, as we demonstrate below.

5   Implementing optimal monetary policy

To implement the optimal monetary policy requires the central bank to possess perfect information about the leads and lags of the inflation rates and the gaps, which is a difficult task. Nonetheless, we can compute the model's implied welfare level under the optimal monetary policy with calibrated parameters, and use this welfare level as a natural benchmark to evaluate the performance of alternative feedback interest rate rules that are feasible to implement based on the information set that the policymakers do possess. We now present our main results based on numerical simulations. The optimal policy is obtained by maximizing the welfare level defined in (52) and (53), subject to the equilibrium conditions (36)-(39).

5.1   The calibration of parameters

We begin with calibration of the model's parameters. The calibrated values are summarized in Table 3. Balanced growth requires the relative risk aversion in consumption to be unity, and thus we set $\sigma = 1$. Following the lead of Hansen (1985), we assume that labor is indivisible, implying that the representative agent's utility is linear in labor hours so that $\omega = 0$. The subjective discount factor is set to $\beta = 0.99$. Thus, with a period in the model corresponds to a

quarter, the annual real interest rate in the steady state is four percent. The empirical evidence surveyed by Taylor (1999) suggests that nominal price contracts on average last for a year. We thus set $\alpha_f = 0.75$ and $\alpha_m = 0.75$ so that the duration of the nominal contracts in the model is on average four quarters. The parameters θ_f and θ_m measure the elasticity of substitution between differentiated goods at the two processing stages. We set both parameters to 10, corresponding to a steady state markup of eleven percent, which is consistent with empirical evidence. We assume that the production functions in the two sectors exhibit constant returns to scale. Following the literature, we set the cost-share of intermediate input in final goods production to $\phi = 0.6$ (see Huang, Liu and Phaneuf (2003) for details in the calibration of ϕ). Finally, we follow the standard business cycle literature and set the AR(1) coefficient ρ_k in the productivity growth process in sector $k \in \{f, m\}$ to be 0.95, and the standard deviations of the innovations to productivity shocks σ_k to be 0.02.

5.2 Intermediate input share and optimal monetary policy

In the central bank's objective function, we have seen that the share of intermediate input in the finished good sector determines how much the policymakers should care about PPI inflation and the marginal cost gap, and therefore is potentially important in determining the welfare levels of alternative policy rules if this objective function is used to evaluate the performance of these policies. We have noted that, although the optimal weights assigned to the variances of CPI inflation and output gap do not depend on ϕ, the weight assigned to the variability of PPI inflation is increasing in ϕ while the weight on the marginal-cost gap in the intermediate good sector is a concave function of ϕ, with an interior peak. The welfare levels will thus be sensitive to the values of ϕ. We now examine the quantitative implications of ϕ on the levels of welfare under the optimal monetary policy.

The welfare measure W defined in (52) is in terms of the utility. The welfare loss measured as a percentage of steady state consumption can be obtained by dividing the utility level W by $U_c(C)C$ (and multiplied by 100). This is the quantitative measure that we use for our experiments. Figure 1 plots the sensitivity of welfare loss to changes in ϕ. The solid line denotes the welfare loss under the optimal monetary policy. For small values of ϕ, the figure shows that the loss is small; as ϕ rises from zero to a moderate level, the loss increases and reaches a peak at $\phi = 0.3$; as ϕ further rises, the welfare loss falls. We consider the range between 0.5 and 0.8 to be a plausible range for the values of ϕ. Then the welfare loss lies between 0.05 and 0.25 percent of steady state consumption. Thus, depending on the share of

intermediate inputs in the final good sector, the tradeoff between the two measures of inflation and the gaps can potentially incur significant welfare losses.

Yet, our main concern is not about how much welfare loss would be incurred under the optimal monetary policy. A more interesting question is how to implement the second-best policy. In light of the central bank's objective function given by (52) and (53), it is difficult to implement the optimal monetary policy since it would require the central bank to possess knowledge about the leads and lags of the inflation rates and the gaps. Nonetheless, we can use the welfare level under the optimal monetary policy as a benchmark to evaluate alternative simple monetary policy rules that are easier to implement.

5.3 Evaluating simple feedback interest rate rules

Simple feedback interest rate rules are often viewed as effective tools to conduct monetary policy. A particularly simple policy rule is the Taylor rule, under which the central bank sets the short-term nominal interest rate in response to fluctuations in CPI inflation and the output gap. Since the Taylor rule ignores other important variables in the objective function, especially the PPI inflation rate, it would be interesting to see how much more welfare loss would be incurred under a Taylor rule than that under the optimal policy. Figure 1 shows that, the welfare loss under the optimal Taylor rule (with the reaction coefficients in front of the targeting variables optimally chosen) is significantly larger than that under the optimal policy.[7] Under the calibrated parameters with $\phi = 0.6$, the optimal Taylor rule incurs a welfare loss of 0.3 percent of consumption, which is about 1.6 times the loss under the optimal policy.

We now investigate whether a simple interest rate rule that includes both CPI inflation and PPI inflation can perform better under the calibrated parameters. Table 4 displays the welfare losses under a set of interest rate rules that allow the short-term rate to respond to, in additional to its own lag, various combinations of fluctuations in CPI inflation, PPI inflation, and the output gap. The losses are expressed as ratios of the actual welfare losses to that under the optimal monetary policy. Evidently, an interest rate rule that targets both CPI inflation and PPI inflation (TR4 in the table) outperforms any rule that excludes either inflation measure (TR2, TR3, TR5, or TR6). Adding the output gap as an additional targeting variable (TR1) does not visibly affect the welfare results.

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[7]We have also included the lagged nominal interest rate in the Taylor rule so as to smooth interest rate fluctuations and to avoid hitting the zero lower bound [for a recent study of the desirability of interest-rate smoothing policies, see, for example, Woodford (1999, 2003b)].

Since both CPI inflation and PPI inflation are readily available in the data, and setting the short-term interest rate to respond to changes in these two measures of inflation brings the welfare level not far from the second-best, an immediate policy implication is that the central bank should be able to construct an "optimal inflation index" that is a weighted average of CPI and PPI inflation, and it can then follow a "modified Taylor rule" that replaces the CPI inflation by the optimal inflation index. Figure 2 shows that, in such an optimal inflation index, the weight on PPI inflation increases with the share of intermediate input; and for all plausible values of ϕ, the PPI weight is between 0.4 and 0.5, far from being negligible. Under calibrated parameters with $\phi = 0.6$, the PPI weight is about 0.46 (see also Table 4). We argue that such a policy rule is as easy to implement as the traditional Taylor rule, and it also brings the welfare level much closer to that under the optimal monetary policy than does the simple Taylor rule or any other rule that excludes PPI inflation as a targeting variable.

6   Optimal interest rate rules under possible Central Bank mis-perception

We have thus far assumed that, in formulating monetary policy, the central bank observes the actual sources of shocks and pays respect to the nominal rigidities in both sectors. It is reasonable, however, to consider situations where the central bank may not know the true sources of shocks or of nominal rigidities. A natural question is then: How much welfare loss would be incurred if the central bank formulates its policy under possible mis-perceptions of the shocks or the nominal rigidities?

To answer this question, we follow two steps of computation. For instance, in the case with possible mis-perceptions of shocks, we first let the central bank simulate the baseline model to find the optimal reaction coefficients in the baseline interest rate rule (i.e., TR1) conditional on its belief about the source of shocks; we then turn on the actual shocks (which may not coincide with the central bank's belief) and compute the welfare loss under the pseudo-optimal interest rate rule. In these experiments, we consider three possible sources of shocks: the shocks may hit just the CPI sector, the PPI sector, or both. Similarly, in the case with possible mis-perception of the source of nominal rigidities, we allow the central bank to formulate a pseudo-optimal interest rate rule conditional on its belief about the price stickiness in each sector, and then compute the welfare loss under such a policy in the baseline economy with sticky prices in

both sectors. This latter case is of particular interest because many commentators make their monetary policy proposals based on a one-sector model with a single source of nominal rigidity.

Table 5 presents the welfare losses under the central bank's mis-perceptions about shocks or nominal rigidities. The welfare loss is normalized to unity if the central bank's belief turns out to be correct. The Table shows that, regardless of the true sources of shocks, the central bank can avoid most of the welfare losses due to mis-perception if it assumes that the shocks hit either the PPI sector alone (e.g., oil shocks) or both sectors. On the other hand, if the central bank incorrectly believes that the shocks hit just the CPI sector, the potential losses would be much greater. The Table also shows that, if the actual economy features sticky prices in both sectors, then formulating monetary policy based on the incorrect belief that there is only a single source of nominal rigidity would incur substantial welfare losses, and this is true regardless of the sources of shocks.

7 Other policy objectives

We have thus far characterized optimal monetary policy, with the objective function facing the central bank being derived from the first principle. In the literature, other policy objectives have also been considered, especially the variance of output gap [e.g., Mankiw and Reis (2002)]. We now examine the implications of introducing the input-output connections on the optimal monetary policy design if the central bank is mainly concerned about the variance of the output gap.

7.1 Extreme inflation-targeting policies

A common view is that the variations of the output gap can be reduced or even eliminated if the monetary authority can achieve price level stability by eliminating fluctuations in CPI inflation [e.g., Goodfriend and King (2001)]. In a similar spirit, we now consider the stabilizing effects of two policies, one sets $\pi_{ft} = 0$ and the other sets $\pi_{mt} = 0$ for all t. We call the first policy an "extreme CPI-inflation targeting regime" and the second policy an "extreme PPI-inflation targeting regime."

Under each policy regime, we use the equilibrium conditions (36)-(39) to compute the variance of the output gap. For convenience, we rewrite the equilibrium conditions here:[8]

$$\pi_{ft} = \beta E_t \pi_{f,t+1} + \kappa_f(\phi \tilde{q}_t + (1-\phi)\sigma \tilde{c}_t), \tag{54}$$

$$\pi_{mt} = \beta E_t \pi_{m,t+1} + \kappa_m(\sigma \tilde{c}_t - \tilde{q}_t), \tag{55}$$

$$\tilde{q}_t = \tilde{q}_{t-1} + \pi_{mt} - \pi_{ft} - \Delta q_t^*, \tag{56}$$

where the term Δq_t^* is the relative price of intermediate goods in the flexible price equilibrium and is given by $\Delta q_t^* = (1-\phi)(\Delta a_{ft} - \Delta a_{mt})$. Denote $e_t = \Delta a_{ft} - \Delta a_{mt}$. For analytical convenience, we assume that $\rho_f = \rho_m = \rho$ so that

$$e_t = \rho e_{t-1} + \varepsilon_t, \tag{57}$$

where $\varepsilon_t = \varepsilon_{ft} - \varepsilon_{mt}$. The distribution assumption about the shocks in the two sectors implies that ε_t has a zero mean and a finite variance given by $\sigma_e^2 = \sigma_f^2 + \sigma_m^2$.

Under the extreme CPI-inflation targeting regime with $\pi_{ft} = 0$, equation (54) implies that $\phi \tilde{q}_t = (\phi - 1)\sigma \tilde{c}_t$. Using this relation, along with (55) and (56), we can eliminate $\tilde{q}_t$ and π_{mt} and obtain a second-order difference equation in $\tilde{c}_t$:

$$\beta E_t \tilde{c}_{t+1} - \left(1 + \beta + \frac{\kappa_m}{1-\phi}\right)\tilde{c}_t + \tilde{c}_{t-1} = \frac{\phi}{(1-\phi)\sigma}(\beta E_t e_{t+1} - e_t). \tag{58}$$

The solution is a second-order autoregressive process (i.e, an AR(2) process) given by

$$\tilde{c}_{ft} = (\rho + \lambda_f)\tilde{c}_{f,t-1} - \rho\lambda_f \tilde{c}_{f,t-2} + \eta_f \varepsilon_t, \tag{59}$$

where $\tilde{c}_{ft}$ denotes the solution of the output gap under the extreme CPI-inflation targeting regime, λ_f is the root of the quadratic polynomial $\beta\lambda^2 - (1 + \beta + \kappa_m/(1-\phi))\lambda + 1 = 0$ that lies within the unit circle, and $\eta_f = \frac{\lambda_f \phi(1-\beta\rho)}{\sigma(1-\beta\rho\lambda_f)}$. Since $\eta_f > 0$, a shock with $\varepsilon_t > 0$, that is, with $\varepsilon_{ft} > \varepsilon_{mt}$, would result in an increase in the output gap. In other words, if the finished good sector's productivity shock dominates, then, under the extreme CPI-inflation targeting regime, the output gap rises; if the intermediate good sector's shock dominates, then the gap falls.

Similarly, the solution of the output gap dynamics under the extreme PPI-inflation targeting regime is an AR(2) process given by

$$\tilde{c}_{mt} = (\rho + \lambda_m)\tilde{c}_{m,t-1} - \rho\lambda_m \tilde{c}_{m,t-2} + \eta_m \varepsilon_t, \tag{60}$$

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[8]The intertemporal Euler equation (38) is omitted here since it only serves to pin down the equilibrium nominal interest rate once the output gap and the inflation rates are solved out.

where $\tilde{c}_{mt}$ denotes the output gap under the extreme PPI-inflation targeting regime, λ_m is the root of the quadratic polynomial $\beta\lambda^2 - (1 + \beta + \kappa_f)\lambda + 1 = 0$ that lies within the unit circle, and $\eta_m = -\frac{\lambda_m(1-\phi)(1-\beta\rho)}{\sigma(1-\beta\rho\lambda_m)}$. Since $\eta_m < 0$, a dominant productivity shock in the finished good sector would result in a fall in the output gap under the extreme PPI-inflation targeting regime.

The policy objective we consider here is to minimize the variance of the output gap. We now compare the implied variance of the gap under the two alternative policy regimes. Following the procedure described in Hamilton (1994, p.58), we use (59) and (60) to obtain

$$\Gamma_j = \frac{(1 + \lambda_j\rho)\eta_j^2\sigma_e^2}{(1 - \lambda_j\rho)[(1 + \lambda_j\rho)^2 - (\rho + \lambda_j)^2]}, \quad j \in \{f, m\}, \tag{61}$$

where $\Gamma_f = \text{Var}(\tilde{c}_{ft})$ and $\Gamma_m = \text{Var}(\tilde{c}_{mt})$ denote the variance of the output gap under each of the two policy regimes.

To see the dependence of the variances of the gap under the two policies on the input-output connections, we plot in Figure 3 the variance of the gap as a function of the parameter $\phi \in [0, 1]$ under each of the two policy regimes, with the rest of the parameters calibrated to their baseline values (see Table 3). The figure shows that the extreme CPI-inflation targeting policy is more effective in stabilizing the fluctuations of the output gap than the extreme PPI-inflation targeting policy. With high values of ϕ, however, the two extreme inflation-targeting policies yield similar volatility of the output gap. If one believes that the primary objective of the central bank should be to stabilize output gap fluctuations, then a policy that maintains consumer price stability seems to be reasonably effective, although for large yet plausible values of the share of intermediate inputs, a policy that maintains producer price stability does almost equally well in achieving output stability.

7.2 Optimal stabilizing inflation-targeting policies

In principle, to achieve the goal of stabilizing output gap fluctuations, the monetary authority does not have to resort to the extreme inflation targeting policies described above. In other words, these rigid extreme policies need not be optimal in the sense of minimizing the variance of the gap. We now characterize the optimal monetary policy that achieves the goal of stabilization and discuss the implementation of the optimal policy through simple feedback interest rate rules.

Under the goal of stabilization, the optimal monetary policy solves the following problem:

$$\text{Min} \quad \Omega \mathrm{E} \sum_{t=0}^{\infty} \tilde{c}_t^2, \tag{62}$$

25

subject to the equilibrium conditions (36) - (39), where Ω is a constant. The solution yields a constant output gap under the optimal stabilizing policy, that is, $\tilde{c}_t = 0$.

Thus, while strict targeting of either CPI inflation or PPI inflation cannot achieve the goal of output-gap stabilization, a policy that targets a mixture of the two measures of inflation rates can eliminate fluctuations in the output gap. The optimal stabilization inflation index obtained here is a weighted average of CPI and PPI inflation rates, with the weights depending on the share of intermediate inputs. In this sense, our results extend those in Mankiw and Reis (2002) to a model with input-output connections.

8 Inflation Accounting: Some Evidence

Our model suggests that, in the presence of two sources of nominal rigidities and the input-output connections between stages of production, a benevolent central bank should take into account variations in both CPI inflation and PPI inflation when conducting monetary policy: both inflation rates appear as a policy goal in the central bank's objective function and a policy instrument that implements optimal monetary policy. The optimal inflation index is a weighted average of CPI and PPI inflation rates, both for the goal of maximizing social welfare and of stabilizing fluctuations in the output gap.

The policy implications of our model contrast those obtained in a standard model with a single source of nominal rigidity, which has been a popular model that guides much of the monetary policy discussion. A natural question is then: Does empirical evidence support the implications of our two-sector model with the kind of input-output connections elaborated in Section 2?

To answer this question, we need to examine empirically the importance of the input-output connections (i.e., the role of ϕ) and of the additional nominal rigidity that we introduce in the CPI sector (i.e, the role of α_f), which are two distinguishing features of our model. In the absence of either feature, one source of nominal rigidity would be shut off, and our model's equilibrium dynamics would reduce to those in a standard model [e.g., Aoki (2001)]. In such an extreme case, there would be no markup variations in the CPI sector. Thus, one way to test the empirical validity of our baseline model against a standard model is to examine whether there are important markup variations in the CPI sector.

To implement such a test, we use the model's implied relation between CPI inflation, PPI inflation, and nominal wage inflation.[9] In particular, our model implies that the consumer price should be a markup over the CPI sector's marginal cost, which is a weighted average of the producer price and nominal wages, as in (15). It follows that the inflation measures are related through

$$\pi_{ft} = \phi \pi_{mt} + (1 - \phi)\pi_{wt} + u_t, \tag{63}$$

where π_{wt} denotes the nominal wage inflation and u_t is a residual that includes exogenous shocks to the CPI sector and (potentially) endogenous variations in the markup as well.

We begin by examining how much of the variations in CPI inflation can be accounted for by the composite of PPI inflation and wage inflation on the right hand side of (63). The composite corresponds to the marginal cost $\hat{v}_{ft}$ in the CPI sector. Using a calibrated value of $\phi = 0.6$ and quarterly inflation data obtained from the U.S. Bureau of Labor Statistics (with a sample period 1983:Q1 - 2003:Q4), we obtain the following variance decomposition:

$$
\begin{aligned}
var(\pi_f) &= var(\hat{v}_f) + var(u) + 2cov(\hat{v}_f, u), \\
1.179 &= 0.663 + 0.525 - 0.008. \tag{64}
\end{aligned}
$$

The fraction of the variance of CPI inflation accounted for by the residual u_t is about 0.44 (i.e., 0.525/1.179), implying important variations in the "inflation residual."

Since the inflation residual contains information about both exogenous shocks to the CPI sector and endogenous variations in the markup, the size of its variance, by itself, does not necessarily imply that there are important markup variations. Yet, if the residual contains only information about shocks, then it should be exogenous. We thus conduct a bi-variate test of the hypothesis that the inflation residual is not Granger-caused by several alternative variables. The variables we use include real GDP, three-month treasury bill rates, and a yield spread (the difference between ten-year treasury constant maturity rates and three-month T-bill rates), all at quarterly frequency. To isolate fluctuations at the business cycle frequency (between six quarters and eight years), we apply the band-pass filter proposed by Baxter and King (1995) to each variable, based on a twelve-quarter centered moving average (the band-pass filter thus reduces the sample size by twenty-four quarters).[10] Table 6 presents the Granger causality test

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[9]We are grateful to Bob King for suggesting that we pursue this line.

[10]As argued by Gali, et al. (2002), the nominal interest rate and the yield spread "can be thought of as a rough measure of the stance of monetary policy, while detrended GDP is just a simple cyclical indicator." Our results do not hinge upon the choice of these variables.

results. The null hypothesis that the inflation residual is not Granger-caused by any of these variables is rejected, with a P-value of less than 0.001 in each case, and the rejection does not depend on the number of lags used in the regressions.

From the inflation accounting exercise, we conclude that there are important variations in the inflation residual, which cannot be completely attributable to exogenous shocks. Our baseline model that features input-output connections between the CPI sector and the PPI sector and nominal rigidities in both sectors provides a plausible interpretation of this empirical finding, whereas a standard model with a single source of nominal rigidities or a multi-sector model with no input-output connections does not.

9   Conclusion

We have presented a framework to evaluate inflation-targeting monetary policy rules. In the model, production of final consumption goods needs to go through two stages of processing, with two sectors interconnected through a vertical chain of production, so that a natural distinction between CPI and PPI arises from the model. We have established a utility-based welfare criterion for a benevolent central bank, so as to provide a useful benchmark for evaluating the performance of alternative monetary policy rules.

The welfare criterion makes it explicit that the central bank should care about not only the variability of CPI inflation and output gap (as a standard one-sector model would suggest), but also the variability of PPI inflation and the real marginal cost in the production of intermediate goods. With the input-output connections, a "cost-push" term arises endogenously in the Phillips-curve equation derived from the optimal pricing decisions of the final good producers, and this term is determined by the same real marginal cost that enters the policy objective. The real marginal cost gap is present in the policy objective since, in our model, fluctuations in the relative price of intermediate goods to final goods have an allocative role. The presence of the endogenous "cost-push" term introduces a tradeoff between the inflation rates and the gaps so that the first-best allocation is not attainable.

With the second-best welfare level as a benchmark, we have evaluated alternative simple interest rate rules. We find that rules that exclude PPI inflation as a targeting variable would typically incur significant welfare losses, while rules that include an optimal inflation index, that is, a weighted average of CPI inflation and PPI inflation, would typically bring the welfare

level close to the second-best. The weight assigned to PPI inflation in the optimal inflation index depends on the share of intermediate goods and is in general non-negligible.

We have further strengthened our case that the central bank should pay respect to nominal rigidities in both the CPI sector and the PPI sector by pointing out that, if the central bank formulates its optimal policy by ignoring the nominal rigidity in any sector, the welfare loss would be large, and this is true regardless of the sources of shocks. When the central bank does not know the actual sources of shocks, we show that it can avoid much of the welfare loss by assuming that the shocks hit both sectors (or the PPI sector alone) and formulate its optimal policy based on this assumption.

To help derive analytical results and to simplify exposition, we have focused on nominal rigidities in the PPI sector and the CPI sector. We do not claim that these are the only sources of nominal rigidities in the actual economy. Clearly, they are not. A more ambitious model should probably take into account of, in addition to these price rigidities, some other sources of nominal or real imperfections, such as nominal wage rigidities. Our analytical results suggest that introducing nominal wage rigidities would unlikely change the main conclusion: because of the presence of sticky prices in both the CPI and the PPI sectors, fluctuations in the sectoral relative price play an important allocative role, even with sticky wages introduced, so that optimal policy should not ignore either inflation rate. In light of the inattention to PPI inflation in both policy practice and academic research, it seems compelling that better understanding of the input-output connections and of the cyclical behavior of PPI inflation should be elevated to the top of research agenda.

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Table 1.

Inflation-targeting around the world

Countries	Year Adopted	Target Variable
Australia	1993	CPI (after 1998)
Canada	1991	CPI (excl. food, energy and taxes)
Finland	1993	CPI (excl. taxes, housing, and interest)
Israel	1991	CPI
New Zealand	1988	CPI (excl. taxes and interest)
Spain	1994	CPI
Sweden	1993	CPI
UK	1992	Retail price index (excl. interest)

Source: Speech by Governor Murray Sherwin, Reserve Bank of New Zealand (1999); Bernanke, et al. (1999); and Leiderman and Svensson (1995).

Table 2.

Cyclical behaviors of alternative measures of inflation

Inflation measures	STD	AR(1)	AR(2)	AR(3)	AR(4)
CPI	1.04	0.95	0.88	0.80	0.72
PPI	1.70	0.91	0.81	0.71	0.60
Core CPI	0.82	0.95	0.88	0.79	0.69
Core PPI	0.94	0.88	0.73	0.58	0.42

Source: Bureau of Labor Statistics and authors' calculation. The data are monthly, from 1970:01 to 2002:12, and are HP-filtered.

Table 3.

Calibrated parameter values

Preferences: $U(C) - V(N)$	$\sigma = -\frac{U_{cc}C}{U_c} = 1, \quad \omega = \frac{V_{nn}N}{V_n} = 0$
Subjective discount factor:	$\beta = 0.99$
Nominal contract duration:	$\alpha_f = 0.75, \ \alpha_m = 0.75$
Composite finished goods: $C = \left[\int Y_f(j)^{\frac{\theta_f - 1}{\theta_f}} dj \right]^{\frac{\theta_f}{\theta_f - 1}}$	$\theta_f = 10$
Composite intermediate goods: $\bar{Y}_m = \left[\int Y_m(i)^{\frac{\theta_m - 1}{\theta_m}} di \right]^{\frac{\theta_m}{\theta_m - 1}}$	$\theta_m = 10$
Finished good production: $Y_f(j) = \bar{Y}_m(j)^{\phi}(A_f N_f(j))^{1-\phi},$	$\phi \in [0.5, 0.8].$
Intermediate good production: $Y_m(i) = A_m N_m(i)$	
Technology shock processes: $\Delta \ln(A_{kt}) = \rho_k \Delta \ln(A_{k,t-1}) + \varepsilon_{kt}, \quad \rho_k = 0.95, \ \sigma_k = 0.02, \ k \in \{f, m\}.$	

Table 4.

Optimal interest rate rules

Policy Rules	Optimal policy coefficients				CPI weight	Welfare loss
	π_{ft}	π_{mt}	$\tilde{c}_t$	r_{t-1}		
TR1	1.68	1.42	0.04	1.12	0.54	1.09
TR2	1.75		0.55	0.82	1	1.64
TR3		2.71	0.62	1.85	0	1.99
TR4	1.68	1.42		1.12	0.54	1.09
TR5	2.06			0.89	1	1.67
TR6		3.18		2.58	0	2.09

Note: The interest rate rules are of the generic form $r_t = a_1\pi_{ft} + a_2\pi_{mt} + a_3\tilde{c}_t + a_4 r_{t-1}$, with the 6 interest rate rules (i.e., TR1 through TR6) each being a special case with appropriate zero-restrictions on the a-coefficients (corresponding to the blank spaces in the table). The first four columns of numbers give the optimal a-coefficients that minimize the welfare loss defined in the text; the fifth column contains the relative weight of CPI-inflation in the optimal rules [i.e., $\frac{a_1}{a_1+a_2}$], and the last column gives the welfare loss under each interest rate rule relative to that under the optimal monetary policy.

Table 5.

Welfare losses when the Central Bank misperceives

the sources of shocks or of nominal rigidities

	Perceived Shocks			Perceived Rigidities		
Actual Shocks	PPI shock	CPI shock	both shocks	PPI rigidity	CPI rigidity	both rigidities
PPI shock	1	6.03	1.00	2.65	1.69	1
CPI shock	1.04	1	1.04	2.64	1.45	1
both shocks	1.00	3.58	1	1.40	1.11	1

Note: The welfare losses under misperceived shocks or nominal rigidities are relative to those under correct perceptions. While we allow "actual" shocks to come from either the PPI sector or the CPI sector (or both), we maintains that prices are sticky in both sectors in the baseline economy.

Table 6.

Bi-Variate Granger Causality Test (1983:Q1 - 2003:Q4)

Variables	Two Lags				Four Lags			
	F-Stat	P-Value	χ^2-Stat	P-Value	F-Stat	P-Value	χ^2-Stat	P-Value
Real GDP	9.57	0.001	20.89	0.001	5.48	0.001	25.79	0.001
Nom. Interest Rate	8.15	0.001	17.78	0.001	11.74	0.001	55.25	0.001
Yield Spread	10.34	0.001	22.55	0.001	9.69	0.001	45.59	0.001

Note: The P-values do not exceed the values reported in the table. Thus, in all cases, the P-values are less than or equal to 0.001.
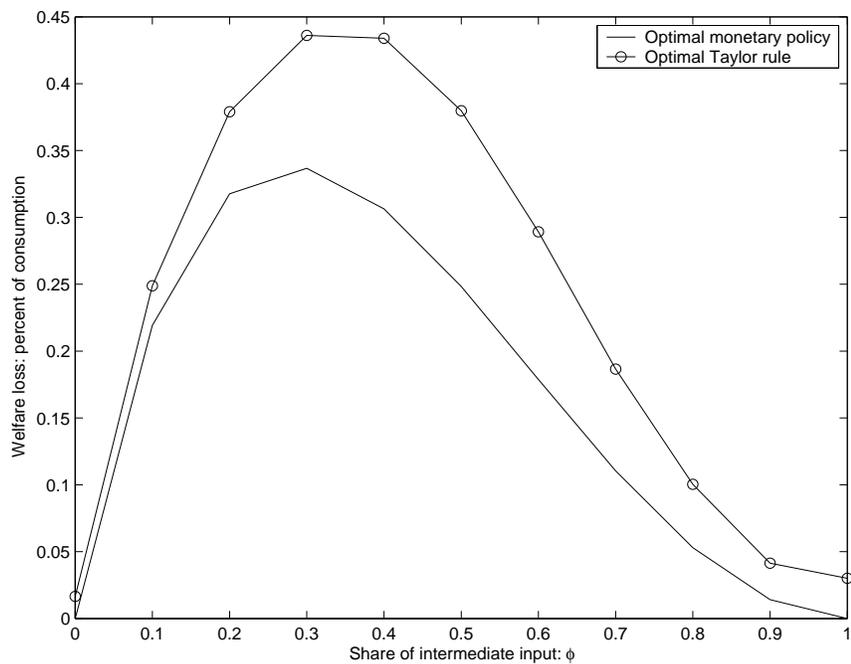
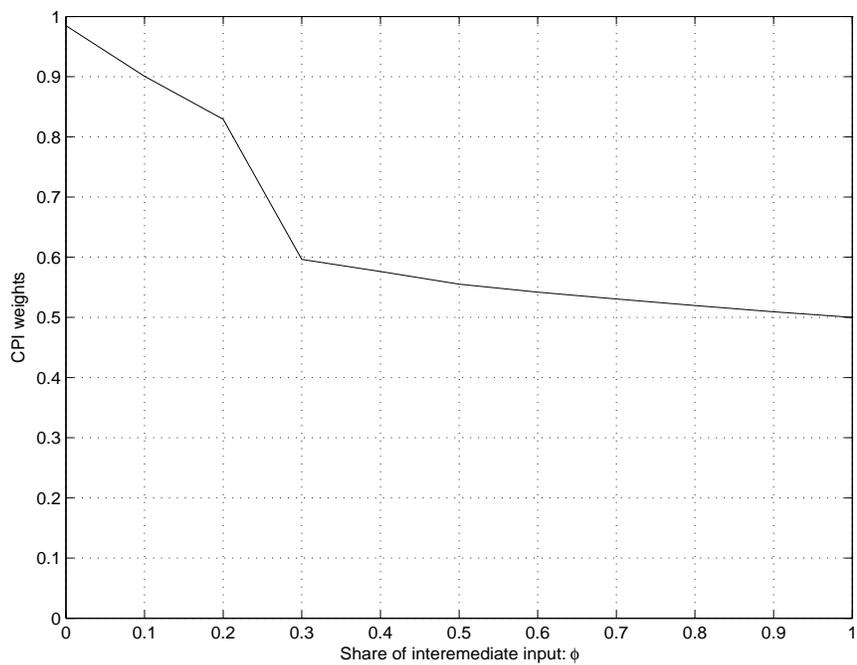Figure 1:—Sensitivity of welfare losses to the share of intermediate input.

Figure 2:—The weight on CPI inflation in the optimal inflation index.
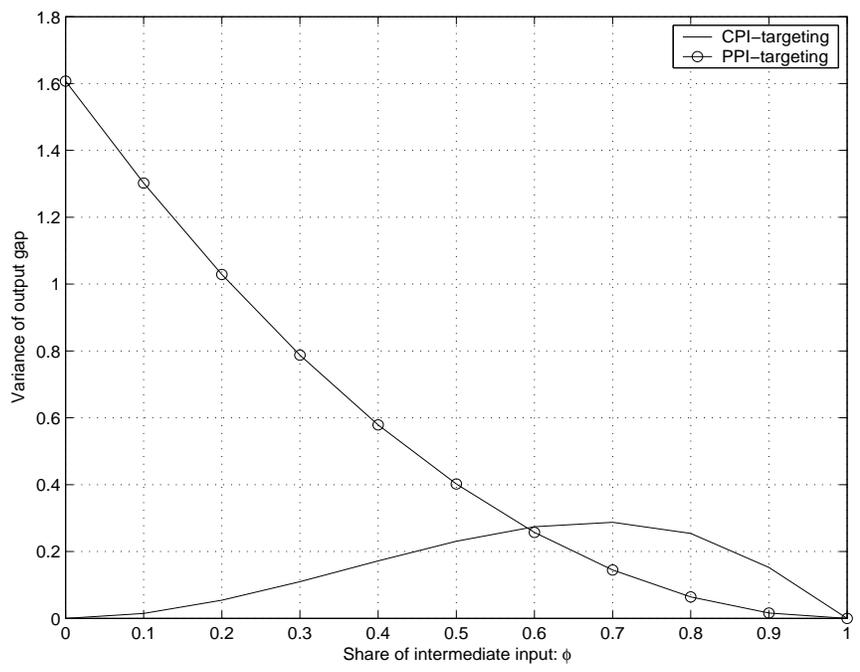
Figure 3:—The variance of output gap under extreme inflation-targeting policies.