ISSN 1936-5330

THE FEDERAL RESERVE BANK *of* KANSAS CITY
ECONOMIC RESEARCH DEPARTMENT

Consumption Amenities and City Population Density

Jordan Rappaport

Revised as of: January 2008

RWP 06-10

RESEARCH WORKING PAPERS

Consumption Amenities and City Population Density

Jordan Rappaport

August 2006: Revised January 2008
(Previously titled: Consumption Amenities and City Crowdedness

RWP 06-10

Abstract: Population density varies widely among U.S. metro areas. A simple, static general equilibrium model demonstrates that moderate differences in metro areas' consumption amenities can cause extremely large differences in their population density. Such amenities are more strongly capitalized into housing prices than into wages. Empirical results suggest that amenities do indeed help to support high density levels and that amenities are becoming a more important determinant of where people choose to live. Matching the empirical correlation between wages and density requires that amenities cause approximately one fifth of the cross-sectional variation in metro population density.

Keywords: Population density, consumption amenities, quality of life, productivity, urban agglomeration

JEL classification: R00, J00, I31

Federal Reserve Bank of Kansas City; 925 Grand Blvd., Kansas City, MO 64198. 816-881-2018 (fax 816-881-2199) Email: jordan.rappaport@kc.frb.org. The views expressed herein are solely those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

1 Introduction

Population density, or "crowdedness," varies hugely across U.S. cities. Among metropolitan areas with a population of at least 100,000 in 2000, the most crowded (New York City) had a population density forty-nine times that of the least crowded (Dothan, Alabama). The second-most crowded (Los Angeles) had a population density twenty times that of the least crowded. Moderate differences in metro areas' total factor productivity can account for this variation (Rappaport, forthcoming). Can plausible differences in metro areas' consumption amenities do so as well? More generally, how important is quality of life relative to productivity in accounting for the distribution of population density across metro areas? And to what extent are amenities capitalized into housing prices versus into wages?

To answer these questions, the present paper lays out and calibrates a simple, static general equilibrium model of city density. Homogenous individuals choose to live and work in one of two local economies. They derive utility from consumption of a traded good, housing, leisure, and a consumption amenity. Firms in each economy produce the traded good and housing using land, capital, and labor. The level of consumption amenities varies exogenously between the two economies. In equilibrium, each economy must offer individuals the same level of utility and provide capital with the same rate of return. The model is a generalization of Rappaport (forthcoming) and is similar to models in Henderson (1974, 1987, 1988), Haurin (1980), Upton (1981), and Haughwout and Inman (2001). The model's equilibrium embeds the compensation for quality-of-life differences that forms the basis of empirical work in Rosen (1979), Roback (1982), Blomquist et al. (1988), Gyourko and Tracy (1989, 1991), Gabriel and Rosenthal (2004), and Chen and Rosenthal (2006).

The paper's methodology combines some of the quantitative elements of the dynamic stochastic general equilibrium (DSGE) literature inaugurated by Kydland and Prescott (1982) with the qualitative elements that characterize more theoretical work. The starting point is the assumption of a structural model, including specific functional forms. As with all theoretical work, the model abstracts from numerous characteristics of potential first-order importance. The functional forms are parameterized using a combination of microeconomic estimation results, national aggregate first moments, and correlations among metro-area aggregate variables. The parameterization includes the choice of baseline values as well as a wide range around these with which to conduct a sensitivity analysis. The sensitivity analysis helps to uncover the workings of the model by characterizing the role of each parameter. In contrast to much of the DSGE literature, which uses data to evaluate the parameterized model, the present paper uses the parameterized model to better understand the data. Could observed outcomes have been generated by processes similar to those in the model? If the model is true, how important is one process relative to another?

The paper finds that plausible differences in consumption amenities can indeed cause the observed large differences in density. Under a baseline calibration, a difference in quality of life valued at 30 percent of average consumption expenditure supports the twenty-fold observed difference in density between the second-most and least-crowded metropolitan areas. This difference in quality of life is within the range of several leading estimates. Much smaller exogenous quality-of-life differences can cause the observed differences in density in the presence of agglomeration economies. Empirical results are consistent with variations in consumption amenities being an important determinant of the distribution of density across metro areas. Density is strongly positively correlated with several subjective rankings of metropolitan-area quality of life. And population growth is strongly positively correlated with several measures of exogenous amenities. Matching an estimate of the correlation between wages and density suggests that variations in quality of life account for approximately one-fifth of the cross-sectional variation in density. The model suggests that high amenity levels are capitalized much more into higher housing prices than into lower wages. Low amenity levels, however, are capitalized approximately evenly into lower housing prices and higher wages.

The paper proceeds as follows. Section 2 describes the paper's empirical motivation: the wide variations in population density and perceived quality of life across U.S. metro areas. Sections 3 and 4 lay out the model and discuss its parameterization. Section 5 describes the model's numerical results, both for a baseline parameterization and for several large perturbations to it. It then discusses the implications of allowing productivity and quality of life to themselves endogenously depend on population density. Section 6 presents empirical results that suggest that variations in quality of life indeed help underpin variations in population density but that variations in productivity are a more important cause. A last section briefly concludes.

2 Empirical Motivation

Quantifying variations in density requires taking a stand on two issues. The first concerns the correct geographic unit to use to make comparisons. Metro areas are used herein because they best correspond to the local economies that are modeled. In particular, metro areas encompass a well-defined labor market in which people both live and work. The second issue concerns how to deal with the unequal distribution of population within any geographic unit. Raw population density—total population divided by total land area—is the most straightforward way to measure metro-area crowdedness. It describes average density as experienced by parcels of metro land. However, heterogeneous settlement patterns make using raw density problematic. Metro areas are constructed as the union of one or more whole U.S. counties. Often, large portions of such counties are primarily agricultural or unoccupied. Hence, the average density experienced by parcels of metro land can be considerably biased downward for the portion of the metro area where most people actually live.

Average density as experienced by residents is instead used herein to measure metro-area crowdedness. It is constructed as a population-weighted average of raw subunit densities (Glaeser and Kahn, 2004; Rappaport, forthcoming). More specifically, the Census Bureau partitions all U.S. counties into subdivisions. These subdivisions are then further partitioned into the portions of any municipalities that lie within them (many municipalities span multiple subdivisions) along with any remaining unincorporated area. In other words, a county subdivision may have a portion of municipality 1, a portion of municipality 2, as well as some unincorporated land. A neighboring county subdivision may have a different portion of municipality 1, which is treated as a separate observation from the portion in the first county subdivision.

The resulting population-weighted average density suggests that metro-area crowdedness in 2000 varied by a multiplicative factor of forty nine (Table 1). Unsurprisingly, the New York City metropolitan area had the highest density with 18.9 thousand persons per square mile (7.3 thousand per square kilometer). The next-most-crowded metro area, Los Angeles, had a weighted density less than half as large. Among people living in metropolitan areas with population of at least 100,000, the median density was experienced by those living in Omaha. That is, at least half of individuals experienced density greater than or equal to that of Omaha, and at least half experienced density less than or equal to that of Omaha.[1]

A second, harder-to-quantify, motivation is the perceived wide variation in quality of life across U.S. localities. Quality of life is meant to connote the direct contribution to utility from local consumption amenities. Equivalently, quality of life can be thought of as a local area's attractiveness to individuals as a place to live abstracting from expected wage and cost-of-living considerations.

An objective way to measure quality of life is to estimate compensating differentials (Rosen, 1979; Roback 1982). The value of a place's quality of life is inferred as the sum of the expected wage sacrifice and cost-of-living premium households accept to live there.

Top-twenty metro area rankings from two compensating differential studies, Blomquist et al. (1988) and Gyourko and Tracy (1991), are shown in Table 2 Panel A. Many of the top-ranked metro areas seem misplaced. For example, few would probably agree that Pueblo Colorado, a small city of approximately 100,000 that lies 40 miles south of Colorado Springs, is the nicest place to live in the United States. Similarly, Macon GA, Bighamton NY, Roanoke VA, Lackawanna PA, Tallahasse Fl, Shreveport LA, Lancaster PA, and Amarillo TX are unlikely to be among most people's choices for the nicest places to live in the United States. Conversely, many metro areas that are typically considered to have high quality of life are ranked poorly. Among 253 urban counties in 1980, Blomquist et al. rank San Francisco County number 105, neighboring Marin County number 142, and New York County (Manhattan) number 216. Among 130 large cities in 1980, Gyourko and Tracy rank Miami number 86, Seattle number 104, and Ann Arbor number 115. Numerous other apparent large misrankings are easily identified.

The most likely explanation for such misrankings is that the estimation of compensating differentials includes a serious omitted-variable bias (Gyourko, Tracy, and Kahn, 1999). In particular, it is difficult to distinguish whether observed differences in wages are attributable to locational compensation or to skill differences. A high-amenity metro area that attracted workers with unobserved high skills and observed high wages would incorrectly be inferred to have low quality of life.

An alternative approach to measuring quality of life is to grade localities using subjective

[1] An alternative measure of the variation in crowdedness, the raw population density of *municipalities* with population of at least 100,000 in 2000, shows a similar forty-five-fold multiplicative spread. Raw density of metro areas varied by a multiplicative factor of 437.

criteria. Top-twenty metro area rankings for two such studies are shown in Table 2 Panel B. Savageau (2000) ranks 327 continental U.S. metro areas in each of seven quality-of-life categories: transportation, education, climate, crime, the arts, healthcare, and recreation. Each of these categories, in turn, is divided into two or more subcategories that can be objectively measured. For example, the transportation category is constructed as a weighted average of daily commute time, public transit revenue-miles, passenger rail departures, interstate highway proximity, nonstop airline destinations, and proximity to other metro areas. The arts category is constructed as a weighted average of number of art museums, museum attendance, per-capita museum attendance, ballet performances, touring artist bookings, opera performances, professional theater performances, and symphony performances. An overall quality-of-life index is then constructed as a weighted average of scores in each of the seven categories. Sperling and Sander (2004) similarly rank 329 continental U.S. metro areas in eight quality-of-life categories.[2]

3 Model

The model uses a static, open-city framework. The world is made up of a national economy and a city economy. The former can be interpreted as the aggregate of numerous city economies. It establishes the reservation level of utility that the city economy must offer mobile individuals. In the parlance of international trade theory, the national economy is "large" and the city economy is "small". That is, outcomes in the national economy affect outcomes in the city economy but *not* vice versa. Results from modelling two interdependent economies would be qualitatively similar.

3.1 Individuals

Individuals derive utility from consumption of a traded good (x), housing (h), leisure, and consumption amenities ($quality$). The level of consumption amenities is assumed to vary exogenously between the two economies. Amenities thereby serve as the model's primary source of crowding.

[2] Savageau (2000) and Sperling and Sander (2004) also include job-opportunity and cost-of-living categories. The overall quality-of-life rankings used herein are recalculated to exclude these.

Let the constant-elasticity-of-substitution (CES) operator, $\sigma_{a,b}(\cdot)$, aggregate a and b according to elasticity and weighting parameters, $\sigma_{a,b}$ and $\eta_{a,b}$:

$$\sigma_{a,b}(a,b) \equiv \left(\eta_{a,b}\, a^{\frac{\sigma_{a,b}-1}{\sigma_{a,b}}} + \left(1-\eta_{a,b}\right) b^{\frac{\sigma_{a,b}-1}{\sigma_{a,b}}} \right)^{\frac{\sigma_{a,b}}{\sigma_{a,b}-1}}$$

Utility in each economy ($i = c, n$) takes a nested CES functional form,

$$U_i = \sigma_{xhl,q}(\, \sigma_{xh,l}(\, \sigma_{x,h}(x_i, h_i)\, ,\, leisure_i)\, ,\, quality_i) \tag{1a}$$

The innermost nesting in (1a) is between the traded good and housing. It has elasticity $\sigma_{x,h}$. The middle nesting is between the resulting traded-good-housing composite and leisure. It has elasticity $\sigma_{xh,l}$. The outermost nesting, between the traded-good-housing-leisure composite and quality of life, has elasticity $\sigma_{xhl,q}$. For each of the three nestings, the assumed calibration determines an associated weighting parameter, $\eta_{x,h}$, $\eta_{xh,l}$, or $\eta_{xhl,q}$.

In the special case where the elasticity of substitution is equal across nestings, (1a) simplifies to a standard CES functional form,

$$U_i = \left(\eta_x x_i^{\frac{\sigma-1}{\sigma}} + \eta_h h_i^{\frac{\sigma-1}{\sigma}} + \eta_l leisure_i^{\frac{\sigma-1}{\sigma}} + \eta_q quality_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{1b}$$

This specialization characterizes the baseline parameterization below. With a unitary elasticity of substitution, $\sigma = 1$, (1b) further reduces to Cobb Douglas utility.

Optimizing behavior by individuals equates the ratio of marginal utility to price *within* each economy. Individuals must each satisfy a budget constraint that their expenditure does not exceed their wage income plus any non-wage income. Except when specified otherwise, non-wage income is assumed to be zero. Perfect mobility by individuals equates utility levels between the city and national economies:[3]

$$U_c = U_n \tag{2}$$

Since quality of life has no natural units, it is measured by individuals' willingness to pay to receive $quality_c$ rather than $quality_n$. Let p_i be the price of housing in terms of the

[3]In the short run, mobility is unlikely to be perfect. Ciccone and Hall (1996), among others, have shown that even long-lagged levels of population are good predictors of current population. Rappaport (2004) shows that even very small frictions to labor and capital mobility imply long transitions following changes in productivity or quality of life. Hence the static equilibria described herein should be interpreted as long-run steady states.

traded good in economy i. Consider the minimum expenditure function required to obtain the national-economy level of utility at the national-economy wage-price vector, $\{w_n, p_n\}$. For present purposes, this expenditure is defined to include the opportunity cost of leisure.

$$\mathrm{e}(w_n, p_n, quality; U_n) \equiv \mathrm{Min}\,(x + p_n\, h + w_n\, leisure) \ \text{ s.t. } \mathrm{u}(x, h, leisure, quality) = U_n.$$

The compensating variation, CV, of $quality_c$ measures an individual's willingness to pay to receive it rather than $quality_n$. It is defined as the negative transfer required for a person facing $\{w_n, p_n, quality_c\}$ to achieve U_n. That is,

$$CV \equiv \mathrm{e}(w_n, p_n, quality_n; U_n) - \mathrm{e}(w_n, p_n, quality_c; U_n)$$

Note that CV is defined to be positive when $quality_c$ exceeds $quality_n$. A normalized measure, $\widetilde{CV}$, divides CV by actual national-economy expenditure, $(x_n + p_n h_n)$.

The compensating variation of $quality_c$ relative to $quality_n$ can differ significantly from the compensating differential, CD, of the same. Using national-economy quantities, CD equals $(p_c - p_n)\,h_n - (w_c - w_n)\,(1 - leisure_n)$ (Rosen, 1979; Roback, 1982). For $quality_c$ close to $quality_n$, CV and CD are approximately equal. But as quality of life in the two economies increasingly differs, CV and CD increasingly differ as well. When $quality_c$ is well above $quality_n$, CD can greatly exceed CV. When $quality_c$ is well below $quality_n$, CD can be much smaller than CV in absolute value. The differences arise primarily because the CD methodology assumes that individuals consume fixed quantities of housing and leisure and that their marginal utility from traded-good consumption is constant. CV, in contrast, allows for substitution and curvature. Under the baseline parameterization below, CV can differ from CD by as much as 15 percentage points.

3.2 Firms

Within each economy ($i = c, n$), perfectly competitive firms employ a constant-returns-to-scale production function that combines land, capital, and labor (D_i, K_i, and L_i) to produce a traded numeraire good and nontraded housing (X_i and H_i). Housing must be consumed in the economy in which it is produced. Aggregate production within each economy is given by

$$X_i = \mathrm{A}_{X,i}\, D_{X,i}^{\alpha_{X,D}}\, K_{X,i}^{\alpha_{X,K}}\, L_{yX,i}^{\alpha_{X,L}} \tag{3}$$

$$H_i = \mathrm{A}_{H,i}\left(\eta_{D,KL} D_{H,i}^{\frac{\sigma_{D,KL}-1}{\sigma_{D,KL}}} + \left(1-\eta_{D,KL}\right)\left(K_{H,i}^{\alpha_{H,K}} L_{H,i}^{\alpha_{H,L}}\right)^{\frac{\sigma_{D,KL}-1}{\sigma_{D,KL}}}\right)^{\frac{\sigma_{D,KL}}{\sigma_{D,KL}-1}} \tag{4}$$

Production of the traded good is Cobb Douglas. The associated assumption of constant factor income shares from traded-good production is consistent with the stylized fact of constant aggregate factor income shares, both cross sectionally and over time (Kaldor, 1961; Gollin, 2002; Willis and Wroblewski, 2007). Aggregate Cobb-Douglas production is also implied by weaker assumptions on firm-level production (Jones, 2005). The factor income share parameters are each assumed to be strictly positive, with $\alpha_{X,D} + \alpha_{X,K} + \alpha_{X,L} = 1$.

Production of housing is characterized by a constant elasticity of substitution between land and an implicit intermediate product of capital and labor. Allowing for a non-unitary elasticity of housing production with respect to land is motivated by numerous empirical studies including McDonald (1981) and Jackson, Johnson, and Kaserman (1984). Modelling the non-land contribution to housing as Cobb Douglas reflects the evidence of constant relative shares of capital and labor in aggregate production. The elasticity of substitution between land and the capital-labor intermediate good is given by $\sigma_{D,KL}$. The weighting parameter $\eta_{D,KL}$, which lies strictly between 0 and 1, calibrates the relative share of factor income accruing to land. The capital-labor intermediate hybrid good is produced with constant returns to scale, $\alpha_{H,K} + \alpha_{H,L} = 1$. These last two coefficients determine the division of factor income between capital and labor.

Total factor productivities, $\mathrm{A}_{X,i}$ and $\mathrm{A}_{H,i}$, are for the moment assumed to be fixed and equal across the two economies (i.e., $\mathrm{A}_{X,c} = \mathrm{A}_{X,n}$ and $\mathrm{A}_{H,c} = \mathrm{A}_{H,n}$). In a later section, productivity will be assumed to endogenously depend on local population density.

Capital and land factor payments are interpreted as being made to foreign owners, who reside neither in the national nor in the city economy. Numerical results are nearly identical if payments to capital and land are instead rebated to individuals on a lump sum basis.

Profit maximization by perfectly competitive firms induces demand such that each of the factors is paid its marginal revenue product. Frictionless intersectoral mobility assures intersectoral factor price equalization within each economy. Capital is additionally assumed to be perfectly mobile across economies, and so its return must be identical across both. Because the present framework is static, this identical capital rent is taken as exogenous. In a dynamic neoclassical framework, it would equal the real interest rate plus the rate of capital depreciation.

3.3 Closure

In addition to the profit and utility maximization conditions, several adding-up constraints must be met. For each of the economies, the land and labor factor markets and the housing market must clear. For the national economy, traded-good consumption must equal traded-good production less any payments to absentee land and capital owners.[4] Land area is predetermined for both economies. For the national economy, population is predetermined as well.

The combined optimization conditions, individual budget constraints, and local adding-up constraints can be reduced to two nonlinear systems of seven equations each. The first system is used to calibrate the utility and housing-production weighting parameters to achieve target national-economy outcomes. It also determines the reservation level of utility that must be met in the city economy. The second system then solves outcomes for the city economy including its population density. In the absence of any sort of increasing returns to scale, the fixed land supply and decreasing marginal utility together suggest that the solution to this combined system is unique.

4 Calibration

A primary purpose herein is to gauge the approximate magnitude of the differences in consumption amenities that are required to match the large observed differences in density across U.S. metro areas. In this spirit and to not imply a false level of precision, parameters are set to round values. The numerical results section, which follows, includes an extensive sensitivity analysis. Table 3 summarizes the calibration, including the sensitivity of results to variations in specific parameters.

[4]Traded-good consumption may differ from production in the city economy via trade with the national economy. Such trade does not relax the national economy's traded-good constraint because the city economy is too small to do so.

4.1 Utility

The calibration of the utility function, (1a), requires parameterizing the elasticities of substitution between the traded good and housing, between the resulting two-way composite and leisure, and between the resulting three-way composite and quality of life. In addition, weighting parameters need to be set that determine the national-economy share of consumption spent on housing and the national-economy share of time devoted to leisure.

The elasticity of substitution between the traded good and housing, $\sigma_{x,h}$, is assumed to equal 0.5. This calibration derives from cross-sectional data on housing prices and the housing share of consumption expenditures. The dots in Figure 1 Panel A plot the housing consumption share against housing prices for 24 large metro areas.[5] The lines represent housing expenditure shares for each of three elasticities of substitution.[6] The line for $\sigma_{x,h}$ equal to 0.50 almost exactly overlays the fitted relationship from a linear regression. This baseline value for $\sigma_{x,h}$ implies a price-elasticity of housing demand that is close to numerous estimates (Goodman, 1988, 2002; Ermisch, Findlay, and Gibb, 1996; Ioannides and Zabel, 2003). For the sensitivity analysis, $\sigma_{x,h}$ is assumed to equal 0.25 and 0.75.[7]

The elasticity of substitution between the traded-good-housing composite and leisure,

[5]The housing share measure is from the Consumer Expenditure Survey. The housing price measure is an index of the rental price of apartments in professionally-managed properties with five or more units. It is constructed by Torto Wheaton Research based on quarterly surveys. The index adjusts for the number of bedrooms per unit and a property's age, but not for other important characteristics such as square footage, parking, and location. Hence the index measures a hybrid of housing prices and housing expenditures. A correctly measured house price should result in a scatter more horizontal than is depicted in Figure 1, in turn implying a higher elasticity of substitution.

[6]For each elasticity, the weighting parameter $\eta_{x,h}$ is chosen so that the implied expenditure share passes through the fitted expenditure share for Pittsburgh from a linear regression of expenditure share on rental price. Pittsburgh's weighted density is close to the population median.

[7]Davis and Ortalo-Magne (2007) argue, based on the housing expenditure shares of renters across metro areas, that $\sigma_{x,h}$ is more likely to be close to 1. But renters are a non-random, minority sample facing a different choice set of housing units. And the inclusion of utility expenses in rental payments likely dampens variation across metro areas. The Consumer Expenditure Survey, the source of the current calibration, is based on both renter and owner expenditures. It calculates the latter as the sum of mortgage interest and charges, property taxes, insurance, maintenance, and repairs. It does *not* impute an equivalent rent, and so addresses a main concern of Davis and Ortalo-Magne. In any case, Davis and Ortalo-Magne's range of estimated housing shares, from 0.20 for Cincinnati up to 0.29 for Miami, is largely consistent with the present calibration.

$\sigma_{xh,leisure}$, is also assumed to equal 0.5. It is calibrated using time diary studies taken in 1965, 1975, 1985, 1993, and 2003 (Robinson and Godbey, 1997; Aguiar and Hurst 2006) and aggregate real wage data for each of these years. The bold line in Figure 1 Panel B plots the average share of weekly hours devoted to leisure by non-retired, working-age men. The remaining lines show optimal values corresponding to the real wage in each of the above years for $\sigma_{xh,leisure}$ equal to each of 0.25, 0.5, and 1.[8] Calibrating $\sigma_{xh,leisure}$ to equal 0.5 exactly matches the total increase in leisure from 1965 to 2003. The implied negative elasticity of labor hours with respect to the real wage is consistent with estimates summarized in Pencavel (1986). In contrast, more recent dynamic estimates, summarized in Blundell and MaCurdy (1999), typically find a positive elasticity of labor hours with respect to the real wage. However, it is difficult to map the intertemporal context of the more recent studies into the present static setting.

The elasticity parameter between the traded-good-housing-leisure composite and quality of life can be set arbitrarily. While $\sigma_{xhl,quality}$ does indeed affect the physical quantity of amenities individuals are willing to trade off, it does not affect the total valuation of those amenities. Different values of $\sigma_{xhl,quality}$ do imply different elasticities of endogenous variables, including population density, with respect to $quality_c$. But consumption amenities have no inherent quantity unit. The elasticities between the endogenous outcomes and the *compensating variation measure* of amenities is independent of $\sigma_{xhl,quality}$. To keep things simple, $\sigma_{xhl,quality}$ is set equal to 0.5, thereby reducing (1a) to its standard CES form, (1b).

Finally, the weighting parameters $\eta_{x,h}$, $\eta_{xh,leisure}$, and $\eta_{xhl,quality}$ need to be calibrated. For a given set of elasticities, $\eta_{x,h}$ is chosen such that national-economy individuals spend 18% of their consumption expenditures on housing. This approximately matches the aggregate U.S. value from 2001 to 2003 based on the Bureau of Labor Statistics' Consumer Expenditure Survey. The sensitivity analysis alternatively assumes national-economy housing expenditure shares of 14% and 22%. The parameter $\eta_{xh,leisure}$ is chosen such that national-economy individuals choose to spend 35% of their time on leisure. This matches the share for 2003 from the time diary studies. The sensitivity analysis alternatively assumes national-economy leisure shares of 20% and 50%. As will become clear, all numerical results are extremely

[8]Biological necessities are assumed to require 9 hours per day, which leaves 105 hours per week of potential leisure. The optimal values assume individuals have no non-wage income. For each elasticity, the weighting parameter $\eta_{xh,leisure}$ is chosen so that the expected leisure share for 1965 matches its actual value.

robust to these latter variations. The lack of natural units for quality of life makes the choice of $\eta_{xhl,quality}$ immaterial.

4.2 Production

The calibration of production requires determining the national-economy factor income share accruing to each of land, capital, and labor in the traded-good and housing sectors. For the housing sector, it also requires determining the elasticity of substitution between land and the capital-labor composite. The rate of return determining capital intensity also needs to be specified.

The land share of factor income derived from the production of the traded good is assumed to be 1.6%. This value is a weighted average across a large number of industries using intermediate input shares estimated by Jorgenson, Ho, and Stiroh (2005).[9] It is nearly identical to the 1.5% land share that Ciccone (2002) suggests is reasonable for the manufacturing sector. Sensitivity analysis is conducted for land factor shares equal to 0.4% and 4.8%. One third of remaining factor income is assumed to accrue to capital; two thirds are assumed to accrue to labor (Gollin, 2002). Because traded-good production is Cobb Douglas, the assumed factor shares hold in both economies.

Non-Cobb-Douglas production in the housing sector implies that factor income shares differ between the two economies. Under the baseline parameterization, land's national-economy factor share is set to 35%. This is slightly below a recent estimate that land accounts for approximately 39% of the implicit factor income attributable to aggregate U.S. housing stock (Davis and Heathcote, 2005).[10] Using microeconomic data, several other researchers have found substantially lower land shares. Based on houses sold in the Knoxville metro area, Jackson, Johnson, and Kaserman (1984) estimate that land accounts for 27% of implicit factor income. Based on houses constructed in new subdivisions of the Portland Oregon metro area, Thorsnes (1997) estimates that it accounts for 17%. But Knoxville is among the least densely populated U.S. metro areas. And new subdivisions tend to be

[9] The industry-specific intermediate input estimates, which are not included in the publication, were kindly provided by the authors.

[10] Davis and Heathcote find that between 1975 and 2004, land accounted for an average of 47% of the sales value of aggregate U.S. housing stock. Adjusting for the fact that structures depreciate but land does not, using the 1.6% rate of structure depreciation suggested by Davis and Heathcote and a 4% required real rate-of-return, gives a 38.8% land factor share.

located at the edges of metro areas. In both cases, land prices are likely to be below average. If the production elasticity of substitution with land is below one as assumed in the baseline calibration, land's factor share would be below average in such places as well. For the sensitivity analysis, the housing land factor share is assumed to equal 20% and 50%. As with traded-good production, one third of remaining factor income is assumed to accrue to capital; two thirds are assumed to accrue to labor.

The elasticity of substitution between land and non-land inputs, $\sigma_{D,KL}$, is assumed to be 0.75. No clear consensus exists on an appropriate value. A survey by McDonald (1981) reports preferred estimates from twelve different studies ranging from 0.36 to 1.13. Updating this research, Jackson, Johnson, and Kaserman (1984) estimate the elasticity to lie somewhere between 0.5 and 1. More recently, Thorsnes (1997) argues that a unitary elasticity of substitution cannot be rejected. For the sensitivity analysis, $\sigma_{D,KL}$ is assumed to equal 0.5 and 1.

Finally, the rent on the services of capital goods, r_K, is set to 0.08, which implicitly represents the sum of a required annual real return plus an annual allowance for depreciation. While some nominal quantities do depend on the parameterization of r_K, all results reported below are completely insensitive to it. This insensitivity reflects that the static model lacks a natural time context.

5 Numerical Results

The model's mechanics are straightforward. The national economy serves to calibrate the utility and production weighting parameters. It also determines the reservation level of utility that city-economy residents must attain. An increase in the city-economy's level of consumption amenities attracts an inflow of labor, putting downward pressure on wages and attracting a complementary inflow of capital. The increase in city-economy population dominates the lower wages to increase housing demand, which in turn puts upward pressure on land prices. Consumption of the traded good, housing, and leisure all fall.

The first subsection below illustrates these mechanics under the baseline calibration. Compensation for higher consumption amenities comes largely via higher housing prices. A second subsection shows how resistance to crowdedness (i.e., the magnitude of the quality-of-life difference required to attain a given increase in density) changes with variations to each of the model's main parameters. Resistance to crowdedness depends most closely on the calibration of the housing production function. A third subsection relaxes the assumptions that quality of life and productivity are completely exogenous. Allowing productivity to depend positively on density can considerably lower resistance to quality-of-life-driven crowding.

5.1 Baseline Calibration

Numerical results from the baseline calibration are shown in Figure 2. Panel A plots the city-economy relative population density (horizontal axis) that is caused by various normalized valuations of city-economy quality of life (vertical axis). In other words, the vertically-plotted $\widetilde{CV}$ should be interpreted as exogenous. The horizontally-plotted relative population density should be interpreted as an endogenous response. Equivalently, for any relative density, the depicted locus gives the required quality-of-life differential.

For example, the $quality_c$ that induces city-economy population density to be one-fourth that of the national economy has a $\widetilde{CV}$ of -0.13. In other words, national-economy residents facing the required $quality_c$ rather than $quality_n$ at the national-economy wage-price vector need a transfer equivalent to 13% of their original traded-good and housing consumption in order to continue to attain U_n. Conversely, the $\widetilde{CV}$ associated with a relative population density of four is 0.18. In this case, national-economy residents facing the required $quality_c$ rather than $quality_n$ at the national-economy wage-price vector could transfer away 18% of their original consumption while still attaining U_n. Whether amenity differences of these magnitudes are plausible is deferred until the second subsection below.

Notice that the required-$\widetilde{CV}$-to-density locus is asymmetric with respect to the origin. The negative $\widetilde{CV}$ required to support a fractional density is smaller in magnitude than the positive $\widetilde{CV}$ required to support the reciprocal multiple density. Equivalently, the $\widetilde{CV}$-to-density locus has a positive second derivative. This asymmetry reflects the increasing marginal cost of crowding as the marginal return to land in production and the marginal utilities from consumption of the traded good, leisure, and especially housing become extremely high.

The remaining panels of Figure 2 plot the relationships of a number of other endogenous outcomes against population density. The desire by individuals to live in high-quality-of-life locales induces an inverse correlation between the traded-good-denominated wage and population density (Panel B). At a one-fourth density, city-economy wages are 4.3% above those in the national economy; at a four-fold density, they are 4.3% below national-economy wages. This counterfactual negative relationship between wages and density will be discussed at length below. Relative land prices vary by an order of magnitude more than do wages (Panel C). They go from 0.18 to 6.1 as relative density goes from one fourth to four.

As the price of land increases, so too does its share of housing factor income (Panel D). At a one-fourth density, land accounts for 26% of housing factor income, which is nearly identical to the micro-based estimate for Knoxville discussed in the calibration section above. At a four-fold density, land accounts for 46% of housing factor income.

At the same time, the residential-demand-driven rise in land prices pulls land out of traded good production into housing production (not shown). As density increases from one fourth to one to four, the percent of city-economy land devoted to housing production increases from 63 to 74 to 83. In other words, the share of the city economy's geographic area devoted to production of the traded good falls as density rises.

In contrast, the actual land factor content of housing—that is, land per unit of housing—falls with density (not shown). At a one-fourth density, the quantity of land per housing unit is approximately three times its national-economy level. At a four-fold density, land per unit housing is approximately two fifths its national-economy level. This low land content is consistent with the construction of increasingly tall apartment buildings as density increases.

The sharply rising price of land causes the price of houses to increase as well (Panel E). But the rise in house prices—from 0.61 to 2.0 as density rises from one fourth to four—is considerably more moderate than the rise in land prices. Housing expenditures, which are what is actually observed, rise by even less (also Panel E).

On the other hand, house prices rise by considerably more than wages fall. As a consequence, compensation for differences in quality of life are capitalized much more into housing prices than into wages. At a one-fourth density, lower housing prices account for 62% of a conventionally-calculated compensating differential. At a four-fold density, higher housing prices account for 81% of such a compensating differential. For comparison, the compensating differential empirical literature is inconclusive on capitalization (e.g., Blomquist et al., 1988; Gyourko and Tracy, 1991). The estimates suggest that some amenities are capitalized primarily into house prices; others are capitalized primarily into wages. For many amenities, house prices and wages move in inconsistent directions, implying a capitalization greater than 100 percent for one and a negative capitalization for the other.

As the price of housing rises, the share of expenditures devoted to housing also rises. The increasing housing consumption share follows directly from the assumed less-than-unitary elasticity of substitution between traded goods and housing, $\sigma_{x,h} < 1$. Because house prices rise with density, the housing consumption share rises with density as well (Panel F). In the model, the housing share rises from 15 percent at a one-fourth density to 24 percent at a four-fold density. This almost exactly matches the range of metro-area housing consumption shares reported in the Consumer Expenditure Survey. Among the 26 metro areas for which the Bureau of Labor Statistics reports data, housing's average share of consumption from 1998 to 2003 ranged from 14 percent (Pittsburgh) up to 24 percent (San Diego).

The actual quantity of housing consumed falls with density, as does traded-good consumption (Panel G). The falling levels of traded and housing consumption offset the rising quality of life, thereby maintaining the reservation level of utility. At a one-fourth density, relative traded and housing consumption are, respectively, 1.06 and 1.36. At a four-fold density, they are 0.92 and 0.64.

Lastly, leisure also falls slightly with density (Panel H). As density rises from one fourth to four, relative leisure falls from 1.04 to 0.94. As is the case with traded and housing consumption, a fall in leisure helps compensate for the rise in quality of life. However, this inverse correlation of leisure with density depends closely on the model's parameterization. On the one hand, the lower wages in dense metros make people poorer and hence want to consume less of everything. On the other hand, lower wages decrease the price of leisure. With a unitary elasticity of substitution with leisure, the two effects exactly offset each other. With a lower elasticity, as under the baseline, the wealth effect dominates.[11]

5.2 Sensitivity Analysis

The wide uncertainty characterizing the baseline parameterization makes sensitivity analysis imperative. Doing so establishes how the model's implied resistance to crowdedness—the value of the quality-of-life difference required to achieve a given difference in city density—depends on its various parameters.

Land is the model's only source of congestion. Changes that increase its implicit factor

[11] Rosenthal and Strange (2008) find that among professional workers, hours worked do indeed increase as density increases. But among non-professional workers, hours decrease with density.

share of national-economy consumption—either by explicitly increasing land's factor share in production or by increasing the expenditure share on land-intensive housing—increase resistance to crowding. The higher the implicit land share of consumption, the greater the difference in quality of life that is required to support a given difference in density. Less obviously, decreasing elasticities of substitution in the production and utility functions increases resistance to crowding at high relative densities but leaves resistance essentially unchanged at low relative densities. Different combinations of parameter values yield a huge range of resistance to crowdedness. A high implicit land share is sufficient for resistance to be strong (large required quality-of-life differences). But a low implicit land share does not guarantee weak resistance.

Figure 3 illustrates the sensitivity of required quality of life to six key model parameters. Resistance to crowding depends closely on land's share of housing factor income. Increasing land's share from 20% through its baseline value of 35% to 50% causes a large counterclockwise rotation of the $\widetilde{CV}$-to-density locus (Panel A). Whereas achieving a one-fourth density under the 20% housing land share requires a $\widetilde{CV}$ of -0.08, doing so with a 50% housing land share requires a $\widetilde{CV}$ of -0.17. Whereas achieving a four-fold density under a 20% housing land share requires a $\widetilde{CV}$ of 0.11, doing so under a 50% housing land share requires a $\widetilde{CV}$ of 0.26. A higher housing land share also results in a moderately larger variation in land prices and a considerably larger variation in housing prices (not shown). Under the low housing land share calibration, relative housing prices rise from 0.78 to 1.46 as density rises from one quarter to four. Under the high housing land share calibration, relative housing prices rise from 0.46 to 2.88.

Required quality-of-life differences are less sensitive to land's share of traded factor income. Increasing land's factor share of traded-good production from 0.4% through its baseline value of 1.6% to 4.8% does cause a counterclockwise rotation of the $\widetilde{CV}$-to-density locus (Panel B). But the rotation is quite small, especially moving from the low calibration to baseline and especially at high relative densities. This insensitivity derives from two partly-offsetting forces. On the one hand, increasing land's share of traded production makes such production more subject to congestion. On the other hand, increasing land's share implies a larger equilibrium amount of land that can be switched from traded to housing production. The ability to pull land out of traded production lessens resistance because of the greater difficulty substituting away from land in housing production. With land's traded factor share equal to 0.4%, the share of city-economy land devoted to housing rises from 86% to 96% as density increases from one quarter to four. With land's traded factor share equal to 4.8%, the share of land devoted to housing rises from 59% to 86%.

Resistance to crowding also increases with housing's share of national-economy consumption expenditure (Panel C). Housing is the more land-intensive good, and so increasing its share of expenditure implicitly increases land's factor share of the national-economy consumption bundle. A high housing share also leaves less land in the traded-good production sector that can be pulled into the housing sector as density increases.

Just as resistance to crowding depends closely on the land share of housing production, it also depends closely on the elasticity with which housing production can be shifted away from land (Panel D). In this case, however, the sensitivity applies only at population densities above one. For $\sigma_{D,KL}$ equal to 1, supporting a four-fold density requires a $\widetilde{CV}$ equal to 0.14. Ratcheting $\sigma_{D,KL}$ down to 0.50 causes the required $\widetilde{CV}$ to nearly double to 0.26. Correspondingly, land and housing prices are considerably higher under the lower elasticity parameterization (not shown). In contrast, supporting a one-quarter density requires approximately the same $\widetilde{CV}$, regardless of $\sigma_{D,KL}$.

To understand this asymmetric sensitivity, realize that the marginal product of land in the production of housing is extremely high in a densely-settled economy (as reflected by the high price of land in such an economy). Hence, the resistance to high levels of crowdedness is quite sensitive to the ability to substitute away from land. But in a sparsely-settled economy, the marginal product of land in the production of housing is quite low. Land is abundant and so its price is low. The capital-labor composite is likely to be relatively abundant as well. Capital is supplied with infinite elasticity; labor is perfectly mobile; and the ability to substitute between capital and labor is high. With this abundance of the housing inputs, there is little sensitivity to the elasticity of substitution with land.

Decreasing the consumption elasticity of substitution between the traded good and housing similarly increases resistance to crowding at high relative densities but not at low ones (Panel E). The increase in resistance as $\sigma_{x,h}$ goes from 0.75 to 0.25 is considerably smaller than the increase in resistance as $\sigma_{D,KL}$ goes from 1 to 0.50. Accommodating individuals' low willingness to substitute away from housing proves easier than accommodating a low technological ability to substitute away from land.

Resistance to crowding is almost completely insensitive to the elasticity of substitution with respect to leisure, $\sigma_{xh,leisure}$ (Panel F) and the leisure share of national-economy time (not shown). As discussed in the calibration section above, resistance is completely insensitive to the elasticity of substitution with respect to amenities, $\sigma_{xhl,quality}$ (not shown).

The sensitivity of resistance to crowding to non-wage income depends on how resistance is measured. Measured by normalized compensating variation, as above, non-wage income has very little effect on resistance (not shown). But measured by units of quality of life, resistance falls as non-wage income increases. Quality of life is a normal good. As people become richer, they demand more of it. Hence, a smaller absolute difference in real amenities offsets the costs associated with a given level of crowding. Non-wage income similarly causes individuals to increase their valuation of a given level of real amenities. This increased valuation dominates the lower quantity of real amenities required to support a given level of crowdedness such that the total valuation of the lower required amenities increases. Resistance as measured by absolute CV thus increases with non-wage income.[12]

Different combinations of the parameterization choices imply huge differences in resistance to crowdedness. A low-resistance combination that pairs together all of the low-land and high-elasticity sensitivity values from Panels A through F—as enumerated in the "Loose" column of Table 3—places a lower bound on plausible resistance to crowdedness (Panel G, dashed line). Moving from a one-quarter to a four-fold density requires $\widetilde{CV}$ to vary only from -0.05 to 0.05. In sharp contrast, a high-resistance combination that pairs together all of the high-land and low-elasticity sensitivity values—as enumerated in the "Tight" column of Table 3—places an upper bound on plausible resistance to crowdedness (Panel G, dashed-dotted line). In this case, moving from a one-quarter to a four-fold density requires $\widetilde{CV}$ to vary from -0.25 all the way to 0.52. This upper-bound range is nearly eight times that of the lower-bound one.

A different combination pairs together the high-land and high-elasticity values (Panel H, dashed-dotted line). Even with a relatively easy ability to substitute away from land, a national-economy consumption bundle with an implicit high land factor share suffices to cause stiff resistance to crowding. Conversely, a low implicit land share does *not* suffice to cause weak resistance. The combination of low-land and low-elasticity values does cause

[12]The comparison of resistance to crowding based on required differences in *quality* and required differences in absolute compensating variation only makes sense between calibrations with identical structural parameters.

weak resistance at densities below one (Panel H, dashed line). But at densities above one, resistance rapidly increases.

Lastly, sensitivity analysis also shows that the much greater capitalization of high levels of quality of life into housing prices rather than into wages is a relatively robust result. Under the base parameterization, higher housing prices account for 81 percent of the conventionally-calculated compensating differential corresponding to an amenity-induced relative density of four. For only three parameterizations discussed above does the house price capitalization share fall below 75 percent. When land's share of housing factor income is low (20 percent in the national economy), the house price capitalization share at a relative density of four falls to 67 percent. Under this parameterization, the crowding of people into the high-amenity economy puts less upward pressure on housing prices and so the downward pressure on wages ends up playing a larger role in restoring equilibrium. Alternatively, when land's share of traded-good factor income is high ($\alpha_{X,D} = 0.048$), the housing-price share of the compensating differential at a relative density of four is just 56 percent. In this case, the pull of land out of traded production into the housing sector lowers the marginal product of its complements, labor and capital. Capital in the traded sector also scales back to maintain its required return, thereby putting further downward pressure on labor's marginal product. Increasing land's share of traded factor income thus increases, in absolute value, the negative elasticity of wages with respect to density thereby increasing wages' share of compensating differentials. Finally, the mixed parameterization that combines a national-economy consumption bundle with a high implicit land share together with high elasticities of substitution implies a housing capitalization share of 67 percent.

At low levels of quality of life, in contrast, the capitalization of compensating differentials divides more evenly between house prices and wages. Under the baseline, lower house prices account for 62 percent of a conventionally-calculated compensating differential at a one-fourth density. The split is relatively close to this for most of the parameterizations discussed above. One exception, on the high side, is the 87 percent house-price share with a low land share of traded production. In this case, the increase in per worker land input as density decreases does little to increase the marginal product of labor. Hence the adjustment to maintain equilibrium must come mostly from housing prices. Conversely, when the land share of traded production is high, labor's marginal product increases rapidly as density decreases thereby causing falling housing prices to account for only 33 percent of a conventionally-calculated compensating differential.

The increase in the house-price share of amenity capitalization with density holds for all parameterizations above. It reflects the sharp increases in marginal product and marginal utility as an input or consumption good becomes scarce compared to the more gradual decreases in marginal product and marginal utility as an input or consumption good becomes abundant. For example, the scarcity of land at a high density and the associated upward pressure on house prices is relatively more important than land's abundance at a low density and the associated downward pressure on house prices. Similarly, the scarcity of labor at a low density is relatively more important than its abundance at a high density.

5.3 The Plausibility of Amenity-Driven Crowding

A motivating question of the present paper is the plausibility that differences in quality of life could cause some of the large observed differences in U.S. metro-area densities. In particular, are the required quality-of-life differences so big as to be unreasonable? Note that this question differs from the question of the importance of quality of life as a determinant of the *distribution* of population density across metro areas. Quality of life may simultaneously be the sole determinant of density in a few varied metro areas but a relatively small contributor to the overall density distribution.

To gauge the plausibility of the required quality-of-life differences requires some estimate of the size of actual quality-of-life differences. The compensating differential literature, limitations notwithstanding, is probably the best source for this. The normalized difference in valuation between the estimated highest-amenity metro area and the estimated lowest-amenity metro area from four leading studies is 31 percent (Bloomquist et al., 1988), 49 percent (Gyourko and Tracy, 1991), 38 percent (Gabriel and Rosenthal, 2004), and 26 percent (Chen and Rosenthal, 2006). While compensating differentials differ from compensating variations, in the present case the two measures imply approximately the same ranges.[13]

Plausible differences in quality of life can support most, though perhaps not all, of the observed differences in population density. Under the base parameterization, the difference

[13] Under the baseline parameterization, the smaller absolute value of compensating differentials relative to compensating variations that correspond to low-amenity estimates approximately offset the larger absolute value of compensating differentials relative to compensating variations that correspond to high-amenity estimates.

in required expenditure between a city economy with relative density equal to that of the most-dense metro area (New York City) and one with relative density equal to that of the least-dense metro area (Dothan, Al) is equivalent to 45% of national-economy consumption. This is within the upper end of estimated compensating differentials (Table 4). Under the base parameterization, the difference in required expenditure between a city economy with relative density equal to that of the second-most-dense metro area (Los Angeles) and one with density equal to that of the least-dense metro area is equivalent to 30% of national-economy consumption. This is within the estimated range of three of the studies and only slightly above that of the fourth.

The required range of quality of life unsurprisingly depends on the model's parameterization. Under the low-resistance parameterization, matching the 49-fold variation between New York and Dothan Al requires just a 13 percent difference in quality of life. This is almost surely within the range of quality-of-life differences across U.S. metro areas. Under the high-resistance parameterization, matching the 20-fold variation between Los Angeles and Dothan requires a 73 percent difference in quality of life. This probably exceeds actual quality-of-life differences across U.S. metro areas.

A different way to judge the plausibility of large, quality-of-life-driven differences in density is to specify a plausible difference in quality of life and then calculate the implied difference in density. The four compensating differential studies discussed above suggest that a 25-percentage-point range is a conservative estimate of the difference in compensating variation between the highest-amenity and lowest-amenity metro area. Table 5 reports the density of a high-amenity city with a compensating variation of 12.5 percent relative to one with a compensating variation of -12.5 percent. Under the baseline parameterization, this assumed difference in quality of life induces a 10.6-fold difference in density, which is just over half of the difference between Los Angeles and Dothan.[14] Under the low-resistance parameterization, a symmetric 25 percent spread supports a relative density range of over one thousand. Under the high-resistance, it supports a relative range of just 2.6. While the latter difference in density is comparatively small, it is nevertheless significant. Among calibrations that set only one of their parameters to its "tight" value and hold the remaining

[14]The convexity of the required-amenity locus implies that a 25-percent amenity spread centered below zero can support a considerably larger difference in density. For example, going from a $\widetilde{CV}$ of -15 percent to a $\widetilde{CV}$ of 10 percent supports a 12.3-fold difference in density.

parameters at their baseline, density differs by a multiplicative factor of at least 7.8.

Together this evidence suggests that it is very plausible that differences in quality of life make a significant contribution to the large differences in population density across U.S. metropolitan areas. The ability of reasonable-sized quality-of-life differences to support large density differences is extremely robust across parameterizations. Large changes from baseline to any single parameter only moderately reduce the strength of quality of life. Even under the least-favorable parameterization, plausible differences in quality of life can more than double population density.

5.4 Endogenous Productivity and Quality of Life

The numerical exercises have so far assumed that productivity and quality of life are exogenous. But a central tenet of urban economic theory is that firms' productivity is likely to increase with the scale and density of aggregate production (Marshall, 1890; Jacobs, 1969). Allowing productivity to depend on density can greatly lessen resistance to amenity-driven crowding.

Increases in density from very low levels may similarly increase quality of life. For example, moving from low to moderate density might facilitate social interaction, allow for greater product variety, and support the provision of public goods. On the other hand, increases in density from very high levels probably decrease quality of life. For example, such higher density might increase traffic, pollution, and other non-priced sources of congestion.

Figure 4 shows some general equilibrium results from allowing TFP to depend on density. Let the elasticity with which density *causes* total factor productivity to increase be denoted by υ_X for the traded good and by υ_H for housing. Each economy ($i = c, n$) produces the traded good with TFP, $A_{X,i} = A_X \cdot density_i^{\upsilon_X}$.[15] A parallel formula holds for TFP in producing housing. To match empirical estimates, which are typically of aggregate agglomeration, the elasticity of TFP with respect to density is assumed to be the same for the traded good and for housing ($\upsilon_X = \upsilon_H = \upsilon$). Estimates of υ range from a lower bound of 0.02 (Combes, Duranton, and Gobillon, 2008) to an upper bound of 0.05 (Ciccone and Hall, 1996; Ciccone, 2002).[16]

[15] More generally, the "exogenous" component, A_X, might vary between economies as well.

[16] Combes, Duranton, and Gobillon estimate that the elasticity of *wages* with respect to density is 0.03. Allowing for aggregate Cobb-Douglas production with a two-thirds labor share of factor income implies a

Agglomerative productivity magnifies quality-of-life differences so that a given difference in observed density can be supported by a smaller compensating variation. Increasing the elasticity of TFP with respect to density (while otherwise maintaining the baseline calibration) causes a clockwise rotation of the $\widetilde{CV}$-density locus (Figure 4 Panel A). For υ equal to 0, which has been the maintained assumption so far, a 20-fold increase in density equivalent to that from Dothan to Los Angeles requires a 30 percentage point change in normalized compensating variation. For υ equal to its lower and upper-bound estimates, the required changes in quality of life are 21 and 7 percentage points, respectively (Table 4). Absent agglomerative productivity, the former of these quality-of-life variations can support only a 7-fold variation in density. Absent agglomerative productivity, the latter quality-of-life variation can support only a 2-fold variation in density.

Correspondingly, a given difference in amenities causes a much greater difference in density. A shift in normalized compensating variation from -12.5 percent to 12.5 percent causes an 10.6-fold increase in density under the baseline. With υ equal to its lower and upper-bound estimates, the corresponding increases are 44-fold and infinite (Table 5). The former implied density range is already near the maximum of what we observe. The implied infinite density range reflects that with sufficiently strong agglomerative forces, the population density of very-low-amenity places will be zero. Even with land prices close to zero, relative productivity and wages in very sparsely-populated places can be too low to allow individuals to attain the nationally-determined reservation utility level.

More generally, with υ equal to its upper-bound estimate of 0.05, resistance to crowding at densities below one is negligible. A density of one quarter follows from just a 2% $\widetilde{CV}$ deficit, which is approximately the lowest quality of life consistent with strictly positive population density. As density decreases below one quarter, the $\widetilde{CV}$-density locus bends back up towards zero. Essentially, higher quality of life must compensate for falls in agglomerative productivity. For example, a density of one sixteenth requires a $\widetilde{CV}$ equal to zero. Of course, a zero compensating variation also supports a unitary relative density. With sufficiently strong agglomeration, the model is characterized by multiple equilibria.

The endogenous increase of productivity with density can reverse the amenity-driven negative correlation between wages and density. As υ increases, the wage-density locus

0.02 elasticity of TFP with respect to density. Estimates of the elasticity with which the *scale* of economic activity increases total factor productivity range from 0.04 to 0.08 (Rosenthal and Strange, 2004).

rotates in a counterclockwise direction (Figure 4 Panel B). Any increase in productivity puts upward pressure on wages. A value of υ equal to its lower-bound estimate of 0.02 is sufficient to cause wages to be essentially flat as density increases. A value of υ equal to its upper-bound estimate of 0.05 causes wages to become strongly increasing with density.

Like productivity, quality of life can be modeled as endogenous. The required $\widetilde{CV}$-to-density loci shown in Figures 2 and 3 above hold regardless of the source of the quality-of-life differences. Allowing quality of life to depend on density is equivalent to adding an extra equation to the current system.

Figure 5 shows one possible dependence of quality of life on density. The required, solid locus gives the combinations of quality of life and density that are consistent with equilibrium for the city economy. The four remaining loci give the quality of life level that each of four city economies would experience at different densities. The vertical differences between the curves reflect differences in quality of life that do not depend on density. These "exogenous" variations in quality of life can be measured by the level of quality of life that would prevail at a unitary density. The circles along the vertical axis thus depict the four exogenous amenity levels. For example, one of the economies has exogenous amenities such that at a unitary density, its quality of life is equal to that of the national economy ($\widetilde{CV}$ at a unitary density equals zero). The "nicest" of the economies has exogenous amenities such that at a unitary density, it has quality of life for which national-economy residents would pay 20% of their income ($\widetilde{CV}$ at a unitary density equals 0.20).

Quality of life is assumed to also have a component that endogenously depends on density. In particular, quality of life endogenously increases as density rises to an intermediate level above one and then endogenously decreases as it rises further. For instance, the economy with exogenous amenities equal to that of the national economy sees its quality of life rise to a maximum $\widetilde{CV}$ of 0.04 at a relative density of 2.8. For the economy with the highest level of exogenous amenities, quality of life rises to a maximum $\widetilde{CV}$ of 0.24 at a relative density of 2.8. If, instead, quality of life did not depend on density, the dashed lines in Figure 5 would be horizontal. In this case, the endogenous component of quality of life would always be zero.

Very little empirical evidence exists on the endogenous response of quality of life to density. A first, obvious problem is that quality of life is not observable. Even if it were, a second challenge would be distinguishing the endogenous relationship from the required one.

This problem of identification is discussed in the next section.

6 Some Empirics: the Importance of Quality of Life

The generalized version of the static model herein has only two possible sources of variation in local density: variation in quality of life and variation in productivity. Within a dynamic context, the model suggests that *changes* in quality of life and productivity are the main source of variations in local growth. An obvious question, then, is how important are the quality-of-life variations relative to the productivity ones? Note that this is essentially a cross-sectional distributional question that differs from whether quality of life could plausibly cause large differences in density among a small subset of metro areas.

Empirical evidence suggests that quality-of-life variations are indeed an important factor helping to underpin the cross-sectional distribution of population density. In particular, density is strongly positively correlated with several quality-of-life indices based on subjective criteria. In addition, population *growth* is strongly positively correlated with several exogenous consumption amenities, which suggests that quality of life is becoming a more important determinant of metro density. Matching the observed positive elasticity of wages with density suggests that cross-sectional variation in quality of life accounts for approximately one fifth of cross-sectional variation in population density.

Figure 5 illustrates two distinct, predicted cross-sectional correlations between density and quality of life. The first is that density should be positively correlated with exogenous amenities. Higher vertical intercepts of the dashed-line, endogenous curves are associated with higher density intersections with the solid, required $\widetilde{CV}$ curve.[17] The second prediction is that density should be positively correlated with overall quality of life, which is the sum of the exogenous and endogenous components. If productivity is the same across local economies, the required locus is identical across them as well. Different levels of exogenous amenities vertically shift the endogenous curve, whatever its slope, thereby identifying the required locus. In Figure 5, the three intersections of the endogenous curves with the required curve illustrate this identification. Each of these three points, along with many other intersections corresponding to other exogenous quality of life levels, lie along the required

[17] Sufficiently steep endogenous curves might also seem to suggest that higher exogenous amenities could be associated with lower density. However, the associated low-density equilibrium would be unstable.

curve and are what would be observed by a researcher.

Of course, productivity also varies across local economies. It can do so exogenously due to fixed local characteristics such as access to raw materials and navigable water. Exogenous differences in productivity vertically shift the required $\widetilde{CV}$ locus. Productivity can also vary endogenously due to variations in local population density. As discussed in the previous section, the positive dependence of productivity on density causes a clockwise rotation of the required locus. If sources of productivity are positively correlated with sources of quality of life (e.g., public transportation infrastructure), the positive correlation of density with quality of life should strengthen. If sources of productivity are negatively correlated with sources of quality of life (e.g., environmental regulation), the positive correlation of density with quality of life should weaken.

Empirically, the correlation of density with exogenous consumption amenities is ambiguous. Coastal proximity seems one obvious exogenous amenity. Density is, indeed, strongly positively correlated with it, even after including measures of proximity to harbors in order to control for productivity differences (Rappaport and Sachs, 2003). But density's correlation with another obvious amenity, nice weather in the form of warm winters and cool summers, can be positive, negative, or zero, depending on the exact empirical specification.

Evidence of a positive correlation of population *growth* with exogenous consumption amenities is much stronger. Growth, like density, is strongly positively correlated with coastal proximity, again controlling for proximity to harbors (Rappaport and Sachs, 2003). Growth is even more strongly and robustly positively correlated with nice weather (Rappaport, 2007b).[18]

Also ambiguous is the correlation between density and overall quality of life. Among the four compensating differential studies referenced above, only one estimates a quality-of-life index with which density is positively correlated (Table 6, Panel A). More specifically, estimated metro-area compensating differentials summed with national median household income give the expenditure required to attain a national reservation level of utility: $\mathrm{e}(quality_i)$, with $\mathrm{e}'(\cdot)$ negative. But only using the Gabriel and Rosenthal estimates does density turn out to be negatively correlated with required expenditure (implying a positive

[18]Some portion of the positive correlation of population growth with nicer weather stems from the advent of air conditioning and increased mobility of the elderly. But Rappaport (2007b) argues that more important than these was the increased valuation of nice weather as a consumption amenity.

correlation of density with overall quality of life). Even then, the magnitude of the negative correlation is considerably smaller than implied by the model's base parameterization. For two of the remaining studies, an absence of correlation between density and quality of life cannot be rejected. The Gyourko and Tracy (1991) estimates are characterized by a statistically-significant negative correlation of density with quality of life.[19]

In contrast, population density is strongly positively correlated with the subjective-criteria, overall quality-of-life rankings by Savageau (2000) and Sperling and Sander (2004) (Table 6, Panels B and C). The Spearman correlation coefficient of density with the former is 0.42 and with the latter is 0.49. Both coefficients statistically differ from zero at the 0.01 level. Density is similarly positively correlated with indices for nearly all of the subsidiary quality-of-life categories. The only exceptions are a small, negative correlation with the Savageau crime index and a moderately strong, negative correlation with the Sperling and Sander health and healthcare index.[20]

To the extent that the subjective-criteria quality-of-life rankings seem more reasonable than the rankings of the compensating-differential literature, there is thus strong evidence that quality-of-life differences help to underpin the distribution of density across U.S. metro areas. The likelihood that many quality-of-life attributes may themselves be the endogenous result of density differences is largely beside the point. However, relatively low, positive correlations of density with subjective climate rankings along with the ambiguous correlation between density and mild weather suggests that amenities may have historically served more as a reinforcing mechanism for productivity-driven crowding rather than as an exogenous impetus.

The strong, positive partial correlation of population *growth* with nice weather suggests that quality of life may be becoming a more important source of people's location decisions. A similar conclusion is reached by Glaeser, Kolko, and Saiz (2001), who find that recent population growth in cities has been positively correlated with several measures of consumption amenities. Shapiro (2006) also concludes that a large portion of the faster growth of high-human-capital cities is attributable to amenities.

[19]Davis and Ortalo-Magne (2007) also argue that the empirical correlation of density with quality of life is negative.

[20]The Spearman correlation coefficient between the two overall indices is 0.65. Correlation coefficients between comparable categories range from 0.77 for climate down to 0.18 for health and healthcare.

Nevertheless, the observed positive correlation between wages and density places an upper bound on the importance of quality of life as a source of local crowdedness. If cross-sectional variations in crowdedness derived primarily from variations in quality of life, the cross-sectional correlation between wages and metro density would be negative. In fact, the wage-density correlation is robustly positive.[21] Using data on U.S. metropolitan areas in 2000, Rappaport (2006) estimates that the elasticity of median individual labor income with respect to person-weighted density lies between 0.10 and 0.20. Using data on French employment areas in 1998, Combes et al. (2008) report an estimated elasticity of mean wages with respect to raw density of 0.05. Using a panel of individual data to control for unobserved skill differences, their estimated elasticity falls to 0.04.[22] These estimates should be interpreted as simple correlations with no implied causality.

On the other hand, the relatively small magnitude of the Combes et al. estimate of the (non-causal) elasticity of wages with respect to density suggests that variations in quality of life do contribute to the observed cross-sectional distribution of density. With exclusively productivity-driven crowding, the elasticity of wages with respect to density under the baseline parameterization is 0.08. Bringing this down to the Combes et al. estimate of 0.04 requires positive co-movement between productivity and quality of life. Increases in quality of life dampen the productivity-driven wage increases. Achieving the benchmark 20-fold variation in density while matching the Combes et al. estimated elasticity requires a 9 percentage point rise in quality of life and a 14 percentage point rise in TFP. Holding productivity constant, this quality-of-life rise can support only a 2.1-fold variation in density. Holding quality of life constant, this productivity rise can support an 8.9-fold variation in density. The increase in quality of life thus accounts for 19 percent of the increase in density (2.1/(2.1+8.9)).[23]

[21] One possible exception is Lee (2005). He finds that the wages of very high-skilled medical workers decrease with city size.

[22] There are several possible reasons for the difference in magnitudes between the Rappaport and Combes et al. aggregate estimates. One is the different unit of observation. The 341 employment districts discretely partition France. The 332 metro areas represent the most-densely-settled quarter of U.S. land area. Another reason is the weighted construction of the U.S. metro densities as described in the empirical motivation section. Both of these reasons imply a dampening of variations in U.S. density in turn implying higher elasticities with respect to density.

[23] An alternative accounting attributes 26 percent of the increase in density to quality of life, log(2.1)/(log(2.1)+log(8.9)).

Of course, this accounting of quality of life's contribution to the cross-sectional density distribution depends on both the parameterization and the estimated wage-density elasticity. Under the low-resistance parameterization, the model implies an exclusively-TFP-driven elasticity of wages with respect to density of only 0.03. In this case, matching a 0.04 estimated elasticity requires productivity-driven-crowding accompanied by some *negative* co-movement with quality of life (e.g., high-productivity manufacturing accompanied by pollution). Under the high-resistance parameterization, the solely-TFP-driven elasticity of wages with density is 0.15. In this case, achieving the benchmark 20-fold variation in density while matching the Combes et al. estimate requires that quality of life account for 26 percent of the density variation. Finally, a higher estimated elasticity of wages with respect to density implies a lower contribution from quality of life. A non-causal wage-density elasticity of 0.06 is consistent with the estimates reported in Ciccone and Hall (1996) and Ciccone (2002). Matching this elasticity under the baseline parameterization implies that quality of life accounts for just a 6 percent share of the variation in density.

The above apportionment of the contribution to variations in density by quality of life and productivity is meant strictly in an accounting sense. Apportioning "credit" for such variations is more difficult because of the interaction between exogenous and agglomerative forces. For example, suppose that agglomerative productivity multiplies exogenous differences in quality of life, as in Section 5.4 above. With the agglomerative elasticity set at its upper bound estimate, a strict accounting shows that TFP will be the substantially larger force underpinning differences in density. Even so, exogenous quality of life differences serve as the catalyst causing differences in density and so are arguably the more important source of variation. Of course, the agglomerative force may alternatively be quality of life and the exogenous force may alternatively be productivity. Ideally, empirical analysis would distinguish among four possible forces: exogenous quality of life, agglomerative quality of life, exogenous productivity, and agglomerative productivity.

Overall, empirical analysis suggests that variations in quality of life do help to underpin the distribution of population density across U.S. metropolitan areas. Either the level of population density or its growth rate is strongly positively correlated with several exogenous amenities. And density is strongly positively correlated with two independent rankings of overall quality of life based on subjective criteria. Matching the observed positive elasticity of wages with respect to density suggests that differences in quality of life account for approximately one fifth of differences in density.

7 Conclusions

Population density differs greatly across U.S. metropolitan areas. A calibrated general equilibrium model suggests that plausible differences in consumption amenities can cause differences in density nearly as large as observed. Under a baseline parameterization, a difference in amenities valued at 30 percent of national-economy consumption can support the observed twenty-fold difference in population density between the second-most-crowded and least-crowded metro areas. Sensitivity analysis shows that resistance to crowdedness depends closely on the housing production function as well as on the implicit land share of national-economy consumption. A high implicit land share is a sufficient, but not necessary, condition for high resistance to crowding.

The model illustrates how several endogenous outcomes co-vary with density. Under the baseline parameterization, wages fall slightly with density, house prices rise moderately, and land prices rise steeply. Compensation for high quality of life is primarily capitalized into land and house prices rather than into wages. In return for enjoying high quality of life, individuals sacrifice small amounts of traded-good and leisure consumption and a large amount of housing consumption.

Empirical analysis finds a strong positive correlation between population density and two subjective-criteria measures of metro-area quality of life. This suggests that amenities help to support the cross-sectional distribution of population density. Strong positive correlations between population growth and several exogenous consumption amenities suggest that quality of life is becoming a more important determinant of where people choose to live. Matching the observed, non-causal elasticity of wages with respect to population density implies, under the baseline parameterization, that cross-sectional variation in quality of life accounts for approximately one-fifth of observed cross-sectional variation in density.

The present, simple model is an ideal platform on which to build a richer framework. A first priority is to allow for the endogenous determination of land size. The elasticity of population density with respect to total population suggests that two thirds of the population attracted to higher productivity and quality of life "spills" over into larger land size rather than into higher density (Table 1, bottom line). A second priority is to introduce heterogeneity among individuals, in terms of skills, wealth, and mobility. Heterogenous skills together with variations in productivity suggests that cities will tend to specialize in some production technologies rather than others as in Duranton and Puga (2001) and Caselli and Coleman (2006). Heterogenous wealth together with variations in quality of life suggests that the rich will outbid the poor to live in high-amenity cities, consistent with Gyourko, Mayer, and Sinai (2006). A third priority is to allow for imperfect individual mobility, which will imply that average realized utility is higher in some metro areas than in others. A fourth priority is to model a full system of cities so as to be able match the observed cross-sectional distribution of density and other endogenous variable.

Even without any further development, the present framework delivers important results. For example, equal productivity growth among all metro areas creates large migrations toward the higher quality-of-life ones (Rappaport, 2007a).

More generally, the present model and results stress the need to better understand the determinants of local quality of life. Along with productivity, it is one of the few fundamental determinants of local population and population density. Any story of different metro outcomes usually maps to a story of different productivity facing firms and different quality of life facing individuals. Agglomeration, in particular, can be seen as being mediated via these two mechanisms. The present model can thus serve as a simple framework to help evaluate local public policy and help predict future local growth.

Acknowledgments

Thank you for advice and feedback to Gilles Duranton, Steven Durlauf, Vernon Henderson, Ping Wang, two anonymous referees, and seminar participants at Washington University. Martina Chura, Taisuke Nakata, Ebiere Okah, and Aarti Singh provided excellent research assistance.

Bibliography

Aguiar, Mark and Erik Hurst (2006). "Measuring Trends in Leisure: The Allocation of Time over Five Decades." Federal Reserve Bank of Boston Working Paper 06-02, January.

Blomquist, Glenn C., Mark C. Berger, and John P. Hoehn (1988). "New Estimates of Quality of Life in Urban Areas." *American Economic Review* 78, 1 (March), pp. 89–107.

Blundell, Richard and Thomas MaCurdy (1999). "Labor Supply: a Review of Alternative Approaches." In *Handbook of Labor Economics* Volume 3A, Orley Ashenfelter and David Card eds. Amsterdam: Elsevier Science Publishers.

Caselli, Francesco and Wilbur John Coleman II (2006). "The World Technology Frontier." *American Economic Review* 96, 3 (June), pp. 499–522.

Chen, Yong and Stuart S. Rosenthal (2006). "Local Amenities and Life Cycle Migration: Do People Move for Jobs or Fun?" Working paper, Syracuse University (June).

Ciccone, Antonio (2002). "Agglomeration Effects in Europe." *European Economic Review* 46, pp. 213–227.

Ciccone, Antonio and Robert E. Hall (1996). "Productivity and the Density of Economic Activity." *American Economic Review* 86, 1 (March), pp. 54–70.

Combes, Pierre-Philippe, Gilles Duranton, and Laurent Gobillon (2008). "Spatial Wage Disparities: Sorting Matters!" *Journal of Urban Economics* (forthcoming).

Davis, Morris A. and Jonathan Heathcote (2005). "The Price and Quantity of Residential Land in the United States." Mimeo, Federal Reserve Board of Governors, (October).

Davis, Morris A. and Francois Ortalo-Magne (2007). "Wages, Rents, Quality of Life." Mimeo, University of Wisconsin, Department of Real Estate and Urban Land Economics, (September).

Duranton, Gilles, and Diego Puga (2001). "Nursery Cities: Urban Diversity, Process Innovation, and the Life Cycle of Products." *American Economic Review* 91, 5 (Dec.), pp. 1454–1477.

Ermisch, J.F., J. Findlay, and K. Gibb (1996). "The Price Elasticity of Housing Demand in Britain: Issues of Sample Selection." *Journal of Housing Economics* 5, 1 (March) pp. 64–86.

Gabriel, Stuart A. and Stuart S. Rosenthal (2004). "Quality of the Business Environment Versus Quality Of Life: Do Firms and Households like the Same Cities?" *The Review of Economics and Statistics* 86, 1 (Feb.), pp. 438–444.

Glaeser, Edward L. and Matthew E. Kahn (2004). "Sprawl and Urban Growth." In *Handbook of Regional and Urban Economics*, Volume 4, eds. J. Vernon Henderson and Jacques-Francoise Thisse. Amsterdam: Elsevier North-Holland, pp. 2481–2527.

Glaeser, Edward L., Jed Kolko, and Albert Saiz (2001). "Consumer City." *Journal of Economic*

Geography 1, 1 (January) pp. 27–50.

Gollin, Douglas (2002). "Getting Income Shares Right." *Journal of Political Economy* 110, 2 (April), pp. 458–474.

Goodman, Allen C. (1988). "An Econometric Model of Housing Price, Permanent Income, Tenure Choice, and Housing Demand." *Journal of Urban Economics* 23, 3 (May) pp. 327–353.

Goodman, Allen C. (2002). "Estimating Equilibrium Housing Demand for 'Stayers'". *Journal of Urban Economics* 51, 1 (January), pp. 1–24.

Gyourko, Joseph, Christopher Mayer, and Todd Sinai (2006). "Superstar Cities." NBER Working Paper 12355, (July).

Gyourko, Joseph and Joseph Tracy (1989). "The Importance of Local Fiscal Conditions in Analyzing Local Labor Markets." *Journal of Political Economy* 97, 5 (Oct.), pp. 1208–1231.

Gyourko, Joseph and Joseph Tracy (1991). "The Structure of Local Public Finance and the Quality of Life." *Journal of Political Economy* 99, 4 (Aug.), pp. 774-806.

Gyourko, Joseph, Joseph Tracy, and Matthew E. Kahn (1999). *Handbook of Regional and Urban Economics*, Volume 3, eds. Paul Cheshire and Edwin S Mills. Amsterdam: Elsevier North-Holland.

Haurin, Donald R. (1980). "The Regional Distribution of Population, Migration, and Climate." *Quarterly Journal of Economics* 95, 2 (Sept.), pp. 293–308.

Henderson, J. Vernon (1974). "The Sizes and Types of Cities." *American Economic Review* 64, 4 (Sept.), pp. 640-656.

Henderson, J. Vernon (1987). "General Equilibrium Modeling of Systems of Cities." In Edwin S. Mills ed., *Handbook of Regional and Urban Economics* Volume 2, pp. 927-958.

Henderson, J. Vernon (1988). *Urban Development: Theory, Fact, and Illusion.* New York: Oxford University Press.

Haughwout, Andrew F. and Robert P. Inman, (2001). "Fiscal Policies in Open Cities with Firms and Households." *Regional Science and Urban Economics* 31, 2-3 (April), pp. 147–180.

Ioannides, Yannis M. and Jeffrey E. Zabel (2003). "Neighborhood Effects and Housing Demand." *Journal of Applied Econometrics* 18, 5 (Sept./Oct.) pp. 563–584.

Jackson, Jerry R., Ruth C Johnson, and David L. Kaserman (1984). "The Measurement of Land Prices and the Elasticity of Substitution in Housing Production." *Journal of Urban Economics* 16, 1 (July), pp. 1–12.

Jacobs, Jane (1969). *The Economy of Cities.* New York: Random House.

Jorgenson, Dale W., Mun S. Ho, and Kevin J. Stiroh (2005). "Growth of U.S. Industries and Investments in Information Technology and Higher Education." In *Measuring Capital in the New*

Economy, eds. Carol Corrado, John Haltiwanger, and Daniel Sichel. Chicago IL: University of Chicago Press.

Jones, Charles I. (2005). "The Shape of Production Functions and the Direction of Technical Change." *Quarterly Journal of Economics* 120, 2 (May), pp. 517–549.

Kaldor, Nicholas (1961). "Capital Accumulation Economic Growth." In F.A. Lutz and D.C. Hague, eds., *The Theory of Capital.* New York: St. Martin's Press, pp. 177-222.

Kydland, Finn E. and Edward C. Prescott (1982). "Time to Build and Aggregate Fluctuations." *Econometrica* 50, 6 (November), pp. 1345–1370.

Lee, Sanghoon Conan (2005). "Ability Sorting and Consumer City." Mimeo, University of British Columbia.

Marshall, Alfred (1890). *Principles of Economics.* London: Macmillan.

McDonald, John F. (1981). "Capital-Land Substitution in Urban Housing: A Survey of Empirical Estimates." *Journal of Urban Economics* 9, 2 (March), pp. 190–211.

Pencavel, John (1986). "Labor Supply of Men: a Survey." In *Handbook of Labor Economics* Volume 1, Orley Ashenfelter and Richard Layard eds. Amsterdam: Elsevier Science Publishers.

Rappaport, Jordan (2004). "Why Are Population Flows so Persistent?" *Journal of Urban Economics* 56, 3 (November), pp. 554–580.

Rappaport, Jordan (2006). "A Productivity Model of City Crowdedness." Federal Reserve Bank of Kansas City, Research Working Paper 06-06, (June).

Rappaport, Jordan (2007a). "Moving to High Quality of Life." Federal Reserve Bank of Kansas City, Research Working Paper 07-02 (March)

Rappaport, Jordan (2007b). "Moving to Nice Weather." *Regional Science and Urban Economics* 37, 3 (May), pp. 375–398.

Rappaport, Jordan (forthcoming). "A Productivity Model of City Crowdedness." *Journal of Urban Economics*, forthcoming.

Rappaport, Jordan and Jeffrey Sachs (2003). "The United States as a Coastal Nation." *Journal of Economic Growth* 8, 1 (March), pp. 5–46.

Roback, Jennifer (1982). "Wages, Rents, and the Quality of Life." *Journal of Political Economy* 90, 6 (Dec.), pp. 1257–1278.

Robinson, John P. and Geoffrey Godbey (1997). *Time for Life: The Surprising Ways Americans Use Their Time.* University Park, PA: The Pennsylvania State University Press.

Rosen, Sherwin (1979). "Wage-Based Indexes of Urban Quality of Life." In Miezkowski and Straszheim, Eds., *Current Issues in Urban Economics.* Baltimore: Johns Hopkins University Press.

Rosenthal, Stuart S. and William C. Strange (2004). "Evidence on the Nature and Sources of Agglomeration Economies." In *Handbook of Regional and Urban Economics*, Volume 4, eds. J Vernon Henderson and Jacques-Francois Thisse. Amsterdam: Elsevier North-Holland, pp. 2118–2171.

Rosenthal, Stuart S. and William C. Strange (2008). "Agglomeration and Hours Worked." *Review of Economics and Statistics, forthcoming.*

Savageau, David (2000). *Places Rated Almanac*. With Ralph D'Agostino. Foster City, CA: IDG Books Worldwide, Inc.

Shapiro, Jesse M. (2006). "Smart Cities: Quality of Life, Productivity, and the Growth Effects of Human Capital." *Review of Economics and Statistics* 88, 2, pp. 324–335.

Sperling, Bert and Peter Sander (2004). *Cities Ranked & Rated*, First Edition. Hoboken, NJ.: Wiley Publishing, Inc.

Thorsnes, Paul (1997). "Consistent Estimates of the Elasticity of Substitution between Land and Non-Land Inputs in the Production of Housing." *Journal of Urban Economics* 42, 1 (July), pp. 98–108.

Willis, Jonathan L. and Julie Wroblewski (2007). "What Happened to the Gains from Strong Productivity Growth?" *Economic Review*, Federal Reserve Bank of Kansas City 92, 1 (First Quarter), pp. 5–23.

Upton, Charles (1981). "An Equilibrium Model of City Size." *Journal of Urban Economics* 10, 1 (July), pp. 15–36.

Table 1: Variations in Population Density

Rank	Metropolitan Area	Density	
1	New York-Nrthrn New Jersey-Long Island, NY-NJ-PA	18.9	
2	Los Angeles-Long Beach-Santa Ana, CA	7.8	
3	San Francisco-Oakland-Fremont, CA	7.2	
4	Chicago-Naperville-Joliet, IL-IN-WI	6.7	
5	Miami-Fort Lauderdale-Miami Beach, FL	5.8	
6	Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	5.2	
7	San Jose-Sunnyvale-Santa Clara, CA	5.1	**6.8**
8	Boston-Cambridge-Quincy, MA-NH	5.0	**times**
9	Salinas, CA	4.7	
10	Washington-Arlington-Alexandria, DC-VA-MD-WV	4.5	
11	Trenton-Ewing, NJ	4.4	
12	Modesto, CA	4.2	
13	Baltimore-Towson, MD	4.0	
14	Oxnard-Thousand Oaks-Ventura, CA	3.9	
15	Milwaukee-Waukesha-West Allis, WI	3.8	
16	Detroit-Warren-Livonia, MI	3.8	
17	Las Vegas-Paradise, NV	3.7	
18	Laredo, TX	3.7	
19	San Diego-Carlsbad-San Marcos, CA	3.7	
20	Santa Cruz-Watsonville, CA	3.7	
:	:	:	
:	:	:	
48	Tampa-St. Petersburg-Clearwater, FL	2.8	
49	Pittsburgh, PA	2.8	
50	**population median** (Omaha-Council Bluffs, NE-IA)	2.8	
51	Lincoln, NE	2.7	**49**
52	St. Louis, MO-IL	2.7	**times**
:	:	:	
:	:	:	
328	Anniston-Oxford, AL	0.5	
329	Morristown, TN	0.5	
330	Ocala, FL	0.5	
331	Bangor, ME	0.4	
332	Dothan, AL	0.4	

share of continental U.S. population: 82.0%
share of continental U.S. land area: 27.7%

elasticity with respect to population: $\varepsilon = 0.34\ (0.02);\ R^2 = 0.39$

Rankings by population density in 2000 of continental U.S. metro areas with population of at least 100,000. Metro area delineations are based on 2003 OMB standard. Density, measured as thousand persons per square mile, is calculated as a population-weighted mean of county-subdivision-place/remainder densities.

Table 2: Ranking Quality of Life

A. Compensating Differential Methodology

Rank	Blomquist, Berger, and Hoehn (1988)	Rank	Gyourko and Tracy (1991)
1	Pueblo, CO	1	Norwalk, CT
2	Norfolk-Virginia Beach-Portsmouth, VA	2	Pensacola, FL
3	Denver-Boulder, CO	3	Gainesville, FL
4	Macon, GA	4	San Diego, CA
5	Reno, NV	5	Stamford, CT
6	Binghamton, NY	6	Columbia, SC
7	Newport News-Hampton, VA	7	Santa Rosa, CA
8	Sarasota, FL	8	Bridgeport, CT
9	West Palm Beach-Boca Raton, FL	9	Tucson, AZ
10	Tuscon, AZ	10	Shreveport, LA
11	Fort Lauderdale-Hollywood, FL	11	Lancaster, PA
12	Fort Collins, CO	12	Modesto, CA
13	Charleston-North Charleston, SC	13	Asheville, NC
14	Salinas-Seaside-Monterey, CA	14	New Orleans, LA
15	Roanoke, VA	15	Fall River, MA
16	Lackawanna, PA	16	Danbury, CT
17	Tallahasee, FL	17	Amarillo, TX
18	Richmond, VA	18	Jacksonville, FL
19	Lexington-Fayette, KY	19	San Francisco, CA
20	Santa Barbara-Santa Maria-Lompoc, CA	20	San Jose, CA

B. Subjective Methodology

Rank	Savageu (2000)	Rank	Sperling and Sander (2004)
1	San Francisco, CA	1	New York, NY
2	Washington, DC-MD-VA-WV	2	Nassau-Suffolk, NY
3	Boston, MA-NH	3	Seattle-Belevue-Everett, WA
4	Seattle-Bellevue-Everett, WA	4	San Francisco, CA
5	Orange County, CA	5	Boston, MA-NH
6	Nassau-Suffolk, NY	6	Ann Arbor, MI
7	San Jose, CA	7	Portland-Vancouver, OR-WA
8	Raleigh-Durham-Chapel Hill, NC	8	Boulder-Longmont, CO
9	Pittsburgh, PA	9	Washington, DC-MD-VA-WV
10	Salt Lake City-Ogden, UT	10	Pittsburgh, PA
11	Denver, CO	11	Atlanta, GA
12	New York, NY	12	Middlesex-Somerset-Hunterdon, NH
13	San Diego, CA	13	Stamford-Norwalk, CT
14	Minneapolis-St. Paul, MN-WI	14	Santa Fe, NM
15	Philadelphia, PA-NJ	15	Corvallis, OR
16	Rochester, NY	16	San Diego, CA
17	Cincinnati, OH-KY-IN	17	Denver, CO
18	Cleveland-Lorain-Elyria, OH	18	Madison, WI
19	Syracuse, NY	19	Santa Barbara-Santa Maria-Lompoc, CA
20	Milwaukee-Waukesha, WI	20	Bergen-Passaic, NJ

Subjective rankings are based on approximately contemporary data. Compensating differential rankings are based on 1980 census data. For Blomquist et al., listed metro areas are location of ranked counties. Subjective rankings are weighted averages of a number of quality-of-life categories. They differ from published summary rankings in that they exclude jobs and cost-of-living categories.

Table 3: Baseline and Alternative Calibrations

Parameter	Base	Low Resistance ("Loose")[1]	High Resistance ("Tight")[1]	Primary Source of Calibration	Sensitivity of Numerical Results
Land Factor Income Share[2] (national economy)					
Traded Good:	**1.6%**	0.4%	4.8%	Jorgenson, Ho, and Stiroh (2005)	medium
Housing:	**35%**	20%	50%	Davis and Heathcote (2005)	high
Housing Production CES ($\sigma_{D,KL}$)	**0.75**	1	0.50	McDonald (1981); Jackson, Johnson, and Kaserman (1984)	high
Required Capital Rent (r_K)	**0.08**			3% real return plus 5% depreciation	none
Utility CES Parameters					
$\sigma_{x,h}$	**0.50**	0.75	0.25	CEX housing consumption shares vs. apartment rents	medium
$\sigma_{xh,leisure}$	**0.50**	1	0.25	Time diary studies vs. real wages	very low
$\sigma_{xhl,quality}$	**0.50**			Arbitrary	none
Consumption Expenditure Shares (national economy)					
Housing	**18%**	14%	22%	Consumer Expenditure Survey	medium
Leisure (share of time)	**35%**	20%	50%	Time Diary Studies	very low

[1]The CES substitution parameters ($\sigma_{D,KL}$, and $\sigma_{x,h}$) have an asymmetric effect on resistance. The "loose" values above are those for which resistance is lower at a relative density of one and above.
[2]Non-land factor income is divided between capital and land on a one-third to two-thirds basis both for the traded good and for housing.

Table 4: Differences in Quality of Life

Estimated Compensating Differential Range	
Bloomquist et al. (1988)	31%
Gyourko and Tracy (1991)	49%
Gabriel and Rosenthal (2004)	38%
Chen and Rosenthal (2006)	26%
Model, Required Compensating Variation	
Most Dense (NYC) to Least Dense (Dothan AL): 49-fold density difference	
Baseline	45%
Low Resistence Parameterization	13%
High Resistence Parameterization	126%
Endogenous productivity, υ_{TFP}= 0.02	34%
Endogenous productivity, υ_{TFP}= 0.05	17%
Second-most Dense (Los Angeles) to Least Dense (Dothan AL): 20-fold density difference	
Baseline	30%
Low Resistence Parameterization	10%
High Resistence Parameterization	73%
Endogenous productivity, υ_{TFP}= 0.02	21%
Endogenous productivity, υ_{TFP}= 0.05	7%

Modeled compensating variation is reported relative to per capita income in the national economy. Estimated compensating differential range is reported relative to national median household income.

Table 5: Density Ranges

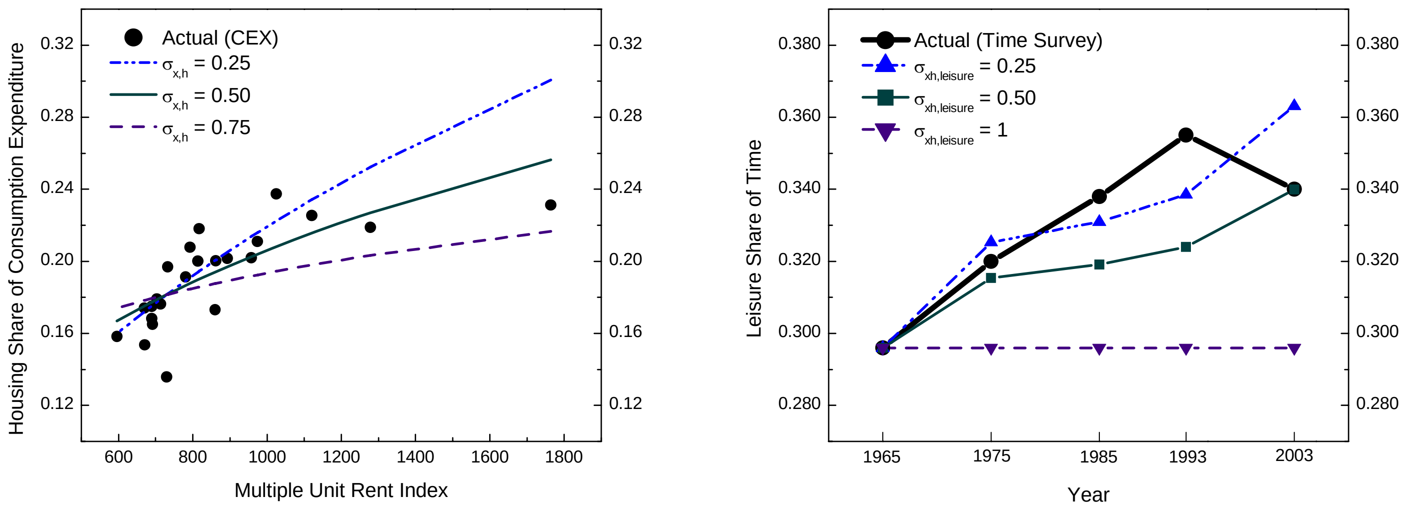
Observed Multiplicative Factors	
Most Dense to Least Dense	48.7
Second-Most-Dense to Least Dense	20.3
Model: Implied Multiplicative Factors from a Symmetric Change in Normalized CV from -12.5% to +12.5%	
Baseline	10.6
Low Resistance Parameterization	1,387
High Resistance Parameterization	2.6
Endogenous Productivity, υ_{TFP}= 0.02	44
Endogenous Productivity, υ_{TFP}= 0.05	infinite

The assumed difference in quality of life is within the estimated ranges of all four compensating differential studies listed in Table 4.

Table 6: Density and Quality of Life

A. Correlation of Density with Required Expenditure	
Modeled Elasticity	
Baseline	-9.2
Low Resistance Parameterization	-3.8
High Resistance Parameterization	-29.0
Endogenous productivity, υ_{TFP}= 0.02	-12.7
Endogenous productivity, υ_{TFP}= 0.05	-28.9
Estimated Elasticity (std. error)	
Blomquist et al. (1988) 130 urban counties, 1980	-0.1 (0.9)
Gyourko and Tracy (1991), 127 metro central cities, 1980	1.5 (0.7)
Gabriel and Rosenthal (2004) 37 metro areas, 1977–to–1995 avg.	-3.7 (1.3)
Chen and Rosenthal (2006) 293 metro areas, 2000	1.2 (0.8)
B. Correlation with Savageau (2000) ranking, 327 metro areas	
Spearman's Rank Correlation (p-value)	
Overall Ranking	0.42 (p=0.00)
Climate	0.10 (p=0.08)
Transportation	0.39 (p=0.00)
Education	0.31 (p=0.00)
Healthcare	0.20 (p=0.00)
Crime	-0.05 (p=0.38)
The Arts	0.49 (p=0.00)
Recreation	0.34 (p=0.00)
C. Correlation with Sperling and Sander (2004) ranking, 329 metro areas	
Spearman's Rank Correlation (p-value)	
Overall Ranking	0.49 (p=0.00)
Climate	0.14 (p=0.01)
Transportation	0.38 (p=0.00)
Education	0.29 (p=0.00)
Health & Healthcare	-0.40 (p=0.00)
Crime	0.18 (p=0.00)
Arts & Culture	0.43 (p=0.00)
Leisure	0.59 (p=0.00)
Attractiveness/Heritage/Ease of Living	0.31 (p=0.00)

Modeled elasticites are measured at a unitary density. The absolute value of elasticity decreases rapidly as density increases. Blomquist et al. and Gyourko and Tracy elasticities are with density in 1990. Remaining estimated elasticities are with density in 2000.



A. Elasticity of Substitution, Housing and Non-Housing

B. Elasticity of Substitution with Leisure

Figure 1. Calibration of Consumption Amenities. Panel A: Dots plot aggregate share of consumption devoted to shelter in each of 24 large metro areas (BLS Consumer Expenditure Survey, 1997–to–2002 average) against Torto-Wheaton multi-unit rental price index (1997–to–2002 average). Lines represent expected housing shares against the price index for each of three elasticity parameters. **Panel B**: Bold line plots actual leisure share of time for each of four years. Remaining lines plot expected leisure share given the real wage in each year (BLS hourly compensation divided by CPI) for each of three elasticity parameters.

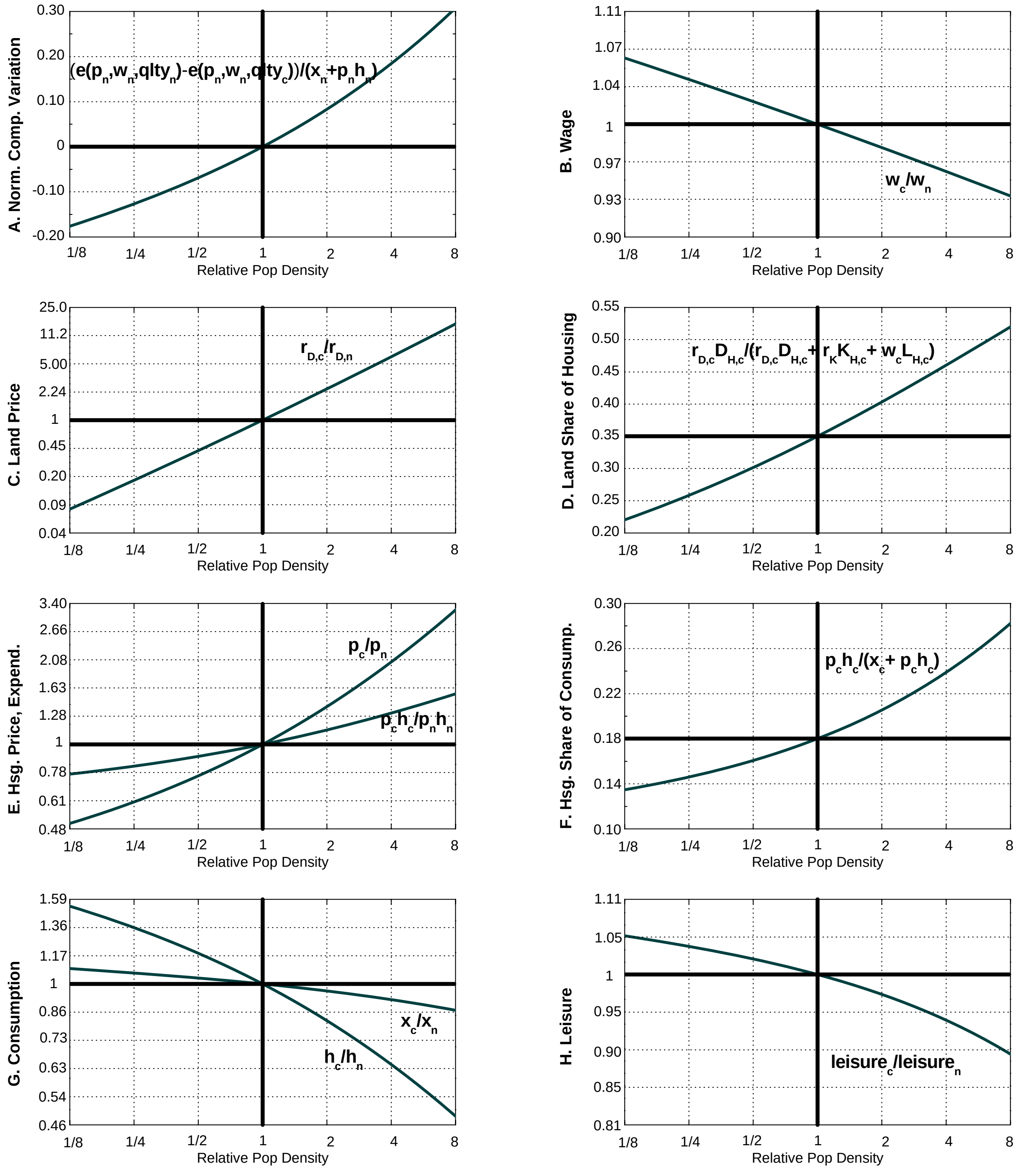


Figure 2. Amenity-Driven Crowding. Panel A shows the difference between city-economy and national-economy quality of life, measured as a compensating transfer to national-economy residents as a share of their income, required to achieve different relative densities under the baseline calibration. Remaining panels show implied ratios of various endogenous variables. Horizontal axes are plotted using a log scale. Vertical axes are also plotted using a log scale, except in panels A, D, and F.

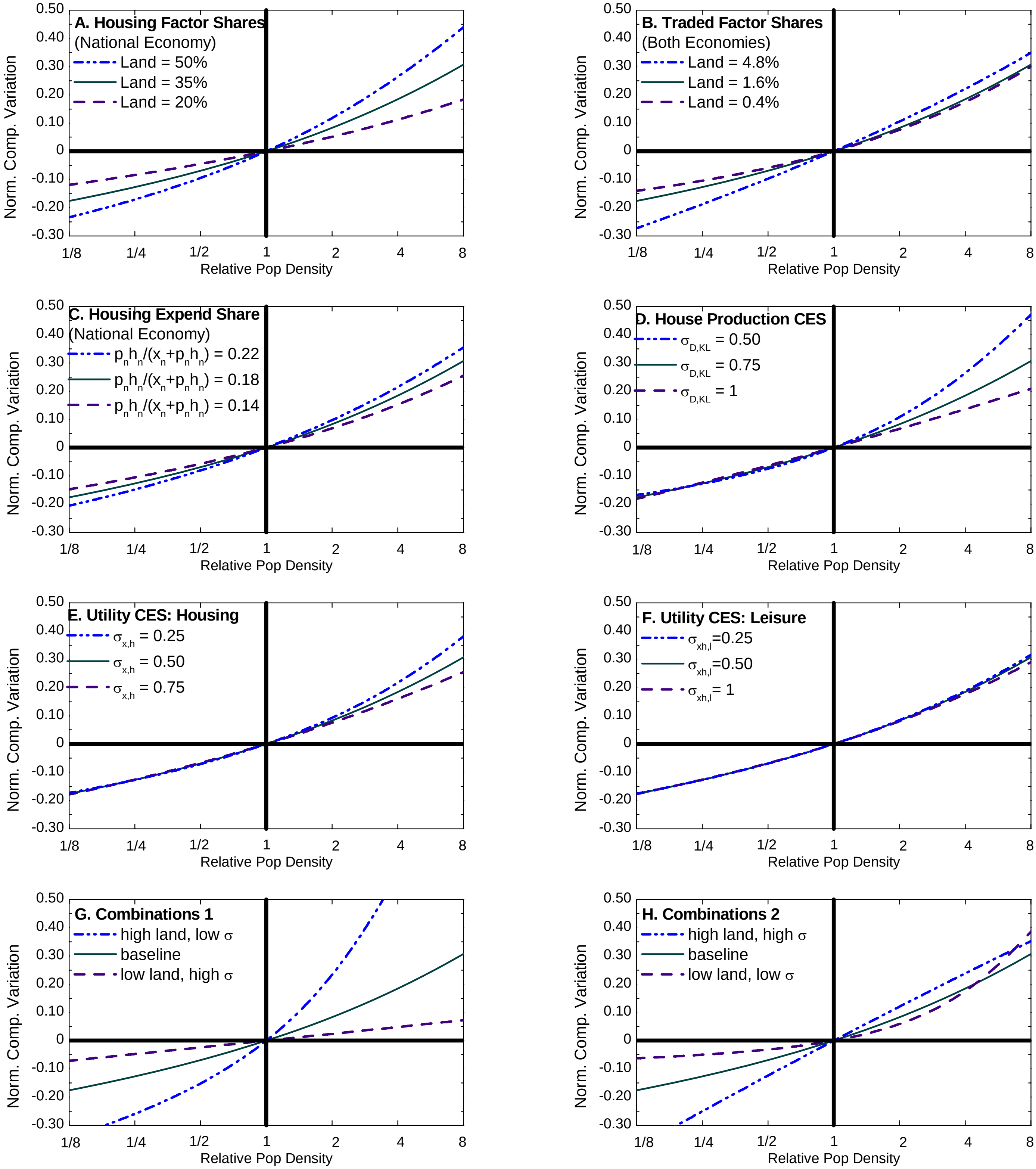


Figure 3. Sensitivity of Amenity-Driven Crowding. Panels show the difference between city-economy and national-economy quality of life required to achieve different relative densities under perturbations from the baseline calibration. Horizontal axes are plotted using a log scale.

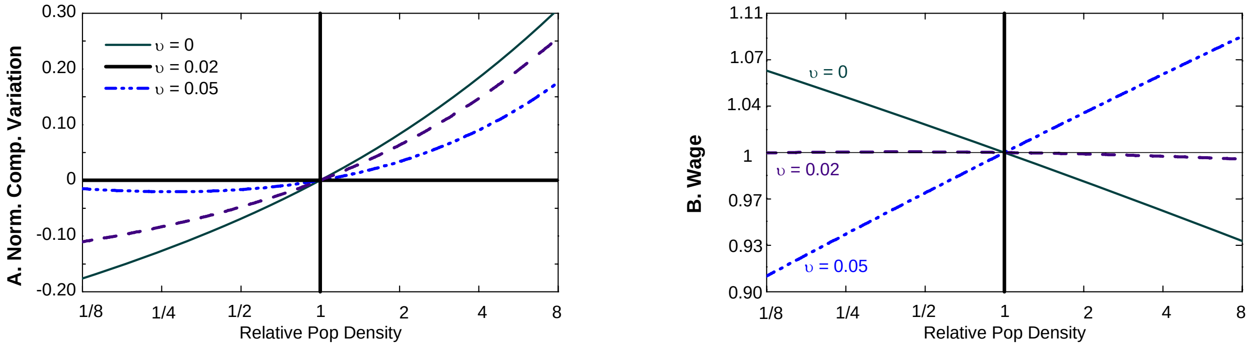


Figure 4. Amenity-Driven Crowding with Endogenous Productivity. Loci assume alternative endogenous TFP elasticities---for both the traded good and housing---with respect to density, υ_{TFP}. All parameters are set at their baseline value. In Panel B, the locus for υ_{TFP} equal to 0.02 lies along the horizontal axis.

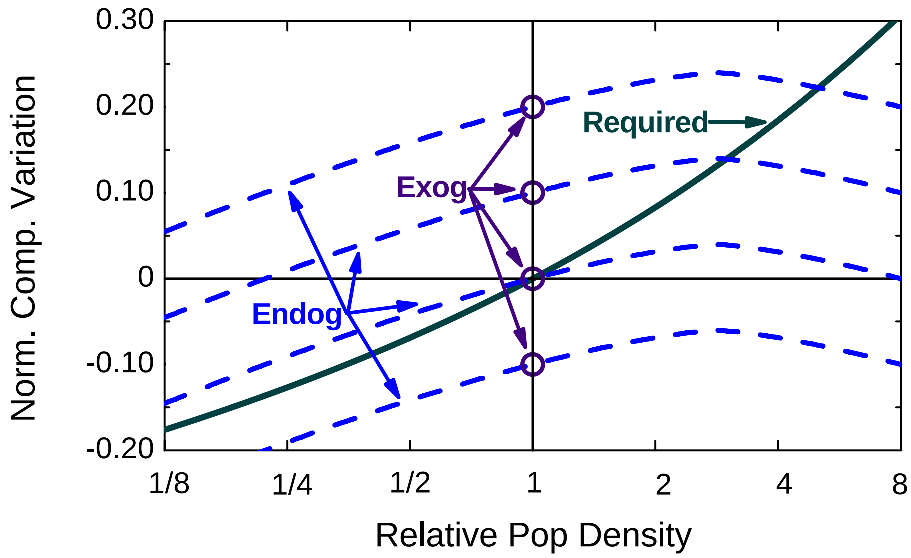


Figure 5. Endogenous Quality of Life. The normalized CV at a unitary density measures exogenous quality of life. The intersection of the exogenous and required locci determines actual density.