ESTIMATING EQUILIBRIUM REAL INTEREST RATES IN REAL TIME

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Abstract

We use a range of simple models and 22 years of real-time data vintages for the U.S. to assess the difficulties of estimating the equilibrium real interest rate in real time. Model specifications differ according to whether the time-varying equilibrium real rate is linked to trend growth, and whether potential output and growth are defined by the CBO's estimates or treated as unobserved variables. Our results reveal a high degree of specification uncertainty, an important one-sided filtering problem, and considerable imprecision due to data uncertainty. Also, the link between trend growth and the equilibrium real rate is shown to be quite weak. Overall, we conclude that statistical estimates of the equilibrium real rate will be difficult to use reliably in practical policy applications.

JEL classification: C5, E4, C3, E52.

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1 Introduction

The equilibrium real interest rate – the rate consistent with stable inflation and output equal to potential – has come to play a key role in monetary policy. For example, using a policy rule such as that suggested by Taylor (1993) to evaluate or guide policy requires an estimate of the equilibrium real rate, or natural rate of interest. According to neoclassical growth theory, the equilibrium real rate is a function of the trend growth rate of output. If the trend growth rate varies over time, so, too, does the natural rate of interest. Some models or theories also link the equilibrium real interest rate to consumer preferences or fiscal policy. Over a number of years, policymakers have recognized the potential for changes in trend growth or other forces to shift the equilibrium real interest rate (see, for example, Greenspan (2000), Meyer (1999), and Poole (2003)).

For the U.S., time–varying estimates of the equilibrium real rate have been generated with various methods.[1] Bomfim (2001) estimates the equilibrium real rate using data from the yield curve for inflation–indexed government securities.[2] A series of recent studies use state–space (or Kalman filter) methods to estimate the equilibrium real rate as an unobserved component in an IS equation relating the output gap to the real rate less the equilibrium real rate.[3] In the Laubach and Williams (2003) formulation, the model consists of such an IS equation, an equation relating the equilibrium real rate to trend growth, and a Phillips curve relating inflation to the output gap. Their model is essentially a time–varying equilibrium rate variant of the specifications used in such studies as Gerlach and Smets (1999) and Smets (1999).[4] As such, the model can also be viewed as a variant of the Rudebusch and Svensson model (1999). Orphanides and Williams (2002) estimate a model much like that of Laubach and Williams, modified such that the equilibrium real rate is no

[1] As is the case with concepts such as trend or potential output and the natural rate of unemployment, various researchers have used more time series–based approaches to estimating the equilibrium real rate. Such approaches could include the Beveridge–Nelson decomposition and band pass or HP filtering. Laubach and Williams (2003) present estimates based on some of these methods. As another example, Brzoza–Brzezina (2003) uses a structural VAR for the real interest rate and inflation to estimate an equilibrium rate.

[2] Bomfim (1997) used the Federal Reserve Board's MPS model to estimate the equilibrium real interest rate.

[3] We should note that the equilibrium real rate estimated with these models corresponds to the rate associated with a medium– or long–term equilibrium in which inflation is stable and output is at potential. Other authors, such as Neiss and Nelson (2003), have constructed so–called *natural* rates of interest, which correspond to the rate of interest that would prevail in a flexible–price economy.

[4] In this specification, the trend and cyclical components of output are decomposed with the approach of Harvey (1985) and Clark (1987). Kuttner (1994) was the first to consider a Phillips curve treating the output gap as a latent variable, using the trend–cycle decomposition of Watson (1986) rather than Harvey and Clark.

longer related to trend growth and simply follows a random walk. Kozicki's (2004) model relies on just an IS equation and a random walk model for the equilibrium real rate, using CBO estimates of potential output to construct the output gap.

These prior studies have found that the equilibrium real interest rate in the United States has varied considerably over time, highlighting the importance of heeding the potential for movements in the real rate in monetary policy analysis. For example, according to the baseline estimates of Laubach and Williams (2003), the equilibrium real rate declined from about 4.5 percent in the mid-1960s to 2.5 percent in the mid-1970s. Given a particular model, the estimates even prove to be robust to some changes in specification — a finding highlighted by Laubach and Williams. However, estimates of the equilibrium real rate do appear to be sensitive to the broad aspects of the model specification. Orphanides and Williams (2002) show their equilibrium real rate estimates to be quite different from those of Laubach and Williams, especially in the 1970s and early 1980s.

Time variation in an equilibrium real rate that is unobserved raises the possibility of potentially severe difficulties in precisely estimating the equilibrium rate in real time. Indeed, the evidence in Orphanides and van Norden (2002) on severe difficulties in estimating the output gap in real time suggests similar real-time difficulties in equilibrium real rate estimation are highly likely.[5] In real time, estimates of the equilibrium rate could be distorted by revisions of source data on output and inflation and the one–sided data filtering on which real–time estimates are necessarily based. Orphanides and Williams (2002) and Laubach and Williams (2003) document that the imprecision associated with one–sided rather than two–sided filtering is considerable. Using data since 1987, Kozicki finds that the combination of data revisions and one–sided filtering makes real-time estimates of the equilibrium real rate highly imprecise.

Building on this prior work, this paper uses a range of models and 22 years of real-time data vintages for the United States to assess the difficulties of estimating the equilibrium real interest rate in real time. We consider versions of the models of both Laubach and Williams (2003) and Kozicki (2004), which differ in whether the time–varying equilibrium real rate is linked to trend growth and whether potential output and growth are defined by

[5]Following the work of Orphanides and van Norden, many other researchers have investigated the difficulties of estimating the output gap in real time, for the U.S. and other economies. Although a comprehensive survey is beyond the scope of this paper, examples include Cayen and van Norden (2004), Gruen, Robinson, and Stone (2002), Kamada (2004), Planas and Rossi (2004), and Runstler (2002). Still other studies have examined the implications for monetary policy. For instance, Rudebusch (2001) considers the implications of output gap and equilibrium real rate mismeasurement for optimal policy rules.

the CBO's estimates or treated as unobserved variables. In light of the likely importance of the one–sided filtering problem, we examine the effectiveness of one potential approach to mitigating the problem, taken from Mise, Kim, and Newbold (2003): extending the available data with simple forecasts of the data, and then constructing smoothed filter estimates at the end of the sample using the forecasted data. As we suggest below, such forward projection might be useful for pushing the data to reasonable endpoints.

Our results highlight a number of difficulties in precisely estimating the equilibrium real rate in real time. First, not surprisingly, the one–sided filtering problem is especially severe, producing revisions in the equilibrium real rate as large as several hundred basis points. In some situations, our proposed approach of using forward projections to extend the data sample and then using two–sided filtering does help to mitigate the end–point imprecision. That said, in light of our findings of an at–best modest payoff to forward projection, our results might be stronger evidence on the need for further investigation than practical relevance. Second, data revisions contribute to imprecision in real time estimates of the equilibrium real rate. Data revisions account for roughly 100-200 basis points of revisions to less recent estimates of the equilibrium real rate, but only 50 basis points or so for more recent estimates. Thus, accounting for data revisions remains critical for historical evaluations of policy.

Through our analysis of various model specifications, we also find a number of other difficulties in estimating the equilibrium real rate — essentially, even putting aside real time considerations, there are many reasons to be concerned about the robustness of equilibrium real rate estimates. Estimates of the equilibrium real rate prove to be highly sensitive to the specification of the real rate equation. For example, whether or not the equilibrium real rate is linked to trend growth or simply follows a random walk can dramatically affect the estimated real rate. Estimates of the real rate can also be highly dependent on the amount of variability allowed in trend growth and the component of the equilibrium real rate determined by forces other than trend growth. We also encounter what amounts to an identification problem, in the form of sensitivity to the initial values of the state space model. Essentially, it is very difficult to decompose the equilibrium real rate into contributions from potential trend growth and other components that may be linked to fiscal policy and consumer preferences.

Ultimately, in light of all of these problems, our results suggest that statistical estimates of the equilibrium real rate will be difficult to use reliably in practical policy applications. Estimates could be useful in historical analyses of the economy and policy, such as that of Orphanides and Williams (2002), with the caveat that different models may well yield very different estimates. But certainly the real time estimation problems make it very difficult to rely on the equilibrium real rate in current policy analysis. In this regard, our findings on real time imprecision echo those of Laubach and Williams (2003), based on just currently available data. In fact, our results suggest that, in real time, the historical mean of the real rate is a more accurate estimate of the equilibrium rate than is the model–based real time estimate.

The paper proceeds as follows. Section 2 describes the models and data we use, along with the details of our approach to estimation. In section 3 we explain our approach to using simple forecasts of the data to extend the sample prior to smoothing. Section 4 presents the results, and section 5 concludes.

2 Data and Models

As detailed in this section, we estimate various versions of the baseline models considered by Laubach and Williams (2003) and Kozicki (2004), using real-time data.[6] This section details the models and estimation approach and then explains the data sources.

2.1 Models

The variables of the models considered include: output, measured as real GDP or GNP, depending on the data vintage; potential or trend GDP, either treated as an unobserved state variable or measured with the CBO's estimate; the output gap; growth in potential or trend GDP; inflation in the GDP chain price index or deflator; the real interest rate, measured in ex post terms as the nominal effective funds rate less inflation; and the equilibrium real interest rate, an unobserved variable.[7] Specifically, we define the variables (all quarterly) and notation as described in Table 1.

[6] For tractability, the models ignore the data revisions. Although it is possible that augmenting the models to include a formulation of the revision process could yield better estimates, we leave the very challenging task of modeling the revision process to future research. The task is made especially challenging by the large number of revisions made to each initial estimate and the variety in the sources of revisions (for example, some reflect conceptual redefinitions, while others reflect more complete source data).

[7] For simplicity, we depart from Laubach and Williams (2003) in using inflation over the past year, instead of a forecast of inflation over the year ahead, to calculate the real interest rate. Using comparable data, our results are quite similar. Of course, other *ex ante* measures of the real rate could be used. But with lagged inflation generally providing a decent forecast of future inflation in the U.S. (see Atkeson and Ohanian (2001), for example), our simple real rate should be comparable to any ex ante measure.

Table 1: Notation

gdp_t	$100 * log(\text{real GDP}_t \text{ or real GNP}_t)$
gdp_t^*	$100 * log(\text{potential or trend real GDP}_t \text{ or GNP}_t)$
y_t	$gdp_t - gdp_t^*$, output gap in t
p_t	GDP chain price index in t, or GDP or GNP deflator in t
π_t	$400 * log(p_t/p_{t-1})$, quarterly inflation in t, annual rate
$\pi_t^{(4)}$	$100 * log(p_t/p_{t-4})$, 4-quarter inflation in t
i_t	nominal federal funds rate, annual rate
r_t	$i_t - \pi_t^{(4)}$, real interest rate in t, annual rate
r_t^*	equilibrium real interest rate in t, annual rate
g_t	potential output growth in t

2.1.1 Models treating trend output as unobserved

We first consider three versions of a model in which trend output and growth are treated as unobserved variables, as in Laubach and Williams (2003). These three specifications include IS and Phillips curve equations that are the same across versions and similar to the constant real rate formulation used in studies such as Gerlach and Smets (1999) and Smets (1999). Moreover, apart from the unobserved component aspects of the model, it is very similar to the model of Rudebusch and Svensson (1999). The particular versions we consider differ in the specification of the behavior of the equilibrium real interest rate.

All three models use the following specification of the IS equation, Phillips curve, and trend–cycle decomposition of GDP:

$$\begin{aligned}
y_t &= a_1 y_{t-1} + a_2 y_{t-2} + \frac{b}{2}(r_{t-1} - r^*_{t-1} + r_{t-2} - r^*_{t-2}) + \epsilon_t, \qquad \sigma_\epsilon \equiv st.dev.(\epsilon_t) \\
\pi_t &= \sum_{i=1}^{8} d_i \pi_{t-i} + f y_{t-1} + \epsilon_{\pi,t}, \qquad \sum_{i=1}^{8} d_i = 1, \qquad \sigma_\pi \equiv st.dev(\epsilon_{\pi,t}) \\
y_t &= gdp_t - gdp_t^* \\
gdp_t^* &= gdp^*_{t-1} + g_{t-1} + \eta_{*,t}, \qquad \sigma_{y*} \equiv st.dev(\eta_{*,t}) \\
g_t &= g_{t-1} + \eta_{g,t}, \qquad \sigma_g \equiv st.dev(\eta_{g,t}) = \lambda_g \sigma_{y*}
\end{aligned} \tag{1}$$

Note that, in this formulation, the parameter λ_g determines the variability of potential growth innovations relative to residual potential output innovations.[8]

[8] This simple model, augmented by an expression for the equilibrium real rate, contains sufficient information to (econometrically speaking) consistently estimate the equilibrium real rate. However, since an expression for monetary policy, r_t, is not included, this partial model could not be used for forecasting or simulation purposes. While inclusion of a policy rate equation could increase the efficiency of the model estimates, it would also introduce several complications. For instance, most policy reaction functions include a term representing the central bank's inflation target, a term that is unobserved and possibly time-varying. With the addition of an unobserved inflation target, consideration of less than perfect knowledge of the

The first two versions of the model, referred to as LW-1 and LW-2, suppose the equilibrium real rate depends on both trend growth and a random walk component representing such forces as fiscal policy and preferences:

$$\begin{aligned} r_t^* &= 4cg_t + z_t \\ z_t &= z_{t-1} + \eta_t, \qquad \sigma_\eta \equiv st.dev.(\eta_t) \\ \sigma_\eta &= \lambda_z \frac{\sqrt{2}}{b}\sigma_\epsilon. \end{aligned} \tag{2}$$

In these two specifications, we take λ_z and λ_g as known parameters. For LW-1, we use the baseline values of Laubach and Williams (2003). For LW-2, we use the values of λ_z and λ_g from the high λ_g specification of Laubach and Williams. That upper value also happens to represent the upper end of the range of the real time estimates of λ_g we obtained by applying the median unbiased estimation approach of Stock and Watson (1998) to each data vintage.[9] Note that the growth rate in the equilibrium rate equation is scaled by 4 in order to annualize the growth rate, as the measured interest rate is stated in annual percentage terms.

The third model, LW-3, supposes that the equilibrium real rate is simply a random walk, as in Orphanides and Williams (2002) and Kozicki (2004):

$$\begin{aligned} r_t^* &= r_{t-1}^* + \eta_{r,t}, \qquad \sigma_r \equiv st.dev.(\eta_{r,t}) \\ \sigma_r &= 0.322, \end{aligned} \tag{3}$$

where the value for σ_r was chosen to equal the estimate of σ_r obtained on average in LW-1 across the 22 vintages of data. This average was very close to the estimates obtained by Laubach and Williams in both their baseline specification (0.340) and in their high λ_g specification (0.332).

In these three models, the parameters to be estimated are a_1, a_2, b, $d_1 \ldots d_8$, f, c (in LW-1 and LW-2 only) , σ_ϵ, σ_π, and σ_{y*}. As noted above, the parameters λ_g and λ_z are fixed at values taken from Laubach and Williams (2003).

inflation target by the private sector, might also be an important historical feature of U.S. economic history as in Kozicki and Tinsley (2003).

[9] Following Laubach and Williams (2003), we estimated λ_g for each data vintage by testing for a break in the growth rate of potential output, using potential output estimated with a version of the LW model restricted to drop the real interest rate terms and make trend growth constant (making the model essentially that of Kuttner (1994)).

2.1.2 Models using CBO estimates of potential output

We also consider two versions of a model in which trend or potential output and potential growth are treated as known data, measured using the CBO's estimate of potential output.[10] That is, both the output gap y_t and trend growth g_t are observed variables. Kozicki (2004) also uses CBO data to measure potential output and growth. The two versions of our CBO data–based model use the same formulation of the so–called IS equation and differ only in the specification of the behavior of the equilibrium real interest rate.

More specifically, in both models the IS equation takes the form

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \frac{b}{2}(r_{t-1} - r^*_{t-1} + r_{t-2} - r^*_{t-2}) + \epsilon_t, \qquad \sigma_\epsilon \equiv st.dev.(\epsilon_t). \tag{4}$$

The first model, referred to as IS/CBO-1, supposes the equilibrium real rate depends on both trend growth and a random walk component representing such forces as fiscal policy and preferences, as in Laubach and Williams (2003):

$$\begin{aligned} r^*_t &= 4cg_t + z_t \\ z_t &= z_{t-1} + \eta_t, \qquad \sigma_\eta \equiv st.dev.(\eta_t) \\ \sigma_\eta &= \lambda_z \frac{\sqrt{2}}{b}\sigma_\epsilon. \end{aligned} \tag{5}$$

The second model, IS/CBO-2, supposes the equilibrium real rate is simply a random walk, as in Orphanides and Williams (2002) and Kozicki (2004):

$$\begin{aligned} r^*_t &= r^*_{t-1} + \eta_{r,t}, \qquad \sigma_r \equiv st.dev.(\eta_{r,t}) \\ \sigma_r &= \lambda_z \frac{\sqrt{2}}{b}\sigma_\epsilon. \end{aligned} \tag{6}$$

For these specifications, the model parameters to be estimated are a_1, a_2, b, c (in version 1 only), σ_ϵ, and λ_z. Note that for IS/CBO-1, the parameter λ_z determines the variability of innovations in the unobserved random walk component of the equilibrium real rate relative to output gap innovations. For IS/CBO-2, λ_z determines relative variability of innovations in the random walk equilibrium real rate to output gap innovations.

2.2 Estimation approach

Our basic estimation strategy follows that of Orphanides and Williams (2002), Laubach and Williams (2003), and Kozicki (2004), among others. For those variance parameters for

[10] See CBO (2001) for details of the CBO's approach to estimating potential GDP.

which maximum likelihood estimation may be subject to the pile-up problem discussed in such sources as Stock and Watson (1998) – λ_z and λ_g – we generally either set them at fixed values taken from Laubach and Williams or estimate them with the median unbiased approach of Stock and Watson. With those coefficients then fixed, we estimate all other parameters of each model by maximum likelihood. However, for some specifications, we also consider estimates in which λ_z and the other model parameters are estimated jointly by maximum likelihood.

For simplicity, we apply the method of Stock and Watson (1998) to estimate λ_z only in the IS/CBO-1 and IS/CBO-2 models, in which the output gap is an observed variable. In this case, we follow Laubach and Williams (2003) in constructing, for each data vintage, Andrews' (1993) sup Wald break test statistic for the constant term in a regression of the (CBO–based) output gap on a constant, two lags of the gap, and the two-quarter average real rate. We then estimate λ_z using Stock and Watson's mapping between λ and the break test statistic. For our variants of the Laubach and Williams (2003) model, in which the output gap is unobserved, we simply rely on the ranges of λ_z and λ_g values provided by Laubach and Williams. As noted above, though, we have constructed real-time estimates of λ_g, and found that the baseline and upper bound values we consider represent some measure of the typical or average value and high value.[11]

In the maximum likelihood implementation, our baseline estimates are based on a simple approach to setting the prior or initial values for the state vector and variance. In the baseline cases, we use the following simple settings:

- LW-1 and LW-2: $y_0^* \sim N(\text{CBO potential at time } 0, 4)$,
  $4g_0 \sim N(\text{CBO potential growth at time } 0, 4)$, $z_0 \sim N(0, 4)$
- LW-3: $y_0^* \sim N(\text{CBO potential at time } 0, 4)$,
  $r_0^* \sim N(\text{CBO potential growth at time } 0, 4)$
- IS/CBO-1: $z_0 \sim N(0, 4)$
- IS/CBO-2: $r_0^* \sim N(\text{CBO potential growth at time } 0, 4)$

The CBO-based prior means are based on the estimates of the output gap and potential growth (annual rate) for the appropriate quarter based on the latest available vintage of

[11]With the tight prior we use in the reported results, across the 22 vintages the estimated λ_g values range from .048 to .069, with an average of .060. These estimates are clearly higher than those of Laubach and Williams (2003), seemingly in part due to the use of a different inflation measure. Using Koopman's (1997) exact initial prior yields larger and more variable λ_g estimates — an average of .080 and a range of .039 to .104.

CBO data. To ensure that the levels of potential output are comparable to the levels of GDP for each of the vintages examined, the priors for the level of potential output are calculated from the CBO gap and the appropriate vintage data for GDP. The prior mean for the equilibrium real rate in the LW-3 and IS/CBO-2 models is based on the same assumed relationship between the equilibrium real rate and potential growth as explicitly represented in the other models ($r_t^* = 4cg_t + z_t$), with $c = 1$ and $z_t = 0$. Prior variances are somewhat arbitrary and represent a tradeoff between the large variance assumptions of diffuse priors and the tighter variance assumptions employed by Laubach and Williams (2003). The variances are large enough to encompass the range of estimates of the equilibrium real rate obtained by Laubach and Williams as well as smoothed estimates of the real funds rate obtained using a bandpass (60) filter or a HP (6400) filter.[12] In addition, prior variances are on the order of sample average standard errors of unobserved states (r^*, g, and gdp^*) as reported in Table 1 of Laubach and Williams.

In light of the non–stationarity of the state vector, we also experimented with diffuse priors (priors with very large variances), but as we discuss in more detail below, we frequently encountered difficulty in getting sensible estimates with diffuse priors. Problems generally arose in specifications which admitted a random walk preference or fiscal policy component (z_t). The problem appears to be one of distinguishing the contributions of trend growth and random walk component z_t in equilibrium real rate fluctuations.

2.3 Data

To examine real-time estimation issues, we consider a time series of data sets — that is, various vintages of data sets. For simplicity, we consider only one vintage per year — specifically, the first quarter vintage, following Kozicki (2004). For each year, the first quarter vintage data are those available on roughly February 15th. In light of the timing of NIPA releases and CBO data updates, each first quarter vintage data set normally includes GDP data through the fourth quarter of the prior year (specifically, the advance, or first estimate of fourth quarter GDP) and the CBO's estimates of potential output published in late January.[13] For example, the 2003:Q1 vintage data set includes GDP data through

[12] See Figures 3 and 4 in Laubach and Williams (2003).

[13] For some vintages, idiosyncracies in the timing of data releases lead to some departures from this convention. Because the CBO's most recent estimated series of potential output is based on NIPA data prior to the benchmark revision of December 2003, for the 2004 vintage we use NIPA data from the 2003:Q4 vintage. Also, publication of fourth quarter data for 1995 was delayed, so 1996:Q1 vintage data only includes observations through 1995:Q3.

2002:Q4. In total, we consider vintages from 1983:Q1 through 2004:Q1 (hereafter, we drop the quarter notation), although, as noted below, our results based on CBO data are shorter.

Our real-time data are taken from two basic sources. NIPA data on output and the price level are taken from the Federal Reserve Bank of Philadelphia's online Real-Time Data Set for Macroeconomists, described in Croushore and Stark (2001). Real-time data on the CBO's estimate of potential output — in vintages from 1991 through 2004 — were provided by Robert Arnold of the CBO. Output is measured with real GDP (in vintages from 1992 onward) or GNP (in vintages prior to 1992). Inflation is measured with the GDP price index (in vintages from 1996 onward), GDP deflator (in the 1992-1995 vintages), or GNP deflator (in vintages prior to 1992).[14]

3 Forward Projection

In light of evidence that much of the real-time imprecision in estimates of unobserved variables is due to one–sided filtering at the end of a sample (see, for example, Orphanides and van Norden (2002), Orphanides and Williams (2002) and Laubach and Williams (2003)), we examine whether an approach suggested by Mise, Kim, and Newbold (2003) might help. Mise, Kim, and Newbold show that the end–of–sample imprecision of estimates of cycle components generated with the Hodrick and Prescott (1997) filter can be reduced by extending the available data with simple, AR model–based forecasts of the data, and then constructing two–sided filter estimates at the end of the sample using the forecasts of the future. In our application, for each data vintage, we use estimated (univariate) AR models to generate forecasts of GDP, inflation, and the real interest rate for 40 quarters beyond the end of the sample. In the case of the CBO–based models, we limit our forecasts to 24 quarters as we actually have available directly from the CBO forecasts of potential output at least six years past the end of the vintage. We use these CBO forecasts to form the trend output growth variable that enters the model. We then append the forecasts to the actual data sample and run the Kalman filter over the augmented sample to obtain smoothed estimates of the equilibrium real interest rate (and the other state variables). Note that the parameters of the state space model to which the Kalman filter is applied are determined by estimates obtained from the actual data sample, not the augmented sample.

More specifically, our forecast models take the form of AR(4) models for output, inflation

[14]The switch from GNP to GDP occurs with the 1992:Q1 vintage. The switch from fixed weights to chain weights occurs with the 1996:Q1 vintage.

(π_t), and the real interest rate (r_t). In the case of the LW models, the output variable in the forecasting model is GDP growth; forecasts of the log level of GDP, the variable that enters the state space model, are obtained by accumulating the forecasts of GDP growth. For the CBO models, the output variable in the forecasting model is the output gap, which is then forecast into the future. Forecasts of potential growth are set to the growth rate of the CBO's projection of potential output. Overall, of course, the forecasting model could be parameterized many different ways; in the interest of parsimony, we've imposed some restrictions that might strike some researchers as arbitrary. But our simple AR model–based approach is sufficient for examining the potential value of the forward projection suggested by Mise, Kim, and Newbold (2003).

To allow for the kind of non-stationarity incorporated in the structural model underlying the estimates of the equilibrium real rate, we estimate the forecasting models with rolling or shortened samples. For each data vintage, we estimate the forecasting equations with just the most recent 15 years of data. The use of a rolling window allows for the possibility of changes over time in the unconditional means of growth and inflation, as well as in the dynamics.

Although there is much in this forward projection approach that might be viewed as ad hoc, we suggest the approach has some conceptual validity, especially with respect to data endpoints. With this approach, we are of course relying on a forecasting model that differs from the structural model — if there were no differences, end–of–sample estimates based on smoothing of forecasted data would be the same as conventional one–sided estimates. But this alternative model might play a useful role in helping to pin down sensible endpoints (see Kozicki and Tinsley (1998, 2001) for discussions of endpoints). The baseline structural model, after all, allows a unit root in trend growth and one or two unit roots in the equilibrium real interest rate (one due to trend growth and the other due to the forces represented in z). So, taken literally, the model implies trend growth and the equilibrium real rate to be unbounded. For many economists, such a model is viewed not as truth but as a convenient shorthand for allowing time variation in trend growth and the equilibrium real rate. The difficulty with the unit root specifications is that the endpoints of the data sample become the endpoints of the model or future data. For example, the sample endpoint might put the real interest rate at zero. Yet the real interest rate is almost sure to revert to some higher level closer to the historical average. In such circumstances, the forecasts generated under our forward projection approach can push the data back to more reasonable — not necessarily exactly right, but more reasonable — endpoints. Thus, the forward projection approach might reduce some of the imprecision associated with one–sided filtering, especially at those times when the sample data end at historically unusual values.

A consequence of the 15-year estimation sample is that forecasts from the AR models may deviate from plausible endpoints. For instance, while the steady state value of the output gap should equal zero, the mean CBO output gap over most 15-year samples deviates from zero, implying that the AR model–based forecasts of the output gap will generally not converge to zero even though the structural model implies they will (for $-1 < a_1 + a_2 < 1$). However, intuition suggests that the implications for the equilibrium real rate will be sensible. The average output gap over the 1980s was negative–a period during which the Federal Reserve was following a policy designed to lower inflation from its elevated level in the late 1970s. Such a policy might reasonably be viewed as one in which the real federal funds rate would be above the equilibrium real rate on average. In other words, forecasts of a negative output gap might reasonably be accompanied by forecasts of the real rate above the equilibrium. This is precisely what the IS equation in the model would imply.

Although intuitively reasonable, some unreported results based on various alternative forecast models shown that forward projection results can be very sensitive to the end points and properties of the forecasting model used. Ultimately, to be consistently useful in practice, the forward projection approach will require considerable care with end point problems. One approach might be to rely on information from other forecast sources, such as Blue Chip or the Survey of Professional Forecasters. Our results are simply meant as a first-pass analysis of the potential value of forward projection.

4 Results

In presenting our findings, we begin by focusing on results based on the latest available data, comparing results across models and raising some issues in equilibrium real rate estimation that appear to be common regardless of data vintage. We then review estimates based on real-time data, highlighting those findings unique to the real-time (as opposed to final vintage) results. We conclude with an examination of the potential for the forward projection approach described above to mitigate the one–sided filtering problem.

4.1 Results for latest available data

As shown in Table 2, across all model specifications our estimates of the parameters of the IS equation and Phillips curve are generally sensible and in line with existing estimates. For example, our current–vintage estimates of the coefficient b on the real interest rate term in the IS equation range between -.058 and -.100, in line with the Laubach and Williams (2003) baseline estimate and the estimate of Rudebusch and Svensson (1999). As reflected in this relatively tight range, using the CBO's estimate of potential output or treating potential output as an unobserved variable yields similar IS equation estimates. For the Phillips curve, the three versions of the Laubach-Williams model (LW-1, LW-2, and LW-3) yield an estimated output gap coefficient of between .186 and .210, which is comparable to estimates from other studies, such as Rudebusch and Svensson, and Smets (1999). Finally, our estimates of the coefficient c relating the equilibrium real rate to trend growth broadly line up with those of Laubach and Williams, indicating a significant link between the equilibrium rate and trend growth. Our estimates of c range from .636 to 1.450, compared to the Laubach-Williams baseline of 1.068.

As to the equilibrium real rate, in some important respects the various models yield broadly comparable estimates. First, as shown in Figure 1, smoothed estimates of the equilibrium real rate generated with each model suggest considerable variation over time in the equilibrium real rate. For example, the equilibrium real rate estimate from the IS/CBO-1 model rises nearly 200 basis points from roughly 1993 to 1998. Second, although the extent of the time-series variation differs across models, at lower frequencies the fluctuations in the estimated real rate are similar across most model specifications. For instance, all of the models show the equilibrium real rate declining from the mid-1960s through the mid-1970s and then rising into the mid-1980s. Finally, over portions of the sample, some of the models yield very similar estimates of the equilibrium real rate. From 1983 through the late 1990s, the real rates implied by the IS/CBO-1 and LW-1 models are generally quite comparable. Similarly, the equilibrium real rate estimated from the IS/CBO-2 model looks much like a smoothed version of the baseline Laubach-Williams (LW-1) estimate.

Broad comparability notwithstanding, at any moment in time the differences across estimates of the equilibrium real rate can be very large — suggesting a high degree of specification uncertainty. As Figure 1 indicates, at any moment in time, estimates can differ by as much as 200 basis points. Even in cases in which two models yield similar estimates over a portion of the sample, the same models can imply very different real rates at other points. For example, despite strong similarity from 1983 to the late 1990s, the real rates implied by the IS/CBO-1 and LW-1 models differ by as much as 150 basis points in the mid-1970s. Overall, these results on specification–related uncertainty corroborate similar findings in Laubach and Williams (2003), for a set of models narrower than that considered in this paper. Although we don't do so for computational simplicity, the "true" degree of uncertainty is even larger once the usual statistical uncertainty associated a given point estimate of the equilibrium real rate is taken into account. Laubach and Williams use Monte Carlo methods to document the considerable statistical uncertainty around the real rate estimate from a given model.

In considering alternative estimation approaches, however, we did encounter one particular, important source of parameter uncertainty. Equilibrium real rate estimates appear to be sensitive to the approach used to estimate the variance associated with the random walk component of the real rate (z_t for the IS/CBO-1 model and r_t^* for the IS/CBO-2 model). We found the equilibrium real rate estimate to be much more volatile when the variance of the random walk component was estimated by maximum likelihood, along with the other parameters, rather than pre-set based on the estimation approach of Stock and Watson (1998). Figure 2 highlights the greater volatility implied by full–MLE estimates of the IS/CBO-1 specification — volatility that many would consider implausible for an equilibrium rate. Reflecting the greater volatility implied by MLE estimates of the innovation variance of the random walk component of r_t^*, using MLE essentially eliminates the differences in the real rate estimates from the IS/CBO-1 and IS/CBO-2 models (Figure 2). The real rate estimates are virtually identical across the models (the same is true of all other vintages), even though the coefficient c on trend growth is essentially 1 in the IS/CBO-1 estimates. As we discuss in more detail below, even when the equilibrium real rate is related to growth, the random walk component often dominates.

Based on this comparison of MLE and median unbiased estimates of the variance of the random walk component of the equilibrium real rate, there doesn't appear to be the sort of pileup problem the Stock and Watson approach is designed to address.[15] Rather, estimation by maximum likelihood yields an estimate above that generated by the Stock and Watson approach. We elaborate on this difficulty in discussing the real time estimation results.

[15]However, attempts to estimate λ_g by maximum likelihood confirmed the importance of the pileup problem for estimates of the variance of the innovation in trend growth.

Note that, in the absence of a pileup problem, it is not clear whether the Stock-Watson approach or MLE should be preferable, as both are consistent.

The differences across models in equilibrium real rate estimates appear to reflect two key forces: (1) differences in estimates of trend growth and (2) a quantitatively weak link between trend growth and the equilibrium real rate. The estimates of trend growth implied by the CBO's measure of potential output are much more volatile (over time) than the trend estimates obtained with the baseline LW-1 specification. Of course, in the LW models, the variability of trend growth is determined by the parameter λ_g. As shown by Laubach and Williams (2003), using values of λ_g higher than in the baseline case makes the estimate of trend growth more volatile, and somewhat more comparable to the CBO–based estimates. The same is evident in our (Figure 1 and Table 1) results for LW-2, which uses the upper-bound λ_g considered by Laubach and Williams rather than the baseline value: the estimated equilibrium real rate is modestly more volatile than in the baseline case. Most economists would probably agree that, in truth, trend growth is either smoothly fluctuating over time or subject to discrete regime changes (that is, constant but subject to very occasional changes in mean). But whether the amount of variation implied by the CBO's estimates of potential output is too high or too low is a matter for debate. Put another way, the statistical uncertainty surrounding estimates of λ_g is large, as evident from the results of Laubach and Williams and our own efforts (described above) to estimate λ_g in real time.

Estimates from the models relating the equilibrium real rate to trend growth and a random walk component (LW-1, LW-2 and IS/CBO-1) show the quantitative link between the equilibrium rate and trend growth to be weak, despite its significance. In these models, much more of the variation in the estimated equilibrium real rate is attributed to the random walk component z_t than to the growth component $4cg_t$ (this is of course less so for LW-2 than LW-1 and IS/CBO-1). Although we omit a chart of z_t and r_t^* in the interest of brevity, the dominance of the random walk component can be seen from the conditional standard deviations of z_t and r_t^* reported in Table 2. For the LW-1, LW-2, and IS/CBO-1 models, the estimated standard deviations of the innovation to z_t (σ_η) are nearly as large as the conditional standard deviations for r_t^* (σ_r). The same result is evident from the estimates in Laubach and Williams (2003).

Before turning to our results on real-time estimates of the equilibrium real rate, we note one other source of uncertainty or difficulty in equilibrium real rate estimation: properly identifying the fluctuations in the real rate attributable to trend growth and the random walk component z representing preferences and fiscal policy. Real rate estimates appear to be sensitive to the specification of the prior on the initial value and variance of the state vector.[16] In some cases, the real rate estimates are reasonably robust, but the decomposition of the equilibrium real rate into contributions from potential growth ($4cg_t$) and the random walk component (z_t) is not. In general, estimates of the coefficient c on trend growth are quite sensitive to the specification of the prior. In our reported results, we used the tight priors described in section 2.2, in the practical interest of obtaining sensible results. In the final available data, using simple diffuse priors (or, for IS/CBO-1, the exact initial conditions approach of Koopman (1997)) generally produced much larger estimates of c and considerably more variable estimates of the equilibrium real rate. Essentially, the problem appears to be one of identification. For example, making the prior variance on z large allows the estimated z_0 to be very negative but offset by a large positive c. Such sensitivity to the prior variance suggests a problem in properly identifying the equilibrium real rate and the contributions of trend growth and the random walk component to equilibrium real rate fluctuations.

4.2 Real-time results

This section uses a relatively long history of real-time data to evaluate how data revisions and end-of-sample filtering problems affect real-time estimates of the equilibrium real rate. We start by evaluating the size and sources of revisions to estimates of the equilibrium real rate and then move to a discussion of robustness and other estimation issues.

Unfortunately, estimation of unobserved economic concepts such as the equilibrium real rate using filtering techniques as in this study tends to result in estimates that are subsequently subject to large revisions. Estimates of current and recent equilibrium real rates are important for real-time policy decisions and uncertainty associated with these estimates complicates decision-making. Estimates of less-recent values of the equilibrium real rate are useful for evaluating past policy. Thus, establishing the degree and sources of imprecision in equilibrium real rate estimates is important for policy.

The main advantage of estimating equilibrium real rates for a collection of vintages of data is that it permits evaluation of the effects of data uncertainty on estimates of the equilibrium real rate. To assess the absolute and relative contributions of data revisions and

[16] Along the same lines, Planas and Rossi (2004) highlight the sensitivity of output gap estimates to priors.

the availability of additional observations to revisions in estimates of the equilibrium real rate, we compare predicted estimates of the equilibrium real rate to smoothed estimates of the equilibrium real rate. From one vintage to the next, revisions to smoothed estimates may be due to data revisions or to the direct effects of additional observations that enter into the two-sided smoothing filter. By contrast, revisions to predicted estimates primarily reflect the effects of data revisions as they are generated using a one-sided filter and do not directly depend on future data. For predicted estimates, although additional observations of later-vintage data may be used to estimate model parameters, in this study the effects on predicted estimates of differences in estimated model parameters due to the additional observations is likely small, since, as discussed below, parameter estimates were generally robust to data revisions and sample.

Figure 3 contrasts the range across vintages of smoothed estimates of the equilibrium real rate (panel a) to the range across vintages of predicted estimates (panel b) for LW-1, and Figure 4 provides comparable ranges for IS/CBO-1. The two lines in each chart represent upper and lower bounds of estimates of the equilibrium real rate across all available vintages. Thus, for example, for 1982:Q4, the upper bound is set to the maximum estimate of the equilibrium rate for that quarter across all 22 vintages of data and the lower bound is the minimum estimate across all vintages. Since each vintage dataset only contains data for observations prior to the data release date (its vintage label), one fewer vintage of data is available for each year after 1982. By 2003, only one vintage of data, 2004, contains observations. A consequence of the gradual reduction in the number of vintages of data is the apparent narrowing of ranges of estimated equilibrium real rates for later time periods.

A comparison of the ranges of predicted estimates to the ranges of smoothed estimates suggests that, as in Orphanides and van Norden (2002), the one-sided filtering problem is the dominant source of revisions to end-of-sample (i.e., real-time) estimates of the equilibrium real rate. The imprecision of one-sided filtering has also been highlighted in such studies as Orphanides and Williams (2002) and Laubach and Williams (2003). In Figures 3 and 4, the ranges of predicted estimates are relatively tight compared to the ranges of smoothed estimates for the last several years, suggesting that data revisions are less important than one-sided filtering for explaining revisions to recent estimates of equilibrium real rates.

Data revisions become more important relative to one-sided filtering concerns when examining revisions to estimates of historical equilibrium real rates. In Figures 3 and 4, the width of range of predicted estimates is closer to the width of the range of smoothed estimates for the first half of the plotted sample. For the less-recent period, when many vintages of data are available, the ranges provide a proxy for the degree of uncertainty in estimates of the equilibrium real rate. In this sense, the 100-200 basis point range of the predicted estimates is an approximation of the uncertainty just due to data revisions. Along the same line, the larger range of the smoothed estimates reflects uncertainty due to both data revisions and one-sided filtering. With this interpretation, uncertainty associated with data revisions is sizable, even though, on the margin, early revisions to real-time estimates of the equilibrium real rate may be largely due to the one-sided filtering problem.

Another approach to characterizing the relative importance of data revisions versus one-sided filtering follows Orphanides and van Norden by comparing the final estimates of the equilibrium real rate to the real-time estimates and a series of quasi-real-time estimates that incorporate the effects of data revisions. As in Orphanides and van Norden, the series of smoothed estimates from the latest available vintage of data is the final estimate and acts as a proxy for the "true" history of the equilibrium real rate. The real-time estimate for a given quarter is defined as the smoothed estimate for that quarter based on the most recent vintage of data that actually would have been available during that quarter. Thus, for example, the real-time estimate for the fourth quarter of 1998 is the smoothed estimate for 1998:Q4 constructed using 1999 vintage data. The quasi-real-time estimate is the predicted estimate for that quarter generated from the model estimated using latest-available data.[17] The predicted estimate is one-sided, just like the real-time estimate, but is based on revised data. Figure 5 shows these three estimates for the baseline Laubach-Williams specification (LW-1) and IS/CBO-1.[18] One observation taken from both panels is that movements in the final estimate lead movements in the real-time and quasi-real-time estimates, as would be expected since the former is obtained from a two-sided filter that uses future information. A second observation is that low frequency variability of real-time and quasi-real-time estimates exceeds that of the final estimates, particularly for the IS/CBO-1 model specification. This second observation was partly responsible for our decision to consider projected-augmented data to address the one-sided filtering problem.

The relative importance of data revisions versus the availability of additional observa-

[17]Here we use predicted estimates from the model is estimated over the full sample of latest-available data. As noted earlier, most model parameters were relatively robust to sample.

[18]Only one real-time estimate is reported per year since only one vintage of data per year was examined.

tions for explaining revisions to estimates of the equilibrium real rate is not clear in Figure 5. For IS/CBO-1, the fact that the quasi-real-time estimates appear closer to the real-time estimates than to the final estimates (panel b), suggests that data revisions are less important than the availability of additional data. However, such a comparison is not as obvious for LW-1(panel a). Figure 6 provides an alternative summary of the relevant information. As in Orphanides and van Norden (2002), we define the total revision to be the difference between the final and real-time estimates of the equilibrium real rate. The part of the revision attributable solely to data revisions is constructed as the difference between the quasi-real-time and real-time estimates. Panel b of Figure 6 confirms our intuition from Figure 4 that for IS/CBO-1, data revisions tend to be a relatively small component of the total revision. However, the same is not true for LW-1, where the size and variability of the data revisions are comparable to those of the total revisions for much of the sample—a somewhat different finding than that of Orphanides and van Norden in their study of real-time estimates of the output gap. Nevertheless, in both panels, as in Orphanides and van Norden, the variability of total revisions is comparable to the variability of the underlying series of interest, here the equilibrium real rate, leading us to conclude that the reliability of the real-time estimates is quite low.

The summary statistics presented in Table 3 quantify the unreliability of the equilibrium real rate estimates and the sometimes material contributions of data revisions. With the LW-1 specification, for example, the volatility of the total real time revision is .78, compared to the volatility of .43 of the final estimate of the equilibrium real rate. For this specification, the standard deviation of the data revision is .45. Under the IS/CBO-1 specification, the standard deviation of the total revision is 1.34, compared to the final estimate's standard deviation of .74 and the volatility of .58 in the data revision.

Because the standard deviation of the final estimate is smaller than the standard deviation of the total revision, a natural question to ask is whether the mean of the real rate would be closer to the equilibrium real rate than the model-based estimates. In fact, this turns out to be the case. Comparing the real-time mean to the final-vintage smoothed estimate of the equilibrium real rate, the mean difference is -0.49 and the standard deviation of the difference is 0.57, considerably smaller than the 0.78 percent standard error for the LW-1 specification.[19]

[19] As was the case for the model-based estimates, in real time, the mean real rate is time-varying, due to data revisions and the availability of additional observations for later vintage data sets, but the degree of

To this point, the discussion of real-time results has focused on two model specifications, LW-1 and IS/CBO-1. Similar results were obtained for the other specifications. In fact, estimation of the models provided evidence of robustness for most model parameter estimates, both across specifications and across vintages. However, estimation using real-time data also revealed some estimation difficulties not apparent in section 4.1's analysis of latest available data. In particular, the tighter prior did not successfully alleviate identification difficulties for all vintages of data, economically questionable results were obtained for some data vintages, and estimated variances of random walk innovations appear to be sensitive to vintage.

Estimates of most model parameters across the different data vintages remained consistent with existing estimates. The three LW specifications were estimated using the 22 vintages of data (1983 through 2004) and the two IS/CBO specifications were estimated using 14 vintages of data (1991 through 2004). Table 4 presents the mean, maximum and minimum of the parameter estimates across vintages.[20] The relatively tight ranges obtained for all model parameters except c provides evidence of robustness to sample as well as to data revisions.

Estimates of c were not robust to alternative data vintages. While, as discussed earlier, a tighter prior helped identify economically plausible estimates of c using latest available data, the same was not true for some of the other vintages. Figure 7 shows estimates of c for each vintage for LW-1, LW-2, and IS/CBO-1. The parameter c does not appear in LW-3 and IS/CBO-2 because in these specifications the equilibrium real rate is modeled as a random walk and is not a function of potential growth. For some vintages in the early 1990s, negative estimates of c were obtained for IS/CBO-1.

The identification problem is clearly revealed in a comparison of IS/CBO-1 estimates of the equilibrium real rate based on 1995 vintage data to those based on 2004 vintage data. In both cases, the estimate of the equilibrium real rate is constructed using the relevant vintage of CBO estimates of potential growth, $g_{CBO,v,t}$, as

$$r^*_{v,t} = c_v * 4 * g_{CBO,v,t} + z_{v,t}$$

where the subscript v on r^*_t, c, g_{CBO}, and z_t is used to denote the dependence of each on

variability is relatively small. The standard deviation of the real-time means is 0.25 percent, with the final vintage estimate of the mean equal to 2.59 percent and the mean of the 22 real-time estimates of the mean equal to 2.34 percent.

[20] Results for each vintage are provided in Appendix tables A1-A5.

data vintage. While for 1995 vintage data, c is estimated to be negative, $c_{1995} = -0.275$, for 2004 vintage data, c is estimated to be positive, $c_{2004} = 1.450$. Consequently, the contributions of potential growth to estimates of the equilibrium real rate ($c_v * 4 * g_{CBO,v,t}$ as shown in Figure 8, panel b) are quite different for the two vintages. While for 1995 vintage data, the contribution of potential growth is small and negligible, the same is not true for 2004 vintage data. Moreover, the contribution of potential growth based on 1995 data is negatively correlated with that based on 2004 data. Despite the large difference in estimates of c, estimates of the equilibrium real rate are similar (Figure 8, panel a). This similarity obtains because contributions of the unobserved random walk components ($z_{v,t}$ as shown in Figure 8, panel c) unwind the differences due to the potential growth component. Moreover, the figure clearly reveals that the random walk component explains at least as much of the variability of the equilibrium real rate as the potential growth component, a point noted earlier with reference to results for latest available data. For both of the charted vintages, the random walk component is at least as important as the potential growth contribution.

Another issue that arose during real-time application of the methodology was sensitivity of estimates of λ_z to vintage. Estimates of λ_z for the IS/CBO models were obtained using the approach of Stock and Watson and are provided in Table 5 with implied estimates of σ_η and σ_r. Although historical CBO estimates of the output gap don't change that much over time, estimates of λ_z were twice as large for some vintages than for others. Smaller differences in implied estimates of σ_η suggest that variability in λ_z across vintages may be related to variability in estimates of b across vintages. Overall, however, sensitivity of estimates of λ_z and σ_η to vintage suggests that confidence intervals and standard errors from single-vintage studies do not adequately incorporate effects of data uncertainty.

A related issue concerns the approach taken to estimate the variance of shocks to the unobserved random walk component of the equilibrium real rate that is not related to potential growth — either representing shocks to preferences or fiscal policy, z_t, in IS/CBO-1, or the shocks to the random walk equilibrium real rate in IS/CBO-2. While the "piling up" problem noted by Stock and Watson motivates use of their proposed median unbiased variance estimator to obtain estimates of the variance of shocks to potential growth, "piling up" difficulties were rarely encountered when estimating the variance of shocks to z_t. As shown in Table 5, maximum likelihood techniques generally provided converged unconstrained estimates of σ_η or σ_r in IS/CBO specifications and these maximum likelihood estimates tended to be larger than Stock-Watson median-unbiased estimates. With larger point estimates obtained using maximum likelihood than using median-unbiased estimation techniques, the downward-bias motivation for using the latter techniques is questionable.

4.3 Projection-augmented real-time results

To assess whether forward projection as suggested by Mise, Kim, and Newbold (2003) might mitigate the one-sided filtering problem associated with real-time estimation of the equilibrium real interest rate, we compare two real-time estimates of the equilibrium rate against the estimate based on final vintage data. The first real-time estimate is the standard one: at each time t (the first quarters of 1983 through 2004), we use the time t vintage data to estimate the model and a filtered estimate of the equilibrium real rate for period $t-1$. The second real-time estimate for period $t-1$ uses the same model estimates but reflects two-sided filtering of a data set extended by appending forecasts obtained from AR(4) models to the actual data sample ending in period $t-1$.[21] We compare these real-time estimates against "truth" defined as the equilibrium real rate estimate based on the final vintage data (and two–sided filtering). In doing so, we tend to discount results for the last few years, because recent estimates of the equilibrium real rate may not be an accurate representation of the truth, as they are based on one-sided filtering and data that will be revised. In taking the currently available estimate as the "truth," we follow Orphanides and van Norden (2002), among others.

Overall, our experiment with forward projection may be characterized as a mixed success. The forward projection appears to be of some benefit to equilibrium real rate estimation based on the LW models, but only small improvements are realized with the IS/CBO models. As shown in Figure 9a, for most years between 1983 and 2000, real-time equilibrium real rate estimates from the LW-1 specification exploiting forward projection are closer to the currently available estimates or "truth" than are the standard real time estimates. The gain is especially large (roughly 200 basis points in mean absolute error) from 1983 through 1987. However, in some years — most notably, 1989-1991 — the estimate exploiting forward projection is less accurate than the simple real-time estimate. For LW-2 gains to forward projection are similar, but for LW-3 improvement in 1983-1986 is much

[21] For LW-1, LW-2, and LW-3structural models, the same 10 years of forecasts are used. In order to take advantage of CBO forecasts with IS/CBO-1 andIS/CBO-2 specifications, the forecast horizon must be constrained to 6 years and forecasts may differ from those used with the LW specifications.

smaller. Improvements with the IS/CBO models are limited. For IS/CBO-1, real-time estimates that use forward projections are closer to final estimates over 1998-2002, but prior to that period tend to track estimates that don't use the forward projections (Figure 9d). For IS/CBO-2, the two sets of real-time estimates are close over the entire sample. Since the forward-projection technique tended to result in larger benefits in the 1980s for the LW models, it is possible that benefits of forward projection will be more apparent further from the end of the last-vintage data sample. Thus, limited availability of real-time CBO data may lead to an understatement of the merits of the projection methodology.

5 Conclusions

Time variation in an equilibrium real interest rate that is unobserved raises the possibility of potentially severe difficulties in precisely estimating the equilibrium rate in real time. In real time, estimates of the equilibrium rate could be distorted by revisions of source data on output and inflation and the one–sided data filtering on which real–time estimates are necessarily based. The results in Orphanides and Williams (2002) and Laubach and Williams (2003) based on last-available data suggest real rate estimates to be highly imprecise in real time, in part due to one–sided filtering.

Building on Kozicki (2004), this paper uses a range of models and 22 years of real-time data vintages for the United States to further assess the difficulties of estimating the equilibrium real interest rate in real time. Our analysis highlights several difficulties in precisely estimating the equilibrium real rate in real time. Of course, one is the one–sided filtering problem. In some situations, our proposed approach (taken from Mise, Kim, and Newbold (2003)) of using forward projections to extend the data sample and then using two–sided filtering helps to mitigate the end–point imprecision. Data revisions present another important challenge to real time estimation. Data revisions can produce large changes over time in given historical estimates of the equilibrium rate, although data revisions are less important for more recent estimates of the real rate. Finally, we encounter a number of other difficulties in estimating the equilibrium that raise concerns about the robustness of equilibrium real rate estimates. Estimates can be highly dependent on the model of the equilibrium rate and the amount of variability allowed in trend output growth, among other things. Moreover, there can be difficulties in identifying the contributions of trend growth and other forces in equilibrium real rate fluctuations.

Ultimately, as suggested by Laubach and Williams (2003), statistical estimates of the equilibrium real rate would appear to be difficult to use reliably in practical policy applications. Estimates could be useful in historical analyses of the economy and policy, such as that of Orphanides and Williams (2002). But certainly the real time estimation problems make it very difficult to rely on model-based estimates of the equilibrium real rate in current policy analysis.

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Table 2: Estimation Results - Latest Available Data

Parameter	Laubach-Williams Version 1	Laubach-Williams Version 2	Laubach-Williams Version 3	IS/CBO Version 1	IS/CBO Version 2
λ_g	0.042	0.110	0.042		
λ_z	0.058	0.047		0.046	0.017
Σa_y	0.925	0.920	0.926	0.909	0.924
b	-0.066	-0.076	-0.058	-0.100	-0.078
	(0.019)	(0.023)	(0.018)	(0.027)	(0.025)
f	0.210	0.186	0.208		
	(0.085)	(0.082)	(0.086)		
c	0.636	0.833		1.450	
σ_ϵ	0.280	0.267	0.312	0.743	0.761
σ_π	0.966	0.976	0.979		
σ_η	0.350	0.232	0.322	0.480	0.233
σ_*	0.659	0.650	0.644		
σ_g	0.028	0.072	0.027		
σ_r	0.357	0.373	0.322	0.485	0.233
Log likelihood	-432.299	-433.304	-432.432	-197.453	-198.221

Notes:
1. The table reports parameter estimates based on the 2004 data vintage, over the sample 1961:Q1 through 2003:Q3. As detailed in section 2.1, in the case of the LW-1, LW-2, and LW-3 models, the values of the parameters λ_g and λ_z are fixed at baseline or high values of Laubach and Williams (2003). For the IS/CBO-1 and IS/CBO-2 models, the values of λ_z are estimated with the Stock and Watson (1998) method. Estimates of λ_z are not comparable in the two IS/CBO models. In IS/CBO-1, λ_z governs the relative variability of innovations to z_t where the equilibrium real rate is related to potential growth and z_t, whereas in IS/CBO-2, λ_z governs the relative variability of innovations to the unobserved random walk equilibrium real rate. The remaining parameters are estimated by maximum likelihood.
2. Standard errors appear in parentheses.
3. Note that, consistent with the model specification, the reported values of σ_g have not been annualized. Comparison with the estimates reported by Laubach and Williams requires multiplying the σ_g estimates in Table 2 by 4.
4. We calculate σ_r as $\sigma_r = \sqrt{16c^2\sigma_g^2 + \sigma_\eta^2}$. For IS/CBO-1, in which σ_g is not a model parameter, we proxy σ_g by the standard deviation of quarterly changes in the growth rate of potential output as measured by the CBO. For IS/CBO-2, $\sigma_r = \lambda_z\sigma_\epsilon\sqrt{2}/b$.

Table 3: Summary Statistics on Revisions in Equilibrium Real Rates

	LW-1 1982-2003	IS/CBO-1 1990-2003
means		
total revision	.32	-.69
data revision	.13	-.10
standard deviations		
final estimate of r^*	.43	.74
total revision	.78	1.34
data revision	.45	.58

Notes:
1. The *total revision* is defined as the final, smoothed estimate of r^* less the real time estimate. The *data revision* is defined as the predicted (unsmoothed) estimate of r^* based on final vintage data less the real time estimate.

Table 4: Summary of Real-Time Estimation Results

Parameter		Laubach-Williams Version 1	Laubach-Williams Version 2	Laubach-Williams Version 3	IS/CBO Version 1	IS/CBO Version 2
Σa_y	mean	0.920	0.914	0.919	0.906	0.916
	min	0.907	0.896	0.906	0.893	0.905
	max	0.953	0.951	0.954	0.923	0.925
b	mean	-0.113	-0.149	-0.112	-0.155	-0.122
	min	-0.228	-0.247	-0.232	-0.197	-0.168
	max	-0.060	-0.066	-0.057	-0.100	-0.078
f	mean	0.190	0.165	0.183		
	min	0.124	0.058	0.121		
	max	0.267	0.244	0.272		
c	mean	0.582	0.776		0.739	
	min	0.432	0.531		-0.546	
	max	0.703	1.141		1.660	
σ_ϵ	mean	0.426	0.516	0.455	0.759	0.780
	min	0.241	0.199	0.275	0.723	0.757
	max	0.712	0.817	0.766	0.815	0.816
σ_π	mean	1.267	1.277	1.269		
	min	0.952	0.962	0.939		
	max	1.610	1.599	1.608		
σ_η	mean	0.316	0.231	0.322	0.477	0.356
	min	0.247	0.062	0.322	0.390	0.233
	max	0.373	0.335	0.322	0.582	0.472
σ_r	mean	0.322	0.355	0.322	0.484	0.356
	min	0.251	0.266	0.322	0.390	0.233
	max	0.376	0.422	0.322	0.584	0.472

Notes:
1. See the notes to Table 2.
2. The table reports summary statistics for parameter estimates based on the available data vintages (1983-2004 for the LW models, 1991-2004 for the IS/CBO models). With a few exceptions in which the starting point is pushed forward a few quarters, the estimation sample begins with 1961:Q1. The reported figures are means, mins, and maxs across vintages.

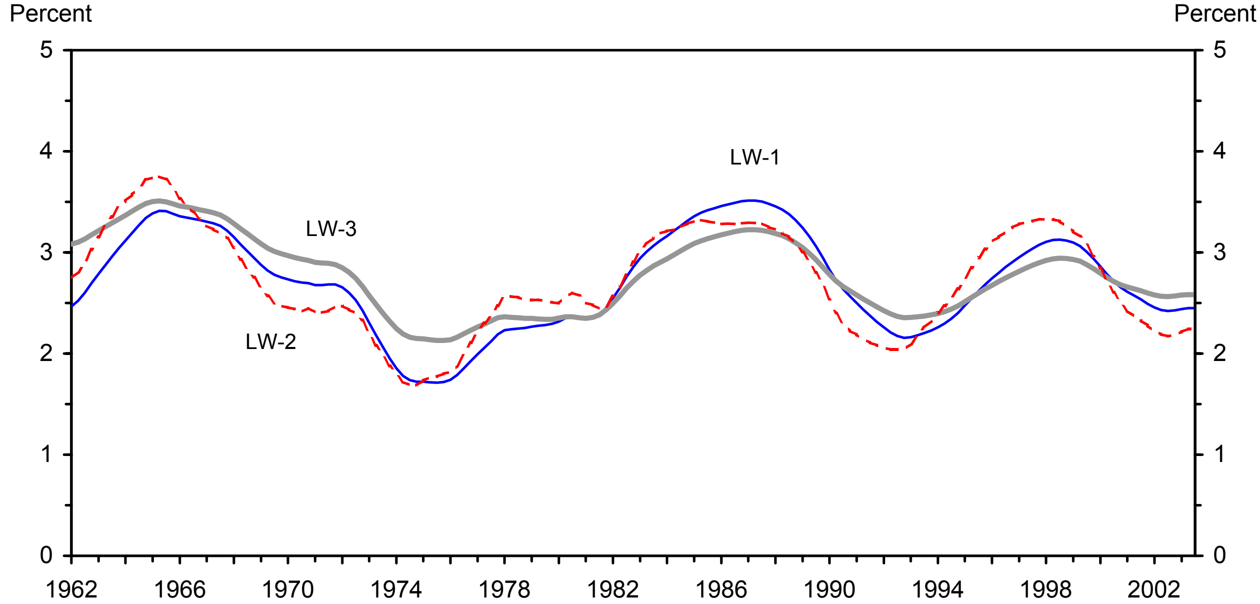
Table 5: Median Unbiased versus Maximum Likelihood Estimates of σ_η and σ_r, IS/CBO models

	IS/CBO-1					IS/CBO-2		
	Median Unbiased			MLE		Median Unbiased		MLE
Vintage	λ_z	σ_η	σ_r	σ_η	σ_r	λ_z	$\sigma_\eta = \sigma_r$	σ_r
1991	0.089	0.534	0.538	0.848	0.848	0.070	0.472	0.832
1992	0.058	0.390	0.390	0.800	0.810	0.057	0.395	0.746
1993	0.078	0.428	0.458	0.756	0.774	0.056	0.402	0.784
1994	0.067	0.413	0.427	0.728	0.750	0.053	0.395	0.747
1995	0.062	0.391	0.394	0.725	0.735	0.047	0.360	0.752
1996	0.044	0.409	0.416	0.711	0.711	0.051	0.434	0.687
1997	0.049	0.423	0.427	0.753	0.753	0.042	0.380	0.702
1998	0.095	0.582	0.584	0.783	0.783	0.030	0.309	0.793
1999	0.087	0.581	0.584	0.808	0.809	0.032	0.355	0.825
2000	0.084	0.547	0.550	0.762	0.763	0.036	0.369	0.767
2001	0.080	0.522	0.527	0.707	0.709	0.026	0.289	0.748
2002	0.065	0.494	0.501	0.753	0.756	0.026	0.296	0.830
2003	0.062	0.490	0.497	0.813	0.815	0.025	0.292	0.850
2004	0.046	0.480	0.485	0.886	0.887	0.017	0.233	0.889

Notes:
1. The results reported in the *Median Unbiased* columns are based on the model estimates reported in the paper (Tables 2 and 4, for example), generated by estimating λ_z (which is one of the coefficients that determines σ_η) with the Stock-Watson median unbiased method and the remaining parameters by maximum likelihood.
2. The results reported in the *MLE* columns are obtained from joint maximum likelihood estimation of the innovation variance σ_η and other model parameters.
3. See the notes to Tables 2 and 4.

Figure 1
Latest Vintage Estimates of the Equilibrium Real Rate

Panel a: Laubach-Williams Models



Panel b: Baseline Laubach-Williams and IS/CBO Models

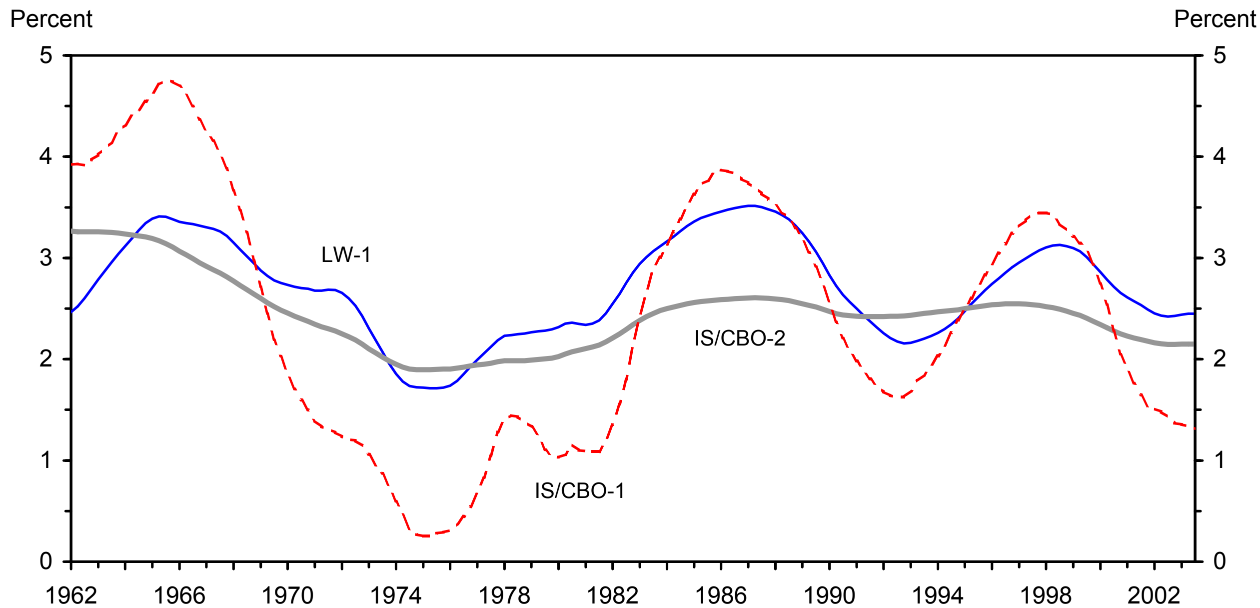


Figure 2

Maximum Likelihood Estimates of the Equilibrium Real Rate

IS/CBO Models

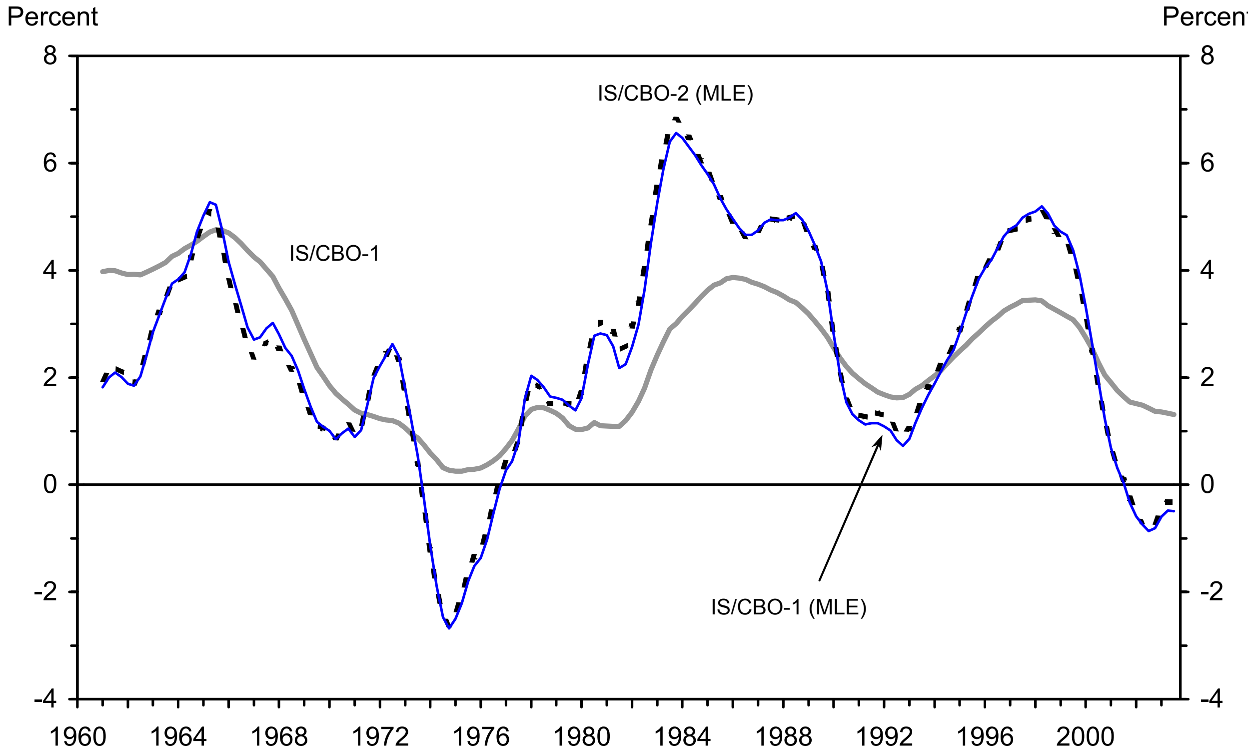
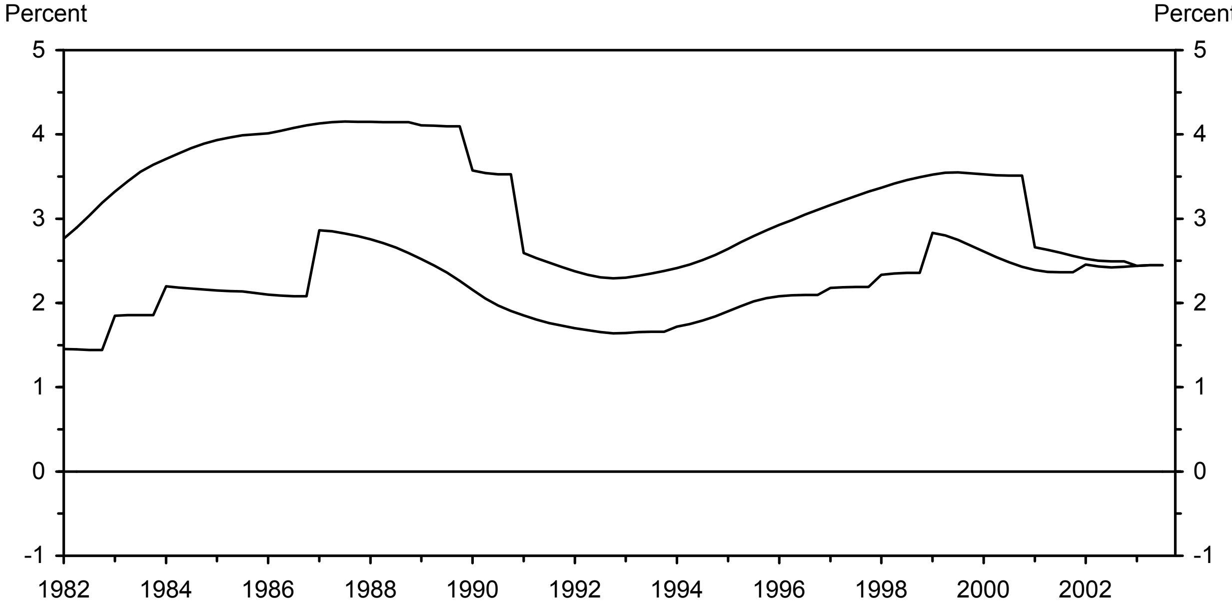


Figure 3
Ranges of Estimates of the Equilibrium Real Rate
Baseline Laubach-Williams Model

Panel a: Range of Smoothed Rates



Panel b: Range of Predicted Rates

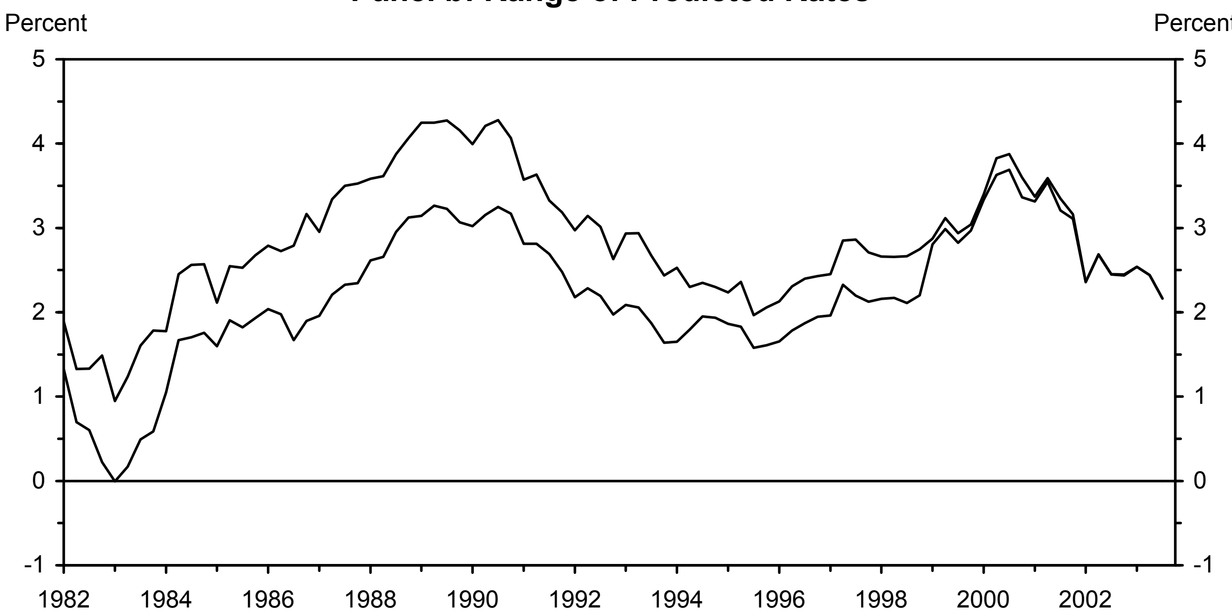
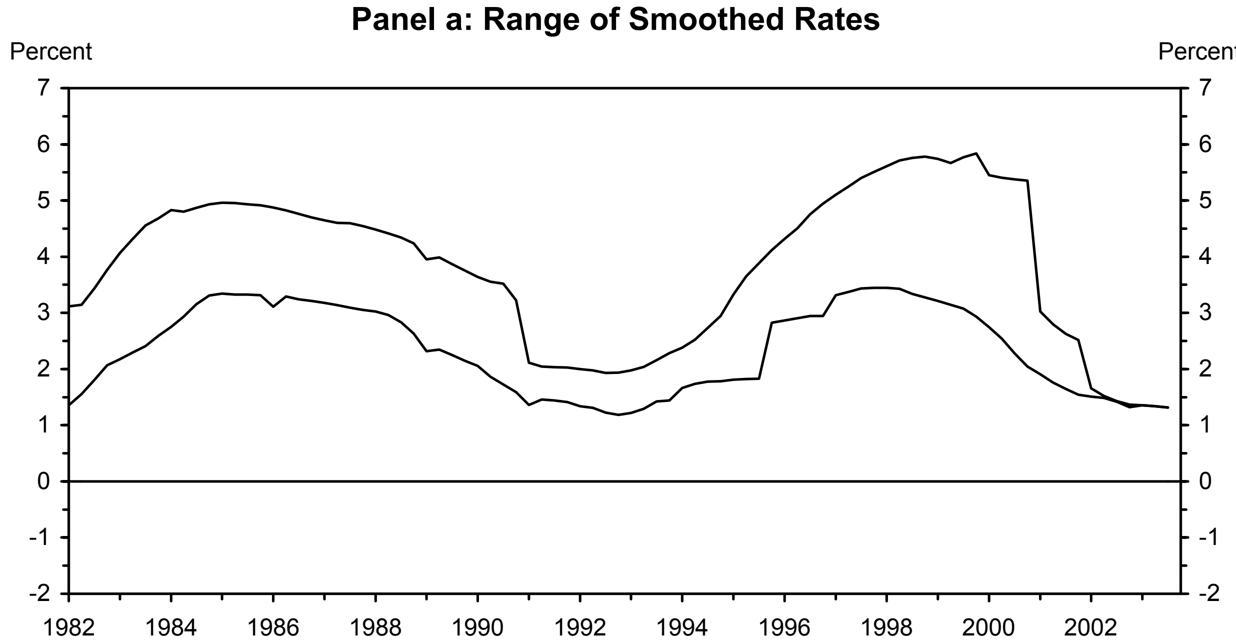


Figure 4
Ranges of Estimates of the Equilibrium Real Rate
IS/CBO-1



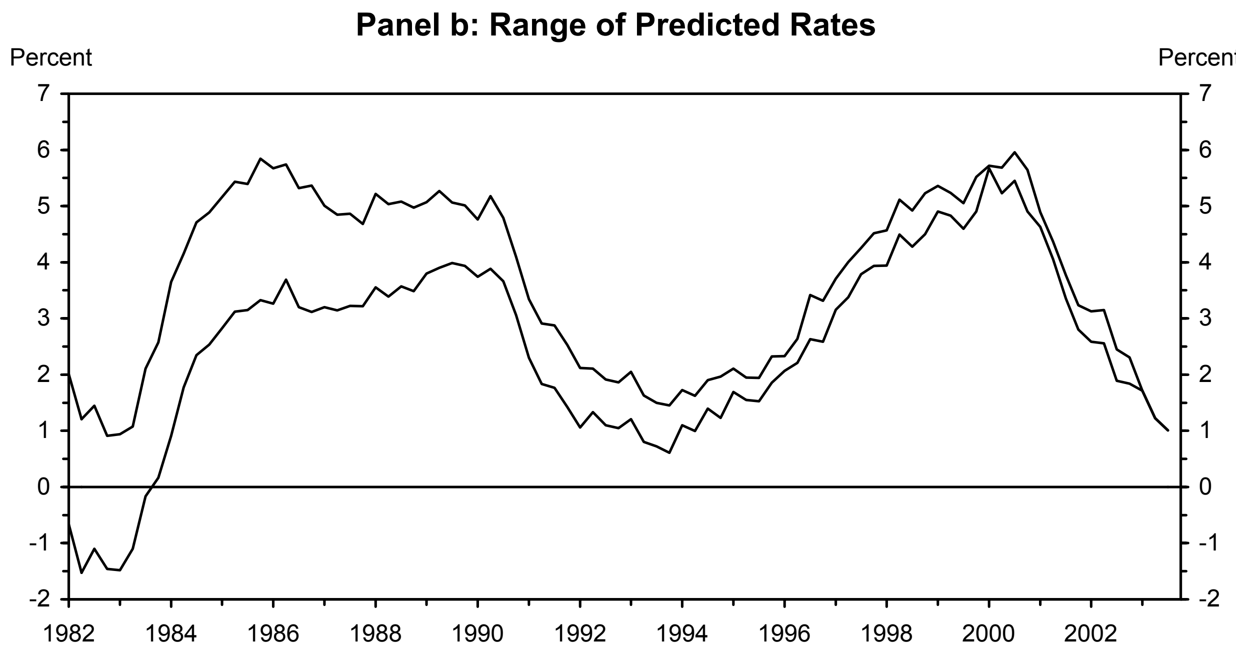
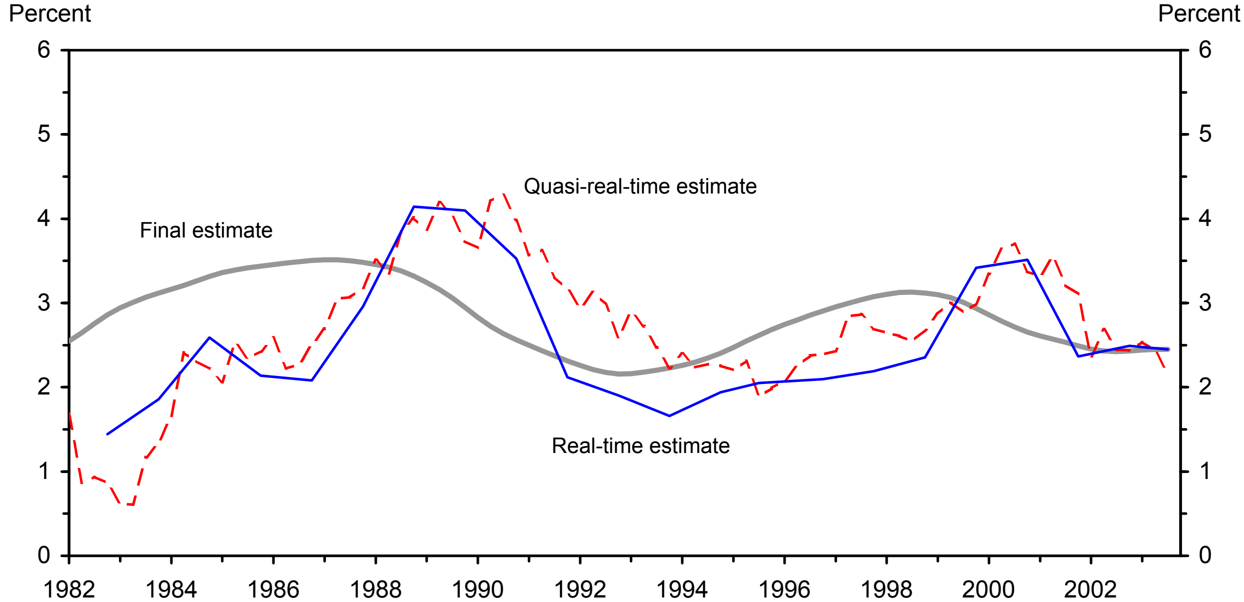


Figure 5
Final vs. Real-Time Estimates of the Equilibrium Real Rate

Panel a: Baseline Laubach-Williams Model



Panel b: IS/CBO-1

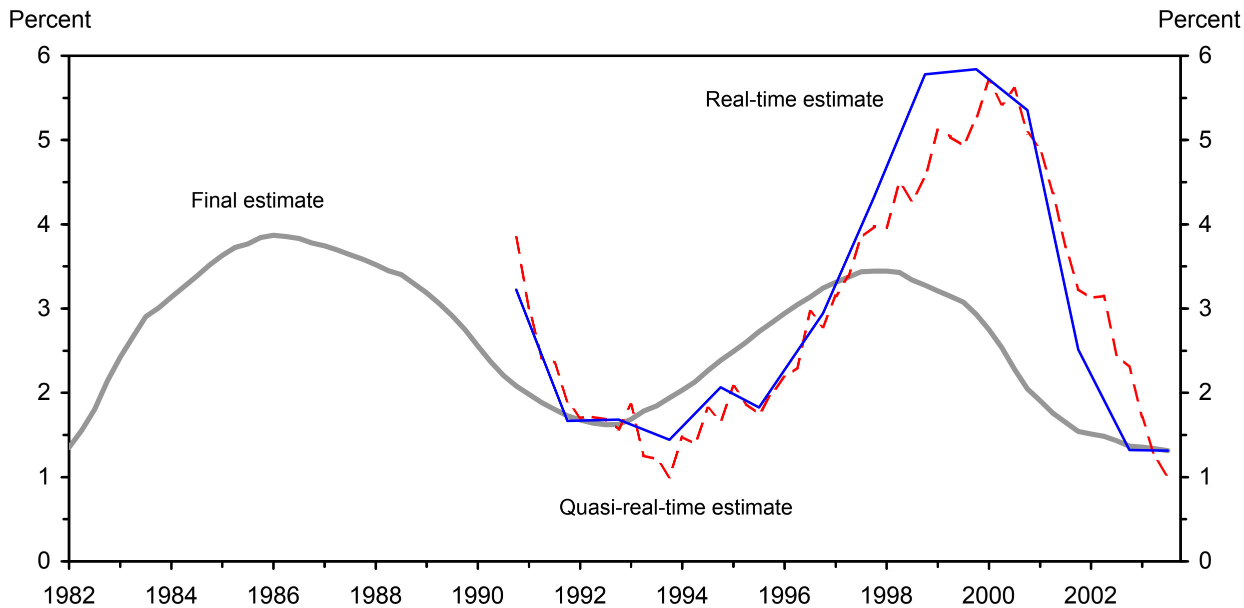
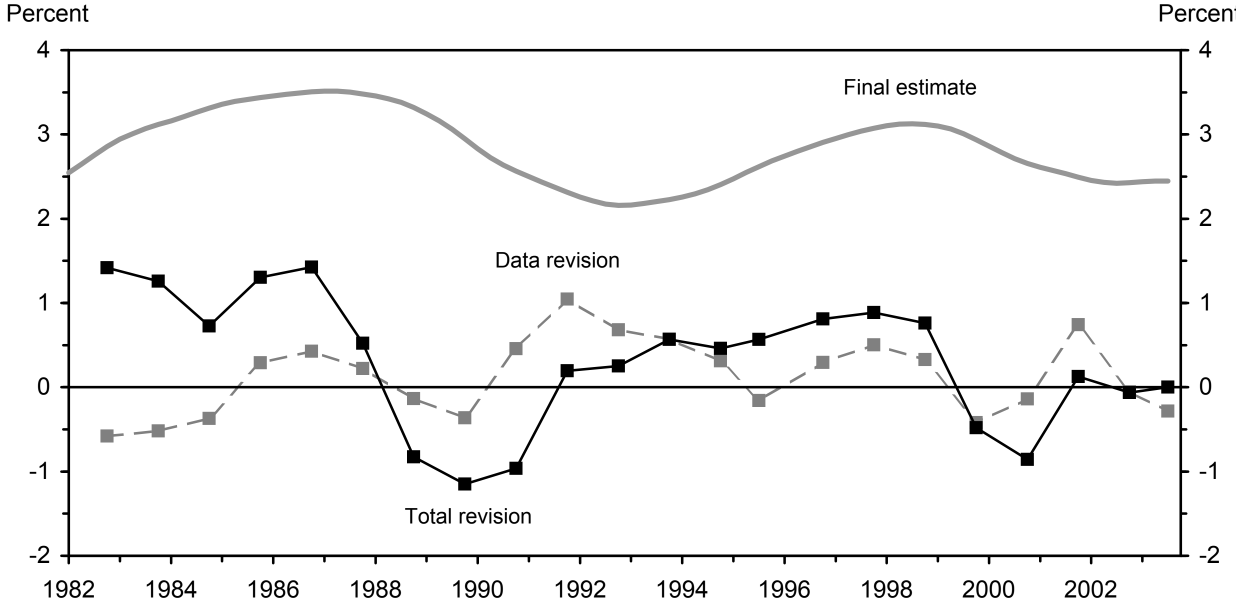


Figure 6
Revisions to Estimates of the Equilibrium Real Rate

Panel a: Baseline Laubach-Williams Model



Panel b: IS/CBO-1

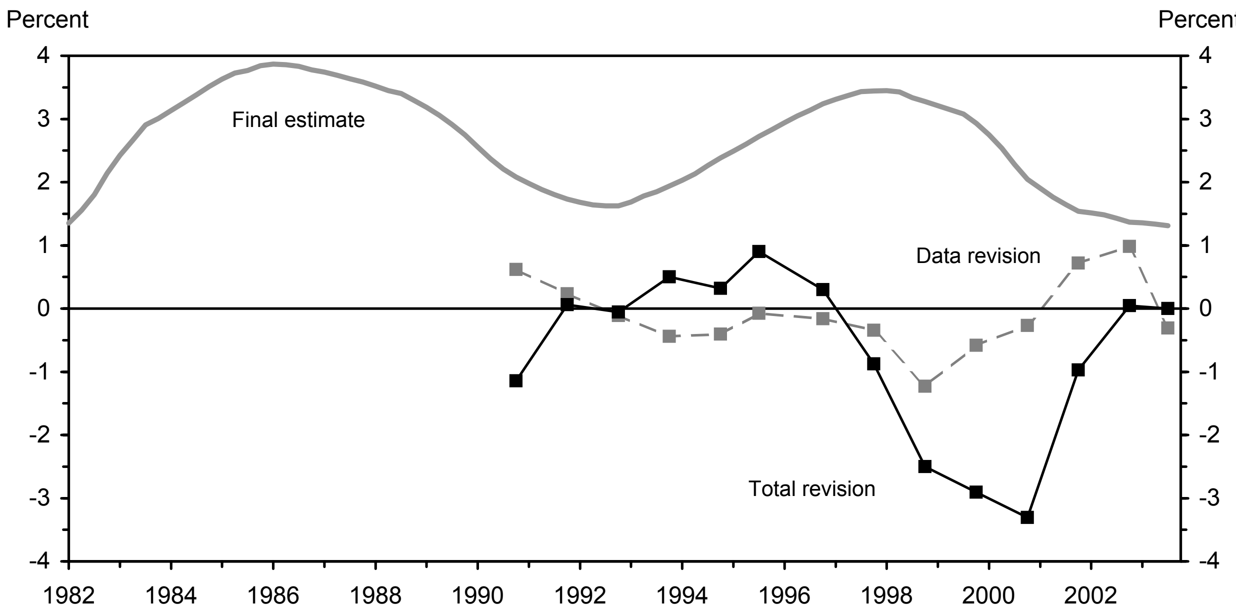


Figure 7
Estimates of c

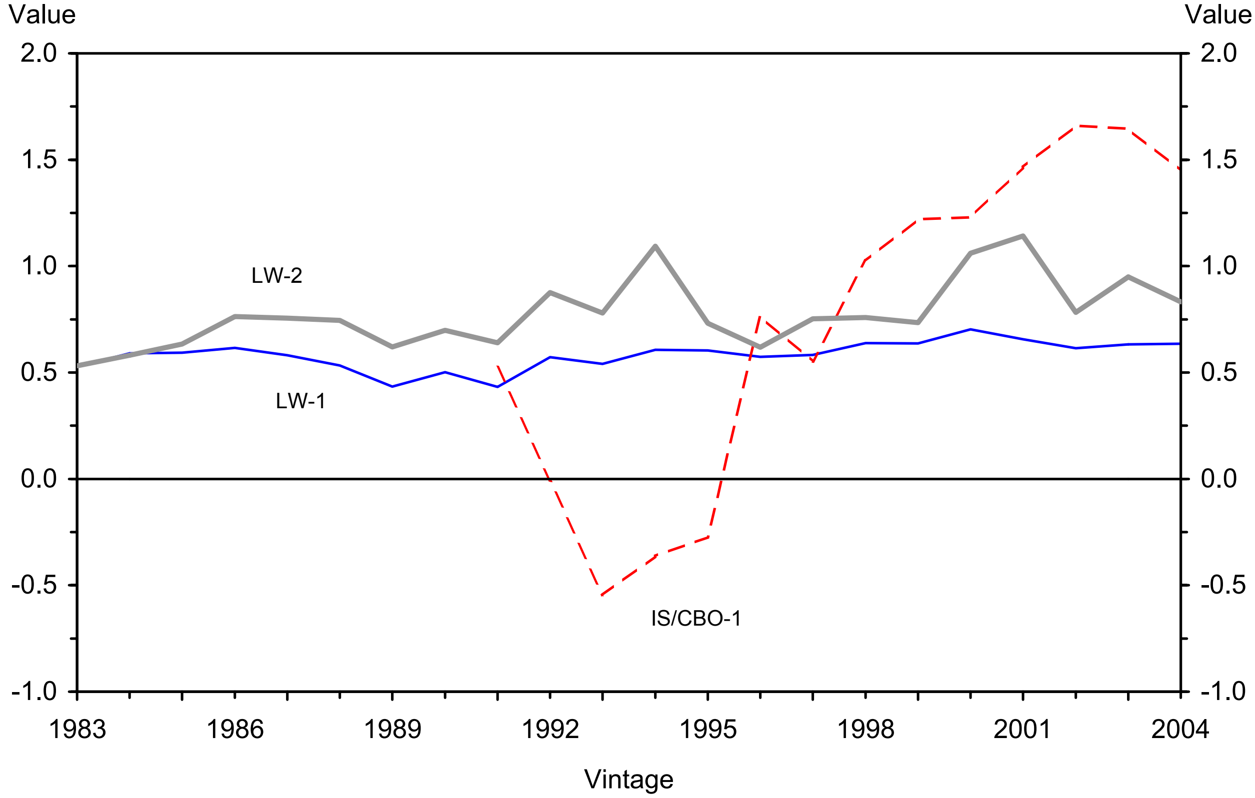
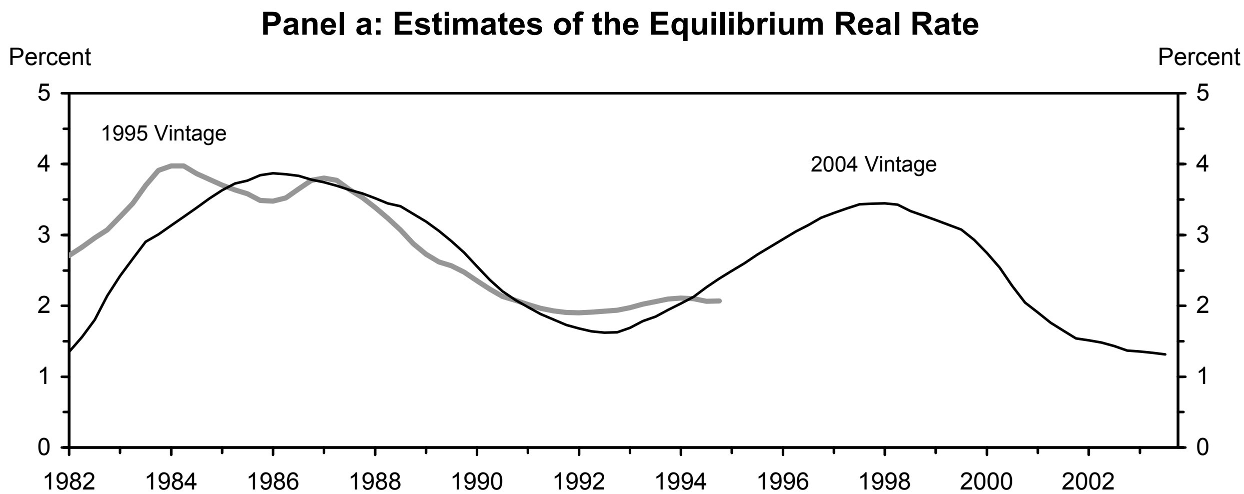
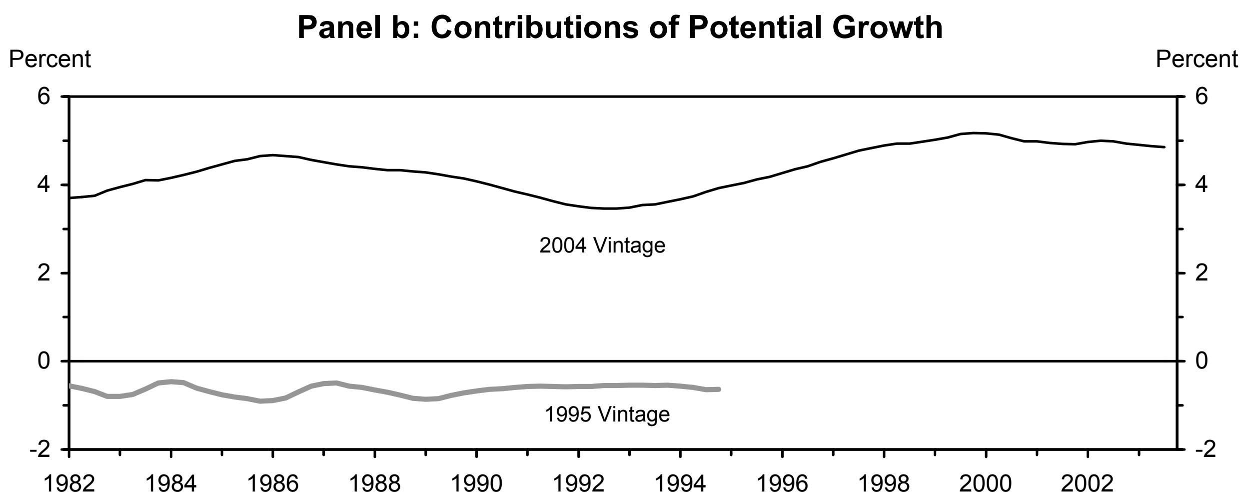


Figure 8
Illustration of the Identification Issue





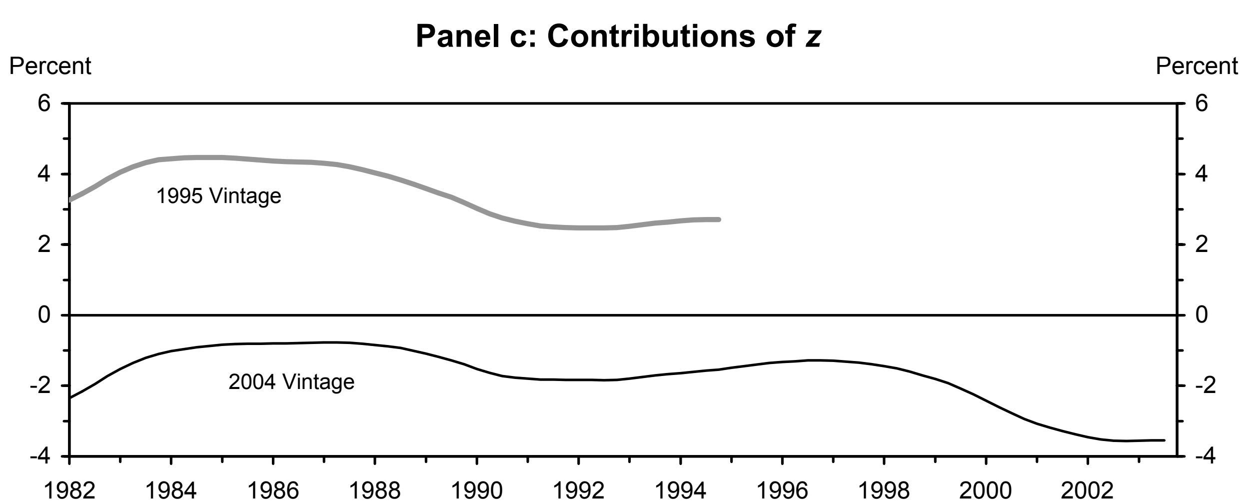


Figure 9a: Real-Time Estimates of the Equilibrium Real Rate

Baseline Laubach-Williams Model

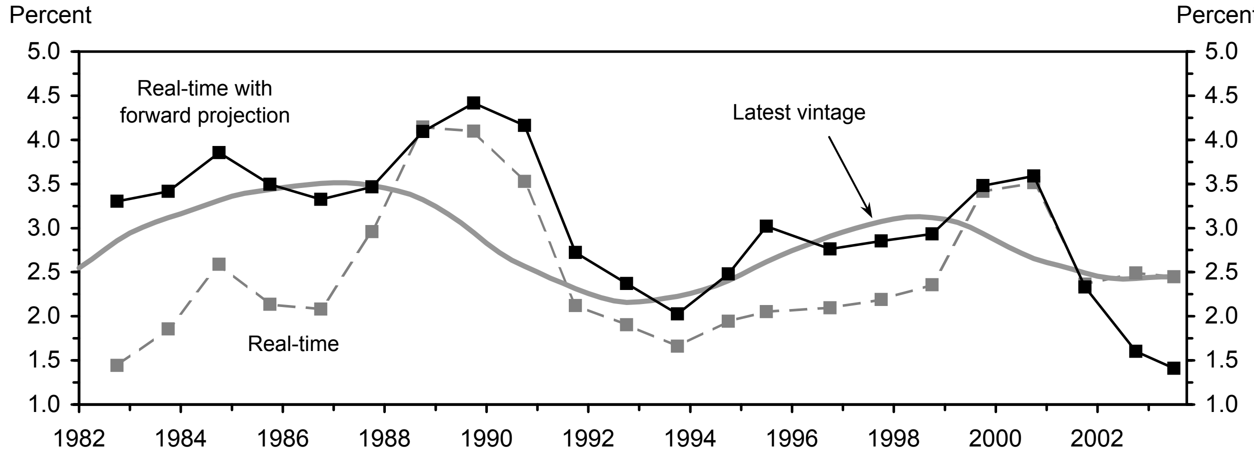


Figure 9b: Real-Time Estimates of the Equilibrium Real Rate

LW-2

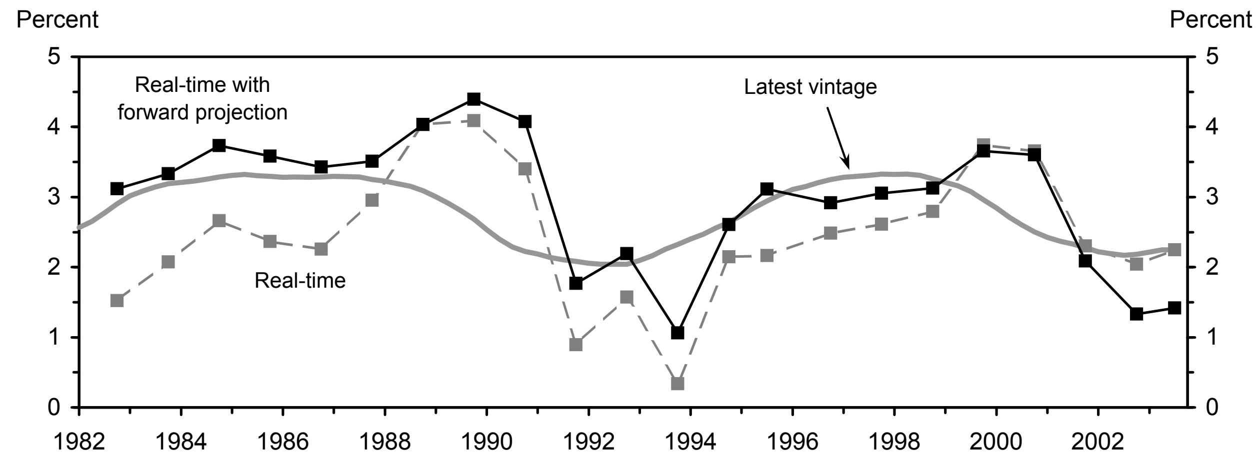


Figure 9c: Real-Time Estimates of the Equilibrium Real Rate

LW-3

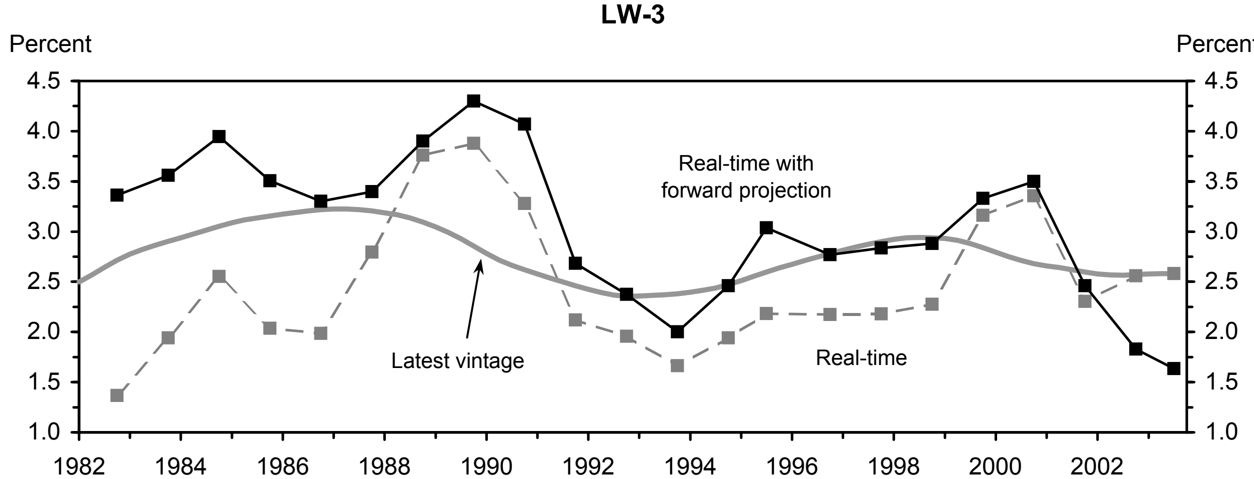


Figure 10a: Real-Time Estimates of the Equilibrium Real Rate
IS/CBO-1

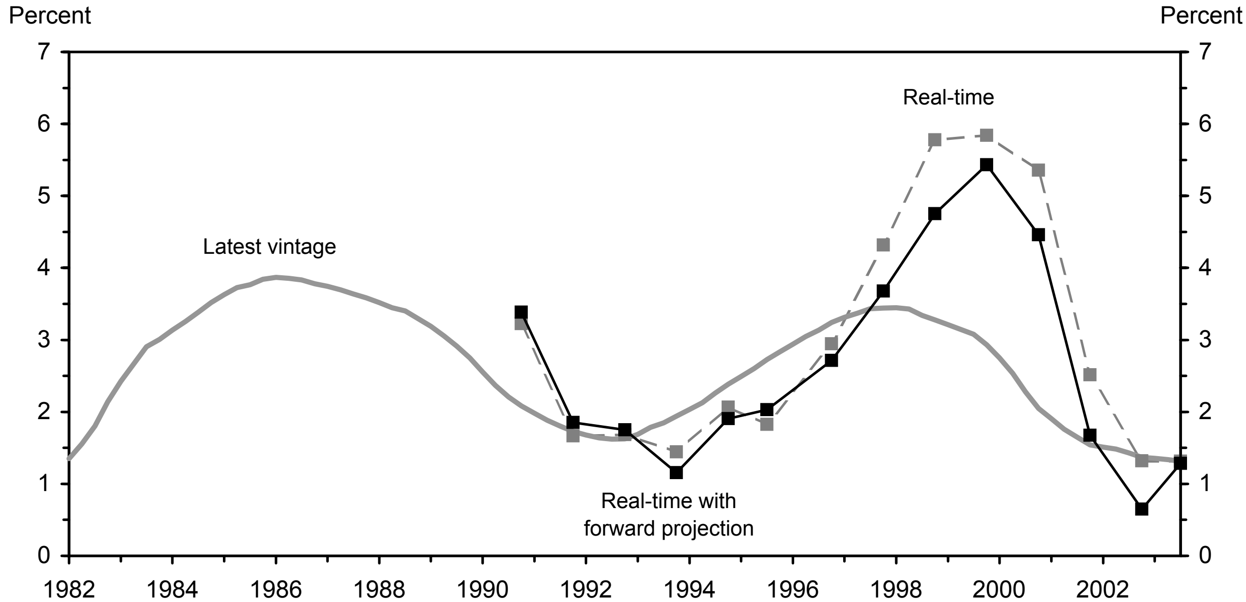
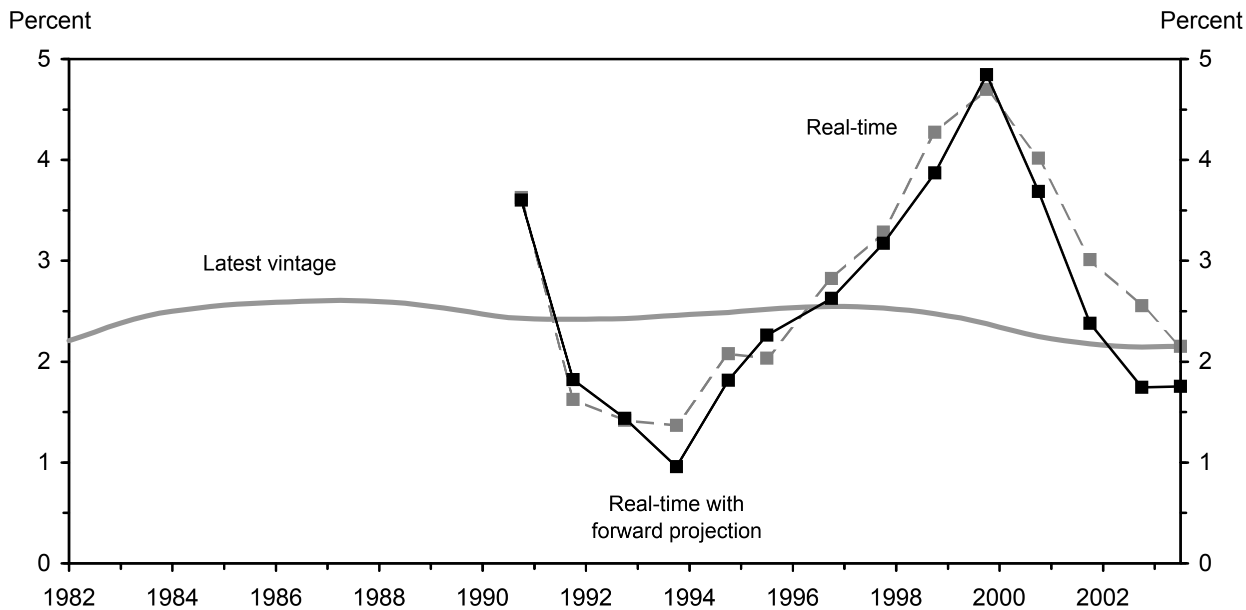


Figure 10b: Real-Time Estimates of the Equilibrium Real Rate
IS/CBO-2



Appendix Tables

Table A1: Estimation results for Laubach-Williams Version 1

Vintage	$a_1 + a_2$	b	c	f	σ_ϵ	σ_η	σ_r
1983	0.953	-0.228	0.527	0.211	0.687	0.247	0.251
1984	0.929	-0.201	0.590	0.124	0.712	0.291	0.294
1985	0.914	-0.190	0.592	0.137	0.674	0.292	0.295
1986	0.915	-0.155	0.616	0.151	0.579	0.307	0.314
1987	0.913	-0.144	0.582	0.172	0.555	0.317	0.323
1988	0.909	-0.128	0.533	0.200	0.528	0.340	0.345
1989	0.907	-0.113	0.434	0.267	0.515	0.373	0.376
1990	0.908	-0.124	0.501	0.234	0.527	0.348	0.352
1991	0.910	-0.112	0.432	0.262	0.491	0.358	0.361
1992	0.908	-0.135	0.573	0.155	0.431	0.262	0.269
1993	0.914	-0.116	0.541	0.169	0.417	0.295	0.301
1994	0.911	-0.124	0.607	0.153	0.431	0.284	0.291
1995	0.913	-0.123	0.603	0.154	0.426	0.284	0.291
1996	0.926	-0.072	0.573	0.199	0.266	0.303	0.311
1997	0.924	-0.067	0.582	0.194	0.266	0.324	0.332
1998	0.926	-0.065	0.638	0.198	0.245	0.312	0.321
1999	0.925	-0.060	0.636	0.205	0.241	0.328	0.337
2000	0.927	-0.068	0.703	0.183	0.271	0.327	0.337
2001	0.924	-0.071	0.657	0.194	0.277	0.321	0.330
2002	0.924	-0.066	0.614	0.208	0.275	0.341	0.348
2003	0.925	-0.065	0.633	0.209	0.277	0.349	0.356
2004	0.925	-0.066	0.636	0.210	0.280	0.350	0.357

Table A2: Estimation results for Laubach-Williams Version 2

Vintage	$a_1 + a_2$	b	c	f	σ_ϵ	σ_η	σ_r
1983	0.951	-0.237	0.531	0.231	0.735	0.206	0.266
1984	0.923	-0.215	0.581	0.169	0.738	0.228	0.294
1985	0.915	-0.215	0.634	0.136	0.684	0.212	0.287
1986	0.911	-0.187	0.763	0.164	0.817	0.290	0.371
1987	0.911	-0.181	0.756	0.175	0.803	0.295	0.376
1988	0.905	-0.168	0.745	0.185	0.798	0.316	0.400
1989	0.900	-0.155	0.620	0.244	0.784	0.335	0.422
1990	0.903	-0.164	0.699	0.218	0.782	0.317	0.400
1991	0.902	-0.157	0.640	0.238	0.773	0.327	0.411
1992	0.907	-0.212	0.876	0.082	0.441	0.138	0.290
1993	0.896	-0.175	0.780	0.146	0.560	0.213	0.315
1994	0.912	-0.247	1.094	0.058	0.229	0.062	0.339
1995	0.901	-0.171	0.731	0.154	0.661	0.258	0.338
1996	0.925	-0.081	0.619	0.156	0.241	0.197	0.313
1997	0.903	-0.139	0.753	0.143	0.679	0.324	0.420
1998	0.923	-0.070	0.759	0.174	0.214	0.202	0.345
1999	0.923	-0.066	0.735	0.183	0.219	0.219	0.354
2000	0.924	-0.095	1.060	0.116	0.199	0.139	0.362
2001	0.919	-0.109	1.141	0.113	0.210	0.128	0.375
2002	0.920	-0.074	0.782	0.186	0.258	0.233	0.367
2003	0.918	-0.084	0.949	0.170	0.271	0.213	0.380
2004	0.920	-0.076	0.833	0.186	0.267	0.232	0.373

Table A3: Estimation results for Laubach-Williams Version 3

Vintage	$a_1 + a_2$	b	f	σ_ϵ	σ_η	σ_r
1983	0.954	-0.232	0.169	0.744	0.322	0.322
1984	0.928	-0.211	0.127	0.766	0.322	0.322
1985	0.915	-0.194	0.121	0.708	0.322	0.322
1986	0.915	-0.156	0.144	0.651	0.322	0.322
1987	0.913	-0.135	0.174	0.594	0.322	0.322
1988	0.908	-0.121	0.197	0.573	0.322	0.322
1989	0.906	-0.105	0.260	0.542	0.322	0.322
1990	0.907	-0.122	0.219	0.542	0.322	0.322
1991	0.909	-0.092	0.272	0.504	0.322	0.322
1992	0.908	-0.160	0.130	0.426	0.322	0.322
1993	0.914	-0.123	0.154	0.424	0.322	0.322
1994	0.912	-0.126	0.140	0.437	0.322	0.322
1995	0.914	-0.132	0.140	0.432	0.322	0.322
1996	0.925	-0.069	0.194	0.293	0.322	0.322
1997	0.921	-0.068	0.189	0.309	0.322	0.322
1998	0.923	-0.066	0.190	0.276	0.322	0.322
1999	0.924	-0.061	0.196	0.275	0.322	0.322
2000	0.927	-0.059	0.195	0.288	0.322	0.322
2001	0.921	-0.068	0.202	0.303	0.322	0.322
2002	0.925	-0.059	0.208	0.305	0.322	0.322
2003	0.926	-0.057	0.208	0.309	0.322	0.322
2004	0.926	-0.058	0.208	0.312	0.322	0.322

Table A4: Estimation results for IS/CBO-1

Vintage	$a_1 + a_2$	b	c	λ_z	σ_ϵ	σ_η	σ_r
1991	0.907	-0.187	0.525	0.089	0.793	0.534	0.538
1992	0.908	-0.172	-0.008	0.058	0.815	0.390	0.390
1993	0.923	-0.197	-0.546	0.078	0.765	0.428	0.458
1994	0.916	-0.175	-0.363	0.067	0.763	0.413	0.427
1995	0.916	-0.169	-0.275	0.062	0.756	0.391	0.394
1996	0.893	-0.123	0.764	0.044	0.801	0.409	0.416
1997	0.897	-0.129	0.552	0.049	0.790	0.423	0.427
1998	0.899	-0.172	1.025	0.095	0.748	0.582	0.584
1999	0.901	-0.157	1.220	0.087	0.745	0.581	0.584
2000	0.909	-0.158	1.229	0.084	0.723	0.547	0.550
2001	0.903	-0.157	1.465	0.080	0.723	0.522	0.527
2002	0.898	-0.137	1.660	0.065	0.734	0.494	0.501
2003	0.903	-0.131	1.645	0.062	0.731	0.490	0.497
2004	0.909	-0.100	1.450	0.046	0.743	0.480	0.485

Table A5: Estimation results for IS/CBO-2

Vintage	$a_1 + a_2$	b	λ_z	σ_ϵ	σ_η	σ_r
1991	0.910	-0.168	0.070	0.805	0.472	0.472
1992	0.909	-0.166	0.057	0.816	0.395	0.395
1993	0.918	-0.155	0.056	0.786	0.402	0.402
1994	0.912	-0.148	0.053	0.776	0.395	0.395
1995	0.912	-0.142	0.047	0.767	0.360	0.360
1996	0.905	-0.133	0.051	0.796	0.434	0.434
1997	0.906	-0.124	0.042	0.794	0.380	0.380
1998	0.916	-0.107	0.030	0.790	0.309	0.309
1999	0.919	-0.101	0.032	0.782	0.355	0.355
2000	0.925	-0.105	0.036	0.757	0.369	0.369
2001	0.924	-0.097	0.026	0.762	0.289	0.289
2002	0.920	-0.095	0.026	0.763	0.296	0.296
2003	0.922	-0.092	0.025	0.758	0.292	0.292
2004	0.924	-0.078	0.017	0.761	0.233	0.233