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The Economics of Two-Sided Payment Card Markets: Pricing, Adoption and Usage

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Abstract

This paper provides a new theory for two-sided payment card markets by positing better microfoundations. Adopting payment cards by consumers and merchants requires a fixed cost, but yields lower marginal costs of making payments. Considering this together with the heterogeneity of consumer income and merchant size, our theory derives card adoption and usage pattern consistent with cross-section and time-series evidence. Our analyses also help explain the observed card pricing pattern, particularly the rising merchant (interchange) fees over time. This is because a private card network, besides internalizing the two-sided market externality, has the incentive to inflate the card transaction value. We show that privately determined card pricing, adoption and usage tend to deviate from the social optimum, and imposing a ceiling on interchange fees may improve consumer welfare.

Keywords: Payment Cards, Two-sided Market, Interchange Fees

JEL Classification: L10, D40, O30

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1 Introduction

1.1 Motivation

As credit and debit cards become increasingly prominent forms of payments, the structure and performance of payment card markets are drawing increasing scrutiny. Many controversial issues are raised about interchange fees – the fees paid to card issuers when merchants accept their credit or debit cards for purchase.¹ Interchange fees are typically set by card networks and in many instances they are considered by competition authorities to be too high.² Particularly, interchange fees in the US are among the highest in the world, and they have been increasing in recent years despite falling costs in the card industry.³

Following the pioneering work of Baxter (1983), an important antitrust literature has discussed the potential anticompetitive effects of the collective determination of interchange fees within payment card systems. However, no systematic theoretical analysis was available until very recently when several formal models of the payment card industry were developed (e.g., Schmalensee 2002, Rochet and Tirole 2003, Wright 2003, 2004). These models aim to provide a more rigorous analysis of pricing and volumes in card payment systems. The framework that they use highlights the existence of common patterns between this industry and other network industries including the Internet, media, video games and software, which have been termed as "two-sided" markets.

The two-sided market theories emphasize the fundamental externality in card payment sys-

¹Visa and MasterCard provide card services through card-issuing banks and merchant-acquiring banks, and are called "four-party" systems. Amex and Discover primarily handle card issuing and acquiring by themselves, and are called "three-party" systems. In both systems, merchants are charged fees for accepting card payments. In a "three-party" system, merchant fees directly go to the card network. In a "four-party" system, merchant fees are split between card issuers and merchant acquiers, with issuers receiving the lion's share called interchange fees. In this paper, we use interchange fees and merchant fees interchangeably.

²Around the world, many competition authorities and central banks have recently taken action on the interchange pricing. Particularly, in Australia, the Reserve Bank of Australia mandated a sizeable reduction in credit-card interchange fees in 2003, and is currently re-evaluating the regulation. Meanwhile, EU, UK, Belgium, Israel, Poland, Portugal, Mexico, New Zealand, Netherlands, Spain and Switzerland have made similar decisions and moves. Action on interchange fees in the US has been mainly driven by private litigation. In 2005, about 50 antitrust cases were filed contesting interchange fees, claiming nearly \$1 trillion damages. Due to the similarity of the actions, they have been consolidated into a single case which is ongoing (Weiner and Wright 2006).

³The current credit card interchange rates, varying by merchants' business type, average approximately 1.75 percent of transaction value in the US. In 2007, American card issuers made \$42 billion (about \$370 per household) from interchange fees (Wang, 2007).

tems. Every card transaction necessarily involves two users: a cardholder and a merchant. Cardholders benefit from their holding a card only if their cards are accepted by a wide range of merchants, and merchants benefit from the card only if a sufficient number of consumers use it. Therefore, it is reasonable for the card network to price differently to cardholders and merchants in order to effectively balance the demand on the two sides of the market.⁴

Meanwhile, an important question is whether interchange fees or card fees in general could be set at the "wrong level." These theories show that, although the socially optimal and privately optimal levels of card fees both depend on the same factors (e.g., issuing costs, acquiring costs, cardholders' and merchants' demand elasticities, market structure, and bargaining power of the parties), they are not equal in general. However, given various complications of the models, including imperfect competition of merchants, there is generally no way to tell that the card fees are systematically too high or too low, as compared with socially optimal levels.

While these two-sided market theories advanced our understanding of the card payment systems, there are weaknesses in their microfoundations. First, these theories typically assume payment cards charge fixed-dollar fees. However, in reality, it is those cards charging proportional fees that have pricing controversies.⁵ Second, they assume a distribution of "convenience benefits" from the use of a payment card for both sides of the market, but do not really explain where those benefits come from. As a result, those benefits are often referred to in nonpecuniary terms, and consumers have a fixed demand for goods invariant of their payment choices. While that assumption may fit other two-sided markets, it does not describe payment demand well, as consumers derive their demand for payment services from their underlying demand to purchase a good or service. In other words, consumers' purchasing power depends on their payment choices, and the monetary benefit has always been the most important consideration for

⁴Payment card systems are not the only case of such two-sided markets. Rochet and Tirole (2003) provide a detailed analysis of other examples, such as the software industry, video games, internet portals, medias, and shopping malls. In all these industries as well, the platforms may price differently to each side of the markets in order to balance the demand, while making a profit overall.

⁵There are four types of general purpose payment cards in the US: (1) credit cards; (2) charge cards; (3) signature debit cards; and (4) PIN debit cards. The first three types of cards are routed over credit card networks and account for 90% of total card purchase volume. They charge proportional fees. In contrast, the PIN debit cards are mainly routed over EFT networks and charge fix dollar fees. In reality, it is the cards charging proportional fees that have pricing controversies.

consumers to choose among different payment options. A similar argument holds for merchant benefits. Third, these theories typically assume merchants engage in a special form of imperfect competition (e.g., Hotelling). While this is handy for considering merchants' business stealing motives for accepting cards, it complicates the overall picture and makes the social welfare analysis difficult. In fact, when one combines the unspecified merchant/consumer benefits from the use of a payment device with the strategic effects of imperfect competition and differences in bargaining power, the welfare analyses in the existing theories are quite formidable.

1.2 A New Approach

In this paper, we provide a new analysis of two-sided payment card markets by positing better microfoundations. First, to be consistent with the reality, we assume payment cards charge proportional fees. Second, the payment methods that consumers use do not yield utility directly, but instead imposes a frictional cost on their purchases. This means the monetary characteristics of payment devices are their only raison d' \hat{e} tre.⁶Third, in contrast to the existing model, we assume a contestable market for merchants, which greatly simplifies the welfare analysis.

With a set of better microfoundations, our approach yields clear and testable hypotheses about the adoption and usage pattern of two-sided payment card markets. Consider the introduction of a payment device with a high fixed but low variable cost of use (for both merchants and consumers). More affluent consumers, with higher levels of consumption and purchases, will choose to adopt the device prior to less affluent consumers. For merchants, facing a similar adoption decision, the larger merchants, or those who sell a higher valued good, will adopt the device earlier than other merchants. Over time, as card adoption/service costs fall and con-

⁶By focusing on the moneyness of payment devices, we might be criticized for overlooking nonmonetary benefits consumers or merchants might derive from their use. We offer three defences. First, the monetary nature of payment devices is arguably their primary purpose. Second, many convenience benefits of payment devices (e.g., protection from theft or saving of time), are closely related with the income and spending of the consumer, and are therefore better captured by our model through the variable cost of use of the payment device. Third, it may be appropriate to model both the monetary and other, direct, benefits of a payment device. But we believe that only by first investigating the adoption pattern of a monetary payment device purely via its derived demand can we understand the circumstances under which sellers of payment devices will choose to employ a strategy of tying a direct benefit (not related to the income of the consumer) to the use of the device, and determining on which side of the market those benefits might be offered. By overlooking the monetary nature of payment devices, one is apt to misunderstand the basic asymmetry between the economic roles of the consumer and the merchant.



Figure 1: Household Credit Card Adoption by Income Quintile

sumer incomes rise, payment cards are then adopted by lower-income consumers and smaller merchants. These predictions are consistent with empirical evidence (see Figures 1 and 2).⁷ In contrast, the literature that overlooks the monetary nature of payment devices does not yield such straightforward empirical conclusions.

In the payment card context, it has previously been pointed out (e.g., Wright 2003) that a merchant serving both cash and card consumers would be competed out of business. However, we find that large merchants who serve both cash and card customers do survive the threat of entry from specialized merchants.⁸ Based on contestable merchants and on payment card adoption costs, our equilibrium is characterized by three categories of merchant sizes. Large merchants adopt payment cards and set a price that is lower than cash-only merchants. So the large merchants attract customers who pay either with cash or the card. Medium size merchants, in contrast, are specialized. Some of them accept only cash. The others accept cards, but set

⁷Data source: Evans and Schmalensee (2005), *Paying with Plastic*, 2nd edition.

⁸They survive the threat of entry because of the presence of fixed card adoption costs for merchants. Because the adoption costs can be spread over a large volume of transactions, and because the variable fees of card use are less than the variable costs of handling cash, large merchants who adopt cards can offer lower prices than cash-only specialized merchants.



Figure 2: Payment Card Share of Transaction Volume by Merchant Type

a price that is higher than the competing cash-only merchants. They attract only consumers that use cards, because cards are cost-saving to them overall. Finally, small merchants are all cash-only merchants. These predictions are broadly consistent with what we observe in reality, but are not implied by the existing theories.

More important, our analyses show that as payment card markets evolve, merchant (interchange) fees increase while card service costs decline, a puzzle pointed out in Hayashi (2005) and Weiner and Wright (2006). This can be explained by the card network's incentives to balance the "two-sided market effects" and the "inflation effect." By "two-sided market effects", we mean the card network needs to attract both merchants and consumers to adopt and use its cards. Meanwhile, the card network may also want to inflate the card transaction value in order to boost its demand, which we call the "inflation effect." Because lowering card fees to consumers but raising them to merchants helps to inflate the value of card transactions, it gives the card network additional incentives to pursue high interchange fees. As card service costs decline over time, the card network is able to further raise interchange fees and extract more profits out of the system. This finding is consistent with Wang (2007), who found the "inflation effect" also exists in mature card markets without adoption externalities.

Using the model, we are able to systematically compare the differences between privately and socially optimal card fees: Because the card network makes profit from serving card users, it does not care about cash users in its pricing decision. Meanwhile, lowering card fees to consumers help inflate the value of card transactions so the network prefers high merchant fees. In contrast, the social planner cares about both card users and cash users, and prefers lower merchant fees because this helps to lower retail prices and raise consumers' real purchases. Therefore, these are the fundamental differences between the private network and the social planner in dealing with the "two-sided market effects" and the "inflation effect."

Meanwhile, we found that under the private network, cards are not over adopted and used as suggested by many existing models. Rather, because the card network charges a high markup, it leads to a lower card adoption and usage compared with the social optimum. Also, we found that cash users are disadvantaged under the private network. However, this is not because they have to subsidize card users as suggested by many existing theories (In fact, cash users are subsidized by card uses because they use less efficient payments), but rather because under the private network, fewer stores serve both card and cash users, so fewer cash users get subsidized by card users who use more efficient payments.⁹

1.3 Road Map

In the next section we lay out our model in greater detail and derive some preliminary results. In section 3 we analyze the equilibria of our model, and compare the outcomes under a monopoly card network, under a Ramsey social planner, and under an "interchange fee ceiling" policy. In section 4 we offer concluding remarks.

⁹Note in this paper, the "cash users" refer to the consumers who use paper payments such as cash or check; the "card users" refer to the consumers who use electronic payments such as debit or credit cards. As empirical evidence shows, electronic payments are more cost efficient than paper payments. It is worth mentioning that if we apply our model to comparing two electronic payments, say credit cards vs. debit cards, we may need to adjust some of the model assumptions accordingly, e.g., some empirical evidence suggests that merchants may incur higher costs serving customers using credit cards than serving those using debit cards.

2 The Model

Our model studies pricing, adoption and usage of monetary payment devices. We first lay out the environment in which only one payment device, referred to as cash, is in use.¹⁰ Then we will consider the introduction of an alternative device, which we refer to as a payment card.

We model the consumers as having generalized Cobb-Douglas preferences across a range of goods. They take prices as given. Each consumer is endowed with income, which is distributed across the population of consumers according to known cumulative distribution function. The merchant side of our model is quite stylized. Each merchant competes in a contestable market for a single good that the merchant sells, and prices are set at the zero profit level.

Consumers and merchants are both presented with the option to adopt a new payment device that offers a lower variable cost of use, but a higher fixed cost relative to the pre-existing alternative. They each make their optimal adoption decision taking the other's choice as given. The model yields a two-sided market under our structure of the heterogeneity of consumer income and merchant size, the fixed adoption costs, and, finally, under the assumption of *price coherence* by merchants that accept both payment devices.¹¹

2.1 Pre-card Market Environment

The economy is composed of a continuum of merchants of measure unity. Each merchant sells a distinct product α in a contestable market. Let c_{α} be non-payment cost for good α . Merchants incur transaction cost τ_m per dollar for accepting cash, which includes handling, safekeeping and fraud expenses. The competition requires zero profit, so the cash price for good α is determined as $p_{\alpha,c}$, where

$$(1 - \tau_m)p_{\alpha,c} = c_\alpha \Longrightarrow p_{\alpha,c} = \frac{c_\alpha}{1 - \tau_m}.$$
(1)

A consumer, indexed by her income I, has generalized Cobb-Douglas preferences across a

 $^{^{10}\}mathrm{Note}$ cash here refers to paper payments broadly, for example, cash and check.

¹¹The assumption of "price coherence" requires that merchants who adopt cards cannot price discriminate based on the consumer's choice of payment method. This assumption is a common one in the payment card literature. A restriction on price discrimination is used in many areas of economics, and is an empirically testable assumption.

variety of goods. She maximizes her utility subject to her income I:

$$U = Max \int_0^{\overline{\alpha}} \alpha \ln x_{\alpha} dG(\alpha) \qquad s.t. \qquad \int_0^{\overline{\alpha}} (1 + \tau_c) p_{\alpha,c} x_{\alpha,I} dG(\alpha) = I.$$

where $\alpha \in (0, \overline{\alpha})$ is the preference parameter distributed with cdf $G(\alpha)$, $x_{\alpha,I}$ is her quantity of demand for good α , and τ_c is the transaction cost to the consumer for using cash.

Therefore, the demand and spending of consumer I on good α can be determined as

$$x_{\alpha,I} = \frac{\alpha I}{(1+\tau_c)p_{\alpha,c}E(\alpha)}, \quad p_{\alpha,c}x_{\alpha,I} = \frac{\alpha I}{(1+\tau_c)E(\alpha)}.$$

Across consumers, the income $I \in (0, \overline{I})$ is distributed with cdf function F(I) and mean E(I). Normalize the aggregate measure of consumer to be unity. At equilibrium, market supply equals demand, so the market quantity and value for product α are as follows:

$$x_{\alpha} = \frac{\alpha E(I)}{(1+\tau_c)p_{\alpha,c}E(\alpha)}, \quad p_{\alpha,c}x_{\alpha} = \frac{\alpha E(I)}{(1+\tau_c)E(\alpha)}.$$

2.2 Card Adoption and Usage

At time T, a payment innovation, referred to as a payment card, is introduced. The payment card service is provided by a monopoly card network, who charges merchants and consumers a proportional fee f_m and f_c respectively.¹² Figure 3 provides an intuitive illustration of the card system. For the card network, the costs of providing the payment card service to merchants and consumers are d_m and d_c respectively. For merchants and consumers, there is a per-period adoption cost k_m (e.g., a fixed cost of renting card-processing equipment) and k_c (e.g., a fixed cost of maintaining bank balance or credit score).¹³ At equilibrium, large merchants and wealthy

¹²For simplicity, we model "three-party" card systems in this paper, but our results can be equally applied to "four-party" systems.

¹³Here we assume merchants and consumers incur zero variable costs for using cards except paying card fees f_m and f_c . This is an innocuous normalization, consistent with the fact that electronic payments are more efficient. For example, merchants may incur variables costs due to cash/check fraud, but their payments are guaranteed when accepting cards.



Figure 3: Illustration of A "Three-Party" Payment Card System

consumers have an advantage in adopting the payment card. This can be shown in the following equilibrium that we construct: Given merchants $\alpha \ge \alpha_0$ accept the card, consumers of income $I \ge I_0$ would like to adopt the card, and vice versa.

2.2.1 Merchants' Choice

Merchants take consumers' card adoption as given when making their card acceptance decision. Given competition in contestable markets, merchants fall into three categories based on their transaction value: (1) Large merchants ($\alpha \ge \alpha_1$) accept payment cards and charge price $p_{\alpha,d} \le p_{\alpha,c}$ so they are patronized by both card and cash customers; (2) Intermediate merchants ($\alpha_0 \le \alpha < \alpha_1$) specialize. They either accept payment cards and charge $p_{\alpha,d}$, where $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \ge p_{\alpha,d} > p_{\alpha,c}$, so they are patronized only by card customers, or they do not accept payment cards and charge $p_{\alpha,c}$ so they only serve cash customers. (3) Small merchants ($\alpha < \alpha_0$) do not accept payment cards and charge $p_{\alpha,c}$, so all customers shop there with cash.

As we will show next, the thresholds α_0 and α_1 are endogenously determined, particularly by the card service fees f_m and f_c charged by the card network. Moreover, because merchants who accept payment cards still have to accept cash, the card network can never charge consumers a card fee higher than cash cost, in other words, $\tau_c \ge f_c$ has to hold. Otherwise, no consumer will ever use a card.

Category (1): $\alpha \geq \alpha_1$ Merchants in this category charge $p_{\alpha,d} \leq p_{\alpha,c}$ and receive revenue from both card and cash customers:

$$p_{\alpha,d}x_{\alpha,d}^{card} = \frac{\alpha[E_{I>I_0}(I-k_c)]}{E(\alpha)(1+f_c)}, \quad p_{\alpha,d}x_{\alpha,d}^{cash} = \frac{\alpha[E_{I$$

where $E_{I>I_0}(I) \equiv \int_{I_0}^{\overline{I}} I dF(I)$.

Contestability requires zero profit so that revenue equals cost,

$$(1 - f_m)p_{\alpha,d}x_{\alpha,d}^{card} + (1 - \tau_m)p_{\alpha,d}x_{\alpha,d}^{cash} = c_\alpha x_{\alpha,d}^{card} + c_\alpha x_{\alpha,d}^{cash} + k_m.$$
(3)

Equations (2) and (3) pin down the price $p_{\alpha,d}$:

$$p_{\alpha,d} = \frac{c_{\alpha} \frac{\alpha[E_{I > I_0}(I - k_c)]}{(1 + f_c)} + c_{\alpha} \frac{\alpha[E_{I < I_0}(I)]}{(1 + \tau_c)}}{(1 - \tau_m) \frac{\alpha[E_{I > I_0}(I - k_c)]}{1 + f_c} + (1 - \tau_m) \frac{\alpha[E_{I < I_0}(I)]}{1 + \tau_c} - k_m E(\alpha)}.$$
(4)

Therefore, merchants $\alpha \geq \alpha_1$ may exist and charge $p_{\alpha,d} \leq p_{\alpha,c} = \frac{c_\alpha}{1-\tau_m}$, where

$$\alpha_1 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+f_c})}.$$
(5)

Note Eq (5) suggests that there is no merchant in this category if $f_m > \tau_m$.

Category (2): $\alpha_0 \leq \alpha < \alpha_1$ Merchants in this category specialize. For each product, there are two merchants. One accepts payment cards and charges $p_{\alpha,d}$, where $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \geq p_{\alpha,d} > p_{\alpha,c}$, so it is patronized only by card customers. The other does not accept payment cards and charges $p_{\alpha,c}$ so it only serves cash customers.

A card merchant in this category receives revenues only from card customers and earns zero

profit, which implies

$$p_{\alpha,d} = \frac{c_{\alpha} \frac{\alpha[E_{I > I_0}(I - k_c)]}{(1 + f_c)}}{(1 - f_m) \frac{\alpha[E_{I > I_0}(I - k_c)]}{1 + f_c} - k_m E(\alpha)}.$$
(6)

Therefore, $\frac{1+\tau_c}{1+f_c}p_{\alpha,c} \ge p_{\alpha,d} > p_{\alpha,c}$ implies that merchants $\alpha_0 \le \alpha < \alpha_1$ are in this group, where α_1 is given in Eq (5) and α_0 is determined by

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})}.$$
(7)

Note Eqs (5) and (7) suggest that if $\frac{1-f_m}{1+f_c} < \frac{1-\tau_m}{1+\tau_c}$, no merchant accepts cards (i.e., no merchant in either category (1) or (2)).¹⁴

Category (3): $\alpha < \alpha_0$ Given $\frac{1-f_m}{1+f_c} \geq \frac{1-\tau_m}{1+\tau_c}$, small merchants $\alpha < \alpha_0$ are in the third category. Due to their small transaction value, accepting payment cards will result $p_{\alpha,d} > \frac{1+\tau_c}{1+f_c}p_{\alpha,c}$. Therefore, they only accept cash.

2.2.2 Consumers' Choice

An individual consumer takes market prices and merchants' card acceptance as given, and decides whether to adopt payment cards or not. Recall merchants in the market fall into different categories according to their sizes relative to the thresholds α_0 and α_1 . A consumer *I*, if not adopting a payment card, may enjoy utility V_c ,

$$V_c = \int_0^{\min(\alpha_1,\overline{\alpha})} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,c}E(\alpha)} dG(\alpha) + \int_{\min(\alpha_1,\overline{\alpha})}^{\overline{\alpha}} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,d}E(\alpha)} dG(\alpha).$$

In contrast, if she adopts a payment card, she enjoys utility V_d ,

$$V_d = \int_0^{\alpha_0} \alpha \ln \frac{\alpha (I - k_c)}{(1 + \tau_c) p_{\alpha,c} E(\alpha)} dG(\alpha) + \int_{\alpha_0}^{\overline{\alpha}} \alpha \ln \frac{\alpha (I - k_c)}{(1 + f_c) p_{\alpha,d} E(\alpha)} dG(\alpha).$$

Therefore, card adoption requires $V_c < V_d$, which implies

¹⁴Since $\tau_c \ge f_c$ has to hold for any consumer ever to use cards, $\frac{1-f_m}{1+f_c} < \frac{1-\tau_m}{1+\tau_c}$ also implies $f_m > \tau_m$.

$$E(\alpha)\ln(\frac{I}{I-k_c}) + \int_{\alpha_0}^{\min(\alpha_1,\overline{\alpha})} \alpha \ln(\frac{p_{\alpha,d}}{p_{\alpha,c}}) dG(\alpha) < E_{\alpha > \alpha_0}(\alpha)\ln(\frac{1+\tau_c}{1+f_c}),\tag{8}$$

where $E_{\alpha > \alpha_0}(\alpha) \equiv \int_{\alpha_0}^{\overline{\alpha}} \alpha dG(\alpha)$.

Equation (8) suggests an adopter's income has to be over the threshold level I_0 :

$$I \ge I_0 = \frac{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)}k_c}{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)} - \exp(\int_{\alpha_0}^{\min(\alpha_1,\overline{\alpha})} \alpha \ln(\frac{p_{\alpha,d}}{p_{\alpha,c}})dG(\alpha)/E(\alpha))}.$$

2.2.3 Two-sided Market Interactions

The interactions between consumers' card adoption and merchants' card acceptance can be summarized as follows. Given $\tau_c \ge f_c$ and $\frac{1-f_m}{1+f_c} \ge \frac{1-\tau_m}{1+\tau_c}$,

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})},\tag{9}$$

$$\alpha_1 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)](\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+f_c})} \quad \text{for} \quad f_m \le \tau_m,$$
(10)

$$I_0 = \frac{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)}k_c}{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)} - \exp\left(\int_{\alpha_0}^{\min(\alpha_1,\overline{\alpha})} \alpha \ln\left(\frac{p_{\alpha,d}}{p_{\alpha,c}}\right) dG(\alpha)/E(\alpha)\right)},\tag{11}$$

$$\left(\frac{p_{\alpha,d}}{p_{\alpha,c}}\right)_{\min(\alpha_1,\overline{\alpha})>\alpha\geqslant\alpha_0} = \frac{\frac{1-\tau_m}{1+f_c}[E_{I>I_0}(I-k_c)]}{\frac{1-f_m}{1+f_c}[E_{I>I_0}(I-k_c)] - \frac{E(\alpha)}{\alpha}k_m},\tag{12}$$

where Eq (12) follows Eqs (1) and (6). To simplify notations, we denote:

$$Z_1 = \left(\frac{1 - f_m}{1 + f_c} - \frac{1 - \tau_m}{1 + f_c}\right), \quad Z_0 = \left(\frac{1 - f_m}{1 + f_c} - \frac{1 - \tau_m}{1 + \tau_c}\right).$$

Accordingly, the card adoption thresholds (9) - (12) can be rewritten as follows:

$$\alpha_0 = \frac{E(\alpha)k_m}{[E_{I>I_0}(I-k_c)]Z_0},$$
(13)

$$\alpha_1 = \frac{Z_0}{Z_1} \alpha_0 \quad \text{for} \quad f_m \le \tau_m, \tag{14}$$

$$I_0 = \frac{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)}k_c}{\left(\frac{1+\tau_c}{1+f_c}\right)^{E_{\alpha > \alpha_0}(\alpha)/E(\alpha)} - \exp\left(\int_{\alpha_0}^{\min(\alpha_1,\overline{\alpha})}\alpha\ln\left(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0Z_0}\right)dG(\alpha)/E(\alpha)\right)}.$$
 (15)

2.3 Monopoly Network and Social Planner

Considering card adoption and usage externalities in the two-sided market, a monopoly card network or the social planner would each set card fees to achieve their respective goals: The former maximizes the network profit and the latter maximizes the consumer surplus.¹⁵

2.3.1 Monopoly Network's Problem

The monopoly card network would like to maximize its profit through card pricing (f_c, f_m) . Note in our model, which abstracts from the "four-party" model to a simpler three-party model, the card network plays the role of both acquiring and issuing. With an assumption of competitive, costless acquiring, the interchange fee would simply equal the merchant service fee. As a result, in our model the interchange fee is simply f_m . The card network solves the following problem:

$$\underset{f_c,f_m}{Max} \frac{E_{\alpha > \alpha_0}(\alpha) E_{I > I_0}(I - k_c)}{E(\alpha)(1 + f_c)} (f_c + f_m - d_m - d_c)$$

s.t. Eqs (13), (14), (15),

¹⁵Note we may alternatively assume the social planner cares about the sum of consumer surplus and network profit. However, this would not change our main results because in that case the social optimum would just be somewhere in between the network-profit-maximizing outcome and the consumer-surplus-maximizing outcome.

$$\tau_c \ge f_c, \quad \frac{1 - f_m}{1 + f_c} \ge \frac{1 - \tau_m}{1 + \tau_c}$$

The objective function clearly shows that the network has incentive to set card fees to internalize the two-sided market externality because both f_c and f_m affect the adoption thresholds α_0 and I_0 . This is consistent with findings in standard "two-sided market" models. However, in addition to that, the objective function also reveals an important "inflation effect" because lowering consumer fee f_c can inflate the card transaction value through the term $\frac{1}{(1+f_c)}$.

To see this more clearly, let us consider a specific example where consumer adoption cost $k_c = 0$, so all consumers own cards for free. As a result, the network would maximize its profit as follows:

$$\underset{f_c,f_m}{Max} \frac{E_{\alpha > \alpha_0}(\alpha)E(I)}{E(\alpha)} \frac{(f_c + f_m - d_m - d_c)}{(1 + f_c)}$$

s.t.
$$\alpha_0 = \frac{E(\alpha)k_m}{E(I)(\frac{1-f_m}{1+f_c} - \frac{1-\tau_m}{1+\tau_c})}$$

$$\tau_c \ge f_c, \quad \frac{1 - f_m}{1 + f_c} \ge \frac{1 - \tau_m}{1 + \tau_c}.$$

This example illustrates the card network's pricing incentives. As shown in the objective function, anything else being fixed, lowering the consumer fee f_c but raising the merchant fee f_m by the same amount would boost the network profit by $\frac{1}{(1+f_c)}$ due to the "inflation effect." However, there is a trade-off because doing so raises the merchant adoption threshold α_0 so negatively affects the term $E_{\alpha>\alpha_0}(\alpha)$. Therefore, the network needs to balance the "inflation effect" and the "merchant adoption effect" in order to maximize its profit. More generally, when $k_c > 0$, the network needs to balance the "inflation effect" and the "two-sided market adoption effects." In either case, the "inflation effect" provides the card network additional incentive to pursue high interchange fee f_m .

2.3.2 Social Planner's Problem

As a payment innovation, card adoption and usage improve social welfare. This can be shown in the following welfare comparison between a cash economy and a card economy.

Recall in a cash economy, an individual consumer I enjoys the utility level $U_{I,c}$:

$$U_{I,c} = \int_0^{\overline{\alpha}} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,c}E(\alpha)} dG(\alpha).$$

In an economy with payment cards, a consumer decides whether to adopt a payment card based on her income. For a card consumer $I \ge I_0$, her utility is

$$(U_{I,d})_{I \ge I_0} = \int_0^{\alpha_0} \alpha \ln \frac{\alpha (I - k_c)}{(1 + \tau_c) p_{\alpha,c} E(\alpha)} dG(\alpha) + \int_{\alpha_0}^{\overline{\alpha}} \alpha \ln \frac{\alpha (I - k_c)}{(1 + f_c) p_{\alpha,d} E(\alpha)} dG(\alpha),$$

while for a cash consumer $I < I_0$, her utility is

$$(U_{I,d})_{I < I_0} = \int_0^{\min(\alpha_1,\overline{\alpha})} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,c}E(\alpha)} dG(\alpha) + \int_{\min(\alpha_1,\overline{\alpha})}^{\overline{\alpha}} \alpha \ln \frac{\alpha I}{(1+\tau_c)p_{\alpha,d}E(\alpha)} dG(\alpha).$$

Therefore, an individual receives different welfare gain depending on her income. For a card consumer $I \ge I_0$, her welfare gain is

$$(U_{I,d} - U_{I,c})_{I \ge I_0} = \int_{\alpha_0}^{\min(\alpha_1,\overline{\alpha})} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) dG(\alpha) + \int_{\min(\alpha_1,\overline{\alpha})}^{\overline{\alpha}} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) dG(\alpha) \qquad (16)$$
$$+ E_{\alpha \ge \alpha_0}(\alpha) \ln(\frac{1+\tau_c}{1+f_c}) + E(\alpha) \ln(\frac{I-k_c}{I}),$$

while for a cash consumer $I < I_0$, her welfare gain is

$$(U_{I,d} - U_{I,c})_{I < I_0} = \int_{\min(\alpha_1, \overline{\alpha})}^{\overline{\alpha}} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) dG(\alpha).$$
(17)

Equations (16) and (17) are intuitive: A card consumer enjoys utility gains from card merchants in both categories (1) and (2), subject to card adoption and usage costs; a cash consumer only benefits from lower prices charged by merchants in category (1) if those merchants exist. Given the above utility measures, the social planner would like to maximize consumer welfare gains subject to the adoption incentive constraints of merchants and consumers as well as the network balanced-budget constraint (Ramsey pricing). The social planner's problem is as follows:

$$\underset{f_{c},f_{m}}{Max} \quad \int_{0}^{\overline{I}} (U_{I,d} - U_{I,c}) dF(I)$$

s.t.
$$Eqs$$
 (4), (12), (13), (14), (15), (16), (17)

$$\tau_c \ge f_c, \quad \frac{1-f_m}{1+f_c} \ge \frac{1-\tau_m}{1+\tau_c}, \quad f_m + f_c \ge d_c + d_m.$$

3 Market Evolution

To explore the implications of our model on card market evolution, we consider an explicit example. Assume $\alpha \in (0,1)$ is uniformly distributed where $E(\alpha) = 1/2$, and $I \in (0,\infty)$ is exponentially distributed where $F(I) = 1 - e^{(-\lambda I)}$ and $E(I) = 1/\lambda$.¹⁶ Note that $E_{\alpha > \alpha_0}(\alpha) = \frac{1-\alpha_0^2}{2}$ and $E_{I>I_0}(I-k_c) = e^{-\lambda I_0}(\frac{1}{\lambda} + I_0 - k_c)$.

3.1 Short-run (Transitional) Dynamics

As expected, two-sided market interactions in our model suggest multiple equilibria. Assume $\alpha_1 < \bar{\alpha} = 1.^{17}$ Eqs (13) and (15) can be rewritten into

$$\alpha_0 = \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)Z_0},$$
(L1)

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)},\tag{L2}$$

¹⁶We also tried other distribution assumptions (e.g., both consumer income and merchant size are exponentially distributed), and the findings are similar.

¹⁷The numerical simulations shows $\alpha_1 < 1$ generally holds.



Figure 4: Interaction of Merchants and Consumers in Card Adoption

where
$$S = \ln \frac{1+f_c}{1+\tau_c} + \frac{Z_0(1+f_c)}{(1-f_m)} (\frac{Z_0}{Z_1} - 1) + \frac{Z_0^2(1+f_c)^2}{(1-f_m)^2} \ln(\frac{\frac{Z_0}{Z_1} - \frac{Z_0(1+f_c)}{(1-f_m)}}{1 - \frac{Z_0(1+f_c)}{(1-f_m)}})$$
 (see Appendix A for the proof).

Characterizing Eq (L1), we have

$$\begin{aligned} \alpha_0|_{I_0 \longrightarrow 0} &\longrightarrow \frac{k_m}{2(\frac{1}{\lambda} - k_c)Z_0} > 0, \quad \alpha_0|_{I_0 \to \infty} \longrightarrow \infty, \\ &\frac{d\alpha_0}{dI_0} > 0, \quad \frac{d^2\alpha_0}{dI_0^2} > 0. \end{aligned}$$

Characterizing Eq (L2), we have

$$I_0|_{\alpha_0 \to 0} \to \frac{\left(\frac{1+\tau_c}{1+f_c}\right)k_c}{\left(\frac{1+\tau_c}{1+f_c}\right) - 1} > 0, \quad \alpha_0|_{I_0 \longrightarrow \infty} \longrightarrow \alpha^u = \left(\frac{\ln\left(\frac{1+\tau_c}{1+f_c}\right)}{\ln\left(\frac{1+\tau_c}{1+f_c}\right) + S}\right)^{1/2} < 1,$$
$$\frac{d\alpha_0}{dI_0} > 0, \quad \frac{d^2\alpha_0}{dI_0^2} < 0.$$

Figure 4 illustrates the interactions between merchants and consumers for card adoption and the resulting transitional dynamics. For a given pricing pair (f_m, f_c) , there exist two steady states with positive levels of card adoption (the no adoption outcome is a steady state as well): a high-adoption equilibrium (I_0^*, α_0^*) and a low-adoption equilibrium (I_0', α_0') . The high equilibrium is stable but the low equilibrium is not. As a result, the card network has incentive to push the card adoption to overcome the low equilibrium. Our analysis suggests if the initial card adoption is high enough, the market will evolve to the high equilibrium. Otherwise, card adoption may fail, and suffer no adoption.

3.2 Long-run Evolution

As shown above, there is one unique high-adoption equilibrium for a given card pricing pair (f_m, f_c) , which is in turn determined by the model parameters $(d_c, d_m, k_c, k_m, 1/\lambda)$. Consequently, the equilibrium market evolution is a series of comparative statics of high-adoption equilibria driven by changing parameter values. Given the complexity of the problem, our following analysis will mainly rely on numerical simulations.

Due to technological progress, card service costs (d_c, d_m) typically decline over time. In the benchmark simulation, we show how the market equilibrium evolves as the card service costs $d_c + d_m$ fall.¹⁸ We then adjust the values of k_c , k_m and $1/\lambda$ to see how the market evolution would also be affected by changing card adoption costs and consumer income.

3.2.1 Monopoly Outcome

At market equilibrium, the card network solves the following problem to maximize its profit:

$$\underset{f_c,f_m}{Max} \quad e^{(-\lambda I_0)} (\frac{1}{\lambda} + I_0 - k_c) (\frac{1 - \alpha_0^2}{1 + f_c}) (f_c + f_m - d_c - d_m)$$

s.t.
$$\alpha_0 = \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda} + I_0 - k_c)Z_0}$$

¹⁸Note only the sum $d_c + d_m$ matters in our analysis because d_c and d_m are not distinguishable from each other.

$$\alpha_1 = \frac{Z_0}{Z_1} \alpha_0 \quad \text{for} \quad f_m \le \tau_m,$$

$$I_{0} = \frac{(\frac{1+\tau_{c}}{1+f_{c}})^{1-\alpha_{0}^{2}}k_{c}}{(\frac{1+\tau_{c}}{1+f_{c}})^{1-\alpha_{0}^{2}} - \exp(2\int_{\alpha_{0}}^{\min(\alpha_{1},1)}\alpha\ln(\frac{(1-\tau_{m})\alpha}{(1-f_{m})\alpha - (1+f_{c})\alpha_{0}Z_{0}})d\alpha)},$$
$$\tau_{c} \ge f_{c}, \quad \frac{1-f_{m}}{1+f_{c}} \ge \frac{1-\tau_{m}}{1+\tau_{c}}.$$

The simulation results, shown in Fig. A1 in the Appendix, are summarized as follows. In the benchmark simulation (Case 1), we set $\tau_m = 0.05$, $\tau_c = 0.05$, $k_m = 160$, $k_c = 160$, $1/\lambda = 10,000$. The results show that as the card service costs $d_m + d_c$ fall, the merchant card fee f_m increases, the consumer card fee f_c decreases, and the card pricing markup $f_c + f_m - d_c - d_m$ increases. More merchants accept card (α_0 decreases), more consumers use card (I_0 decreases), but fewer card merchants are patronized by cash users (α_1 increases). As a result, the card users gain more welfare, cash users gain less welfare, and the total consumer welfare increases. In monetary terms, merchants and consumers spend more on card services to pay for an increasing card transaction value, and the card payment spending to card sales ratio decreases.¹⁹ Meanwhile, cash transaction value declines and the cash cost to sales ratio is fixed at $\tau_m + \tau_c$. In total, the society pays less for payment services, and the total payment spending to total sales ratio decreases.

We then study the effects of k_m and k_c . In the simulation of Case 2, we reduce both k_c and k_m by equal proportion compared to the benchmark case, e.g., set $k'_c = k'_m = 128$ so $k'_c/k_c = k'_m/k_m = 0.8$. The results show, compared to the benchmark case, the network now charges higher card fees f_m and f_c , so the card markup is higher. In spite of higher card fees, the lower adoption costs induce more merchants and consumers to adopt card compared to the benchmark case. Consequently, the card consumers enjoy more welfare gains. In monetary terms, merchants and consumers now spend more on card adoption and service fees to pay for a

¹⁹The card service spending includes both card adoption costs and service fees for consumers and merchants.

higher card transaction value, and the card payment spending to card sales ratio is higher than the benchmark case, while cash costs and cash transaction value are lower. The total payment spending to sales ratio could be higher or lower than the benchmark case depending on the value of card service costs $d_m + d_c$.

In Case 3, we raise the consumer income $1/\lambda' = 12500$ so that $\lambda'/\lambda = 0.8$. The simulation results are equivalent to Case 2 in terms of card pricing, adoption and welfare gains. Meanwhile, a higher income leads to more spending on card and cash payment services, and more card and cash transaction value than Case 2, while the payment spending to sales ratios are the same.

In fact, it is not totally a surprise that the growth of income $(1/\lambda)$ and the decline of adoption costs (k_c, k_m) have equivalent effects on card pricing and adoption. A formal proof is as follows.

Proposition 1 Under the parameter values $(1/\lambda, k_c, k_m)$ and $(\theta/\lambda, \theta k_c, \theta k_m)$, the network profit maximization yields the same card prices and adoption rates.

Proof. See Appendix A for the proof.

3.2.2 Social Optimum

The social planner maximizes the consumer surplus subject to adoption incentive constraints of merchants and consumers as well as the network balanced-budget constraint:

$$\underset{f_c,f_m}{Max} \quad \int_0^\infty (U_{I,d} - U_{I,c}) \lambda e^{-\lambda I} dI$$

s.t.
$$(U_{I,d} - U_{I,c})_{I \ge I_0} = \int_{\alpha_0}^{\min(\alpha_1, 1)} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) d\alpha + \int_{\min(\alpha_1, 1)}^{\overline{\alpha}} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) d\alpha + (\frac{1 - \alpha_0^2}{2}) \ln(\frac{1 + \tau_c}{1 + f_c}) + \frac{1}{2} \ln(\frac{I - k_c}{I}),$$

$$(U_{I,d} - U_{I,c})_{I < I_0} = \int_{\min(\alpha_1, \overline{\alpha})}^{\overline{\alpha}} \alpha \ln(\frac{p_{\alpha,c}}{p_{\alpha,d}}) d\alpha,$$

$$\begin{split} (\frac{p_{\alpha,c}}{p_{\alpha,d}})_{\overline{\alpha} \ge \alpha \ge \min(\alpha_1,\overline{\alpha})} &= \frac{\frac{1-f_m}{1+f_c}e^{-\lambda I_0}(\frac{1}{\lambda}+I_0-k_c) + \frac{1-\tau_m}{1+\tau_c}[\frac{1}{\lambda}-e^{-\lambda I_0}(\frac{1}{\lambda}+I_0)] - \frac{k_m}{2\alpha}}{\frac{1-\tau_m}{1+f_c}e^{-\lambda I_0}(\frac{1}{\lambda}+I_0-k_c) + \frac{1-\tau_m}{(1+\tau_c)}[\frac{1}{\lambda}-e^{-\lambda I_0}(\frac{1}{\lambda}+I_0)]}, \\ (\frac{p_{\alpha,c}}{p_{\alpha,d}})_{\min(\alpha_1,\overline{\alpha}) > \alpha \ge \alpha_0} &= \frac{(\frac{1-f_m}{1+f_c})e^{-\lambda I_0}(\frac{1}{\lambda}+I_0-k_c) - \frac{k_m}{2\alpha}}{\frac{1-\tau_m}{1+f_c}e^{-\lambda I_0}(\frac{1}{\lambda}+I_0-k_c)}, \\ \alpha_0 &= \frac{k_m}{2e^{(-\lambda I_0)}(\frac{1}{\lambda}+I_0-k_c)Z_0}, \\ \alpha_1 &= \frac{Z_0}{2I_1}\alpha_0 \quad \text{for} \quad f_m \le \tau_m, \\ I_0 &= \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2}-\exp(2\int_{\alpha_0}^{\min(\alpha_1,1)}\alpha\ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha-(1+f_c)\alpha_0Z_0})d\alpha)}, \end{split}$$

$$\tau_c \ge f_c, \quad \frac{1 - f_m}{1 + f_c} \ge \frac{1 - \tau_m}{1 + \tau_c}, \qquad f_m + f_c \ge d_c + d_m.$$

The simulations are done using the same parameterization as before. The results, shown in Fig. A2 in the Appendix, are summarized as follows. As before, in the benchmark simulation (Case 1), we set $\tau_m = 0.05$, $\tau_c = 0.05$, $k_m = 160$, $k_c = 160$, $1/\lambda = 10000$. The results show that as the card service costs $d_c + d_m$ fall, both the merchant card fee f_m and the consumer card fee f_c decrease, and the card pricing markup stays at zero. More merchants accept payment cards (α_0 decreases), more consumers use cards (I_0 decreases), and more card merchants are patronized by cash users (α_1 decreases). As a result, both card users and cash users gain more welfare. In monetary terms, merchants and consumers spend less on card adoption and service fees to pay for a higher card transaction value, so the card payment spending to card sales ratio decreases. Meanwhile, cash transaction value declines and the cash cost to sales ratio is fixed at $\tau_m + \tau_c$. In total, the society pays less for payment services, and the total payment spending to total sales ratio decreases.

We then study the effects of k_c and k_m . In the simulation of Case 2, we reduce both k_c

and k_m by equal proportion compared to the benchmark case, e.g., set $k'_c = k'_m = 128$ so $k'_c/k_c = k'_m/k_m = 0.8$. The results show, compared to Case 1, the social planner now charges lower merchant fee f_m but higher consumer fee f_c , and the card markup remains zero. Under the lower adoption costs, more merchants and consumers adopt card compared to the benchmark case, and the consumers, both the card users and cash users, enjoy more welfare gains. In monetary terms, merchants and consumers now spend less on card adoption and service fees to pay for a higher card transaction value, and the card payment spending to card sales ratio is lower than the benchmark case, so are cash costs and cash transaction value. Consequently, the total payment spending to sales ratio is lower than the benchmark case.

In the simulation of Case 3, we raise the consumer income $1/\lambda' = 12500$ so that $\lambda'/\lambda = 0.8$. As we found before in the monopoly network case, this yields an equivalent change as Case 2 in terms of card pricing, adoption and welfare gains. Meanwhile, a higher income leads to more spending on card and cash payment services, and higher card and cash transaction value than Case 2, while the payment spending to sales ratios are the same.

3.2.3 Comparing Monopoly Outcome and Social Optimum

Several important differences stand out as we compare the monopoly outcome and the social optimum. Figure A3 in the Appendix compares the simulation results of Case 1 between the monopoly outcome and the social optimum. We find that under the same parameterization, the monopoly network charges a much higher interchange fee f_m than the social planner. As a result, both merchant and consumer card adoption are lower, and fewer card merchants serve cash customers. These lead to lower welfare gains to both card consumers and cash consumers. In monetary terms, the monopoly network requires merchants and consumers to spend more on card adoption and service fees to pay for a lower card transaction value than the social optimum, so the card payment spending to card sales ratio is higher. Meanwhile, cash costs and transaction value are higher than the social optimum.

As the card service costs $d_m + d_c$ fall, the monopoly card network raises the interchange fee f_m to merchants but lowers card fee f_c to consumers, and raises the card pricing markup $f_c + f_m - d_c - d_m$. Meanwhile, fewer card merchants are patronized by cash users (α_1 increases) so cash users gain less welfare. In contrast, the social planner would lower card fees to both merchants and consumers, and keep the card markup at zero. Meanwhile, more card merchants are patronized by cash users (α_1 decreases), and both card users and cash users gain more welfare.

Furthermore, Figs. A1 and A2 show when the card adoption costs (k_c, k_m) fall or the mean consumer income $(1/\lambda)$ rises, the monopoly network charges higher card fees f_m and f_c , and raises the card markup. Consequently, merchants and consumers spend more on card adoption and service fees to pay for a higher card transaction value, and the card payment spending to card sales ratio becomes higher. In contrast, the social planner lowers card fee to merchants and keeps the card markup at zero. Meanwhile, merchants and consumers spend less on card adoption and service fees to pay for a higher card transaction value, and the card payment spending to card sales ratio becomes lower.

What cause the fundamental differences between the monopoly outcome and social optimum? The answer lies on their different objectives. The card network makes its profit from providing card services, so it only cares about card users but not cash users. Moreover, lowering card fees to consumers help inflate the value of card transactions, so the card network prefers high interchange fees. As card service costs decline over time, the card network is able to further raise interchange fees to extract more profits out of the system. In contrast, the social planner maximizes the consumer surplus, so it cares about both card users and cash users, and cares about consumers' real purchases rather than their nominal spending. Therefore, the monopoly card pricing and output are very different from the social optimum, a finding in contrast with previous literature, including Schmalensee (2002) and Wright (2003, 2004).

3.2.4 Interchange Fee Applications

Our model provides a general framework to study the pricing, adoption and usage of payment devices. Particularly, it sheds light on some controversial issues surrounding the payment card interchange fees. Why have interchange fees increased in recent years? Given the card service cost $d_c + d_m$ is decreased over time, as would occur with technological progress, our model suggests card fees would increase for merchants but decrease for consumers. As we explained earlier, this can be understood as attempting to maximize card network profit.

Also, our model shows that monopoly card network tends to pursue higher interchange fee than the social planner. In light of recent debates and actions on interchange regulations, our model provides a natural framework for conducting policy experiments. For example, we may conduct a simulation using the same parameterization as Case 1, where we set $\tau_m =$ 0.05, $\tau_c = 0.05$, $k_m = 160$, $k_c = 160$, $1/\lambda = 10000$. We then compare the outcomes under a monopoly card network with and without a binding interchange ceiling, where the ceiling is set as $f_m \leq 0.03$.

The simulation results, shown in Fig. A4 in the Appendix, are consistent with what we expect. Compared with Case 1, once a binding interchange ceiling is imposed, the monopoly card network then has to charge higher fee to the consumer side but the overall card markup is suppressed. As a result, merchant card adoption is higher, and consumer card adoption is lower. Meanwhile, the percentage of category (1) merchants becomes higher, and both card users and cash users enjoy higher welfare gains. In monetary terms, merchants and consumers now spend less on card adoption and service fees to pay for a lower nominal card transaction value, and the card payment spending to card sales ratio decreases. At the same time, cash transaction value rises while the cash cost to sales ratio is fixed at $\tau_m + \tau_c$. In total, the society pays less for payment services, and the total payment spending to total sales ratio decreases.²⁰

4 Conclusion

This paper provides a new two-sided market theory to study the pricing, adoption and usage of payment devices. Using this framework, we are able to study the evolution of payment card

²⁰Note although our policy experiment shows that imposing an interchange fee ceiling may improve consumer welfare, it may still be a challenge for policy makers to set the "correct" level of the ceiling given all kinds of additional complications in reality. See Wang (2007) for more discussions.

markets both in the short run and in the long-run, and also shed new light on related competition policy issues.

The benefits of adopting a payment innovation–a payment card–in our model are pecuniary. Adopting payment cards by consumers and merchants requires a fixed cost, but yields lower marginal costs of making payments. Considering this together with the heterogeneity of consumer income and merchant size, our theory yields some unique insights different from the existing literature. One of our first result shows that in equilibrium, high-income consumers and highvalue or volume merchants are likely to adopt card devices earlier than others. Meanwhile, three types of merchants exist in the market: Some only serve cash customers, some only serve card customers, and some serve both. These findings fit well with empirical evidence. In another difference with existing models, cash users in our model benefit when a store has sales paid for both by cards and cash. In other words, cash users are "subsidized" by card users.

Our analyses show that as payment card markets evolve, the interchange fees increase over time. This is because a monopoly card network, besides internalizing the two-sided market externality, has the incentive to inflate the value of card transactions. Lowering card fees to consumers but raising them to merchants help inflate the card transaction value, so the card network prefers high interchange fees. As card service costs decline over time, the card network is able to further raise interchange fees to extract more profits out of the system. In contrast, our model shows that the social planner would prefer lower interchange fees than the monopoly network, and imposing a ceiling on interchange fees may improve consumer welfare. These findings provide some support for the ongoing investigations that competition authorities around the world are taking on the payment card interchange fees.

There are many avenues for further research. Wang (2007) examines four-party systems in mature card markets without adoption externalities. Further extensions to this environment would be useful. Models of the vertical restraints imposed by card networks would also be of interest in this model and its extensions. Finally, modeling consumer credit constraints and the provision of credit via the payment device is another important direction for future research.

Appendix A: Proofs

Derivation of Equation (L2)

Recall Eq. (15):

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(2\int_{\alpha_0}^{\frac{Z_0}{Z_1}\alpha_0} \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0 Z_0}) d\alpha)}.$$

Note that

$$\int \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha - (1+f_c)\alpha_0 Z_0}) d\alpha$$

= $\frac{\alpha^2}{2} (\ln(1-\tau_m)) - \frac{1}{2} \alpha^2 \ln((1-f_m) - \frac{\alpha_0 Z_0(1+f_c)}{\alpha})$
+ $\frac{\alpha_0 Z_0(1+f_c)}{2(1-f_m)} \alpha + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2} \ln(\alpha - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}).$

Therefore, we derive

$$\begin{split} &\int_{\alpha_0}^{\frac{Z_0}{Z_1}\alpha_0} \alpha \ln(\frac{(1-\tau_m)\alpha}{(1-f_m)\alpha-(1+f_c)\alpha_0Z_0})d\alpha \\ &= \frac{\alpha_0^2 Z_0^2(1+f_c)}{2(1-f_m)Z_1} + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\ln((\frac{Z_0}{Z_1}\alpha_0) - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}) \\ &\quad + \frac{1}{2}\alpha_0^2\ln\frac{(1+f_c)}{1+\tau_c} - \frac{\alpha_0^2 Z_0(1+f_c)}{2(1-f_m)} - \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\ln(\alpha_0 - \frac{\alpha_0 Z_0(1+f_c)}{(1-f_m)}) \\ &= \frac{1}{2}\alpha_0^2\ln\frac{1+f_c}{1+\tau_c} + \frac{\alpha_0^2 Z_0(1+f_c)}{2(1-f_m)}(\frac{Z_0}{Z_1} - 1) + \frac{\alpha_0^2 Z_0^2(1+f_c)^2}{2(1-f_m)^2}\{\ln(\frac{(\frac{Z_0}{Z_1}) - \frac{Z_0(1+f_c)}{(1-f_m)}}{1-\frac{Z_0(1+f_c)}{(1-f_m)}})\}. \end{split}$$

Equation (15) can then be rewritten into

$$I_0 = \frac{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} k_c}{(\frac{1+\tau_c}{1+f_c})^{1-\alpha_0^2} - \exp(S\alpha_0^2)},$$
 (L2)

where $S = \ln \frac{1+f_c}{1+\tau_c} + \frac{Z_0(1+f_c)}{(1-f_m)} (\frac{Z_0}{Z_1} - 1) + \frac{Z_0^2(1+f_c)^2}{(1-f_m)^2} \ln(\frac{\frac{Z_0}{Z_1} - \frac{Z_0(1+f_c)}{(1-f_m)}}{1 - \frac{Z_0(1+f_c)}{(1-f_m)}}).$

Proof of Proposition 1

Proposition 1 Under the parameter values $(1/\lambda, k_c, k_m)$ and $(\theta/\lambda, \theta k_c, \theta k_m)$, the network profit maximization yields the same card prices and adoption rates.

Proof. Let us first prove if $(f_c^*, f_m^*, I_0^*, \alpha_0^*)$ maximizes the network profit under the parameterization $(1/\lambda, k_c, k_m)$, then $(f_c^*, f_m^*, \theta I_0^*, \alpha_0^*)$ maximizes the network profit under the parameterization $(\theta/\lambda, \theta k_c, \theta k_m)$.

This can be shown by constructing a contradiction. Assume $(f_c^*, f_m^*, I_0^*, \alpha_0^*)$ generates the maximal network profit π^* under the parameterization $(1/\lambda, k_c, k_m)$. Then, under the parameterization $(\theta/\lambda, \theta k_c, \theta k_m), (f_c^*, f_m^*, \theta I_0^*, \alpha_0^*)$ satisfies the constraints and offers profit $\theta \pi^*$. Assume $(f_c^*, f_m^*, \theta I_0^*, \alpha_0^*)$ does not maximize the profit, then there is another choice $(f_c', f_m', I_0', \alpha_0')$ that offers profit $\pi' > \theta \pi^*$. However, $(f_c', f_m', I_0', \theta, \alpha_0')$ also satisfies the constraint and offers a profit $\pi'/\theta > \pi^*$ under the parameterization $(1/\lambda, k_c, k_m)$. This contradicts the assumption that π^* is the maximal profit under the parameterization $(1/\lambda, k_c, k_m)$.

Therefore, under the parameter values $(1/\lambda, k_c, k_m)$ and $(\theta/\lambda, \theta k_c, \theta k_m)$, the monopoly network requires the same card prices (f_c^*, f_m^*) and adoption rates $(e^{(-\lambda I_0^*)}, 1 - \alpha_0^*)$. Note that under the parameterization $(1/\lambda, k_c, k_m)$, consumer card adoption rate is $1 - F_{\lambda}(I_0^*) = e^{(-\lambda I_0^*)}$; under the parameterization $(\theta/\lambda, \theta k_c, \theta k_m)$, consumer card adoption rate is again the same, $1 - F_{\lambda/\theta}(\theta I_0^*) = e^{(-\lambda I_0^*)}$. Appendix B: Simulations (Figs. A1-A4)

The simulations are conducted for market equilibrium outcomes under a monopoly card network, under a Ramsey social planner, and under an "interchange fee ceiling" policy. There are three cases of parameterization as follows.

	k_m	k_c	$1/\lambda$	τ_m	τ_c	$d_m + d_c$
Case 1	160	160	10,000	0.05	0.05	(0, 0.05)
Case 2	128	128	10,000	0.05	0.05	(0, 0.05)
Case 3	160	160	12,500	0.05	0.05	(0, 0.05)

Simulation Parameterization (Cases 1-3)

The simulation results are presented in the attached Figs A1-A4 as follows.

- Figure A1 compares monopoly outcomes between Cases 1, 2 and 3.
- Figure A2 compares socially optimal outcomes between Cases 1, 2 and 3.
- Figure A3 compares monopoly outcome and social optimum for Case 1.
- Figure A4 compares monopoly outcomes for Case 1 with and without a binding

interchange ceiling, where the ceiling is set as $f_m \leq 0.03$.



Figure A1: Monopoly Network (Case 1 vs. Cases 2 & 3)





Figure A2: Social Optimum (Case 1 vs. Cases 2 & 3)





Figure A3: Monopoly Outcome vs. Social Optimum (Case 1)



Figure A4: Monopoly Network with and without An Interchange Ceiling (Case 1)



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