

**SPECIFIC FACTORS MEET  
INTERMEDIATE INPUTS: IMPLICATIONS  
FOR STRATEGIC COMPLEMENTARITIES  
AND PERSISTENCE**

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## **Abstract**

A central challenge to monetary business-cycle theory is to find a solution to the problem of persistence and delay in the real effects of monetary shocks. Previous research has identified separately specific factors and intermediate inputs as two promising mechanisms for generating the persistence and delay in a staggered price-setting framework. Models based on either of these two mechanisms have also been used in the design of optimal monetary policy.

By examining a staggered price model that features both specific factors and intermediate inputs, the author finds an offsetting interaction between the two individually promising mechanisms, which leads to a cancellation of much of the impact of each in propagating monetary shocks. This finding posits a challenge to the search for robust monetary transmission mechanism and design of optimal monetary policy.

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## 1. Introduction

The search for a monetary business-cycle theory that can account for the relationship between money and other economic activity has long been a challenge. In meeting the challenge, considerable effort has been made in the past decade to develop optimization-based sticky price models, such as those described in Goodfriend and King (1997). These models, however, have encountered difficulties in explaining some of the empirical regularities concerning the volatility and co-movement of nominal and real aggregates [e.g., King and Watson (1996)].

In an important analysis, Chari, Kehoe, and McGrattan (2000) stress a substantial difficulty faced by optimization-based staggered price models in explaining the persistence (and delay) in the response of real economic activity to nominal disturbances. Earlier studies suggested a promise of staggered price-setting for generating persistence. Yet, these authors demonstrate that, when the rules for setting prices are derived endogenously, staggered price mechanism, by itself, cannot generate much persistence. The problem is that, endogenous price-setting, even if in a staggered fashion, does not generate a long period of endogenous nominal inertia from a short period of exogenous nominal price stickiness that is not too much greater than suggested by the empirical evidence of Bils and Klenow (2004), Bils, Klenow, and Kryvtsov (2003), and Klenow and Kryvtsov (2003). They argue that a central challenge to monetary business cycle theory is to find a solution to the persistence problem, but conclude that “mechanisms to solve the persistence problem must be found elsewhere.”

The failure of staggered price mechanism in generating persistence can be traced to the lack of strategic complementarities between price-setter’s behavior in this class of models. This manifests the potential importance of real features of the economy for propagation of nominal shocks, a view that also has a tradition in the literature.<sup>1</sup> The past decade has indeed witnessed a surge in interest in incorporating various real features into a staggered price-setting framework to enhance the model. Two real features relevant for many industries in most modern economies, specific factor inputs and produced intermediate inputs, have been separately identified as promising for generating strategic complementarities in pricing

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<sup>1</sup>The use of the term “real features” is meant to be sentimental. It is referred to various market imperfections that impede the response of marginal production cost to changes in output, which Ball and Romer (1990) call “real rigidities”, as well as various supply-side features that allow a more elastic response of output to changes in demand without much increase in marginal cost, which Dotsey and King (2001) call “real flexibilities”.

and persistence in the real effects of nominal shocks.<sup>2</sup> Models based on either of these two mechanisms have also been used in the design of optimal monetary policy.<sup>3</sup>

Incidentally, little work has been done to investigate jointly the two individually promising mechanisms. By examining in this paper a staggered price model that features both specific factors and intermediate inputs, I identify an offsetting interaction between the two mechanisms, which leads to a cancelation of much of the impact of each in propagating monetary shocks. This is perhaps surprising given that intermediate inputs have been found to be reinforcing with some other real features in the previous literature.<sup>4</sup> To my knowledge, this paper is the first to show a detrimental interaction between two separately helpful real features for propagating nominal shocks. It posits a challenge to the search for robust monetary transmission mechanism and design of optimal monetary policy.

To drive this point home, I construct a dynamic stochastic general equilibrium staggered price model, featuring jointly specific factors and intermediate goods, with a constant elasticity of substitution between these two sources of production inputs, as consistent with the empirical evidence of Basu (1995) and Rotemberg and Woodford (1996, 1999). My primary objective is to investigate the nature of the interaction between the two real features for the transmission of nominal shocks. In carrying out this investigation, I first establish a closed-form equilibrium relation to decompose analytically the roles of the two mechanisms and of their interaction in generating strategic complementarities in pricing. I then discuss the intuition for a positive relationship between the degree of strategic complementarities and the amount of persistence

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<sup>2</sup>See, among others, Kimball (1995), Rotemberg (1996), Rotemberg and Woodford (1997), Gust (1997), Ascari (2001), Bergin and Feenstra (2000, 2001), Edge (2002), Woodford (2003), and Huang and Liu (2001, 2002, 2003). The idea that factor specificities might be important for the real effects of nominal shocks can be traced back at least to Blanchard and Fischer (1989, Chapter 8) and Ball and Romer (1990, 1991) [see, also, Dixon and Rankin (1994) for an early survey]. The idea on the relevance of intermediate inputs for the real effects of nominal shocks can be traced back to an even earlier date to Means (1935), Gordon (1981, 1990), and Blanchard (1983, 1987), and has been revisited recently by Basu (1995), Clark (1999), Hanes (1999), and Linnemann (2000).

<sup>3</sup>See Woodford (2003) and Huang and Liu (2004), among others.

<sup>4</sup>In particular, Bergin and Feenstra (2000) find a nonlinear interaction between intermediate inputs and translog preferences that goes beyond the sum of the individual contributions of the two real features to generating persistence. Dotsey and King (2001) consider intermediate inputs together with variable capacity utilization and labor supply variability along the employment margin (in addition to the hours-worked margin), and they find that, not only the three real features separately contribute to generating persistence, but “their effects on persistence are mutually reinforcing.”

and delay in the real effect of a nominal shock. I later derive an analytical solution for aggregate dynamics to formally establish this positive linkage. In particular, I obtain a necessary and sufficient condition for the response of real aggregate output to be hump-shaped, solve analytically for the timing of the peak, and characterize both the condition and the timing by the degree of strategic complementarities along with parameters governing the shock process.

These analytical results imply that the presence of intermediate inputs significantly weakens the impact of specific factors in propagating a nominal shock, while with the presence of specific factors the use of intermediate inputs in production serves even to dampen the real effect of the shock. This detrimental interaction between the two individually promising mechanisms leads to a reduction in the degree of strategic complementarities in pricing and persistence in the response of real aggregate output, and diminishes the possibility of a hump in the impulse response function or shifts the timing of the hump to an earlier date. Numerical simulations lead to similar conclusions drawn from analytical results.

To understand this negative interaction, note that the basic idea of introducing real features of the economy to magnify the real effect of a nominal shock rests upon the intuition that these features may serve to dampen the desired price responses to changes in aggregate demand condition. Mechanisms that either impede the response of marginal cost to changes in output, or allow a more elastic response of output to changes in demand without causing too much variation in marginal cost, are potential candidates.

To see why specific factors are such a candidate, consider a nominal shock, such as a shock to nominal money supply. Under staggered price-setting, the shock will cause a shift in factor demand schedules and a change in real income, which will shift factor supply schedules. The two schedule shifts reinforce to guide factor and good prices to adjust to the direction of the shock. With factor specificities, the demand for a factor input that is specific to a firm depends directly on the demand facing the firm for its output, and thus inversely on the relative price of the output, which, given prices charged by other firms, is determined by the factor price to the extent it accounts for the firm's marginal cost.<sup>5</sup> Thus, any change in the relative good price due to a movement in the factor price would cause a counter-forcing shift in the factor demand schedule, which would offset partially the shift induced by the shock when holding constant the relative good price, resulting in smaller movements in the factor and good prices in the first place. It is through this negative feedback between adjustments in factor prices and

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<sup>5</sup>This is in contrast to the case with homogenous factor inputs, where the demand for a common factor input depends on the demand for aggregate output, and thus is independent of any relative output price.

adjustments in good prices that factor specificities serve to restrain these price adjustments to magnify the real effect of the shock.

Intermediate inputs are also such a candidate. Since movements in factor prices lead to changes in good prices only to the extent they result in variations in marginal cost, and since variations in marginal cost resulting from given movements in factor prices are scaled down by the presence of intermediate inputs, a greater share of intermediate inputs in production implies smaller adjustments in good prices to given movements in factor prices. Further, a given shift in factor demand or supply schedules has a smaller effect on factor prices, the greater is the elasticity of factor demand, which I show is an increasing function of the share of intermediate inputs.

These two individually promising mechanisms are, however, counter-forcing when in joint presence. On the one hand, the presence of an intermediate input attenuates the feedback restraint embodied in factor specificities through the two aforementioned channels that make itself individually promising. First, with a larger share of intermediate inputs, given movements in factor prices lead to smaller variations in marginal cost and smaller changes in good prices, and thus the resulting counter-forcing shift in factor demand schedules, as induced by factor specificities, is smaller. Second, a larger share of intermediate inputs gives rise to a greater factor demand elasticity, and thus the offsetting effect on factor price adjustments of a given counter-forcing shift in factor demand schedules is smaller. On the other hand, such attenuating effect is generally strong that it dominates the individual promise of intermediate inputs and thus, with the presence of specific factors in the first place, the use of intermediate inputs in production serves even to dampen the real effect of a nominal shock.

The rest of the paper is organized as follows. Section 2 sets up the model and defines an equilibrium. Section 3 presents the decomposition, provides some intuitions behind it, and uses it to obtain analytical results and conduct numerical simulations to assess the consequence of the offsetting interaction between specific factors and intermediate inputs for strategic complementarities and persistence. Section 4 further details the results, by first deriving an analytical solution for equilibrium dynamics to establish analytical linkages between the degree of strategic complementarities and the amount of persistence in output impulse response, the likelihood of a hump on the impulse response, and the timing of the hump, and then conducting numerical simulations to confirm the conclusions drawn from these analytical results. Section 5 concludes. Most derivations and proofs are relegated to the Appendix.

## 2. A Model of Specific Factors and Intermediate Inputs with Staggered Pricing

The model features a representative household and a continuum of firms indexed on the unit interval  $[0, 1]$ , each of which produces a differentiated good. At each date  $t$ , a representative distributor combines all differentiated goods  $\{Z_{i,t}\}_{i \in [0,1]}$  into a composite good  $Z_t$  such that  $Z_t = [\int_0^1 Z_{i,t}^{(\theta-1)/\theta} di]^{\theta/(\theta-1)}$ , where  $\theta \in (1, \infty)$  is the elasticity of substitution between the individually differentiated goods. The distributor takes the prices  $\{P_{i,t}\}_{i \in [0,1]}$  of the individual goods as given and chooses the bundle of the goods to minimize the cost of fabricating a given quantity of the composite good. It sells the composite good to the household or firms at its unit cost  $\bar{P}_t = [\int_0^1 P_{i,t}^{1-\theta} di]^{1/(1-\theta)}$ . So it is assumed that the distributor cannot discriminate its selling price between the household and the firms, or across different firms. The demand for a type  $i$  good is given by

$$Z_{i,t} = \left( \frac{P_{i,t}}{\bar{P}_t} \right)^{-\theta} Z_t. \quad (1)$$

Units of the composite good purchased by the household can either be consumed directly or be converted using linear technologies into a continuum of types of investment goods, while the firms purchase the composite good for use as an intermediate input in the production of the differentiated goods.

### 2.1. Household

The household has a continuum of members, each of which possesses a differentiated labor skill. In each period  $t$ , the household derives utility from its total consumption, while it cares about the dis-utility of each of its members resulting from supplying to firms their differentiated labor skills. The objective of the household is to maximize

$$E_t \sum_{s=0}^{\infty} \beta^s \left[ U(C_{t+s}) + \int_0^1 V(L_{h_i,t+s}) di \right], \quad (2)$$

where  $E_t$  is the conditional expectation operator and  $\beta \in (0, 1)$  is the household's subjective discount factor. The arguments  $C_t$  and  $L_{h_i,t}$  denote respectively the household's consumption and quantity of labor of type  $i$  supplied in period  $t$ . The period-utility function,  $U$ , and the period-disutility function,  $V$ , are strictly increasing and strictly decreasing, respectively, and both are strictly concave and twice continuously differentiable.

At every date there is available for trade a complete set of one-period, state-contingent nominal bonds, which the household can use to transfer its nominal wealth across dates and states of the world. The no-arbitrage condition then implies the existence of a unique set of stochastic discount factors, which can be used to determine at any date the nominal present

value of a nominal quantity in any future date and state. Denote by  $D_{t,t+1}$  the stochastic discount factor from date  $t + 1$  to  $t$ . The nominal price at  $t$  of a one-period bond that pays off one unit of nominal account in a particular state of the world at  $t + 1$  is equal to  $D_{t,t+1}$  times the probability that this particular state will indeed be realized at  $t + 1$  conditional on the information available at  $t$ . Other financial claims can similarly be priced. In particular, a one-period bond issued at date  $t$  that pays off one unit of nominal account in all states of the world at  $t + 1$  has a nominal value at  $t$  of  $E_t D_{t,t+1}$ , and thus a gross nominal interest rate of  $(E_t D_{t,t+1})^{-1}$ . In general, if the random quantity  $B_t$  represents the household's holdings at  $t$  of the one-period, state-contingent nominal bonds, then this portfolio has a nominal value at  $t$  of  $E_t(D_{t,t+1}B_t)$ .

The household's budget constraint in period  $t$  requires that its expenditures on consumption and investment plus asset accumulation do not exceed its disposable income during the same period, that is,

$$\bar{P}_t C_t + \bar{P}_t \int_0^1 I_{i,t} di + E_t(D_{t,t+1}B_t) - B_{t-1} \leq \int_0^1 R_{i,t}^k K_{h_i,t-1} di + \int_0^1 W_{i,t} L_{h_i,t} di + \Pi_t, \quad (3)$$

where  $I_{i,t}$  and  $K_{h_i,t-1}$  denote the quantity of investment good of type  $i$  that the household obtains in period  $t$  and its stock of capital of type  $i$  as of date  $t - 1$ ,  $R_{i,t}^k$  and  $W_{i,t}$  are nominal rental rate on capital of type  $i$  and nominal wage rate paid to labor of type  $i$  in period  $t$ , and  $\Pi_t$  represents the household's claim to firms' profits in period  $t$ .

The household maximizes (2) subject to (3), a law of motion  $I_{i,t} = K_{h_i,t} - (1 - \delta)K_{h_i,t-1}$ , for all  $i \in [0, 1]$ , where  $\delta \in [0, 1]$  is a depreciation rate common to all types of capital, and a borrowing constraint  $B_t \geq -B$ , for some large positive number  $B$ , which serves to prevent the household from playing Ponzi schemes without bound. The household takes its initial capital stocks  $\{K_{h_i,-1}\}_{i \in [0,1]}$  and debt position  $B_{-1}$ , as well as all prices, wages, and capital rental rates as given in solving the utility-maximization problem.

## 2.2. Firms

A firm that produces a type  $i$  good has the following technology

$$Z_{i,t} = \left\{ \tilde{\phi}^{\frac{1}{e}} X_{i,t}^{\frac{e-1}{e}} + (1 - \tilde{\phi})^{\frac{1}{e}} \left[ \tilde{\alpha} K_{f_i,t}^\alpha L_{f_i,t}^{1-\alpha} \right]^{\frac{e-1}{e}} \right\}^{\frac{e}{e-1}} - F, \quad (4)$$

where  $X_{i,t}$ ,  $K_{f_i,t}$ ,  $L_{f_i,t}$  represent the firm's inputs of the intermediate good, capital, and labor, respectively, and  $F$  is a real fixed cost which is common to all firms. The parameter  $e \in [0, \infty)$  corresponds to the elasticity of substitution between the primary factors and the intermediate input,  $\tilde{\phi} \in [0, 1)$  and  $\alpha \in [0, 1)$  will help determine the share of material cost in the production



of gross output and the share of capital cost in the value-added inputs, respectively, and  $\tilde{\alpha}$  is a constant given by  $\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ . The specification in (4) implies a unit elasticity of substitution between labor and capital, which is a common assumption in the literature.

All firms are input-price takers, but are imperfect competitors in output markets, where they set prices for their products for  $N > 1$  periods in a staggered fashion and supply at these prices whatever quantities of the goods prescribed by the demand schedule (1). To be specific, all firms are divided into  $N$  equally measured cohorts, where firms in cohort 1 set new prices in periods  $0, N, 2N, \dots$ , firms in cohort 2 set new prices in periods  $1, N+1, 2N+1, \dots$ , and so on. At each date  $t$ , if it is the time that a firm  $i$  can set a new price, then it chooses  $P_{i,t}$  for its product for periods  $t$  through  $t+N-1$  to maximize

$$\mathbb{E}_t \sum_{s=0}^{N-1} D_{t,t+s} [(1+\tau)P_{i,t}Z_{i,t+s} - Q(Z_{i,t+s}) - T_t], \quad (5)$$

where  $D_{t,t+s} = \prod_{r=1}^s D_{t+r-1,t+r}$  denotes the  $s$ -period stochastic discount factor from date  $t+s$  to  $t$ , for all  $s > 0$ , with  $D_{t,t} = 1$ ,  $\tau$  is a flat rate at which the firm's output is subsidized, and  $T_t$  is an indirect business tax common to all firms. Here  $Q(Z_{i,t})$  represents the total cost of  $i$  at  $t$  for producing  $Z_{i,t}$ , which can be obtained by choosing  $X_{i,t}$ ,  $K_{f,i,t}$ , and  $L_{f,i,t}$  to minimize  $\bar{P}_t X_{i,t} + R_{i,t}^k K_{f,i,t} + W_{i,t} L_{f,i,t}$ , subject to (4), taking  $\bar{P}_t$ ,  $R_{i,t}^k$ , and  $W_{i,t}$  as given. This total cost is given by

$$Q(Z_{i,t}) = Q_{i,t}[Z_{i,t} + F], \quad \text{where} \quad Q_{i,t} = \left\{ \tilde{\phi} \bar{P}_t^{1-e} + (1-\tilde{\phi}) \left[ (R_{i,t}^k)^\alpha W_{i,t}^{1-\alpha} \right]^{1-e} \right\}^{\frac{1}{1-e}}. \quad (6)$$

The implied demands for the intermediate input, capital, and labor are, respectively,

$$X_{i,t} = \tilde{\phi} \left[ \frac{Q_{i,t}}{\bar{P}_t} \right]^e [Z_{i,t} + F], \quad (7)$$

$$K_{f,i,t} = \alpha \left[ \frac{Q_{i,t}}{R_{i,t}^k} \right] \left[ 1 - \tilde{\phi} \left( \frac{Q_{i,t}}{\bar{P}_t} \right)^{e-1} \right] [Z_{i,t} + F], \quad (8)$$

$$L_{f,i,t} = (1-\alpha) \left[ \frac{Q_{i,t}}{W_{i,t}} \right] \left[ 1 - \tilde{\phi} \left( \frac{Q_{i,t}}{\bar{P}_t} \right)^{e-1} \right] [Z_{i,t} + F]. \quad (9)$$

We note that even if a firm cannot set a new price at a given date, it would still need to solve the cost-minimization problem, and thus (6)-(9) must hold for all firms in all periods. Taking the demand schedule (1) and the cost function (6) as given, the solution to the profit-maximization problem (5) is obtained as

$$P_{i,t} = \mu \frac{\mathbb{E}_t \sum_{s=0}^{N-1} D_{t,t+s} \bar{P}_{t+s}^\theta Z_{t+s} Q_{i,t+s}}{\mathbb{E}_t \sum_{s=0}^{N-1} D_{t,t+s} \bar{P}_{t+s}^\theta Z_{t+s}}, \quad (10)$$

where  $\mu \equiv \theta(\theta - 1)^{-1}(1 + \tau)^{-1}$  is the steady-state effective markup of price over marginal cost.<sup>6</sup>

Equation (10) says that a firm's optimal price is a markup over a weighted average of its marginal costs during the periods in which its currently chosen price will remain in effect. Using (7)-(10), it can be verified that, in steady state, the share of payment to intermediate inputs in total production cost is equal to  $\phi = \tilde{\phi}\mu^{1-e}$ .

### 2.3. Market Clearing and Equilibrium

I have thus far assumed implicitly that a differentiated good is produced using a specific type of capital and a specific type of labor (together with an intermediate input), and that nevertheless there is neither monopoly power of the household nor monopsony power of the firms in the factor markets.<sup>7</sup> To help connecting with the literature, it would be helpful to allow the specification of the model to be flexible enough to nest the scenarios with homogeneous capital or\and labor as special cases. I therefore introduce two binary variables,  $\omega_1$  and  $\omega_2$ , each of which can take on values 0 and 1, corresponding respectively to the case without and with capital specificities and the case without and with labor specificities.<sup>8</sup> The market

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<sup>6</sup>Allowing a production subsidy  $\tau = (\theta - 1)^{-1}$  helps connect to the recent literature on monetary policy rules, where a subsidy is often assumed to eliminate steady-state monopolistic distortions.

<sup>7</sup>One way to justify this assumption is to think of each point on the unit interval as consisting of a large number of firms that use the same type of capital and labor to produce one type of good, and each member unit in the household as consisting of a large number of investors and workers who supply the same type of capital and labor. Since there is a large number of participants on both demand and supply sides in each of the factor markets, it is not theoretically objectionable to rule out the possibility of any collective behaviors in these marketplaces and assume that all factor prices are determined in a competitive fashion. An alternative approach is to think of the economy as consisting of  $N$  divisions (say, equally divided on the unit interval), with a large number of firms and household members in each division who supply goods (to the whole economy) and factors (only to the local firms) that are homogenous within division but differentiated across divisions, and with prices for goods being set in a staggered fashion across divisions. One can then argue that competition in factor markets within division would eliminate any monopoly or monopsony power in these markets. One can interpret the divisions as regions, cities, industries, sectors, etc.

<sup>8</sup>My consideration of the all-or-nothing extremes regarding factor specificities is only meant to be pathological. A more realistic specification would be to allow for the possibility of conversions of different "types" of factors subject to converting costs, where "types" may be identified by specialties, regions, or attachments to industries or firms, so that factor prices may vary across suppliers-demanders for some periods following a shock, but have a tendency to converge in the long run. Since it is mainly these short-run dynamics of factor prices that matter for the determination of short-run responses of goods prices to shocks, which are what matter for the analysis in this paper, my choice to consider the opposite extremes allows to capture effectively the essential ingredients of factor specificities without an explicit discussion of the converting costs alluded to.

clearing conditions for capital and labor in period  $t$  can then each be expressed compactly as

$$\omega_1 K_{f_i,t} + (1 - \omega_1) \int_0^1 K_{f_i,t} di = \omega_1 K_{h_i,t-1} + (1 - \omega_1) \int_0^1 K_{h_i,t-1} di, \quad (11)$$

$$\omega_2 L_{f_i,t} + (1 - \omega_2) \int_0^1 L_{f_i,t} di = \omega_2 L_{h_i,t} + (1 - \omega_2) \int_0^1 L_{h_i,t} di, \quad (12)$$

for all  $i \in [0, 1]$ , where I have assumed that at any date capitals available for firms to rent are accumulated by the household during the previous period. The market clearing condition for the composite good is given by  $C_t + \int_0^1 I_{i,t} di + \int_0^1 X_{i,t} di = Z_t$ , where the sum of the first two terms on the left hand side corresponds to real GDP or real aggregate spending,  $Y_t$ . The bond market clearing condition is standard.

An equilibrium for this model economy consists of allocations  $C_t$ ,  $B_{i,t}$ ,  $K_{h_i,t}$ , and  $L_{h_i,t}$ , for the household, allocations  $X_{i,t}$ ,  $K_{f_i,t}$ , and  $L_{f_i,t}$ , and prices  $P_{i,t}$ , for a firm  $i$ , for all  $i \in [0, 1]$ , together with stochastic discount factors  $D_{t,t+1}$ , prices  $\bar{P}_t$ , capital rental rates  $\{R_{i,t}^k\}_{i \in [0,1]}$ , and wages rates  $\{W_{i,t}\}_{i \in [0,1]}$ , that satisfy the following conditions: (i) taking capital rental rates and wage rates, as well as all prices but its own as given, each firm's allocations and prices solve its profit-maximization problem; (ii) taking capital rental rates and wage rates, as well as all prices as given, the household's allocations solve its utility-maximization problem; (iii) markets for bonds, capital, labor, and the composite good clear; (iv) total production subsidy is equal to total indirect business tax.

### 3. Analytics of Micro-Foundations

In this section, I derive an aggregate supply relation and use this relation to investigate the micro-foundation underlying the roles of specific factors and intermediate inputs and, in particular, of their interaction in generating strategic complementarities between the price-setting decisions by firms that supply differentiated goods, and persistence in the real effects of nominal disturbances. To help achieve analytical transparency, I follow a common approach to approximate the equilibrium conditions by a log-linear system. In what follows, I use lowercase letters to denote the log-deviations of corresponding level variables from their steady-state values. To help obtain closed-form solutions, I shall assume until further notice that aggregate capital is constant with no depreciation.

#### 3.1. Aggregate Supply

Using log-linearized equilibrium conditions, I obtain (see the Appendix for details)

$$p_t = \left( \frac{1 - \Gamma}{N} \right) \sum_{s=0}^{N-1} E_t \bar{p}_{t+s} + \left( \frac{\Gamma}{N} \right) \sum_{s=0}^{N-1} E_t \tilde{y}_{t+s}, \quad (13)$$

where I have set  $\beta = 1$  to simplify notations. Here,  $\bar{p}_t$  and  $\tilde{y}_t$  correspond to the price level and nominal aggregate spending, and  $p_t$  denotes the price set at time  $t$  by a firm that can choose a new price at  $t$ . Since firms are identified by the timing of their price-setting decisions, I have dropped the index  $i$  in the expression of the individual prices. The parameter  $\Gamma$  is given by

$$\Gamma = (1-\phi) \left[ \frac{\sigma(1-\alpha)}{\xi+\alpha} + \frac{\mu^{1-\mathbf{I}_F} - \phi}{\mu - \phi} \right] \left[ \frac{1-\alpha}{\xi+\alpha} + \frac{e(\mu-1)\phi}{\mu-\phi} \right]^{-1} \left[ 1 + \frac{\theta\mu^{-\mathbf{I}_F}(1-\phi)}{\frac{1+\xi}{\alpha(1+\xi)\omega_1 + \xi(1-\alpha)\omega_2} - 1 + e\phi} \right]^{-1} \quad (14)$$

where  $\mathbf{I}_F$  is an indicator function that takes on the value of 1 if  $F > 0$  and 0 if  $F = 0$ , and  $\sigma = -CU''/U'$  and  $\xi = LV''/V'$  denote steady-state relative risk aversions in consumption and hours worked, respectively. Clearly,  $\Gamma > 0$  for all admissible values of the model's parameters. In what follows, I will only present the results for the case with no fixed cost since the results for the case with fixed cost are strikingly similar.<sup>9</sup>

Equation (13) prescribes the optimal pricing behavior of a firm that can only reset its price once every  $N$  periods. Could the firm reset its price at every date, it would prefer a “desired” price  $p_t^*$  at time  $t$  given by

$$p_t^* = (1-\Gamma)\bar{p}_t + \Gamma\tilde{y}_t. \quad (15)$$

The fact that the firm can only change its price infrequently implies that the actual price it sets for its entire contract duration will only approximate its desired price in each of the contract periods on an average basis. This can be seen by combining (13) and (15), which yields

$$p_t = \frac{1}{N} \sum_{s=0}^{N-1} E_t p_{t+s}^*. \quad (16)$$

Note that  $p_t$  so chosen at time  $t$  must remain in effect for all periods from  $t$  through  $t+N-1$ , and it is in this sense that the mere existence of nominal price contracts by itself already implies a certain degree of nominal price stickiness. Although I shall not attempt here to say anything new about the underlying reasons for the existence of such contracts, but rather treat their existence as a structural feature of the environment in which firms sell their products, it is important to note, as many empirical studies indicate, that the length of nominal contracts observed in actual economies on average is too short to, by itself, explain the persistent real effects of nominal disturbances [e.g., Taylor (1999), Bils and Klenow (2004), Bils, Klenow, and Kryvtsov (2003), and Klenow and Kryvtsov (2003)]. Thus a useful model must explain when a firm may choose not to change its price by much even if it has the chance to adjust the price.

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<sup>9</sup>These results are available upon request from the author.

The answer to this question hinges upon the magnitude of  $(1 - \Gamma)$ , which determines how a firm's desired price,  $p_t^*$ , depends upon the level of prices charged by other firms,  $\bar{p}_t$ , under an arbitrarily specified stochastic process for the nominal spending,  $\tilde{y}_t$ . Pricing decisions are *strategic complements* (*substitutes*) if a firm's *desired* price would vary in the same (opposite) direction as does the level of prices charged by other firms.<sup>10</sup> It can be seen from (15) that strategic complementarities (substitutability) exist if and only if  $\Gamma < 1$  ( $\Gamma > 1$ ). To see the significance of strategic complementarities for persistence in the real effects of nominal disturbances, note that  $\Gamma$  measures the response of  $p_t^*$  to variations in  $\tilde{y}_t$ , given the response of  $\bar{p}_t$ . If  $\Gamma < 1$ , then a one percent change in  $\tilde{y}_t$  will induce a less than one percent change in  $p_t^*$ , so long as the change in  $\bar{p}_t$  is less than one percent. Hence, a firm will price less aggressively if other firms do. As all firms price less aggressively, the response of real output becomes more persistent. Such a linkage between strategic complementarities and persistence will be made more transparent in Section 4. The general conclusion is that, a smaller value of  $\Gamma$  implies a greater degree of strategic complementarities between the price-setting decisions by firms and a more persistent effect of nominal disturbances on real economic activity.

The question then is: How would the presence of specific factors and intermediate inputs affect the magnitude of  $\Gamma$ ? To answer this question, one can appeal to the right-hand side of (14), which decomposes the effect on  $\Gamma$  of intermediate inputs into four multiplicative terms and summarizes that of specific factors into the last one. Indeed, with homogeneous capital and labor inputs (i.e., with  $\omega_1 = \omega_2 = 0$ ), this last term is equal to 1. Since the first three terms are each decreasing in  $\phi$ , a larger share of intermediate inputs tends to lower  $\Gamma$  through lowering these terms. This explains why a number of studies find that the presence of intermediate inputs with factor homogeneity helps generate strategic complementarities and persistence.<sup>11</sup> Similarly, since the last term is smaller than 1 if  $\omega_1$  and  $\omega_2$  are not both 0, the allowance for factor specificities tends to lower  $\Gamma$  through factoring in this less-than-unit term. This explains why several studies find that the presence of specific factors helps increase the degree of strategic complementarities and persistence.<sup>12</sup>

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<sup>10</sup>This notion of strategic complementarities (substitutability) is similar to that used in Woodford (2003). For a more general characterization of strategic complementarities (substitutability) in a somewhat broader context, see, among others, Haltiwanger and Waldman (1985), Cooper (1999), and the references cited therein.

<sup>11</sup>See, for example, Bergin and Feenstra (2000), Huang and Liu (2001, 2003), and Dotsey and King (2001).

<sup>12</sup>See Rotemberg and Woodford (1997), Gust (1997), Ascari (2001), Edge (2002), Woodford (2003), and Huang and Liu (2003), among others.

More importantly, the last term on the right-hand side of (14) captures the effect on  $\Gamma$  of the interaction between specific factors and intermediate inputs. It shows that these two individually promising mechanisms are counter-forcing when in joint presence. To see this, note that this last term is an increasing function of  $\phi$ . Thus, the presence of intermediate inputs reduces the influence of factor specificities on  $\Gamma$  and thus on strategic complementarities and persistence. This reduction is driven by two mutually reinforcing forces: the one in the numerator,  $(1-\phi)$ , is at work regardless of the value of  $e$ , while the one in the denominator,  $\phi$ , is at work as long as  $e > 0$ , and this force is stronger, the greater is  $e$ . In fact, as I will show below, such negative effect is so strong that it generally dominates the effects of intermediate inputs on  $\Gamma$  as via the first three terms on the right-hand side of (14). As a consequence, with the presence of specific factors in the first place, the use of intermediate inputs in production serves even to increase the magnitude of  $\Gamma$ , and thus to decrease the degree of strategic complementarities in pricing and dampen the real effect of nominal disturbances.

### 3.2. Some Intuitions

Before examining in more details the negative interaction between the two separably helpful real features of the economy, I provide first some intuitions for why  $\Gamma$  can be decomposed into the four terms as of the right-hand side of (14), which is the key to understanding why the two mechanisms are individually helpful, but offsetting when in joint presence. To carry through the intuitions in a transparent way without loss of insights, I shall base my discussions here on the case with no capital or production subsidy. Setting  $\alpha = \tau = 0$  (along with  $F = 0$ ), equation (14) simplifies to

$$\Gamma = (1 - \phi) \left( \frac{\sigma}{\xi} + 1 \right) \left[ \frac{1}{\xi} + \frac{e\phi}{\phi + \theta(1 - \phi)} \right]^{-1} \left[ 1 + \frac{\theta(1 - \phi)}{\xi^{-1} + e\phi} \right]^{-1}, \quad (17)$$

where I have set  $\omega_2 = 1$  to capture labor specificities.

Recall that  $\Gamma$  determines a firm's desired price response to variations in nominal spending, given the price responses of other firms. It can be seen from rewriting (15) into  $p_t^* - \bar{p}_t = \Gamma y_t$ , where  $y_t$  denotes changes in real GDP, that  $\Gamma$  also links the firm's desired relative price change to variations in real income, or real aggregate demand. As mentioned before, these variations would lead to reinforcing shifts in the household's labor supply schedule and the firm's labor demand schedule, causing the real wage faced by the firm to adjust in accordance with the last three terms in (17).

To see that intermediate inputs are an individually promising mechanism, first note that a movement in the real wage induces a change in the firm's desired relative price only to the

extent such movement causes a variation in the firm's marginal cost. Since the variation in marginal cost resulting from a given movement in the real wage is scaled down by the share of intermediate inputs in production, a greater share of intermediate inputs implies a smaller adjustment in the desired relative price to a given movement in the real wage. More specifically, the desired relative price change is  $(1 - \phi)$  fraction of the real wage adjustment since, with  $\phi$  share of intermediate inputs in production, a one percent change in real wage results in only a  $(1 - \phi)$  percent change in real marginal cost. This is why  $(1 - \phi)$  shows up as the first term in (17), which illustrates the first channel by which intermediate inputs are individually helpful.

The second channel that makes intermediate inputs individually helpful has to do with the fact that the real wage adjusts according to the middle two terms in (17), in addition to the last term. As a matter of fact, these middle two terms characterize the equilibrium wage adjustment for the case with labor homogeneity. Since a one percent change in real income causes a  $\sigma/\xi$  percent shift in the labor supply schedule,  $\sigma/\xi$  shows up as one component of the second term.<sup>13</sup> Since a one percent change in real aggregate demand causes a one percent shift in the labor demand schedule, 1 shows up as another component of the second term.<sup>14</sup> In sum, the term  $(\sigma/\xi + 1)$  summarizes the extents of the two schedule shifts.<sup>15</sup> The next term is the inverse of the sum of the absolute values of the elasticities of labor supply,  $1/\xi$ , and of labor demand,  $-e\phi/[\phi + \theta(1 - \phi)]$ .<sup>16</sup> This is an inverse relationship, since the more elastic

<sup>13</sup>Note that the relative risk aversion in consumption,  $\sigma$ , determines how fast the household's marginal utility of consumption falls (rises) as its income rises (falls), and the relative risk aversion in hours worked,  $\xi$ , determines how rapidly its marginal dis-utility of working falls (rises) as its hours worked fall (rise). Therefore,  $\sigma/\xi$  measures the desired change in the household's labor supply with respect to a change in its real income at any given wage rate. This can be seen more transparently from rewriting the log-linearized labor supply schedule (34) in the Appendix as  $l = (1/\xi)(w - \bar{p}) - (\sigma/\xi)y$ .

<sup>14</sup>This can be seen more transparently by combining (33), (35), (37), and (38) in the Appendix, while setting  $\alpha = \tau = F = \omega_2 = 0$ , to derive the labor demand schedule  $l = -\{e\phi/[\phi + \theta(1 - \phi)]\}(w - \bar{p}) + y$ , with the real aggregate demand,  $y$ , rather than total sales,  $z$ , as a shift variable. Note that, with labor homogeneity, there is only an economy-wide prevailing real wage, the change in which will cause a change in the difference between  $z$  and  $y$  [this can be seen by combining (33) and (35), recognizing that all  $w_i$  are equal to some economy-wide  $w$ ].

<sup>15</sup>Note that this term is strictly decreasing in  $\phi$  in the presence of a fixed cost, as is clear from (14).

<sup>16</sup>See Footnotes 13 and 14. To understand this expression for labor demand elasticity, note that, any variations in the real wage would create an incentive for the firm to substitute between intermediate inputs and labor. By definition, as long as both of these two factor inputs are used in production, a larger  $e$  implies that it is easier to substitute between them. On the other hand, the larger is  $\phi$ , the more important are materials relative to labor in production, and thus the more sensitive is the demand for labor to variations in the real wage, as long as there is some degree of substitutability between the two factors. This is why this labor demand elasticity is increasing in  $e$  for  $\phi > 0$  and increasing in  $\phi$  for  $e > 0$ . In contrast, it is decreasing in  $\theta$  for  $e > 0$  and  $\phi \in (0, 1)$ .

the labor supply schedule or the labor demand schedule is, the smaller is the equilibrium wage adjustment resulting from a given shift in either of these two schedules. Since the absolute value of the labor demand elasticity is increasing in the share of intermediate inputs, a given shift in the labor demand or supply schedule has a smaller effect on the real wage, the greater is the share of intermediate inputs in production. This illustrates the second channel by which intermediate inputs are individually helpful.

The mechanism that makes labor specificities an individually promising real feature has to do with the fact that the real wage adjusts also according to the last term in (17), in addition to the middle two terms. As is clear from the decomposition, this last term captures the effect of labor specificities on the adjustment in the real wage faced by the firm. Since the firm employs a specific type of labor, the demand for this labor input depends directly on the demand facing the firm for its output, and thus inversely on the relative price of the output which, given the prices charged by other firms, is determined by the real wage to the extent it accounts for the firm's marginal cost.<sup>17</sup> Thus, any change in the relative price due to a movement in the real wage would cause a counter-forcing shift in the labor demand schedule, which would offset partially the shift induced by variations in real aggregate demand when holding constant the relative price, resulting in smaller movements in the real wage and in the relative price at the first place. It is through this negative feedback between adjustments in the real wage and adjustments in the desired relative good price that labor specificities serve to restrain the adjustments in the wage and the price to magnify the real effect of the nominal disturbance. The larger is the price elasticity of demand for goods,  $\theta$ , the greater is

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This is so since a larger  $\theta$  implies a greater elasticity of substitution between the individually differentiated goods, and thus the firm can rely less on substitution between the composite intermediate input and labor while more on substitution between the individual goods. As a consequence, when  $\theta$  increases, the firm's demand for labor becomes less sensitive to variations in the real wage. This tension between these two dimensions of factor substitutions exists if only if both intermediate inputs and labor are used in production and there is some degree of substitutability between them. This is why  $\theta$  enters this elasticity only if  $\phi \neq 0$  or 1 and  $e > 0$ .

<sup>17</sup>To see this more clearly, combine (33) and (43), while setting  $\alpha = 0$  and  $N = 1$ , to derive the desired relative price response by a type  $i$  firm to changes in its real wage cost as  $p_i^* - \bar{p} = (1 - \phi)(w_i - \bar{p})$ , and combine (33), (37), and (38), while setting  $\alpha = \tau = F = 0$  and  $\omega_2 = 1$ , to derive the schedule of demand for a type  $i$  labor as  $l_i = -e\phi(w_i - \bar{p}) - \theta(p_i^* - \bar{p}) + z$ , with the total sales,  $z$ , as a shift variable. Note that, with labor specificities, there is a prevailing real wage for each type of labor, and an individual change in one prevailing wage does not affect the difference between  $z$  and  $y$  [again, this can be seen by combining (33) and (35), recognizing that  $w_i$  may be different from  $w_j$  for  $i \neq j$ ]. Also note that, in the case with homogenous labor input, the labor demand schedule is dependent of the demand for aggregate output, but independent of any relative output price (see Footnote 14).



the counter-forcing shift resulting from a given change in the relative price; and, the greater is the labor supply elasticity,  $\xi^{-1}$ , the smaller is the offsetting effect of a given counter-forcing shift on the adjustment in the real wage. This is why this last term is decreasing in  $\theta$  but increasing in  $\xi^{-1}$ .<sup>18</sup>

More importantly, this very last term in (17) also captures the offsetting interaction between the two individually promising mechanisms. On the one hand, the presence of intermediate inputs attenuates the feedback restraint embodied in labor specificity and thus its impact on the real wage adjustment. This is done through the two channels that make intermediate inputs themselves individually promising. First, with  $\phi$  share of intermediate inputs in production, a one percent movement in the real wage leads to only a  $(1 - \phi)$  percent change in the firm's real marginal cost. Thus, the firm's desired relative price change in response to a given movement in the real wage is attenuated by a factor of  $\phi$ , and so is the resulting counter-forcing shift in the labor demand schedule as induced by labor specificity (see Footnote 17). This is why in this last term,  $\theta$  is multiplied by  $(1 - \phi)$ . Second, given  $e > 0$ , a larger  $\phi$  gives rise to a greater labor demand elasticity, imputed to which a smaller offsetting effect on the real wage adjustment of a given counter-forcing shift in the labor demand schedule (see Footnotes 14-17). This is why in this last term,  $e\phi$  is added to  $\xi^{-1}$ . These two sources of attenuation reinforce to weaken the power of labor specificities in generating strategic complementarities and persistence.<sup>19</sup> On the other hand, such negative interaction is so strong that its effect generally dominates the effects of intermediate inputs on  $\Gamma$  as via the first three terms on the right-hand side of (17) that capture their individual promise. As a result, with the presence of labor specificities in the first place, the use of intermediate inputs in production serves even to increase the magnitude of  $\Gamma$ , and thus to decrease the degree of strategic complementarities and persistence.

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<sup>18</sup>This is to say, a greater labor supply elasticity reduces the contribution of labor specificities to lowering  $\Gamma$ . If we set  $\phi = 0$  in (17), then  $\Gamma = (\sigma + \xi)/(1 + \theta\xi)$ , which is an increasing function of  $\xi^{-1}$  provided that  $\theta\sigma > 1$ . This is in contrast to the case with labor homogeneity, where  $\Gamma$  is given by  $(\sigma + \xi)$ , which is clearly decreasing in  $\xi^{-1}$ . Thus, in the absence of intermediate inputs in production, a higher elasticity of labor supply implies a larger degree of persistence in the case with labor homogeneity, but a smaller degree of persistence in the case with labor specificities. It can be shown that, in the case with intermediate inputs and labor homogeneity,  $\Gamma$  is decreasing, invariant, or increasing with  $\xi^{-1}$ , if  $(\sigma e - 1)\phi$  is less than, equal to, or larger than  $\theta(1 - \phi)$ . Since the use of intermediate inputs also reduces the contribution of labor specificities to lowering  $\Gamma$ , when both these two real features are present,  $\Gamma$ , and thus the degree of persistence, become less sensitive to the magnitude of labor supply elasticity, as will be illustrated by the results to be reported in the subsequent sections.

<sup>19</sup>It is worth noting that this last term in (17) is an increasing function of not only  $\phi$ , but also  $e$  for any  $\phi > 0$ . This should be in contrast with the third term in (17), which is an decreasing function of  $e$  for any  $\phi > 0$ .

### 3.3. Analytical Result

In this section, I further illustrate the consequence of the negative interaction between specific factors and intermediate inputs in the light of (14). The following proposition shows that this interaction not only leads to a reduction in the influence of factor specificities on strategic complementarities and persistence, but generally turns intermediate inputs into a detrimental device. To help exposition, I present here the analytical result only for the case with no capital, and leave the discussion on the quantitative implications of the more general result for the case with capital to the next section.

**Proposition 3.1:** *Set  $\alpha = F = 0$ , and  $\omega_2 = 1$ . Fixed  $\sigma \geq 0$ ,  $\xi > 0$ ,  $\theta > 1$ . Then, for any  $e > \underline{e} \equiv \mu[\xi^2\theta + \xi(1 - \mu)]^{-1}$ ,  $\Gamma(\phi)$  is  $C^1$  on  $[0, 1)$ , and there exist  $0 < \phi^* < \phi^{**} < 1$  such that*

$$\Gamma'(\phi) > 0 \quad \text{if } \phi \in [0, \phi^*), \quad \text{"="} \quad \text{if } \phi = \phi^*, \quad \text{"<"} \quad \text{if } \phi \in (\phi^*, 1), \quad (18)$$

$$\Gamma(\phi) > \Gamma(0) \quad \text{if } \phi \in (0, \phi^{**}), \quad \text{"="} \quad \text{if } \phi = \phi^{**}, \quad \text{"<"} \quad \text{if } \phi \in (\phi^{**}, 1), \quad (19)$$

where  $\Gamma'(0)$  is defined as the right-hand derivative of  $\Gamma(\phi)$  at  $\phi = 0$ .

According to the proposition, under fairly general parameter restrictions, the presence of intermediate inputs reduces the contribution of labor specificities to lowering  $\Gamma$  so significantly that the effect more than offsets their contribution to lowering  $\Gamma$  under labor homogeneity in a local and in a global sense with respect to the intermediate input share, unless the share exceeds two successive threshold values,  $\phi^*$  and  $\phi^{**}$ . The parameter values under which the proposition holds largely cover their empirically plausible ranges. For example, the long-run average markup of price over marginal cost is empirically small, suggesting a value of  $\underline{e}$  close to  $\xi^{-2}\theta^{-1}$ .<sup>20</sup> In the case with no production subsidy and a markup of 5%, corresponding to a value of  $\theta$  of 21, the proposition holds for all  $e > 0.0005$  if  $\xi = 10$ , for all  $e > 0.002$  if  $\xi = 5$ , and for all  $e > 0.05$  even if  $\xi$  is as small as 1 (corresponding to an hours-worked elasticity as large as 1). Even for a markup as large as 11%, corresponding to a value of  $\theta$  as small as 10, these lower bounds on  $e$  are only increased by a factor of about 2, and thus stay small in general. In the case with a production subsidy that eliminates steady-state monopolistic distortions, these lower bounds on  $e$  are even smaller. According to the recent estimates by Basu (1995) and Rotemberg and Woodford (1996, 1999), a value of  $e$  in the range of 0.36 and 0.69 can be empirically plausible.

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<sup>20</sup>While most studies have a markup of about 11%, the recent studies by Basu and Fernald (1997b, 2000) suggest it can be as small as 5% once controlling for capacity utilization rates.

### 3.4. Quantitative Implications

I turn now to assessing the quantitative implications of the more general result for the case with capital. To save space, from now on and throughout the rest of the paper, I shall continue to focus my discussion and analysis for the case with capital but without production subsidy.<sup>21</sup>

Figure 1 plots  $\Gamma$  against the share of intermediate inputs,  $\phi$ , under factor specificities. In generating the figure,  $\alpha$  is set to 0.33, as is standard in the literature.<sup>22</sup> The elasticity of substitution between differentiated goods,  $\theta$ , is set equal to 10, as in Chari *et al.* (2000). There are four panels in the figure, each under a different combination of values for the relative risk aversion in consumption,  $\sigma$ , and in hours worked,  $\xi$ . Though some studies suggest that the value of  $\sigma$  can be as small as 0 or as large as 30, the general consensus is that it is between 1 and 10 [e.g., Prescott (1986), Mehra and Prescott (1985, 1988), and Kocherlakota (1996)]. The value of  $\xi$  is set in the range from 5 to 20, corresponding to intertemporal hours-worked elasticity of 20% to 5%, as to be consistent with the empirical evidence.<sup>23</sup> The consensus in the literature about the value of  $e$ , the elasticity of substitution between primary factors and intermediate inputs, is that it is between 0 and 1.<sup>24</sup> For tractability, however, an extreme value of  $e$ , either 0 or 1, is often assumed in the existing studies.<sup>25</sup> I take here a diagnostic approach and thus each panel displays  $\Gamma$  for five different values of  $e$ , ranging from 0 to 1.

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<sup>21</sup>The results for the cases with production subsidy or\and without capital are qualitatively similar and quantitatively more striking. These results are not reported here but available upon request from the author.

<sup>22</sup>In the light of the evidence that long-run profits are close to zero [e.g., Basu and Fernald (1997a)],  $\alpha$  should correspond closely to the share of cost of capital in total value added in the National Income and Product Account (NIPA), with an implied value of about one third. For the case with a fixed cost in the production function (4), and given the assumption that total production subsidy is equal to total lump-sum tax,  $\alpha$  can be formally calibrated to 0.33 using consistent NIPA data, by setting the steady-state ratio of fixed cost to gross output equal to  $(\theta - 1)^{-1}$  (so that steady-state profits are zero and there are no incentives for firms to enter or exit industries in the long run).

<sup>23</sup>See, for example, Pencavel (1986), Altonji (1986), Ball (1990), and Card (1994). Similar results have been obtained for greater labor supply elasticities, such as those suggested by MaCurdy (1983), Mulligan (1998), Kimmel and Kniesner (1998), and Rupert, Rogerson, and Wright (2000). See Footnote 18.

<sup>24</sup>See, for example, the aforementioned estimates by Basu (1995) and Rotemberg and Woodford (1996, 1999). In the case with  $\mu > 1$ , restricting  $e$  to being less than 1 has a theoretical advantage in addition to its seeming empirical merit: for any  $\phi$  between 0 and 1, the corresponding  $\tilde{\phi}$  also lies between 0 and 1. If  $\mu = 1$ , however, then  $\tilde{\phi} = \phi$  regardless of the value of  $e$ .

<sup>25</sup>For the Leontief specification (i.e.,  $e = 0$ ), see, for example, Rotemberg and Woodford (1995) and Woodford (2003). For the Cobb-Douglas specification (i.e.,  $e = 1$ ), see, among others, Basu (1995), Bergin and Feenstra (2000, 2001), Linnemann (2000), Hillberry and Hummels (2002), Ambler et al. (2002), Huang and Liu (2003), and Yi (2003). Basu and Kimball (1997) consider both specifications.

As can be seen from the figure, if  $e$  is set to 0, then  $\Gamma$  is almost invariant to changes in  $\phi$ , implying that the reduction in the impact of factor specificities on  $\Gamma$  due to the presence of intermediate inputs is significant enough to essentially cancel out the impact of the latter on  $\Gamma$  under factor homogeneity [see, also, Woodford (2003)]. In contrast, in all cases with  $e > 0$ , as  $\phi$  rises from 0,  $\Gamma$  keeps increasing until  $\phi$  reaches a threshold value,  $\phi^*$ , then  $\Gamma$  starts to decrease, but will stay above its value at  $\phi = 0$  until  $\phi$  reaches another threshold value,  $\phi^{**}$ . This implies that, locally for all  $\phi \leq \phi^*$  and globally for all  $\phi \leq \phi^{**}$ , the presence of intermediate inputs reduces the contribution of factor specificities to lowering  $\Gamma$  so significantly that the effect more than offsets their contribution to lowering  $\Gamma$  under factor homogeneity. In consequence, as  $\phi$  rises from 0 to  $\phi^*$ , the degree of strategic complementarities and persistence will keep declining, and will then start to increase as  $\phi$  rises further, but will continue to be smaller than in the case without intermediate inputs, until  $\phi$  rises above  $\phi^{**}$ . This pattern of  $\Gamma$  in varying with  $\phi$  is consistent with what is suggested by Proposition 3.1.

Two observations at this point are worth mentioning. First, in all panels of the figure, the threshold value  $\phi^*$  lies between 0.4 and 0.7 or in its close vicinity, which conforms to an empirically reasonable range for the share of payment to intermediate inputs in total production cost.<sup>26</sup> The threshold value  $\phi^{**}$ , on the other hand, lies far beyond this range and is often close to 1. Second, when  $\phi$  takes on values in this range, the magnitude of  $\Gamma$  is often several times greater than its value at  $\phi = 0$  even for small or moderate  $e$ . In the lower-right panel, for instance, the value of  $\Gamma$  is about 0.13 if  $\phi = 0$ , while if  $\phi$  takes on values in its empirically plausible range  $\Gamma$  can be as large as 0.27 for  $e = 0.1$ , 0.34 for  $e = 0.2$ , 0.46 for  $e = 0.5$ , and 0.58 for  $e = 1$ .<sup>27</sup> Recall that a value of  $e$  between 0.36 and 0.69 is empirically plausible in the

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<sup>26</sup>With markup pricing,  $\phi$ , which measures the cost share, equals the share of intermediate inputs in gross output times the steady-state markup. Jorgenson, Gollop, and Fraumeni (1987) estimate the revenue share of materials in total U.S. manufacturing output of at least 50 percent over the period 1947-1979. A similar figure can be obtained using more recent data covering 1958-1996 for 459 4-digit SIC U.S. manufacturing industries from the NBER Manufacturing Productivity Database (constructed by Eric J. Bartelsman, Randy A. Becker, and Wayne B. Gray, 2000), with much of it derived from the U.S. Annual Survey of Manufacturing. Huang, Liu, and Phaneuf (2004) using data in the 1998 Annual Input-Output Table of the Bureau of Economic Analysis (BEA, 1998) estimate the ratio of “total intermediate” to “total industry output” for the manufacturing sector of 0.6. Incidentally, Nevo (2001) finds the share of raw materials in the U.S. food industry (SIC 20) of also about 0.6, based on the Annual Survey of Manufacturers over the period 1988-1992. The recent historical study by Hanes (1999) indicates that the input-output structure in U.S. economy was less sophisticated in the interwar period than in the postwar period, suggesting a possibly smaller value of  $\phi$  before World War II.

<sup>27</sup>A value of  $\Gamma$  in the range of 0.10 to 0.15 is required to generate substantial strategic complementarities to explain roughly the observed degree of sluggishness of aggregate price adjustments in response to variations in

light of the estimates by Basu (1995) and Rotemberg and Woodford (1996, 1999). Together, these observations suggest that the consequence of the offsetting interaction between specific factors and intermediate inputs for strategic complementarities and persistence is significant for empirically plausible parameter values.

#### 4. Implications for Macro-Dynamics

In this section, I solve analytically for equilibrium dynamics to make transparent how  $\Gamma$  determines the response of real aggregate output to variations in nominal aggregate expenditure or shocks in money supply. I then derive a necessary and sufficient condition for the response to be hump-shaped, solve analytically for the timing of the peak, and characterize the condition and the timing by  $\Gamma$  along with parameters governing a shock process. Together, these establish the positive relationship between the degree of strategic complementarities in pricing and the amount of persistence and delay in the real effect of a nominal shock. This relationship, when coupled with those results derived in the previous section, indicates a potentially significant consequence of the offsetting interaction between specific factors and intermediate inputs for propagation of nominal shocks. Finally, I simulate the model to confirm conclusions drawn from analytical results. In particular, I demonstrate, using impulse response functions, how the interaction between the two individually promising features reduces the degree of persistence in the response of real aggregate output, and diminishes the possibility of a hump in the impulse response function or shifts the timing of the hump to an earlier date.

##### 4.1. Impulse Response Functions: Closed-Form Solutions

I begin by deriving closed-form solutions for equilibrium dynamics. Substituting into (13) the equation defining the price level,  $\bar{p}_{t+s} = \frac{1}{N} \sum_{r=0}^{N-1} p_{t+s-r}$ , for  $s = 0, \dots, N-1$ , collecting terms and rearranging, I obtain

$$p_t = \frac{1 - \Gamma}{N - 1 + \Gamma} \sum_{s=-N+1}^{N-1} b_s \mathbb{E}_t p_{t+s} + \frac{\Gamma}{N - 1 + \Gamma} \sum_{s=0}^{N-1} \mathbb{E}_t \tilde{y}_{t+s}, \quad (20)$$

where  $b_s$  equals  $(N - |s|)/N$  for  $s \neq 0$  and 0 for  $s = 0$ .

To help sharpen the results, I will set  $N$  equal to 2 throughout this subsection, so that there is a minimum amount of exogenous nominal stickiness. In accordance, (20) simplifies to a second order difference equation in  $p_t$ . Applying to it the standard methods for solving 

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nominal expenditure under the observed frequency of price adjustments in an economy such as that of the U.S. See, for example, Rotemberg and Woodford (1997).

linear difference equations and the law of iterated expectations, I obtain a recursion in  $p_t$ ,

$$p_t = ap_{t-1} + \frac{2a\Gamma}{1-\Gamma} \sum_{i=0}^{\infty} a^i (\mathbb{E}_t \tilde{y}_{t+i} + \mathbb{E}_t \tilde{y}_{t+i+1}), \quad (21)$$

where the autoregressive coefficient  $a$  is given by

$$a = \frac{1 - \sqrt{\Gamma}}{1 + \sqrt{\Gamma}}. \quad (22)$$

Denote by  $\mu_t$  the growth rate in nominal aggregate spending  $\tilde{y}_t$ . Combining (21) and its lagged version with the relation  $\tilde{y}_t = \bar{p}_t + y_t$ , and using again the equation defining the price level, I obtain a recursion in real GDP,  $y_t$ , as

$$y_t = ay_{t-1} + \frac{1+a}{2} \mu_t - \frac{a\Gamma}{1-\Gamma} \sum_{i=0}^{\infty} a^i \left[ 2 \sum_{j=1}^i (\mathbb{E}_t \mu_{t+j} + \mathbb{E}_{t-1} \mu_{t+j-1}) + \mathbb{E}_t \mu_{t+i+1} + \mathbb{E}_{t-1} \mu_{t+i} \right], \quad (23)$$

with the understanding that  $\sum_{j=1}^0 (\mathbb{E}_t \mu_{t+j} + \mathbb{E}_{t-1} \mu_{t+j-1}) \equiv 0$ .

The autoregressive coefficient  $a$  in (23) is the key to determining how an initial response of  $y_t$  to an innovation in  $\mu_t$  will evolve over time. As will become more clear below, a bigger  $a$  implies a larger degree of persistence in output response and a greater possibility of a hump in the impulse response function. In the light of (22),  $a$  is a monotone decreasing function of  $\Gamma$ , and is smaller than 1 in absolute value for all  $\Gamma > 0$ . If  $\Gamma \geq 1$ , then  $a \in (-1, 0]$ , and there is no endogenous persistence in output dynamics. If  $\Gamma < 1$ , then  $a \in (0, 1)$ , and output response is endogenously persistent. That is to say, persistence exists if and only if there are strategic complementarities between the pricing decisions by firms. The smaller  $\Gamma$  is, the larger is  $a$ , and the greater is the degree of persistence.

To make more transparent the dependance of output persistence on  $a$  and thus on  $\Gamma$ , I proceed now to further simplify (23). To do so, I need to specify a nominal spending growth rule. To choose a specification that has some empirical appeal, note that, under the quantity theory of money with a constant velocity, growth in nominal expenditure corresponds to growth in nominal money supply. Hence, I consider a stationary ARMA(1,1) specification to allow the empirically observed high-order dynamics in the response of money growth to an exogenous monetary policy shock, as in Edge (2000). Specifically, I consider the following process for  $\mu_t$ ,

$$\mu_t = \rho \mu_{t-1} + \epsilon_t + \varphi \epsilon_{t-1}, \quad (24)$$

where  $\epsilon_t$  is a white noise process. I assume a zero steady-state growth in nominal expenditure, corresponding to a zero steady-state inflation rate, so there is no constant term in (24). While

stationarity of  $\mu_t$  requires  $|\rho| < 1$  so that the autoregressive component of (24) can have a MA( $\infty$ ) representation, invertibility of the moving average component of (24) requires  $|\varphi| < 1$ .

Under the specification in (24), and given that  $|a| < 1$  and  $|\rho| < 1$ , I can solve from (23) the MA( $\infty$ ) representation for  $y_t$ ,

$$y_t = \varphi_0 \epsilon_t + \sum_{i=1}^{\infty} (\varphi_0 a^i + \varphi_1 a_i) \epsilon_{t-i}, \quad (25)$$

where the coefficients  $\varphi_0$ ,  $\varphi_1$ , and  $a_i$  are given by

$$\begin{aligned} \varphi_0 &= \frac{(1-\rho)(1+a)^2 + (1-\varphi)(1-a^2)}{4(1-a\rho)}, \\ \varphi_1 &= \frac{(1-\rho)(\rho+\varphi)(1+a)^2}{4(1-a\rho)}, \\ a_i &= \sum_{j=0}^{i-1} a^{i-j-1} \rho^j, \quad \forall i \geq 1. \end{aligned} \quad (26)$$

We note that if  $a \neq \rho$ , then  $a_i = (a^i - \rho^i)/(a - \rho)$  for all  $i \geq 1$ . It can be verified that the coefficients of the infinite-order moving average process (25) are absolutely summable, so the infinite sequence in (25) generates a well-defined covariance-stationary process.

With (25), I can now derive analytically the impulse response function of real GDP following an innovation in the growth rate of nominal expenditure. The effects of  $\epsilon_t$  on  $y_{t+i}$  are given by

$$\frac{\partial y_t}{\partial \epsilon_t} = \varphi_0, \quad \frac{\partial y_{t+i}}{\partial \epsilon_t} = \varphi_0 a^i + \varphi_1 a_i, \quad \forall i \geq 1. \quad (27)$$

Thus, if a one percent shock occurs in  $\epsilon_t$ , then there will be an immediate output response  $y_t = \varphi_0$  at time  $t$ , and subsequent responses  $y_{t+i} = \varphi_0 a^i + \varphi_1 a_i$  at time  $t+i$ , for all  $i \geq 1$ . How persistent the responses are depends on how large are the responses at time  $t+i$  relative to the initial response at time  $t$ . This is the concept of dynamic contract multipliers, given by

$$\frac{\partial y_{t+i}/\partial \epsilon_t}{\partial y_t/\partial \epsilon_t} = a^i + \frac{\varphi_1}{\varphi_0} a_i, \quad \forall i \geq 0, \quad (28)$$

assuming  $\varphi_0 \neq 0$ , and with the understanding that  $a_0 \equiv 0$ . Greater contract multipliers imply more persistent output responses. It is worth noting that, both the actual impulse responses (27) and the dynamic contract multipliers (28) depend only on  $i$ , the length of time separating the shock ( $\epsilon_t$ ) and the observed value of the output response ( $y_{t+i}$ ). They do not depend on  $t$ ; that is, they do not depend on the date when the shock occurs or the dates of the observations themselves.

With the closed-form solutions for output dynamics, I can now state the main results of this subsection that the degree of persistence and the likelihood of a hump on the impulse

response function of real GDP depend positively on  $a$  and thus negatively on  $\Gamma$ . Without loss of generality, attention in the rest of this subsection will be restricted to the case in which there is some degree of strategic complementarities and endogenous persistence, that is, to the case with  $\Gamma \in (0, 1)$  [thus with  $a \in (0, 1)$ ]. I consider  $(\rho, \varphi) \in [0, 1]^2$  in the light of the empirical evidence provided in Edge (2000).

**Proposition 4.1:** *The values of the actual impulse responses (27) and the dynamic contract multipliers (28) are strictly positive and strictly increasing in  $a$  (thus strictly decreasing in  $\Gamma$ ).*

Proposition 4.1 says that the response of real GDP to a positive (negative) innovation in the growth rate of nominal expenditure is positive (negative) on impact, as well as in all periods following the innovation. Given that  $a \in (0, 1)$  and  $\rho \in [0, 1)$ , (26) and (27) imply that  $\lim_{i \rightarrow \infty} (\partial y_{t+i} / \partial \epsilon_t) = 0$ , so that the effect will eventually die out. In fact, as will be shown in the Appendix, once the response starts to level off or decline, it will keep declining forever and, therefore, will approach zero monotonically from that point onward. Yet, the proposition says that the effect will die out more gradually, the greater is  $a$ . Further, it does not preclude the possibility that the response may first increase before starting to decrease; that is, it does not rule out the possibility of a hump-shaped impulse response function. What the above observations do suggest is that there can be at most one hump in the impulse response function, the existence of which requires that the response in the immediate subsequent period following the innovation is greater than the response on impact.

Inspecting (26) and (28) reveals that a hump is more likely to occur, the greater is  $a$  (or the smaller is  $\Gamma$ ). This is so since, first, as will be shown in the Appendix,  $(\varphi_1/\varphi_0)a_i$  is increasing in  $a$  (strictly increasing in  $a$ , unless  $\rho = \varphi = 0$ ), and second, provided that  $a + \rho > 1$ ,  $a_i$  is strictly increasing in  $i$  for small  $i$ . The following proposition characterizes the necessary and sufficient condition for the existence of a hump in the impulse response function of real GDP.

**Proposition 4.2:** *The impulse response function of real GDP is hump-shaped if and only if*

$$\Gamma \leq \left[ \frac{-(1-\rho)(2-\rho-\varphi) + \sqrt{(1-\rho)^2(2-\rho-\varphi)^2 + 8(\rho+\varphi)(1-\rho)(1-\varphi)}}{4(1-\varphi)} \right]^2. \quad (29)$$

As will be shown in the Appendix, (29) is the necessary and sufficient condition for the inequality  $\partial y_{t+1} / \partial \epsilon_t \geq \partial y_t / \partial \epsilon_t$ . As will also be shown there, there is generically no flat portion



on the impulse response function of real GDP and, therefore, whenever this inequality holds it holds mostly as a strict inequality, which, in light of the above discussions, is the necessary and sufficient condition for the impulse response function to be hump-shaped.

Needless to say, a precondition for the possibility of a hump is for the upper bound in (29) to be strictly positive.<sup>28</sup> Whether or not this precondition holds depends upon the values of  $\rho$  and  $\varphi$ . In the extreme case that  $\rho = \varphi = 0$  (i.e., in the case that growth in nominal expenditure follows a white noise process), the upper bound is zero and thus (29) will never be met and a hump can never occur. But, as long as  $\rho$  and  $\varphi$  are not both 0, the upper bound is larger than 0 and thus a hump is possible. In particular, there is the possibility of a hump if growth in nominal expenditure follows a stationary AR(1) process [i.e., if  $\rho \in (0, 1)$  and  $\varphi = 0$ ],<sup>29</sup> an invertible MA(1) process [i.e., if  $\rho = 0$  and  $\varphi \in (0, 1)$ ],<sup>30</sup> or a stationary ARMA(1,1) process with an invertible moving average component [i.e., if  $(\rho, \varphi) \in (0, 1)^2$ ]. In such a case, whether a hump will actually occur depends upon how small is  $\Gamma$  (or how large is  $a$ ).<sup>31</sup>

<sup>28</sup>It can be shown that for  $(\rho, \varphi) \in [0, 1]^2$  this upper bound always lies in  $[0, 1]$ .

<sup>29</sup>It is worth noting that neither the dynamic contract multipliers in (28) nor the upper bound in (29) are monotone in  $\rho$ , which measures the degree of persistence in an AR(1) nominal expenditure growth. For instance, it can be verified that  $(\partial y_{t+1}/\partial \epsilon_t)/(\partial y_t/\partial \epsilon_t)$  is strictly increasing in  $\rho$  for  $\rho \in (0, \rho^*)$ , but is strictly decreasing in  $\rho$  for  $\rho \in (\rho^*, 1)$ , where  $\rho^* = [2 - \sqrt{2(1-a)}]/(1+a)$  is strictly between 0 and 1 for all  $a \in (0, 1)$ . It can also be shown that the upper bound in (29) is small as is  $\rho$  close to either 0 or 1, but it is much larger for moderate values of  $\rho$ . These observations suggest that a more persistent AR(1) nominal expenditure growth does not necessarily imply a larger degree of output persistence, or a greater likelihood of a hump in the impulse response function.

<sup>30</sup>It can be verified that both the dynamic contract multipliers in (28) and the upper bound in (29) are strictly increasing in  $\varphi$ . Therefore, the bigger is the moving average coefficient in a MA(1) nominal expenditure growth process, the larger is the degree of output persistence, and the greater is the likelihood of a hump in the impulse response function.

<sup>31</sup>I focus here on output dynamics. As I show elsewhere, inflation dynamics are given by

$$\pi_t = (1 - \varphi_0)\epsilon_t + \sum_{i=1}^{\infty} \left[ \varphi_0(1-a)a^{i-1} + \varphi_1(a_{i-1} - a_i) + (\rho + \varphi)\rho^{i-1} \right] \epsilon_{t-i}.$$

The general result is that the impulse response function of inflation is always hump-shaped in such a Taylor-type sticky-price (or sticky-wage) model, as long as  $\Gamma < 1$ . This is true even with a random-walk money process (i.e., even with  $\rho = \varphi = 0$ ). This stands in contrast to results which would be obtained in a Calvo-type sticky-price (or sticky-wage) model. For instance, with a random-walk money process, neither the impulse response function of output nor the impulse response function of inflation can be hump-shaped in the latter model, regardless of how small a  $\Gamma$  the model is embodied with. Interested readers are referred to Huang (2004) for details.

Related to the possibility of a hump is the issue concerning the timing of the hump. The following proposition shows that, when the impulse response function of real GDP is indeed hump-shaped, the magnitude of  $a$  (and thus of  $\Gamma$ ) may also affect when the hump is to occur.

**Proposition 4.3:** *Suppose there is a hump in the impulse response function of real GDP following an innovation in the growth rate of nominal expenditure at time  $t$ . The time at which the hump occurs is  $t + 1 + [i^*]$ , where*

$$i^* = \begin{cases} 0.5, & \text{if } \rho = 0, \\ \frac{a}{1-a} - \frac{\varphi_0}{\varphi_1} a, & \text{if } \rho > 0 \text{ and } a = \rho, \\ \frac{\log \varphi_1 + \log(1-\rho) - \log[(a-\rho)\varphi_0 + \varphi_1] - \log(1-a)}{\log a - \log \rho}, & \text{if } \rho > 0 \text{ and } a \neq \rho, \end{cases} \quad (30)$$

and  $[i^*]$  denotes the largest integer not exceeding  $i^*$ .

The assumption of a hump in the impulse response function of real GDP implies that  $i^*$  in (30) is well-defined and positive (see the Appendix). It can also be verified that  $i^*$  is increasing in  $a$  (and thus decreasing in  $\Gamma$ ). Therefore, a greater  $a$  (or a smaller  $\Gamma$ ) tends to induce a more delayed peak in a hump-shaped impulse response function.

The results presented in this subsection so far have further detailed the messages conveyed in Section 3. Since the degree of persistence, and the likelihood and timing of a hump are all increasing in  $a$  and thus decreasing in  $\Gamma$ , the effect on  $\Gamma$  derived from the negative interaction between specific factors and intermediate inputs has a negative consequence for propagation of nominal shocks. To get a quantitative feel about this consequence, I plot in Figures 2-3 the normalized impulse response of real GDP to a nominal expenditure growth shock [given by the dynamic contract multipliers (28)] under factor specificities. The figures are generated for the cases with AR(1) and ARMA(1,1) nominal expenditure growth processes, respectively. In generating the figures, I have set  $\sigma = 1$ ,  $\xi = 20$ ,  $\alpha = 0.33$ , and  $\theta = 21$ , which are all empirically plausible values. I consider each period in the model as corresponding to one quarter of a year, and thus, with  $N = 2$ , the length of each price contract is equal to two quarters. Given the quarterly frequency, I consider for the AR(1) process an autoregressive coefficient of  $\rho = 0.57$ , as in Chari *et al.* (2000), and I add in a moving average component with a coefficient of  $\varphi = 0.93$  for the ARMA(1,1) process, as in Edge (2000). Displayed in each figure are four panels, corresponding in a clockwise order to the cases with  $e = 0.1$ ,  $e = 0.2$ ,  $e = 0.5$ , and  $e = 1$ . Five impulse response functions are plotted in each panel, corresponding to the cases

with  $\phi = 0$  (solid line),  $\phi = 0.3$  (dashed line),  $\phi = 0.5$  (line with circles),  $\phi = 0.7$  (line with stars), and  $\phi = 0.9$  (broken line with dots).

As these figures make clear, the consequence of the negative interaction between specific factors and intermediate inputs is to reduce the degree of persistence in the response of real GDP to a nominal expenditure growth shock (in both figures), diminish the likelihood of a hump on the impulse response (Figure 2), and shift to an earlier date the timing of the peak on a hump-shaped impulse response (Figure 3). Although I have only displayed the figures for one set of empirically plausible parameter values (other than  $e$  and  $\phi$ ), similar results have been obtained under other reasonable parameter values (not reported here).

#### 4.2. Impulse Response Functions: Numerical Simulations

The assumption of a constant aggregate capital maintained thus far has allowed me to obtain closed-form solutions to deliver the main messages of this paper in a highly transparent way. As a robustness check to the main findings, I now relax this assumption and solve numerically a fully specified monetary business cycle model with variable aggregate capital, whereby money is introduced using the standard money-in-the-utility approach. Note that, up to this point, I have not assumed any specific form of the utility function, because the analytical results obtained so far do not hinge upon such a specification. To conduct numerical simulations in this subsection, I do need to specify a functional representation for the household's utility. I assume that its period utility can be represented by the following parameterized function:

$$\frac{1}{\nu} \log[bC^\nu + (1-b)(M/\bar{P})^\nu] + \frac{\lambda}{\eta} \int_0^1 (1 - L_{h_i})^\eta di, \quad (31)$$

where  $M/\bar{P}$  denotes its real money balances. Its budget constraint in period  $t$  is accordingly modified as

$$\begin{aligned} & \bar{P}_t C_t + \bar{P}_t \int_0^1 I_{i,t} \left[ 1 + \frac{\psi}{2} \left( \frac{I_{i,t}}{K_{h_i,t-1}} \right)^2 \right] di + E_t(D_{t,t+1} B_t) - B_{t-1} + M_t - M_{t-1} \\ & \leq \int_0^1 R_{i,t}^k K_{h_i,t-1} di + \int_0^1 W_{i,t} L_{h_i,t} di + \Pi_t + TR_t. \end{aligned} \quad (32)$$

Here  $0.5\psi I_{i,t} (I_{i,t}/K_{h_i,t-1})^2$  is a capital adjustment cost with a scale parameter  $\psi$ , and  $TR_t$  is a lump-sum transfer to the household. As are  $\{K_{h_i,-1}\}_{i \in [0,1]}$  and  $B_{-1}$ , the household's initial holdings of money,  $M_{-1}$ , is taken as given.

An equilibrium can be defined similarly as in Section 2, with three modifications. First, money market clears, while money growth follows a process such as the one specified in (24). Second, the market clearing condition for the composite good now also takes into account of

total capital adjustment cost. Third, total production subsidy plus lump-sum transfer is equal to total indirect business tax plus newly created money.

To compute an equilibrium, I first substitute out a number of variables and reduce the log-linearized equilibrium conditions to  $N + 2$  equations, including a pricing equation,  $N$  Euler equations for capitals, and an Euler equation for money. Once I have these equations, I compute a Markov equilibrium in which prices and allocations are functions of the state of the economy. The state variables are lagged prices, the beginning-of-period capital stocks, and the money growth rate. The decision variables are current prices, investments, and consumption. The details of the computation procedure and the Matlab code are available upon request.

Figures 4-5 report the simulated impulse response of real GDP, normalized by its initial response, to a money growth shock under factor specificities. The size of the shock is chosen so that the money stock increases by one percent four quarters after the shock. In conducting the simulations, I have set  $N = 4$  (so the length of each price contract is equal to four quarters), along with  $\alpha = 0.33$ ,  $\beta = 0.99$ ,  $\theta = 10$ ,  $\nu = -1.56$ ,  $b = 0.94$ ,  $\delta = 0.02$ , and  $e = 1$ , which are all widely used parameter values in the literature.<sup>32</sup> Figure 4 is generated under an AR(1) money growth process ( $\rho = 0.57$ ) used in Chari *et al.* (2000), which is also in the line with the evidence presented in King (1992), and Figure 5 is generated under an ARMA(1,1) money growth process ( $\rho = 0.53$  and  $\varphi = 0.93$ ) used in Edge (2000). There are four panels in each figure, corresponding in a clockwise order to the cases with the labor supply elasticity of 1, 0.5, 0.25, and 0.1. The labor supply elasticity is linked to  $(1 - L)/L$  divided by  $(1 - \eta)$ . Thus these cases correspond to the values of  $\eta$  of  $-1$ ,  $-3$ ,  $-7$ , and  $-19$ , respectively, given a steady-state leisure-labor ratio of 2. There are five output responses in each panel, corresponding to the cases with the intermediate input share of 0 (solid line), 0.2 (dashed line), 0.4 (line with circles), 0.6 (line with stars), and 0.8 (broken line with dots). In each case, the capital adjustment cost parameter  $\psi$  is adjusted so that the initial response of total investment is as 2.3 times large as that of real GDP, in accordance with the empirical evidence of Leeper, Sims, and Zha (1996).

Once again, as is evident from the figures, the consequence of the negative interaction between specific factors and intermediate inputs is to reduce the degree of persistence in the response of real GDP to a money growth shock, and to diminish the likelihood of a hump in the impulse response function or shift to an earlier date the timing of the peak on a hump-shaped impulse response. The results under other reasonable parameter values are similar, as I find in

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<sup>32</sup>The values of  $\nu$  and  $b$  can be drawn from the money demand literature. See Chari *et al.* (2000) for details.

a sensitivity analysis (not reported here). In sum, the numerical simulations conducted here conform to the basic findings elaborated by the analytical results obtained earlier.

## 5. Concluding Remarks

A central challenge to monetary business-cycle theory is to find a solution to the problem of persistence and delay in the response of real economic activity to nominal disturbances. In meeting this challenge, various real features of the economy have been proposed to enhance the staggered price mechanism, which was found unable by itself to solve the persistence problem. Two such features, specific factors and intermediate inputs, have been separately found helpful. Models based on either of the two have also been used in the design of optimal monetary policy. Yet these two individually promising mechanisms have not been investigated jointly.

The current paper represents some initial attempt in taking on this issue. I examine here the interaction between specific factors and intermediate inputs in a staggered price-setting framework that features jointly these two sources of production inputs. My main finding is on an offsetting interaction between these two individually promising mechanisms, which leads to a cancelation of much of the impact of each in propagating nominal shocks.

While this finding manifests a kind of challenge in search for robust monetary transmission mechanism and design of optimal monetary policy, it can be viewed as a useful step along the road. One natural extension is to examine the interaction of the two real features under state-dependent pricing, instead of time-dependent pricing. Dotsey and King (2001, 2005) demonstrate that these pricing rules may have different implications for different real features. Although the nature of the interaction between these two sources of production inputs is unlikely to change under state-dependent pricing, as the intuitions illustrated in Section 3 suggest, the quantitative implications of such negative interaction under a state-dependent pricing rule is certainly an issue worth investigating.

More broadly speaking, issues concerning the robustness of mechanisms have only started to receive attention [e.g., Dotsey and King (2001, 2005)]. As Basu (2005) observes, in attempts to solve the persistent problem, “The standard paper in this literature takes a workhorse model, and then adds to it a scattering of the mechanisms that have been proposed to enhance the model,” while less attention has been paid to the interactions among the mechanisms. This perhaps is harmless if the individual mechanisms are only independent or reinforcing. But this paper shows that this is not always the case, and individually promising mechanisms can be counter-forcing with a significant negative consequence. As Basu points out, an easy solution of a set of robust mechanisms probably does not exist. Future research should both enhance our

understanding and help our search for robust monetary transmission mechanism and design of optimal monetary policy.

## Appendix. Derivations and Proofs

**Derivation of (13)-(14):** Using the steady-state versions of (6) and (10), it is straightforward to verify that the log-linear form of the marginal cost function is given by

$$q_{i,t} = \phi \bar{p}_t + (1 - \phi)[\alpha r_{i,t} + (1 - \alpha)w_{i,t}], \quad (33)$$

where recall that  $\phi = \tilde{\phi}\mu^{1-e}$  represents the share of payment to the intermediate input in total production cost in the steady state. The log-linearized labor supply equation is

$$w_{i,t} - \bar{p}_t = \sigma y_t + \xi l_{h_i,t}, \quad (34)$$

where I have replaced  $c_t$  with  $y_t$  on the right-hand side of (34) given that aggregate capital is constant with no depreciation.

To log-linearize the market clearing conditions for goods, capital, and labor, I need to first calibrate the fixed cost  $F$ . This can be done by setting steady-state profit equal to zero, while using the steady-state pricing relation and the balanced-budget condition that aggregate production subsidy is financed by aggregate indirect business tax. I also want to allow the case with no fixed cost. This gives rise to a unified representation of  $F = (\mu^{\mathbf{I}F} - 1)Z$ . Then using (1), (7)-(9), and the steady-state pricing relation, I obtain the log-linear forms of the aforementioned market clearing conditions as, respectively,

$$z_t = \frac{\mu - \mu^{\mathbf{I}F}\phi}{\mu - \phi} y_t + \frac{e\mu^{\mathbf{I}F}\phi}{\mu - \phi} \int_0^1 (q_{i,t} - \bar{p}_t) di, \quad (35)$$

$$0 = \omega_1(q_{i,t} - r_{i,t} + g_{i,t}) + (1 - \omega_1) \int_0^1 (q_{i,t} - r_{i,t} + g_{i,t}) di, \quad (36)$$

$$l_{h_i,t} = \omega_2(q_{i,t} - w_{i,t} + g_{i,t}) + (1 - \omega_2) \int_0^1 (q_{i,t} - w_{i,t} + g_{i,t}) di, \quad (37)$$

where

$$g_{i,t} = \frac{(1 - e)\phi}{1 - \phi} (q_{i,t} - \bar{p}_t) + \mu^{-\mathbf{I}F} [-\theta(p_{i,t} - \bar{p}_t) + z_t], \quad (38)$$

and I have used the long-linear form of the price index  $\bar{p}_t = \int_0^1 p_{i,t} di$  in simplifying (35). I have also used the fact that, in the case with factor homogeneity, all household members are faced with economy-wide factor prices and thus makes identical factor supply decisions.

From (33)-(38), the real marginal cost facing firm  $i$  at  $t$  can be solved as

$$\begin{aligned}
& \{(1 + \xi) - [\alpha(1 + \xi)\omega_1 + \xi(1 - \alpha)\omega_2](1 - e\phi)\} (q_{i,t} - \bar{p}_t) \\
= & (1 - \phi) \left[ \sigma(1 - \alpha) + (\xi + \alpha) \frac{\mu^{1-\mathbf{I}_F} - \phi}{\mu - \phi} \right] y_t \\
& - (1 - \phi) [\alpha(1 + \xi)\omega_1 + \xi(1 - \alpha)\omega_2] \theta \mu^{-\mathbf{I}_F} (p_{i,t} - \bar{p}_t) \\
& + \left\{ (1 - \phi)(\xi + \alpha) \frac{e\phi}{\mu - \phi} + [\alpha(1 + \xi)(1 - \omega_1) + \xi(1 - \alpha)(1 - \omega_2)](1 - e\phi) \right\} \int_0^1 (q_{i,t} - \bar{p}_t) di.
\end{aligned} \tag{39}$$

Integrating (39) from 0 to 1 yields

$$\int_0^1 (q_{i,t} - \bar{p}_t) di = (1 - \phi) \left[ \frac{\sigma(1 - \alpha)}{\xi + \alpha} + \frac{\mu^{1-\mathbf{I}_F} - \phi}{\mu - \phi} \right] \left[ \frac{1 - \alpha}{\xi + \alpha} + \frac{e(\mu - 1)\phi}{\mu - \phi} \right]^{-1} y_t. \tag{40}$$

Substituting (40) back into (39) results in

$$\begin{aligned}
q_{i,t} - \bar{p}_t = & (1 - \phi) \left[ \frac{\sigma(1 - \alpha)}{\xi + \alpha} + \frac{\mu^{1-\mathbf{I}_F} - \phi}{\mu - \phi} \right] \left[ \frac{1 - \alpha}{\xi + \alpha} + \frac{e(\mu - 1)\phi}{\mu - \phi} \right]^{-1} y_t \\
& - \frac{\theta \mu^{-\mathbf{I}_F} (1 - \phi)}{\frac{1 + \xi}{\alpha(1 + \xi)\omega_1 + \xi(1 - \alpha)\omega_2} - 1 + e\phi} (p_{i,t} - \bar{p}_t).
\end{aligned} \tag{41}$$

Equation (41) relates a firm's marginal cost to its relative price in addition to aggregate output. This is so unless both capital and labor are homogenous inputs, then all firms face the same marginal cost which is independent of firms' relative prices but only of aggregate output. Suppose at time  $t$  firm  $i$  can set a new price. This new price,  $p_{i,t}$ , once set, will be in effect for  $N$  periods. That is,  $p_{i,t+s} = p_{i,t}$  for  $s = 0, \dots, N - 1$ . I can therefore write (41) for the firm's entire price contract duration as

$$\begin{aligned}
q_{i,t+s} = & (1 - \phi) \left[ \frac{\sigma(1 - \alpha)}{\xi + \alpha} + \frac{\mu^{1-\mathbf{I}_F} - \phi}{\mu - \phi} \right] \left[ \frac{1 - \alpha}{\xi + \alpha} + \frac{e(\mu - 1)\phi}{\mu - \phi} \right]^{-1} y_{t+s} \\
& + \left[ 1 + \frac{\theta \mu^{-\mathbf{I}_F} (1 - \phi)}{\frac{1 + \xi}{\alpha(1 + \xi)\omega_1 + \xi(1 - \alpha)\omega_2} - 1 + e\phi} \right] \bar{p}_{t+s} \\
& - \frac{\theta \mu^{-\mathbf{I}_F} (1 - \phi)}{\frac{1 + \xi}{\alpha(1 + \xi)\omega_1 + \xi(1 - \alpha)\omega_2} - 1 + e\phi} p_{i,t}, \quad \text{for } s = 0, \dots, N - 1.
\end{aligned} \tag{42}$$

Substituting (42) into the log-linear form of the optimal pricing rule (10),

$$p_{i,t} = \frac{1}{N} \sum_{s=0}^{N-1} \mathbb{E}_t q_{i,t+s}, \tag{43}$$

where I have set  $\beta = 1$  for simplicity, and rearranging, give rise to equations (13) and (14), where  $\tilde{y}_{t+s} = \bar{p}_{t+s} + y_{t+s}$  denotes nominal aggregate spending in period  $t + s$ . Note that I have dropped the individual firms' index  $i$  on the left-hand side of (13) since firms are completely identified by the time at which they can set a new price. QED

**Proof of Proposition 3.1:** Consider the case with a production subsidy that eliminates steady-state monopolistic distortions. In this case,  $\Gamma$  is given by

$$\Gamma(\phi) = (\sigma + \xi)\mathcal{D}(\phi)^{-1}(1 - \phi)(1 + \xi e\phi), \quad (44)$$

where  $\mathcal{D}(\phi) \equiv 1 + \xi\theta + \xi(e - \theta)\phi$ , which is clearly positive for all  $\phi \in [0, 1)$ . Thus  $\Gamma(\phi)$  is  $C^1$  on  $[0, 1)$ . Its first order derivative is  $\Gamma'(\phi) = (\sigma + \xi)\mathcal{D}(\phi)^{-2}\mathcal{F}(\phi)$ , where

$$\mathcal{F}(\phi) = \xi^2 e(\theta - e)\phi^2 - 2\xi e(1 + \xi\theta)\phi + (\xi^2\theta e - 1). \quad (45)$$

Given that  $e > \underline{e}$ , if  $e = \theta$ , then  $\mathcal{F}(\phi)$  is a negatively-sloped straight line that crosses the  $\phi$ -axis strictly between 0 and 1. If  $e \neq \theta$ , then  $\mathcal{F}(\phi)$  is a quadratic function, and there are two real roots to the equation  $\mathcal{F}(\phi) = 0$ , one of which is always strictly between 0 and 1. In the case that  $e > \theta$ ,  $\mathcal{F}(\phi)$  is strictly concave and the other root is smaller than 0; while in the case that  $e < \theta$ ,  $\mathcal{F}(\phi)$  is strictly convex and the other root is greater than 1. These together show that there exists  $\phi^* \in (0, 1)$  such that (18) holds.

I next use (44) to obtain  $\Gamma(\phi) - \Gamma(0) = (\sigma + \xi)(1 + \xi\theta)^{-1}\mathcal{D}(\phi)^{-1}\mathcal{G}(\phi)$ , where

$$\mathcal{G}(\phi) = -\xi e(1 + \xi\theta)\phi^2 + (\xi^2\theta e - 1)\phi. \quad (46)$$

Clearly,  $\mathcal{G}(\phi)$  is a strictly concave function. There are two real roots to the equation  $\mathcal{G}(\phi) = 0$ , one of which is always 0 while the other of which is always smaller than 1. Given that  $e > \underline{e}$ , this second root is greater than 0. This combined with the previous paragraph implies that there exists  $\phi^{**} \in (\phi^*, 1)$  such that (19) holds.

The proof for the case with steady-state monopolistic distortions is much more involved, and is thus omitted here but available upon request from the author. QED

**Proof of Proposition 4.1:** Given that  $a \in (0, 1)$  and  $(\rho, \varphi) \in [0, 1)^2$ , it is clear from (26) that  $\varphi_0$  and  $a_i$  are strictly positive and  $\varphi_1$  is positive (strictly positive unless  $\rho = \varphi = 0$ ). It is also clear from the first two equations in (26) that

$$\begin{aligned} \frac{\partial \varphi_0}{\partial a} &= \frac{(1 - \rho^2) + a(1 - \rho)[2(1 + \varphi) - a(1 + \rho)] + (1 - \rho\varphi)(1 - a)^2}{4(1 - a\rho)^2}, \\ \frac{\partial \varphi_1}{\partial a} &= \frac{(1 - \rho)(\rho + \varphi)(1 + a)[2 + \rho(1 - a)]}{4(1 - a\rho)^2}, \\ \frac{\partial(\frac{\varphi_1}{\varphi_0})}{\partial a} &= \frac{2(1 - \rho)(1 - \varphi)(\rho + \varphi)}{[(1 - \rho)(1 + a) + (1 - \varphi)(1 - a)]^2}. \end{aligned} \quad (47)$$

It follows that  $\varphi_0$  is strictly increasing in  $a$ , and  $\varphi_1$  and  $\varphi_1/\varphi_0$  are increasing in  $a$  (strictly increasing in  $a$  unless  $\rho = \varphi = 0$ ). Finally, the third equation in (26) reveals that  $a_1 = 1$  and  $a_i$  is strictly increasing in  $a$  for all  $i > 1$ . QED



**Proof of Proposition 4.2:** I first show that, once the response of real GDP to an innovation in the growth rate of nominal expenditure starts to level off or decline, it will keep declining forever. Formally, I shall prove that, if, for some  $t \geq 0$  and some  $i \geq 1$ , it is true that

$$\frac{\partial y_{t+i-1}}{\partial \epsilon_t} \geq \frac{\partial y_{t+i}}{\partial \epsilon_t}, \quad (48)$$

then it must be true for any  $j \geq i$  that

$$\frac{\partial y_{t+j}}{\partial \epsilon_t} > \frac{\partial y_{t+j+1}}{\partial \epsilon_t}. \quad (49)$$

To prove (49) for all  $j \geq i$ , it suffices to prove it for  $j = i$ . Using (27), I can write (48) as

$$\varphi_0(1-a)a^{i-1} \geq \varphi_1(a_i - a_{i-1}), \quad (50)$$

and write (49) for the case with  $j = i$  as

$$\varphi_0(1-a)a^i > \varphi_1(a_{i+1} - a_i). \quad (51)$$

If  $\rho = 0$ , then from (26), (51) reduces to  $(a\varphi_0 + \varphi_1)(1-a)a^{i-1} > 0$ , which clearly holds. I thus only need to prove (51) for the case with  $\rho > 0$ . Using (26), (50) is equivalent to

$$\begin{aligned} [(a-\rho)\varphi_0 + \varphi_1](1-a)a^{i-1} &\leq \varphi_1(1-\rho)\rho^{i-1}, & \text{if } a < \rho, \\ i &\geq \frac{1}{1-a} - \frac{\varphi_0}{\varphi_1}a, & \text{if } a = \rho, \\ [(a-\rho)\varphi_0 + \varphi_1](1-a)a^{i-1} &\geq \varphi_1(1-\rho)\rho^{i-1}, & \text{if } a > \rho, \end{aligned} \quad (52)$$

and (51) is equivalent to

$$\begin{aligned} [(a-\rho)\varphi_0 + \varphi_1](1-a)a^i &< \varphi_1(1-\rho)\rho^i, & \text{if } a < \rho, \\ i &> \frac{a}{1-a} - \frac{\varphi_0}{\varphi_1}a, & \text{if } a = \rho, \\ [(a-\rho)\varphi_0 + \varphi_1](1-a)a^i &> \varphi_1(1-\rho)\rho^i, & \text{if } a > \rho. \end{aligned} \quad (53)$$

Note that, since  $\rho > 0$ , we have  $\varphi_1 > 0$ . It is then straightforward to show that each of the three weak inequalities in (52) implies in order each of the three strict inequalities in (53).

The above result and the fact that the output response will die out eventually (see the paragraph following Proposition 4.1) imply that, (i) there can be at most one hump on the impulse response function, and (ii) the impulse response function is indeed hump-shaped if and only if the response in the immediate subsequent period following the innovation is greater than

the response on impact, that is, if and only if  $\partial y_{t+1}/\partial \epsilon_t \geq \partial y_t/\partial \epsilon_t$ .<sup>33</sup> In light of (26) and (28), this necessary and sufficient condition for a hump is equivalent to

$$\frac{(1-\rho)(\rho+\varphi)(1+a)}{(1-\rho)(1+a)+(1-\varphi)(1-a)} \geq 1-a, \quad (54)$$

which, in light of (22), is equivalent to

$$2(1-\varphi)\sqrt{\Gamma}^2 + (1-\rho)(2-\rho-\varphi)\sqrt{\Gamma} - (1-\rho)(\rho+\varphi) \leq 0. \quad (55)$$

Now view the left side of (55) as a function of  $\sqrt{\Gamma}$ , and denote it by  $\mathcal{J}(\sqrt{\Gamma})$ . Note that this is a strictly convex function, given that  $\varphi \in [0, 1)$ . There exist two real roots to the equation  $\mathcal{J}(\sqrt{\Gamma}) = 0$ , the smaller of which is negative, while the larger of which is nonnegative and equal to the nonnegative square root of the upper bound in (29). It follows that, (55) holds if and only if (29) does. QED

**Proof of Proposition 4.3:** The hump shall occur at time  $t+i$  for some  $i \geq 1$  such that

$$\frac{\partial y_{t+i-1}}{\partial \epsilon_t} \leq \frac{\partial y_{t+i}}{\partial \epsilon_t} > \frac{\partial y_{t+i+1}}{\partial \epsilon_t}, \quad (56)$$

or, in light of (27), such that

$$\varphi_0 a^{i-1} + \varphi_1 a_{i-1} \leq \varphi_0 a^i + \varphi_1 a_i > \varphi_0 a^{i+1} + \varphi_1 a_{i+1}. \quad (57)$$

I now break into three cases.

Case 1 :  $\rho = 0$

It is easy to verify that, for all  $i \geq 1$ , the second inequality in (57) holds if and only if  $(a\varphi_0 + \varphi_1)(1-a)a^i > 0$ . This strict inequality always holds. Thus the hump can only occur at time  $t+1$ .

Case 2 :  $\rho > 0$  and  $a = \rho$

I derive from (57) the following two inequalities,

$$\varphi_0 a^{i-1} + (i-1)\varphi_1 a^{i-2} \leq \varphi_0 a^i + i\varphi_1 a^{i-1} > \varphi_0 a^{i+1} + (i+1)\varphi_1 a^i. \quad (58)$$

Using (58) to solve for  $i$ , I obtain

$$i-1 \leq \frac{a}{1-a} - \frac{\varphi_0}{\varphi_1} a < i. \quad (59)$$

---

<sup>33</sup>As will be shown in the proof of Proposition 4.3, there is generically no flat portion in the impulse response function and, therefore, whenever this inequality holds it holds mostly as a strict inequality.

Case 3 :  $\rho > 0$  and  $a \neq \rho$

I rewrite (57) into the following two inequalities,

$$\begin{aligned} \left(\varphi_0 + \frac{\varphi_1}{a - \rho}\right) a^{i-1} - \left(\frac{\varphi_1}{a - \rho}\right) \rho^{i-1} &\leq \left(\varphi_0 + \frac{\varphi_1}{a - \rho}\right) a^i - \left(\frac{\varphi_1}{a - \rho}\right) \rho^i \\ &> \left(\varphi_0 + \frac{\varphi_1}{a - \rho}\right) a^{i+1} - \left(\frac{\varphi_1}{a - \rho}\right) \rho^{i+1}. \end{aligned} \quad (60)$$

Using (60) to solve for  $i$ , I obtain

$$i - 1 \leq \frac{\log \varphi_1 + \log(1 - \rho) - \log[(a - \rho)\varphi_0 + \varphi_1] - \log(1 - a)}{\log a - \log \rho} < i. \quad (61)$$

The assumption of a hump implies  $\varphi_1/\varphi_0 \geq 1 - a$ , which in turn implies  $(a - \rho)\varphi_0 + \varphi_1 > 0$ . It can then be verified that the middle terms in (59) and (61) are well defined and positive. Meanwhile, the fact that these middle terms are positive guarantees the existence of strictly positive integers  $i$  that satisfy (59) and (61). Let  $i^*$  be defined as in (30). Then the hump shall occur at time  $t + i$  for  $i^* < i \leq 1 + i^*$ , that is, it shall occur at  $t + i = t + [1 + i^*] = t + 1 + [i^*]$ . Generically,  $i^*$  so defined is not an integer, so there is generically no flat portion in the impulse response function. This implies, as can be easily verified, that whenever the weak inequality in (56) holds, it holds mostly as a strictly inequality. QED

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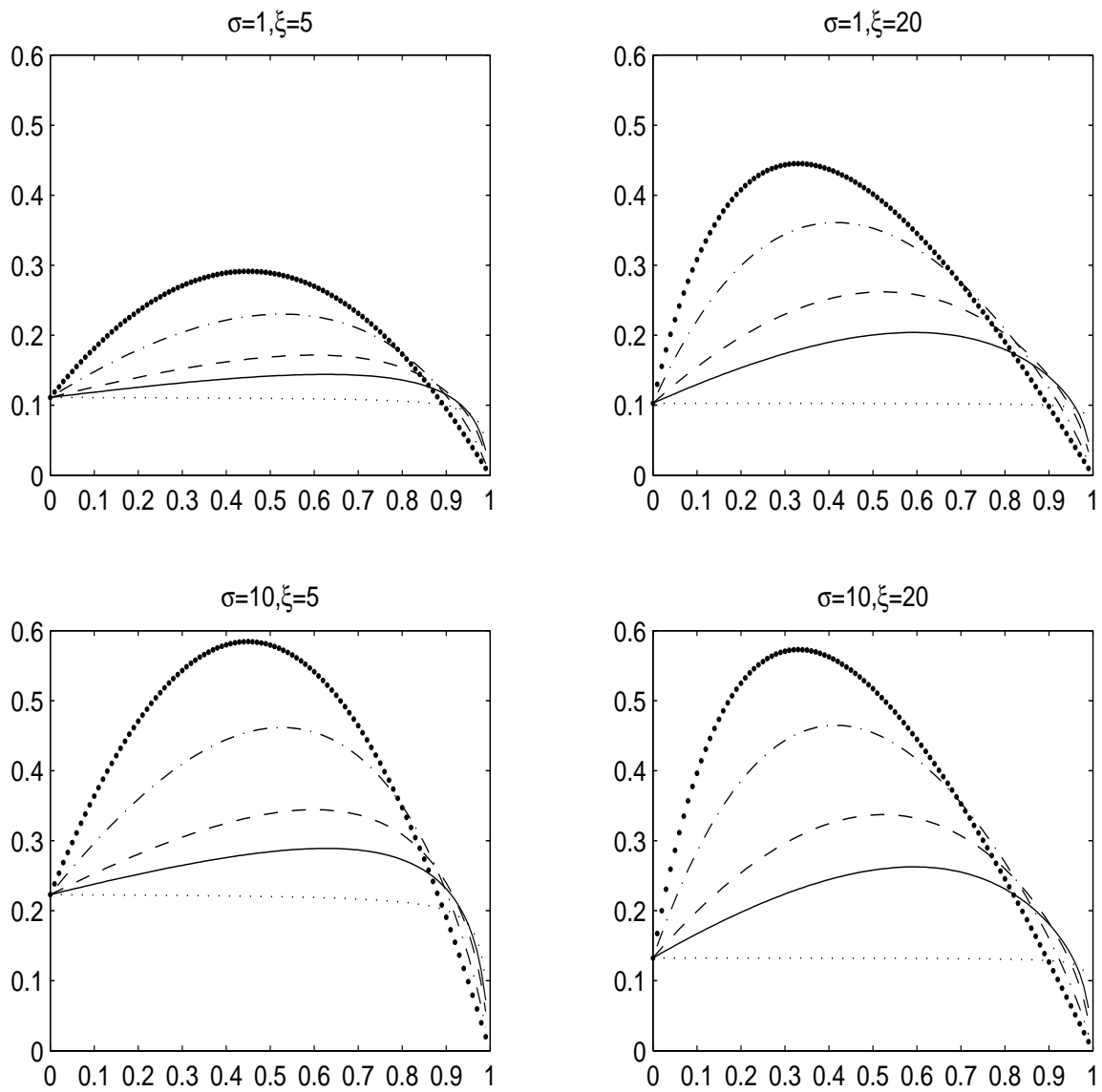


Figure 1.  $\Gamma$  as a function of  $\phi$ : In each panel, the five curves, from bottom to top, correspond in order to the cases with  $e = 0, 0.1, 0.2, 0.5,$  and  $1,$  respectively.



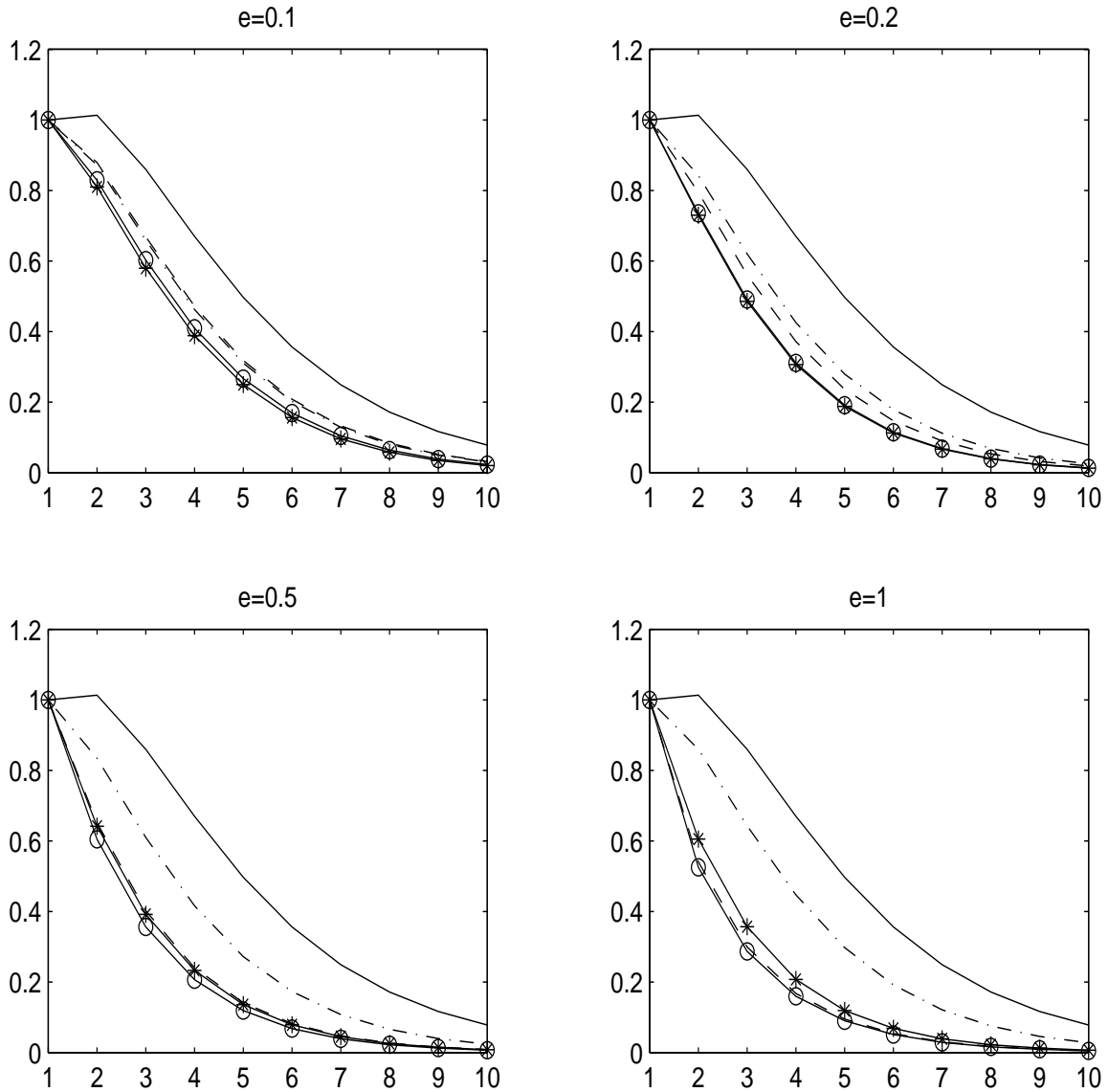


Figure 2. The normalized impulse response of real GDP (in quarters after a nominal expenditure growth shock). Each panel plots five impulse response functions, corresponding to the cases with  $\phi = 0$  (solid line),  $\phi = 0.3$  (dashed line),  $\phi = 0.5$  (line with circle),  $\phi = 0.7$  (line with star), and  $\phi = 0.9$  (broken line with dot), respectively. With an AR(1) nominal expenditure growth process ( $\rho = 0.57$  and  $\varphi = 0$ ). (The length of each price contract is equal to two quarters.)

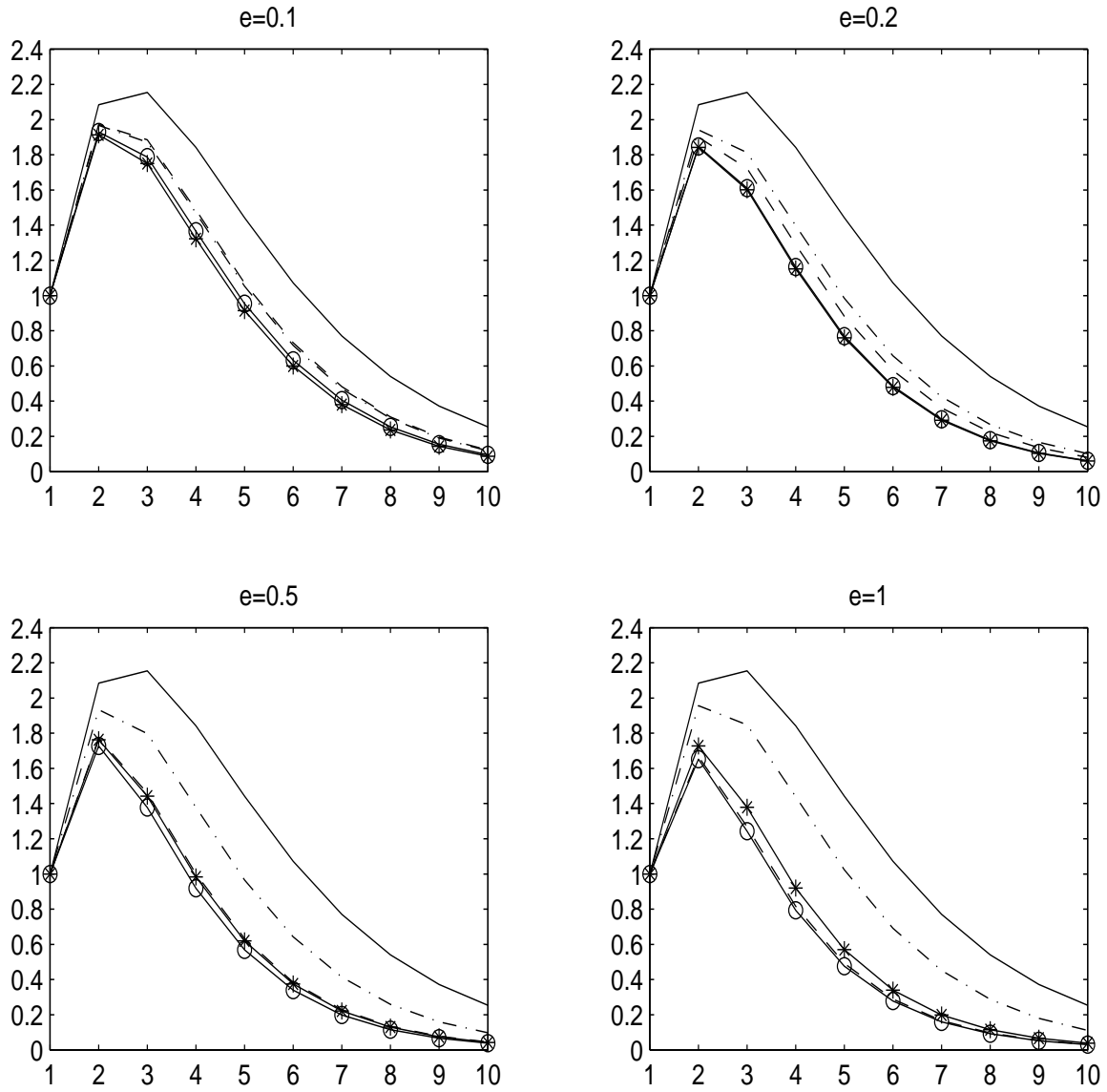


Figure 3. The normalized impulse response of real GDP (in quarters after a nominal expenditure growth shock). Each panel plots five impulse response functions, corresponding to the cases with  $\phi = 0$  (solid line),  $\phi = 0.3$  (dashed line),  $\phi = 0.5$  (line with circle),  $\phi = 0.7$  (line with star), and  $\phi = 0.9$  (broken line with dot), respectively. With an ARMA(1,1) nominal expenditure growth process ( $\rho = 0.57$  and  $\varphi = 0.93$ ). (The length of each price contract is equal to two quarters.)

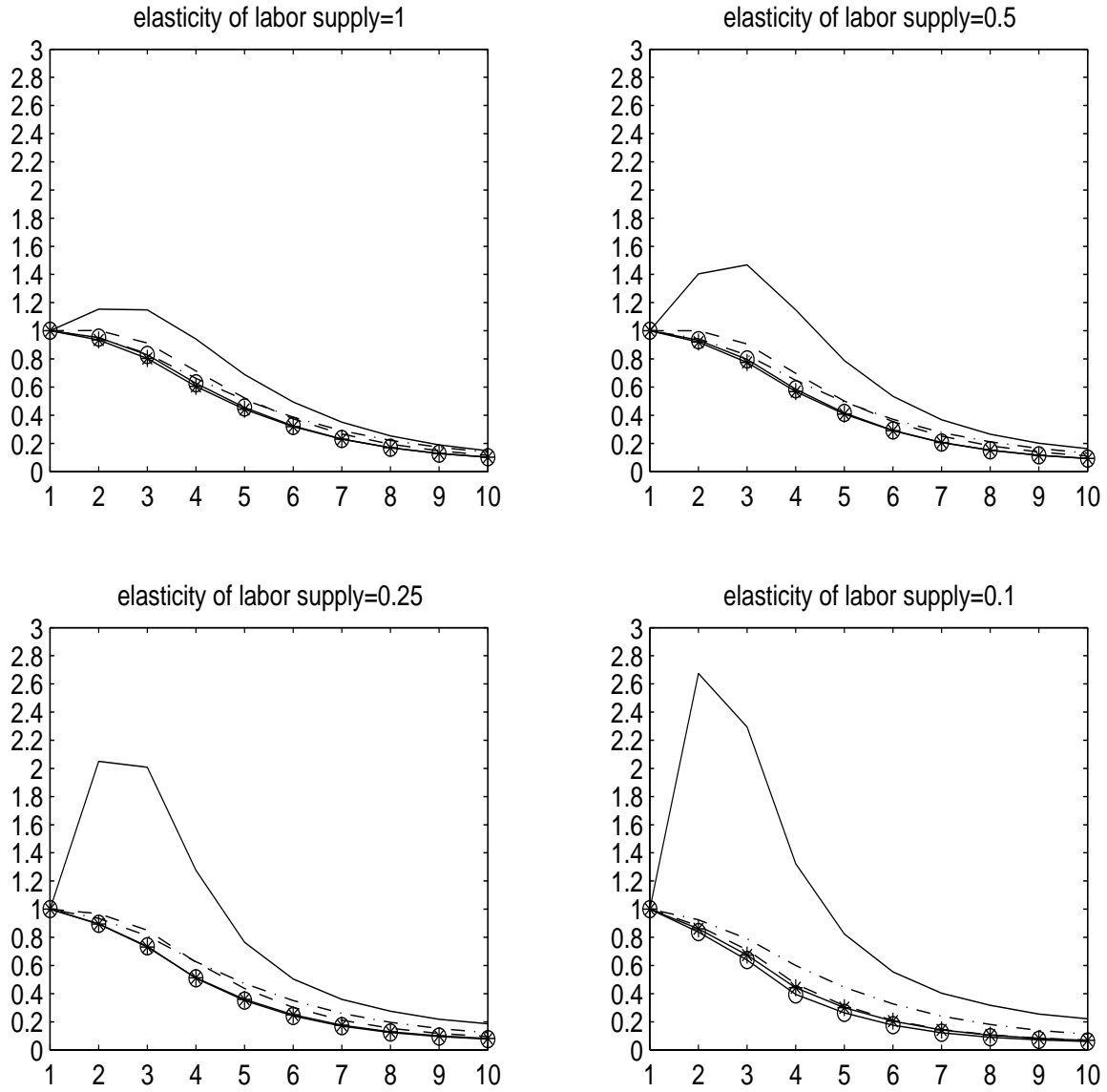


Figure 4. The normalized impulse response of real GDP (in quarters after a money growth shock). Each panel plots five impulse response functions, corresponding to the cases with

$\phi = 0$  (solid line),  $\phi = 0.2$  (dashed line),  $\phi = 0.4$  (line with circle),  $\phi = 0.6$  (line with star), and  $\phi = 0.8$  (broken line with dot), respectively.

With an AR(1) money growth process ( $\rho = 0.57$  and  $\varphi = 0$ ).

(The length of each price contract is equal to four quarters.)

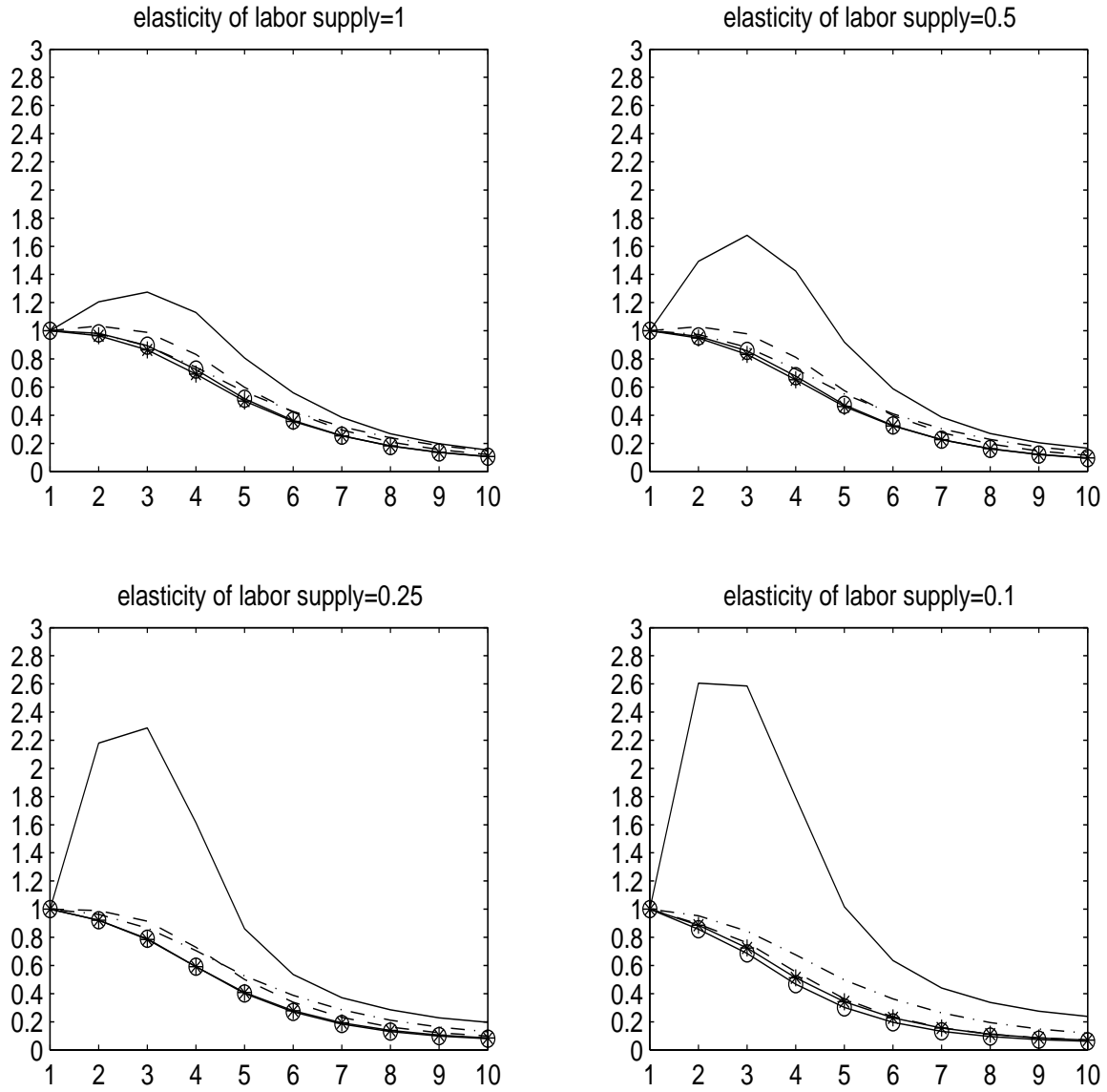


Figure 5. The normalized impulse response of real GDP (in quarters after a money growth shock). Each panel plots five impulse response functions, corresponding to the cases with  $\phi = 0$  (solid line),  $\phi = 0.2$  (dashed line),  $\phi = 0.4$  (line with circle),  $\phi = 0.6$  (line with star), and  $\phi = 0.8$  (broken line with dot), respectively. With an ARMA(1,1) money growth process ( $\rho = 0.53$  and  $\varphi = 0.93$ ). (The length of each price contract is equal to four quarters.)