ISSN 1936-5330

The Federal Reserve Bank *of* Kansas City
Research Working Papers

To Sell or to Borrow? A Theory of Bank Liquidity Management

Michal Kowalik

December 2014
RWP 14-18

Research Working Papers

To sell or to borrow?

A Theory of Bank Liquidity Management*

Michał Kowalik†

December 2014

Abstract

This paper studies banks' decision whether to borrow from the interbank market or to sell assets in order to cover liquidity shortage in presence of credit risk. The following trade-off arises. On the one hand, tradable assets decrease the cost of liquidity management. On the other hand, uncertainty about credit risk of tradable assets might spread from the secondary market to the interbank market, lead to liquidity shortages and socially inefficient bank failures. The paper shows that liquidity injections and liquidity requirements are effective in eliminating liquidity shortages and the asset purchases are not. The paper explains how collapse of markets for securitized assets contributed to the distress of the interbank markets in August 2007. The paper argues also why the interbank markets during the 2007-2009 crisis did not freeze despite uncertainty about banks' quality.

JEL: G21, G28

Keywords: banking, liquidity, interbank markets, secondary markets.

*This paper was completed when I was an economist in the Banking Research Department at the Federal Reserve Bank of Kansas City. I would like to thank Viral Acharya, Javed Ahmed, Gaetano Antinolfi (discussant), Mitchell Berlin, Jose Berrospide, Lamont Black, Ricardo Correa, Darell Duffie, Itay Goldstein, Andy Jobst (discussant), Pete Kyle (discussant), David Martinez-Miera, Nada Mora, Chuck Morris, Rafael Repullo, Christian Riis (discussant), Javier Suarez, Razvan Vlahu and participants of the System Committee Meeting on Financial Structure and Regulation, 2013 ECB workshop on Money Markets, the FDIC/JFSR 2012 Banking Research conference, 2012 Fall Midwest Macro, 2013 MFA, 2013 IBEFA Seattle and seminars at CEMFI, the FRB of Kansas City and of Philadelfia, the Federal Reserve Board, National Bank of Poland and the University of Vienna for their valuable comments. The views expressed herein are those of the author and do not necessarily represent those of the Federal Reserve Bank of Boston, Federal Reserve Bank of Kansas City or the Federal Reserve System.

†Federal Reserve Bank of Boston, 600 Atlantic Ave, Boston, MA, 02210, tel.: 617-973-6367, email: michal.kowalik@bos.frb.org

One of the most significant features of modern banks is their ability to turn individual illiquid loans into tradable securities and use them as source of liquidity. At the same time these banks still use cash reserves and interbank markets as sources of liquidity. On the one hand, reliance on tradable securities reduces banks' cost of liquidity management. On the other hand, as the 2007-09 financial crisis showed reliance on such securities exposed the whole financial system to shocks to credit risk embedded in these securities. Banks that relied on such securities became the center of events in August 2007, when markets for securitized assets showed signs of stress, which immediately spread to interbank markets hampering banks' ability to manage their liquidity. We provide a theory of bank liquidity management that captures the above mentioned trade-off and is consistent with recent empirical evidence about robustness of interbank market after the initial shock in August 2007.

In our model, we analyze the banks' choice between cash reserves, unsecured interbank borrowing and asset sales in case of exposure to liquidity and credit risk. After each bank allocates its endowment between a risky asset and cash reserves, it receives private signals about the quality of its asset and its liquidity need. Next, each bank decides how to cope with its liquidity need: use its cash reserves, borrow/lend on the interbank market or sell/buy the asset on a secondary market. Liquidity is then reallocated between illiquid banks, liquid banks and outside investors using the interbank and secondary markets.

This very simple and generic setup á la Diamond-Dybvig (1983) generates powerful results. First, despite asymmetric information about the quality of banks' assets affecting both the interbank and secondary markets, the illiquid banks with the high-quality asset (the good banks) prefer to borrow rather than sell. Borrowing is more attractive for the good banks than selling because of a lower adverse selection cost. Borrowing has a lower adverse selection cost than selling, because the share of the bad borrowing banks to the good borrowing banks is lower than the share of the sold bad assets to the sold good assets. When borrowing, both the good and the bad banks borrow the same amount because borrowing is costly since it has to be repaid. The good banks sell only a portion of their asset needed to cover their liquidity needs, whereas the bad banks sell all of it to profit from asymmetric information. In other words, the bad illiquid banks contaminate the secondary market to a higher degree than the interbank market. The result that the good banks prefer to borrow rather than sell is reminiscent of the pecking order theory by Myers and Majluf (1984).

Second, we study the consequence of the good illiquid banks' preference to borrow in equilibrium with the endogenous asset price and interbank loan rate. Generally, in equilibrium the average quality of borrowing banks is higher than those of selling. Because the good illiquid banks borrow the quality of the sold asset is lower than if there were no interbank market. In the polar case, in which there is enough interbank loans for *all* illiquid banks, in equilibrium the secondary market breaks down as in Akerlof (1970). Because the good illiquid banks prefer to borrow, they go to the interbank market. All bad illiquid banks follow them to profit from asymmetric information. In turn, no illiquid bank sells its asset. Although the bad banks contaminate the interbank market, no good illiquid bank has a desire to deviate and sell anticipating that the adverse selection cost of selling would be higher than of borrowing, since the bad banks would contaminate the secondary market even more than the interbank market. Such an equilibrium exhibits features of an interbank market that is stressed but not frozen (using the language of Afonso, Kovner and Schoar (2011)). There is enough liquidity for all illiquid banks, although the interbank loan rate is elevated and the asset price is very low. This is consistent with the recent empirical evidence discussed later.

Finally, we also provide conditions for which the liquidity transfer breaks down resulting in bankruptcy of illiquid banks. This occurs when the quality of the bad asset is sufficiently low and there is not enough loans for all illiquid banks. Because there is not enough loans for all illiquid banks, some illiquid banks are forced to sell. These are the bad banks, because the good banks are less reluctant to sell. However, if the quality of the bad asset is so low that selling even all of the asset does not generate enough cash for a bad bank to become liquid, these banks come back to the interbank market to look for missing liquidity. Because there is not enough interbank loans for all illiquid banks, some of them banks are rationed and go bankrupt. The crucial assumption for this liquidity rationing to occur is that the liquidity on the interbank market is fixed and cannot be adjusted immediately. This is a reasonable assumption in the context of an acute liquidity shock, which our model intends to captures (see also Freixas, Martin and Skeie (2009)).

We close the model by allowing the banks to choose ex ante the distribution of their initial endowment between cash and the risky asset. The banks trade off the cost of holding cash, which is a lower investment in the risky asset, with the benefit of holding cash, which is twofold. A liquid bank can earn a positive net return on interbank lending (a speculative motive). An illiquid bank can lower its liquidity need (a precautionary motive). There are two reasons why banks' choices of cash holdings can be socially inefficient. If there were no bank failures when banks held no cash, any positive cash holdings due to the two above motives are socially wasteful, because they lower the investment in the risky but productive asset. In case banks' cash holdings lead to failures of illiquid banks, the banks may carry too low cash reserves from the welfare perspective, because they do not internalize the social cost of their failures.

Our model provides several policy implications. First, form of liquidity requirements depends on the banks' choice of ex ante cash holdings. If zero cash holdings do not result in bankruptcies of illiquid banks, the socially optimal are zero and the banks should be required to invest all of their endowment in the risky asset. If banks' choices of cash holdings lead to bankruptcies, the socially optimal cash holding should be sufficiently high to prevent these bankruptcies. Second, credible commitment by a central bank to provide liquidity on the interbank market induces socially optimal cash reserves. In cases without bankruptcies for zero cash holdings, the central bank floods the interbank market with liquidity to the point where speculative and pre-cautionary motives do not matter, inducing banks to invest all their endowment in the risky asset. In case with bankruptcies, the social planner avoids them simply by providing the missing liquidity. Finally, simple asset purchases by the central banks are ineffective, because low asset price is due to adverse selection and not lack of liquidity on the secondary market.

Our model is well suited for thinking about the effect of uncertainty surrounding quality of banks assets on their liquidity management. First, our paper aims at explaining how uncertainty about quality of banks' assets affects liquidity redistribution via the interbank and secondary market. This helps us to understand the conditions for effectiveness of different policy tools in addressing liquidity shortages. Second, we view our paper as a model of acute liquidity shocks, during which access to liquidity sources other than cash, interbank lending, asset sales and central bank is impossible. In that sense, our model explains well differing performance of interbank and secondary markets after the breakout of the recent financial crisis in August 2007, which was triggered by an increase of uncertainty about riskiness of the mortgage-backed securities owned by banks.

Existing empirical evidence provides some support for our interpretation of events in August 2007. First, as Acharya, Afonso and Kovner (2013) show the U.S. banks exposed to the shock at the ABCP market increase their interbank borrowing in the first two weeks of August 2007. This is consistent with our argument about how an acute shock from a secondary market may transmit to the interbank market. Second, Kuo, Skeie, Youle, and Vickrey (2013) point out that contrary to the conventional wisdom the term interbank markets for which the counterparty risk is an important determinant of borrowing costs did not freeze during the 2007-2009 crisis (see also Afonso, Kovner and Schoar (2011) for similar evidence on the Fed Funds market). Specifically, the volume actually increased right after the start of the crisis in August 2007 despite of the jump in the spread between the 1- and 3-month LIBOR and OIS (Figures 1 and 2). This supports our argument that the interbank market served as an important source of liquidity during an acute shock originating from the secondary market.[1] The jump in the volume might have been also facilitated by the Federal Reserve's liquidity injections, which occurred immediately after the start of the crisis.[2]

Literature Review. The paper is related to an extensive literature on bank liquidity and shares many common features with other papers. Our contribution is to model the efficiency trade-off that arises from banks reliance on the secondary and interbank markets at the same time: tradable securities, on the one hand, improve efficiency by lowering reliance on cash and interbank market, but, on the other hand, they might lead to vulnerabilities as the recent crisis have shown. Because the coexistence of the secondary and interbank markets is crucial for individual banks and the financial system as a whole, our paper bridges a gap between two strands of literature on

[1] Acharya, Afonso and Kovner (2013) discuss also banks' strategies to deal with the ABCP shock in a longer horizon.

[2] Brunetti, di Filippo and Harris (2011) show interestingly that liqiudity injection by the European Central Bank were ineffective in the fall 2007.

bank liquidity that are concerned with only one of these two markets at a time.

The first strand of literature concerns liquidity provision through interbank markets. The paper closest to ours is the work of Freixas, Martin and Skeie (2011) who provide a model of an interbank market which is affected by uncertainty about distribution of liquidity shocks. Their model captures variation in banks' liquidity needs during the 2007-2009 crisis. They provide a theoretical justification for the Federal Reserve's interest rate management and an explanation how it contributed to the stability of Fed Funds market. Our work complements their work by emphasizing an effect of uncertainty about quality of banks' tradable assets on the liquidity distribution through the interbank markets. We provide conditions justifying liquidity injections in case of shocks to fundamentals and also provide a reason for why the interbank markets did not freeze.

Freixas and Holthausen (2005), in the context of cross-border interbank markets, and Heider, Hoerova and Holthausen (2009), in the context of recent crisis, also examine role of asymmetric information about banks' quality on the interbank market. Both papers show how an interbank market can freeze, when the highest quality banks stop borrowing due to severe adverse selection. The interbank market does not freeze in our model, because we model an acute stress event, in which the highest quality banks experience the lowest adverse selection cost of purchasing liquidity at the interbank markets.

Allen and Gale (2000) and Freixas, Parigi and Rochet (2000) study contagion through interbank markets. Rather than vulnerability of interbank networks, our paper can be viewed as a model of contagion through different markets to which the banks are exposed.

Our paper is also related to papers focused on liquidity hoarding on the interbank markets. Acharya and Skeie (2011) show how moral hazard due to too high leverage impairs banks' ability to roll over debt, increases liquidity hoarding and decreases interbank lending. Ashcraft, McAndrews and Skeie (2011) provide another model of precautionary liquidity hoarding against liquidity shocks. Gale and Yorulmazer (forthcoming) study liquidity hoarding in a model combining speculative and precautionary motives.

The second strand of literature studies vulnerability of liquidity provision through secondary markets for banks' assets. The two closest papers to ours are Malherbe (forthcoming) and Bolton, Santos and Scheinkman (2011), which both analyze the effect of asymmetric information about quality of banks' assets on liquidity distribution in the banking system. Malherbe (forthcoming) models self-fulfilling collapse of liquidity provision in an Akerlof (1970) spirit. Bolton, Santos and Scheinkman (2011) are interested in the timing of asset sales on secondary markets. The common feature of these two papers with ours is the source of adverse selection on the secondary markets: uncertainty whether the selling banks trade due to illiquidity or quality reasons. Using the same source of adverse selection, Brunnermeier and Pedersen (2009) show how secondary markets dry up, when funding necessary to keep the secondary market liquid depends on the liquidity of the very same market.

Other related papers study the impact of fire sales on intermediaries' liquidity. Fecht (2004) using the Diamond (1997)-extension of Diamond and Dybvig (1983) shows how fire sales impact banks linked by a common secondary market. Martin, Skeie and von Thadden (2012) show how reliance on securitization might increase probability of bank runs when the investors from whom banks borrow value banks collateral very low.

The remainder of the paper is organized as follows. Section 1 describes the setup. In Section 2 and 3, we derive optimal bank behavior and equilibria for given cash reserves. Section 4 and 5 describe ex ante equilibria and welfare. Section 6 discusses policy implications. The Appendix contains proofs of the results.

1 Setup

There are three dates, t=0,1,2, and one period. At t=0 each bank decides how to split one unit of its endowment between a risky asset and cash reserves (called also cash or reserves) in order to maximize its return at t=2. The endowment belongs to the bank, i.e., we abstract from any debt except the interbank debt incurred at t=1 (this simplifies algebra considerably without affecting the results). At t=1 each bank receives two signals about the return structure of its asset and its liquidity need. After signals are revealed, the interbank market for loans and secondary market for banks' asset open, where the banks can manage their liquidity needs, in addition to using their cash reserves. At t=2 the asset's returns are realized and payments are made.

The banks. At t=0 there is a continuum of mass one of identical banks. Each bank can invest fraction $\lambda \in [0; 1]$ of endowment in cash that returns 0 in net terms and $1 - \lambda$ in the asset.[3]

At t=1 each bank receives a private signal about the return structure of its asset. With probability q the bank learns that the asset is good and returns $R > 1$ at t=2 with certainty (we call such a bank a "good bank"). With probability $1 - q$ the bank learns that the asset is bad ("bad bank") and has the following return structure: it pays R at t=2 with probability $p > 0$ and 0 otherwise, where $pR < 1$. It holds that $[q + (1 - q)p] R > 1$.

At t=1 each bank receives also a private signal about its liquidity need. We model the liquidity need (shock) as a need to inject cash into the bank in an amount of $d < 1$.[4] We call banks hit by the liquidity shock illiquid, and the other banks liquid. With probability π the bank is liquid, and with $1 - \pi$ illiquid. If an illiquid bank cannot generate enough liquidity to pay d it becomes bankrupt and sells all of its asset. Proceeds from such a sale accrue towards the payment of d. Finally, we assume that shocks to the asset's returns and liquidity are independent.

The exact underpinnings of the liquidity need d are not crucial for the model. As the liquidity need d is currently modelled, we could interpret it as payment tied to some contingent liabilities (such as derivatives or committed lines of credit) to which the banks committed before t=0. Alternatively, we could model the banks' liquidity needs using a Diamond-Dybvig-style framework as proposed in similar papers on the commercial banks' liquidity management such as Freixas and Holthausen (2005) and Freixas et al (2011). Section 8 provides more details on how we could use a Diamond-Dybvig setup to (i) endogenize the size and incidence of liquidity shocks, (ii) relax the assumption about the independence of liquidity and asset return shocks, (iii) create a reason for the interbank market, and (iv) add debt incurred at t=0. In the baseline model, we deliberately abstract from explicit modelling of liquidity shock in order to focus on details of banks' liquidity management and ease on notation and algebra without affecting our main results.

[3] Alternatively to model a short-term liquidity management, we could assume that the bank starts out with a unit of an asset and has some spare liquidity and decides what fraction of this spare liqudity to preserve and to "consume" (see Gale and Yorulmazer (2011)).

[4] Restricting $d < 1$ is done for algebraic convenience. Allowing for $d \geq 1$ would add more cases in which selling banks could not become liquid by selling as well as those in which the banks would never be solvent when borrowing. Because these cases are fairly strightforward we concentrate on the case $d < 1$.

The only crucial assumption here is that the occurrence of the liquidity need d is not perfectly correlated with the realization that the asset is bad. As long as we maintain this assumption, which we think is plausible, we preserve asymmetric information about the quality of banks's assets at t=1 needed for our main results.

The interbank and secondary market. In modelling the interbank market we follow the literature (e.g. Freixas and Holthausen (2005) and Freixas et al (2011)): interbank lending is unsecured, diversified, and competitive with banks acting as price takers. Assumption about diversification of interbank loans at each lending bank makes default risk on interbank loans algebraically tractable, because each lender will be exposed to average risk on the credit market (Freixas and Holthausen (2005)).

In modelling the secondary market we also follow the literature (e.g. Akerlof (1970), Bolton, Santos and Scheinkman (2011) or Malherbe (2014)). The banks can sell their assets to banks and to outside investors, which we think of as institutions that have a tolerance for long-term assets such as pension funds, mutual funds, sovereign wealth funds and hedge funds. The outside investors are competitive and are able to absorb any amount of assets that appear on the market (Malherbe (2014)). Finally, the asset sale under the asymmetric information scenario occurs at a single price (e.g. Akerlof (1970), Bolton, Santos and Scheinkman (2011) and Malherbe (2014)).

Two remarks are in order. First, the only crucial assumption for our main result that interbank markets can be sustained during an acute liquidity shock despite uncertainty about the quality of banks' assets is that the cost of accessing funding immune to adverse selection (such as insured deposits) is prohibitively high for the good illiquid banks. If access to such funding were cheaper than funding using interbank borrowing or asset sales, the good illiquid would cope with their liquidity shock using insured deposits and leave the secondary and interbank markets leading to their collapse in spirit of Akerlof (1970).[5] However, our assumption is not restrictive, because we think of our model as a discussion of an acute liquidity shock, which might be impossible to cope with at short notice using funding sources other than cash, interbank borrowing, asset sales and

[5]Empirical evidence cited in introduction points out that even after the initial shock the interbank markets performed well despite of increasing usage of insured deposits (Achrya et al.). This might be explained as follows. When the initial shock hits, the market participants face high uncertainty about the individual banks' quality. As the time passes, they learn more about the individuals banks so that the source of adverse selection vanishes and the interbank markets may operate smoothly again (Afonso et al. (2011)).

borrowing from the central bank.

Finally, one important funding alternative that we do not consider is secured borrowing. In our model, the banks could borrow in secured manner using cash or the risky asset as collateral. However, we can neglect secured borrowing without loss of generality, because it is not more attractive than the funding sources we consider. This happens for two reasons. First, secured borrowing against cash held by banks as reserves at t=1 cannot be cheaper than using cash directly, because both are equivalent and cash reserves are directly available to the bank at no additional cost. Second, secured borrowing against the risky asset is the same as unsecured borrowing in our model. The reason is that in our simple model the probability of individual bank's default is exactly the same as probability that its asset pays 0 (given that the illiquid banks will use cash as the first line of defense against liquidity shocks). Hence, secured borrowing does not create any advantage for the borrower or lender over unsecured borrowing.

2 Case of Perfect Information

As a benchmark we solve the model when all agents observe liquidity shocks and asset quality of individual banks. We solve the model backwards starting at t=1 after all the realization of the shocks. Because the case of perfect information is simple to analyze, we keep additional notation at minimum and postpone formalization of the liquid and illiquid banks' decision problems to the next section, where we analyze the case of asymmetric information. Here and throughout the paper, we focus on the case in which the cash reserves from t=0 are not higher than the liquidity need, $\lambda \leq d$, because holding $\lambda > d$ would never be optimal in equilibrium when storage of cash yields 0 in net terms. We can characterize the equilibria on the interbank and secondary markets at t=1 as follows.

Lemma 1. *Assume there is a perfect information about the individual banks' liquidity needs and asset quality. The equilibrium price of the good and bad asset at t=1 is equal to their fundamental value R and pR,* respectively. *If the bad banks can generate enough liquidity to cope with their liquidity shock, $pR(1-\lambda)+\lambda \geq d$, in equilibrium at t=1 each illiquid bank is indifferent between selling its asset and borrowing from other banks. The equilibrium interbank loan rate for the good* *and bad banks is equal to* 1 and $\frac{1}{p}$, *respectively. If for some* $\lambda > 0$ *the bad banks cannot generate enough liquidity to cope with their liquidity shocks,* $pR(1-\lambda)+\lambda < d$, *in equilibrium at t=1 the good illiquid banks are indifferent between selling their asset and borrowing from other banks, and the BI banks are bankrupt. The equilibrium interbank loan rate for the good bank is 1, and nobody lends to the BI banks at any loan rate.*

Proof: Because the competitive outside investors observe the quality of the banks' assets and can absorb any quantity, they pay the full expected return on these assets in equilibrium: R for the good asset and pR for the bad asset. Now we analyze the interbank market clearing. We start with the case in which the bad banks can cope with their liquidity shock. That occurs when sum of cash from their reserves λ and proceeds from selling all of their asset at price pR, $pR\,(1-\lambda)$, is not lower than their liquidity need d, i.e., $pR(1-\lambda)+\lambda \geq d$. The banks with excess cash want to lend to the good and bad banks as long as the loan rates on loans for these banks are not lower than the break even loan rates on loans for these banks. The break even loan rates are the ones that just compensate for the risk of lending to such banks. Hence, this break even loan rate is 1 for the good and $\frac{1}{p}$ for the bad banks. As it turns out these break even loan rates are the highest loan rates at which the illiquid banks would be willing to borrow given the equilibrium prices of their asset. By selling all of their assets the illiquid banks can realize the full return on their asset already at t=1. Specifically, their payoff from selling is then $R(1-\lambda)+\lambda-d > 0$ for the good banks and $pR(1-\lambda)+\lambda-d \geq 0$ for the bad banks. If $R_{D,G}$ and $R_{D,B}$ are the interbank loan rates for loans to the good and bad banks respectively, then as long as these loan rates are not lower than 1, the banks borrow the amount of liquidity shortfall $d-\lambda$, which yields a payoff of $R(1-\lambda)-R_{D,G}\,(d-\lambda)$ for the good banks and $p\,[R(1-\lambda)-R_{D,B}\,(d-\lambda)]$ for the bad banks. Comparing payoffs from selling and borrowing for the good and bad banks shows that the illiquid banks would be willing to borrow as long as the loan rates are not higher than the break even loan rates: $R_{D,G} \leq 1$ for the good banks and $R_{D,B} \leq \frac{1}{p}$ for the bad banks. Hence, it must follow that the interbank market for the loans to the good banks clears at 1 and at $\frac{1}{p}$ for the bad banks. Of course, at such equilibrium loan rates and asset prices, the illiquid banks are indifferent between selling and borrowing. If we have that $pR(1-\lambda)+\lambda < d$ or $\lambda \in \left[0; \frac{d-pR}{1-pR}\right)$ (which can occur only if $pR < d$), the equilibrium on the market for interbank loans to the good banks is the same as in the previous case. The bad illiquid banks go bankrupt, because they cannot raise enough liquidity to cope with their liquidity shock by selling all of their assets and nobody lends to them because the value of their asset $pR(1-\lambda)$ is not enough to repay the loan in the amount of $d-\lambda$ even at the break even loan rate $\frac{1}{p}$.$\blacksquare$

Lemma 1 established three benchmark results against which we will discuss the main results of our paper. First, under perfect information it does not matter how banks cope with their liquidity shocks in equilibrium. Arbitrage forces make the illiquid banks indifferent between selling and borrowing in resemblance to Modigliani and Miller (1958). Second, despite of fixed amount of cash reserves held by banks at t=1, there is no cash-in-the-market effect on the interbank market in equilibrium. The reason is that on the frictionless secondary market the banks can sell their asset at no cost, meaning that the illiquid banks do not find interbank borrowing more attractive than selling. Third, the bad illiquid banks fail only when they have too little cash reserves λ carried from t=0. Using Lemma 1 we can solve for the optimal choice of cash reserves and risky asset at t=0.

Lemma 2. *The optimal choice of cash reserves at t=0 is 0.*

Proof: In the Appendix.

The intuition behind Lemma 2 is simple. In case $pR \geq d$, the bank is never bankrupt at t=1. Because the illiquid banks can always sell their assets at t=1 and realize the asset full return already at t=1, there is no need to hold cash which is less productive than the risky asset. The same logic applies even when $pR < d$. The reason is that by holding cash to save itself from bankruptcy at t=1 when the bank becomes bad and illiquid is too costly in expectation when compared with a loss of return on the risky when the bank is good or bad and liquid. Finally, the fact that the optimal choice of λ is 0 does not mean that the interbank market is not active at t=1. It is possible that the banks can sell their asset and lend out the excess cash from these proceeds to the borrowing banks.

In the rest of the paper we analyze the case of asymmetric information.

3 Banks' liquidity management under asymmetric information

Because we solve the model backwards, we start at t=1. The crucial difference between the cases of perfect and asymmetric information is that under the asymmetric information the asset price and the loan rate depend on the agents' expectations about the quality of the sold assets and the types of borrowing banks. In turn, the quality of the sold assets and the types of borrowing banks are outcomes of the optimal individual banks' liquidity management decisions, which depend on the anticipated difference between the asset price and the loan rate. Hence, in the case of asymmetric information we have to first pin down the optimal choice of banks' liquidity management decisions for given asset price and loan rate. After that we can solve for an equilibrium in which the asset price and the loan rate reflect these optimal banks' choices and are consistent with agents' expectations about these choices. Such an approach contrasts with the case of perfect information, where the equilibrium price of the asset was independent of the banks' liquidity management choices and set a clear benchmark for banks' lending and borrowing decisions. For exposition purposes, this section characterizes optimal behavior of liquid and illiquid banks for given asset price and loan rate, and the next section presents the characterization of an equilibrium at t=1.

After the liquidity and asset return shocks are realized at t=1, there are four types of banks: good and liquid (GL), bad and liquid (BL), good and illiquid (GI), and bad and illiquid (BI). Each type of the bank manages its liquidity by looking for the best use for its cash reserves, for the optimal amount l of lending or borrowing on the interbank market (with $l < 0$ meaning borrowing), and for the optimal amount S of asset to be sold or bought selling or buying the asset on the secondary market. Each bank takes the interbank loan rate R_D, the expected fraction $\widehat{p}$ of banks repaying their interbank loans at t=2, and the asset price P as given when deciding how to optimally manage its liquidity.

We can write the liquidity management decision problem of all four types of banks in a compact form after undertaking the following simplifications. First, the liquid and illiquid banks differ only in their liquidity need. We use an indicator function μ that takes value 1 if the bank is illiquid and

0 otherwise to capture when a bank has to pay d or not. Second, the good and bad banks differ only in the probability of success of their asset, p_i, where for the bad bank ($i = B$) $p_B = p$ and the good bank ($i = G$) $p_G = 1$. Finally, without any loss of generality we ignore the possibility that the banks can buy other banks' asset, because asset purchase is never better than storing cash in equilibrium. The competitive outside investors bid up the asset price to its expected return resulting in a net return of 0, which is exactly the same as return on cash. Moreover, additional liquidity supplied by banks on the secondary market has no influence on the asset price in presence of the outside investors with deep pockets.

The liquidity management decision problem of a bank reads

$$\max_{l,S} \begin{cases} p_i[(1-\lambda-S)R + (SP+\lambda-l-\mu d) + \widehat{p}R_D l] + (1-p_i)[(SP+\lambda-l-\mu d) + \widehat{p}R_D l], \text{ if } l > 0, \\ p_i \max[0; (1-\lambda-S)R + (SP+\lambda-l-\mu d) + R_D l] + (1-p_i)\max[0; (SP+\lambda-l-\mu d) + R_D l], \text{ if } l \leq 0 \end{cases} \tag{1}$$

$$\text{s.t. } S \in [0; 1-\lambda], SP + \lambda - \mu d \geq l.$$

The first line of the objective function in program (1) is bank's expected return at t=2 when it lends on the interbank market ($l > 0$). The first term is the return in case the asset succeeds with probability p_i. $(1-\lambda-S)R$ is the return on the remaining asset after selling S units. $SP+\lambda-l-\mu d$ is the excess cash left after the bank carries cash λ from $t = 0$, receives SP from selling S of the asset at a price P, lends l at the interbank market and pays d if it is illiquid ($\mu = 1$). $\widehat{p}R_D l$ is the expected return on the interbank loans at t=2. The return on the interbank loans is deterministic because we assumed that each lending bank has a diversified interbank loan portfolio. The second term is the return in the case the bank's asset pays 0, which comprises only of the excess cash and return on the interbank loans. The second line of the objective function in program (1) is bank's expected return at t=2 when it borrows on the interbank market ($l < 0$; we add the case of $l = 0$ here). The expected return for $l \leq 0$ differs from the case with $l > 0$ for two reasons. First, the return on interbank lending $\widehat{p}R_D$ has to be substituted with R_D, the loan rate the borrowing bank pays on its loan. Second, we have to include max-operators in the expected return to take into account that illiquid banks might not be able to raise enough liquidity when they borrow (in which case they go bankrupt and limited liability limits their downside). We do not have to do this in the case when they lend, because lending can only occur when they pay d and do not go bankrupt in the first place. The objective function in program (1) takes into account that the banks would never borrow and lend at the same time, because lending is never more profitable than borrowing ($\widehat{p}R_D \leq R_D$). In the last line of program (1) there are two constraints under which the bank maximizes its expected return. The first constraint represents the amount of the asset available for sale. The second constraint limits the amount the bank can lend on the interbank market. The highest amount of lending equals to the sum of cash carried from t=0, λ, and cash raised by selling S of the asset at price P at t=1, diminished by the payment of d in case of the illiquid bank. There is no need for including a lower bound on lending, because it arises endogenously from solving (1).

Before we present the solution to program (1) we state some technicalities. We denote with l_{iL} and S_{iL} the optimal choices of lending and selling for the good ($i = G$) and the bad ($i = B$) liquid banks, and, similarly, l_{iI} and S_{iI} for the illiquid banks. We do not report the banks' optimal decisions for $\widehat{p}R_D < 1$ because they are not relevant in equilibrium. In the case when a bank is indifferent between selling or not, we assume that it sells all of its asset. We state our solution to program (1) in two steps, separately for the liquid and illiquid banks.

Lemma 3: *Assume that $\widehat{p}R_D \geq 1$. The liquid bank's optimal lending decision is $l_{iL} = S_{iL}P + \lambda$ for $\widehat{p}R_D > 1$ and $l_{iL} \in [0; S_{iL}P + \lambda]$ for $\widehat{p}R_D > 1$, where $i = B, G$. The liquid bank's optimal selling decision is: $S_{iL} = 1 - \lambda$ for $P\widehat{p}R_D \geq p_i R$, and $S_{iL} = 0$ for $P\widehat{p}R_D < p_i R$, for $i = B, G$. If $P \geq \frac{R}{\widehat{p}R_D}$, both, the good and bad, illiquid banks sell their asset. If $P \in \left[\frac{pR}{\widehat{p}R_D}; \frac{R}{\widehat{p}R_D}\right)$, only the bad banks sell their asset. If $P < \frac{pR}{\widehat{p}R_D}$, none of the bank sells its asset.*

Proof: in the appendix.

The main observation from Lemma 1 is that the GL banks are less willing to sell their asset than the BL banks for a given price P and loan rate R_D, i.e., when the investors cannot distinguish between the quality of banks' asset. Because the good asset is worth more than the bad asset, asset's price P has to be high enough to convince the GL banks to sell their asset and invest the proceeds in interbank lending. Because we assumed upfront that asset purchases are not profitable for the banks, the liquid banks would never borrow, because there are no profitable opportunities to invest the borrowings into.

The following Proposition reports optimal choices of illiquid banks and provides the foundations for the most important result of our paper.

Proposition 1: *Assume that* $\widehat{p}R_D \geq 1$.

1. For $P(1-\lambda)+\lambda > d$ *an illiquid bank with the asset for which* $p_i \geq \widehat{p}$ *chooses*

$$\begin{cases} l_{iI} = -(d-\lambda) \text{ and } S_{iI} = 0 \text{ for } R_D < \frac{R}{P} \\ l_{iI} = 0 \text{ and } S_{iI} = \frac{d-\lambda}{P} \text{ for } R_D \in \left(\frac{R}{P}; \frac{p_i}{\widehat{p}}\frac{R}{P}\right) \\ l_{iI} = (1-\lambda)P + \lambda - d \text{ and } S_{iI} = 1-\lambda \text{ for } R_D > \frac{p_i}{\widehat{p}}\frac{R}{P} \end{cases}$$

An illiquid bank with the asset for which $p_i < \widehat{p}$ *chooses*

$$\begin{cases} l_{iI} = -(d-\lambda) \text{ and } S_{iI} = 0 \text{ for } R_D < \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)} \\ l_{iI} = (1-\lambda)P + \lambda - d \text{ and } S_{iI} = 1-\lambda \text{ for } R_D > \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)} \end{cases}$$

Both types of banks are indifferent between the alternatives when R_D *hits the respective threshold. In addition, assume* $p_G = 1 \geq \widehat{p} > p_B$. *If* $P \geq \frac{R}{R_D}$, *both, the good and bad, illiquid banks sell their asset. If* $P \in \left[\frac{p_B R}{\widehat{p}R_D} + \frac{d-\lambda}{1-\lambda}\left(1-\frac{p_B}{\widehat{p}}\right); \frac{R}{R_D}\right)$, *only the bad illiquid banks sell their asset. If* $P < \frac{p_B R}{\widehat{p}R_D} + \frac{d-\lambda}{1-\lambda}\left(1-\frac{p_B}{\widehat{p}}\right)$, *none of the illiquid banks sells its asset.*

2. If $P(1-\lambda)+\lambda \leq d$ *each illiquid bank* i *chooses* $l_{iI} = -(d-\lambda)$ *and* $S_{iI} = 0$ *for* $R_D < \frac{1-\lambda}{d-\lambda}R$ *. For* $R_D \geq \frac{1-\lambda}{d-\lambda}R$ *each illiquid bank* i *is indifferent between borrowing and selling.*

Proof: in the appendix.

The first result in Proposition 1 is that no illiquid bank carries excess cash till t=2. Borrowing banks borrow only an amount that is needed to cover the liquidity shortfall $d-\lambda$, because borrowing is generally more costly than using own cash reserves. Lending banks sell all of their assets and lend out all remaining cash after paying d because the return on lending is generally higher than return on storing cash.

The second result is that the illiquid banks' options to cope with its liquidity shock depend on the anticipated asset price P. The asset price P determines whether the illiquid bank can use selling instead of borrowing to cope with the liquidity shock. If P is such that $P(1-\lambda)+\lambda > d$, the illiquid bank has enough cash to pay d just by using its cash reserves λ and selling all of its asset $1-\lambda$ for a price P. Hence, instead of borrowing the illiquid bank can cope with the liquidity shortfall $d-\lambda$ by selling. In this case, the higher the loan rate R_D compared with the asset's price P, the more attractive is selling for each type of bank i. However, if $P(1-\lambda)+\lambda<d$, the bank has to borrow, because it does not have enough cash to pay d only by selling the asset and using its cash reserves from t=0. For loan rates such that the borrowing bank has a positive return at t=2 ($R_D < \frac{1-\lambda}{d-\lambda}R$), the bank prefers to borrow, because by selling its return is 0 at t=2 (either it is bankrupt if $P(1-\lambda)+\lambda<d$ or earns nothing if $P(1-\lambda)+\lambda=d$). For any other loan rates the illiquid bank is indifferent between borrowing or selling, because it makes either zero return or is bankrupt.

The third result is again that the bad banks are more willing to sell than the good bank for given price and loan rate. In other words, the price P has to be high enough to convince the good illiquid bank to sell its asset. We state this result for $p_G = 1 \geq \widehat{p} > p_B = p$, because it is the only relevant case in the equilibrium (an equilibrium in which the GI banks sell and the BI banks borrow ($\widehat{p} = p_B = p$) cannot exist, because the BI banks would then prefer to sell their asset for a price R). For the illiquid banks the diverging preference for selling and borrowing occurs for two reasons. The first reason is similar to the case of liquid banks as in Lemma 3. The good bank is less reluctant to sell its asset when the investors cannot distinguish between the quality of the project and pay a indiscriminate price P. Second, the bad banks are more willing to sell than the good banks, because the bad bank's loan portfolio would be less risky than the bank itself ($p_B < \widehat{p}$) making lending and selling more attractive for such a bank than keeping its asset.

Proposition 1 provides intuition for our main result that in equilibrium with asymmetric information high-quality banks borrow rather than sell. The intuition comes from comparing the amounts of assets sold and of interbank borrowing chosen by the illiquid banks with different asset quality. When the banks with different asset quality ($p_i \geq \widehat{p}$ and $p_i < \widehat{p}$) borrow at the same time, they demand exactly the same amount, $d-\lambda$, because for both types of banks borrowing is costly (when they borrow one unit they have to repay more than one unit). However, when these banks sell at the same time, they sell different amounts. The banks with low asset quality ($p_i < \widehat{p}$) always sell all of its asset $1-\lambda$, whereas the banks with higher asset quality ($p_i \geq \widehat{p}$) might want to sell only an amount $\frac{d-\lambda}{P}$.[6] The banks with higher asset quality sell less of it, because they anticipate that their asset is undervalued due to presence of banks with lower asset quality, who can sell their project for at least what it is worth. The readiness of the banks with low asset quality to sell more of their assets will lead to a higher adverse selection cost on the secondary market than on the interbank market, where all banks borrow the same amount. In turn, the good banks will prefer to borrow in equilibrium rather than sell.

4 Equilibria on the interbank and secondary market

After determining banks's optimal liquidity management choices for given asset price and loan rate, we turn to finding the equilibrium asset price and loan rate. As argued earlier the equilibrium asset price and loan rate have to reflect the optimal banks' choices and be consistent with the expectations of all agents about these choices. Hence, there is a feedback between the equilibrium asset price and loan rates, banks' optimal choices and agents' beliefs about these choices. We use the concept of perfect Bayesian equilibrium as in Freixas and Holthausen (2005). We do not define our perfect Bayesian equilibrium explicitly in order to save on notation.

We have to note that there might be multiple equilibria because of the nature of the perfect Bayesian equilibrium concept. An equilibrium, in which at least some illiquid banks borrow, might coexist with an equilibrium in which none of the illiquid banks borrows, once we impose sufficiently pessimistic off-equilibrium beliefs about the quality of borrowing banks. Proof of Proposition 2 contains set of parameters for which the Cho-Kreps intuitive criterion eliminates the equilibrium, in which none of the banks borrows. In what follows, we restrict ourselves to these parameters and focus on discussion of the equilibrium in which at least some illiquid banks borrow.

In order to simplify the exposition of the results, we split the discussion of the equilibria at t=1 into two sections. In Section 4.1 we discuss the case in which the equilibrium price P^* is such that $P^*(1-\lambda)+\lambda > d$ and there is no liquidity shortage on the interbank market. In Section 4.2 we

[6] In fact, there is no equilibrium in which both good and bad illiquid banks sell all of its asset in order to lend excess cash on the interbank market. In such a case, there would be no demand for interbank loans and the interbank market would not clear.

discuss the opposite case. In both sections we discuss only the equilibrium choices of the illiquid banks, because they are the object of our interest.

4.1 Equilibria without liquidity shortage

Because of the feedback between the asset price and loan rates, optimal banks' choices and agents' beliefs, a process of finding equilibria would normally be a tedious task. However, given the GI banks' preference to borrow and our focus on equilibria with at least some banks borrowing, we can narrow the set of possible equilibria to those in which the good banks borrow. We pin down specific equilibria by first assuming specific banks' liquidity management choices in equilibrium, and then writing down equilibrium conditions consistent with these choices. Finally, we check whether the asset price and loan rate that are solution to these conditions fulfill all the requirements for existence of this particular equilibrium.

The precise process for finding the such equilibria is quite complex due the discrete distribution of bank types. Discrete distribution of bank types means that we would have to incur a significant notational burden in order to write down one set of equilibrium conditions that would include all possible equilibrium outcomes as solutions.[7] In this section, we present only equations describing an equilibrium, in which all GI banks borrow and the BI banks are indifferent between borrowing and selling. Such an equilibrium is the one that most closely resembles the most important result of our paper, and the one that contrasts the most with the equilibrium in the case of perfect information described in Lemma 1.

The equilibrium, in which all GI banks borrow and the BI banks are indifferent between borrowing and selling, is described by the following equations:

$$P^* = pR, \tag{2}$$

[7] If we work with a continuous distribution of asset quality, we are able to write down a single set of equilibrium equations at t=1. However, even for a uniform distribution the set of equilibrium eqautions has no analytical solution at t=1.

$$
\begin{aligned}
&\pi q\lambda+\pi\left(1-q\right)\left(\lambda+P^{*}\left(1-\lambda\right)\right)+\left(1-\pi\right)\left(1-q\right)\sigma^{*}\left(\lambda+P^{*}\left(1-\lambda\right)-d\right) \qquad (3)\\
= \;&\left(1-\pi\right)\left(q+\left(1-q\right)\left(1-\sigma^{*}\right)\right)\left(d-\lambda\right),
\end{aligned}
$$

$$
\widehat{p}^{*}=\frac{q}{q+\left(1-q\right)\left(1-\sigma^{*}\right)}+\frac{\left(1-q\right)\left(1-\sigma^{*}\right)}{q+\left(1-q\right)\left(1-\sigma^{*}\right)}p, \tag{4}
$$

$$
R_{D}^{*}=\frac{R}{P^{*}+\left(\frac{\widehat{p}^{*}}{p}-1\right)\left(P^{*}-\frac{d-\lambda}{1-\lambda}\right)}, \tag{5}
$$

where P^{*}, σ^{*}, R_{D}^{*}, and $\widehat{p}^{*}$ are the equilibrium values of the asset price, fraction of the BI banks that sell, loan rate and expected share of borrowing banks that repay their interbank loans at t=2.

Equation (2) states that the equilibrium price P^{*} is equal to the expected return on the bad asset. This reflects the investors' anticipation that only the bad banks sell.[8] Equation (3) describes the interbank market clearing. Its left hand side is the total supply of loans by the GL banks providing its cash reserves λ (the first term) as well as all BL banks and a fraction σ^{*} of the BI banks, which sell all of their asset at a price P^{*} (the last two terms). The right hand side of equation (3) is the demand for loans by all GI banks and a fraction $1-\sigma^{*}$ of the BI banks. Equation (4) provides the anticipated share of borrowing banks that repay their interbank loans at t=2, given that all GI banks and only the fraction $1-\sigma^{*}$ of the BI banks borrow. Equation (5) provides the equilibrium loan rate for which the BI banks are indifferent between borrowing and selling (see Proposition 1).

After solving the system of equations (2)-(5) for P^{*}, R_{D}^{*}, $\widehat{p}^{*}$ and σ^{*} we have to check that this is indeed an equilibrium (details can be found in the proof of Proposition 1). First, we need to check whether the solution fulfills the conditions describing the anticipated equilibrium choices of the liquid and GI banks. Lemma 3 and Proposition 1 provide us with intuition for why these conditions will be fulfilled in an equilibrium in which the BI banks are indifferent between borrowing and selling and only the bad banks sell.[9] Second, the critical variable for the existence of all our equilibria is the amount of cash reserves λ, because it determines whether the interbank

[8] This equation describes trivially secondary market clearing, because the demand for the asset is perfectly elastic given that the investors have deep pockets.

[9] The BL banks sell, because their asset is priced correctly and they find it profitable to lend. For the GL banks the price is too low to sell but they will lend, given that it is profitable to do so. The BI banks borrow, because they are more willing to do so than the BI banks, which are indifferent between borrowing and selling.

market clears under the anticipated banks' borrowing choice. In this type of equilibrium, we can check whether under given λ such equilibrium exists, by checking whether the equilibrium fraction σ^* of the selling BI banks is within the $[0;1]$-interval. Once σ^* is outside of it, it means that the interbank market cannot clear under the assumed banks' choices and the solution cannot constitute an equilibrium.

We report our results in a set of two claims. First, we state our main result on the separation of banks between the interbank and secondary market (Proposition 2). Second, in Lemma 4 we provide monotonicity results on our equilibrium variables.

Proposition 2: *Suppose that the parameters are such that the equilibrium asset's price P^* is such that $P^*(1-\lambda)+\lambda > d$.*

For sufficiently low cash reserves λ only some of the GI banks borrow, and the rest of the GI banks and all BI banks sell. The share of borrowing GI banks increases with λ until it reaches 1.

For intermediate cash reserves λ all GI banks and some of the BI banks borrow, and the rest of BI banks sells. The share of borrowing BI banks increases with λ until it reaches 1.

For sufficiently high cash reserves λ all GI and BI banks borrow.

Proof: in the appendix.

Proposition 2 states the most important result of the paper. In equilibrium, for a given λ the average bank borrowing on the interbank market is of higher quality than the average bank selling on the secondary market. Proposition 2 shows how the results from Proposition 1 for given asset price and loan rate materialize in equilibrium when these prices are endogenous. The main intuition behind our result is that the adverse selection cost of selling is higher than the adverse selection cost of borrowing from the perspective of the good illiquid banks. As argued earlier, the reason for this discrepancy is that the share of the sold bad asset in the total amount of sold asset is higher than the share of bad borrowing banks in the total amount of borrowing banks. Hence, when the GI anticipate that they would face a higher adverse selection cost on the secondary market than on the interbank market, the good banks prefer to incur it on the interbank market.

Proposition 2 shows also that the average quality of borrowing and selling banks decreases with cash reserves λ. This result is driven by interplay of the "cash-in-the-market" effect and asymmetric information on the interbank market. On the one hand, GI banks' preference for borrowing attracts the BI banks to the interbank market. Because the lending banks cannot distinguish between the bad and good banks, the borrowing BI banks receive a subsidy from the borrowing GI banks. On the other hand, the fixed amount of cash reserves from t=0 and cash raised from asset sale by lending banks puts a limit on the supply of loan supply. Hence, for sufficiently low cash reserves λ there is not enough interbank loans for all illiquid banks and some of them have to sell. Because the BI banks are more willing to sell than the GI banks, they are the first ones to leave the interbank market when supply of loans becomes scarce (as cash reserves λ fall into intermediate level). Eventually, for low supply of interbank loans the GI banks are also forced to sell. On the contrary, when liquidity on the interbank market is abundant all illiquid banks switch to the interbank market abandoning the secondary market all together.

Limited supply of interbank loans gives a rise to a liquidity premium on the interbank market reflected in the equilibrium loan rate (the cash-in-the-market effect), which does not arise on the secondary, where liquidity is abundant. The liquidity premium on the interbank market arises due to an implicit assumption about limits to arbitrage, i.e., the outside investors with abundant liquidity cannot lend on the interbank market. First, we think of this assumption as plausible, because the outside investors such as pension or hedge funds are not interested in unsecured interbank lending at short maturities, and the banks themselves might not be able to mobilize sufficient liquidity to arbitrage away the liquidity premium at a short notice (see Freixas et al. (2011)). Second, our result about the resiliency of the interbank market in times of uncertainty has nothing to do with our assumption about limits to arbitrage. In fact, limits to arbitrage are making it harder to obtain this result because we are discouraging banks from borrowing by making it more expensive due to the liquidity premium. If we allowed for a free flow of liquidity between the interbank and secondary market, then for any amount of banks' cash reserves λ we would obtain the result that we obtain for sufficiently high cash reserves in Proposition 1: all illiquid banks would borrow and none of them would sell.

To close up the discussion of equilibria without liquidity shortage we provide monotonicity results for equilibrium price, loan rate and total lending volume as functions of cash reserves λ.

Lemma 3: *Suppose that parameters of the model are such that there are equilibria in which* $P^* (1 - \lambda) + \lambda > d$.

1. Equilibrium price P^* *is decreasing in* λ *until all of the GI banks leave the secondary market, when* P^* *reaches* pR. From then on $P^* = pR$.

2. R_D^* *is increasing in*λ *until all of the GI banks leave the secondary market, then it can be increasing or decreasing until all of the BI banks leave the secondary market, and equals* $\frac{1}{q+(1-q)p}$ *when all liquid banks borrow.*

3. Total lending volume is decreasing in λ *until all of the GI banks leave the secondary market, can be non-monotonic (first increasing and then decreasing) until all of the BI banks leave the secondary market, and decreasing when all liquid banks borrow.*

Proof: Proofs are straightforward.

Equilibrium price P^* of the asset decreases with cash reserves as long as some of the GI banks sell, reflecting the results of Proposition 1. When the cash reserves increase, the equilibrium asset price falls, because more of the GI banks leave the secondary market, whereas all of the BI banks sell. The changes in the equilibrium loan rate R_D^* reflect the interplay between asymmetric information and limited interbank loan supply. On the one hand, the equilibrium loan rate increases with cash reserves, because more of the BI banks can borrow and the loan rate reflects decreasing expected quality of borrowing banks (measured by $\widehat{p}^*$). On the other hand, the loan rate decreases with λ, because supply of interbank loans becomes more abundant and the liquidity premium decreases.[10] Total lending volume is generally decreasing in λ because each illiquid bank's loan demand $d - \lambda$ decreases. However, for intermediate cash reserves total lending volume can be non-monotonic. The reason is that increasing share of the borrowing BI banks increases the total loan demand.

[10] In two knife-edge cases for $\lambda = \lambda_1$ and $\lambda = \lambda_2$ as defined in the proof of Proposition 1 R_D^* is indeterminate, because the demand and supply of interbank loans are inelastic (see also Freixas, Martin and Skeie (2011) for a similar result). In addition, in a case with continuous distribution of asset quality the interpplay between asymmetric information and limited loan supply gives a clear-cut inverse-U-shape of R_D^* as a function of λ.

4.2 Equilibrium with liquidity shortage

In the previous section we found equilibria in which the equilibrium price of the asset is so high that a selling bank can always achieve a positive return (and become liquid) by selling all of its asset. However, there is nothing in the model that would guarantee that equilibria from Proposition 1 always exists. The interbank market attracts higher quality banks, resulting in an equilibrium asset price that is affected by much higher share of the bad asset sold. If the expected return on the bad asset is sufficiently low, the equilibrium asset price might fall so low that the selling illiquid banks cannot raise enough liquidity to become liquid.

In this section we turn to equilibria in which the equilibrium asset price might be so low that a selling illiquid bank earns zero return or cannot raise enough liquidity to become liquid, $P^* (1 - \lambda) + \lambda \leq d$. We first provide conditions under which we can have $P^* (1 - \lambda) + \lambda \leq d$, i.e., the equilibria from Proposition 2 do not exist.

Lemma 4: *Once we have that $pR < d$, equilibria from Proposition 2 do not exist ($P^* (1 - \lambda) + \lambda \leq d$) for low λ if the liquidity need d is high and the share of good banks q is low, or for intermediate λ if the liquidity need d and the share of good banks q are high. Otherwise, equilibria from Proposition 2 always exist.*

Proof: In the Appendix.

The necessary condition for $P^* (1 - \lambda) + \lambda \leq d$ is that the expected return on the bad asset is lower than the liquidity need d, $pR < d$. Otherwise, pR is so high that the lowest equilibrium asset price is never lower than the liquidity need d. Fig. 3 presents the results of Lemma 4 under $pR < d$. First, $P^* (1 - \lambda) + \lambda \leq d$ obtains for low cash reserves λ when liquidity need d is high and the share of good banks q is sufficiently low. For such d and q the expected return on the sold asset is so low, that without significant cash reserves λ a bank cannot raise enough liquidity by selling even all of its asset. Second, $P^* (1 - \lambda) + \lambda \leq 0$ can also obtain for intermediate λ when the liquidity d and the share of the good banks q are high. Here the result obtains because for some intermediate λ the share of selling good banks is very low after almost all of them switched to borrowing. Hence if liquidity need is high enough and the equilibrium price is close to pR, equilibria from Proposition 2 might not exist. Later we will discuss the role of the interbank market, limits to arbitrage and asset fire sales in the results of Lemma 4 and subsequent Proposition.

The following Proposition presents equilibrium outcomes when $P^* (1 - \lambda) + \lambda \leq d$.

Proposition 3: *Suppose that parameters of the model are such that there are equilibria in which* $P^* (1 - \lambda) + \lambda \leq d$.

1. If there is sufficient liquidity on the interbank market for all illiquid banks to borrow, all illiquid banks become liquid by borrowing.

2. If there is no sufficient liquidity on the interbank for all illiquid banks to borrow, the following equilibria arise. For sufficiently low R *and cash reserves* λ *such that* $P^* (1 - \lambda) + \lambda < d$ *there is an equilibrium with liquidity shortage. In such an equilibrium only some of the illiquid banks become liquid by borrowing, and the rest of the illiquid banks sells their assets and goes bankrupt. The equilibrium loan rate is* $\frac{1-\lambda}{d-\lambda}R$ *and asset's price is lower than* $\frac{d-\lambda}{1-\lambda}$. *Otherwise there is an equilibrium in which* $P^* (1 - \lambda) + \lambda = d$ *without bank bankruptcies and all illiquid banks being indifferent between borrowing and selling. The equilibrium loan rate is* $\frac{1-\lambda}{d-\lambda}R$ *and the equilibrium asset's price is* $\frac{d-\lambda}{1-\lambda}$.

Proof: In the Appendix.

We describe focus first on the more interesting equilibrium with liquidity shortage. Let's assume that the anticipated asset price is too low to make banks liquid by selling, $P^* (1 - \lambda) + \lambda < d$. Per Proposition 1 for such an asset price *each* illiquid bank, regardless of its asset quality, would like to borrow for *any* loan rate R_D for which it can repay the interbank loan at t=2, $R_D < \frac{1-\lambda}{d-\lambda}R$. Otherwise, each of these bank is indifferent between borrowing and selling, because either it cannot repay its loan for $R_D > \frac{1-\lambda}{d-\lambda}R$ or borrowing delivers payoff of 0 for $R_D = \frac{1-\lambda}{d-\lambda}R$. The liquid banks supply loans only if the loan rate is sufficiently low so that the illiquid banks can repay their loans at t=2, $R_D \leq \frac{1-\lambda}{d-\lambda}R$. As long as there is enough interbank loans for all illiquid banks, they can all become liquid by borrowing and there are no defaults in equilibrium. As soon as there is not enough interbank loans for all illiquid banks to borrow, the interbank market clears at the highest loan rate at which the liquid banks lend and the illiquid are indifferent between selling and borrowing, $R_D = \frac{1-\lambda}{d-\lambda}R$. Then the available interbank loans are assigned randomly to all of the liquid banks and only some of them get the loans. The rest of the illiquid banks becomes bankrupt, because the equilibrium asset price is too low for them to become liquid.[11] Of course, this too low an asset price arises endogenously.

There is also an equilibrium in which $P^*(1-\lambda)+\lambda = d$. This equilibrium arises due to our assumption that in case $P^*(1-\lambda)+\lambda < d$ the bankrupt banks, independently of their asset quality, sell all of their assets, and that the available interbank loans are randomly assigned to the illiquid banks. These two assumptions lead to a higher share of the good asset on the secondary market than in the equilibria in which $P^*(1-\lambda)+\lambda > d$ for two reasons. First, the good bankrupt banks sell $1-\lambda$ instead of $\frac{d-\lambda}{P}$. Second, the share of the selling bankrupt good banks is the same as their share in the whole of illiquid banks, q, because they were randomly assigned loans. This share is higher than the share when they decide when to borrow on their own. Given these assumptions it is possible for some λ neither the equilibria from Proposition 2 nor the equilibria with liquidity shortage arise. If the banks' selling decisions are like in equilibria with liquidity shortage, the asset price might be such that $P(1-\lambda)+\lambda > d$ for some λ, because the share of the good asset sold by the bankrupt good banks is too high. However, then the good banks would again turn to borrowing as in equilibria without liquidity shortage. This in turn would depress the price of the asset to the level in which $P(1-\lambda)+\lambda < d$. Hence, in such a region there must be an equilibrium in mixed strategies for all types of banks in which any of them is indifferent between selling and borrowing and the equilibrium asset price is such that $P^*(1-\lambda)+\lambda = d$.

4.3 The role of interbank market

After stating the most important results of the paper it is worthwhile to discuss the interplay of the interbank and secondary markets for managing liquidity. Under perfect information we obtained an equivalence of Modigliani-Miller theorem: illiquid banks are indifferent between selling and borrowing.

However, the results under asymmetric information are starkly different. The illiquid banks,

[11] Instead of rationing the share of borrowing illiquid banks we could ration the size of the interbank loan. In such a case each illiquid bank would borrow less than $d-\lambda$ and sell its asset to come up with the missing cash. Given that all illiquid banks would do the same they would all default in an equilibrium with liquidity shortage. Once we assume that defaults are socially costly, rationing the loan size size is less inefficient than the assumed rationing of the number of borrowing banks which limits the number of defaulting banks. For that reason we stick to the latter form of rationing.

specifically the good illiquid banks, prefer to borrow rather than sell. The good illiquid banks anticipate that, if they sell, the bad illiquid banks will flood the secondary market with their bad asset to dispose it at a price higher than the value of their asset. At the same time, the bad illiquid banks anticipate that the good illiquid banks would prefer to borrow, so they try to borrow too. This time the good banks are ready to put it up with the adverse selection cost because borrowing is costly and the bad illiquid banks just borrow the same amount as the good banks.

In equilibrium, the outcome is that the good illiquid banks generally prefer to borrow and sell only when there is not enough liquidity on the interbank market. In extreme case, this preference for borrowing can destroy the quality of the sold asset so much that all illiquid banks borrow and go bankrupt if there is not enough loans for all of them. Indeed, we can show that for some λ close to λ_2 such that we have an equilibrium in which banks go bankrupt due to liquidity shortage on the interbank market, $P^* (1 - \lambda) + \lambda < d$, there would be no bankruptcies, if the banks could only sell the asset. The reason is that a higher share of the good banks would sell when there were no interbank market.[12]

The striking feature is that the presence of the interbank market disrupts the liquidity management to such extent that despite of abundant liquidity on the secondary market some banks can go bankrupt due to lack of liquidity. This results relies on our assumption that in a very acute shock it might be impossible to mobilize additional private liquidity to arbitrage away the liquidity premium. Had this been possible we would have less extreme but still powerful effect coming from the interbank market. In such a case the interbank market would attract all of the illiquid banks leaving only the bad liquid banks ready to sell their asset. In effect, making the secondary market useless for the banks' liquidity management decisions.

[12] In case without interbank market the equilibrium asset price is that same as in the equilibrium that the intuitive criterion eliminates (P^e as in the proof of Proposition 2). We can show that P^e is decreasing in λ and reaches its minimum value pR for $\lambda = d$. Given that $P^* = pR$ for all $\lambda \in [\lambda_1; d]$. Then it has to be that $P^e > P^*$ for all $\lambda \in [\lambda_1; d]$. Equilibria with liquidity shortage arise when $P^* = pR < \frac{d-\lambda_2}{1-\lambda_2}$. Then by continuity argument there must be some λ close to λ_2 for which $pR < \frac{d-\lambda}{1-\lambda} < P^e$. We cannot show it for λ, because given the amount of the bad asset sold it is still possible that for some very low R there are $\lambda \in [0; d)$ such that $P^e < \frac{d-\lambda}{1-\lambda}$.

4.4 Social welfare

Social welfare at t=1 is the sum of expected profits of all four types of banks at t=2 and the sum of payments d by the illiquid banks. By including these payments in welfare calculation we make them welfare neutral if they are made.[13] We also assume that missing a payment by a bankrupt bank costs $\tau \geq 0$ units of welfare per unit of missed payment.

Lemma 5: *Social welfare at t=1 in equilibria in which $P^* (1 - \lambda) + \lambda \geq d$ is equal to the expected value of the asset and cash kept by all banks, $(1 - \lambda) (q + (1 - q) p) R + \lambda$. Social welfare at t=1 in equilibria in which $P^* (1 - \lambda) + \lambda < d$ is lower than $(1 - \lambda) (q + (1 - q) p) R + \lambda$* if $\tau > 0$.

Proof: In the appendix.

In equilibria with $P^* (1 - \lambda) + \lambda \geq d$ the highest possible social welfare at t=1 is achieved, because the full value of banks is reached. First, all illiquid banks obtain enough liquidity to pay d and none of them fails. Second, transfer of liquidity on the interbank and secondary markets are welfare-neutral, because it just redistributes cash reserves between banks in need of cash and other agents that have it in excess. Specifically, the cash-in-the market effect on the interbank market is welfare neutral.

In equilibria with $P^* (1 - \lambda) + \lambda < d$ the highest possible social welfare at t=1 for a given λ cannot be achieved if $\tau > 0$. Selling banks that cannot borrow raise only $P^* (1 - \lambda) + \lambda$ of cash by selling, which is not enough to pay the full d. Hence, each of those bankrupt banks misses the payment of $d - (P^* (1 - \lambda) + \lambda)$, which costs the society τ per missed unit of payment.

5 Optimal choice of cash reserves

To close the model we solve for optimal choice of cash reserves at t=0. At t=0 the banks maximize their expected t=2-profits by choosing λ consistent with an equilibrium that they anticipate at t=1. The following result summarizes the optimal choice of λ at t=0.

Lemma 6: *When the parameters are such that $P^* (1 - \lambda) + \lambda > d$* for any $\lambda \in [0; d)$ at t=1,

[13] If the payments d were not added to welfare, their execution by banks would be socially costly, because they would reduce banks' returns. Hence, it would be socially beneficial to close all illiquid banks and sell their assets without making the payment d.

the optimal choice of cash reserves λ at t=0 increases when the asset return R in case of success R decreases. For some intermediate R there are multiple equilibria: the banks choose either not to hold any or positive cash reserves. When the parameters are such that there exists an equilibrium with liquidity shortage at t=1 for some λ, for some intermediate R the banks choose at t=0 such λ that an equilibrium with liquidity shortage at t=1 occurs.

Proof: In the appendix.

In case of equilibria without liquidity shortage, we are able to obtain an analytical solution at t=0, which we present in the Lemma above. In case parameters are such that $P^* (1 - \lambda) + \lambda \leq d$, it is impossible to obtain analytical solutions at t=0. In the proof of Lemma 6 we provide two numerical examples in which we vary R to understand the optimal choice of λ at t=0. In both cases we find qualitatively similar result as in the case where no equilibria with liquidity shortage arise.

In case of when $P^* (1 - \lambda) + \lambda > d$ for any $\lambda \in [0; d)$ at t=1, the bank's choice of cash reserves at t=0 depends on the profitability of the asset (see Fig. 5). The less profitable the asset is the more cash the bank wants to invest in at t=0. At t=0 the bank faces the following trade-off. On the one hand, higher cash reserves reduce the expected return at t=2 because the bank invests less in the long-term asset. On the other hand, the bank anticipates that cash reserves might be valuable at t=1 for speculative and pre-cautionary reasons. If the bank is liquid, cash might be used to earn positive net return on interbank lending due to the cash-in-the-market effect. If the bank is illiquid, cash reduces the need to acquire costly cash through either interbank market or selling (if the bank becomes good and illiquid). Hence, as the long-term asset becomes less profitable, the bank invests more in cash at t=0.

In consequence, the banks at t=0 would also choose such λ that later at t=1 they might fail due to liquidity shortage. The reason is that for some intermediate R (and d is such that liquidity shortage might occur) the profitability of the long-term asset is such that it is perfectly rational for the banks to choose cash reserves for which they might fail at t=1.

There is possibility of multiple equilibria in the choice of cash reserves λ at t=0. The reason is that same as in Malherbe (2014): coordination failure at t=0. Multiple equilibria due to coordination failure arise, because the equilibrium asset price at t=1 might be decreasing in cash reserves λ, and at t=0 the banks prefer low (high) cash reserves λ when they anticipate a high (low) asset price at t=1. On the one hand, if a bank expects all other banks to have zero cash reserves λ, it also chooses zero cash reserves. Carrying positive cash reserves would not be optimal, because if all other banks have no cash reserves, the asset price at t=1 will be high. If the asset price at t=1 is expected to be high, it is better to have only the risky asset and no cash reserves. However, if each bank expects all other banks to have high cash reserves, it also chooses high cash reserves λ. The logic is exactly the opposite as in the case of the previous equilibrium.

Multiple equilibria arise only for some intermediate R. There are no multiple equilibria for high R for the following reason. The value of the bad asset is so high, that by selling their bad asset the bad banks can provide so much interbank loans that only bad illiquid banks are forced to sell. In such a case the equilibrium asset price at t=1 is pR and constant for any λ. Hence, the banks' choice of λ at t=0 does not influence the equilibrium asset price, which is needed to for the multiple equilibria to arise. There are also no multiple equilibria for low R, because then the return on the asset is too low. For sufficiently low R the highest equilibrium asset price is so low that the banks prefer to hold positive cash reserves.

We do not put much emphasis on multiple equilibria in the choice of cash reserves in our setting for three reasons. First, they arise only for certain range of parameters. Second, existence of multiple equilibria is sensitive to assumption of abundant liquidity on the secondary market. Once we allow for fire sales of assets, the equilibrium asset price at t=1 might actually be increasing in cash reserves. The reason is that as cash reserves increase the banks need to sell less reducing the fire sale discount. If this is the case, multiple equilibria seize to exist

6 Welfare analysis

Social welfare for a given optimal choice of cash reserves λ at t=0 is equal to social welfare provided in Lemma 5 corresponding to this particular choice of λ. In general, banks' privately optimal choice of cash reserves is not socially efficient. This occurs for two reasons depending on whether these choices lead to an equilibrium with or without liquidity shortage.

First, the socially efficient choice of cash is 0 when it does not lead to an equilibrium with liquidity shortage. Zero cash reserves are socially optimal, because the investment in the long-term productive asset is the highest and there are no bank bankruptcies. However, the banks as price-takers do not internalize that their choice of cash reserves has influence on the asset price and loan rate. Specifically, they do no take into account that higher cash reserves lower the equilibrium asset price, reducing the value of the asset as an insurance against liquidity shocks vis-a-vis interbank loans financed through cash reserves.

Social optimality of zero cash reserves in equilibria without bank bankruptcies arises due to our asymmetric (and unrealistic) treatment of the interbank and secondary market: we do not allow for the cash-in-the-market effect on the secondary market (asset fire sales). Allowing for asset fire sales would push the socially optimal cash reserves towards positive values. This would happen, if for zero cash reserves, the high amount of asset sold by the illiquid banks would result in a fire sale discount for which equilibria with liquidity shortage and bank bankruptcies occur. In addition, asset fire sales that do not cause bank bankruptcies would strengthen illiquid banks' preference for the interbank market, which is the main result of our paper. Given that discussion of asset fire sales has quite intuitive welfare implications and does not affect our model's positive implications we refrained from modelling them explicitly.

Second, the banks might choose cash reserves such that an equilibrium with liquidity shortage occurs. The reason is that they do not internalize the social cost of their bankruptcy. In a numerical example in Lemma 6 the banks may choose zero cash reserve instead of cash reserves that guarantee enough interbank loans for all banks. If the social cost of banks' bankruptcy τ is high enough, these zero cash reserves are socially inefficient. The banks choose zero cash reserve for sufficiently high R, because they profit from high return on the productive risky asset and do not incur their social cost of bankruptcy.

7 Policy Implications

In this section we discuss several policy implications, i.e., tools that are at the social planner's disposal to address the above mentioned inefficiencies. Given that our model is very simple and does not allow for more elaborate discussion of the social cost of using these tools, we only qualitatively discuss differences between these different tools.

7.1 Liquidity requirements

As can be seen from the discussion about reasons for inefficient cash reserve choices, the form of liquidity requirements differs dramatically depending on the equilibrium outcome. In case there are no equilibria with liquidity shortage and the banks choose positive cash reserves, the social planner could increase welfare by making the banks hold *less* cash than they want. Specifically, the social planner would restrict cash reserves to zero (see Malherbe (2014) for a similar result). In case the banks choose zero cash reserves and equilibria with liquidity shortage arise, the social planner could increase welfare by making banks hold *more* cash than the banks want. The social planner would then impose a liquidity requirement guaranteeing enough interbank loans for all illiquid banks.

Our discussion suggests that banks' liquidity in form of cash reserves (or very liquid but low yielding assets) is socially valuable during crises, but socially costly during normal times. Ultimately, the answer about the exact form of the liquidity requirement would have to be answered in a model with aggregate uncertainty with respect to crucial parameters of the model. Introduction of aggregate uncertainty would allow for both types of equilibria to occur with some positive probability.[14] In such a case the social planner's choice of liquidity requirement would depend on the expected cost of bank bankruptcies.

7.2 Interventions on the interbank market

We discuss now a possibility that the social planner acts as a central bank that has a possibility to lend to individual banks. We preserve here the assumption that the central bank cannot distinguish between the good and bad illiquid banks. In our stylized model, central banks' loans to individual banks could be implemented via discount window lending (albeit at a market interest rate without any add-ons) to individual banks or central bank appearing as another lender on the interbank

[14] Aggregate uncertainty with respect to only one parameter might not be enough in our model to encompass both types of outcomes.

market with additional liquidity lending at the market loan rate. Any such intervention would have exactly the same effect on the equilibrium outcomes and central bank's holding of interbank loans. Finally, the central bank acting on the interbank market ends up exactly with the same expected return on a unit lent as any other lenders, $\widehat{p}R_D$.

We first discuss the case in which the banks take positive cash reserves and there are no equilibria with liquidity shortage. From previous discussions we know that the banks see no value in holding cash if the return on interbank lending and therefore the cost of borrowing are equal to the return on storage. Central bank has the possibility to flood the interbank market with liquidity to the point, where the interbank loan rate drops to its break even level for lending to the good and bad banks, $\frac{1}{q+(1-q)p}$. If the central bank's promise to do so is credible, then there is no reason for the banks to hold cash reserves and they choose optimally zero cash reserves. The central bank could implement such a policy by credibly committing itself to flood the interbank market at t=1 if the equilibrium asset price and loan rate are different from the ones which arise for zero cash reserves. However, the issue with this policy is its credibility. Once, the banks choose cash reserves λ, the central bank's intervention itself has no welfare benefits. Because there are no bank bankruptcies at t=1, liquidity redistribution and, therefore, additional central bank lending would be welfare neutral. Hence, if we introduced additional cost of central bank's intervention at t=1 (such as political backlash), such a policy would not be credible.

We now turn to the case in which the banks take zero cash reserve for which there is an equilibrium with liquidity shortage and inefficient bank bankruptcies. Contrary to the previous case, lending to illiquid banks that cannot obtain loans and go bankrupt is a welfare enhancing policy. Hence, the credibility issue is less pronounced. Here the banks would again choose zero cash reserve, because the central bank would guarantee that there would be enough liquidity for all illiquid banks.

It is now important to observe that central bank's lending in case of equilibria with liquidity shortage leads in our model to a higher social welfare than a minimum liquidity requirement discussed earlier. Although both policies eliminate inefficient bankruptcies, minium liquidity requirement leads to lower investment in the long-term productive project than a credible promise of central bank's lending. In absence of additional cost of central bank's lending and of liquidity requirements, in model with aggregate uncertainty the social planner would like to keep the cash in banks at zero and lend to them in times of crisis.

7.3 Asset purchases

Given that in our model there are no fire sales, simple asset purchases by the central bank do not work as means to increase social welfare. Adding additional liquidity on the secondary market has no bite, because there is already unlimited amount of it. Specifically, in case of equilibria with liquidity shortage, asset purchases by the central bank are ineffective, because they do not attract more of the good banks to the secondary market.

Ineffectiveness of asset purchases gains on importance, because it is also robust to introduction of asset fire sales. In a model with asset fire sales the liquidity shortage on the interbank market would arise due to adverse selection and fire sales on the secondary market. However, asset purchases could only address the asset price decline due to fire sales and would not eliminate liquidity shortage due to adverse selection. In fact, asset purchases eliminating fire sales could even lead to further price decline due to adverse selection. The reason is that an increase in liquidity on the secondary market could attract more of the riskier banks to the secondary market (who are more willing to sell than the GI banks). However, an increase in a share of riskier banks would put the downward pressure on the price due to falling asset quality and still trap the banks in an equilibrium with liquidity shortage.

The ineffectiveness of central bank's asset purchases in an equilibrium with liquidity shortage has to be contrasted with its lending on the interbank market. Central bank's lending and asset purchases are in a narrow sense a similar action, because both increase the supply of liquidity. The effect of an increase in liquidity supply differs due to different motives for which the illiquid banks use both markets to purchase liquidity. In an adverse selection environment the interbank market is a preferable source of liquidity for good banks. Hence, liquidity shortage on the interbank market is not due to too much adverse selection but to too little liquidity, whereas the reason why banks cannot obtain enough liquidity on the secondary market is exactly opposite.

8 Discussion

8.1 Modelling of liquidity shock d

We describe elements of a Diamond-Dybvig setup that would allow us to (i) endogenize the size and incidence of liquidity shocks, (ii) relax the assumption about the independence of liquidity and asset return shocks, (iii) create a reason for the interbank market, and (iv) add debt incurred at t=0.

A simple Diamond-Dybvig setup allows for debt incurred by banks at t=0 and liquidity needs that come from impatient consumers. Such a setup would be augmented by two assumptions. First, the size of impatient consumers' liquidity need at t=1 would be different across banks as in Freixas et al. (2011) to generate liquid and illiquid banks. It has to be noted that as in original Diamond-Dybvig model the impatient consumers' liquidity needs would be uncorrelated with realization of banks' asset return at t=1. Second, consumers would not be able to observe the true return on banks' assets. This assumption would keep the possibility of equilibria with liquidity shortage occurring. In an equilibrium with liquidity shortage at t=1 the patient consumers would run on the banks that could not obtain sufficient liquidity, and the run would be indiscriminate given that the consumers could not observe the true asset returns. Hence, we would be able to maintain our assumption on liquidity shocks not being perfectly correlated with the realization bad return. If we allowed for consumers to get an imperfect signal about the return on asset of their banks, we would increase a chance that the patient consumers (a severe liquidity shock) would hit a bank with worse fundamentals. Hence, we would endogenize the assumption that liquidity shocks might be correlated with banks' quality in form an informational-based run. Such an extended setup would still generate the same results: first, the willingness of good banks to borrow (because there is nothing in the consumer behavior that would change that) and the existence of equilibria with and without liquidity shortage would also prevail.

9 Conclusion

The paper provides a generic model of liquidity provision in which the illiquid banks can choose between cash, interbank borrowing and asset sales as sources of liquidity. We show that asymmetric information has quite dramatic effect on the performance of the interbank and secondary markets. Under perfect information we obtain Modigliani-Miller-type result, in which the illiquid banks are indifferent between selling or borrowing. Under asymmetric information, we show that banks prefer to borrow rather than sell, because the adverse selection cost of selling is higher than of borrowing. We then obtain two equilibrium results. First, if there are no bank bankruptcies, liquidity redistribution is welfare neutral despite banks preferring one liquidity source than the other. Second, it is possible that banks' inability to become liquid by selling their assets forces them to borrow, resulting in liquidity shortage on the interbank market and socially inefficient bank failures.

The paper main contribution is to provide a novel explanation for a transmission of a shock from the secondary market for bank assets to the interbank markets. As such the paper can be viewed as an interpretation of events in August 2007. We also argue why, contrary to conventional wisdom, the interbank markets did not freeze despite collapse of securitization markets. We argue that in case of an acute stress the interbank markets provide outside liquidity at the lowest adverse selection cost.

References

Acharya, V., G. Afonso, A. Kovner. 2013. "How Do Global Banks Scramble for Liquidity? Evidence from the Asset-Backed Commercial Paper Freeze of 2007." Working Paper, FRBNY and NYU Stern.

Acharya, V., D. Skeie. 2011. "A Model of Liquidity Hoarding and Term Premia in Inter-Bank Markets," Journal of Monetary Economics, 58, pp. 436-447.

Afonso, G., A. Kovner, A. Schoar. 2011. "Stressed, Not Frozen: The Federal Funds Market in the Financial Crisis'" Journal of Finance, 66. pp. 1109-1139.

Akerlof, G. 1970. "The Market for Lemons: Quality Uncertainty and the Market Mechanism," Quarterly Journal of Economics, 82, pp. 488-500.

Allen, F., D. Gale. 2000. "Financial Contagion,' The Journal of Political Economy, 108, pp. 1–33.

Ashcraft, A., J. McAndrews, D. Skeie. 2011. "Precautionary Reserves and the Interbank Market," Journal of Money, Credit and Banking, 43, pp. 311–348.

Brunnermeier, M., L. H. Pedersen. 2009. "Market Liquidity and Funding Liquidity," Review of Financial Studies, 22, pp. 2201-2238.

Brunetti, C., M. di Filippo, J. H. Harris. 2011. "Effects of Central Bank Intervention on the Interbank Market during the Subprime Crisis," Review of Financial Studies, 24, pp. 2053-2083.

Basel Committee on Banking Supervision. 2013. "Basel III: the Liquidity Coverage Ratio and liquidity risk monitoring tools."

Bolton, P., T. Santos, J. Scheinkman. 2011. "Outside and Inside Liquidity," The Quarterly Journal of Economics, 126, pp. 259–321.

Diamond, D., P. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity," Journal of Political Economy, 91, pp. 401–419.

Diamond, D. 1997. "Liquidity, Banks, and Markets," Journal of Political Economy, 105, pp. 928–956.

Farhi, E., J. Tirole. 2012. "Collective Moral Hazard, Maturity Mismatch, and Systemic Bailouts," American Economic Review, 102(1), pp. 60–93

Fecht, F. 2006. "On the Stability of Different Financial Systems," Journal of the European Economic Association, 2, pp. 969–1014.

Freixas, X., C. Holthausen. 2005. "Interbank Market Integration under Asymmetric Information," Review of Financial Studies, 18, pp. 459-490.

Freixas, X., A. Martin, D. Skeie. 2011. "Bank Liquidity, Interbank Markets, and Monetary Policy," Review of Financial Studies, 24, pp. 2656-2692.

Freixas, X., B. Parigi, J.-C. Rochet. 2000. “Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank," Journal of Money, Credit, and Banking, 32, pp. 611–638.

Gale, D, T. Yorulmazer. Forthcoming. "Liquidity Hoarding," Theoretical Economics.

Heider, F., M. Hoerova, C. Holthausen. 2009. "Liquidity hoarding and interbank market spreads: the role of counterparty risk," ECB working paper.

Keeton, W. 1979. "Equilibrium Credit Rationing," New York: Garland Press.

Kuo, D., D. Skeie, T. Youle, J. Vickrey. 2013. "Identifying Term Interbank Loans from Fedwire Payments Data," Federal Reserve Bank of New York Working Paper.

Malherbe, F.. Forthcoming. "Self-fulfilling Liquidity Dry-Ups," Journal of Finance.

Martin, A., D. Skeie, E.-L. von Thadden. 2012. "The Fragility of Short-Term Secured Funding Markets," working paper.

Myers, S., N. Majluf, 1984. "Corporate financing and investment decisions when firms have information that investors do not have," Journal of Financial Economics, 13, 187-221.

Perotti, Enrico, Javier Suarez. 2011. "A Pigovian Approach to Liquidity Regulation," International Journal of Central Banking, vol. 7, pp. 3-41

Sarkar, A. 2009. "Liquidity risk, credit risk, and the federal reserve’s responses to the crisis," Journal of Financial Markets and Portfolio Management, 23, 335-348.

Tirole, J. 2011. "Illiquidity and All Its Friends," Journal of Economic Literature, 49(2): 287–325.

Rochet, J.-C. 2004. "Macroeconomic Shocks and Banking Supervision," Journal of Financial Stability, vol. 1, p. 93-110.

10 Proofs

Proof of Lemma 3

We make the following observation that the liquid banks will never borrow, because the proceeds from borrowing have no use for these banks: they have no payment d to make nor there are profitable opportunities to invest them into (asset purchases have return on storage so borrowing to buy would be waste of resources). After rewriting (1) for $\mu = 0$ the liquid bank's decision problem reads:

$$\max_{l,S}(1-\lambda-S)p_iR+(SP+\lambda-l)+\widehat{p}R_Dl, \text{ s.t. } S\in[0;1-\lambda],\ l\in[0;SP+\lambda].$$

The bank's expected return is linear in l and in S. Then we have that for $\widehat{p}R_D > 1$ $l_{iL} = S_{iL}P+\lambda$, and for $\widehat{p}R_D = 1$ $l_{iL} \in [0; S_{iL}P+\lambda]$ for $i = B, G$.

For $\widehat{p}R_D \geq 1$ after taking into account the optimal lending decisions the bank's expected return becomes

$$(1-\lambda)p_iR+\widehat{p}R_D\lambda+(P\widehat{p}R_D-p_iR)S.$$

The first two terms in the last expression are constant. Inspection of the last term delivers the second result concerning the optimal selling decision: $S_{iL} = 1-\lambda$ for $P\widehat{p}R_D \geq p_iR$, and $S_{iL} = 0$ for $P\widehat{p}R_D < p_iR$, for $i = B, G$. The third result about the banks' willingness to sell for a given P, R_D and $\widehat{p}$ follows from the fact that $p_B < p_G = 1$.

Proof of Proposition 1

As in the body of Lemma 4 we split the discussion into two cases depending on the price P.

In the first case the anticipated price is such that $P(1-\lambda)+\lambda > d$. As noted in the text we concentrate on the case when $\widehat{p}R_D \geq 1$. If $\widehat{p}R_D = 1$, the bank lends out all excess cash as assumed in the text, implying that the constraint $SP+\lambda-l \geq d$ binds. If $\widehat{p}R_D > 1$, the bank finds optimal to exhaust the constraint $SP+\lambda-l \geq d$. If this constraint were slack, then the bank would have excess cash to store at a gross return of 1 till t=2. However, holding excess cash is not profitable, because either the bank borrows too much, which is costly ($R_D > 1$ due to $\widehat{p}R_D > 1$ and $\widehat{p} \leq 1$), or lend too little if it has spare cash after paying d (it looses $\widehat{p}R_D > 1$ on every unit of excess cash stored).

Substituting $l = SP + \lambda - d$ into (1) gives us

$$\max_S \begin{cases} p_i(1-\lambda)R - \widehat{p}R_D\,(d-\lambda) + S(P\widehat{p}R_D - p_iR), \text{ if } S > \frac{d-\lambda}{P}, \\ p_i[(1-\lambda)R - R_D\,(d-\lambda) + S(PR_D - R)], \text{ if } S \leq \frac{d-\lambda}{P}. \end{cases} \tag{6}$$

We make two observations: both parts of the last expression are linear in S, and the expected return has a kink at $S = \frac{d-\lambda}{P}$. These observations together with $S \in [0; 1-\lambda]$ imply that the illiquid bank's optimal selling decision S_{iI} is one of three possibilities: 0, $\frac{d-\lambda}{P}$ or $1-\lambda$. To find which of these possibilities is optimal we compare the values of the expected return for each of these possibilities by inserting them into the corresponding part of the expected return (6):

$$\begin{cases} p_i[(1-\lambda)R - R_D\,(d-\lambda)], \text{ if } S = 0, \\ \frac{p_iR}{P}[P\,(1-\lambda) + \lambda - d], \text{ if } S = \frac{d-\lambda}{P}, \\ \widehat{p}R_D[P\,(1-\lambda) + \lambda - d], \text{ if } S = 1-\lambda. \end{cases}$$

For each of these optimal selling decisions there is a corresponding optimal lending/borrowing decision given by the fact that $l = SP + \lambda - d$. We have to note that for our comparison between these three values makes sense only if $(1-\lambda)R - R_D\,(d-\lambda) > 0$. Otherwise borrowing would always be dominated by selling under $P\,(1-\lambda) + \lambda > d$. We skip the details of the algebra from comparing of these three values and only report the results in Lemma 2.

In order to prove the result on the willingness to sell by banks we make several observations. First, the only interesting case if when $p_G \geq \widehat{p} > p_B$. This occurs because of the binary nature of the distribution of type i. $\widehat{p}$ will be the average probability of borrowing banks' repaying their loans. Hence, in an economy with two types we cannot have that $p_G > p_B \geq \widehat{p}$. In addition, the case $p_B \geq \widehat{p} > p_G$ will not occur in our equilibria, so we choose to ignore it. Second, the good bank does not sell at all and borrows for $R_D < \frac{R}{P}$, and the bad bank sells for $R_D \geq \frac{R}{P+\left(\frac{\widehat{p}}{p_i}-1\right)\left(P-\frac{d-\lambda}{1-\lambda}\right)}$. Solving these two inequalities for P delivers that the good illiquid bank does not sell for $P < \frac{R}{R_D}$

and the bad illiquid bank sells for $P \geq \frac{p_B R}{\widehat{p} R_D} + \frac{d-\lambda}{1-\lambda}\left(1 - \frac{p_B}{\widehat{p}}\right)$. The inequality delivering our result

$$\frac{p_B R}{\widehat{p} R_D} + \frac{d-\lambda}{1-\lambda}\left(1 - \frac{p_B}{\widehat{p}}\right) < \frac{R}{R_D}$$

is equivalent to $R(1-\lambda) > R_D(d-\lambda)$. This proves our result, because our result holds whenever borrowing delivers positive return for the bank (as explained at the end of previous paragraph).

In the second case we have that $P(1-\lambda) + \lambda \leq d$. We can simplify (**??**) by observing that for $P(1-\lambda) + \lambda \leq d$ the bank's return at t=2 when the asset pays zero is never positive. The reason is that the bank has to borrow ($l < 0$) and the lowest possible R_D is 1 if $\widehat{p} R_D \geq 1$. In such a case, when the asset pays zero the highest possible return at t=2 is not positive:

$$(SP + \lambda - l - d) + R_D l \leq (SP + \lambda - l - d) + l \leq SP + \lambda - d \leq P(1-\lambda) + \lambda - d \leq 0.$$

Hence, (**??**) boils down to

$$\max_{l,S} \begin{cases} p_i[(1-\lambda-S)R + (SP+\lambda-l-d) + R_D l], \text{ if } (1-\lambda-S)R + (SP+\lambda-l-d) + R_D l \geq 0, \\ 0, \text{ otherwise} \end{cases}$$

$$\text{s.t. } S \in [0; 1-\lambda], SP + \lambda - l \geq d.$$

Whether the bank gets a positive return at t=2 depends on the amount it sells and borrows. Hence, we have to find optimal lending and selling decision as well as the condition for which the bank achieves a non-negative return when the asset pays at t=2 simultaneously.

Assume the bank can achieve a positive return at t=2. If it is the case, the bank borrows the least possible amount, $l = SP + \lambda - d$, because it has to borrow and it is costly to do so (as we will show we will always have $R_D > 1$ in equilibrium). Substituting $l = SP + \lambda - d$ into the first part of the last expression for the expected return yields

$$p_i[(1-\lambda)R - R_D(d-\lambda) + S(PR_D - R)]. \tag{7}$$

Because (7) is linear in S, the optimal selling decision depends on the sign of $PR_D - R$. If $PR_D - R \leq 0$, the bank does not sell or is indifferent how much it sells. In either case (7) reads $p_i[(1-\lambda)R - R_D(d-\lambda)]$ and is not negative for $R_D \leq \frac{1-\lambda}{d-\lambda}R$. For such R_D it holds that $PR_D - R < 0$, because

$$PR_D - R \leq P\frac{1-\lambda}{d-\lambda}R - R \leq \frac{R}{d-\lambda}\left[P(1-\lambda)+\lambda-d\right] < 0.$$

Hence, if $R_D < \frac{1-\lambda}{d-\lambda}R$, the illiquid bank borrows $l_{iI} = -(d-\lambda)$ and sells nothing $S_{iI} = 0$ and can repay its interbank loans at t=2. If $R_D = \frac{1-\lambda}{d-\lambda}R$, the bank is indifferent between borrowing and selling, because both deliver return of 0.

Now we show that for $R_D > \frac{1-\lambda}{d-\lambda}R$ (7) is always negative, so that the illiquid bank always receives a payoff 0 for such loan rates. First, $R_D > \frac{1-\lambda}{d-\lambda}R$ implies that the first two terms in (7) are negative, $(1-\lambda)R - R_D(d-\lambda) < 0$. This also implies that the sign of the highest value of (7) depends on the value of its third term, $S(PR_D - R)$. If $PR_D - R \leq 0$, (7) is maximized for $S = 0$, but then (7) boils down to the sum of its first two terms, which is negative. If $PR_D - R > 0$, (7) is maximized for $S = 1-\lambda$. After inserting $S = 1-\lambda$ into (7) we have that

$$\begin{aligned} & p_i[(1-\lambda)R - R_D(d-\lambda) + (1-\lambda)(PR_D - R)] \\ = \quad & p_i R_D[P(1-\lambda)+\lambda-d] < 0. \end{aligned}$$

This concludes our proof that the illiquid bank receives always the payoff 0 for any $R_D > \frac{1-\lambda}{d-\lambda}R$. In such a case the illiquid bank is indifferent between borrowing or selling. This concludes the proof of Lemma 2.

Proof of Proposition 2

We postpone the discussion of the multiple equilibria till the end of the proof and concentrate now on equilibria, in which the banks borrow. We also present the derivation of the conditions for which $\lambda + P^*(1-\lambda) > d$ in the proof of Lemma 4.

We construct equilibria one at a time. This is dictated by the discrete distribution of types. We start with an equilibrium in which all illiquid banks borrow. Then we proceed to an equilibrium, in which all the GI banks borrow and some of the BI banks have to sell. Finally, we discuss an equilibrium, in which all BI banks and some of the GI banks have to sell.

We denote as P^*, R_D^* and $\widehat{p}^*$ the equilibrium values of price, loan rate and the expected fraction of borrowing banks that will repay their loans at t=2. The share of the banks that repay their interbank loans at t=2, $\widehat{p}$, equals to the expected probability of success of the asset held by the borrowing banks. The reason is that, as shown in Lemma 2, a borrowing bank does not hold any cash reserves till t=2 so the probability of bank's default is equal to the probability of its asset paying 0.

We construct the equilibria in the following way. We first assume a certain equilibrium, in which requires us to stipulate certain banks' optimal choices. Then we write down equilibrium equations that are consistent with these optimal choices, and derive P^*, R_D^* and $\widehat{p}^*$. Finally, we check if the assumed optimal choice of the banks are indeed optimal under the derived P^*, R_D^* and $\widehat{p}^*$. This requires checking two things. First, we need to make sure that the conditions from Lemmata 1 and 2 relating to the assumed banks' optimal choices are satisfied for the derived P^*, R_D^* and $\widehat{p}^*$. Second, we have to check whether there is enough cash reserves λ carried from t=0 for the interbank market to clear.

Equilibrium in which all illiquid banks borrow Let's assume that we have an equilibrium, in which all illiquid banks borrow. Using the assumption that all illiquid banks borrow, we can provide some more structure to our equilibrium. First, in such equilibrium the GL banks do not sell. This is implied by the assumption that the GI banks borrow. From Lemma 2 the GI banks borrow if $R > PR_D$, which implies that $R > PR_D \geq P\widehat{p}R_D$. $R \geq P\widehat{p}R_D$ implies per Lemma 1 that the GL do not sell. Second, only the BL banks sell and the equilibrium price P^* equals pR. This follows because none of the other types of banks sell and outside investors' deep pockets imply that they bid up the price till it hits the anticipated return on the purchased asset. Third, in such an equilibrium the liquid banks have to supply enough liquidity for all illiquid banks on the interbank market. Total demand for interbank loans is $(1-\pi)(d-\lambda)$, where each of $1-\pi$ illiquid banks demands a loan equal to its liquidity shortfall $d-\lambda$. Total supply of interbank loans is $\pi q\lambda + \pi(1-q)(\lambda + P(1-\lambda))$, where $\pi q\lambda$ is the sum of cash reserves provided by all GL banks, and $\pi(1-q)(\lambda + P^*(1-\lambda))$ is the sum of cash reserves λ and cash raised from selling the asset by all BL banks. Hence, there is enough liquidity on the interbank market for all illiquid banks, when total supply is not lower than total demand:

$$\pi\left[q\lambda + (1-q)(\lambda + P^*(1-\lambda))\right] \geq (1-\pi)(d-\lambda),$$

or (using $P^* = pR$)

$$\lambda \geq \max\left[0; \frac{(1-\pi)d - (1-q)\pi pR}{1-(1-q)\pi pR}\right] \equiv \lambda_2.$$

We split the discussion of equilibria into two cases: $\lambda > \lambda_2$ and $\lambda = \lambda_2$.

For $\lambda > \lambda_2$ there is excess supply of interbank loans. Hence, the interbank market clears only if the lending banks are indifferent between lending and storing cash, i.e., $\widehat{p}^* R_D^* = 1$. Given that in equilibrium all illiquid banks borrow, the expected fraction of borrowing banks repaying their loans at t=2 is $\widehat{p}^* = q + (1-q)p$ implying an equilibrium loan rate $R_D^* = (q + (1-q)p)^{-1}$. Using Lemma 1 we confirm that for the derived P^*, $\widehat{p}^*$ and R_D^* the liquid banks are indifferent between lending and cash storage, the GL banks do not sell (because $P^*\widehat{p}^* R_D^* = pR < R$) and the BL sell all of their asset (because $P^*\widehat{p}^* R_D^* = pR \geq pR$). Using Lemma 2 we also confirm that borrowing is optimal for the GI and BI banks. For the GI banks it holds that $R > P^* R_D^* = \frac{pR}{\widehat{p}^*}$, because $p < \widehat{p}^*$. For the BI banks it holds that $R_D^* < \frac{R}{P^* + \left(\frac{\widehat{p}^*}{p} - 1\right)\left(P^* - \frac{d-\lambda}{1-\lambda}\right)}$, which is equivalent to $p < \widehat{p}^*$ in equilibrium.

For $\lambda = \lambda_2$ the supply and demand for loans are equal. Hence, the equilibrium loan rate is indeterminate because both supply and demand are inelastic for certain ranges of R_D. The liquid banks lend for R_D^* such that lending is at least as profitable as cash storage, $(q + (1-q)p) R_D^* \geq 1$ and such that the illiquid banks can repay their loans when the asset pays, $R(1-\lambda) - (d-\lambda) R_D^* \geq 0$. From Lemma 2 we know that the first banks to withdraw from the interbank market are the BI banks. Hence, the upper bound on the loan rate for which all illiquid banks borrow is given by the BI banks' indifference condition from Lemma 2, i.e., $R_D^* \leq \frac{R}{P^* + \left(\frac{\widehat{p}^*}{p} - 1\right)\left(P^* - \frac{d-\lambda}{1-\lambda}\right)}$ for $P^* = pR$ and $\widehat{p}^* = q + (1-q)p$. The upper bound on R_D^* for which all illiquid banks borrow is lower than the upper bound for which the liquid banks lend, $\frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)} < \frac{1-\lambda}{d-\lambda}R \Leftrightarrow \lambda + P^*\left(1-\lambda\right) > d$. Hence, the interbank market clears for any $R_D^* \in \left[\frac{1}{\widehat{p}^*}; \frac{R}{P^*+\left(\frac{\widehat{p}^*}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}\right]$. It is again easy to check that P^*, R_D^* and $\widehat{p}^*$ satisfy the conditions from Lemmata 1 and 2 for which the stipulated banks' choices are optimal.

Equilibrium in which all GI banks and only some of the BI banks borrow Again we first assume that such an equilibrium exists: all GI banks and only some of the BI banks borrow. From Lemma 2 we know that such an equilibrium can exist, because the GI banks prefer to borrow for some loan rates for which the BI banks do not. Again, when the GI banks borrow, the GL banks do not sell. Hence, as above, the equilibrium price is still $P^* = pR$. The equilibrium equations, (2)-(5), were provided and explained at the beginning of Section 3.1, hence we provide here only their solution.

Solving the interbank market clearing equation for σ^* delivers

$$\sigma^* = \frac{(1-\pi)\,d - \pi\,(1-q)\,P^* - \lambda\left(1-(1-q)\,\pi P^*\right)}{(1-\pi)\,(1-q)\,(1-\lambda)\,P^*}$$

Hence, the equilibrium exists for $\sigma^* \in (0;1)$. It can be easy verified that $\sigma^* > 0$ for $\lambda < \lambda_2$ and $\sigma^* < 1$ for $\lambda > \lambda_1 \equiv \max\left[\frac{(1-\pi)d-(1-q)pR}{1-(1-q)pR}; 0\right]$, where $\lambda_1 \leq \lambda_2$. It is again straightforward to check that all the required conditions from Lemmata 1 and 2 are satisfied.

Using the above condition we can also characterize an equilibrium for $\lambda = \lambda_1$ when $\sigma^* = 0$. The interbank market clears because the amount of loans supplied by all liquid banks and selling BI banks is the same as demand for loans from all GI banks. No BI banks borrow. The construction of the equilibrium loan rate, which is again indeterminate, is similar to the case $\lambda = \lambda_2$. The interbank market clears for such loan rates that the GI banks want to borrow and the BI banks want to sell, $R_D^* \in \left[\frac{R}{P^*+\left(\frac{1}{p}-1\right)\left(P^*-\frac{d-\lambda}{1-\lambda}\right)}; \frac{1}{p}\right]$, where $\widehat{p}^* = 1$ and $P^* = pR$.

Equilibrium with only some GI banks borrowing For $\lambda \in [0;\lambda_1)$ the supply of loans from all liquid and BI banks is lower than the demand for loans from all GI banks. Hence, the interbank market clears when only some of the GI banks sell. Because only the GI banks borrow we have $\widehat{p}^* = 1$. Moreover, for this equilibrium to arise the GI banks have to be indifferent between borrowing and selling. In fact, Lemma 2 implies for $p_G = \widehat{p}^* = 1$ that the GI banks are indifferent between all three options listed in case $p_G = \widehat{p}^*$. Hence, in equilibrium it has to be that $R = P^* R_D^*$. In addition, $R = P^* R_D^*$ together with Lemma 1 implies that the GL banks are also indifferent between keeping and selling all. Denote γ_S the fraction of the GI banks selling only $\frac{d-\lambda}{P}$ and γ_A the fraction of the GI banks selling all, and γ the fraction of the GL banks selling all. After taking into account that $\widehat{p}^* = 1$, the equilibrium conditions are

$$\begin{aligned} & \pi q \left[(1-\gamma)\lambda + \gamma\left(\lambda + P^*(1-\lambda)\right)\right] \\ & +\pi(1-q)\left(\lambda + P^*(1-\lambda)\right) \\ & +(1-\pi)(1-q)\left(\lambda + P^*(1-\lambda) - d\right) \\ & +(1-\pi) q \gamma_A \left(\lambda + P^*(1-\lambda) - d\right) \\ = \; & (1-\pi) q (1-\gamma_S-\gamma_A)(d-\lambda) \end{aligned} \tag{8}$$

$$\begin{aligned} P^* = \; & \frac{\pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]}{(1-q)(1-\lambda) + \pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]} R \\ & + \frac{(1-q)(1-\lambda)}{(1-q)(1-\lambda) + \pi q \gamma (1-\lambda) + (1-\pi) q \left[\gamma_S \frac{d-\lambda}{P} + \gamma_A (1-\lambda)\right]} pR, \end{aligned} \tag{9}$$

$$R_D^* = \frac{R}{P^*} \tag{10}$$

(8) is the market clearing condition for the interbank market. The left hand side is the supply of interbank loans, provided by a fraction $1-\gamma$ of the GL banks that lend only cash reserves, as well as by a fraction γ of the GL banks, all bad banks, and a fraction γ_A of the GI banks that sell all of their asset and lend out all of its cash (after covering paying d in case of illiquid banks). The right hand side is the demand for interbank loans by a fraction $1-\gamma_S-\gamma_A$ of the GI banks. (9) is the equilibrium price paid by investors, who anticipate that a fraction γ_S of the GI banks sell only $\frac{d-\lambda}{P}$ as well as all bad banks, a fraction γ of the GL banks, and fraction γ_A sell all of their asset. (10) is the loan rate that guarantees that the GI banks are indifferent between borrowing and selling

and the GL banks between keeping and selling all. Moreover, it is easy to check Lemma 2 to see that the BI banks prefer to sell all and lend out under (10). Despite the fact that there are more unknowns than equations, P^* is uniquely determined by (8)-(10)

$$P^* = \frac{R((1-\pi)d-\lambda)}{(1-\pi)d-\lambda+(1-p)(1-q)(1-\lambda)R}, \tag{11}$$

implying the uniqueness of R_D^* as well.

Multiple equilibria In addition to the above equilibria, there is also a perfect Bayesian equilibrium in which all GI banks sell and none of the banks borrows. First, we show that such an equilibrium can exist. Second, we show when an intuitive criterion can eliminate it.

The equilibrium price P^e is

$$P^e = \frac{(1-\pi)\,q\frac{d-\lambda}{P^e}}{(1-q)\,(1-\lambda)+(1-\pi)\,q\frac{d-\lambda}{P^e}}R + \frac{(1-q)\,(1-\lambda)}{(1-q)\,(1-\lambda)+(1-\pi)\,q\frac{d-\lambda}{P^e}}pR. \tag{12}$$

The investors buying the asset anticipate that all bad banks sell all of their assets and the GI banks sell only the amount $\frac{d-\lambda}{P^e}$ they need to cover their liquidity shortfall $d-\lambda$. Hence, the aggregate amount of the bad asset on the market is $(1-q)\,(1-\lambda)$ and of the good asset $(1-\pi)\,q\frac{d-\lambda}{P}$. The above equation has two solutions for P^e that have opposite signs. To see this, denote $a = (1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda)$ and $b = 4\,(1-q)\,(1-\lambda)\,q\,(1-\pi)\,(d-\lambda)\,R > 0$. Then the solution to (12) reads $\frac{a\pm\sqrt{b+a^2}}{2(1-q)(1-\lambda)}$. Because $b > 0$ we have for any a that $a-\sqrt{b+a^2} < 0$ and $a+\sqrt{b+a^2} > 0$. Hence, the equilibrium price P^e is the positive solution:

$$\begin{aligned} P^e \;\; &= \;\; \frac{1}{2\,(1-q)\,(1-\lambda)}\left[(1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda)\right. \\ &\left. +\sqrt{4\,(1-q)\,(1-\lambda)\,q\,(1-\pi)\,(d-\lambda)\,R + ((1-q)\,(1-\lambda)\,pR - q\,(1-\pi)\,(d-\lambda))^2}\right]. \end{aligned} \tag{13}$$

Such an equilibrium can be supported by arbitrary and sufficiently negative off-equilibrium beliefs that rule out borrowing. For example, we can impose an off-equilibrium belief such that any bank that borrows is deemed to be a bad bank. Under such beliefs the lowest loan rate for which liquid banks are willing to lend is $\frac{1}{p}$. Now we can show that under such a loan rate no GI bank borrows. Selling delivers the GI banks a payoff of $R\left(1-\lambda-\frac{d-\lambda}{P^e}\right)$. Because the GI banks sell a positive amount of the asset it has to be that the equilibrium price is higher than the expected return on the bad asset $P^e > pR$. Hence, the GI banks' payoff from selling is higher than $R\left(1-\lambda-\frac{d-\lambda}{pR}\right) = R\left(1-\lambda\right)-\frac{d-\lambda}{p}$, which is exactly the payoff when the GI banks would decide to borrow an amount $d-\lambda$ at the loan rate $\frac{1}{p}$. Hence this proves the existence of the stipulated equilibrium. Of course, it is obvious that all bad banks want to sell, given that they are subsidized by the good banks.

However, the intuitive criterion can eliminate such an equilibrium for certain parameters. This criterion puts some discipline on the off-equilibrium beliefs. It requires that in an equilibrium when a deviation is profitable for one type but not for the other, agents when building their beliefs cannot associate this deviation with the type for which it is not profitable for any of their beliefs. If an equilibrium does not satisfy this criterion it should be discarded (Bolton and Dewatripont (2005)). We will apply this criterion to show that our equilibrium with selling which is supported by any beliefs for which that a borrowing bank is not good can be discarded.

For our purposes it suffices to show that for which parameter constellations the only bank that would like to deviate is a GI bank and not the BI bank for the most favorable off-equilibrium belief about which bank borrows. The most favorable off-equilibrium belief upon seeing a borrowing bank is that it is a GI bank. Hence, a competitive loan rate R_D would be 1 for such a bank. First, we show that the GI bank would deviate. For a loan rate of 1 the deviating GI bank's payoff is $R\left(1-\lambda\right)-\left(d-\lambda\right)$. Then this must be higher than the payoff from selling in equilibrium because $P^e < R$ due to selling bad banks: $R\left(1-\lambda-\frac{d-\lambda}{P^e}\right) < R\left(1-\lambda-\frac{d-\lambda}{R}\right) = R\left(1-\lambda\right)-\left(d-\lambda\right)$. Second, we show for which parameters no BI bank wants to deviate. We have to show when the BI banks' payoff from selling for P^e, $P^e\left(1-\lambda\right)+\lambda-d$, is not lower than the payoff from deviating to borrowing for $R_D=1$, $p\left(R\left(1-\lambda\right)-\left(d-\lambda\right)\right)$. We can show that inequality

$$P^e\left(1-\lambda\right)+\lambda-d \geq p\left(R\left(1-\lambda\right)-\left(d-\lambda\right)\right)$$

is equivalent to

$$D - dE \geq \lambda \left(D - E \right), \tag{14}$$

where

$$\begin{aligned} D &= R \left[q \left(1 - \pi \right) - p \left(1 - q \right) \right], \\ E &= q \left(1 - \pi \right) + \left(1 - \overline{p} \right), \\ \overline{p} &= q + \left(1 - q \right) p. \end{aligned}$$

Any time (14) holds the equilibrium with no borrowing does not exist, because we can find such "reasonable" off-equilibrium beliefs, for which the GI banks want to deviate from the equilibrium and the BI banks do not. The region where it happens depends on λ. In order to narrow the set of possible equilibria, we opt out to find such parameters for which (14) holds for all $\lambda \in [0; d]$. Because $d \in (0; 1)$ we have that $D - dE > D - E$. From this follows that (14) holds for all $\lambda \in [0; d]$ when $D \geq dE$. To see this we consider four cases. First, when $D - E > 0$, then (14) becomes $\lambda \leq \frac{D-dE}{D-E}$. We require that $\frac{D-dE}{D-E} \geq d$, which is equivalent to $D > 0$, but this has to hold when $D - E > 0$, because $E > 0$. Hence, $D - E > 0$ means that (14) holds for all λ. Second, for $D - E = 0$ we can use similar arguments to show that (14) holds for all λ. Third, when $D - dE \geq 0 > D - E$, (14) holds for all λ, because its right-hand side is non-negative and the left hand side negative. Fourth, the last case is when $D - dE < 0$, then (14) becomes $\lambda \geq \frac{D-dE}{D-E} > 0$. Hence, (14) holds either only for some $\lambda \in (0; d)$ or for none if $\frac{D-dE}{D-E} > d$. Summarizing we have that (14) holds for all λ for $D - dE \geq 0$ or

$$\pi \leq 1 - \frac{pR \left(1 - q \right) + d \left(1 - \overline{p} \right)}{q \left(R - d \right)}.$$

This inequality holds for some $\pi \in (0; 1)$ when $1 - \frac{pR(1-q)+d(1-\overline{p})}{q(R-d)} > 0 \Leftrightarrow q \left[(1-p)R - dp \right] > pR + (1-p)\, d$. In turn, this inequality holds for some $q \in (0; 1)$ when $d < (1 - 2p)\, R$. To see this realize that t his inequality holds for some $q \in (0; 1)$when $(1-p)R - dp > 0$ and $\frac{pR+(1-p)d}{(1-p)R-dp} < 1$, which are equivalent to $d < \frac{1-p}{p} R$ and $d < (1-2p)\, R$ respectively. But we can show that $1 - 2p < \frac{1-p}{p}$

for any p, implying our claim. Hence, (14) holds for all $\lambda \in [0; d]$ when

$$\pi \in \left(0; 1 - \frac{pR(1-q) + d(1-\overline{p})}{q(R-d)}\right], \ q \in \left(\frac{pR + (1-p)d}{(1-p)R - dp}; 1\right] \text{ and } d < (1-2p)R.$$

It is straightforward to see that as p converges to zero, the above set of parameters expands and converges to:

$$\pi \in \left(0; 1 - \frac{d(1-q)}{q(R-d)}\right], \ q \in \left(\frac{d}{R}; 1\right] \text{ and } d < R.$$

Given that for p close to 0 we have to have $qR > 1$, hence, the last two inequalities are satisfied in such a case. Then the first inequality gives also a broad range of π for which it is satisfied. Moreover, it does so for the most interesting case of lower π, i.e., a low fraction of liquid banks.

Proof of Lemma 4

Equilibria from Proposition 1 exist when $(1-\lambda)P^* + \lambda > d$. We now formally derive conditions under which this inequality holds for P^* derived in Proposition 1. In order to do so we define the difference $(1-\lambda)P^* + \lambda - d$ as a function f of λ and find out for which λ values of this function are positive by solving $f(\lambda) > 0$. Given P^* derived in Proposition 1 we have that

$$f(\lambda) = \begin{cases} (1-\lambda)\frac{R((1-\pi)d-\lambda)}{(1-\pi)d-\lambda+(1-p)(1-q)(1-\lambda)R} + \lambda - d, \text{ for } \lambda \in [0; \lambda_1) \text{ and } \lambda_1 > 0, \\ (1-\lambda)pR + \lambda - d, \text{ for } \lambda \in [\max[0; \lambda_1]; d]. \end{cases}$$

One immediate observation that we makes is that $f(d) = (1-d)pR + d - d = (1-d)pR > 0$. We split the discussion of the solution to the inequality $f(\lambda) > 0$ into two cases: $\lambda_1 \leq 0$ and $\lambda_1 > 0$.

First, we discuss the case $\lambda_1 \leq 0$, which is equivalent to $d \leq \frac{1-q}{1-\pi}pR$. Then $f(\lambda) = (1-\lambda)pR + \lambda - d$ for $\lambda \in [0; d]$. Moreover, f is strictly increasing in λ, because $pR < 1$. Solving $(1-\lambda)pR + \lambda - d > 0$ with respect to λ delivers $\lambda > \max[0; \lambda_{UB}]$, where $\lambda_{UB} = \frac{d-pR}{1-pR}$. Summarizing our findings delivers that $f(\lambda) > 0$ and hence existence of equilibria from Proposition 1 if

$$\lambda \in \begin{cases} (\lambda_{UB}; d], \text{ for } \lambda_1 \leq 0 \text{ and } \lambda_{UB} > 0 \\ (0; d], \text{ for } \lambda_1 \leq 0 \text{ and } \lambda_{UB} \leq 0 \end{cases} \tag{15}$$

Second, we discuss the case $\lambda_1 > 0$, in which f consists of two parts as originally defined. We first outline the strategy of the proof. To find out for which λ $f(\lambda) > 0$ we will first show that f is strictly decreasing and concave for $\lambda \in [0;\lambda_1)$, given that we already know that for $\lambda \in [\lambda_1; d]$ f is increasing. This implies that f achieves the lowest value for $\lambda = \lambda_1$. Hence, $f(\lambda_1)(1-\lambda_1)+\lambda_1 > d$ will imply that equilibria from Proposition 1 exist for all $\lambda \in [0; d]$, and $f(\lambda_1)(1-\lambda_1)+\lambda_1 \leq d$ will imply that for some λ around λ_1 these equilibria do not exist. To be more precise, the sign of the lowest value $f(\lambda_1)$ will determine for which λ our equilibria exist as reported here: Case $f(\lambda_1) < 0$, $f(0) > 0$ and $\lambda_1 > 0$

$$\lambda \in \begin{cases} [0; d], \text{ if } f(\lambda_1) > 0 \text{ and } \lambda_1 > 0 \\ [0; d] \setminus \{\lambda_1\}, \text{ if } f(\lambda_1) = 0 \text{ and } \lambda_1 > 0 \\ [0; \lambda_{LB}) \cup (\lambda_{UB}; d], \text{ if } f(\lambda_1) < 0,\ f(0) > 0 \text{ and } \lambda_1 > 0 \\ (\lambda_{UB}; d], \text{ if } f(0) \leq 0 \text{ and } \lambda_1 > 0 \end{cases} \tag{16}$$

Fig. **??** illustrates the solution (16).

Hence, we only have to show that f is strictly decreasing and concave for $\lambda \in [0; \lambda_1)$. First, we show that f is strictly decreasing by showing that the first derivative of f for such λ is negative. The derivative of f for $\lambda \in [0; \lambda_1)$ is

$$\frac{[(1-\pi)d - \lambda + (1-p)(1-q)(1-\lambda)R]^2 + R\left[d(1-\pi)\left(2\lambda - d(1-\pi)\right) - (1-p)(1-q)(1-\lambda)^2 R - \lambda^2\right]}{[(1-\pi)d - \lambda + (1-p)(1-q)(1-\lambda)R]^2}.$$

The sign of this derivative is determined by the sign of its nominator, which can be rewritten as a quadratic polynomial, $A\lambda^2 + B\lambda + C$, where $\overline{p} = q + (1-q)p$ and

$$A = -[1 + (1-p)(1-q)R](\overline{p}R - 1) < 0,$$

$$B = 2[d(1-\pi) + (1-p)(1-q)R](\overline{p}R - 1),$$

$$C = -(1-p)(1-q)\overline{p}R^2 + 2d(1-p)(1-q)R(1-\pi) + d^2(1-\pi)^2(R-1).$$

Now we show that the above polynomial is always negative for all λ. In order to show it, we establish two claims. First, $A < 0$ because $\overline{p}R > 1$. Second, the determinant of the polynomial, $B^2 - 4AC$, is negative. After some algebra we have that

$$B^2 - 4AC = -4(1-p)(1-q)R^2(\overline{p}R - 1)(1 - d(1-\pi))^2 < 0.$$

Hence, two claims imply that the nominator, and therefore the derivative of f, are negative for all λ (not only for $\lambda \in [0; \lambda_1)$). This proves our claim that f for $\lambda \in [0; \lambda_1)$ is strictly decreasing. Hence, it follows immediately that f for $\lambda_1 > 0$ has a minimum at $\lambda = \lambda_1$. Second, we show that f is strictly concave by showing that the second derivative of f is negative. The second derivative of f for $\lambda \in [0; \lambda_1)$ is

$$-\frac{2(1-p)(1-q)R^2(1-d(1-\pi))^2}{[(1-\pi)d - \lambda + (1-p)(1-q)(1-\lambda)R]^3} < 0.$$

The sign obtains because of both nominator and denominator of the last expression are positive under our assumptions. This completes our proof.

We can solve each of the four conditions that delimit different cases of (15) and (16), $\lambda_1 = 0$, $f(\lambda_1) = 0$, $f(0) = 0$, and $\lambda_{UB} = 0$, for d, which delivers respectively, $d = \frac{1-q}{1-\pi}pR$, $d = \frac{qpR}{qpR + \pi(1-pR)}$, $d = \frac{\overline{p}-\pi}{1-\pi}R$, and $d = pR$. Then we can plot these conditions in a $(d; q)$-diagram, where all four conditions cross at $q = \pi$.

After combining all the claims so far we obtain the formal version of Proposition 1.

Proposition 1: *Suppose that the parameters are such that the equilibrium asset's price P^* is such that $P^*(1-\lambda) + \lambda > d$ which occurs for λ such as in (15) or (16). Then there exist thresholds for cash reserve λ, λ_1 and λ_2, for which the following results obtain:*

For $\lambda \in [0; \min[\lambda_1; \lambda_{LB}])$ and $\min[\lambda_1; \lambda_{LB}] > 0$ only some of the GI banks borrow, and the rest of the GI banks and all BI banks sell. The share of GI banks borrowing increases with λ.

For $\lambda = \lambda_1 > \max[0; \lambda_{LB}]$ all GI banks borrow, and all BI banks sell.

For $\lambda \in [\max[0; \lambda_1; \lambda_{UB}]; \lambda_2)$ and $\lambda_2 > \max[0; \lambda_1; \lambda_{UB}]$ the GI banks and some of the BI banks borrow and do not sell and the rest of BI banks sell. The share of BI banks borrowing increases with λ.

For $\lambda \in [\max[0; \lambda_2; \lambda_{UB}]; d)$ *the GI and BI banks borrow and do not sell.*

Proof of Proposition 3

Now we are looking for equilibria when $P^*(1-\lambda)+\lambda-d \leq 0$. Again, we will construct our equilibria by first assuming existence of a specific equilibrium and then by finding conditions for which it exists.

Equilibrium with liquidity shortage We start with the equilibrium with liquidity shortage in which only some of the illiquid banks obtain enough liquidity to pay d. This means that the anticipated price P^* has to fulfill $P^*(1-\lambda)+\lambda-d<0$ and that there is not enough liquidity on the interbank market for all banks to borrow, $\lambda<\lambda_2$. From Lemma 2 we know that once $P^*(1-\lambda)+\lambda-d<0$, all illiquid banks prefer to borrow when $R_D<\frac{1-\lambda}{d-\lambda}R$ and are indifferent between selling and borrowing when $R_D \geq \frac{1-\lambda}{d-\lambda}R$. At the same time the liquid banks lend for $R_D \in \left[\frac{1}{\bar{p}}; \frac{1-\lambda}{d-\lambda}R\right]$. For any $R_D>\frac{1-\lambda}{d-\lambda}R$ the loan supply is zero because no borrower would repay its loans at t=2. For any $R_D<\frac{1}{\bar{p}}$ lending is not profitable, because it does not compensate for the anticipated risk. Because there is not enough liquidity for all the illiquid banks on the interbank market, this market clears for $R_D^*=\frac{1-\lambda}{d-\lambda}R$. Then all illiquid banks are indifferent between borrowing and selling, and only a fraction of the illiquid banks get loans. Those banks that cannot obtain interbank loans cannot raise enough liquidity from the secondary market to pay d and become insolvent.

The share of the illiquid banks that get interbank loans is given by the total supply of loans divided by the loan size, $\frac{\pi[\lambda+(1-q)(1-\lambda)P^*]}{d-\lambda}$. Hence, the share of the illiquid banks that cannot pay d is the rest of the illiquid banks, $\nu=1-\pi-\frac{\pi[\lambda+(1-q)(1-\lambda)P^*]}{d-\lambda}$. We assume that the insolvent banks dump all of their asset on the market. Proceeds from sale go directly towards the payment of d. By selling all of their assets we are actually reducing the set of parameters for which an equilibrium with liquidity shortage occurs, because all of the good asset on the market pushes price upwards.

The equilibrium price is given by

$$P^* = \frac{\nu q\left(1-\lambda\right)}{\nu\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}R + \frac{\nu\left(1-q\right)\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}{\nu\left(1-\lambda\right)+\pi\left(1-q\right)\left(1-\lambda\right)}pR. \tag{17}$$

At the same time, because the loans are allocated randomly to the pool of illiquid banks, by law of large numbers we have that the average quality of the insolvent banks is the same as the average quality of the entire population of illiquid banks, $\overline{p}$. That means that for the equilibrium loan rate $R_D^* = \frac{1-\lambda}{d-\lambda}R$ the lending banks break even ($\overline{p}\frac{1-\lambda}{d-\lambda}R > 1 \Leftrightarrow \overline{p}R > 1 > \frac{d-\lambda}{1-\lambda}$).

Equation (17) has two solutions of the form

$$P_{1,2}^* = \frac{F \pm \sqrt{F^2+G}}{2\left(1-q\right)\left(1-\lambda\right)\pi},$$

where·

$$\begin{aligned} F &= d(1-q\pi)+(1-q)\pi(\overline{p}R(1-\lambda)-\lambda)-\lambda, \\ G &= 4(1-q)R\pi(1-\lambda)\left[(\overline{p}+p(1-q)\pi)\lambda - d(\overline{p}-q\pi)\right]. \end{aligned}$$

For the solution to exist we have to have $F^2+G \geq 0$. Whenever $F^2+G<0$, there cannot be an equilibrium with the liquidity shortage. After some cumbersome, and therefore omitted, algebra we can show that one of the solutions to (17) is lower, and the other higher, than the average quality of the asset in the banking system, $\overline{p}R$. Hence, the equilibrium price is the one that is lower than $\overline{p}R$, and is given by

$$P^* = \frac{F - \sqrt{F^2+G}}{2\left(1-q\right)\left(1-\lambda\right)\pi}. \tag{18}$$

By implicitly differentiating (17) we can show that the equilibrium price (18) decreases with λ. Moreover, we can show that for $\lambda = \lambda_2$ (18) is equal to pR. These two properties imply that for $\lambda \in (\lambda_1; \lambda_2)$ (18) is higher than the equilibrium price in an equilibrium without liquidity shortage, which is pR in this interval. This implies that for certain λ there are no equilibria in which $P^*\left(1-\lambda\right)+\lambda > d$ or $P^*\left(1-\lambda\right)+\lambda < d$, and therefore, there are no equilibria in pure strategies in which banks prefer to either sell or borrow. We discuss this case below.

With that we turn to the existence of the equilibrium with liquidity shortage. Such an equilibrium exists whenever there are some $\lambda \in [0; \lambda_2)$ for which $P^* (1-\lambda) + \lambda < d$. We re-write hits inequality as

$$\frac{F - \sqrt{F^2 + G}}{2(1-q)(1-\lambda)\pi} < \frac{d-\lambda}{1-\lambda}$$

or

$$\frac{\sqrt{F^2+G}}{(1-q)} > (p + (1-p)q) R + \frac{d(1-(2-q)\pi)}{(1-q)\pi} - \left((p+(1-p)q)R + \frac{1-(1-q)\pi}{(1-q)\pi}\right)\lambda \quad (19)$$

The right hand side of (19) is non-positive for some $\lambda \geq \frac{(p+(1-p)q)R + \frac{d(1-(2-q)\pi)}{(1-q)\pi}}{(p+(1-p)q)R + \frac{1-(1-q)\pi}{(1-q)\pi}} \equiv \widehat{\lambda}$. Hence, the equilibrium with liquidity shortage exists for any $\lambda \in \left[\max\left[\widehat{\lambda}; \lambda_{LB}; 0\right]; \min[\lambda_2; \lambda_{UB}]\right)$ if $\widehat{\lambda} < \min[\lambda_2; \lambda_{UB}]$, because the left hand side of (19) is always non-negative. For $\lambda \in \left(\max[\lambda_{LB}; 0]; \min\left[\widehat{\lambda}; \lambda_2; \lambda_{UB}\right]\right.$ if $\widehat{\lambda} > \max[\lambda_{LB}; 0]$, the right hand side of (19) is positive. Then by squaring both side of the inequality (19) and simplifying, it becomes quadratic with a solution that it holds for $\lambda < \lambda_{FR,1}$ or $\lambda > \lambda_{FR,2}$ and λ such that $\lambda \in \left(\max[\lambda_{LB}; 0]; \min\left[\widehat{\lambda}; \lambda_2; \lambda_{UB}\right]\right)$ if $\widehat{\lambda} > \max[\lambda_{LB}; 0]$. $\lambda_{FR,1}$ and $\lambda_{FR,2}$ solve the inequality with a equality sign, where

$$\lambda_{FR,1,2} = \frac{H - / + \sqrt{H^2 - 4IJ}}{2I},$$

and

$$\begin{aligned} H &= d[2 - \pi - qR(1-(2-q)\pi] - R[p + (1-(1-q)\pi)(\overline{p} - pq)], \\ I &= 1 - R[\overline{p} - q\pi(1-q)(1-p)], \\ J &= d[d(1-\pi) - R[\overline{p}(1-(1-q)x) - q\pi]]. \end{aligned}$$

Expressions for $\lambda_{FR,1}$ and $\lambda_{FR,2}$ are very complicated, and it is impossible to obtain clear cut conditions for the existence of the equilibria with liquidity shortage. We will resort to some numerical examples to show that such an equilibrium exists. In our numerical examples for parameters such that $H^2 - 4IJ \geq 0$ gave us that $\lambda_{FR,1} \leq \lambda_{FR,2}$. In such a case we can set

$$\lambda_{FR} = \lambda_{FR,2} = \frac{H + \sqrt{H^2 - 4IJ}}{2I}.$$

In fact, once we have parameters such that λ_{FR} exists, we can provide some concrete conditions under which equilibria with liquidity shortage exist. We can show algebraically that $\lambda_{FR} = \lambda_2 = \lambda_{UB}$ for $R = \widehat{\widehat{R}} = \frac{d\pi}{p((1-d)(1+q\pi)-\pi(1-2d))}$. In fact, we can show that for $R < \widehat{\widehat{R}}$ we have that $\lambda_{FR} < \lambda_2 < \lambda_{UB}$, and for $R > \widehat{\widehat{R}}$ we have that $\lambda_{FR} > \lambda_{UB}$ and $\lambda_2 > \lambda_{UB}$. Moreover, we know that a necessary condition for $P^*(1-\lambda)+\lambda \leq d$ is $pR(1-\lambda_1)+\lambda_1 \leq d$ or $R \leq \frac{d\pi}{p((1-d)(1+q\pi)-\pi(1-2d))} \equiv \widehat{R}$. It is easy to show that $\widehat{R} > \widehat{\widehat{R}}$, which is equivalent to $1 > \pi$. Then, we get the result that for $R \in \left[\widehat{\widehat{R}}; \widehat{R}\right)$ we can have only an equilibrium in which $P^*(1-\lambda) + \lambda = d$. Once $R < \widehat{\widehat{R}}$ an equilibrium with liquidity shortage will for sure exist for $\lambda \in (\max[0; \lambda_{FR}]; \lambda_2)$.

Equilibrium with $P^*(1-\lambda)+\lambda-d = 0$ As argued earlier there is a possibility that for some λ there are no equilibria in which the price is such that $P^*(1-\lambda)+\lambda < d$ or $P^*(1-\lambda)+\lambda > d$. Hence, for such λ in equilibrium it must hold that $P^*(1-\lambda)+\lambda = d$ or $P^* = \frac{d-\lambda}{1-\lambda}$.

When the illiquid banks anticipate $P^* = \frac{d-\lambda}{1-\lambda}$, they all would like to borrow for $R_D < \frac{1-\lambda}{d-\lambda}R_D$. Because we are in the case that $\lambda < \lambda_2$, then there is not enough liquidity on the interbank market for all illiquid banks. Hence, the interbank market clears for $R_D^* = \frac{1-\lambda}{d-\lambda}R_D$. Given that $R_D^* = \frac{1-\lambda}{d-\lambda}R_D$ and $P^* = \frac{d-\lambda}{1-\lambda}$ all illiquid banks are indifferent between selling or borrowing. The good illiquid banks are also indifferent between selling all $(1-\lambda)$ or only $\frac{d-\lambda}{P^*}$ of the asset, because $\frac{d-\lambda}{P^*} = 1-\lambda$. For notational simplicity we assume the good illiquid banks that do not get the interbank loans sell all of their asset.

Because all illiquid banks are indifferent between selling or borrowing we have an equilibrium in mixed strategies. We denote as γ_M (β_M) the share of the good (bad) illiquid banks that borrow. Hence, the equilibrium in mixed strategies is described by the following two equations:

$$\frac{d-\lambda}{1-\lambda} = \frac{q(1-\pi)(1-\gamma_M) + p[(1-q)(1-\pi)(1-\beta_M) + \pi(1-q)]}{q(1-\pi)(1-\gamma_M) + (1-q)(1-\pi)(1-\beta_M) + \pi(1-q)} R\ . \qquad (20)$$

and

$$\frac{\pi\left[\lambda+(1-q)(1-\lambda)\frac{d-\lambda}{1-\lambda}\right]}{d-\lambda}=(1-\pi)(\gamma_M q+\beta_M(1-q)).$$

The first equation describes the secondary market, where the equilibrium price $\frac{d-\lambda}{1-\lambda}$ equals to the expected quality of the sold asset. The second equation describes the clearing of the interbank market, where the left (right) hand side is the total loan supply (demand). The solution to these two equations reads

$$\gamma_M=\frac{-d^2(1-\pi)+d\overline{p}R(1-\pi)-d\lambda(\overline{p}R(1-\pi)+\pi-2)-(R(p+(1-p)q(1-\pi))1-\lambda)+\lambda)\lambda}{(-1+p)qR(-1+x)(-1+\lambda)(-d+\lambda)}$$

and

$$\beta_M=\frac{d^2(1-\pi)-dR(\overline{p}-\pi)+d\lambda(R(p+(1-p)q-\pi)+\pi-2)+\lambda(\overline{p}R(1-\lambda)+\lambda)}{(1-p)(1-q)R(1-\pi)(d-\lambda)(1-\lambda)}.$$

We can show the following properties of γ_M and β_M, for which we omit the algebra. First, γ_M and β_M cross at λ_{FR}. Second, $\gamma_M>0$ and $\beta_M=0$ for $\lambda=\lambda_{LB}\geq 0$. Third, the expected probability of repayment of interbank loans

$$\widehat{p}^*=\frac{q\gamma_M}{q\gamma_M+(1-q)\beta_M}+\left(1-\frac{q\gamma_M}{q\gamma_M+(1-q)\beta_M}\right)p \tag{21}$$

converges to $\overline{p}$ as λ converges to λ_{FR}. Moreover, the first two properties of γ_M and β_M imply that $\gamma_M>\beta_M$ for $\lambda\in[\lambda_{LB};\lambda_{FR})$, and, therefore, $\widehat{p}^*$ given by (21) is higher than $\overline{p}$ for $\lambda\in[\lambda_{LB};\lambda_{FR})$. Moreover, these properties show a continuity of the equilibrium variables at the boundaries of the intervals, λ_{LB} and λ_{FR}, where $P^*(1-\lambda)+\lambda=d$.

Equilibrium with all illiquid banks borrowing It is possible that we have $pR(1-\lambda_2)+\lambda_2<d$, implying that $\lambda_2<\lambda_{UB}$. However, because for $\lambda\geq\lambda_2$ there is enough liquidity on the interbank market for all the banks to borrow, we have an equilibrium in which there is enough interbank loans for all illiquid banks and $P^*(1-\lambda)+\lambda<d$ in which $R_D^*\in\left[\frac{1}{p};\frac{1-\lambda}{d-\lambda}R\right]$ and $P^*=pR$. In fact, this equilibrium is the limit of equilibria with liquidity shortage when λ converges to λ_2

from the left. To see this, just take the limit of v, the expression for the amount of banks that cannot get interbank loans, when λ converges to λ_2 from the left:

$$\lim_{\lambda \to_- \lambda_2} v = \lim_{\lambda \to_- \lambda_2} \left(1 - \pi - \frac{\pi \left[\lambda + (1-q)(1-\lambda) P^* \right]}{d - \lambda} \right) = 0,$$

where P^* is given by (18) and we know that for $\lambda = \lambda_2$ (18) equals to pR.

Proof of Lemma 5

For the case $P^*(1-\lambda)+\lambda > d$ social welfare is a sum of profits for each type of bank, $\Pi(\lambda; R_D; P; \widehat{p})$, and payments made by illiquid banks, $d(1-\pi)$, for given λ and the corresponding equilibrium values R_D^*, P^*, and $\widehat{p}^*$. Hence, social welfare at t=1 is $SW_1 = \Pi(\lambda; R_D^*; P^*; \widehat{p}^*) + d(1-\pi)$ where

$R_D = R_D^*$, $P = P^*$, $\widehat{p} = \widehat{p}^*$, and

$$\Pi\left(\lambda; R_D; P; \widehat{p}\right) = \begin{cases} \pi q\left[\left(1-\gamma\right)\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\gamma\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \pi\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)+ \\ \left(1-\pi\right)q\left[\left(1-\gamma_S-\gamma_L\right)\left(R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right)+\gamma_S R\left(1-\lambda-\frac{d-\lambda}{P}\right)\right]+ \\ +\left(1-\pi\right)q\gamma_L\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right)+\left(1-\pi\right)\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right), \\ \text{for } \lambda\in[0;\lambda_1) \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right)\right], \\ \text{for } \lambda=\lambda_1 \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)\left(1-\beta\right)p[(1-\lambda)R-R_D\left(d-\lambda\right)]\right]+ \\ \left(1-\pi\right)\beta\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda-d\right), \\ \text{for } \lambda\in(\lambda_1;\lambda_2) \\ \\ \pi\left[q\left[R\left(1-\lambda\right)+\widehat{p}R_D\lambda\right]+\left(1-q\right)\widehat{p}R_D\left(\left(1-\lambda\right)P+\lambda\right)\right]+ \\ \left(1-\pi\right)\left[q\left[R\left(1-\lambda\right)-R_D\left(d-\lambda\right)\right]+\left(1-q\right)p[(1-\lambda)R-R_D\left(d-\lambda\right)]\right], \\ \text{for } \lambda\in[\lambda_2;d] \end{cases} \tag{22}$$

We can show that for all $\lambda \in [0; d]$

$$\Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) = \left(1-\lambda\right)\left(q+\left(1-q\right)p\right)R+\lambda-d\left(1-\pi\right).$$

Hence, social welfare in equilibria with $P^*\left(1-\lambda\right)+\lambda > d$ is $SW_0 = \left(1-\lambda\right)\left(q+\left(1-q\right)p\right)R+\lambda$.

For $P^*(1-\lambda)+\lambda = d$ all illiquid banks pay d. The banks' profits are given by

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) && (23)\\
= \; & \pi\left[q\left[R(1-\lambda)+\widehat{p}^* R_D^*\lambda\right]+(1-q)\,\widehat{p}^* R_D^*\left((1-\lambda)P^*+\lambda\right)\right]\\
& +(1-\pi)\,q\left[\gamma_M\left(R(1-\lambda)-R_D(d-\lambda)\right)+(1-\gamma_M)\left((1-\lambda)P^*+\lambda-d\right)\right]\\
& +(1-\pi)(1-q)\left[\beta_M p\left(R(1-\lambda)-R_D(d-\lambda)\right)+(1-\beta_M)\left((1-\lambda)P^*+\lambda-d\right)\right],
\end{aligned}$$

where $R_D^* = \frac{1-\lambda}{d-\lambda}R$, $P^* = \frac{d-\lambda}{1-\lambda}$ and $\widehat{p}^*$ is given by (21). We can show that (24) equals to

$$(1-\lambda)\left(q+(1-q)\,p\right)R+\lambda-d(1-\pi).$$

Hence, again we have that social welfare is $SW_0 = (1-\lambda)\left(q+(1-q)\,p\right)R+\lambda$.

For $P^*(1-\lambda)+\lambda < d$ only a fraction $(1-\pi-v)$ of illiquid banks pay d, a fraction of v pays $P^*(1-\lambda)+\lambda < d$ and only the liquid banks have positive profits. We can show that

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) && (24)\\
= \; & \pi\left[q\left[R(1-\lambda)+\widehat{p}^* R_D^*\lambda\right]+(1-q)\,\widehat{p}^* R_D^*\left((1-\lambda)P^*+\lambda\right)\right]
\end{aligned}$$

where $R_D^* = \frac{1-\lambda}{d-\lambda}R$, P^* as given by (18), $\widehat{p}^* = q+(1-q)\,p$ and $\nu = 1-\pi-\frac{\pi[\lambda+(1-q)(1-\lambda)P^*]}{d-\lambda}$. Using the expressions for the equilibrium price (17) and for the amount of illiquid banks that cannot become liquid v we can show that (24) equals to

$$\begin{aligned}
& \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right)\\
= \; & (1-\lambda)\left(q+(1-q)\,p\right)R+\lambda-\left[(1-\pi)-\nu\right]d-v\left((1-\lambda)P^*+\lambda\right).
\end{aligned}$$

This implies the following social welfare

$$
\begin{aligned}
SW_{LS} &= \Pi\left(\lambda; R_D^*; P^*; \widehat{p}^*\right) \\
&\quad + \left[(1-\pi) - \nu\right] d + v\left((1-\lambda) P^* + \lambda\right) && (25)\\
&\quad - v\left[d - (1-\lambda) P^* + \lambda\right] \tau && (26)\\
&= SW_0 - v\left[d - (1-\lambda) P^* + \lambda\right] \tau \leq SW_0, && (27)
\end{aligned}
$$

because $\tau \leq 0$.

Proof of Lemma 6

At t=0 each bank chooses optimal λ anticipating a certain equilibrium at t=1 as derived in Proposition 1 and 2 for this given λ or an interval of λ. In looking for the optimal choice of λ we will assume again assume that the bank anticipates an equilibrium at t=1 for a given choice of λ or its region, and then we will maximize the bank's profits at t=0 and see whether this choice of λ for the anticipated equilibrium at t=1 is consistent with this anticipated equilibrium.

Using $\Pi\left(\lambda; R_D; P; \widehat{p}\right)$ to denote the banks' expected profit at t=0 as a function of λ, P, R_D, the bank solves the following problem at t=0 taking as given P, R_D and $\widehat{p}$:

$$\max_{\lambda\in[0;d]} \Pi\left(\lambda; R_D; P; \widehat{p}\right),$$

where the functional form of $\Pi\left(\lambda; R_D; P; \widehat{p}\right)$ depends on the anticipated equilibrium as in (22)-(24).

We tackle first the most general case in which $\lambda_2 > \lambda_1 > 0$, which occurs for some $R < \frac{(1-\pi)d}{(1-q)p}$.

At the beginning observe that at t=0 any choice of $\lambda \geq \lambda_2$ guarantees that an illiquid bank will pay d at t=1 regardless of whether $P^*(1-\lambda) + \lambda$ is higher or lower than d, because then there is enough liquidity on the interbank market for all banks, so that no illiquid bank needs to sell. Now, it is clear that none of the banks will choose $\lambda > \lambda_2$, because taking more than λ_2 would be waste of resources. For any $\lambda > \lambda_2$ there would be excess supply of liquidity on the interbank market, implying that the banks keep too much of cash reserves.

The anticipated values of equilibrium variables at t=1 for $\lambda = \lambda_2$ have to be such that the first order condition with respect to λ holds with equality at t=0. Otherwise, the bank would either prefer to set λ smaller or bigger than λ_2. The first order condition is given by deriving the fourth expression in (22) with respect to λ:

$$\pi \left[q\left(-R+\widehat{p}R_D\right)+(1-q)\,\widehat{p}R_D\left(-P+1\right)\right]+(1-\pi)\left[q\left(-R+R_D\right)+(1-q)\,p\left(-R+R_D\right)\right].$$

In equilibrium for $\lambda = \lambda_2$ at t=1 the banks expect that $\widehat{p}^* = q+(1-q)\,p$ and $P^* = pR$, but R_D^* is indeterminate. Hence, it is given by the binding first order constraint as in Freixas et al. (2011) after inserting $\widehat{p}^*$ and P^* into it: $R_D^* = \frac{qR+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$.

This choice of λ and equilibrium values of R_D^*, $\widehat{p}^*$ and P^* constitute an equilibrium at t=0, if $R_D^* = \frac{qR_G+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)}$ is in the interval $\left[\frac{1}{q+(1-q)p};\frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR}\frac{d-\lambda_2}{1-\lambda_2}}\right]$ as determined in the proof of the Proposition 1. We can see that is always holds that $\frac{qR+(1-\pi)(1-q)pR}{(q+(1-q)p)(1-\pi(1-q)pR)} > \frac{1}{q+(1-q)p}$ (which is equivalent to $(q+(1-q)\,p)\,R>1$). $\frac{[q+(1-\pi)(1-q)p]R}{(q+(1-q)p)(1-\pi(1-q)pR)} \leq \frac{1}{[q+(1-q)p]-\frac{q(1-p)}{pR}\frac{d-\lambda_2}{1-\lambda_2}}$ holds for $R \leq \underline{R}$, where $\underline{R}$ is given by solving the last inequality for R with equality sign

$$\underline{R} = \frac{dq^2\pi - p^2\left(1-q\right)\left(1+d\left(1-\pi\right)\left(1+q\pi\right)\right)+pq(1-d(1+2\left(1-q\right)\pi -}{p(q^2(1-\pi+q\pi-d(1-(2-q)\pi))-p\left(1-q\right)q(2-(1-2q)\pi+\left(1-q\right)\pi^2 d(2-\pi)(1+\left(1-q\right)\pi))+p^2}$$

Hence, the bank chooses at t=0 $\lambda = \lambda_2$ for $R \leq \underline{R}$.

Now we split the discussion into cases in which $P^*\left(1-\lambda\right)+\lambda > d$ and $P^*\left(1-\lambda\right)+\lambda \leq d$.

The case $P^*\left(1-\lambda\right)+\lambda > d$ **for all** $\lambda \in [0;d)$ For $\lambda \in (\lambda_1;\lambda_2)$ we use the similar procedure as for $\lambda = \lambda_2$. The derivative of (22) with respect to λ is

$$\begin{aligned}
&\pi\left[q\left(-R+\widehat{p}R_D\right)+(1-q)\,\widehat{p}R_D\left(-P+1\right)\right]+\\
&(1-\pi)\left[q\left(-R+R_D\right)+(1-q)\left(1-\beta\right)p\left(-R+R_D\right)\right]+\\
&(1-\pi)\,\beta\left(1-q\right)\widehat{p}R_D\left(1-P\right)
\end{aligned}$$

The difference to the case with $\lambda = \lambda_2$ is that this time equilibrium values of P, R_D, σ and $\widehat{p}$ at t=1 are functions of λ. Hence, the equilibrium choice of λ, λ^*, and corresponding equilibrium variables at t=1 is an interior solution to a system of five equations: the binding above first order condition, and equations (2)-(5). Because equilibrium values of λ, R_D, σ and $\widehat{p}$ are complicated objects, we do not providing them (the reader is more than welcome to ask the author for the Mathematica code that offers the solutions). The solution to this system of equations is an equilibrium if the equilibrium value of λ, λ^*, is between $(\lambda_1; \lambda_2)$. This occurs for $R \in \left(\underline{R}; \overline{R}\right)$, where $\overline{R}$ is the solution of an equation in which λ_1 is equal to the chosen λ^*. Again we refrain from providing the exact form of $\overline{R}$. Hence, the bank chooses optimally some $\lambda^* \in (\lambda_1; \lambda_2)$ for $R \in \left(\underline{R}; \overline{R}\right)$.

For $\lambda = \lambda_1$ again we apply the same procedure as for $\lambda = \lambda_2$. The first order condition with respect to λ (obtained for the second expression in (22)) equals zero for $R_D^* = \frac{qR}{1-(1-q)pR}$, which has to be in the interval $\left[\frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}}; \frac{1}{p}\right]$. It always holds that $\frac{qR}{1-(1-q)pR} \leq \frac{1}{p}$, and $\frac{qR}{1-(1-q)pR} \geq \frac{1}{1-\frac{1-p}{pR}\frac{d-\lambda_1}{1-\lambda_1}}$ is equivalent to $R \geq \overline{R}$. Hence, the bank chooses optimal $\lambda = \lambda_1$ for any $R \geq \overline{R}$.

For $\lambda \in [0; \lambda_1)$ (this interval is not empty for $d > \frac{p(1-q)R}{1-\pi}$) the derivative of the first expression in (22) with respect to λ reads

$$\begin{aligned}&(R_D - R)\, q\left(1 - \pi\gamma - (1-\pi)(\gamma_S + \gamma_A)\right) + \\ &(1-P)\left[R_D\left(1 - q + \pi q\gamma + (1-\pi)\, q\gamma_S\right) + \frac{R}{P}(1-\pi)\, q\gamma_S\right].\end{aligned}$$

We have to evaluate the sign of this derivative at the equilibrium arising at t=1. Using the equilibrium loan rate (10) the above derivative boils down to

$$\frac{R}{P^*}(1 - P^*)\,. \tag{28}$$

The expression (28) implies the following. If the anticipated equilibrium price P^* at t=1 is below 1, then the bank will choose any $\lambda \geq \lambda_1$, because (28) is negative, and, therefore, the bank's profits at t=0 are increasing in λ. If the anticipated price P^* at t=1 is higher than 1, then the bank will choose $\lambda = 0$ at t=0. If the anticipated price is 1, then the bank is indifferent between any $\lambda \in [0; \lambda_1)$. At the same time from Proposition 1 we know that the equilibrium price at t=1 given by (11) is decreasing in λ. Hence, as depicted in Fig. 4 we might have either a unique or multiple equilibria depending on the parameters.

First, a unique equilibrium exists when the equilibrium price (11) is lower than 1 for all $\lambda \in [0;\lambda_1)$ (it can never be above 1 for all $\lambda \in [0;\lambda_1)$, because it converges to $pR < 1$ for λ converging to λ_1 from the left). This occurs when the highest possible price at t=1, which obtains for $\lambda = 0$, is not higher than 1. Then the bank will not choose any $\lambda \in [0;\lambda_1)$. Hence, then we have a unique equilibrium given by one of the optimal λ derived for the cases in which $\lambda \geq \lambda_1$. Formally, this occurs when the expression (11) for $\lambda = 0$ is not higher than 1:

$$\frac{Rd(1-\pi)}{d(1-\pi)+(1-p)(1-q)R} \leq 1.$$

This holds for any R if $d \in \left(\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right]$ and $p < \frac{1}{2}$ or for $R \in \left(\frac{1}{\overline{p}}; \widetilde{R}\right)$ if $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, where $\widetilde{R} \equiv \frac{d(1-\pi)}{d(1-\pi)-(1-q)(1-p)}$ (under our assumptions on d and π it holds that $\widetilde{R} > \frac{1}{\overline{p}}$).

Second, when $R \geq \widetilde{R}$ and $d > \max\left\{\frac{p(1-q)R}{1-\pi}; \frac{(1-p)(1-q)R}{1-\pi}\right\}$, then we have multiple equilibria as seen in Fig. 4. One equilibrium is $\lambda^* = 0$, because then the equilibrium price (11) is higher than 1 and choice of $\lambda = 0$ is consistent with that price because (28) is negative for that price. Another equilibrium is a choice of $\lambda^* \geq \lambda_1$ as in the case of the above unique equilibrium. In such a case the equilibrium price is $pR < 1$, which is consistent with the choice of $\lambda \geq \lambda_1$ and negative sign of (28) for such a price. This equilibrium leads to lower profits and is less efficient than the one with $\lambda^* = 0$. Observe that the expected profit of the banks boils down to $(1-\lambda)\overline{p}R + \lambda - d(1-\pi)$ (and social welfare is $(1-\lambda)\overline{p}R + \lambda$) as shown previously. Hence, lower choice of λ implies higher profits and welfare.

It has to be noted that although $P = 1$ nullifies (28) it is an unstable equilibrium. The reason is that if we take any arbitrarily small perturbation around $P = 1$ the bank would prefer to set either $\lambda^* = 0$ or $\lambda^* \geq \lambda_1$, given that the equilibrium loan rate would adjust and the interbank market would always clear (see also Malherbe (2014)).

There is also a possibility of two additional cases: $\lambda_2 \leq 0$, which occurs for $R \geq \frac{(1-\pi)d}{\pi(1-q)p}$, and $\lambda_2 > 0 \geq \lambda_1$, which occurs for $R \in \left[\frac{(1-\pi)d}{(1-q)p}; \frac{(1-\pi)d}{\pi(1-q)p}\right)$. In case $\lambda_2 \leq 0$, there is enough liquidity for any illiquid bank on the interbank market for any $\lambda > 0$. Hence, the optimal choice of λ at t=0 is 0 and we always have an equilibrium in which all illiquid banks lend. In case $\lambda_2 > 0 \geq \lambda_1$, we need to use the above results for the case when $\lambda^* \in (\lambda_1; \lambda_2)$. The banks choose λ_2 optimally for

$R \in \left[\frac{(1-\pi)d}{(1-q)p}; \min\left[\underline{R}; \frac{(1-\pi)d}{\pi(1-q)p}\right]\right)$ and $\lambda^* \in [0; \lambda_2)$ for $R \in \left[\max\left[\underline{R}; \frac{(1-\pi)d}{(1-q)p}\right]; \frac{(1-\pi)d}{\pi(1-q)p}\right)$. IN addition, here we can have multiple equilibria if $\widetilde{R} < \frac{(1-\pi)d}{\pi(1-q)p}$.

Given the algebraic complexity of the solution in the body of the Lemma we just report the qualitative results.

The case $P^*(1-\lambda) + \lambda \leq d$ Finally, we analyze the case when there is a potential for an equilibrium with liquidity shortage for some λ. Here, it is impossible to obtain such clear cut conditions for the optimal choice of λ as in the case when $P^*(1-\lambda) + \lambda > d$ for all $\lambda \in [0; d)$. The reason is that it is very hard to obtain analytically clear cut conditions for the existence of equilibria at t=1. However, as highlighted in the proof of Proposition 2, once we have parameters such that a solution λ_{FR} exists, we can obtain such conditions. In what follows we use two numerical examples to provide some insight on the existance of equilibria in which $P^*(1-\lambda) + \lambda \leq d$ and the optimal choice of λ in such cases.

Example 1 We use values $p = 0.1$, $q = 0.7$, $\pi = 0.4$ and $d = 0.5$, and we vary R. We know that for $pR(1-\lambda_1) + \lambda_1 > d$ or $R > \widehat{R} \approx 3.636$ we always have $P^*(1-\lambda) + \lambda > d$ for any $\lambda \in [0; d)$. Hence, we are interested in the cases in which $R \in \left(\frac{1}{q+(1-q)p}; \widehat{R}\right]$, where $\frac{1}{q+(1-q)p} \approx 1.37$. We proceed in such a way that we first pin down which equilibria exist at t=1 for given λ and R, and then we look for optimal λ at t=0 given these equilibria at t=1.

As shown in the proof of Proposition 2, for $R \in \left[\widehat{\widehat{R}}; \widehat{R}\right]$ there is no equilibrium with liquidity shortage. However, because for such R we can have $pR(1-\lambda_1) + \lambda_1 < d$ for some λ it must mean that there exists an equilibrium with $P^*(1-\lambda) + \lambda = d$ at t=1. In fact, at t=1 for a given λ we have an equilibrium with only some good banks borrowing for $\lambda \in [0; \lambda_{LB})$, where P^* is given by (11), an equilibrium with illiquid banks being indifferent between borrowing and selling and $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\lambda_{LB}; \lambda_{UB}]$, as well as an equilibria in which all GI banks borrow and some or all BI banks borrow for $\lambda \in (\lambda_{UB}; d)$. We have that $\lambda_{LB} > 0$, for two reasons. First, we know from Lemma 4 that λ_{LB} exists once $R \leq \widehat{R}$. Second, solving $\lambda_{LB} > 0$ for R reveals that it holds for $R > \frac{d(1-\pi)}{p(1-q)+\pi-q} \approx 0.9$.

Now we can look for the optimal choice of λ anticipating a certain equilibrium at t=1. First, the bank does not find optimal to choose any $\lambda \in [0; \lambda_{LB})$. From the previous analysis we know that it occurs for $R < \widetilde{R} = 10$. Because $\widetilde{R} = 10 > \widehat{R}$, we obtain our claim. Second, the bank find optimal to choose $\lambda = \lambda_2$ when anticipating equilibria in which all the GI banks borrow at t=1. From the previous analysis this happens for $R < \underline{R} \approx 4.1$. Because $\underline{R} \approx 4.1 > \widehat{R} \approx 3.636$, we obtain our claim. Third, we are left with the optimal choice of λ at t=1 when the banks anticipate an equilibrium in which $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\lambda_{LB}; \lambda_{UB}]$. Deriving (23) with respect to λ we obtain

$$\begin{aligned}
&\pi \left[q\left[\widehat{p}R_D - R\right] + (1-q)\,\widehat{p}R_D\,(1-P)\right] \\
&+(1-\pi)\,q\left[\gamma_M\,(R_D - R) + (1-\gamma_M)\,(1-P)\right] \\
&+(1-\pi)\,(1-q)\left[\beta_M p\,(R_D - R) + (1-\beta_M)\,(1-P)\right].
\end{aligned}$$

Inserting all the equilibrium values for R_D, P, $\widehat{p}$, γ_M and β_M into the last expression yields a following expression

$$\begin{aligned}
&1 - \overline{p}R + \frac{(1-d)\,(\overline{p}-\pi)\,R}{d-\lambda} + \frac{(1-d)^2\,(1-q)\,(1-q\pi)}{(1-\lambda)\,(d\,(1-q)+q)} \\
&+\frac{qR(\overline{p}-q\pi) - d(1-p(1-q)^2R - q((1-q)(1-\pi)R+\pi))}{(d(1-q)+q)(d(1-q)+q\lambda)},
\end{aligned} \tag{29}$$

which becomes a polynomial of the third degree in λ, once we take out a common denominator. Therefore, an analytical solution for λ and providing conditions when such a solution might not be in $[\lambda_{LB}; \lambda_{UB}]$ is not possible. In our numerical example, we can show that the only real root of the above equation is above λ_{UB} for all $R \in \left[\widehat{\widehat{R}}; \widehat{R}\right]$, implying that the bank's profits are increasing in λ for all $\lambda \in [\lambda_{LB}; \lambda_{UB}]$. However, we know that once $\lambda > \lambda_{UB}$ the bank would like to choose λ_2. Hence, in our example for all $R \in \left[\widehat{\widehat{R}}; \widehat{R}\right]$ we have that the bank would always choose $\lambda = \lambda_2$.

Now we take up the case when $R \in \left(\frac{1}{p}; \widehat{\widehat{R}}\right)$. We know that this time an equilibrium with liquidity shortage exists. At t=1 for a given λ we have an equilibrium with only some good banks borrowing for $\lambda \in [0; \lambda_{LB})$ if $\lambda_{LB} > 0$, where P^* is given by (11), an equilibrium with illiquid banks being indifferent between borrowing and selling and $P^* = \frac{d-\lambda}{1-\lambda}$ for $\lambda \in [\max[0; \lambda_{LB}]; \lambda_{FR}]$ if $\lambda_{FR} > 0$, an equilibrium in which some illiquid bank become insolvent for $\lambda \in (\max[\lambda_{UB}; \lambda_{FR}]; \lambda_2)$

as well as an equilibrium in which all illiquid banks can borrow on the interbank market for $\lambda \in [\lambda_2; d)$.

From the previous analysis we know that in all equilibria in which all illiquid banks are solvent the bank chooses the optimal λ as λ_2. In our numerical example again the expression (29) is always positive. Now, we are looking into optimal choice of λ when the bank anticipates an equilibrium with liquidity shortage at t=1. Deriving (24) with respect to λ we obtain

$$\pi \left[q \left[\widehat{p} R_D - R \right] + (1-q)\, \widehat{p} R_D \left(1 - P\right) \right]. \tag{30}$$

Again after inserting equilibrium values for R_D, P and $\widehat{p}$, we obtain a derivative whose sign determines the optimal choice of λ. Due to complexity of (18) this derivative becomes also very complex. In our numerical example however, its value for all $R \in \left(\frac{1}{p}; \widehat{\widehat{R}}\right)$ is positive, implying again that the bank would like to choose λ_2.

Example 2 We use values $p = 0.1$, $q = 0.7$, $\pi = 0.85$ and $d = 0.9$, and we vary R again. Observe that there are so many liquid banks (high π) that $\lambda_2 \leq 0$ is for all $R \geq \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. For such R we always have that the banks can take $\lambda = 0$ and there is enough interbank loans at t=1 after the bad illiquid bank sell there loans. Hence, we are interested in the case $R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. Then for $\lambda \in [0; \lambda_2)$ the banks have to sell at t=1 to become liquid.

Next, we can show that for $R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$ and all $\lambda \in [0; \lambda_2)$ we have that $P^*(1-\lambda) + \lambda \leq d$. To see this, we make the following observations based on previously established claims. First, $\lambda_2 < \lambda_{UB}$ holds for $R < \widehat{\widehat{R}} \approx 9.11$. Hence, it holds for all the interesting cases for $R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. Second, we can use expressions (15) and (16). Expression (15) applies when $\lambda_1 < 0$ or $R > \frac{1-\pi}{1-q}\frac{d}{p} = 4.5$. Moreover, we have just seen that $\lambda_2 < \lambda_{UB}$ in our interval of interest. Hence, from (15) our claim follows for $R \in \left(\frac{1-\pi}{1-q}\frac{d}{p}; \frac{1-\pi}{1-q}\frac{d}{p\pi}\right)$. Once we have that $R \leq \frac{1-\pi}{1-q}\frac{d}{p}$, we can use expression (16). Here we can show that the highest value $f(0) = -0.45 < 0$, which obtains for the highest R in this interval, $R = \frac{1-\pi}{1-q}\frac{d}{p}$, given that $f(0)$ is increasing in R. Given the last observation and $\lambda_2 < \lambda_{UB}$ expression (16) also gives us that we have $P^*(1-\lambda) + \lambda \leq d$ for all $R \leq \frac{1-\pi}{1-q}\frac{d}{p}$.

In fact, in our numerical example we have only an equilibrium with liquidity shortage for all

$R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. In our numerical example, $\lambda_{FR} < 0$ or does not exist for all $R < \frac{1-\pi}{1-q}\frac{d}{p\pi} \approx 5.29$. Hence, we could only find examples in which we had to take a look only at the derivative (30) evaluated at the equilibrium values at t=1. In all of our examples this derivative evaluated at the equilibrium values at t=1 is increasing in λ. This means that we have a possibility of multiple equilibria that occur in a similar way to the ones in the case without liquidity shortage when $\lambda \in [0; \lambda_1)$. Multiple equilibria occurs, because given that each bank has to look for the highest profits at the corners of the interval $[0; \lambda_2)$, it has to also take into account what other banks do. As in Malherbe (2014) this might lead to coordination problems and multiple equilibria in the choice of λ at t=0. We could find examples in which the derivative is always negative for R close to the upper bound on the interval $\frac{1-\pi}{1-q}\frac{d}{p\pi}$, implying that the bank would like to choose optimally $\lambda = 0$. This is sensible given that for $R > \frac{1-\pi}{1-q}\frac{d}{p\pi}$ this is also the optimal choice of λ. As R decreases, the derivative can become negative for low λ and positive for high λ, indicating multiple equilibria. As R decreases further the derivative becomes positive for all λ, indicating optimal choice of λ_2.

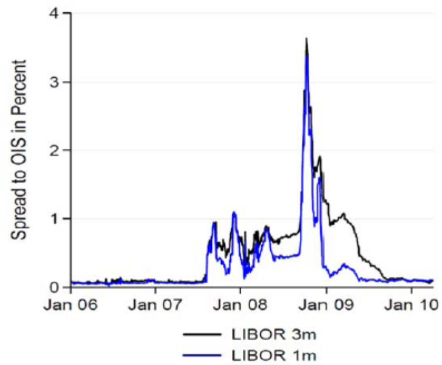

Figure 1: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 1 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts the spread between the 1- and 3-month Libor and OIS.

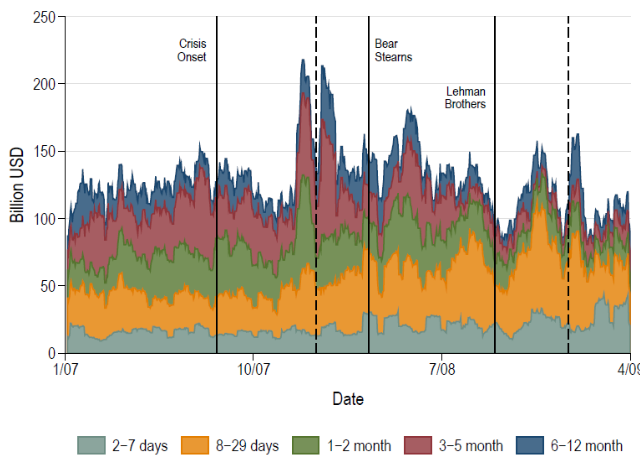

Figure 2: Source: Kuo, Skeie, Youle, and Vickrey (2013). This figure (Figure 5 in Kuo, Skeie, Youle, and Vickrey (2013)) depicts maturity-weighted volume of term interbank loans orginiated between January 2007 and March 2009.

Figures

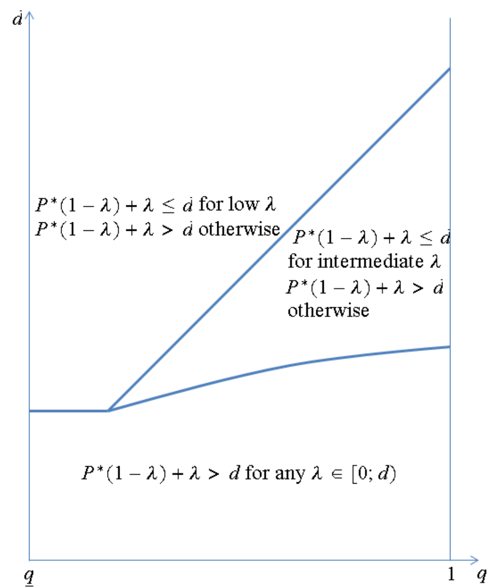

Figure 3: Results of Lemma 4

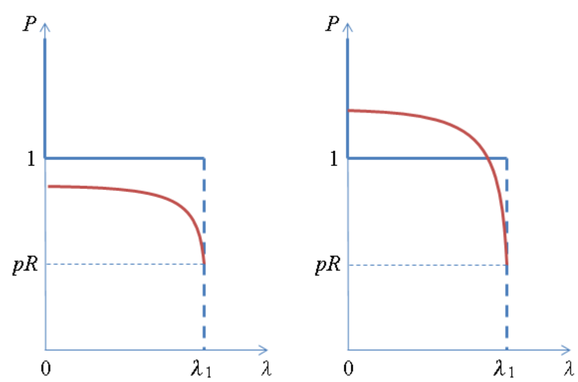

Figure 4: The case of $\lambda \in [0; \lambda_1)$. The blue step-wise curve is the optimal choice of λ at t=0 based on the aniticipated price at t=1. The dashed part represents that $\lambda = \lambda_1$ does not belong to the interval $[0; \lambda_1)$. The red solid curve is the equilibrium price at t=1 as a function of λ. The crossing points of these two curves pin down the equilibrium values of λ and P. The left panel represents the case in which the bank does not choose any $\lambda \in [0; \lambda_1)$ as optimal. The right panel show the case with multiple equilibria in which the optimal $\lambda = 0$ or the optimal $\lambda \geq \lambda_1$ as derived earlier.

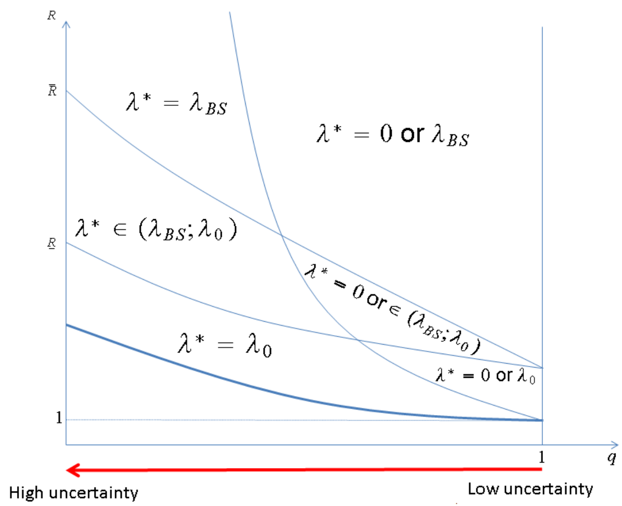

Figure 5: Lemma 6 in case of equilibria without liquidity shortage for any λ.