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# Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates

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# OPTIMAL MONETARY POLICY UNDER COMMITMENT WITH A ZERO BOUND ON NOMINAL INTEREST RATES

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**Abstract:** We determine optimal monetary policy under commitment in a forward-looking New Keynesian model when nominal interest rates are bounded below by zero. The lower bound represents an occasionally binding constraint that causes the model and optimal policy to be nonlinear. A calibration to the U.S. economy suggests that policy should reduce nominal interest rates more aggressively than suggested by a model without lower bound. Rational agents anticipate the possibility of reaching the lower bound in the future and this amplifies the effects of adverse shocks well before the bound is reached. While the empirical magnitude of U.S. mark-up shocks seems too small to entail zero nominal interest rates, shocks affecting the natural real interest rate plausibly lead to a binding lower bound. Under optimal policy, however, this occurs quite infrequently and does not imply positive average inflation rates in equilibrium. Interestingly, the presence of binding real rate shocks alters the policy response to (non-binding) mark-up shocks.

**Keywords:** nonlinear optimal policy, zero interest rate bound, commitment, liquidity trap, New Keynesian

**JEL classification:** C63, E31, E52

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# 1 Introduction

The low levels of nominal interest rates experienced over the last years in major world economies has generated considerable interest in how monetary policy should be conducted in the presence of a zero lower bound on nominal interest rates. Nevertheless, there exists no rigorous treatment of the optimal policy design problem under the standard conditions of uncertainty and rational expectations. Intuition on how monetary policy should be conducted had to be built from models without bound (e.g., Clarida, Galí and Gertler (1999) and Woodford (2003)), or from models with the bound but either deterministic (e.g., Jung, Teranishi, and Watanabe (2005) and Eggertsson and Woodford (2003)) or with backward-looking expectations (e.g., Kato and Nishiyama (2005)).

This paper studies optimal monetary policy under commitment in a stochastic and forward-looking New Keynesian model along the lines of Clarida, Galí and Gertler (1999) and Woodford (2003), but takes explicitly into account that nominal interest rates cannot be set to negative values.<sup>1</sup>

In our model the lower bound on nominal interest rates will occasionally be reached due to adverse shocks hitting the economy.<sup>2</sup> As a result, we are able to study how monetary policy should be conducted when interest rates are still positive but there is the possibility of reaching the lower bound in the near future. In addition, having a fully stochastic setup allows us to calibrate the model to the U.S. economy and to assess the quantitative implications of the zero lower bound on nominal interest rates.

Two *qualitatively* new features of optimal policy emerge from our analysis.

First, we find that nominal interest rates may have to be lowered more aggressively in response to shocks than what is instead suggested by a model without lower bound. Such ‘preemptive’ easing of nominal rates is optimal because agents anticipate the possibility of shocks leading to zero nominal

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<sup>1</sup>In principle negative nominal rates are feasible, e.g., if one is willing to give up free convertability of deposits and other financial assets into cash or if one could levy a tax on money holdings, see Buiter and Panigirtzoglou (2003) and Goodfriend (2000). However, there seems to be no general consensus on the applicability of such policy measures.

<sup>2</sup>Private agents will rationally anticipate this possibility.

rates in the future and reduce already today their output and inflation expectations correspondingly.<sup>3</sup> Such expectations end up amplifying the adverse effects of shocks and thereby trigger a stronger policy response. A similar finding for backward-looking models is reported by Kato and Nishiyama (2005) and Orphanides and Wieland (2000).

Second, the presence of shocks that lead to zero nominal rates alters also the optimal policy response to non-binding shocks. This occurs because the policymaker cannot affect the average real interest rate in any stationary equilibrium, therefore, faces a ‘global’ policy constraint. The inability to lower nominal and real interest rates as much as desired requires that optimal policy increases rates less (or lowers rates more) in response to non-binding shocks, compared to the policy that would instead be optimal in the absence of the lower bound.

There are also a number of *quantitative* results regarding optimal monetary policy for the U.S. economy emerging from this analysis.

First, the zero lower bound appears inessential in dealing with mark-up shocks, i.e., variations over time in the degree of monopolistic competition between firms.<sup>4</sup> More precisely, the empirical magnitude of mark-up shocks observable in the U.S. economy for the period 1983-2002 is too small for the lower bound on nominal rates to be reached. This would remain the case even if the true variance of mark-up shocks were threefold above our estimated value.

Second, the shocks to the ‘natural’ real rate of interest may cause zero nominal rates, but this happens relatively infrequently and is a feature of optimal policy.<sup>5</sup> Based on our estimates for the 1983-2002 period, in the U.S. economy the bound would be reached on average one quarter every 17 years under optimal policy. Once zero nominal interest rates are observed, they are expected to endure not more than 1 to 2 quarters. Moreover, the average welfare losses entailed by the zero lower bound seem to be rather small.

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<sup>3</sup>Expectations are reduced because once the lower bound is reached inflation and output become negative.

<sup>4</sup>These shocks are often called ‘cost-push’ shocks, e.g., Clarida et al. (1999).

<sup>5</sup>The natural real rate is the real interest rate associated with the optimal use of productive resources under flexible prices.

The latter results, however, are sensitive to the size of the standard deviation of the estimated natural real rate process. In particular, we find that zero nominal rates would occur much more frequently and generate higher welfare losses if the real rate process had a somewhat larger variance.

Third, as argued by Jung, Teranishi, and Watanabe (2005) and Eggertsson and Woodford (2003) optimal policy reacts to zero nominal interest rates by generating inflationary expectations in the form of a commitment to let future output gaps and inflation rates increase above zero. The policymaker thereby effectively lowers the real interest rates that agents are confronted with.

Since reducing real rates using inflation promises is costly (in welfare terms), the policymaker has to trade-off the welfare losses generated by too high real rates with those stemming from higher inflation rates. We find that the required levels of inflation and the associated positive output gap are very moderate. A negative 3 standard deviation shock to the natural real rate requires a promise of an increase in the annual inflation rate in the order of 15 basis points and a positive output gap of roughly 0.5%.

Finally, while the optimal policy response to shocks through the promise of above average output and inflation may in principal generate a ‘commitment bias’, the quantitative effects turn out to be negligible. This holds not only for our baseline calibration but also for a range of alternative model parameterizations that we look at. It suggests that optimal policy for the U.S. economy implements an average inflation rate of zero even when taking directly into account the zero lower bound on nominal interest rates.<sup>6</sup>

The remainder of this paper is structured as follows. Section 2 briefly discusses the related literature. Thereafter, section 3 introduces the model and the policy problem. Section 4 presents our calibration for the U.S. economy. The solution method we employ is described in section 5. Section 6 presents the main results. We then discuss in section 7 the robustness of our findings to various parameter changes, and briefly conclude in section 8. Our strategy for identifying the historical shocks and numerical algorithm are discussed in the appendix.

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<sup>6</sup>Zero inflation is optimal because it minimizes the price dispersion between firms with sticky prices and we abstract from the money demand distortions associated with positive nominal interest rates.

## 2 Related Literature

A number of recent papers study the implications of the zero lower bound on nominal interest rates for optimal monetary policy.

Most closely related is Kato and Nishiyama (2005) who consider a stochastic backward-looking model with an occasionally binding zero lower bound constraint. Jung et al. (2005) and Eggertsson and Woodford (2003) consider forward-looking models under perfect foresight and analytically derive optimal targeting rules.<sup>7</sup> In this paper we consider instead a fully stochastic setup which requires solving the model numerically to obtain the rational expectations equilibrium.

A related set of papers focuses on optimal monetary policy in the absence of credibility. In a companion paper of ours, Adam and Billi (2004), we derive the optimal discretionary policy with zero lower bound in a stochastic forward-looking model. Eggertsson (2005) analyzes discretionary policy and the role of nominal debt policy as a way to achieve credibility.

The performance of simple monetary policy rules is examined by Fuhrer and Madigan (1997), Wolman (2005), and Coenen, Orphanides and Wieland (2004). A main finding of this set of papers is if the targeted inflation rate is close enough to zero, simple policy rules formulated in terms of inflation rates, e.g., the Taylor rule (1993), can generate significant real distortions. Reifschneider and Williams (2000) and Wolman (2005) show that simple policy rules formulated in terms of a price level target can considerably reduce these real distortions. Benhabib et al. (2002) study the global properties of Taylor-type rules showing that these might lead to self-fulfilling deflation that converges to a low inflation or deflationary steady state. Evans and Honkapohja (2005) study the properties of global Taylor rules under adaptive learning, showing the existence of an additional steady state with even lower inflation rates.

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<sup>7</sup>Eggertsson and Woodford (2003) also consider a simple stochastic setup where the economy never falls into a liquidity trap: the economy is either unexpectedly in a situation with zero nominal interest rates and reverts back to positive nominal rates with a fixed probability each period or it is in a situation where nominal rates are positive already and the zero lower bound never binds in the future.

The role of the exchange rate and monetary-base rules in overcoming the adverse effects of a binding lower bound on interest rates is analyzed, e.g., by Auerbach and Obstfeld (2005), Coenen and Wieland (2003), McCallum (2003), and Svensson (2003).<sup>8</sup>

### 3 The Monetary Policy Problem

We consider a simple and well-known monetary policy model of a representative consumer and firms in monopolistic competition facing restrictions on the frequency of price adjustments (Calvo (1983)). Following Rotemberg (1987), this is often referred to as the ‘New Keynesian’ model, that has frequently been studied in the literature, e.g., Clarida, Galí and Gertler (1999) and Woodford (2003).

We augment this otherwise standard monetary policy model by explicitly imposing the zero lower bound on nominal interest rates. We thus consider the following problem:

$$\max_{\{y_t, \pi_t, i_t\}} -E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda y_t^2) \quad (1)$$

s.t.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t \quad (2)$$

$$y_t = E_t y_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + g_t \quad (3)$$

$$i_t \geq -r^* \quad (4)$$

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t} \quad (5)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \quad (6)$$

$$u_0, g_0 \text{ given} \quad (7)$$

where  $\pi_t$  denotes the inflation rate,  $y_t$  the output gap, and  $i_t$  the nominal interest rate expressed as deviation from the interest rate consistent with the zero inflation steady state.

Under certain conditions the monetary policy objective (1) can be interpreted as a quadratic approximation to the utility of the representative

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<sup>8</sup>Further articles dealing with the relevance of the zero lower bound can be found in the special issues of the Journal of Japanese and International Economies Vol. 14, 2000 and the Journal of Money Credit and Banking Vol. 32 (4,2), 2000.

household.<sup>9</sup> The welfare weight  $\lambda > 0$  is then given by

$$\lambda = \frac{\kappa}{\theta}$$

where  $\theta > 1$  denotes the price elasticity of demand for the goods produced by monopolistic firms. Equation (2) is a forward-looking Phillips curve summarizing, up to first order, profit-maximizing price setting behavior by firms, where  $\beta \in (0, 1)$  denotes the discount factor and  $\kappa > 0$  is given by

$$\kappa = \frac{(1 - \alpha)(1 - \alpha\beta) \sigma^{-1} + \omega}{\alpha} \frac{1}{1 + \omega\theta}$$

with  $\alpha \in (0, 1)$  denoting the share of firms that cannot adjust prices in a given period,  $\sigma > 0$  the household's intertemporal elasticity of substitution, and  $\omega > 0$  the elasticity of a firm's real marginal costs with respect to its own output level.<sup>10</sup> Equation (3) is a linearized Euler equation summarizing, up to first order, households' intertemporal maximization. The shock  $g_t$  captures the variation in the 'natural' real interest rate and is usually referred to as a real rate shock, i.e.,

$$g_t = \sigma(r_t - r^*) \tag{8}$$

where the natural real rate  $r_t$  is the real interest rate consistent with the flexible price equilibrium, and  $r^* = 1/\beta - 1$  is the real rate of the deterministic zero inflation steady state.<sup>11</sup> The requirement that nominal interest rates have to remain positive is captured by constraint (4). Finally, equations (5) and (6) describe the evolution of the shocks, where  $\rho_j \in (-1, 1)$  and  $\varepsilon_{j,t} \sim iiN(0, \sigma_j^2)$  for  $j = u, g$ .<sup>12</sup>

### 3.1 How much non-linearity?

Instead of the fully nonlinear model, we study linear approximations to firms' and households' first order conditions, i.e., equations (2) and (3),

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<sup>9</sup>This requires steady output to be efficient, e.g., thanks to the existence of an output subsidy that neutralizes the distortions from monopolistic competition, and the output gap to be defined as the difference between the actual output level and the efficient level, see chapter 6 of Woodford (2003) for details.

<sup>10</sup>See chapter 3 in Woodford (2003) for further details.

<sup>11</sup>The shock  $g_t$  summarizes all shocks that under flexible prices generate time variation in the real interest rate, therefore, it captures the combined effects of preference shocks, productivity shocks, and exogenous changes in government expenditure.

<sup>12</sup>As shown subsequently, this specification of the shock processes is sufficiently general to describe the historical sequence of shocks in the U.S. economy for the period 1983:1-2002:4 that we consider.



respectively, and a quadratic approximation to the objective function, i.e., equation (1). This means that the only nonlinearity that we take account of is the one imposed by the zero lower bound (4).<sup>13</sup>

Clearly, this modelling approach has advantages and disadvantages. One disadvantage is that for the empirically relevant shock support and the estimated value of the discount factor the linearizations (2) and (3) may perform poorly at the lower bound. Yet, this depends on the degree of nonlinearity present in the economy, an issue about which relatively little is empirically known.

A paramount advantage of our approach is that one can economize in the dimension of the state space. Higher-order approximations to the equilibrium conditions would require an additional state variable to keep track over time of the higher-order effects of price dispersion, as shown by Schmitt-Grohé and Uribe (2004). Computational costs would become prohibitive with such an additional state.<sup>14</sup> A further advantage of focusing solely on the nonlinearities induced by the lower bound is that one does not have to parameterize higher order terms when calibrating the model. This seem important, given the lack of empirical evidence on this matter.

Finally, the simpler setup implies that our results remain more easily comparable to the standard linear-quadratic analysis without lower bound that appears in the literature, as the only difference consists of imposing equation (4).

## 4 Calibration to U.S. Economy

To assess the quantitative importance of the zero lower bound for monetary policy, we assign parameter values for the coefficients appearing in equations (1) to (6) by calibrating the model to the U.S. economy.

Table 1 summarizes our baseline parameterization. The values for  $\alpha, \theta, \sigma, \omega$  (and  $\kappa, \lambda$ ) are taken from table 6.1 in Woodford (2003). The parameters of

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<sup>13</sup>Technically, this approach is equivalent to linearizing the first order conditions of the nonlinear Ramsey problem around the first best steady state except for the non-negativity constraint for nominal interest rates, that is kept in its original nonlinear form.

<sup>14</sup>Our model has 4 state variables with continuous support and it takes already 39 hours to obtain convergence on a Pentium 4 with 2.6 GHz.

the shock processes and the discount factor are estimated using U.S. data for the period 1983:1-2002:4, following the approach of Rotemberg and Woodford (1998). The implied steady state real interest rate for this parameterization is 3.5% annually. Details of the estimation and reasons for the sample period chosen are given in appendix A.1.

The identified historical shock series are shown in figure 3. Mark-up shocks do not display any significant autocorrelation and have a standard deviation of approximately 0.61% annually.<sup>15</sup> Real rate shocks, however, are rather persistent. As one would expect, the natural real rate seems to fall during recessions, e.g., at the beginning of the 1990s and at the start of the new millennium. The implied annual standard deviation of the natural real rate, as implicitly defined in equation (8), is equal to 1.63% annually.<sup>16</sup>

The robustness of our findings to various assumptions regarding the parameterization of the model is considered in section 7.

## 5 Solving the Model

This section illustrates how we solve the optimal policy problem and derive the associated rational expectations equilibrium. While some insights can be gained from an analytical approach, the presence of forward-looking variables and of an occasionally binding constraint require to rely on numerical solution methods. We first describe our numerical solution approach and then derive analytical results using first order conditions.

### 5.1 Numerical solution approach

An important complication for solving the model is that the policymaker's maximization problem fails to be recursive, since constraints (2) and (3) involve forward-looking variables. For this reason we cannot directly resort to dynamic programming techniques. To obtain a dynamic programming

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<sup>15</sup>This lack of autocorrelation contrasts with Ireland (2004) who uses data starting in 1948:1. Extending our sample back to this date would also lead to highly persistent mark-up shocks. But our identification of shocks requires the absence of structural breaks, so we restrict attention to the shorter sample period.

<sup>16</sup>When using instead the period 1979:4-1995:2 as in Rotemberg and Woodford (1998), which includes the volatile years 1980-1982, we find an annual standard deviation of 2.57% for the natural real rate.

formulation we rewrite the policy problem (1)-(7) by recursifying the Lagrangian of the infinite horizon problem, as in Marcet and Marimon (1998).

This involves the following four steps. First, write down the Lagrangian for problem (1)-(7) and let  $\gamma_t^1$  and  $\gamma_t^2$  denote the Lagrange multipliers for constraints (2) and (3), respectively. Since these constraints are forward-looking, some terms involve period  $t$  Lagrange multipliers  $\gamma_t^i$  ( $i = 1, 2$ ) multiplied by period  $t+1$  choice variables  $\pi_{t+1}$  and  $y_{t+1}$ . Second, relabel the Lagrange multipliers in these terms as  $\mu_{t+1}^i$  and define the transition equations  $\mu_{t+1}^i = \gamma_t^i$ . Third, collect all terms dated  $t$  and add  $-\mu_0^1\pi_0 + \mu_0^2\frac{1}{\beta}(\sigma\pi_0 + y_0)$  in period zero, defining  $\mu_0^i = 0$  ( $i = 1, 2$ ). This delivers the following infinite horizon Lagrange problem

$$\begin{aligned} \min_{\{\gamma_t^1, \gamma_t^2\}} \max_{\{y_t^1, \pi_t^2, i_t\}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t (-\pi_t^2 - \lambda y_t^2 \right. \\ \left. + \gamma_t^1 (\pi_t - \kappa y_t - u_t) - \mu_t^1 \pi_t \right. \\ \left. + \gamma_t^2 (-y_t - \sigma i_t + g_t) + \mu_t^2 \frac{1}{\beta} (\sigma \pi_t + y_t) \right] \quad (9) \end{aligned}$$

subject to

$$\begin{aligned} i_t &\geq -r^* \\ \mu_{t+1}^1 &= \gamma_t^1 \\ \mu_{t+1}^2 &= \gamma_t^2 \\ u_{t+1} &= \rho_u u_t + \varepsilon_{u,t+1} \\ g_{t+1} &= \rho_g g_t + \varepsilon_{g,t+1} \\ \mu_0^1 &= 0 \\ \mu_0^2 &= 0 \\ u_0, g_0 &\text{ given} \end{aligned} \quad (10)$$

Then, defining the one-period return function as

$$\begin{aligned} h(y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) &\equiv -\pi_t^2 - \lambda y_t^2 \\ &+ \gamma_t^1 (\pi_t - \kappa y_t - u_t) - \mu_t^1 \pi_t \\ &+ \gamma_t^2 (-y_t - \sigma i_t + g_t) + \mu_t^2 \frac{1}{\beta} (\sigma \pi_t + y_t). \end{aligned} \quad (11)$$

the infinite horizon problem (9) can be written in recursive form

$$\begin{aligned} W(\mu_t^1, \mu_t^2, u_t, g_t) &= \min_{(\gamma_t^1, \gamma_t^2)} \max_{(y_t, \pi_t, i_t)} \{ h(y_t, \pi_t, i_t, \gamma_t^1, \gamma_t^2, \mu_t^1, \mu_t^2, u_t, g_t) \\ &+ \beta E_t W(\mu_{t+1}^1, \mu_{t+1}^2, u_{t+1}, g_{t+1}) \} \end{aligned} \quad (12)$$

where optimization is subject to the set of constraints (10), which now involve only backward-looking transition equations. Equation (12) is a generalized Bellman equation, requiring maximization with respect to the controls  $(y_t, \pi_t, i_t)$  and minimization with respect to the Lagrange multipliers  $\gamma_t^1 \leq 0$  and  $\gamma_t^2 \geq 0$ . In our numerical approach we approximate the value function that solves the recursive functional equation (12) so to obtain the associated optimal policy functions for  $(y_t, \pi_t, i_t)$  and  $(\gamma_t^1, \gamma_t^2)$ . The numerical algorithm used is described in detail in appendix A.3.<sup>17</sup>

Note that the reformulated problem (12) has two additional state variables  $(\mu_t^1, \mu_t^2)$ , i.e., the lagged Lagrange multipliers, bringing the total number of state variables up to four. The additional states can be interpreted as ‘promises’ that have to be kept from past commitments, leading to deviations from purely forward-looking policy whenever their values differ from zero. This can be seen from the expression of the one-period return function  $h(\cdot)$  given by equation (11). In particular,  $-\mu_t^1 + \frac{\sigma}{\beta}\mu_t^2 > 0$ , e.g., indicates a promise from past commitments to generate today higher inflation rates than what purely forward-looking policy would imply. Likewise,  $\frac{1}{\beta}\mu_t^2 > 0$  indicates a promise from past commitments to generate today larger output gaps than what purely forward-looking policy suggests.<sup>18</sup>

## 5.2 Targeting rule

Some insights about the solution to the optimal policy problem can also be gained from the first-order conditions of problem (9). As shown in appendix A.2, these can be used to derive the ‘targeting rule’

$$\left( y_t + \frac{\kappa}{\lambda}\pi_t + \frac{\kappa}{2\lambda}\mu_t^1 - \frac{1 + \kappa\sigma}{2\lambda\beta}\mu_t^2 \right) \cdot (i_t + r^*) = 0 \quad (13)$$

which, provided  $i_t > -r^*$ , specifies a relationship between current output, inflation, and the lagged Lagrange multipliers that has to be satisfied under optimal policy in equilibrium. Interestingly, this relationship is linear despite the policy problem being nonlinear. For the case where the lower bound has

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<sup>17</sup>Our solution approach is complementary to that of Christiano and Fisher (2000) and Aiyagari et al. (2002), which uses the first order conditions of problem (12) to solve for the optimal policy functions.

<sup>18</sup>Clearly, inverting the signs of the inequalities implies inflation and output gap promises of opposite direction.

not been reached in any period  $j \leq t$ , the targeting rule (13) simplifies to<sup>19</sup>

$$y_t - y_{t-1} + \frac{\kappa}{\lambda} \pi_t = 0 \quad (14)$$

This is the targeting rule derived, e.g., in Clarida et al. (1999) for a model that abstracts from the existence of a lower bound. Even though this simplified targeting rule (14) can describe optimal policy in a model either with or abstracting from the lower bound, satisfying it might require setting interest rates differently, depending on whether or not the model abstracts from the bound. This is the case because inflation and output are forward-looking processes. Expected future output and inflation depend on whether or not the model accounts for the possibility of the zero nominal interest rate constraint being binding in the future.

## 6 Optimal Policy with Lower Bound

This section describes the optimal policy with a lower bound on nominal interest rates for the calibration to the U.S. economy shown in table 1. Throughout the paper variables are expressed in terms of percentage point deviations from deterministic steady state values. Interest rates and inflation rates are expressed in annualized percentage deviations, while the real rate shock and the mark-up shock are stated in quarterly percentages.

### 6.1 Optimal Policy Functions

Figure 4 presents the optimal responses of  $(y, \pi, i)$  and the Lagrange multipliers  $(\gamma^1, \gamma^2)$  to both a mark-up shock and a real rate shock.<sup>20</sup> The responses of the Lagrange multipliers are of interest because they represent commitments regarding future inflation rates and output gaps, as explained in the previous section. Depicted are the optimal policy responses both for the case of the zero lower bound being imposed in the model (solid line) and for the case of interest rates allowed to become negative (dashed line with circles).

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<sup>19</sup>This is shown in appendix A.2.

<sup>20</sup>The state variables not shown on the  $x$ -axes are set to their (unconditional) average values. Policies are shown for a range of  $\pm 4$  unconditional standard deviations of both the mark-up shock and the real rate shock.

The left-hand panel of figure 4 shows that the optimal response to mark-up shocks is virtually unaffected by the presence of the zero lower bound.<sup>21</sup> Independently of whether the bound is imposed or not in the model, a negative mark-up shock lowers inflation, increases output, and leads to a promise of future inflation, as indicated by the values  $\gamma_t^1 < 0$  and  $\gamma_t^2 = 0$  (recall from section 5.1 that  $-\mu_{t+1}^1 + \frac{\sigma}{\beta}\mu_{t+1}^2 > 0$  implies higher inflation in  $t + 1$  than what purely forward-looking policy would suggest then). Future inflation ameliorates the deflationary effect of the shock through the expectational channel present in equation (2). Overall, however, the required interest rate changes in response to mark-up shocks are rather small, implying that mark-up shocks do not plausibly lead to a binding lower bound.

The situation is quite different if we consider the policy response to a real rate shock, as depicted on the right-hand panel of figure 4. Without zero lower bound in the model these shocks do not generate any policy trade-off: the required real rate can be implemented through appropriate variations in the nominal rate alone. Once the lower bound is considered, sufficiently negative real rate shocks cause the bound to be binding, so promising future inflation remains the only instrument for implementing reductions in the real rate. The values  $\gamma_t^1 < 0$  and  $\gamma_t^2 > 0$  associated with negative real rate shocks (implying  $-\mu_{t+1}^1 + \frac{\sigma}{\beta}\mu_{t+1}^2 > 0$ ) reveal that once the lower bound is reached the policymaker indeed commits to future inflation as a substitute for nominal rate cuts. Yet, inflation is a costly instrument (in welfare terms) and it would be suboptimal to completely undo the output losses generated by negative real rate shocks. As a result, there is a negative output gap, some deflation, and nominal interest rates are at their lower bound. All these features are generally associated with the concept of a ‘liquidity trap’.

Figure 5 depicts the optimal interest rate response to the current value of the real rate shocks in greater detail. This reveals that nominal interest rates have to be reduced more aggressively than is the case when nominal rates are allowed to become negative.<sup>22</sup> A stronger interest rate reduction is

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<sup>21</sup>The optimal reaction to mark-up shocks is different with or without the bound, but the difference is quantitatively small for the calibrated parameter values. We will come back to this point in section 6.4.

<sup>22</sup>Kato and Nishiyama (2005) found a similar effect with a backward-looking AS curve, which suggests that our result is robust to the introduction of lagged inflation terms into the ‘New Keynesian’ AS curve. Using different models, Orphanides and Wieland (2000)

optimal because the possibility of additional shocks in the future generating a binding lower bound places downward pressure on expected future output and inflation, since these variables become negative once the bound is reached, see the right-hand panel of figure 4. The reduced output and inflation expectations amplify the effects of negative real rate shocks in equation (3), thereby require that the policymaker lowers nominal rates faster. As a result, zero nominal rates are reached much earlier than suggested by a model without lower bound.

This amplification effect via private sector expectations points towards an interesting complementarity between policy decisions and private sector expectations formation, that may be of considerable importance for actual policy making. Suppose, e.g., that agents suddenly assign a larger probability to the lower bound being binding in the future. Since this lowers output and inflation expectations, policy would reduce the nominal interest rate and cause the economy to move into the direction of the expected change. The existence of possible sunspot fluctuations, however, is an issue that has to be explored in future work.

## 6.2 Dynamic Response to Real Rate Shocks

Figure 6 displays the mean response of the economy to real rate shocks of  $\pm 3$  unconditional standard deviations.<sup>23</sup> With our baseline calibration of table 1 the annual ‘natural’ real rate, i.e., the real interest rate consistent with the efficient use of productive resources, stands temporarily at +8.39% and -1.39%, respectively; the interesting case being the one where full use of productive resources requires a negative real rate.

As argued by Krugman (1998), negative real rates are plausible even if the marginal product of physical capital remains positive. For instance 

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and Reifschneider and Williams (2000) also report more aggressive easing than in the absence of the zero bound.

<sup>23</sup>Since in this nonlinear model certainty equivalence fails to hold, instead of the more familiar deterministic impulse responses we discuss results in terms of the implied ‘mean dynamics’ in response to shocks. The mean dynamics in this and other graphs are the average responses computed for  $10^5$  stochastic simulations. The initial values for the other states are set equal to their *unconditional* average values. Setting them to *conditional* average values consistent with the real rate shock process does not make a noticeable difference.

agents may require a large equity premium, e.g., historically observed in the U.S., or the price of physical capital may be expected to decrease.

Figure 6 shows that in response to a negative real rate shock annual inflation rises by about 15 basis points for up to 3 or 4 quarters and then returns to a value close to zero. Similarly, output increases slightly above potential from the second quarter and slowly returns to potential. Getting out of a ‘liquidity trap’ induced by negative real-rate shocks, therefore, requires that the policymaker promises to let future output and inflation increase above zero for a substantial amount of time.<sup>24</sup> The qualitative feature of this finding is also reported in Eggertsson and Woodford (2003), and in a somewhat different way in Auerbach and Obstfeld (2005). Our results tend to clarify, however, that the required amount of inflation and the output boom are rather modest.

One should note that ex-post there would be strong incentives to increase nominal interest rates earlier than promised, since this would bring both inflation and output closer to their target values. The feasibility of the optimal policy response, therefore, crucially depends on the policymaker’s credibility. Whether policymakers can and may want to credibly commit to such policies is currently subject of debate, e.g., Orphanides (2004).<sup>25</sup>

### 6.3 Frequency of Zero Nominal Rates and Welfare Implications

In this section we discuss the frequency with which zero nominal rates can be expected and the related welfare implications.

For the calibration to the U.S. economy, under optimal policy zero nominal interest rates occur rather infrequently, namely in about one quarter every 17 years on average. Moreover, they tend to prevail for rather short periods of time (roughly 1.4 quarters on average). Figure 7 displays the probability with which zero nominal rates occur for  $n$  quarters, conditional

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<sup>24</sup>Promised inflation and output gaps are positive as long as  $-\gamma_t^1 + \frac{\sigma}{\beta}\gamma_t^2 > 0$  and  $\gamma_t^2 > 0$ , respectively, see section 5.1.

<sup>25</sup>Interestingly, the Bank of Japan recently announced explicit macroeconomic conditions that have to be fulfilled before it may consider abandoning its current zero interest rate policy.



on the nominal rate being at zero in quarter one. The likelihood that zero nominal rates persist for more than 4 quarters is merely 1.8%.

Since the lower bound is reached rather infrequently, possible biases for average output and inflation emerging from the nonlinear policy functions are expected to be small. In fact, our simulations show that for the calibration at hand there are virtually no average level effects for these variables.<sup>26</sup>

Finally, as one would expect, the average welfare effects generated by considering a zero lower bound in the model are rather small. The additional welfare losses of the zero lower bound are roughly 1% of those generated by the stickiness of prices alone.<sup>27</sup> Since the zero lower bound is reached rather infrequently, the conditional welfare losses associated with being at the lower bound can nevertheless be quite substantial.

## 6.4 Global Implications of Binding Shocks

This section reports a qualitatively new finding that stems from the presence of negative real rate shocks leading to zero nominal rates. It turns out that these binding shocks alter the optimal policy response to non-binding shocks, i.e., positive real rate shocks and mark-up shocks of both signs. In this sense, the existence of a lower bound has ‘global’ implications on the shape of the optimal policy functions.

For the baseline parameterization of the U.S. economy given in table 1, however, these global effects are quite weak, since the lower bound is reached rather infrequently. To illustrate the global effects, we thus assume in this section that the variance  $\sigma_g^2$  of the innovations  $\varepsilon_{g,t}$  is threefold that implied by the baseline calibration.<sup>28</sup>

Figure 8 illustrates the mean response of the real rate to a  $\pm 3$  standard deviation real rate shock under optimal policy. The upper panel shows

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<sup>26</sup>Average output and inflation deviate less than 0.01% from their steady state values.

<sup>27</sup>We compute unconditional welfare losses under optimal policy, evaluating objective (1), both with and without lower bound in the model. Welfare losses are obtained averaging the discounted losses across 1000 simulations, of the initial states  $(u_0, g_0, \mu_0^1, \mu_0^2)$  from their stationary distributions, each 1000 periods long.

<sup>28</sup>This value is roughly consistent with the estimated variability of real rate shocks in the period 1979:4-1995:2, i.e., the time span considered by Rotemberg and Woodford (1998). In fact, the unconditional variance of the real rate shocks for 1979:4-1995:2 is about 2.5-fold that for the period 1983:1-2002:4.

the case with lower bound and the lower panel depicts the case without bound in the model. While in the latter case the policy reaction is perfectly symmetric, accounting for the bound creates a sizeable asymmetry: the real rate reduction in response to a negative shock is much weaker than the corresponding increase in response to a positive shock.<sup>29</sup>

Equation (3), however, implies that the policymaker is unable to affect the average real rate in any stationary equilibrium.<sup>30</sup> Therefore, the less strong real rate decrease for a binding real rate shock has to be compensated with a less strong real rate increase (or a stronger real rate decrease) in response to other shocks. A close look at figure 8 reveals that this is indeed the case: the real rate increase with the lower bound falls slightly short of the one implemented without bound in the model.

Moreover, it is optimal to undo the asymmetry by trading-off across all shocks, e.g., also across mark-up shocks. This is illustrated in figure 9 which plots the economy’s mean response to  $\pm 3$  standard deviation mark-up shocks. The left-hand panel illustrates the response when the zero lower bound is considered and the right-hand panel depicts the case without bound in the model. Clearly, the mean reactions change considerably once the lower bound is accounted for. Real rates are now lowered more (increased less) in response to negative (positive) mark-up shocks. This is the case even though mark-up shocks do not lead to zero nominal rates.

## 7 Sensitivity Analysis

We now analyze the robustness of our findings to a number of variations in our baseline calibration. Particular attention is given to the sensitivity of the results to changes in the parameterization of the shock processes.

### 7.1 More Variable Shocks

We estimate the shock processes using data for a time period that most economists would consider to be relatively ‘calm’ especially when confronted

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<sup>29</sup>With negative shocks expected inflation has to be used to reduce the real interest rate which is a costly instrument in welfare terms.

<sup>30</sup>This can be seen by taking unconditional expectations of equation (3), imposing stationarity, and noting that  $E[g_t] = 0$ .

with the more ‘turbulent’ 1960s and 1970s. Since one cannot exclude that more turbulent times might lie ahead, it seems of interest to study the implications of optimal policy with more variable mark-up and real rate shocks. In this regard, this section considers the sensitivity of our findings to an increase of the shock variances  $\sigma_u^2$  and  $\sigma_g^2$  above the values in table 1.

Increasing the variance of mark-up shocks we find that the results are remarkably stable. This holds even if setting the variance of  $\sigma_u^2$  threefold above its estimated value. Average output and (annual) inflation are virtually unaffected. Moreover, zero nominal rates still occur with the same frequency and persistence as for the baseline parameterization of table 1.

The picture changes somewhat increasing the variance of real rate shocks. While average output remains virtually unaffected, average inflation and the average frequency and persistence of zero nominal rates do change, albeit to different extents. This is illustrated in the first three panels of figure 10, that show the implications of increasing the variance of real rate shocks up to threefold above that of the baseline calibration.<sup>31</sup> Average inflation and the average persistence of zero nominal rates change only in minor ways. Instead, as real rate shocks become more variable, zero nominal rates occur much more often.

Moreover, as can be observed in the lowest panel of figure 10, the additional welfare losses generated by considering the zero lower bound in the model increase markedly with the variance of the real rate shock process. While for the baseline calibration the additional average losses of the zero lower bound over and above those generated by the stickiness of prices is in the order of 1%, once the variance of real rate innovations is threefold the additional losses surge to roughly 33%. This shows that the welfare effects of the zero lower bound are fairly sensitive to the variance of the assumed real rate process.

One should note that the effects of the variability of shocks on the average level of output and inflation differ considerably from those reported in earlier contributions. Uhlig (2000), e.g., reports negative level effects for both variables when analyzing optimal policy in a backward-looking model. Clearly,

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<sup>31</sup>See footnote 28.

the gains from promising positive values of future output and inflation cannot show up in a backward-looking model. Similarly, Coenen, Orphanides and Wieland (2004) report negative level effects for a forward-looking model considering Taylor-type interest rate rules, rather than optimal policy as in this paper. Moreover, unlike suggested by Summers (1991), our results do not justify that it is necessary to target positive inflation rates so as to safeguard the economy against hitting the zero lower bound.

## 7.2 Lower Interest Rate Elasticity of Output

Our benchmark calibration of table 1 assumes an interest rate elasticity of output of  $\sigma = 6.25$ , which seems to lie on the high side for plausible estimates of the intertemporal elasticity of substitution.<sup>32</sup> Therefore, we also consider the case  $\sigma = 1$  that corresponds to log utility in consumption, and constitutes the usual benchmark parameterization in the real business cycle literature. Table 2 presents the parameters values implied by assuming  $\sigma = 1$  instead of  $\sigma = 6.25$ . Note that the values of  $\kappa$  and  $\lambda$  are also changed, as they depend on the intertemporal elasticity of substitution.<sup>33</sup> To estimate the shock processes, we follow the same procedure as for the baseline calibration. Details of the estimation are given in appendix A.4.

Overall, our findings seem robust to the change in the intertemporal elasticity of substitution. In particular, the level effects on average output and inflation remain negligible. Moreover, required inflation in response to a negative 3 standard deviation real rate shock is still in the order of 15 basis points annually. Even more importantly, the additional welfare losses generated by considering the zero bound in the model are rather small and in the order of less than one-half percent of the losses generated by the stickiness of prices alone.

Respect to the baseline, however, the lower bound is reached more frequently, namely in about one quarter every 5 years on average. Zero nominal rates occur more often because the variance of the real rate shock process implied by the parameterization in table 2 is about 45% higher than in our

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<sup>32</sup>As argued by Woodford (2003), a high elasticity value may capture non-modeled interest-rate-sensitive investment demand.

<sup>33</sup>See equations (2.19) and (2.22) in chapter 6 of Woodford (2003).

baseline.<sup>34</sup> However, binding shocks now generate lower additional welfare losses: the steeper slope  $\kappa$  of the Phillips curve implies that inflation reacts more strongly to output. As a result, the required amount of inflation can be generated with less positive output gaps, implying lower welfare losses.

## 8 Conclusions

In this paper we determine optimal monetary policy under commitment taking directly into account the zero lower bound on nominal interest rates and assess its quantitative importance for the U.S. economy. One of the main findings is that, given the historical properties of the estimated shock processes for the U.S. economy, the zero lower bound seems neither to impose large constraints on optimal monetary policy nor to generate large additional welfare losses. Furthermore, we show that the existence of the zero lower bound might require to lower nominal interest rates more aggressively in response to adverse shocks than what is suggested instead by a model without lower bound.

Our findings raise a number of further issues. First, the omission of fiscal policy clearly constitutes a shortcoming; the study of the potential role of fiscal policy in ameliorating adverse welfare effects entailed by the lower bound seems to be of interest. Second, given the widespread belief among academics and practitioners that lagged inflation is a major determinant of inflation, an issue that should be addressed is the robustness of our findings to the introduction of lagged inflation in the Phillips curve.

Finally, the central bank's credibility is key to our results. The use of expected inflation is unavailable to a discretionary policymaker, as there is no incentive to implement promised inflation *ex-post*. As a result, the case for preemptive easing is even stronger with discretionary policy making because the use of current interest rates is the only available policy instrument. Consequently, the zero lower bound on nominal interest rates is binding more often and generates considerably higher welfare losses. This is shown in a companion paper, see Adam and Billi (2004), where we analyze the implications of relaxing the assumption of policy commitment.

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<sup>34</sup>Mark-up shocks also play a less marginal role, a negative shock in the order of 4 standard deviations now leads to zero nominal rates.

## A Appendix

### A.1 Identification of historical shocks (baseline calibration)

To identify the historical shock processes we apply the procedure of Rotemberg and Woodford (1998). In particular, we first construct output and inflation expectations by estimating expectation functions from the data. We plug these expectations along with actual values of the output gap and inflation into equations (2) and (3), then identify the shocks  $u_t$  and  $g_t$  with the equation residuals.

We measure the output gap by linearly detrended log real GDP, and inflation by the log quarterly difference of the implicit deflator.<sup>35</sup> Using quadratically detrended GDP or HP(1600)-filtered GDP leaves the estimated parameters of the shock processes virtually unchanged. Detrended output is depicted in figure 1. For the interest rate we use the quarterly average of the fed funds rate in deviation from the average real rate for the whole sample, which is approximately equal to 3.5% (in annual terms). Based on this latter estimate we can set the quarterly discount factor as shown in table 1.

The expectation terms in equations (2) and (3) are constructed from the predictions of an unconstrained VAR in output, inflation, and the fed funds rate with three lags. Estimating expectation functions in such a way is justified as long as there are no structural breaks in the economy. Since our sample period, 1983:1-2002:4, starts after the disinflation policy under Federal Reserve Chairman Paul Volcker, monetary policy is expected to have been reasonably stable, see Clarida et al. (2000). A VAR lag order selection test based on the Akaike information criterion for a maximum of 6 lags suggests the inclusion of 3 lags. A Wald lag exclusion test indicates that the third lags are jointly significant at the 2% level. The correlations of the VAR residuals are depicted in figure 2. Substituting the expectations in equations (2) and (3) with the VAR predictions one can identify the shocks  $u_t$  and  $g_t$ . The implied shock series are shown in figure 3.

Fitting univariate AR(1) processes to these shocks delivers the following

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<sup>35</sup>The data is taken from the web site of the Bureau of Economic Analysis: [www.bea.gov](http://www.bea.gov).

OLS estimates:

$$\rho_u = 0.129 \quad (0.113)$$

$$\rho_g = 0.919 \quad (0.050)$$

$$\sigma_u = 0.153$$

$$\sigma_g = 1.091$$

where numbers in brackets indicate standard errors. A univariate AR(1) describes the shock processes  $u_t$  and  $g_t$  quite well. In particular, there is no significant autocorrelation left in the innovations  $\varepsilon_{i,t}$  ( $i = u, g$ ). Moreover, when estimating AR(2) processes the additional lags remain insignificant.

The estimated value of  $\rho_u$  is insignificant at conventional significance levels. For this reason we set  $\rho_u = 0$  and let the standard deviation of the innovations  $\varepsilon_{u,t}$  match the standard deviation of the identified mark-up shocks, which is approximately equal to 0.61% annually.

Although real rate shocks seem quite persistent, the persistence drops considerably once one uses actual future values to identify output and inflation expectations in equations (2) and (3), which amounts to assuming perfect foresight. The estimated autoregressive coefficient for the real rate shocks then drops to  $\rho_g = 0.794$ , indicating that better forecasts than our simple VAR-predictions would most likely lead to a reduction in the estimated persistence. Moreover, when using VAR-predictions but considering the period 1979:4-1995:2, as in Rotemberg and Woodford (1998), the point estimate falls to  $\rho_g = 0.827$ . For these reasons we set  $\rho_g = 0.8$  in our calibration.<sup>36</sup> The standard deviation  $\sigma_g$  of the innovations  $\varepsilon_{g,t}$  in table 1 equates the unconditional standard deviation of the calibrated real shock process to the standard deviation of the identified shock values.

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<sup>36</sup>This value cannot be rejected at the 1% significance level when using estimates based on the VAR-expectations. In an earlier version of this paper, which is available upon request, we used instead the point estimates for  $\rho_u$  and  $\rho_g$ .

## A.2 Derivation of the targeting rule

Let  $\mathcal{L}$  denote the Lagrangian stated in equation (9). The first-order conditions with respect to  $(y_t, \pi_t, i_t)$  are then

$$\frac{\partial \mathcal{L}}{\partial y_t} = -2\lambda y_t - \kappa \gamma_t^1 - \gamma_t^2 + \frac{1}{\beta} \mu_t^2 = 0 \quad (15)$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = -2\pi_t + \gamma_t^1 - \mu_t^1 + \frac{\sigma}{\beta} \mu_t^2 = 0 \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial i_t} \cdot (i_t + r^*) = \gamma_t^2 \cdot (i_t + r^*) = 0, \quad \gamma_t^2 \geq 0, \quad i_t \geq -r^* \quad (17)$$

Eliminating Lagrange multipliers  $(\gamma_t^1, \gamma_t^2)$  delivers the optimality condition (13) stated in the text. If  $i_j > -r^*$  for all  $j \leq t$ , from (17) it follows that  $\gamma_j^2 = \mu_{j+1}^2 = 0$  ( $j \leq t$ ). Equation (15) for period  $t - 1$  then implies  $\frac{\kappa}{2\lambda} \mu_t^1 = -y_{t-1}$  and (13) simplifies to (14).

## A.3 Numerical algorithm

We use the collocation method to numerically approximate the value function solving the generalized Bellman equation (12) and obtain the corresponding optimal policy functions.<sup>37</sup> Kato and Nishiyama (2005) in earlier work use the collocation method for solving a standard Bellman equation of a backward looking model.

We discretize the state space  $S \equiv (\mu^1, \mu^2, u, g) \subset R^4$  into a set of  $N$  collocation nodes  $\aleph = \{s_n | n = 1, \dots, N\}$ , where  $s_n \in S$ . One then interpolates the value function over these collocation nodes by choosing basis coefficients  $c_n$  ( $n = 1, \dots, N$ ) such that

$$W(s_n) \approx \sum_{n=1, \dots, N} c_n \zeta(s_n) \quad (18)$$

at each node  $s_n \in \aleph$ , where  $\zeta(\cdot)$  is a four dimensional cubic spline function. Equation (18) is an approximation to the left-hand side of (12). Then to evaluate the right-hand side of (12) one has to approximate the expected value  $EW(t(s_n, x_1, x_2, \varepsilon))$ , where  $t(\cdot)$  denotes the state transition function,  $x_1 = (\gamma^1, \gamma^2)$  and  $x_2 = (y, \pi, i)$  are the vectors of controls, and  $\varepsilon = (\varepsilon_u, \varepsilon_g)$  are the multivariate normal innovations of the shock processes. Assuming

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<sup>37</sup>See chapter 11 in Judd (1998) and chapters 6 and 9 in Miranda and Fackler (2002) for more detailed expositions.



normality of the innovations, the expected value function can be approximated by Gaussian-Hermite quadrature, which involves discretizing the shock distribution into a set of quadrature nodes  $\varepsilon_m$  and associated probability weights  $\omega_m$  ( $m = 1, \dots, M$ ).<sup>38</sup>

Substituting the collocation equation (18) for the value function  $W(t(\cdot))$ , the right-hand side of (12),  $RHS_c(\cdot)$ , can be approximated as

$$\begin{aligned} RHS_c(s_n) \approx & \inf_{x_1} \sup_{x_2} \{h(s_n, x_1, x_2) \\ & + \beta \sum_{m=1, \dots, M} \sum_{n=1, \dots, N} \omega_m c_n \zeta_j(t(s_n, x_1, x_2, \varepsilon_m))\} \end{aligned} \quad (19)$$

at each node  $s_n \in \aleph$ . The minimization/maximization problem (19) can be implemented using standard Newton methods, taking into account that  $i \geq -r^*$ . This delivers  $RHS_c(\cdot)$  and the policy functions  $x_{1c}(\cdot)$  and  $x_{2c}(\cdot)$  at the collocation nodes. Using the collocation technique one can then approximate  $RHS_c(\cdot)$  by a new set of basis coefficients  $c'_n$  ( $n = 1, \dots, N$ ) such that

$$RHS_c(s_n) = \sum_{n=1, \dots, N} c'_n \zeta(s_n) \quad (20)$$

at each node  $s_n \in \aleph$ .

Equations (18), (19), and (20) together define the iteration

$$c \rightarrow \Phi(c) \quad (21)$$

where  $c$  is the initial vector of basis coefficients in (18) and  $\Phi(c)$  the vector of basis coefficient  $c'$  in (20). The fixed point of equation (12) satisfies  $c^* = \Phi(c^*)$ . To solve for this fixed point the algorithm proceeds as follows:

- Step 1 Choose the degree of approximation  $N$  and  $M$ , and collocation and quadrature nodes. Guess an initial basis coefficient vector  $c^0$ .
- Step 2 Iterate on equation (21) and update the basis function coefficient vector  $c^k$  to  $c^{k+1}$ .
- Step 3 Stop if  $|c^{k+1} - c^k|_{\max} < \tau$ , where  $\tau > 0$  is a tolerance level and  $|\cdot|_{\max}$  denotes the maximum absolute norm. Otherwise repeat step 2.

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<sup>38</sup>See chapter 7 in Judd (1998) for details.

Once convergence is achieved and the (approximate) fixed point  $c^*$  is found, one needs to assess the accuracy of the solution. Define the residual function

$$R_{c^*}(s) = RHS_{c^*}(s) - \sum_{n=1, \dots, N^\varepsilon} c_n^* \zeta(s)$$

where  $s \in \mathbb{N}^\varepsilon = \{s_n | n = 1, \dots, N^\varepsilon\}$  is a grid of nodes for which  $\mathbb{N}^\varepsilon \cap \mathbb{N} = \emptyset$ . Then compute the maximum absolute approximation error

$$e^{abs} = \max_{s \in \mathbb{N}^\varepsilon} |R_{c^*}(s)|$$

and the maximum relative approximation error

$$e^{rel} = \max_{s \in \mathbb{N}^\varepsilon} \left| R_{c^*}(s) / \sum_{n=1, \dots, N^\varepsilon} c_n^* \zeta(s) \right|$$

For the baseline parameterization we set  $N = 6875$  and  $M = 9$ , with relatively more nodes placed into the area of the state space where the policy functions display kinks. The support of the discretization is chosen so as to cover  $\pm 4$  unconditional standard deviations of the exogenous states  $u$  and  $g$ , and to insure that in a long simulation of  $10^6$  periods all values for  $\mu^1$  and  $\mu^2$  fall inside the chosen support. Since the latter can only be verified ex-post, i.e., after having obtained the solution, some experimentation is necessary.

Our initial guess for  $c^0$  is consistent with the solution of the problem without lower bound. The tolerance level is set to  $\tau = 1.49 \cdot 10^{-8}$ , i.e., the square root of machine precision. Convergence is reached after about 39 hours on a Pentium IV with 2.6 GHz. The approximation errors are  $e^{abs} = 0.0021$  and  $e^{rel} = 0.0027$ , where  $\mathbb{N}^\varepsilon$  contained more than 75000 nodes.

#### A.4 Identification of historical shocks (RBC calibration)

We re-estimated the shock processes using the parameters from table 2 together with VAR-based expectations, following the procedure described in appendix A.1. The autocorrelation coefficient for the mark-up shocks now turns out to be statistically significant at the 1% level. Therefore, in table 2 we set  $\rho_u$  equal to its point estimate. The point estimate (standard deviation) of the autocorrelation of the real rate shocks is now  $\rho_g = 0.882$  (0.059). Since we still cannot reject  $\rho_g = 0.8$  at conventional significance levels, we

keep this value of the baseline parameterization. As before, the standard deviation  $\sigma_g$  of the innovation  $\varepsilon_{g,t}$  is chosen so as to match the standard deviation of the estimated real rate shocks.

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Parameter	Economic interpretation	Assigned value
$\beta$	quarterly discount factor	$\left(1 + \frac{3.5\%}{4}\right)^{-1} \approx 0.9913$
$\sigma$	real rate elasticity of output	6.25
$\alpha$	share of firms keeping prices fixed	0.66
$\theta$	price elasticity of demand	7.66
$\omega$	elasticity of firms' marginal costs	0.47
$\kappa$	slope of the Phillips curve	0.024
$\lambda$	weight on output in the loss function	$\frac{0.048}{4^2} = 0.003$
$\rho_u$	AR-coefficient mark-up shocks	0
$\rho_g$	AR-coefficient real rate shocks	0.8
$\sigma_u$	s.d. mark-up shock innovations (quarterly %)	0.154
$\sigma_g$	s.d. real rate shock innovations (quarterly %)	1.524

Table 1: Parameter values (baseline calibration)

Parameter	Economic interpretation	Assigned value
$\beta$	quarterly discount factor	$\left(1 + \frac{3.5\%}{4}\right)^{-1} \approx 0.9913$
$\sigma$	real rate elasticity of output	1
$\alpha$	share of firms keeping prices fixed	0.66
$\theta$	price elasticity of demand	7.66
$\omega$	elasticity of firms' marginal costs	0.47
$\kappa$	slope of the Phillips curve	0.057
$\lambda$	weight on output in the loss function	0.007
$\rho_u$	AR-coefficient mark-up shocks	0.36
$\rho_g$	AR-coefficient real rate shocks	0.8
$\sigma_u$	s.d. mark-up shock innovations (quarterly %)	0.171
$\sigma_g$	s.d. real rate shock innovations (quarterly %)	0.294

Table 2: Parameter values (RBC calibration)

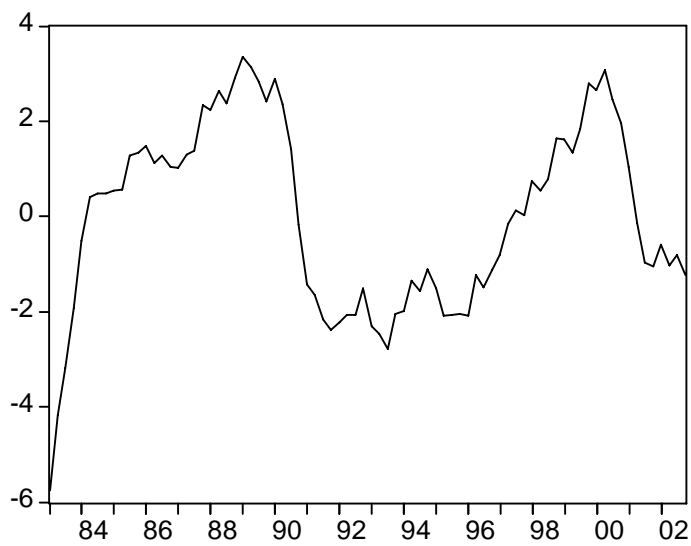


Figure 1: Detrended U.S. output



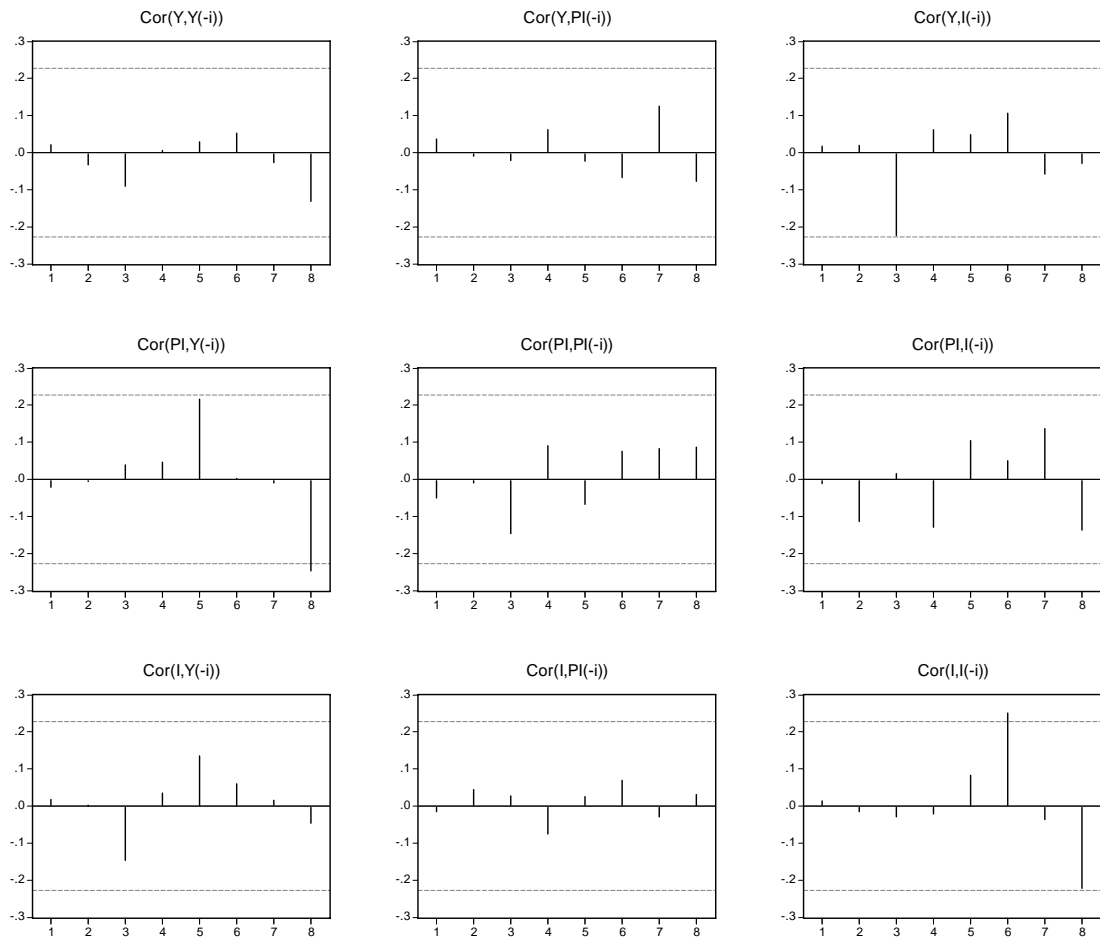


Figure 2: Residual autocorrelations with 2 s.d. error bounds for an unrestricted VAR in GDP, inflation, and fed funds rate

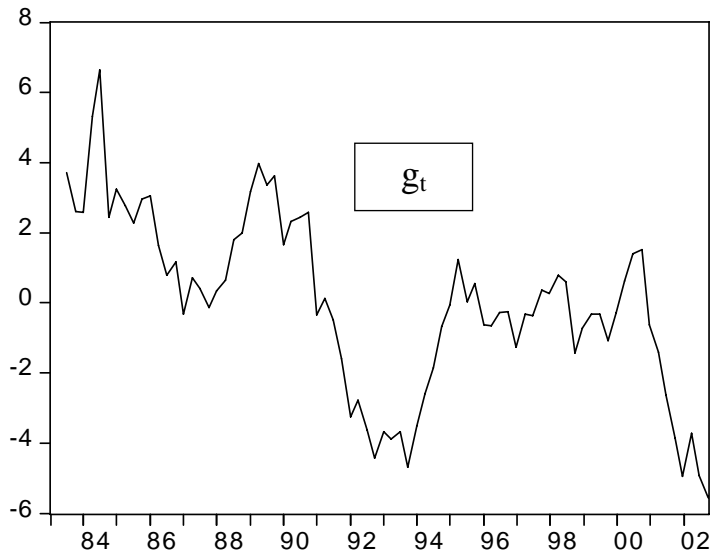
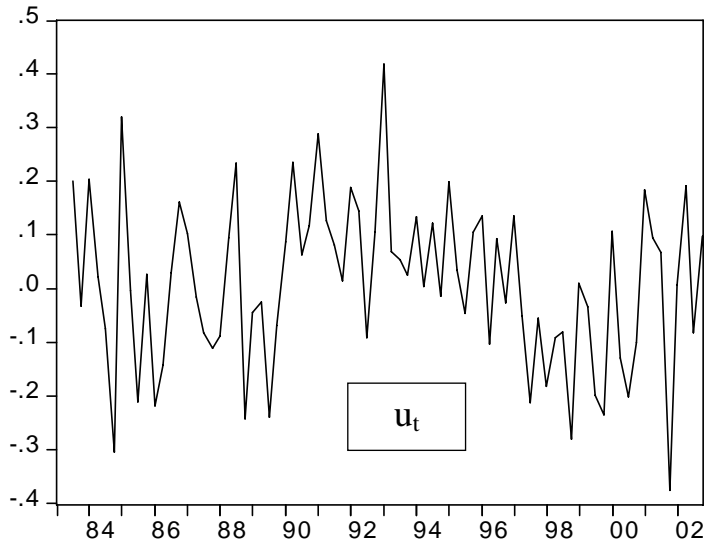


Figure 3: Identified shock processes

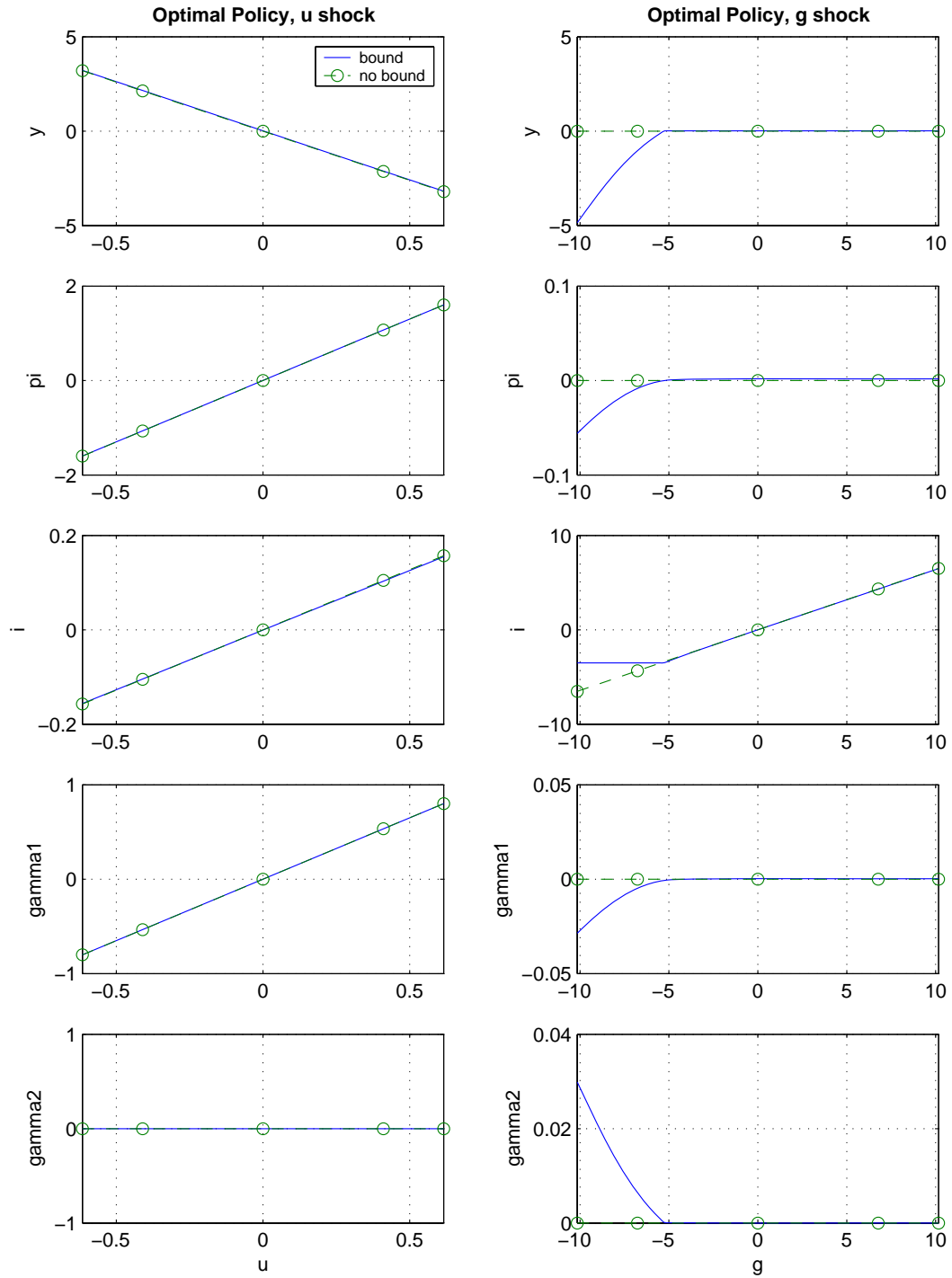


Figure 4: Optimal policy responses (baseline calibration)

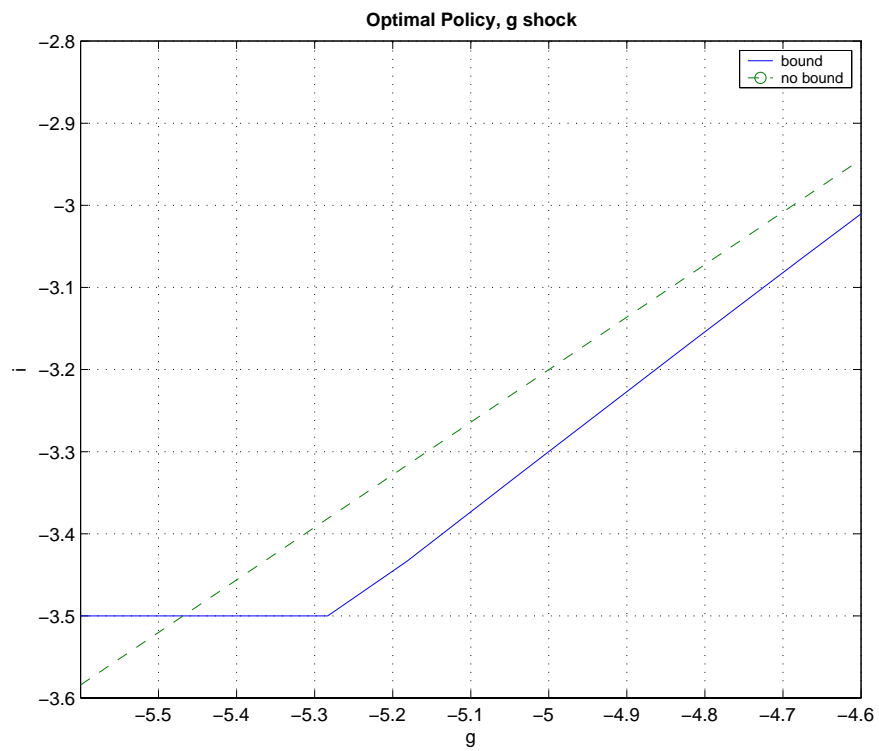


Figure 5: More aggressive easing with lower bound (baseline calibration)

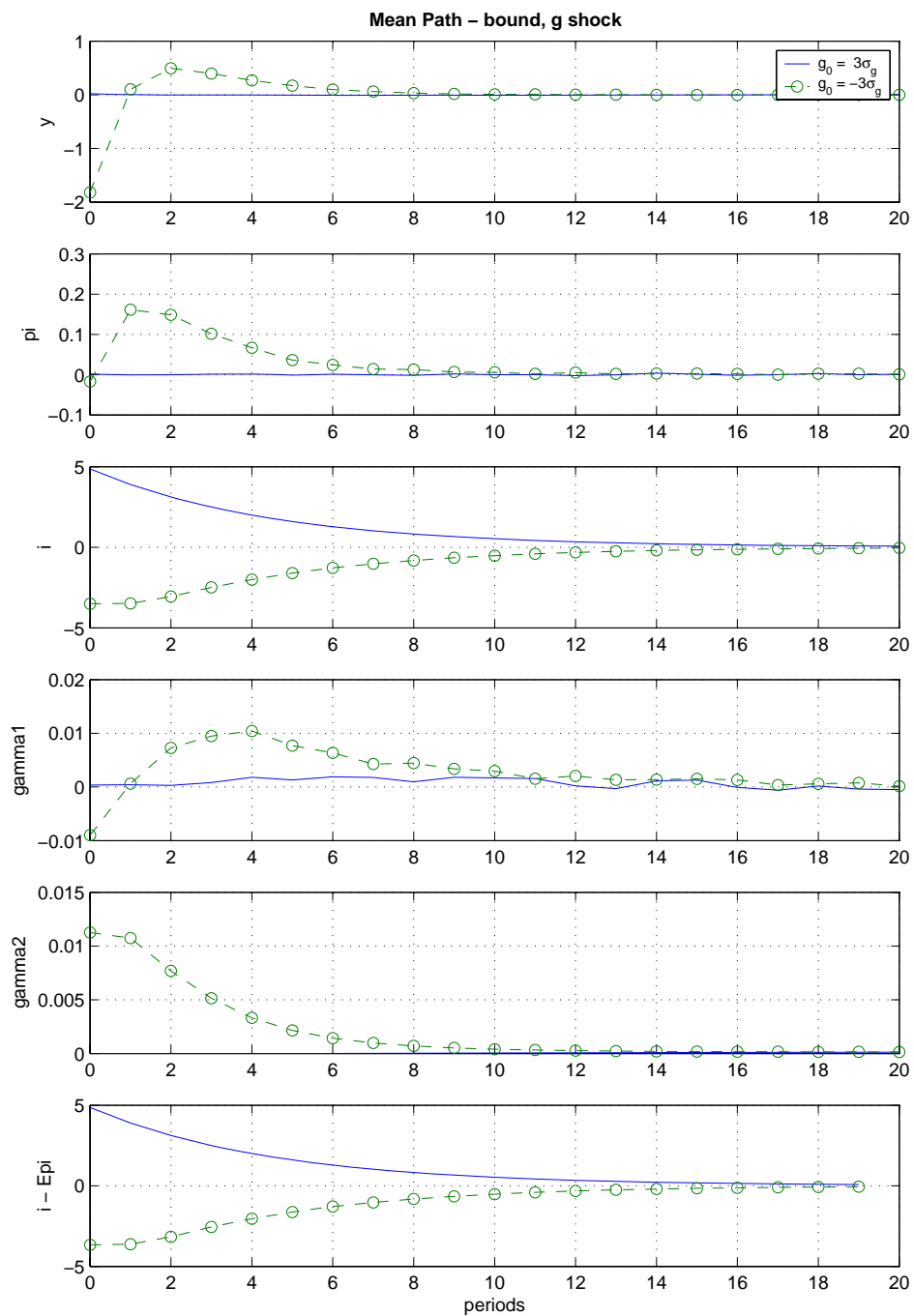


Figure 6: Mean response to  $\pm 3$  s.d. real rate shocks (baseline calibration)

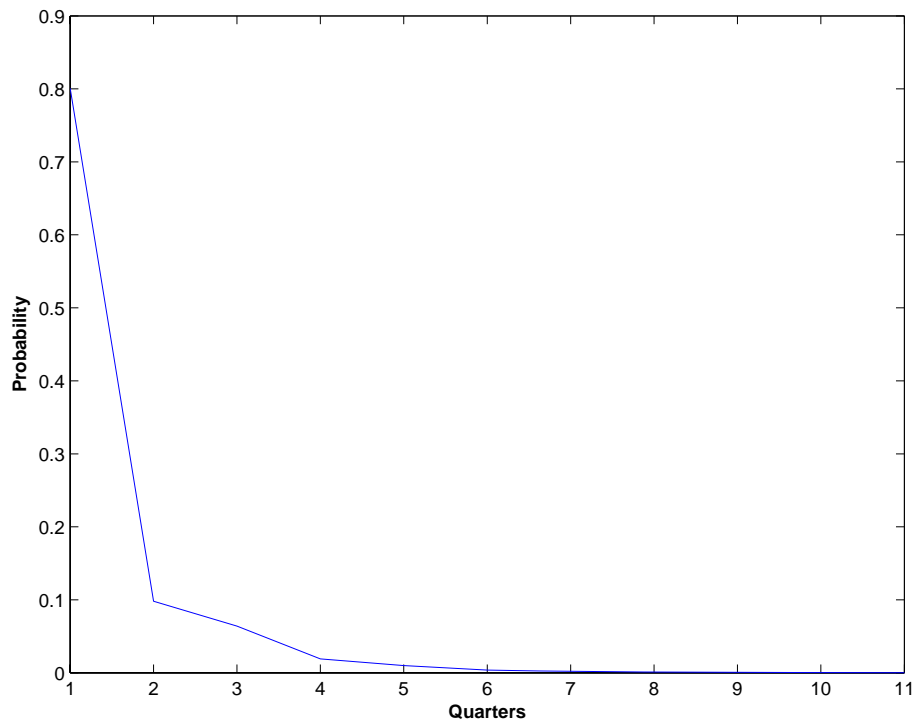


Figure 7: Persistence of zero interest rates (baseline calibration)

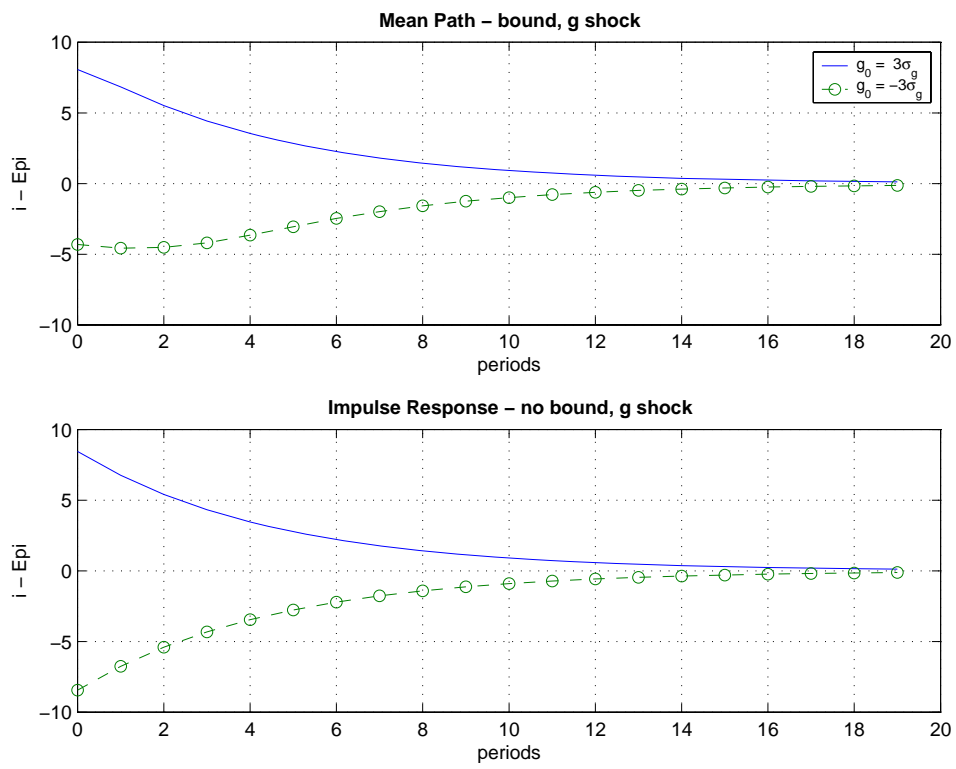


Figure 8: Asymmetric real rate response with lower bound (3-fold variance of real rate shocks)

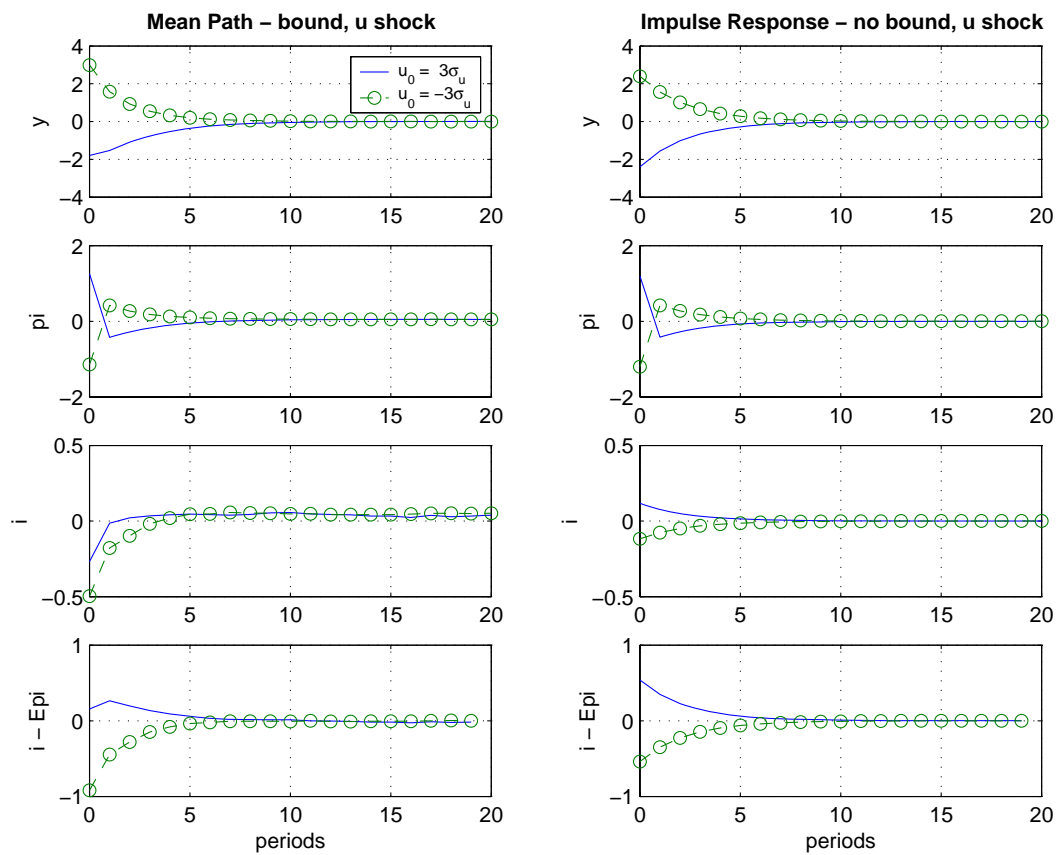


Figure 9: Mean response to  $\pm 3$  s.d. mark-up shock (3-fold variance of real rate shocks)



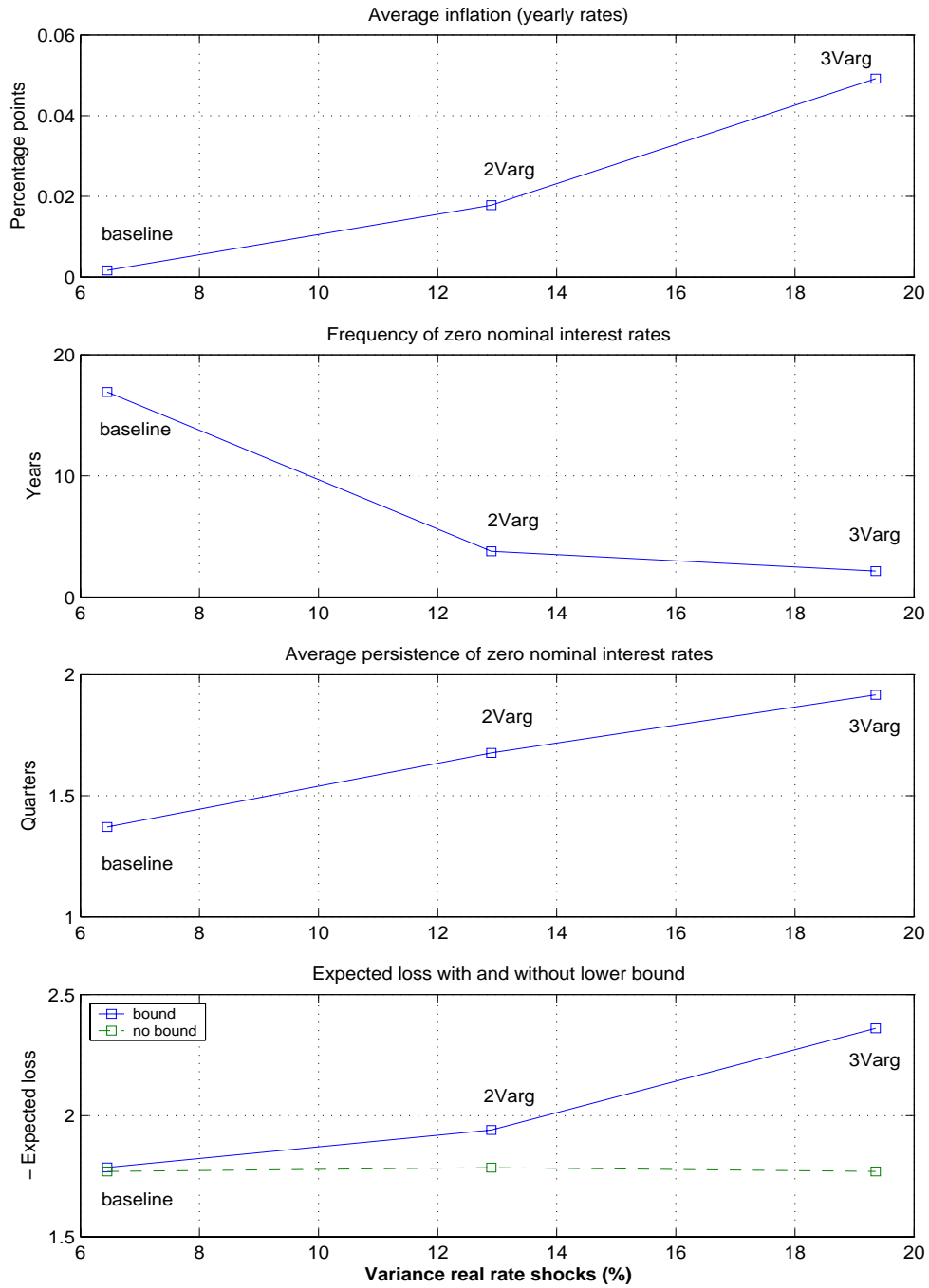


Figure 10: Sensitivity to the variance of real rate shocks