HETEROGENEITY, REDISTRIBUTION, AND THE FRIEDMAN RULE

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Abstract

We study monetary models with non-degenerate stationary distribution of money holdings. We find that the Friedman rule does not typically maximize ex-post social welfare. An increase in the rate of growth of the money supply has two effects: the standard distortionary, or rate-of-return, effect makes money a less desirable asset for all moneyholders. A second, redistributive effect, creates a transfer from one type of agent to the other. An increase in the rate of growth on money away from the Friedman rule can produce a rate-of-return effect that dominates the standard effect.

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1 Introduction

Until 1951, the Federal Reserve System of the US explicitly pegged nominal interest rates on the Treasury's debt obligations. In March 1951, the Treasury-Fed Accord ended this explicit arrangement, ostensibly freeing the Fed to pursue an independent monetary policy. It was natural for economists to ask: how should an independent yet benevolent central bank conduct monetary policy? Milton Friedman (1969) offered a simple and yet deep answer (the Friedman rule): since money is an asset, the central bank ought to change the stock of outstanding money at a rate that causes the real rate of return on money to equal the real return rate on other physical assets. Over the next three decades, researchers have studied Friedman's dictum using the two main workhorse models for monetary theory, the infinitely-lived representative agent (ILRA) model and the overlapping generations (OG) model. In the context of infinitely-lived-*representative*-agent models, when lump-sum taxes and transfers are available, the Friedman rule is the optimal monetary policy.[1] The seminal reference on the Friedman rule in OG models is Wallace (1984). Wallace shows that once heterogeneity among agents is explicitly considered, it may be impossible for the central bank to settle on *one* monetary policy rule, including the Friedman rule, that benefits every agent.

By construction, monetary policy cannot have redistributive effects in representative-agent models. Yet these effects are known to be quantitatively

[1] See, for instance, Woodford (1990) and Ljunqvist and Sargent (2000). Note that Chari, Christiano and Kehoe (1996) and Correia and Teles (1996) extend this to the case in which other distortionary taxes are present.

significant and important (see, for example, Erosa and Ventura, 2002). For agents holding a disproportionate amount of money, their welfare is clearly negatively related to inflation. In contrast, agents holding relatively less money can realize welfare gains from higher inflation. This paper shows the following: If monetary policy can have redistributive effects then, a monetary policy that sets long-run nominal interest rates to zero – that is the Friedman rule – does not typically maximize ex-post social welfare, and in some cases, it does not maximize ex-ante welfare. Indeed, a necessary condition for the Friedman rule to be suboptimal ex-post is that changes in the rate of growth of the money supply have redistributive effects.

We study different monetary environments in which heterogeneity among agents produces a long-run non-degenerate distribution of money holdings. In particular, we study: 1) the random-matching model of money due to Lagos and Wright (2002, hereafter, LW), 2) a turnpike model of the type introduced by Townsend (1980), and 3) an OG model with stochastic relocation as in Schreft and Smith (1997) and Smith (2002). In each model, agents have heterogeneous money holdings in equilibrium. In the LW frameworks the heterogeneity comes from differences in agents' preferences. In the turnpike environment it arises from different endowment patterns. In the OG model agents alive in the same period may be from different generations. Our results are robust to these various ways of obtaining heterogenous money holdings in equilibrium.

In each of the models we study, an increase in the rate of growth of the money supply away from the Friedman rule has two effects. First, the standard distortionary, or rate-of-return, effect is to make money a less desirable asset thereby decreasing the utility of all agents. Second, the redistributive effect is a transfer from one type of agent to the other. We show, for each environment, examples for which the redistributive effect dominates the rate-of-return effect when the rate of growth of the money supply is not too high.

Markets are assumed to be incomplete, implying it is impossible to undo transfers by means of non-distortionary fiscal policy. Deviating from the Friedman rule therefore produces multiple Pareto optimal yet non-comparable allocations. This assumption is crucial for our results. Indeed, if the central bank can levy type-specific lump-sum taxes, it is always best to implement the Friedman rule. This is because it is possible to offset any redistribution induced by monetary policy with an appropriate lump-sum tax or transfer. Type-specific lump-sum taxes and transfers are not the only way redistribution effects can be undone. Bhattacharya, Haslag, and Russell (2004), Haslag and Martin (2003), and da Costa and Werning (2003) describe other mechanisms which produce the same result.

In the models we consider, the policymaker who chooses the rate of growth of the money supply is faced with different types of agents and can assign different weights to each type. Hence, we consider *ex-post* social welfare. Since it is possible to specify a social welfare function which puts enough weight on the type that benefits from such a deviation, it follows *ex-post* social welfare may not be maximized at the Friedman rule.[2]

[2]Under a different approach we could have appealed to a political economy criterion. For example, we could assume agents vote on their preferred policy. We can show, for each of the economies we study, examples where at least 50 percent of the agents prefer a deviation from the Friedman rule. Alternatively, we could assume agents influence the

Our work is part of a growing literature studying environments (with heterogeneity) in which the Friedman rule is not optimal (see, for example, Levine (1991), Molico (1999), Deviatov and Wallace (2001), Smith (2002a,b), Edmonds (2002), Green and Zhou (2002), Albanesi (2003)). These papers consider an ex-ante welfare criterion and argue the Friedman rule might not be optimal from such a standpoint. In other words, their analysis assumes that agents "pick their preferred monetary policy under a "veil of ignorance", before knowing their true identity" [Ljungqvist and Sargent (2000)]. In contrast, we present results using a ex-post welfare criterion and can therefore better capture the "conflict of interest" between different types of agents that a benevolent policymaker has to consider. Our paper is also related to recent work that studies the impact of agent heterogeneity on monetary policy. Kocherlakota (2002) shows monetary policy should react to the degree of heterogeneity in the economy. Berentsen, Camera, and Waller show a one shot deviation from the Friedman rule might increase ex-ante welfare in a search economy of the type introduced by Lagos and Wright (2002).

The rest of the paper proceeds as follows. Section 2, 3, and 4 describe the search and matching economy, the turnpike economy, and the overlapping generations economy, respectively. Section 5 concludes.

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choice of monetary policy by spending resources lobbying. For example, the "distance" between a group's preferred policy and the chosen policy could depend on the ratio of resources this group spends lobbying to the total resources spent lobbying. The Friedman rule will not be chosen for any such rule that implements a policy strictly between different group's preferred policies. In this paper, however, we limit ourselves to showing Pareto incomparable allocations can arise.

2 A search economy

This section considers a search model of the type introduced by LW.[3] Time is discrete and there is a continuum of mass 1 of infinitely-lived agents. Each period is divided in two sub-periods. It is assume that there are two types of goods: special goods which are traded in a decentralized market during the first sub-period and general goods which are traded in a centralized market during the second sub-period.

We consider each market in turn, starting with the centralized market. As in LW, preferences in the centralized market are assumed to be quasi-linear, so that the utility from consuming an amount X and producing an amount H is given by $U(X) - AH$, where U is twice continuously differentiable, $U' > 0$, $U'' < 0$, and $A > 0$. As in LW, is also assumed U is unbounded and $U'(X^*) = 1$ for $X^* \in (0, \infty)$ with $U(X^*) > X^*$.

In the decentralized market, agents only consume and produce a subset of the goods. Agents do not produce the goods they like to consume. We assume there are no double coincidence of wants and denote by σ the probability of a single coincidence of wants. Unlike the standard LW model, agents also differ in how much they value special goods relative to general goods. The utility derived by a type-α agent from consuming an amount x and producing an amount h is given by $\alpha u(x) - c(h)$, where $\alpha > 0$, u and c are at least thrice continuously differentiable, $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $u'' < 0$, $c'' \geq 0$, and $u(\bar{q}) = c(\bar{q})$ for some $\bar{q} > 0$.

A central bank can expands or contracts the money supply via lump-sum

[3] We are indebted to Randy Wright for showing us how to work out the example in this section.

transfers or taxes, denoted by τ, during the centralized market. The money supply evolves according to $M_t = (1+z)M_{t-1}$. Hence, $\tau = zM_{t-1}$.

Let ϕ denote units of consumption good per unit of money (the inverse of the price level). Then we define $W_\alpha(m, \phi)$ to be the value function for a type α agent entering the centralized market with money m, and $V_\alpha(m, \phi)$ be the value function for this agent entering the decentralized market with money m. The problem of an agent in the centralized market is

$$W_\alpha(m, \phi) = \max_{X,H,m^+} \{U(X) - AH + \beta V_\alpha(m^+, \phi)\}$$

subject to

$$X = \omega H + \phi(m + \tau - m^+), \tag{1}$$

where ω denotes the real wage, and m^+ is the money carried out of the market. We assume ω is fixed; for example, because of a linear technology.

Assuming an interior solution, we can substitute for H to get

$$W_\alpha(m, \phi) = \max_{X,m^+} \left\{ U(X) - \frac{A}{\omega} \left[X - \phi(m + \tau - m^+) \right] + \beta V_\alpha(m^+, \phi) \right\}.$$

The first order conditions for X and m^+ are, respectively,

$$U'(X) = \frac{A}{\omega}, \tag{2}$$

$$\frac{A\phi}{\omega} = \beta V'_\alpha(m^+, \phi). \tag{3}$$

Also notice $W_\alpha(m, \phi)$ is linear since $W'_\alpha(m, \phi) = \frac{A\phi}{\omega}$ for all m. As in LW, the cost of producing H is linear, X and m^+ are independent of m. Hence, if there is only one type, all agents consume the same amount and leave the market with the same money holdings. With more than one type, however, m^+ may depend on α as can be seen from (3).

We now turn to the decentralized market. Let the joint distribution of money and types in this market be given by $F(\tilde{m}, \tilde{\alpha})$. The above analysis shows F is degenerate conditional on types. In other words, all type-α agents exit the centralized market with the same amount of money m_α^+ and thus enter the decentralized market with the same amount m_α. Hence, it is enough to know the distribution of types $G(\tilde{\alpha})$.

We can write the value function for an type-α agent entering the decentralized market with money m as

$$V_\alpha(m, \phi) = \sigma \int \{-c[q_{\tilde{\alpha}}(m_{\tilde{\alpha},\phi})] + W_\alpha[m + d_{\tilde{\alpha}}, \phi]\} \, dG(\tilde{\alpha})$$
$$+\sigma \{\alpha u[q_\alpha(m, \phi)] + W_\alpha[m - d_\alpha(m, \phi), \phi]\} + (1 - 2\sigma)W_\alpha(m, \phi).$$

This expression states that with probability σ, the agent is a seller who produces a quantity $q_{\tilde{\alpha}}(m_{\tilde{\alpha}}, \phi)$ of special goods in exchange for $d_{\tilde{\alpha}}$ units of money. With probability σ the agent is a buyer who consumes $q_\alpha(m, \phi)$ units of special goods acquired with $d_\alpha(m, \phi)$ units of money. In particular, we have assumed, as will be verified below, that the terms of trade q and d depend on the buyer's but not the seller's money balances. Now take the partial derivative of the above expression with respect to m:

$$\begin{aligned} V'_\alpha(m, \phi) = \ & \sigma \textstyle\int +W'_\alpha[m + d_{\tilde{\alpha}}(m_{\tilde{\alpha}}, \phi), \phi] dG(\tilde{\alpha}) + \sigma\alpha u'[q_\alpha(m, \phi)]q'_\alpha(m, \phi) \\ & +\sigma[1 - d'_\alpha(m, \phi)]W'_\alpha[m - d_\alpha(m, \phi), \phi] + (1 - 2\sigma)W'_\alpha(m, \phi). \end{aligned}$$

Recall $W'_\alpha(m, \phi) = \frac{A\phi}{\omega}$ for all m, so we can write

$$V'_\alpha(m, \phi) = \sigma\alpha u'[q_\alpha(m, \phi)]q'_\alpha(m, \phi) + [1 - \sigma d'_\alpha(m, \phi)]\frac{A\phi}{\omega}. \tag{4}$$

Hence, $V'_\alpha(m, \phi)$ depends on an agent's own type, α, and money holding, m, but not on other agents type and money holdings.

As in LW, we assume the terms of trade are determined by the generalized Nash solution where the buyer has bargaining power θ and the threat points are given by the continuation values. First, note for any real d and any type α, $W_\alpha(m+d,\phi) - W_\alpha(m,\phi) = d\frac{A\phi}{\omega}$. It follows that the terms of trade (q,d) between a buyer of type α with money holding m and a seller of any type is given by

$$\max_{q,d} \left[\alpha u(q) - d\frac{A\phi}{\omega}\right]^{\theta} \left[-c(q) + d\frac{A\phi}{\omega}\right]^{1-\theta}$$

subject to $d \leq m$. Thus, as claimed above, the terms of trade do not depend on the seller's type.

As in LW, it can be shown that in any equilibrium it must be the case that $d = m$. In order to find q, we take the partial derivative of the above expression with respect to q and set it equal to zero. This implies $q = q_\alpha(m,\phi)$ is the solution to

$$m\frac{A\phi}{\omega} = g_\alpha(q), \tag{5}$$

where $g_\alpha(q)$ is defined as

$$g_\alpha(q) \equiv \frac{\theta\alpha u'(q)c(q) + (1-\theta)\alpha u(q)c'(q)}{\theta\alpha u'(q) + (1-\theta)c'(q)}.$$

For example, if the buyer has all the bargaining power, so $\theta = 1$, this expression reduces to $g_\alpha(q) = c(q)$.

In the general case, implicit differentiation yields

$$q'_\alpha(m,\phi) = \frac{A\phi}{\omega g'_\alpha(q)}. \tag{6}$$

We can substitute this expression, as well as $d'_\alpha(m,\phi) = 1$, into $V'_\alpha(m,\phi)$, given by equation (4). Then, with equation (3), we get

$$\frac{A\phi}{\omega} = \beta\left\{\sigma\alpha u'(q_\alpha^+)\frac{A\phi^+}{\omega g'_\alpha(q_\alpha^+)} + (1-\sigma)\frac{A\phi^+}{\omega}\right\},$$

where we use the superscript $+$ to denote next period. Since we focus on steady states, we know q is constant and $\phi = (1+z)\phi^+$. The above expression then reduces to

$$1 + z = \beta \left[\sigma \frac{\alpha u'(q_\alpha)}{g'_\alpha(q_\alpha)} + (1 - \sigma) \right]. \tag{7}$$

This expression determines the equilibrium value of q_α for an agent of type α. From equation (5) we get $m_\alpha = \frac{\omega g_\alpha(q_\alpha)}{A\phi}$.

We can simplify expression (7) further. Define $\beta \equiv 1/(1+r)$ and the nominal interest rate $(1+i) = (1+z)(1+r)$. Then we can write

$$1 + \frac{i}{\sigma} = \frac{\alpha u'(q_\alpha)}{g'_\alpha(q_\alpha)}.$$

The price ϕ can be obtained through the money market clearing condition $\int m_\alpha dG(\alpha) = M$, since the q_α's are determined by equation (7).

Now let $\hat{m}_\alpha$ denote the money with which an agent of type α enters the centralized market. $\hat{m}_\alpha$ will depend on the type of meeting the agent was in during the previous decentralized market. In that market, the agent might have been either a seller, or a buyer, or no trade occurred. Hence,

$$\hat{m}_\alpha = \begin{cases} 0 & \text{with probability } \sigma, \\ m_\alpha & \text{with probability } 1 - 2\sigma, \\ m_\alpha + m_{\tilde{\alpha}} & \text{with probability } \sigma G(\tilde{\alpha}). \end{cases}$$

Since $\hat{m}_\alpha$ varies across agents according to their type, so must H_α. Indeed, we can rewrite equation (1) as follows

$$\omega H_\alpha = X - \phi(\hat{m}_\alpha + \tau - m_\alpha^+).$$

A similar expression must hold for the average $\hat{m}_\alpha$ across type α agents, which is given by $\bar{m}_\alpha = (1-\sigma)m_\alpha + \sigma M$. Hence, we have

$$\omega \bar{H}_\alpha = X - \phi(\bar{m}_\alpha + \tau - m_\alpha^+)$$

$$= X - \phi(\sigma + z)(M - m_\alpha), \tag{8}$$

where we have made use of the fact that $m_\alpha^+ = (1+z)m_\alpha$, in steady state, and $\tau = zM$.

Note that X is the same across type and is thus independent of the rate of growth of the money supply z. Assuming ϕ is fixed for a moment, for any type α holding less than the average money balances M an increase in z will reduce expected hours $\bar{H}_\alpha$. Since H enters the utility function linearly this increases expected utility. Clearly the opposite is true for any type holding more than the average money balances.

This can be illustrated by a simple example. Assume preferences are $\ln(X) - H$ in the centralized market and $\alpha_i \ln(x) - h$ in the decentralized market, with $\alpha_i \in \{\alpha_L, \alpha_H\}$, $\alpha_L < \alpha_H$. The two types L and H have equal mass. We also assume $\theta = 1$ so buyers make take-it-or-leave-it offers. Under this assumption, equation (5) implies

$$m\frac{\phi}{\omega} = q.$$

It is easy to verify that equation (7) becomes

$$1 + z = \beta\left[\sigma\alpha\frac{1}{q_\alpha} + (1-\sigma)\right].$$

Thus the expressions for q_α and m_α are given by

$$q_\alpha = \frac{\alpha\beta\sigma}{[1 + z - \beta(1-\sigma)]}$$

and

$$m_\alpha = \frac{\omega}{\phi}\frac{\alpha\beta\sigma}{[1 + z - \beta(1-\sigma)]}.$$

The money market clearing condition is

$$M = \frac{1}{2}\frac{\beta\sigma}{[1+z-\beta(1-\sigma)]}\frac{\omega}{\phi}\left(\alpha_L+\alpha_H\right).$$

This determines the price ϕ. With a little algebra we can write equation (8) for type L as

$$\bar{H}_{\alpha_L} = 1-\frac{1}{2}\frac{\beta\sigma(\sigma+z)}{[1+z-\beta(1-\sigma)]}\left(\alpha_H-\alpha_L\right).$$

Let $\Gamma \equiv (\sigma+z)/\left[1+z-\beta(1-\sigma)\right]$. Since $\alpha_H > \alpha_L$, average hours for type L will decrease if $\partial\Gamma/\partial z > 0$.

$$\frac{\partial\Gamma}{\partial z} = \frac{1+z-\beta(1-\sigma)-(\sigma+z)}{[1+z-\beta(1-\sigma)]^2},$$

which is positive for $\beta \in (0,1)$.

We can summarize these result in the following proposition.

Proposition 1 *Increasing the rate of the money supply creates a transfer from types with high α to types with low α which can make types with low α better off.*

If ϕ is kept fixed, it is easy to see how the transfer operates; a higher z allows types with low α to work less. Of course, in equilibrium, ϕ will decrease, which makes all types worse off. However, for α sufficiently low, the redistributive effect more than compensate for the rate-of-return effect. Agents with such low values of α are made better off by a deviation from the Friedman rule. Hence, different values of z give Pareto incomparable allocations. It is easy, by putting sufficient weight on the utility of types with a low α to write a social welfare function that is not maximized at $z = \beta - 1$.

This is true despite the fact that, as in LW, it can be shown from equation (7) that the Friedman rule generates the second best equilibrium in terms of efficiency. Further, if $\theta = 1$, so that there is no hold up problem associated with bargaining, then equation (7) yields $g_\alpha(q) = c(q)$ whenever $z = \beta$. This implies the first best, $\alpha u'(q_\alpha) = c'(q_\alpha)$.

Because increasing z reduces the utility of both types of agents, the Friedman rule would be optimal if it were possible to achieve the transfer from types with high α to types with low α through other, less distorting, means. For example, assume type-specific lump-sum taxes and subsidies are available. It is possible to implement a transfer, by taxing high-α types and subsidizing low-α types, without increasing z. Hence a necessary condition for the Friedman rule to be suboptimal ex-post is that changes in the rate of growth of the money supply have redistributive effects.

3 A Turnpike Model

This section studies a version of the turnpike model developed in Townsend (1980) and described in Ljungqvist and Sargent (2000). Time is indexed by $t = 0, 1, 2, ...$, there is a single, perishable consumption good and a countably infinite number of infinitely-lived agents. There are two types of agents differing in their endowment patterns. Specifically, type-E agents are endowed with 1 unit of the consumption good at even dates and nothing at odd dates. Type-O agents are endowed with 1 unit of the consumption good at odd dates and nothing at even dates. Each type-O agent is endowed with M_0 units of fiat money at date 0.

We restrict market participation in two ways. First, at each date t, there is a single pairing of one type-E and one type-O populating a market. Second, a type-E will be paired with the specific type-O agents only once.[4] These restrictions, combined with the absence of any common agent or intermediary, eliminates the possibility of debt issues. In what follows, the utility function u is assumed to be CRRA and is described as

$$u(c) = \frac{c^{1-\rho}}{1-\rho} \tag{9}$$

where c is consumption.

3.1 The type E agent's problem

The problem of type E agents can be written recursively as follows:

$$v(m) = \max u(c) + \beta u(c') + \beta^2 v(m''),$$

subject to

$$c + \pi m' = 1 + \tau + m, \tag{10}$$

$$c' + \pi m'' = \tau + m', \tag{11}$$

where c is consumption in even periods and c' is consumption in odd periods, m is the amount of real money balances the agent holds at the beginning of an even period, τ is a real lump-sum money transfer from the government which is positive if the money supply grows and negative if it shrinks, and π is the inflation rate (the ratio of tomorrow's price to today's price; in this stationary environment, it is also the rate of growth of the money supply).

[4] We define $I_E = [1_E, 2_E, ...]$ and $J_O = [1_O, 2_O, ...]$. Let $i_E \in I_E$ and $j_O \in J_O$. Then i_E is paired with j_O only once.

We know if $\pi > \beta$, then $m = m'' = 0$. The first order conditions imply

$$\pi u'(c) = \beta u'(c'), \tag{12}$$

which using (9) yields

$$\frac{c'}{c} = \left(\frac{\beta}{\pi}\right)^{\rho}, \tag{13}$$

while (11) yields

$$c + \pi c' = I \tag{14}$$

where $I \equiv 1 + \tau + \pi\tau$. It is easy to verify that

$$c = \frac{I}{1 + \pi \left(\frac{\beta}{\pi}\right)^{\rho}}, \tag{15}$$

$$c' = \left(\frac{\beta}{\pi}\right)^{\rho} \frac{I}{1 + \pi \left(\frac{\beta}{\pi}\right)^{\rho}}. \tag{16}$$

3.2 The type O agent's problem

Similarly, the problem of type O agents can be written as

$$v(\bar{m}) = \max u(\bar{c}) + \beta u(\bar{c}') + \beta^2 v(\bar{m}''),$$

subject to

$$\bar{c} + \pi \bar{m}' = \tau + \bar{m}, \tag{17}$$

$$\bar{c}' + \pi \bar{m}'' = 1 + \tau + \bar{m}', \tag{18}$$

where $\bar{c}$ is consumption in even periods and $\bar{c}'$ is consumption in odd periods, $\bar{m}$ is the real money balances the agent holds at the beginning of an even period, and τ and π are as defined above.

We know if $\pi > \beta$, then $\bar{m}' = 0$. Standard aforedescribed arguments yield

$$\bar{c} = \frac{I}{\pi + \left(\frac{\pi}{\beta}\right)^{\rho}}, \tag{19}$$

$$\bar{c}' = \left(\frac{\pi}{\beta}\right)^{\rho} \frac{I}{\pi + \left(\frac{\pi}{\beta}\right)^{\rho}}. \tag{20}$$

We can combine equations (15) and (19) to get

$$\bar{c}\left[\pi + \left(\frac{\pi}{\beta}\right)^{\rho}\right] = c\left[1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}\right].$$

Feasibility requires that $c + \bar{c} = 1$, and so

$$c = \frac{\pi + \left(\frac{\pi}{\beta}\right)^{\rho}}{\pi + \left(\frac{\pi}{\beta}\right)^{\rho} + 1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}},$$

$$c' = \frac{1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}}{\pi + \left(\frac{\pi}{\beta}\right)^{\rho} + 1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}}.$$

It can be shown that $c \rightarrow c'$ as $\pi \rightarrow \beta$. Moreover, the gap between c and c' increases as π increases. Why is there a gap between c and c'? The reason has to do with the odd-even endowment pattern. If the return on money is less than $1/\beta$, agents prefer to consume a unit of good today than to consume tomorrow goods they have bought with the cash equivalent of a unit of goods today. Hence they consume more at dates when they receive their endowments than at other dates. As π increases, the return to money falls thereby increasing the gap between odd and even period consumption.

3.3 Evaluating type E agents' welfare

Consider the start of an even date. At such a date, the type E agent would prefer a contraction in the money growth rate while a type O agent would prefer no inflation. A contraction would enable the type E agent to get a high return on his investment in money and he enjoys that in a period in which he gets a high endowment. Neither agent likes positive inflation though.

Formally, the welfare of a type E agent is given by

$$U^E = \sum_{t=0}^{\infty} \beta^{2t} \left[u(c) + \beta u(c')\right] = \frac{1}{1-\beta^2} \left[u(c) + \beta u(c')\right]. \tag{21}$$

For future use, note that

$$\frac{\partial u(c)}{\partial \pi} = \left[\pi + \left(\frac{\beta}{\pi}\right)^{\rho}\right]^{-\rho} \rho \frac{\left[1+\pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]\left[1+\frac{1}{\pi}\left(\frac{\pi}{\beta}\right)^{\rho}\right]}{\left[\pi + \left(\frac{\pi}{\beta}\right)^{\rho} + 1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]^{2-\rho}}$$

and

$$\frac{\partial u(c')}{\partial \pi} = -\left[1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]^{-\rho} \rho \frac{\left[1+\pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]\left[1+\frac{1}{\pi}\left(\frac{\pi}{\beta}\right)^{\rho}\right]}{\left[\pi + \left(\frac{\pi}{\beta}\right)^{\rho} + 1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]^{2-\rho}}. \tag{22}$$

Since

$$\frac{\partial U^E}{\partial \pi} = \frac{1}{1-\beta^2}\left[\frac{\partial u(c)}{\partial \pi} + \beta\frac{\partial u(c')}{\partial \pi}\right], \tag{23}$$

we have

$$\frac{\partial U^E}{\partial \pi} = 0 \Leftrightarrow \left[\pi + \left(\frac{\beta}{\pi}\right)^{\rho}\right]^{-\rho} - \beta\left[1 + \pi\left(\frac{\beta}{\pi}\right)^{\rho}\right]^{-\rho} = 0. \tag{24}$$

Below we prove the welfare of type-E agents is not maximized at $\pi = \beta$.

Proposition 2 *If $\rho = 1$, or $u(c) = ln(c)$, then $\partial U^E/\partial\pi = 0 \Leftrightarrow \pi = 1$. If $\rho < 1$, then $\partial U^E/\partial\pi = 0$ for some value of $\pi > 1$. Conversely, if $\rho > 1$, then $\partial U^E/\partial\pi = 0$ for some value of $\pi \in (\beta, 1)$.*

Proof. These results are immediate from equation (24). ■

Proposition 2 shows type-E agents benefit if the central bank chooses a money growth rate greater than that prescribed by the Friedman rule. Thus, if the central bank puts enough weight on the welfare of type-E agents, it will choose $z > \beta$.

As in the economy of the previous section an increase in the rate of growth of the money supply has two effects. First, it reduces the utility of all agents as it makes their consumption more volatile (c deviates more from c', which hurts any risk-averse agent). On the other hand, it creates a transfer from type-O to type-E agents. If the money stock does not grow too fast, the value of the transfer to type-E agents exceeds the cost in terms of volatility of consumption.

Also, as in the previous economy, the Friedman rule would be optimal if it were possible to make transfers that are less distorting. The type of transfer described in the previous section can be implemented if type-specific lump-sum transfers are feasible. Hence, again, a necessary condition for the Friedman rule to be suboptimal ex-post is that changes in the rate of growth of the money supply have redistributive effects.

4 A OG Model with random relocation

We consider a model economy in which money is valued because of limited communication across two spatially separated locations. Only a succinct description of the economic environment is provided; the interested reader is referred to Schreft and Smith (2002) and Bhattacharya, Haslag, and Russell (2004) for more details.

Time is discrete and denoted by $t = 1, 2...$ The world is divided into two spatially separated locations. Each location is populated by a continuum of agents of unit mass. Agents live for two periods and receive an endowment of ω units of the single consumption good when young and nothing when old. There also is an initial old generation whose members are endowed with an amount of cash M_0. Only old-age consumption is valued. Let c_t denote old-age consumption of the members of the generation born at date t; their lifetime utility is given by $u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}$, where $\rho \in (0, 1)$.

After receiving their endowment and placing it into a bank, agents learn whether they must move to the other location or not. Let α denote the probability that an individual will be relocated. We assume a law of large number holds so α is also the measure of agents that are relocated. α is the same on both islands so that moves across location are symmetric. Movers redeem their bank deposits in the form of money as this is the only way for them to acquire goods in the new location. In contrast, nonmovers redeem their deposits in the form of goods. Goods deposited in the bank can be used to acquire money from old agents belonging to the previous generation or put into storage. Each unit of the consumption good put into storage at date t yields $x > 1$ units of the consumption good at date $t + 1$, where x is a known constant.

The CB can levy lump-sum taxes τ on the endowment of agents by collecting the tax in the form of money balances removed from the economy. In contrast, a lump-sum subsidy is received in the form of a money injection. The money supply evolves according to $M_{t+1} = (1 + z) M_t$ and z is chosen by the CB in a manner that will be explained below. We assume $x \geq \frac{1}{1+z}$

implying that money is a bad asset. Let p_t denote the time t price level; in steady states, $p_{t+1} = (1+z)\,p_t$. Also, since we focus on steady-states, we drop the time subscript in what follows.

Agents deposit their entire after-tax/transfer endowments with a bank. The bank chooses the gross real return it pays to movers, d^m, and to non-movers, d^n. In addition, the bank chooses values m (real value of money balances) and s (storage investment) respectively. These choices must satisfy the bank's balance sheet constraint

$$m + s \leq \omega - \tau. \tag{25}$$

Banks behave competitively, so they take as given the return on their investments. In particular, the return on real money balances is p_t/p_{t+1}. If $x > p_t/p_{t+1}$ banks will want to hold as little liquidity as possible since money is dominated in rate of return. If $x = p_t/p_{t+1}$, banks are indifferent between money and storage. In this case, we consider the limiting economy as $p_t/p_{t+1} \to x$.

Banks must have sufficient liquidity to meet the needs of movers. This is captured by the following expression:

$$\alpha d^m(\omega - \tau) \leq \frac{m}{1+z}. \tag{26}$$

A similar condition for non-movers, who consume all the proceeds from the storage technology, is given by

$$(1-\alpha)d^n(\omega - \tau) \leq xs. \tag{27}$$

Banks maximize profits. Because of free entry, banks choose in equilibrium their portfolio in a way that maximizes the expected utility of a representative depositor. The bank's problem is written as

$$\max_{d^m, d^n} \quad \frac{(\omega-\tau)^{1-\rho}}{1-\rho}\left\{\alpha\left(d^m\right)^{1-\rho}+(1-\alpha)\left(d^n\right)^{1-\rho}\right\} \tag{28}$$

subject to equations (25), (26), and (27).

Let $\gamma \equiv \frac{m}{\omega-\tau}$ denote the bank's reserve-to-deposit ratio. Then, since equations (25), (26), and (27) hold with equality, the bank's objective function is to choose γ to maximize

$$\frac{(\omega-\tau)^{1-\rho}}{1-\rho}\left\{\alpha^{\rho}\left[\frac{\gamma}{1+z}\right]^{1-\rho}+(1-\alpha)^{\rho}\left[(1-\gamma)x\right]^{1-\rho}\right\}. \tag{29}$$

Bhattacharya, Guzman, Huybens, and Smith (1997) show that the reserve to deposit ratio chosen by the bank is given by

$$\gamma = \frac{1}{\left[1+\frac{1-\alpha}{\alpha}\left\{(1+z)\,x\right\}^{\frac{1-\rho}{\rho}}\right]} \tag{30}$$

and that it increases as $1+z$ decreases. For the initial old, consumption is equal to the real value of money balances. Let M_0 denote the quantity of nominal money balances held by a member of the initial old generation. Then, $c_1^0 = \frac{M_0}{p_1}$, where $p_1 = \frac{(1+z)M_0}{\gamma(\omega-\tau)}$. Note, in equilibrium, the reserve-to-deposit ratio and the lump-sum tax are functions of the money growth rate. At a steady state, the central bank maximizes the following objective function

$$W(z) = (1-\beta)\frac{\left(\frac{M_0}{p_1}\right)^{1-\rho}}{1-\rho}+\beta\frac{\{\Omega(z)\}^{1-\rho}}{1-\rho}\Gamma(z)$$

where $\Omega(z) := \omega-\tau(z)$, and $\Gamma(z) := \alpha^{\rho}\left[\frac{\gamma(z)}{1+z}\right]^{1-\rho}+(1-\alpha)^{\rho}\left[(1-\gamma(z))x\right]^{1-\rho}$. This allows us to find the rate of growth of the money supply chosen by the central bank under different assumptions about the weight of the initial old generation and, in a steady state, all other generations. For example, if $\beta = 0$, then the central bank only considers the utility of the initial old. Conversely, as $\beta \to 1$, the weight of the initial old goes to zero and so the central bank maximizes the utility of a representative generation (in steady states) and completely ignores the initial old.

Proposition 3 *The optimal rate of growth of the money supply is given by*

$$z = 1 + \frac{1-\beta}{\beta}\frac{\gamma}{\alpha^{\rho}},$$

where γ is computed from (30), along with the constraint that $1 + z \geq \frac{1}{x}$.

Proof. See Appendix. ■

It is straightforward to see that if $\beta \to 1$, then $z \to 1$. As $\beta \to 0$, in the limit the weight is all on the initial old; the constraint $1 + z \geq \frac{1}{x}$ eventually binds and the central bank implements the Friedman rule.

Bhattacharya, Haslag, and Russell (2004) and Haslag and Martin (2003) study how an increase in the rate of growth of the money supply away from the Friedman rule creates a transfer from agents who hold money to those who do not. Indeed this effect may dominate the negative effects of a higher money growth rate and can render the Friedman rule suboptimal. As in the previous economies, deviations from the Friedman rule also come at a cost here since the difference in consumption between movers and nonmovers increases as the rate of growth of the money supply increase. When the money stock does not grow too fast, the value of the transfer exceeds the cost created by the volatility in consumption. Edmonds (2002) has a comparable result. Again, the sub-optimality of the Friedman rule hinges crucially on the assumption that it is not possible to undo the transfers created by changes in the rate of growth of the money supply. Bhattacharya, Haslag and Russell (2004) show that when these transfers can be undone, the Friedman rule is once again optimal. Hence, once again, a key component of the explanation for why the Friedman rule is suboptimal (ex-post) is that changes in the rate of growth of the money supply have unremovable redistributive effects. Note also that unlike in the two economies studied above, here the Friedman rule is additionally sub-optimal ex-ante as shown in Smith (2002).

5 Summary and conclusion

In this paper, we consider steady state monetary policy in several alternative economic environments: two economies with infinitely-lived agents and an overlapping-generations economy. We provide examples where the Friedman rule is not the ex-post welfare maximizing monetary policy in these economies. To varying degrees, these results are known in the literature. Certainly this is true in the overlapping generations economy. Our aim is to explain why the welfare maximizing policy is not the Friedman rule in each case. Indeed, our main contribution is to offer one common explanation to account for the shared monetary policy results.

An key characteristic of the models we considered is that agents have heterogenous money holding. This heterogeneity arises from differences in agents' preferences in the Lagos-Wright (2002) framework (as well as in the money-in-the-utility-function economy). It is generated by differences in endowment patterns in the turnpike economy. Finally, differences in the age of agents living during the same period are responsible for this heterogeneity in the OG framework (old agents may hold money while young agents do not). In each case, a change in the rate of growth of the money supply has a redistributive effect. The intuition for our results in all three models studied in this paper can be described as follows: Suppose the money supply doubles, then we expect the price level to approximately double. In a representative agent model, this would have no effect on welfare. In a model where agents have heterogenous money holdings, the increase in the price level will hurt those holding small amounts of money less than those holding large amounts, even if the newly-issued money is distributed equally. In fact, the increase in the price level might benefit those who hold little money.[5] In each case, we provided an example where an increase in the rate of growth of the money supply creates Pareto incomparable allocations.

Heterogeneity, therefore, plays an important role in explaining why the Friedman rule does not maximize ex-post steady-state welfare in a variety of economic settings. A second important assumption in each of the economies considered is that markets are incomplete so agents, or the monetary authority, are unable to undo the redistribution caused by an increase in the rate of growth of the money supply away from the Friedman rule. For example, in each of the environments we consider, the Friedman rule would be optimal if type-specific lump-sum taxes were available.

[5] We thank an anonymous referee for suggesting this intuition.

Appendix

Proof of proposition 3

For future reference, note that, in steady states,

$$-\tau = \frac{M_t - M_{t-1}}{p_t} = -m\left(\frac{z}{1+z}\right)$$

$$m = \gamma\left(\omega - \tau\right) = \frac{\gamma\omega\left(1+z\right)}{\left(1+z\right)-\gamma z}$$

and hence,

$$\frac{M_0}{p_1} = \frac{M_1}{p_1}\frac{M_0}{M_1} = m\frac{1}{1+z} = \frac{\gamma\omega}{\left(1+z\right)-\gamma z}.$$

Substitute M_0/p_1 into the central bank's objective function. Take the derivative with respect to z, obtaining

$$\begin{aligned}&\left(\frac{\gamma\omega}{1+z-z\gamma}\right)^{-\rho}\left[\frac{\gamma'\omega\left(1+z-z\gamma\right)-\gamma\omega\left(1-\gamma-z\gamma'\right)}{\left(1+z-z\gamma\right)^2}\right]\\&+\frac{\beta}{1-\beta}\frac{\Omega^{1-\rho}\Gamma}{1+z}\left[\frac{1+z}{\Omega}\frac{\partial\Omega}{\partial z}+\frac{1}{1-\rho}\frac{1+z}{\Gamma}\frac{\partial\Gamma}{\partial z}\right]=0\end{aligned}$$

It can be verified that

$$\frac{1}{1-\rho}\frac{1+z}{\Gamma}\frac{\partial\Gamma}{\partial z} = -\gamma$$

and

$$\frac{1+z}{\Omega}\frac{\partial\Omega}{\partial z} = \frac{\gamma+(1+z)z\gamma'}{1+z-\gamma z}.$$

After rearranging, we get

$$\begin{aligned}&\left(\frac{\gamma\omega}{1+z-z\gamma}\right)^{-\rho}\omega\left[\frac{\gamma'\left(1+z\right)-\gamma\left(1-\gamma\right)}{\left(1+z-z\gamma\right)^2}\right]\\=\quad&-\frac{\beta}{1-\beta}\Gamma\frac{\left(\omega\left(1+z\right)\right)^{1-\rho}}{\left(1+z-z\gamma\right)^{1-\rho}}\frac{\left(1-\gamma\right)z}{\rho\left(1+z-z\gamma\right)}\end{aligned}$$

which simplifies to

$$\gamma^{1-\rho} = -\frac{\beta}{1-\beta}\Gamma\left(1+z\right)^{1-\rho} z$$

Note $\Gamma = \alpha^\rho \left(\frac{\gamma}{1+z}\right)^{1-\rho} + (1-\alpha)^\rho \left[(1-\gamma)\,x\right]^{1-\rho} = \left(\frac{\alpha}{\gamma}\right)^\rho \left[\frac{\gamma}{(1+z)^{1-\rho}} + \frac{1-\gamma}{x^{1-\rho}}\right]$. From the bank's maximization we have, $\alpha^\rho \frac{1}{1+z}\left(\frac{\gamma}{1+z}\right)^{-\rho} - \left(\frac{1-\alpha}{1-\gamma}\right)^\rho x^{1-\rho} = 0$, so that

$$\gamma = \frac{\beta}{1-\beta} z\alpha^\rho$$

which can be rewritten as

$$1+z = 1 + \frac{1-\beta}{\beta}\frac{\gamma}{\alpha^\rho}.$$

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