Credit Spreads and Interest Rates: A Cointegration Approach

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Abstract

This paper uses cointegration to model the time-series of corporate and government bond rates. We show that corporate rates are cointegrated with government rates and the relation between credit spreads and Treasury rates depends on the time horizon. In the short-run, an increase in Treasury rates causes credit spreads to narrow. This effect is reversed over the long-run and higher rates cause spreads to widen. The positive long-run relation between spreads and Treasurys is inconsistent with prominent models for pricing corporate bonds, analyzing capital structure, and measuring the interest rate sensitivity of corporate bonds.

1. Introduction

Credit spreads, the difference between corporate and government yields of similar maturity, are a fundamental tool in fixed income analysis. Credit spreads are used as measures of relative value and it is common for corporate bond yields to be quoted as a spread over Treasuries. In this paper, we use a cointegration approach to provide an alternative model of credit spreads and analyze how credit spreads respond to interest rate movements. We find that corporate rates and government rates are cointegrated and the relation between credit spreads and Treasury rates depends on the time horizon. Over the short-run, credit spreads are negatively related to Treasury rates. Initially, spreads narrow because a given rise in Treasuries produces a proportionately smaller rise in corporate rates. Over the long-run, however, this relation is reversed. A rise in Treasury rates eventually produces a proportionately larger rise in corporate rates. This widens the credit spread and induces a positive relation between spreads and Treasury rates.

These results are interesting for several reasons. First, they have important implications for models of capital structure and for models of pricing corporate debt. For example, the capital structure model of Leland and Toft (1996) and the bond pricing models of Longstaff and Schwartz (1995) and Merton (1974) contain a common prediction: in equilibrium, an increase in the risk free rate will decrease a firm's credit spread. This prediction is inconsistent with our finding of a positive long-run relation between credit spreads and Treasury rates. In addition, since the models do not specify the dynamics of the adjustment process, they cannot capture the distinction between the short-run and long-run behavior that we observe in the data. Second, our results question the inference drawn from empirical studies of credit spreads. Duffee (1998) and Longstaff and Schwartz (1995), for example, report that *changes* in credit spreads are negatively related to *changes* in Treasuries. This result is sometimes interpreted as suggesting that the level of equilibrium credit spreads is negatively related to the level Treasury rates and therefore consistent with the above models. However, by analyzing changes, their methodology focuses on the short-run behavior and has little ability to detect long-run positive relation between spreads and rates. Third, our findings have implications for managing the interest rate risk of corporate bonds. Chance (1990) and others have argued that the presence of default risk shortens the effective duration of corporate bonds. While the negative short-run relation is consistent with this logic, the positive long-run response implies that corporate bonds are eventually more sensitive to interest rate movements than otherwise similar Treasury bonds. Finally, our empirical results contribute to understanding the time series process of credit risk. This has implications for term structure models of corporate yields, the pricing of credit derivatives, and methods for measuring credit risk.

The essence of a cointegration relationship among two variables is that they share a common unit root process. When this occurs, it is possible to construct a stationary variable from a linear combination of the two non-stationary variables. If the two variables, x_{1t} and x_{2t}, are cointegrated, then the error-correction term, $x_{1t} - 8x_{2t}$, is stationary and the cointegrating vector is $(1,-8)$. Intuitively, 8 measures the long-run relation between x_{1t} and x_{2t}; when x_{1t} and x_{2t} are cointegrated, 8 can be viewed as the slope coefficient in the regression of x_{1t} on x_{2t}. Since $x_{1t} - 8x_{2t}$ is stationary, cointegration implies that corporate and government yields cannot drift arbitrarily far apart and the dynamic path of corporate yields is related to $x_{1t} - 8x_{2t}$, or the deviation from its long-run equilibrium level.

Cointegration provides an attractive methodology for our analysis. It provides a flexible functional form for modeling non-stationary variables and it is straightforward to construct impulse response functions showing the dynamic effects of interest rate shocks. In addition, the cointegration vector provides a direct test of economic hypotheses. For example, if equilibrium corporate spreads are negatively related to Treasury rates, then *8* must be less than one. When this occurs, a 1% increase in Treasury rates will lead to a less than 1% increase in corporate rates.[1] Thus, over the long-term, higher rates would be associated with lower credit spreads.

We use two approaches to analyze the relation between credit spreads and Treasury rates. Our first approach follows the cointegration model Johansen and Juselius (1990) to analyze the long-run relation. Using monthly bond yields from 1960 to 1997, we find that a 1% increase in 10-year Treasury rates generates long-term increases of 1.028% for Aaa rates and 1.178% for Baa rates. Our second approach emphasizes the short-run dynamics. We use our error-correction estimates to construct impulse response functions. These functions trace out the adjustment path of corporate rates to Treasury shocks and distinguish between short-term and long-term relations. With this approach, we find that a 1% rise in the Treasury rate has asymmetric short and long-run effects. In the short-term, the Aaa and Baa spreads fall 34 and 47 basis points, respectively. Over the long-term, however, the effect is reversed. The Aaa spread eventually returns to its initial level while the Baa spread rises by 17 basis points. These point estimates are very close to the long-run estimates from our cointegration model.

The distinction between the short-run and long-run response of credit spreads to interest rate movements has important implications for theoretical models. The predictions of these models are equilibrium or long-term predictions and should be evaluated with long-run cointegration

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[1]To simplify the language, we use the convention that a 1% increase refers to a one unit increase. For example, if the interest rate is 5%, a 1% increase will change it to 6%, not 5.05%.

estimates. Our results show the long-term relation is positive and therefore inconsistent with the models of Merton (1974), Kim, Ramaswamy, and Sundaresan (1993), Longstaff and Schwartz (1995), and Leland and Toft (1996).

We also find that yields on Aaa, Baa, and Treasury bonds are jointly cointegrated with two cointegrating vectors. However, we find that rates in one credit class do *not* provide additional information about rates in the other class. This evidence supports the approach in Duffie and Singleton (1996) of modeling individual credit classes separately.

Our approach to analyzing the dynamics of credit risk differs from previous empirical studies of credit spreads. For example, Sarig and Warga (1989), Litterman and Iben (1991), and Helwege and Turner (1998) analyze the shape of the term structure of risky debt, but do not examine how it changes over time. Duffee (1998) focuses on the effects from call options embedded in corporate bonds and shows these options induce a negative relation between corporate and Treasury yields. His analysis of credit spreads, however, relies on a simple VAR approach that excludes error correction terms. As we show in section 3, analyzing cointegrated variables with simple VARs can generate misleading inferences. Bernanke (1983), Keim and Stambaugh (1986), and Davis (1992) examine credit spreads, but their focus is on using spreads to explain the behavior of macro-economic and financial variables.

We subjected our cointegration analysis to several specification checks. Following Konishi, Ramey, and Granger (1993), we introduced a variety of stationary macro variables into our error-correction regressions. The macro variables were generally insignificant and did not reduce the magnitude or significance of the error-correction coefficients. Controlling for the heteroskedasticity in rates due to the 1979-1982 change in monetary policy operating procedures reduced the significance of results, but did not alter our conclusions. Finally, our results did not change when using the Engle and Granger (1988) cointegration test, which is more robust to problems of spurious cointegration.

Since our long-run results are inconsistent with theoretical models, we analyze, in considerable detail, an example where higher rates can lead to increased credit spreads. Following Merton (1974) we use an options approach to value corporate debt and determine credit spreads. However, we extend his approach to allow the value of the firm's assets to be affected by a change in interest rates. In this case, we show that increasing the risk free rate can increase the credit spread.

The remainder of the paper is as follows. Section 2 discusses the theory and existing empirical evidence on the relation between credit risk and risk free rates. Section 3 describes the cointegration methodology. Section 4 describes the data and provides summary statistics. Section 5 presents our bivariate cointegration results and Section 6 presents our multivariate cointegration results. Section 7 concludes.

2. The long-run relation between credit spreads and the risk free rate

A. Theoretical Models

The relation between the risk premium for corporate debt and the risk free interest rate is an important component of the capital structure model of Leland and Toft (1996) and the corporate debt pricing models of Merton (1974), Kim, Ramaswamy, and Sundaresan (1993), and Longstaff and Schwartz (1995). The comparative statics of these models predict that equilibrium credit spreads are negatively related to the risk free rate. Unfortunately, it is difficult to provide a convincing intuitive explanation for this negative relation. While it is possible that a ‘flight to quality’ could induce a temporary negative relation between corporate and government rates, it seems more likely that high nominal rates would be associated with a high risk premium for corporate debt. For example, the model in Bernanke and Gertler (1989) implies that higher interest rates, all else constant, will increase agency problems for borrowers. This increases credit spreads because it widens the gap between internal and external financing costs.

Since our long-run empirical results are inconsistent with the bond pricing and capital structure models, we analyze how these models might be modified to generate a positive relation between spreads and rates. We focus on what appears to be the most promising avenue, allowing changes in rates to *directly* affect firm value. Models with indirect effects, such as Longstaff and Schwartz (1995) do not capture the patterns we observe in the data. We emphasize that our analysis is only suggestive. Precise modeling of these relations is difficult and not addressed in this paper.

To provide an example where spreads and rates can be positively related we rely on Merton (1974). We use an options framework, where the evolution of firm value is described by the diffusion process, $dV = uV dt + sV dZ$. In this framework, changes in the risk free rate have no effect on firm value. The intuition for this result is that the drift term u is perfectly correlated with the risk free rate. Higher values for the risk free rate imply higher discount rates, but these are offset by higher future cash flows, or higher values of u. In a Black-Scholes-Merton world, these two effects exactly offset each other and thus preserve firm value.

The effect of an increase in rates is shown in Figure 1, which plots expected firm value against time. Since the current value of the firm is held constant, increased rates cause the future value to rotate up from the solid line P_0 to the dashed P_1. The future value is higher because of the rise in future cash flows; the current value is unchanged because of the offsetting rise in the discount rate.

Figure 1 also illustrates the intuition from the Merton (1974) model. Assume that the firm defaults if its value V falls below a predetermined threshold value, K. This is shown by the horizontal line in the figure. It is clear that when the expected return rises, the firm value moves away from the threshold and the default probability falls. Accordingly, an increase in rates should lower the firm's credit spread.

However, this is not the only way to view an increase in the risk free rate. An increase in rates could trigger a drop in firm value. All else constant, the lower firm price implies a higher expected return, or an increase in the drift term u. In Figure 1, the firm value shifts down from V_0 to V_1. The growth rate is higher, but the firm value is lower and now closer to the default threshold. In this scenario, an increase in rates could increase the likelihood of default and thereby increase the firm's credit spread.

This same principle can also be illustrated more formally with examples. Consider a hypothetical firm whose only assets are risk free bonds. Assume the market value of the risk free bonds is \$100 and the firm has issued a zero coupon bond with a face value of \$90, due in one year. Following Merton (1974), we know the equity in the firm can be valued as a call option on the value of the firm's assets, with a strike price of \$90. Since the total value must be partitioned between debt and equity, the value of the debt is the difference between the total firm value and the value of the equity. The debt value is equivalent to holding the firm's entire assets and selling a call option on the assets with a strike price of \$90.

To value the debt and equity components, assume the asset volatility is 10 percent and the continuously compounded risk free rate is 5 percent per year. To simplify the calculations, assume the firm's assets are 5-year zero coupon bonds and the term structure is flat. Based on these assumptions, the Black-Scholes-Merton value of the equity is $14.63 and the debt is $85.37. Since the face value of debt is $90, the continuously compounded expected return to the bonds is ln(90/85.37) or 5.28 percent. Since the risk free rate is 5 percent, this corresponds to a credit spread of 28 basis points.

Now consider the effect of an exogenous parallel shift of the yield curve to 7 percent.[2] The value of the call option rises to 16.23 and the value of the debt drops to 83.77. The expected return on the bond rises to 7.17 percent but the credit spread falls to 0.17 percent. Consistent with Merton (1974), Longstaff and Schwartz (1995), and Leland and Toft (1996), an increase in rates has lowered the firm's credit spread. These values are summarized in the first two columns of Table 1.

An important assumption of this example is that changes in the risk free rate do not effect the value of the firm's assets. This assumption is open to question. For example, while Leland and Toft (1996) assume that changes in the risk free rate do not effect firm assets, they also caution "While we have performed the standard ceteris paribus comparative statics, it should be observed that the firm value may itself change with changes in the default-free interest rate." [3]

Although incorporating the effect of interest rates on firm values is a challenging extension

[2]Strictly speaking, our examples require that the yield curve be flat and non-stochastic at 5 percent, and then be flat and non-stochastic at 7 percent.

[3]See footnote 25, on page 1003.

of the option models, it is easy to incorporate into our example. Since the current value of the five-year bonds is $100, then the face value (or future value) of the bonds must be $128.40. When rates rise from 5 percent to 7 percent, the current value of the firm's assets falls from $100 to $90.48. Incorporating the effect of interest rates on firm value requires only recalculating the call option value based on the lower firm value. Using the 7 percent interest rate and the $90.48 asset value, the Black-Scholes-Merton the value of the equity falls to $7.70 and the debt to $82.78. The expected return on the bond rises to 8.36 percent, yielding a credit spread of 1.36 percent. These values are shown in columns 3 and 4 of Table 1. In this case, an increase in rates has increased the credit spread.

An advantage of this approach is that we can analyze the effect of credit quality on the relation between credit spreads and the risk free rate. For example, consider changing the face value of the debt from $90 to $85. All else constant, the lower strike price makes it more likely the debt holders will be repaid in full, and corresponds to a reduction in credit risk.

To evaluate the sensitivity to credit quality, we need only recalculate the credit spreads using the $85 strike price. Using the 5 percent risk free rate, the value of the equity rises to $19.20. The yield on the debt falls to 5.07 percent and the credit spread is 7 basis points. As expected, the lower credit risk reduces the credit spread, which falls from 28 to 7 basis points. If the risk free rate rises to 7 percent, the firm value again falls to $90.48. With the $85 face value of debt, the equity falls to $11.59 and the debt is worth $78.89. The debt yield is 7.46 percent with a spread of 46 basis points. These values are summarized in columns 5 and 6.

Comparing the credit spreads for the $85 and $90 strike prices, it is clear that the lower credit quality debt is more sensitive to changes in the risk free rates. If the face value of the debt is

$85, the 2% rise in risk free rates causes the credit spread to widen by 39 basis points; if the face value is $90, then the spread widens by 108 basis points.

B. Empirical-Based Models

The relation between credit risk and the risk free rate is also an important component of empirical-based models for pricing risky debt. For example, Jarrow, Lando, and Turnbull (1997) develop a pricing model based on the probability transition matrix governing the evolution of future debt ratings. Das and Tufano (1996) extend this approach by allowing separate stochastic processes for both the default rate and the recovery rate. A characteristic of both models is that the correlations between important parameters are specified exogenously. Jarrow, Lando, and Turnbull assume that the credit spread is uncorrelated with the risk free rate, while Das and Tufano assume a negative correlation between spreads and recovery rates.

While these models can incorporate different empirical assumptions, they do assume that the probability transition matrix is independent of the level of interest rates. Although independence seems like a reasonable assumption, our finding of a positive long-run relation between spreads and rates suggests that higher rates increase the risk of default and, therefore, increase the likelihood of downgrades. This would imply that the probability transition matrix is not independent of the level of interest rates.[4]

C. Empirical Evidence

Cornell and Green (1991), Fridson and Kenney (1994), Longstaff and Schwartz (1995), and Duffee (1998) document a significant negative relation between changes in credit spreads and

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[4]Empirically, we find evidence of such a relation. Using Moody's quarterly data from 1973:Q1 to 1997:Q4, the correlation between the 10-year Treasury rate and the ratio of rating downgrades to ratings upgrades is 0.28, significant at the 1 percent level.

changes in Treasury rates. There are, however, two reasons to question whether these results imply a negative long-run relationship between the levels of Treasury rates and credit spreads. First, the empirical specifications in these studies focus on changes and do not incorporate equilibrium relationships between the variables. This is important because the predictions of the theoretical models are long-run or equilibrium predictions. Since the models do not specify the transition path from one equilibrium to another, it is questionable to draw inference about the equilibrium spread from the short-run dynamics. Second, estimates from these studies on the relation between credit spreads and Treasury rates will be biased and inconsistent if corporate and Treasury rates are cointegrated. As the next section shows, estimation with cointegration techniques solves both problems.

3. A cointegration model of risky and risk free debt

In this section we provide a cointegration framework to analyze the relation between corporate and Treasury bond yields. The advantage of this approach is that it incorporates the long-run relationship between the corporate and risk-free rates into the short-run dynamics of the empirical model. This framework also provides a direct test of whether credit spreads are negatively related to Treasury rates over the long-run.

Cointegration is based on the idea that while a set of variables are individually nonstationary, a linear combination of the variables might be stationary. While the variables are individually unbounded, the existence of a stationary combination implies that the variables cannot drift arbitrarily far apart. Intuitively, it is the long-run equilibrium relationship that links the cointegrated variables together. Cointegration also implies the short-term movements of the variables will be affected by the lagged deviation from the long-run relationship between the variables.

An alternative view of cointegration is that two variables are cointegrated when both are driven by the same unit root process. If corporate rates can be modeled as the sum of the risk free Treasury rate and a risk premium, it is clear both Treasury and corporate rates share a common process. Since both are driven by the same stochastic trend, they cannot evolve independently and the levels of the variables will be linked together.

To present this formally, consider the vector representation $X_t = \mu_t + \varepsilon_t$, where $X_t = \{X_{1t}, X_{2t}\}$ represents two data vectors, $\mu_t = \{\mu_{1t}, \mu_{2t}\}$ represents two stochastic trends, and $\varepsilon_t = \{\varepsilon_{1t}, \varepsilon_{2t}\}$ represents two i.i.d. error terms. If there is a stationary linear combination of the two variables, then there exists a 2×2 non-zero matrix *B* such that $B\mu_t = 0$. The test for cointegration is therefore based on the rank of *B*. In the two variable case, there can be at most one independent linear combination of X_{1t} and X_{2t} that is stationary. In this case, if the rank of *B* equals one, then the variables (X_{1t}, X_{2t}) are said to be cointegrated.

Assuming ΔX_t is stationary, the short-term dynamics of two cointegrated variables are captured in an error-correction model.

$$\Delta X_{1,t} = a_{10} + \gamma_1 (X_{1,t-1} - \lambda X_{2,t-1}) + \sum_{i=1}^{k} a_{i,11} \Delta X_{1,t-i} + \sum_{i=1}^{k} a_{i,12} \Delta X_{2,t-i} + \varepsilon_{1,t} \qquad \textbf{(1)}$$

$$\Delta X_{2,t} = a_{20} + \gamma_2(X_{1,t-1} - \beta X_{2,t-1}) + \sum_{i=1}^{k} a_{i,21}\Delta X_{1,t-i} + \sum_{i=1}^{k} a_{i,22}\Delta X_{2,t-i} + \varepsilon_{2,t} \tag{2}$$

In this model, the cointegrating vector is said to be $(1,-\beta)$, and the linear combination $X_{1,t} - \beta X_{2,t}$ is stationary. The economic interpretation of $X_{1,t} - \beta X_{2,t}$ is that it represents the deviation from the long-run equilibrium relationship between X_1 and X_2. In the error-correction model, this deviation affects the short-term behavior of ΔX_t, with the error-correction coefficients, γ_1 and γ_2, describing how quickly X_1 and X_2 respond to the deviation.

It is well known that existence of cointegration between X_1 and X_2 causes the time series behavior of X to differ from a conventional vector autoregression. Equations (2) and (3) can be written in matrix form as

$$\Delta X_t = A_0 + \Pi X_{t-1} + A_1 \Delta X_{t-1} + \ldots + A_k \Delta X_{t-k} + \varepsilon_t, \tag{3}$$

where A_0 is a (1×2) vector of intercepts and $A_1 \ldots A_k$ are (2×2) matrices of coefficients on lagged ΔX. The important characteristic distinguishing cointegration models from VAR models is whether $\Pi = 0$. If this restriction holds, then ΔX_t can be represented by a VAR. However, if the rank of Π exceeds zero, the elements of Π are non-zero. In this case, the series are cointegrated and the lagged X should be included in the regression. The VAR approach, which omits the lagged levels of X, can generate misleading inferences because it neglects the long-run relation between the integrated variables.

The tests for cointegration involve estimating the rank of Π. For an $n\times 1$ vector of $I(1)$ variables, X_t, the cointegration model can be written as,

$$\Delta X_t = A X_{t-1} + A(L)\Delta X_{t-1} + \varepsilon_t, \tag{4}$$

where $A(L)$ is a p-th order matrix polynomial in the lag operator and ε_t is a vector of i.i.d. error terms. Johansen (1988) shows that the number of cointegrating vectors, k, equals the rank of A. He provides two likelihood ratio tests for determining the rank of A, based on the number of nonzero eigenvalues in A. The first test, the maximal eigenvalue test, is really a sequence of tests. After sorting the estimated eigenvalues of A in descending order, the k-th statistic provides a test of the null hypothesis that the $\text{rank}(A) = k$ against the alternative that the $\text{rank}(A) = k+1$. The second test statistic, the trace statistic, is the running sum of the maximal eigenvalue statistics. The k-th trace statistic provides a test of the null hypothesis that the $\text{rank}(A) \le k$ against the alternative that the $\text{rank}(A) > k$. Critical values for these test statistics are provided in Osterwald-Lenum (1992).

Using cointegration to analyze corporate and Treasury rates has two attractive features. First, estimates of the cointegration vector tell us about the credit spread and its relation with Treasury rates. To see this, partition X_t into $X_{1,t}$, the corporate rate, and $X_{2,t}$, the Treasury rate. If the credit spread is uncorrelated with the Treasury rate over the long-term, then the cointegrating vector should be (1, -1). Alternatively, suppose the estimated vector is $(1, -\lambda)$. All else constant, a one unit rise in the government rate implies a λ unit rise in the equilibrium corporate rate.[5] Thus, $\lambda < 1$ implies that a rise in government rates will eventually be associated with a decline in the credit

[5]An alternative interpretation of λ comes from the Engle and Granger (1988) cointegration test. In their model, λ is the slope coefficient from the regression of the corporate rate against the Treasury rate. Under cointegration, they show λ is a consistent estimate of the long-run relation between the two variables.

spread because the corporate rate increases by less than the government rate. Alternatively, $8 > 1$ implies that an increase in government rates will ultimately increase the spread. A second attractive feature is that cointegration can distinguish between short-run and long-run behavior. It is straight forward to construct impulse response functions that capture both the short-term dynamics and the long-run relation between spreads and Treasurys.

4. Data and summary statistics

A. Data description

Our data contain monthly averages of daily rates for 10-year constant maturity Treasury Bonds and Moody's Aaa and Baa seasoned bond indices. We selected these series because of their long history. The data cover the period January 1960 to December 1997, for a total of 456 observations. Other corporate bond indices are available, but they cover much shorter periods. Similarly, only the 10-year government bond series has a relatively long history. The 30-year constant maturity index starts only in 1977 and the 20-year constant maturity index is unavailable between 1987 and 1992.

The Moody's indices are constructed from an equally weighted sample of yields on 75 to 100 bonds issued by large non-financial corporations. To be included in the indices, each bond issue must have a face value exceeding $100 million, a liquid secondary market, and an initial maturity of greater than twenty years. Each data series was obtained from the Board of Governors of the Federal Reserve System, release G.13.

Our Aaa and Baa series contain some callable bonds. The embedded option gives the issuer the right to repurchase the bonds and may affect the relation between credit spreads and interest rates. Duffee (1998) argues that these options induce a negative relation between spreads and non-callable Treasuries because a decline in the Treasury yield will increase the value of the option.[6] To exclude these effects, Duffee constructs corporate indices that include only noncallable bonds. While this sampling procedure controls for the callability, it unfortunately limits the data available for analysis. Few corporations issued non-callable debt prior to the mid-eighties, so Duffee's analysis is limited to 1985 through 1995, a period of generally declining rates. In contrast, our indices cover a 38 year period and contain a much richer set of interest rate dynamics.

The bias introduced from callable bonds in our sample is difficult to quantify. Over our sample period, Bliss and Ronn (1998) document that many Treasury bonds also contained embedded call options. As a result, the presence of call options in the Treasury bond series should partially offset the impact of the calls in the corporate series. In addition, assuming callability induces a negative relation between spreads and rates, then our estimates of 8 in the cointegrating vector (1, -8) will be biased downward. Thus, to the extent the callability of corporate bonds is greater than that of government bonds, the true value of 8 for non-callables will be even more positive than reflected in our estimates.

An alternative bias, which goes in the opposite direction, comes from tax differentials. In many states, income received from corporate bonds is subject to state income tax while income from Treasury bonds is exempt. This difference will cause the estimated *8* to be higher than the true *8*. To see this, view the after-tax corporate return as the sum of the after-tax Treasury return

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[6] While the decline in rates will raise the intrinsic value of the option, it should also be noted lower rates imply a higher present value of the strike price. In addition, as an empirical matter, lower rates tend to be associated with a lower volatility. These factors will reduce the negative relation between spreads and Treasury yields.

plus a risk premium, $r_c(1 - J_c) = r_g(1 - J_g) + r_p$. Solving for the pre-tax corporate return yields $r_c = r_p/(1 - J_c) + r_g(1 - J_g)/(1 - J_c)$. Since $J_c > J_g$, a 1% increase in r_g will be associated with a more than 1% increase in r_c. Assessing the magnitude of this bias is difficult because it depends on the fraction of corporate bonds held in tax exempt accounts and the state income tax rate of the marginal investor.

B. Summary statistics

Table 2 contains summary statistics for interest rates, spreads, and changes in spreads. Over the 1960 - 1997 period, the 10-year government rates averaged 7.46 percent, Aaa rates averaged 8.145 percent, and Baa rates averaged 9.147 percent. The mean monthly changes in rates are close to zero for each series. The Aaa - 10-year spreads (Aaa10) averaged 0.684 percent over the sample period, while the Baa - 10-Year spreads (Baa10) averaged 1.689 percent. The standard deviations are 0.38 percent for the Aaa10 spread and 0.65 percent for the Baa10 spread. Figure 2 presents this information graphically. Over the 1960-1997 period, the spreads range from -0.10 to 1.52 percent for the Aaa bonds, and from 0.40 to 3.81 for the Baa bonds.

Table 3 presents autocorrelations for the Baa, Aaa, and 10-year Treasury rates. For the first four lags, the autocorrelation coefficients are greater than 0.95 for each series. The high degree of persistence is consistent with the presence of a unit root. Table 4 reports augmented Dickey-Fuller and Phillips-Perron unit root tests. Using between one and six lags, both tests fail to reject the presence of a unit root for corporate or government rates at the 5 percent level. In addition, the Dickey-Fuller and Phillips-Perron tests for the first differences (not reported) are significant at the 1 percent level. Thus, the levels of the interest rates appear nonstationary while the changes appear stationary. These results are consistent with the conclusions of a number of studies on unit roots in nominal interest rates.[7]

The notion that interest rates are nonstationary is not without controversy. If taken literally, the presence of a unit root implies that nominal interest rates may be negative. In addition, it can be argued that interest rates follow a highly persistent, but stationary, time series process. In such a case, it is well known that test statistics for unit roots have low power against near unit root alternatives. For example, the test statistics in Table 4 cannot reject the null hypothesis that the interest rates are stationary with a first-order autocorrelation coefficient of 0.99. However, even if the interest rates are stationary, Granger and Swanson (1996) argue that cointegration techniques are appropriate for highly persistent variables.

The impact of using changes or levels to analyze the relation between credit spreads and Treasuries is reflected in Figures 3 and 4. Figure 3 plots the relation between the change in the Baa spread and the change in the 10-year constant maturity Treasury yield, while Figure 4 plots the levels of these variables. These figures show a strong negative relation between the changes, but also a clear positive relation between the levels. Since the theoretical models are based on the relation between levels, an inference drawn from an analysis of changes will be misleading.

5. Bivariate Cointegration Results

[7]See Rose (1988), Hall, Anderson and Granger (1992), and Konishi, Ramey and Granger (1993) for short-term rates, and Mehra (1994) and Campbell and Shiller (1987) for long-term rates.

Table 5 presents the results of tests for cointegration between government and corporate bond rates. Following Johansen and Juselius (1990) our estimates show that both corporate series are cointegrated with the government rates. For the Aaa series, the first maximal eigenvalue statistic is significant at the 1 percent level. This statistic rejects the null hypothesis that there are no cointegrating vectors in favor of the alternative hypothesis that there is one cointegrating vector. The second eigenvalue statistic, however, does not support the existence of two cointegrating vectors. The test statistic of 2.56 does not reject the null hypothesis that there is one cointegrating vector. The results for the Baa series are very similar. The results for both series are based on using two lags of the data in the estimation. The lag length was determined by the Schwartz Criteria.

Given the existence of cointegration between the Aaa and Treasury bond series, and between the Baa and Treasury bond series, Table 6 reports the corresponding cointegrating vectors. The Aaa vector is (1, -1.028) and the Baa vector is (1, -1.178). Following Johansen and Juselius (1990), Table 6 also provides Wald and likelihood ratio tests of the hypothesis that $8 = 1$. For the Baa series, both tests strongly reject the hypothesis that $8 = 1$. The *p*-values for both tests are less than 1 percent. For the Aaa series, however, we cannot reject the null hypothesis that $8 = 1$. The *p*-values for the Wald and Likelihood Ratio tests rise to 56 and 60 percent.

The result that 8_{Aaa} is insignificantly greater than one while 8_{Baa} is significantly greater than one has two interesting implications. First, since both values exceed one, it implies that a 1% increase in Treasury rates will ultimately generate an increase in corporate rates of more than 1%. Thus, as interest rates rise, credit spreads will eventually widen. This is consistent with the summary statistics in Das and Tufano (1996), but inconsistent with the predictions of Merton

(1974), Longstaff and Schwartz (1995), and Leland and Toft (1996). Second, the Baa bonds exhibit a greater long-run sensitivity to interest rate movements than Aaa bonds. This is inconsistent with a commonly held view that increased credit risk will make corporate bonds less interest rate sensitive. For example, the models by Chance (1990), Longstaff and Schwartz (1995), and Leland and Toft (1996) predict that increased default probabilities will shorten the effective duration of corporate bonds.

An alternative way to interpret the cointegrating relationship is to estimate equation (2), the error-correction regression. Cointegration implies the coefficient on the error-correction term will be negative and significant, with the size of the coefficient measuring the sensitivity of corporate rates to the error-correction term. Using the estimated cointegrating vectors from Table 6, Table 7 presents estimates of the error-correction model. For the Aaa rates, the coefficient on the error-correction term is -0.059 with a t-statistic of -2.13. For the Baa rates, the error-correction coefficient is -0.043 with a t-statistic of -2.47. As expected, both the error-correction coefficients are negative. All else constant, a widening of last month's credit spreads implies a narrowing of the spread this month.

A more accurate description of the adjustment process from interest rate shocks comes from the impulse response functions. Figure 5 shows the short and long run impact of a 100 basis point increase in the Treasury rate.[8] Initially, the Aaa rate rises by only 66 basis points and the Baa rate by only 53 basis points. This implies that the Aaa spread falls by 34 basis points and the Baa

[8]Impulse response functions require an identifying assumption about the contemporaneous relationship between corporate and government rates. We assume that a change in the government rate has a contemporaneous impact on corporate rates, but that a change in the corporate rate has no contemporaneous impact on the government rate.

spread falls by 47 basis points. Gradually, these declines are reversed. The Baa spread returns to its original level after about a year and then continues to rise, leveling off at 17 basis points above its pre-shock level. The Aaa spread eventually returns to its initial level.

An important implication of our results is they offer little support for the theoretical models of Merton (1974), Kim, Ramaswamy, and Sundaresan (1983), Longstaff and Schwartz (1995), and Leland and Toft (1996). The predictions of these models rely on the equilibrium or long-run behavior, not on the short-term dynamics. While our short-run negative relation is similar to Longstaff and Schwartz (1995) and Duffee (1998), the negative relations do not persist. Figure 5 shows the initial negative effect is reversed and the long-run relation between spreads and Treasurys is very similar to the estimates of 8 in Table 5.

To examine the sensitivity of our results, we conducted six robustness checks. First, we tested for cointegration using an alternative method developed by Engle and Granger (1988). In the presence of near unit roots, Gonzalo and Tae-Hwy (1998) show the Johansen test tends to find spurious cointegrating relationships while the Engle-Granger test is much less sensitive to this problem. Our cointegration results remain unchanged using the Engle-Granger test. Second, we examined the effect of increasing the lag lengths. Including additional lags had no effect on the cointegration results. In the error-correction regressions, the additional lags increased the standard errors but the point estimates were largely unaffected. Third, we examined the sensitivity of our results to heteroskedasticity associated with the 1979-1982 change in monetary policy operating procedures. We reestimated the cointegration model after transforming the data according to three volatility periods: 1960:1 to 1979:9, 1979:10 to 1982:11, and 1982:12 to 1997:12. With this GLS transformation, the cointegration results are unchanged, but the estimates of 8 in the cointegrating vectors fall to 0.972 for the Aaa and 1.12 for the Baa. The qualitative results, however, remain the same-- λ_{Baa} remains significantly greater than 1.0 and λ_{Aaa} remains insignificantly different from 1.0.

Our fourth check was to examine the sensitivity of our results due to data limitations. One issue is maturity. In our sample, the corporate bonds have longer maturities than the Treasuries. This raises the possibility that our results might reflect a positive relation between interest rates and the slope of the term structure. However, during 1953-1987, the slope of the yield curve between 10 and 20 years was uncorrelated with the 20-year bond rate. A second issue is data aggregation. Our indices are based on the monthly average of daily values. To examine the effects of this aggregation, we replicated the cointegration analysis with end-of-month rates. We selected end-of-month values for the 10-year constant maturity Treasurys used the Lehman Brothers Aaa and Baa series from the Fixed Income Database at the University of Houston.[9] Over the 1973:1-1997:10 period, the results from the end-of-month data indicate a slightly stronger long-run effect. Using the averaged data, the cointegrating vectors are .94 and 1.13 for the Aaa and Baa series. With end-of-month data, the vectors increased to 1.03 and 1.18.

Our final robustness check was to include macro-economic variables in the error-correction regressions. Since we use monthly data, we examined the following series: the growth in U.S. industrial production, the growth in the NAPM (National Association of Purchasing Managers) index, the growth in non-farm employment, the ratio of leading economic indicators to lagging economic indicators, and the Stock-Watson alternative experimental recession index XRI-2 series. The XRI-2 series was obtained from the NBER database, while the other series were obtained from the Board of Governors' FAME database. Including these variables, one at a time, had little

[9]The corporate series were generously provided by Arthur Warga.

effect on the error-correction coefficients. For the Baa regressions, the growth in industrial production, the NAPM index, and non-farm employment were all insignificant. The ratio of the leading to lagging economic indicators was significant at the 2 percent level and the Stock-Watson index was significant at the 6 percent level. The magnitudes of the corresponding error-correction coefficients, however, increased to -0.056 and -0.047, with t-statistics of -2.97 and -2.64. The results for the Aaa series were similar. Only the ratio of the leading to lagging economic indicators was significant. Including this variable increased the magnitude of the error-correction coefficient to -0.074, with a t-statistic of -2.28.

6. Multivariate Cointegration Results

In section 5 we showed that the long-term relation between credit spreads and Treasuries differed across credit classes. In particular, an increase in government rates induced a larger increase in the Baa credit spread than in the Aaa spread. In this section we continue this investigation by examining whether rates in one credit class contain information about the level and short-term dynamics of rates in the other credit class.

We analyze this issue by estimating cointegrating vectors and error-correction models for the Aaa, Baa, and Treasury rates together. With the two corporate rates in the system, it is possible that the rate in one credit class affects the short-run dynamics of the rate in the other class as well as its long-run equilibrium level. In addition, because there are three variables in the system, there is the possibility that the system contains two cointegrating vectors. In this case, proper specification of the error-correction model requires an error-correction term for each cointegrating vector.

Table 8 shows that over the 1960-1997 period, there is evidence of two cointegrating vectors among the three interest rates. Using Johansen's procedure, the maximum eigenvalue statistic for one cointegrating vector is 15.1 and the corresponding trace statistic is 17.6. Both statistics are significant at the 5 percent level, rejecting the null hypothesis that there is only one cointegrating vector among the three series. For two cointegrating vectors, the maximum eigenvalue and trace statistics fall to 2.48. Neither statistic is significant and therefore the tests do not reject the null that the series contain two cointegrating vectors. The two error correction terms are: ECT1: *Baa* + 0.1205*Aaa* - 1.3004*Treasury* and ECT2: -0.6208*Baa* + *Aaa* - 0.2920*Treasury*.

To determine whether rates in one credit class provide information about rates in the other class, we estimate error-correction models with two error-correction terms. With multiple error-correction terms, however, inference can be difficult because of the possibility that the error-correction terms may cancel each other out. For example, suppose that in the Baa error correction regression the estimated coefficients on the lagged changes in the Aaa rates are insignificant but the estimated coefficients on the error correction terms are significant and equal to -1 for ECT1 and 0.1205 for ECT2. Given these results, it would be incorrect to conclude that the Aaa rate provides information about the Baa rate since the impact of the Aaa rates across the error correction terms sum to zero $((-1\times0.1205) + (0.1205\times1))$.

To avoid the problem of offsetting error correction terms, we construct linear combinations of the two terms.[10] These combinations eliminate either the Aaa rate or the Baa rate. The

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[10] For a given number of cointegrating vectors, any vectors that span the space of the system variables are also cointegrating vectors. The Johansen procedure identifies the cointegrating vectors by assuming that the vectors are orthogonal. As long as the variables span

resulting vectors are ECT-NoAaa: 1.075*Baa* - 1.265*Treasury* and ECT-NoBaa: 1.075*Aaa* - 1.099*Treasury*. For the Aaa rate, we then estimate the error correction model using ECT-NoBaa and either ECT1 or ECT2; for the Baa rate we use ECT-NoAaa and either ECT1 or ECT2. With this transformation we can test whether rates in one credit class provide information about rates in the other class. For example, in the Aaa regression, any effect from the Baa rates will be reflected in the lagged *) Baa* coefficients or one of the original error-correction terms (ECT1 or ECT2), depending on which one is included in the regression.

The error correction estimates are presented in Tables 9 and 10. Table 9 shows that information on Baa rates do not help explain Aaa rates—the coefficients on the lagged changes in the Baa rates and on ECT1 or ECT2 are not significant. In addition, the R^2 is the same as for the bivariate cointegration results shown in Table 8. Similarly, Table 10 shows that for the Baa equation the coefficients on the lagged changes in the Aaa rates and on ECT1 or ECT2 are not significant and that the R^2 is the same as in the bivariate equation. Overall, the results suggest that corporate rates do not provide useful information across credit classes

This finding has two implications for modeling the term structure of corporate rates. First, it offers support for models that rely on a single credit quality factor. Examples include Merton (1974), Duffie and Singleton (1996), Madan and Unal (1998), and Duffie (1998). If the Baa rates independently affected the Aaa rates, then the models would benefit from incorporating information across credit classes. Second, our results offer support for the approach suggested in Duffie and Singleton. They propose modeling the yield on risky debt as a composite function of a

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the same space, however, the cointegrating vectors need not be orthogonal. Thus, any linear combination of the two cointegrating vectors are also cointegrating vectors.

short-rate process, a liquidity process, and a process governing expected loss in the event of default. Since these processes differ across credit classes, they suggest a parsimonious approach is to estimate a separate term structure model for each credit class and avoid untangling the components of the composite. Our finding of no significant effects across credit classes supports Duffie and Singleton's view of modeling the classes independently.

7. Conclusion

This study uses cointegration methodology to analyze the short-run and long-run relation between corporate bond rates and Treasury rates. We find evidence of cointegration and a distinction between short-run and long-run behavior. In the short-run, a rise in Treasury rates is associated with a decline in credit spreads. In the long-run, however, a rise in Treasury rates will increase credit spreads. A 1% rise in the Treasury rate is associated with a 1.028% rise in Aaa rates and a 1.178% rise in Baa rates. The positive long-run or equilibrium relation between credit spreads and Treasurys is inconsistent with predictions from the capital structure model of Leland and Toft (1996) and the corporate debt pricing models of Merton (1974), Kim, Ramaswamy, and Sundaresan (1993), and Longstaff and Schwartz (1995). The comparative statics of these models predict that equilibrium credit spreads are negatively related to the risk free rate.

Evidence of a positive long-term relation between spreads and Treasury rates also implies that the effective duration of corporate bonds is greater than otherwise similar Treasury bonds. This has significant risk management implications because it is commonly assumed that credit risk causes the effective duration of corporate bonds to be less than Treasurys of similar maturity.

In addition, we find that Aaa, Baa, and Treasury rates are jointly cointegrated. The Baa rate, however, does not appear to contain information important for determining the Aaa rate and vice versa. This finding suggests that models of the term structure of corporate rates are unlikely to be improved by incorporating information across credit classes.

Table 1
Option-Based Values for Debt and Equity

These calculations assume the only assets of the firm are 5-year zero coupon government bonds. The volatility of the assets is 10 percent per year and the debt maturity is one year.

	Firm value is not affected by) R		**Firm Value is affected by) R**		**Firm Value is affected by) R**	
Interest Rate (percent)	**5**	**7**	**5**	**7**	**5**	**7**
Firm Value	$100.00	$100.00	$100.00	$90.48	$100.00	$90.48
Face value of debt	$90.00	$90.00	$90.00	$90.00	$85.00	$85.00
Equity	$14.63	$16.23	$14.63	$7.70	$19.20	$11.59
Debt	$85.37	$83.77	$85.37	$82.78	$80.80	$78.89
Expected return on debt (percent)	5.28	7.17	5.28	8.36	5.07	7.46
Credit spread (percent)	0.28	0.17	0.28	1.36	0.07	0.46

Table 2
Summary Statistics

The statistics are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's and the 10-year Treasury series is a constant maturity series from the Board of Governors.

	Aaa	Baa	10-year	) Aaa	) Baa	) 10-yr	Aaa10	Baa10
Mean	8.14	9.15	7.45	.005	.004	.002	.684	1.69
Std. Dev.	2.61	2.97	2.57	.242	.219	.307	.377	.644
Median	8.04	8.90	7.25	.010	.000	.001	.700	1.66
Percentile (10%)	4.42	5.02	4.17	-.220	-.220	-.320	.210	.920
Percentile (90%)	12.1	13.6	11.5	.250	.220	.330	1.21	2.55

Table 3
Sample Autocorrelations

The estimates are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's and the 10-year Treasury series is a constant maturity series from the Board of Governors. The Box-Ljung Q-Statistic tests the null hypothesis that the series is not serially correlated. This statistic is distributed $P^2(n)$, where n is the number of lags. The null hypothesis is rejected at a significance level of less than 0.1 percent for all lags.

	Aaa		Baa		10-year	
Lag	Coefficient	Q-Statistic	Coefficient	Q-Statistic	Coefficient	Q-Statistic
1	.993	453	.995	454	.991	451
2	.983	898	.988	903	.977	890
3	.974	1335	.979	1345	.965	1320
4	.966	1767	.971	1781	.954	1740
5	.958	2192	.963	2210	.943	2152
6	.948	2608	.954	2633	.930	2553

Table 4
Unit Root Tests for Levels of Interest Rates

The columns labeled "Dickey-Fuller" are the results from augmented Dickey-Fuller tests. The null hypothesis is that the series contains a unit root. The percentage *p*-values (in parentheses) are approximate asymptotic *p*-values calculated using the method described in MacKinnon (1991). The estimates are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's and the 10-year Treasury series is a constant maturity series from the Board of Governors.

	10-year Treasury		Aaa Corporate		Baa Corporate	
Lags	Dickey-Fuller	Phillips-Perron	Dickey-Fuller	Phillips-Perron	Dickey-Fuller	Phillips-Perron
1	-1.94 (31)	-1.59 (49)	-1.75 (41)	-1.50 (53)	-1.57 (50)	-1.29 (63)
2	-1.61 (48)	-1.61 (48)	-1.49 (54)	-1.52 (52)	-1.43 (57)	-1.34 (61)
3	-1.70 (43)	-1.61 (48)	-1.56 (50)	-1.51 (53)	-1.49 (54)	-1.36 (60)
4	-1.65 (46)	-1.61 (48)	-1.59 (49)	-1.51 (53)	-1.53 (52)	-1.39 (59)
5	-1.90 (33)	-1.63 (47)	-1.73 (42)	-1.53 (52)	-1.59 (49)	-1.41 (58)
6	-1.74 (41)	-1.64 (46)	-1.62 (47)	-1.54 (51)	-1.59 (49)	-1.43 (57)

Table 5
Cointegration Results

This table uses Johansen's (1988) maximum likelihood method to estimate of the rank of Π for the corporate and government rates in the two variable regression

$$\Delta X_t = \Pi X_{t-1} + A(L)\Delta X_{t-1} + \varepsilon_t$$

The corporate rates are the Aaa and Baa series from Moody's, and the government rate is the 10-year constant maturity Treasury series from the Board of Governors. The estimates are based on monthly data from 1960:1 to 1997:12.

Panel A: Aaa and Treasury rates

	Maximal Eigenvalue Statistic		Trace Statistic	
Eigenvalues	Statistic	5% critical value	Statistic	5% critical value
.050	23.0	14.07	25.6	15.41
.006	2.63	3.76	2.63	3.76

Panel B: Baa and Treasury rates

	Maximal Eigenvalue Statistic		Trace Statistic	
Eigenvalues	Statistic	5% critical value	Statistic	5% critical value
.055	25.6	14.07	28.2	15.41
.006	2.56	3.76	2.56	3.76

Table 6
Estimates of Cointegrating Vectors

This table reports estimates of 8 in the cointegrating vector (1, -8) for the corporate and government rates using Johansen’s (1988) maximum likelihood method. The corporate rates are the Aaa and Baa series from Moody's, and the government rate is the 10-year constant maturity Treasury series from the Board of Governors. The estimates are based on monthly data from 1960:1 to 1997:12. Both the Wald and likelihood ratio tests have a chi-square distribution with two degrees of freedom.

		Wald Test		Likelihood Ratio Test	
Corporate Rate Series	8	H_o: 8 = 1	*p*-value (percent)	H_0: 8=1	*p*-value (percent)
Aaa	1.028	1.18	56	1.02	60
Baa	1.178	31.98	0	12.79	0

Table 7
Estimates of the Error-Correction Model

This table estimates the coefficients of the bivariate error-correction models

$$\Delta Aaa_t = a_{10} + \gamma_1 (Aaa_{t-1} - \lambda 10yr_{t-1}) + \sum_{i=1}^{2} a_{i,11} \Delta Aaa_{t-i} + \sum_{i=1}^{2} a_{i,12} \Delta 10yr_{t-i} + \varepsilon_{1,t}$$

$$\Delta Baa_t = a_{20} + \gamma_2 (Baa_{t-1} - \lambda 10yr_{t-1}) + \sum_{i=1}^{2} a_{i,21} \Delta Baa_{t-i} + \sum_{i=1}^{2} a_{i,22} \Delta 10yr_{t-i} + \varepsilon_{1,t} \; .$$

The error-correction terms were estimated using Johansen's (1988) maximum likelihood method. The estimates are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's, and the 10-year constant maturity Treasury series is from the Board of Governors.

	Dependent Variable			
	Aaa		Baa	
Term	Coefficient	t-statistic	Coefficient	t-statistic
Constant	.033	1.98	.019	1.76
Δ Aaa$_{t-1}$	.005	0.05		
Δ Aaa$_{t-2}$	-.058	-0.60		
Δ Baa$_{t-1}$			.218	2.64
Δ Baa$_{t-2}$			.049	0.68
Δ 10yr$_{t-1}$	.401	5.14	.273	5.17
Δ 10yr$_{t-2}$	-.168	-2.13	-.146	-2.92
Error-correction	-.059	-2.13	-.043	-2.47
λ	1.028		1.178	
R^2	.27		.33	

Table 8
Tests for Multiple Cointegrating Vectors

This table uses Johansen's (1988) maximum likelihood method to estimate of the rank of Π for the Aaa, Baa, and 10-year Treasury rates in the regression

$$\Delta X_t = \Pi X_{t-1} + A(L)\Delta X_{t-1} + \epsilon_t$$

The Aaa and Baa series are from Moody's, and the 10-year constant maturity Treasury series is from the Board of Governors. The estimates are based on monthly data from 1960:1 to 1997:12.

	Maximal Eigenvalue Statistic		Trace Statistic	
Eigenvalues	Statistic	5% critical value	Statistic	5% critical value
.060	28.0	20.97	45.5	29.68
.033	15.1	14.07	17.6	15.41
.005	2.48	3.76	2.48	3.76

Table 9
Estimates of the Multivariate Error-Correction Model for the Aaa Rate

This table estimates the coefficients of the error-correction model

$$\Delta Aaa_t = a_{j0} + \gamma_{j,1} ECTj + \gamma_{j,2} ECT\text{-}NoBaa + \sum_{i=1}^{2} a_{i,j1} \Delta Aaa_{t-i}$$
$$+ \sum_{i=1}^{2} a_{i,j2} \Delta Baa_{t-i} + \sum_{i=1}^{2} a_{i,j3} \Delta 10yr_{t-i} + \varepsilon_{1,t},$$

where *j=1,2* indexes whether the equation uses the Johansen error correction terms ECT1 or ECT2 as defined in the text and ECT-NoBaa is the linear combination of ECT1 and ECT2 that eliminates the Baa term. The Aaa and Baa series are from Moody's, and the 10-year constant maturity Treasury series is from the Board of Governors. The estimates are based on monthly data from 1960:1 to 1997:12.

Independent Variable	Coefficient	t-statistic	Coefficient	t-statistic
Constant	.038	1.92	.038	1.92
ΔAaa_{t-1}	.031	.23	.031	.23
ΔAaa_{t-2}	-.086	-.68	-.086	-.68
ΔBaa_{t-1}	-.026	-.22	-.026	-.22
ΔBaa_{t-2}	.052	.47	.052	.47
$\Delta 10yr_{t-1}$	.403	5.10	.403	5.10
$\Delta 10yr_{t-2}$	-.172	-2.16	-.172	-2.16
$ECT1_{t-1}$	.015	.40	--	--
$ECT2_{t-1}$	--	--	-.024	-.40
$ECT\text{-}NoBaa_{t-1}$	-.072	-1.37	-.048	-1.57
R^2	.27		.27	

Table 10
Estimates of the Multivariate Error-Correction Model for the Baa Rate

This table estimates the coefficients of the error-correction model

$$\Delta Baa_t = a_{j0} + \gamma_{j,1} ECTj + \gamma_{j,2} ECT\text{-}NoAaa + \sum_{i=1}^{2} a_{i,j1} \Delta Aaa_{t-i} + \sum_{i=1}^{2} a_{i,j2} \Delta Baa_{t-i} + \sum_{i=1}^{2} a_{i,j3} \Delta 10yr_{t-i} + \epsilon_{1,t},$$

where $j=1,2$ indexes whether the equation uses the Johansen error correction terms ECT1 or ECT2 as defined in the text and ECT-NoAaa is the linear combination of ECT1 and ECT2 that eliminates the Aaa term. The Aaa and Baa series are from Moody's, and the 10-year constant maturity Treasury series is from the Board of Governors. The estimates are based on monthly data from 1960:1 to 1997:12.

Independent Variable	Coefficient	t-statistic	Coefficient	t-statistic
Constant	.017	.97	.017	.97
Δ Baa$_{t-1}$	.194	1.86	.194	1.86
Δ Baa$_{t-2}$	.030	.32	.030	.32
Δ Aaa$_{t-1}$	.039	.34	.039	.34
Δ Aaa$_{t-2}$	.054	.49	.054	.49
Δ 10yr$_{t-1}$	.261	3.81	.261	3.81
Δ 10yr$_{t-2}$	-.174	-2.52	-.174	-2.52
ECT1$_{t-1}$	.060	0.16	--	--
ECT2$_{t-1}$	--	--	.007	.16
ECT-NoAaa$_{t-1}$	-.100	-.27	-.040	-2.45
R^2	.33		.33	

Figure 1
Relation Between Firm Value, Expected Return, and Default Threshold

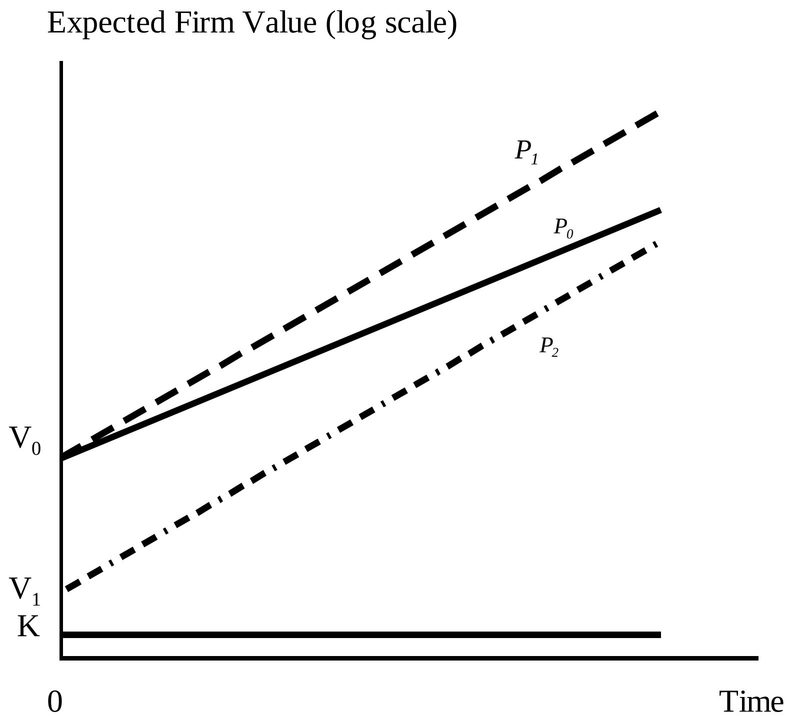


Figure 2

Spreads of Aaa and Baa Corporate Yields over the 10-year Treasury Yield

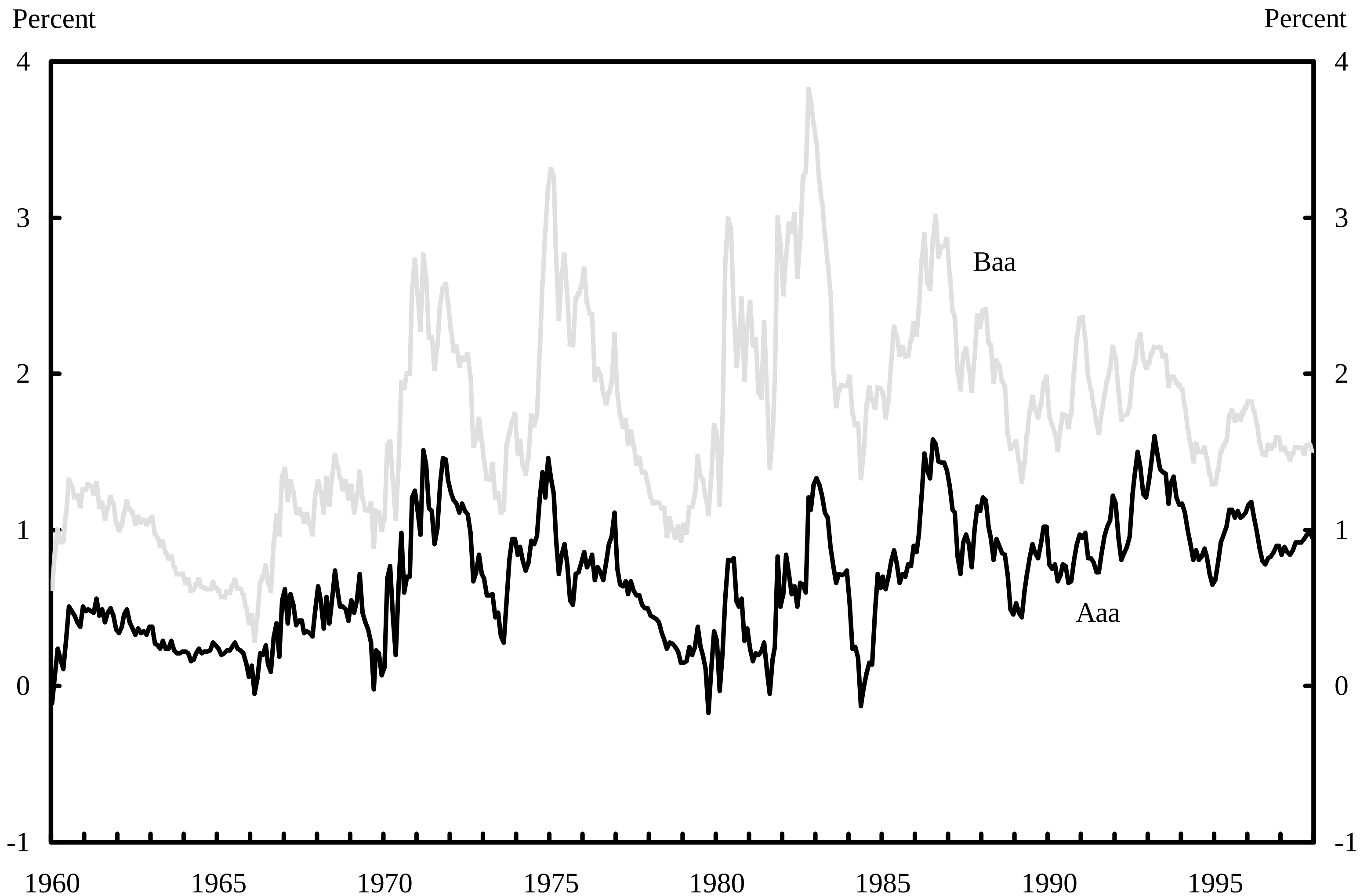


Figure 3

Changes in Baa Corporate Spread and 10-year Treasury Yield

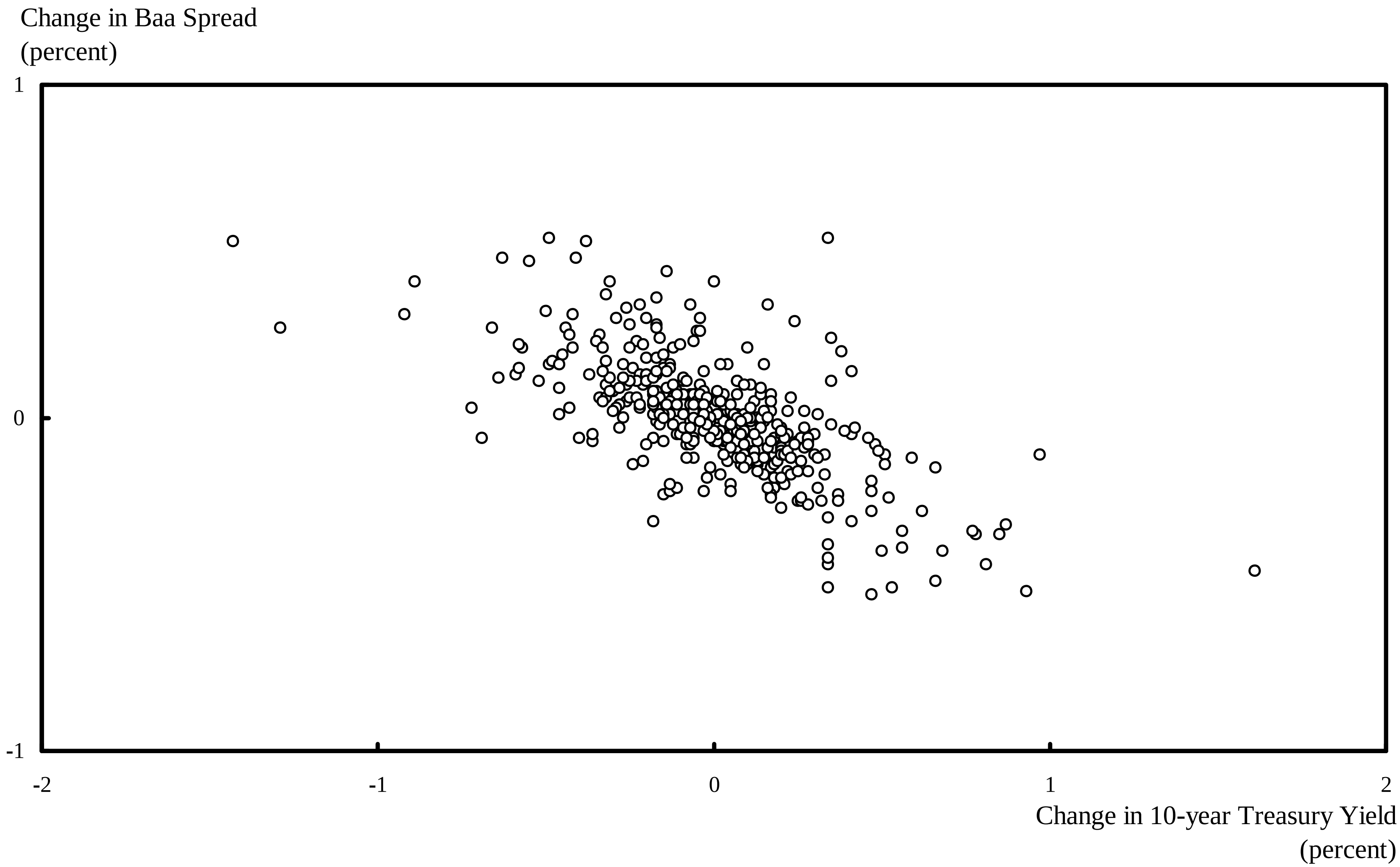


Figure 4

Baa Corporate Spread and 10-year Treasury Yield

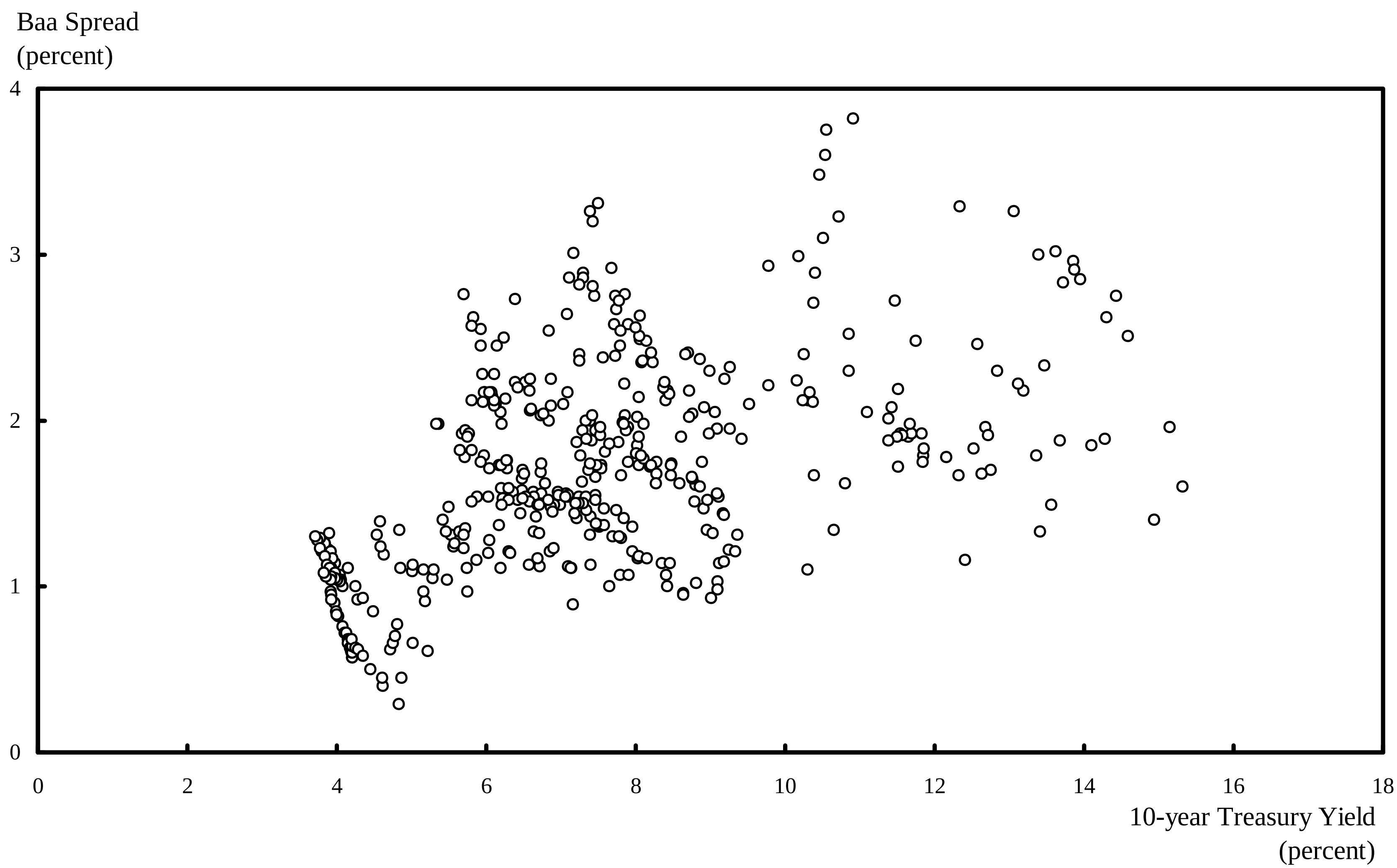
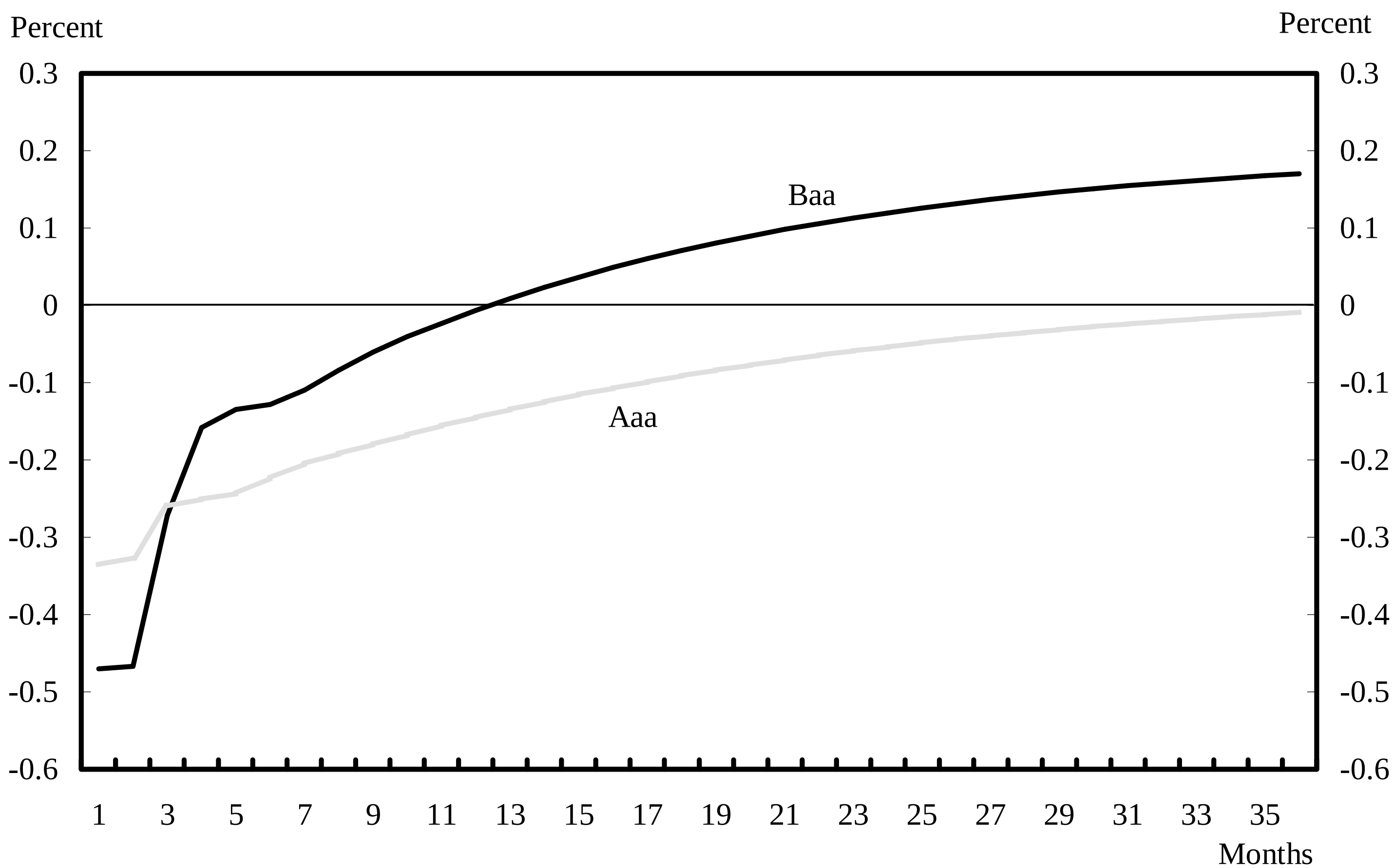


Figure 5

Response of Aaa and Baa Spreads to 1 Percentage Point Change in 10-year Treasury Bond Rate



References

Bernanke, B., 1983, “The Determinants of Investment: Another Look,” *American Economic Review*, 73, 71-75.

Bernanke, B. And M. Gertler, 1989, “Agency Costs, Net Worth, and Business Fluctuations,” American Economic Review, v79,14-31.

Bliss, R. and E. Ronn, 1998, “Callable U.S. Treasury Bonds: Optimal Calls, Anomalies, and Implied Volatilities,” *The Journal of Business*, v71, 211-252.

Campbell, J. and R. Shiller, 1987, "Cointegration and Tests of Present Value Models", *Journal of Political Economy*, v95, 1062-1088.

Chance, Donald, 1990, “Default Risk and the Duration of Zero Coupon Bonds,” *Journal of Finance*, v45, 265-274.

Cornell, B. and K. Green, 1991, “The Investment Performance of Low Grade Bonds Funds,” *Journal of Finance*, v46, 29-48.

Cumby, R. and M. Evans, 1997, “The Term Structure of Credit Risk: Estimates and Specification Tests,” working paper, Georgetown University.

Das, Sanjiv and Peter Tufano, 1996, “Pricing Credit-Sensitive Debt when Interest Rates, Credit Ratings and Credit Spreads are Stochastic,” *The Journal of Financial Engineering*, v5, 161-198.

Davis, E., 1992, “Credit Quality Spreads, Bond Market Efficiency And Financial Fragility,” *The Manchester School*, LX Supplement, 21-46.

Duffee, G., 1998, “Treasury Yields and Corporate Bond Yield Spreads: An Empirical Analysis,” *Journal of Finance*, v53, 2225-2242.

Duffie, D., and K. Singleton, 1995, “An Econometric Model of the Term Structure of Interest Rate Swap Yields,” *Journal of Finance*, v52, 1287-1321.

Duffie, D., and K. Singleton, 1996, “Modeling Term Structures of Defaultable Bonds,” working paper, Stanford University.

Duffie, D., 1998, “Defaultable Term Structure Models With Fractional Recovery of Par,” working paper, Stanford University.

Engle, R. and C.W. J. Granger. "Co-Integration and Error Correction: Representation, Estimation, and Testing," *Econometrica*, March 1987, v55, 251-276.

Fridson, M. and J. Kenney, 1994, "How do Changes in Yields Affect Quality Spreads?" *Extra Credit,* July/August, 4-13.

Gonzolo, J., and L., Tae-Hwy, 1988, "Pitfalls in Testing for Long Run Relationships," *Journal of Econometrics*, September, v16, 129-54.

Granger, C. W. J. and P. Newbold. 1974. "Spurious Regressions in Econometrics," *Journal of Econometrics* v2, 111-20.

Granger, C. and P. Swanson, 1996, "Future Developments in the Study of Cointegrated Variables," *Oxford Bulletin of Economics and Statistics*, v58, 537-53.

Hall, A., H. Anderson, C. Granger, 1992, "A Cointegration Analysis of Treasury Bill Yields,", The Review of Economics and Statistics, v74, 116-126.

Helwege, J. and C. Turner, 1998, "The Slope of the Credit Yield Curve for Speculative-Grade Issuers," forthcoming, the *Journal of Finance*.

Jarrow, R., D. Lando, and S. Turnbull, 1997, "A Markov Model of the Term Structure of Credit Spreads," *Review of Financial Studies*, v10, 481-523.

Johansen, S., 1988, "Statistical Analysis of Cointegrating Vectors," *Journal of Economic Dynamics and Control*, v12, 231-254.

Johansen, S. and K. Juselius, 1990, "Maximum Likelihood Estimation and Inference on Cointegration--With Applications to the Demand for Money," *Oxford Bulletin of Economics and Statistics*, v52, 169-210.

Keim, D., and R. Stambaugh, 1986, "Predicting Returns in the Bond and Stock Markets," *Journal of Financial Economics*, v17, 357-390.

Kim, I., K. Ramaswamy, and S. Sundaresan, 1983, "Does Default Risk in Coupons Affect the Valuation of Corporate Bonds?: A Contingent Claims Model," *Financial Management* v22, 117-131.

Konishi, T., V. Ramey, and C. Granger, 1993, "Stochastic Trends and Short-Run Relationships Between Financial Variables and Real Activity," National Bureau of Economic Research Working Paper: 4275.

Leland, H. and K. Toft, 1996, "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, v51, 987-1020.

Litterman, R., and T. Iben, 1991, "Corporate Bond Valuation and the Term Structure of Credit Spreads," *Journal of Portfolio Management*, v17, 52-64.

Longstaff, F. and E. Schwartz, 1995, “A Simple Approach to Valuing Risky Fixed and Floating Rate Debt,” *Journal of Finance* v50, 789-820.

MacKinnon, James, 1991, “Critical Values for Cointegration Tests,” *Long--Run Economic Relationships*, R.F. Engle and C.W.J. Granger, eds., London, Oxford, pp 267-276.

Madan, D. and H. Unal, 1998, “A Two-Factor Hazard-Rate Model for Pricing Risky Debt in a Complex Capital Structure,” working paper, University of Maryland.

Mehra, Y., 1994, "An Error-Correction Model of the Long-Term Bond Rate" *Economic Quarterly*, Federal Reserve Bank of Richmond, v80, 49-67.

Merton, R., 1974, “On the Pricing of Corporate Debt: The Risk Structure of Interest Rates,” *Journal of Finance*, v29, 449-470.

Nielsen, S., and and E. Ronn, 1996, “The Valuation of Default Risk in Corporate Bonds and Interest Rate Swaps,” working paper, University of Texas.

Osterwald-Lenum, M., 1992, “A Note with Quantiles of the Asymptotic Distribution of the Maximum Likelihood Cointegration Rank Test Statistics, *Oxford Bulletin of Economics and Statistics*, v54, 461-472.

Rose, A., 1988, "Is the Real Interest Rate Stable?" *Journal of Finance*, v43, 1095-1112.

Sarig, O., and A. Warga, 1989, “Some Empirical Estimates of the Risk Structure of Interest Rates,” Journal of Finance, v44, 1351-1360.