Monetary Policy in an Estimated Optimization-Based Model with Sticky Prices and Wages

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Abstract

This paper serves two purposes. First, it provides estimates of an optimization-based equilibrium model with sticky prices and wages. Second, the estimated model is used to analyze the welfare properties of various interest rate rules for conducting monetary policy. As shown by Erceg et al. (1999), an important feature of this model is that it involves a tradeoff between the variances of price and wage inflation and the output gap. This tradeoff implies that it is desirable for the monetary authority to respond to more than inflation, output, and past interest rates when setting the current interest rate. Indeed, the welfare optimal policy can be approximated with responses to both price and wage inflation and the past interest rate. By contrast, rules that call for a strong response to either detrended output or the output gap result in a much lower level of welfare.

1 Introduction

Most recent work involving monetary policy rules has focused exclusively on the responses of interest rates to inflation, output, or past interest rates, e.g. the recent volume by Taylor (1999a).[1] Focus on these three variables has been sufficient for the models, and typically ad hoc loss functions, used in these analyses. In the context of optimization-based models that incorporate only one nominal rigidity, stabilizing inflation at zero is Pareto optimal, and this equilibrium can be approximately achieved with the simple policy of responding strongly to both current inflation and the past level of the interest rate. However, once the welfare function of households involves a variance tradeoff, e.g., due to the existence of a second nominal rigidity, it is no longer clear that policymakers can confine themselves to looking solely at inflation and interest rates (and perhaps output). In this paper, we estimate a model that incorporates both nominal price and wage rigidities to analyze whether in practice interest rate rules restricted to respond to only inflation, output, and past interest rates are approximately optimal in a class of simple rules.

Our model features monopolistic competition and staggered price setting in both product and labour markets. Households maximize utility by choosing consumption and setting wages in a staggered fashion. Firms maximize profits by choosing prices in a staggered fashion. This extension of the standard optimizing model used in recent analyses of monetary policy is compelling for at least three reasons. First, evidence on staggered *wage* setting is at least as persuasive as evidence on staggered *price* setting. Second, as demonstrated in Erceg (1997), staggered wage setting generates a flat marginal cost schedule at the individual firm level, and hence persistent output effects of monetary shocks. Third, explicit modelling of the wage setting behaviour of households allows us to estimate directly the elasticity of labour supply, which figures prominently in household welfare and thus plays an important role in the evaluation of policy rules.[2]

[1]The most notable exception in the analysis of optimal policy rules are models for small open economies that incorporate exchange rates, e.g. Batini and Haldane (1999). In the structural VAR literature that seeks to identify the effects of exogenous monetary policy shocks, and thus provides implicit models of interest rate setting, e.g. Bernanke and Mihov (1998), monetary aggregates and commodity prices are also included.

[2]Our model is a variant of the one used by Erceg et al. (1999), where we have incorporated decision lags in the consumption and wage choices of households and the pricing decisions of firms. They show that nominal price and wage stickiness together imply a variance tradeoff between price and wage inflation and the output gap.

An important first step in our analysis is to estimate our model using data for the U.S. economy. Understanding the practical implications of various interest rate rules requires that we obtain estimates of the structural parameters and shocks. To date, however, there exists only a few studies which estimate optimization-based models for monetary policy evaluation. In light of the large recent literature on monetary policy rules, we find the paucity of empirical structural models to be troubling in view of the Lucas (1976) critique. It is compelling to believe that agents will understand the nature of any new policy regime offered in these analyses. Furthermore, since the underlying structural parameters in an optimization-based model play a crucial role in the welfare analysis of alternative rules, it is vital that the values of these parameters have some empirical validation from macroeconomic time series. Our estimation model is unique among those employed to analyze interest rate rules - whether based on explicit optimizing foundations or not - in that we utilize data on both prices and wages to obtain estimates of the relevant inputs to our welfare analysis.[3]

We adopt and extend the estimation strategy of Rotemberg and Woodford (1997). The essence of this approach is to obtain estimates of the structural parmeters based on impulse response functions to an exogenous monetary policy shock, and estimates of the structural shocks to replicate the remaining time series features of our endogenous variables. In addition, we show that Rotemberg and Woodford's methods are an example of minimum distance estimation, which provides us with standard errors for our estimates. The advantage of this approach versus directly specifying stochastic processes for the shocks and estimating the model by maximum likelihood, e.g. Kim (1999) and Ireland (1997,1999), is that it clarifies which moments of the data are crucial for determining the structural parameters.

The second part of our analysis focuses on the welfare properties of simple interest rate rules. One contribution from this analysis is to clarify the role of output in interest rate rules. Most non-optimizing models build in a tradeoff between the variances of inflation and the measure of the output gap that policymakers are assumed to care about. Therefore, it is optimal in these models for the interest rate to respond to the output gap. Conversely, the standard optimizing model with only sticky prices has no such tradeoff.[4] Stabilizing

[3] In recent work, Kim (1999) estimates an optimization-based model that embeds both sticky prices and wages, but he does not use data on wages in the formation of the likelihood function.

[4] In non-optimizing models, the output gap is constructed, both conceptually and empirically, as deviations of output from a *smooth* trend; whereas, in optimizing models, the notion of potential output is different,

the output gap, i.e. stabilizing *only inefficient fluctuations* in output, can be achieved by stabilizing inflation because dispersions in output across firms is caused solely by inflation in the presence of sticky prices. Erceg et al. (1999) argue that the presence of a variance tradeoff in a model with both sticky prices and wages reintroduces a role for the output gap in interest rate rules and, in particular, that monetary policy can nearly achieve the welfare optimal outcome by responding to both inflation and the output gap. In contrast, we find that under our estimates for the structural parameters and shocks, a strong interest rate response to the output gap can lead to severely sub-optimal outcomes.

Looking ahead to our most important results, we obtain estimates of the intertemporal elasticity of substitution in consumption and the elasticity of labour supply that are in line with evidence from panel data. Our estimate of the markup in goods markets is higher than the labour markup and both are below twenty percent. The key implication of our estimates for policy is that near optimal outcomes can be achieved by having the interest rate respond to both price and wage inflation, as well as the lagged interest rate. The optimal response to output, whether measured as a deviation from the steady state, i.e. detrended output, or as the output gap, i.e. the deviation of output from its Pareto optimal level, is negligible. Our estimated version of a sticky price and wage model does not overturn a striking result obtained by Rotemberg and Woodford (1999) in a model with only nominal price rigidities; namely, that substantial inertia in interest rate setting is desirable. Furthermore, contrary to the conclusions of Erceg et al. (1999), having the monetary authority respond to only inflation and the lagged interest rate does not lead to a substantial decline in welfare.

The plan of the remainder of the paper is as follows. Section 2 introduces the model. Section 3 presents our estimation methodology and results. Section 4 decribes the welfare function of the respresentative household. Section 5 analyzes the welfare properties of simple interest rate rules. Section 6 concludes. An appendix provides approximations to the model equations and the welfare function.

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since it is identified as the Pareto optimal, or efficient, level of output, which in general could be very *volatile*. Both output itself and the efficient level of output are assumed to evolve around (the same) deterministic steady-state path, which in practice is taken to be a linear trend.

2 Model

In this section we introduce a structural model of price inflation, wage inflation and output determination similar to the model developed in Erceg et al. (1999). Real effects of monetary policy in this model are due to imperfect competition and staggered price and wage setting in goods and labour markets.

The economy consists of a continuum of households and firms, and there is a continuum of differentiated, perishable goods and differentiated kinds of labour services. Each household is the monopolistic supplier of one kind of labour service, and consumes a CES aggregate of all the differentiated goods. The household sets a nominal wage for its labour services, and supplies as many hours as are demanded at its chosen wage. Each firm is the monopolistic producer for one good, and uses a CES aggregate of households' labour services in the production process. The firm sets a price for its good, and satisfies demand at this price. Because the analysis focusses on the effects of monetary policy at the business cycle horizon, capital accumulation is not modelled.

Household i's utility is defined over the index C_t^i, where

$$C_t^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

z denotes a specific good, and $\theta > 1$ parameterizes the elasticity of substitution in the household's preferences between the various goods. As θ gets large, goods become ever closer substitutes, whereas if θ approaches 1 from above, goods are less and less substitutable. Hence θ also measures the market power of each of the firms located on the interval [0,1], with market power decreasing in θ.

The "consumption-based price index" is defined as

$$P_t \equiv \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}} \tag{2}$$

The price index P_t denotes the minimum amount the household has to spend to obtain one unit of the composite good C_t defined as in (1). Maximizing the index (1) for a given level of consumption expenditure, the household allocates consumption across individual products according to

$$c_t^i(z) = \left[\frac{p_t(z)}{P_t} \right]^{-\theta} C_t^i. \tag{3}$$

Household i is the sole supplier of labour services h^i, and its objective is to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t(u(C_t^i;\xi_t)-v(h_t^i;\zeta_t))\right] \tag{4}$$

subject to a demand schedule for its labour services and the budget constraint

$$E_t[\delta_{t,t+1}A_{t+1}^i]\leq A_t^i+W_t^ih_t^i+\Pi_t-P_tC_t^i \tag{5}$$

Within each period, the household derives utility $u(\cdot;\xi_t)$ from consumption C_t^i as defined in (1), while supplying hours h_t^i reduces utility, as indicated by the function $v(\cdot;\zeta_t)$. In the budget constraint, P_t denotes the price index defined in (2), and A_t denotes the nominal value of the household's holdings of financial assets at the beginning of period t. W_t^i is the hourly wage that household i charges, and Π_t the household's share in firms' profits, which we assume are distributed lump-sum to households. $\delta_{t,\tau}$ is a stochastic discount factor, pricing in period t assets whose payoffs are in period τ. Financial markets are assumed to be complete, and in particular there exists a riskless one-period nominal bond, the gross return on which is given by $R_t\equiv(E_t\delta_{t,t+1})^{-1}$. The stochastic disturbance ξ_t is interpreted as preference or "demand" shock, while ζ_t is a disturbance to labour supply. The household's choice variables are consumption and hours or, given the demand function for its labour services, its wage.

Firm z is the monopolistic supplier of good z, which it produces according to the production function

$$y_t(z)=e^{\eta_t}\bar{K}^aH_t(z)^{1-a} \tag{6}$$

where η_t denotes a stochastic technology disturbance, the capital stock employed by each firm is fixed at $\bar{K}$, and the firm's labour input is a CES aggregate of different households' labour services

$$H_t(z)=\left[\int_0^1 h_t^i(z)^{\frac{\phi-1}{\phi}}di\right]^{\frac{\phi}{\phi-1}} \tag{7}$$

The parameter $\phi>1$ characterizes the elasticity of substitution between the various types of labour services. The wage index W_t is defined as

$$W_t\equiv\left[\int_0^1(W_t^i)^{1-\phi}di\right]^{\frac{1}{1-\phi}} \tag{8}$$

Maximizing the index (7) for a given level of wage payments, firm z allocates demand for individual labour services according to

$$h_t^i(z)=\left[\frac{W_t^i}{W_t}\right]^{-\phi}H_t(z). \tag{9}$$

Aggregate demand for output is defined as $Y_t = C_t + G_t$, where $C_t \equiv \int_0^1 C_t^i di$, and G_t is an exogenously given component of demand for output, which is assumed to be determined one period ahead. Assuming that G_t is allocated across the different goods by maximizing an index defined analogous to the consumption index (1), the demand faced by firm z is given by

$$y_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} Y_t. \tag{10}$$

Analogously, by integrating (9) across firms, the demand for its labour services faced by household i is

$$h_t^i = \left[\frac{W_t^i}{W_t}\right]^{-\phi} H_t \tag{11}$$

where $H_t \equiv \int_0^1 H_t(z)dz$.

We now characterize households' utility-maximizing consumption and wage decisions, and firms' profit-maximizing price choices. Because we wish to use solution methods for linear rational expectations models, the equilibrium conditions we use are log-linear approximations to the exact, nonlinear first order conditions of households and firms. For reasons discussed in Woodford (1999a) the welfare analysis later on is facilitated by log-linearizing around the efficient steady state, i.e. the steady state corresponding to a situation without market power and nominal rigidities in goods and labour markets. The efficient steady state level of output is determined by the condition that households' marginal rate of substitution between labour and consumption equal marginal product of labour, i.e.

$$\frac{v_h(H(\bar{Y}); 0)}{u_c(\bar{Y} - \bar{G}; 0)} = (1-a)(\bar{Y}/\bar{K})^{-\frac{a}{1-a}} \tag{12}$$

where $\bar{Y}$ and $\bar{G}$ denote the steady state values of output and exogenous demand respectively. The presence of market power of households and firms implies that, absent some offsetting policy, the steady state output level is below this efficient level of output. To justify log-linearizing the exact equilibrium conditions around the efficient steady state, below we will have to assume that tax policies are in place which offset the inefficiencies caused by imperfect competition in goods and labour markets. Furthermore, we log-linearize around a steady state in which there is zero price and wage inflation.

Households are assumed to choose their consumption purchases two periods ahead, i.e. C_t^i is chosen in $t-2$.[5] The decision lag for consumption implies that the household's Euler

[5] Although this choice of decision lag is somewhat arbitrary, it is no more arbitrary than choosing to

equation takes the form

$$E_t u_c(C^i_{t+2};\xi_{t+2}) = E_t \lambda^i_{t+2} P_{t+2} \tag{13}$$

where λ^i_t denotes household i's marginal utility of income at date t. Since households are free to take investment decisions each period with immediate effect, λ_t has to satisfy

$$\lambda_t = \beta E_t[R_t \lambda_{t+1}] \tag{14}$$

Dropping the superscript i implicitly assumes that, because of complete markets, households insure themselves against all idiosyncratic risk, and therefore the path of consumption is identical across households. Let $\hat{\lambda}_t$ denote the percentage deviation of $\lambda_t P_t$ from its steady state value. Then the log-linear approximation of (14) is

$$\hat{\lambda}_t = E_t[\hat{R}_t - \pi_{t+1} + \hat{\lambda}_{t+1}] \tag{15}$$

$$= \sum_{T=t}^{\infty} E_t[\hat{R}_T - \pi_{T+1}] \tag{16}$$

where $\hat{R}_t$ is the percentage deviation of the interest rate from its steady state value consistent with zero inflation. The log-linear approximation of the Euler equation (13) is therefore

$$-\tilde{\sigma} E_t[\hat{C}_{t+2} - \tilde{\xi}_{t+2}] = \sum_{T=t+2}^{\infty} E_t[\hat{R}_T - \pi_{T+1}] \tag{17}$$

where $\hat{C}_t \equiv (C_t - \bar{C})/\bar{C}$ denotes the percentage deviation of consumption from its steady state value $\bar{C}$, $\tilde{\sigma} \equiv -u_{cc}(\bar{C})\bar{C}/u_c(\bar{C})$, and $\tilde{\xi}_t \equiv -(u_{c\xi}(\bar{C})/u_{cc}(\bar{C})\bar{C})\xi_t$ is the disturbance to the marginal utility of consumption.

Log-linearizing aggregate demand around the steady state yields

$$\hat{Y}_t = s_c \hat{C}_t + \tilde{G}_t \tag{18}$$

where $\hat{Y}_t \equiv (Y_t - \bar{Y})/\bar{Y}$, $\tilde{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, and $s_c \equiv \bar{C}/\bar{Y}$. By substituting from the log-linearized aggregate demand equation for C_t, the Euler equation can be written as

$$\hat{Y}_t = -\sigma^{-1} E_{t-2} \sum_{T=t}^{\infty} [\hat{R}_T - \pi_{T+1}] + \hat{G}_t \tag{19}$$

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specify our model at a quarterly frequency - or, for that matter, *any* frequency - in the absence of compelling evidence to the contrary. As in Rotemberg and Woodford (1997), we choose a two quarter lag to match the timing of the maximum impact of a monetary policy shock on output in our model to that in the VAR. Instead, we could introduce and estimate a free parameter that captures the average decision lag of households due to, e.g., time-to-build constraints.

where $\sigma \equiv \tilde{\sigma}/s_c \equiv -u_{cc}(\bar{C})\bar{Y}/u_c(\bar{C})$, and $\hat{G}_t \equiv \tilde{G}_t + s_c E_{t-2}\tilde{\xi}_t$. Equation (19) is the model's "IS equation".

The assumption for wage and price adjustment we use is Rotemberg and Woodford's (1997) variant of Calvo's (1983) staggered price setting. Each period a fraction $1-\lambda$ of households is chosen at random and independent of their individual histories, and is being offered the opportunity to set a new wage. Hence, from the perspective of an individual household, the wage set in period t applies with probability 1 in period t, with probability λ it applies in period $t+1$, with probability λ^2 in period $t+2$ and so forth. Rotemberg and Woodford assume furthermore that at the end of period $t-1$, a fraction γ^w of those households who choose a new wage can apply this wage beginning at date t, the remaining fraction $1-\gamma^w$ applies this wage beginning at date $t+1$. Let W_t^1 denote the wage chosen in $t-1$ by those households whose wage comes into effect in period t, and let W_t^2 denote the wage chosen in $t-2$ by those households whose wage comes into effect in t. The aggregate wage level is then given by

$$W_t = [\lambda W_{t-1}^{1-\phi} + (1-\lambda)\gamma^w (W_t^1)^{1-\phi} + (1-\lambda)(1-\gamma^w)(W_t^2)^{1-\phi}]^{\frac{1}{1-\phi}} \tag{20}$$

The wage W_t^1 is chosen to maximize

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left[\lambda_T(1+\tau_w)W_t^1\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T - v\left(\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T;\zeta_T\right)\right] \tag{21}$$

Since the wage chosen at the end of period $t-1$ will apply at time t with probability 1, at time $t+1$ with probability λ and so forth, the household discounts utility in future periods *conditional* on W_t^1 still applying by $(\lambda\beta)^{T-t}$. Marginal utility of income at any point in time is the same across households. Therefore, the household's utility from charging wage W_t^1 in period T is given by the product of marginal utility of income and earnings (the first term in brackets) less the disutility from supplying $(W_t^1/W_T)^{-\phi}H_T$, the number of hours demanded at wage W_t^1 and aggregate wages and hours W_T and H_T (the second term in brackets). τ_w denotes a subsidy for employment. By choosing $\tau_w = (\phi-1)^{-1}$, the effect of imperfect competition in labour markets on the steady state output level can be offset.

The first-order condition for W_t^1 can be expressed as

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T\left[v_h\left(\left(\frac{W_t^1}{W_T}\right)^{-\phi}H_T;\zeta_T\right) - \frac{\phi-1}{\phi}\lambda_T P_T(1+\tau_w)\frac{W_t^1}{P_T}\right] = 0. \tag{22}$$

Households choose their nominal wage in period $t-1$ such that the discounted sum of expected future real wages $(1+\tau_w)W_t^1/P_T$ equals the discounted sum of expected future marginal rates of substitution between consumption and leisure $v_h(h^1_{t,T};\zeta_T)/(\lambda_T P_T)$ times a markup $\frac{\phi}{\phi-1}$, where we used $h^1_{t,T}$ as shorthand for the number of hours supplied in period T at wage W^1_t.

In the Appendix we derive a log-linear approximation to this first-order condition. Using this log-linear approximation as well as the corresponding relation for W^2_t and the log-linear approximation of the wage index (20), we obtain the following law of motion for the rate of wage inflation $\pi^w_t \equiv \log(W_t/W_{t-1})$:

$$\pi^w_t = (1-\psi^w)E_{t-2}\pi^w_t + \psi^w\left[\kappa^w(\hat{Y}_t - \hat{Y}^w_t) - \frac{\kappa^w(1-a)}{\omega+\sigma(1-a)}(\hat{w}_t + \nu_{t-1}) + \beta E_{t-1}\pi^w_{t+1}\right]. \quad (23)$$

The parameter $\omega \equiv v_{hh}(\bar{H};0)\bar{H}/v_h(\bar{H};0)$ measures the elasticity of the disutility of labour supply at the steady state level of hours $\bar{H}$. The coefficient

$$\kappa^w \equiv \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}\;\frac{\omega+\sigma(1-a)}{(1+\phi\omega)(1-a)}$$

describes the elasticity of wage inflation with respect to the gap between actual output $\hat{Y}_t$ and

$$\hat{Y}^w_t \equiv \frac{1-a}{\omega+\sigma(1-a)}E_{t-1}\left[\frac{\omega}{1-a}\eta_t + \omega\tilde{\zeta}_t + \sigma\hat{G}_t\right], \quad (24)$$

the level of output consistent with stable wage inflation. The coefficient $\psi^w \equiv \gamma^w\lambda/(1-\gamma^w(1-\lambda))$ equals 1 for $\gamma^w = 1$, the case in which all wage adjustments are effective the following period. The term $\hat{w}_t \equiv \log(W_t/P_t)$ denotes the percentage deviation of the real wage from its steady state. Positive deviations of the real wage from steady state reduce wage inflation. Finally,

$$\nu_{t-1} \equiv E_{t-1}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) - E_{t-2}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1})$$

is the revision from $t-2$ to $t-1$ in expectations of the long-term real interest rate in period t. Such revisions reduce wage inflation because they raise the returns households expect from their future earnings.

Price adjustment by firms is modelled analogous to wage adjustment by households. Each period a fraction $1-\alpha$ of firms is chosen at random and independent of their individual histories, and is being offered the opportunity to adjust their price. At the end of period $t-1$, a fraction γ^p of those who choose a new price can apply this price beginning at date t, the remaining fraction $1-\gamma^p$ applies this price beginning at date $t+1$. Let p^1_t denote the price chosen in $t-1$ by those firms whose price comes into effect in period t, and let

p_t^2 denote the price chosen in $t-2$ by those firms whose price comes into effect in t. The aggregate price level is then given by

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha)\gamma^p (p_t^1)^{1-\theta} + (1-\alpha)(1-\gamma^p)(p_t^2)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{25}$$

The price p_t^1 is chosen to maximize

$$E_{t-1} \sum_{T=t}^{\infty} \alpha^{T-t} \delta_{t,T} \left[(1+\tau_p) p_t^1 \left(\frac{p_t^1}{P_T} \right)^{-\theta} Y_T - W_T \left(\left(\frac{p_t^1}{P_T} \right)^{-\theta} \frac{Y_T}{e^{\eta_T}} \right)^{\frac{1}{1-a}} \right] \tag{26}$$

Since the price chosen at the end of period $t-1$ will apply at time t with probability 1, at time $t+1$ with probability α and so forth, the firm discounts future profits *conditional* on p_t^1 still applying by $\alpha^{T-t}\delta_{t,T}$, where $\delta_{t,T}$ is the stochastic discount factor introduced in (5). The first term in brackets denotes revenues in period T at price p_t^1, the second term the firm's labour cost implied by the level of output that is demanded in period T at price p_t^1. τ_p denotes a subsidy for producing output. By choosing $\tau_p = (\theta-1)^{-1}$, the effect of imperfect competition in goods markets on the steady state output level can be offset.

The first-order condition with respect to p_t^1 can be written as

$$E_{t-1} \sum_{T=t}^{\infty} \alpha^{T-t} \delta_{t,T} \left(\frac{p_t^1}{P_T} \right)^{-\theta} Y_T \cdot \left[(1+\tau_p) p_t^1 - \frac{\theta}{\theta-1} (1-a)^{-1} e^{\frac{-\eta_T}{1-a}} W_T \left(\left(\frac{p_t^1}{P_T} \right)^{-\theta} Y_T \right)^{\frac{a}{1-a}} \right] = 0. \tag{27}$$

Firms set the price in period $t-1$ such that the price, adjusted for the subsidy, equals a weighted average of expected future marginal cost at the level of output demanded at price p_t^1, times a markup $\frac{\theta}{\theta-1}$.

A log-linear approximation to this first-order condition is derived in the Appendix. Using this log-linear approximation as well as the corresponding relation for p_t^2 and the log-linear approximation of the price index (25), the law of motion for the rate of price inflation $\pi_t \equiv \log(P_t/P_{t-1})$ is given by

$$\pi_t = (1-\psi^p) E_{t-2}\pi_t + \psi^p \left[\kappa^p (\hat{Y}_t - \hat{Y}_t^p) + \frac{\kappa^p(1-a)}{a} \hat{w}_t + \beta E_{t-1}\pi_{t+1} \right]. \tag{28}$$

The coefficient

$$\kappa^p \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{a}{1-a+\theta a}$$

denotes the elasticity of price inflation with respect to the gap between actual output $\hat{Y}_t$ and

$$\hat{Y}_t^p \equiv a^{-1} E_{t-1} \eta_t, \tag{29}$$

the level of output consistent with stable price inflation. The coefficient $\psi^p \equiv \gamma^p \alpha/(1 - \gamma^p(1-\alpha))$ equals 1 for $\gamma^p = 1$, the case in which all price adjustments are effective the following period. Unlike in the wage inflation equation, positive deviations of the real wage from steady state increase price inflation.

In addition to the IS and wage and price inflation equations, a fourth structural equation is necessary to determine the paths of the four endogenous variables $\{\hat{Y}_t, \pi_t, \pi_t^w, \hat{R}_t\}$. For the estimation of this model, monetary policy is assumed to be described by a feedback rule for the one-period nominal interest rate of the form

$$\hat{R}_t = \sum_{k=1}^{3} \mu_k \hat{R}_{t-k} + \sum_{k=0}^{2} \psi_k \hat{w}_{t-k} + \sum_{k=0}^{2} \phi_k \pi_{t-k} + \sum_{k=0}^{2} \theta_k \hat{Y}_{t-k} + \epsilon_t \tag{30}$$

To summarize, the model consists of the IS equation (19), the wage inflation equation (23), the price inflation equation (28), and the feedback rule for the interest rate (30). Except for stochastic disturbances, wage and price inflation in this model are predetermined one period ahead, output two periods. The structural disturbances of the model are $\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p$, and ϵ_t. The first three of these shocks are themselves predetermined one period ahead, and so are wage and price inflation and output. The model parameters are the structural parameters, $\beta, \sigma, \omega, a, \alpha, \theta, \gamma^p, \lambda, \phi, \gamma^w$, and the parameters of the feedback rule (30).

3 Estimation

This section discusses and presents results of estimation of the model parameters and the shocks. We adopt the estimation strategy of Rotemberg and Woodford (1997), which we motivate as an example of minimum-distance estimation. The estimation process has three steps. The first step is to construct and estimate a vector autoregression (VAR) for the model's four endogenous variables. This provides estimates of the interest rate rule (30). The second and third steps are to choose the model's structural parameters and structural shocks, respectively, based on subsets of the first and second moments of our data series as captured by the VAR. In particular, the structural parameters are chosen so that the responses of the endogenous variables in the model to an exogenous monetary policy shock,

ϵ_t, match as closely as possible the responses estimated from the VAR. Given the estimates of the VAR and the structural parameters, the shock processes are chosen so that the model responses of output, inflation, and the real wage to perturbations in the three unidentified shocks in the VAR match exactly the responses of those variables in the VAR to the shocks.[6] We elaborate on this approach below before turning to a discussion of the results. First, we describe our data.

3.1 Data

Our data set is for the United States. It is comprised of quarterly observations on real (chain-weighted) GDP, the GDP deflator, compensation per hour in the nonfarm business sector, and the federal funds rate.[7] Because we wish to identify the historical interest rate rule from the VAR, it is important that the VAR be estimated over a sample period in which policy can be characterized by an interest rate rule with constant coefficients. Several empirical studies of U.S. monetary policy have identified a change in policy behaviour around the beginning of the Volcker chairmanship in 1979 (e.g. Clarida et al., 1998). By contrast, policy since the disinflation of the early 1980s has displayed a high degree of stability in the sense of being well described by a rule like (30). We therefore choose a sample period ranging from 1980:1 to 1997:4.

We present empirical results in terms of the real wage instead of wage inflation because we find impulse responses of the real wage more convenient to interpret and, in other work, the effects of monetary policy on wages is measured as effects on real wages, not wage inflation. Given our definition of variables in the previous subsection, the two are linked by $\pi_t^w = \hat{w}_t - \hat{w}_{t-1} + \pi_t$.

To express the data conformable with the theoretical series $\{\hat{Y}_t, \pi_t, \hat{w}_t, \hat{R}_t\}$ of the model, real GDP is logarithmized and a linear trend is removed, inflation is computed as log first differences of the GDP deflator, the real wage is computed as the logarithm of compensation per hour deflated by the GDP deflator and a linear trend is removed, and the federal funds rate is expressed at quarterly rate. Let $\{y_t, \pi_t, w_t, r_t\}$ denote these series, which are

[6] In actuality, since the shock processes must be specified in order to solve for the rational expectations equilibrium, and because the shocks are constructed from estimates of the structural parameters, the structural parameters and shocks are determined jointly.

[7] Quarterly values of the federal funds rate are computed as within-quarter averages of (effective) daily rates.

conformable with their theoretical counterparts up to a constant.

3.2 Identification and Estimation of the VAR

The theoretical model implies that, because they are predetermined, output, inflation, and the real wage are not contemporaneously affected by an interest rate innovation, while the form of the interest rate rule (30) allows for contemporaneous feedback from output, inflation, and the real wage to the interest rate. This is sufficient to identify the parameters of the historical interest rate rule and the series of interest rate innovations $\{\epsilon_t\}$. Let $Z_t = (r_t, w_{t+1}, \pi_{t+1}, y_{t+1})'$, and let $\bar{Z}_t = (Z_t', Z_{t-1}', Z_{t-2}')'$. The reason for defining Z_t in this manner is that the elements of Z_t all belong to the period t information set, since output, inflation, and the real wage are predetermined. The structural form of a VAR(3) in Z_t can then be written as

$$T\bar{Z}_t = m + A\bar{Z}_{t-1} + \bar{e}_t \tag{31}$$

where T is an identity matrix with a lower triangular 4 by 4 submatrix in the upper left corner, the first four rows of A contain coefficients, and the last eight rows of the VAR are identities. Accordingly, the last eight elements of $\bar{e}_t$ are zeros. The first four elements are mutually orthogonal, so that the first four diagonal elements of the covariance matrix V of $\bar{e}_t$ are distinct from zero, and all remaining elements of V are zero. Under our identifying assumption, the first row of A contains the coefficients of the historical interest rate rule (30), and the first element of $\bar{e}_t$ is ϵ_t.[8]

The recursive structure of the VAR allows us to estimate the equations in (31) by OLS. Table 1 shows estimates of the reaction function (30). While it is difficult to interpret estimates from a reduced form equation such as (30), it is worthwhile to point out that the sum of coefficients on lagged federal funds rates is 0.65, implying that monetary policy exhibited a great deal of inertia over this period.[9] As we will see below, and for essentially the reasons articulated by Woodford (1999b), an *even greater* degree of inertia in interest rate setting is desirable. The solid lines in the panels of Figure 1 shows the estimated

[8] As pointed out above, the series $\{y_t, \pi_t, w_t, r_t\}$ are conformable with their theoretical counterparts up to constants. By including the constant m in the VAR, the coefficients in the first row of A can be interpreted as the coefficients in (30).

[9] Our estimate of inertial policy *does not* necessarily imply that the Federal Reserve had a smoothing motive in setting interest rates. The lagged interest rates could simply be proxies for missing variables which themselves exhibit a high degree of serial correlation (see Amato and Laubach, 1999).

Table 1: Estimates of Reaction Function in VAR

μ_1	0.49	μ_2	-0.04	μ_3	0.2
ψ_0	-0.06	ψ_1	-0.02	ψ_2	0.01
ϕ_0	0.08	ϕ_1	0.04	ϕ_2	0.56
θ_0	0.59	θ_1	0.04	θ_2	-0.46
$\sum_{k=1}^{3} \mu_k$	0.65	σ_ϵ	0.78		

impulse responses of output, inflation, the real wage, and the interest rate to an exogenous increase in the interest rate of one percent. The dashed lines are two standard deviation confidence intervals.[10] Due to our identifying assumption, output, inflation, and the real wage do not respond during the quarter of the interest rate innovation. Output hardly responds during the following quarter, then falls in the second quarter after the innovation, before gradually returning to its original level over the following eight quarters. Inflation initially reacts faster than output to the innovation, but then oscillates between negative and near zero values, returning more slowly to its original level. The real wage responds very little at first, before turning negative in quarters three and four after the shock and positive thereafter, again slowly returning to its original level.[11] A caveat to these results is that the uncertainty around all four impulse response functions is considerable, although the wide confidence intervals do not necessarily imply that all parameter estimates based on matching the impulse response functions will be similarly uncertain.

3.3 Estimation of Structural Parameters

The information about the second moments of the series is summarized in the matrices T, A, and V. To illustrate how the structural parameters and shocks are obtained from estimates of these matrices, some notation may be helpful. Let L_1 denote a quadratic form describing the differences between the four variables' impulse responses to a monetary policy shock

[10] The algorithm used for bootstrapping the confidence intervals is the one of Berkowitz and Kilian (1996).

[11] Remarkably little work has been done on the response of wages to exogenous monetary policy shocks. Two notable exceptions are Christiano et al. (1997) and Leeper et al. (1996). Using different identifying assumptions and econometric methods, both sets of authors nonetheless find a weak response of the real wage to a monetary policy shock similar to what we find up to five quarters after the shock. As in Figure 1, these studies also report periods in which the response is positive, while the hypothesis of no response in *any* period cannot be rejected at standard levels.

implied by the model and those estimated from the VAR, and let L_2 denote a quadratic form describing these same differences between the impulse responses to $\bar{e}_{2t}, \bar{e}_{3t}$, and $\bar{e}_{4t}$. Furthermore, let P_1 denote the vector of structural parameters, i.e. $\sigma, \omega, a, \alpha, \theta, \gamma^p, \lambda, \phi, \gamma^w$, and let P_2 denote the vector of parameters characterizing the processes $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$.[12] Note that the structural disturbances $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$, because of their interpretation as shocks to aggregate demand and potential output, do not necessarily have to be i.i.d., but may follow a more complex process.[13] With this notation, the problem of estimating the structural parameters and shock parameters can be described as

$$\min_{P_1, P_2} L_1(P_1, P_2) + L_2(P_1, P_2) \tag{32}$$

A first observation is that, if the structural disturbances $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$ are exogenous and orthogonal to $\{\epsilon_t\}$, the impulse responses to the monetary policy shock do not contain any information about the structural disturbances, and hence L_1 depends on P_1 only. Second, Rotemberg and Woodford show that for *any* choice of P_1, the structural disturbances can be chosen such as to perfectly match the impulse response functions to innovations in these shocks implied by the model to the VAR's impulse responses to $\bar{e}_{2t}, \bar{e}_{3t}$, and $\bar{e}_{4t}$, i.e.

$$\min_{P_2} L_2(P_1, P_2) = 0 \;\forall P_1.$$

These two observations suggest to estimate parameters P_1 by minimizing L_1, and then compute P_2 such that $L_2 = 0$. Furthermore, write $L_1(P_1) \equiv g(P_1; \mathbf{Y}_T)' g(P_1; \mathbf{Y}_T)$, where the distance function $g(\cdot, \cdot)$ is a vector-valued function containing the differences between the model's and the VAR's responses of all four endogenous variables to a monetary policy shock and $\mathbf{Y}_T$ contains a history of the data. Then the estimator of P_1 obtained from minimizing $L_1(P_1)$ is a minimum distance estimator with an identity weighting matrix.

Turning to identification of the structural parameters, note that β can be recovered from the first moments of the data. Since β^{-1} is the steady-state gross real rate of return in our model, and the average *ex post* real interest rate in our sample is one percent (on a quarterly basis), we set β equal to 0.99. Unfortunately, inspection of the model equations (19), (23), and (28), reveals that not all of the other parameters are separately identified.

[12] As discussed below, we can obtain an estimate of β from the first moments of the data as captured in the vector m.

[13] For illustrative purposes, the specification of P_2 presupposes that some assumption about the functional form of these processes has been made.

The three parameters α, θ, and γ^p appear in the model only through κ^p and ψ^p in the price inflation equation (28); therefore, at most two of these parameters can be estimated. Likewise, we can estimate only two of the three parameters λ, ϕ, and γ^w, since they appear in the model only through κ^w and ψ^w in the wage inflation equation (23). Based on several survey studies, we follow Rotemberg and Woodford by setting $\alpha \equiv 0.66$, which implies that prices remain unchanged on average for three quarters. Similarly, we impose $\lambda \equiv 0.66$. Although γ^p, γ^w, and ω are each identified (given values for α and λ), the ratio γ^w/γ^p and ω are not separately well-determined from the data. Since the value of ω has much stronger implications for the welfare analysis to follow, we fix $\gamma^w \equiv \gamma^p$, which has the interpretation of imposing equal measures of exogenous rigidity in prices and wages (under the assumption $\alpha \equiv \lambda$).[14] Of course, since it is ψ^p and ψ^w that are separately identified, and not γ^p and γ^w, fixing γ^w/γ^p is somewhat artificial. We could change the values for α and λ, thereby getting different estimates for γ^p and γ^w, without affecting the fit of our model. Finally, following Rotemberg and Woodford (1997), we set a equal to 0.25. Due to the presence of monopolistic competition, a equals one minus the product of labour's share in firm z and firm z's price markup. Our choice of a will prove to be consistent with a steady-state labour share of 0.63. Given these values, the remaining parameters $\sigma, \omega, \gamma^p, \kappa^p$, and κ^w are estimated by the minimum distance method described above. We seek to minimize the difference between the model's and the VAR's responses for all four endogenous variables during quarters 1 to 5 following a monetary policy shock in quarter 0.

The estimates for $\sigma, \omega, \gamma^p, \kappa^p$, and κ^w, and the implied values for γ^w, θ, and ϕ, are displayed in Table 2 (standard errors are in parentheses).[15] The estimate of σ implies an

[14]The unrestricted estimate of ω is -0.1 and γ^w/γ^p is 0.3. However, there is only a 0.1 percent difference between the objective attained in unrestricted estimation compared to imposing the restriction $\gamma^w/\gamma^p = 1$. The parameter γ^p is largely determined by the quarter one response of inflation to a shock, but the ratio γ^w/γ^p affects the entire path response of the real wage to shocks, as does the parameter ω. Increasing γ^w/γ^p *ceteris paribus* has the effect of strengthening the response of the real wage to a monetary shock, especially early on (since, in this case, wages respond more quickly to the shock than prices), while increasing ω *ceteris paribus* has the opposite profile on the response of the real wage, with a relatively bigger impact in later quarters (since workers are less willing to substitute labour over time, which is especially binding in the presence of sticky wages in the short term).

[15]Standard errors are calculated from the asymptotic covariance of the minimum distance estimator. An estimate of the covariance matrix of the distance function $g(P_1, \mathbf{Y}_T)$ can be obtained from the covariance matrix of the impulse response functions estimated from the VAR under the hypothesis that our structural model is correctly specified. As in the calculation of the standard errors displayed in Figure 1, we estimate

Table 2: Estimates of Structural Parameters

Parameter	*Estimate*	*Standard Error*
β	0.99	0.001
σ	0.26	0.09
ω	0.2	0.69
γ^p	0.56	0.11
κ^p	0.019	0.004
κ^w	0.035	0.012
θ	6.27	1.95
ϕ	8.48	6.03

elasticity of intertemporal substitution of consumption of 3.9. This is larger than what has been found in the non-durable consumption literature and what is typically assumed in the real-business cycle literature (e.g. values between one-half and one), but it is smaller than Rotemberg and Woodford's estimate of 6.25. However, since the variable C in our model - as in Rotemberg and Woodford's - proxies for all interest-rate sensitive components of output, and not just non-durable consumption, a value higher than two appears justified. The standard error of σ is 0.09, indicating that this value is fairly well-determined by the data, as one would expect from the closeness of fit of the model's output response to the VAR's.

If wages were flexible, our estimate of ω would imply a Frisch elasticity of labour supply of 5.0, which is about half the size of Rotemberg and Woodford's estimate.[16] The plausibility of our estimate is difficult to determine from the micro panel data literature, since the functional forms used in that literature are based on first-order conditions derived in a setting with flexible wages. Nonetheless, our estimate is only slightly larger than the highest estimate presented by Mulligan (1998). The standard error of ω is quite large, but standard-sized confidence intervals still rule-out a wide range of interesting cases.

this covariance matrix using the algorithm of Berkowitz and Killian (1996). The Jacobian of $g(P_1, \mathbf{Y}_T)$ with respect to P_1 is evaluated numerically at the parameter estimates.

[16]The elasticity of labour supply is not separately identified in Rotemberg and Woodford's model, even though a similar quantity implicitly appears in their parametrization. Instead, they derive an estimate of this elasticity based on their estimate of σ and calibrated values for a and the elasticity of the average real wage with respect to variations in output that are orthogonal to preference and technology shocks.

The estimate of κ^p implies a steady-state markup of prices over marginal cost of 19%, which is quite similar to Rotemberg and Woodford's value of 15%. Finally, the estimate of κ^w implies a steady-state markup of the real wage over the marginal rate of substitution of 13%, which, as with our estimate of the steady-state price markup, is neither so low nor so high to be regarded as implausible.

Figure 2 presents the impulse responses of the four endogenous variables to a monetary policy shock in the model (solid lines) and the VAR (dashed lines). Overall, over the first five quarters after the shock, the responses of the model closely match those of the VAR. The main discrepancies are in the inflation and real wage responses, primarily from the fact that the model cannot replicate, for any parameter values, the hump in inflation three to four quarters after the shock and the hump in the real wage two quarters after the shock.

3.4 Construction and Estimation of Shocks

Rotemberg and Woodford (1997) provide a convenient method for choosing the shock processes, given estimates of the structural parameters. Let $\tilde{Z}_t$ denote the model's predictions for the series of endogenous variables, while $\bar{Z}_t$ denotes the actual observations of the variables over the sample. Later, when the model is used for simulations, $\bar{Z}_t$ and $\tilde{Z}_t$ will clearly not coincide. For the purpose of estimating the model under the historical policy rule, however, one would wish $\tilde{Z}_t$ to match $\bar{Z}_t$ as closely as possible. The law of motion for $\bar{Z}_t$ can be obtained by premultiplying the VAR by T^{-1}, which yields the reduced form

$$\bar{Z}_t = b + B\bar{Z}_{t-1} + T^{-1}\bar{e}_t. \tag{33}$$

After quasi-differencing and leading by one period, the model's IS equation can be written as

$$M'\tilde{Z}_t = (M' + N')E_{t-1}\tilde{Z}_{t+1} + \hat{G}_{t+1} - E_{t-1}\hat{G}_{t+2} \tag{34}$$

while the equation for wage inflation, after leading by one period, can be written as

$$P'E_{t-1}\tilde{Z}_t + R'E_t\tilde{Z}_{t+1} = \hat{Y}^w_{t+1} + \frac{\sigma(1-a)}{\omega + \sigma(1-a)}E_t(\hat{G}_{t+2} - \hat{G}_{t+1}) \tag{35}$$

and, similarly, the equation for price inflation, after leading by one period, can be written as

$$V'E_{t-1}\tilde{Z}_t + W'E_t\tilde{Z}_{t+1} = \hat{Y}^p_{t+1} \tag{36}$$

where M, N, P, R, V, and W are vectors containing the structural parameters. Suppose it were possible to choose $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$ such that the law of motion for $\{\tilde{Z}_t\}$ implied by equations (34)-(36) coincides exactly with the law of motion for $\{\bar{Z}_t\}$ implied by (33). In this case, the model-consistent, i.e. rational expectations, in equations (34)-(36) coincide with the expectations implied by the VAR, i.e. $E_t \tilde{Z}_{t+k} = B^k \tilde{Z}_t$. Conversely, by substituting $\bar{Z}$ for $\tilde{Z}$ and the expectations implied by the VAR for the model-consistent expectations in equations (34)-(36), one can solve for the processes $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$ with the property that $\tilde{Z}_t = \bar{Z}_t \; \forall t$. In fact, given the identification of the series $\{\epsilon_t\}$ with $\{\bar{e}_{1t}\}$, all that is required is that the model's implied values for output and price and wage inflation match perfectly those in the data, since then the estimated interest rate rule implies that the model's predicted interest rate is also identical to the historical process. The processes that achieve this are given by

$$
\begin{aligned}
[\hat{G}_{t+1}, \hat{Y}_{t+1}^w, \hat{Y}_{t+1}^p]' &= C\bar{Z}_{t-1} + D\bar{e}_t \qquad (37)\\
C &= \begin{bmatrix} M' - N'B(I-B)^{-1} \\ P' + R'B - \frac{\sigma(1-a)}{\omega+\sigma(1-a)}(N'B - M'(I-B)) \\ V' + W'B \end{bmatrix} B \\
D &= \begin{bmatrix} M' \\ R'B + \frac{\sigma(1-a)}{\omega+\sigma(1-a)}(M'(I-B) + N'B^2(I-B)^{-1}) \\ W'B \end{bmatrix} U.
\end{aligned}
$$

Note that the processes $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$ so defined depend on the entire vector $\bar{e}_t$, which implies that they are not orthogonal to $\bar{e}_{1t}$. However, orthogonality of $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$ to the monetary policy shock is both a requirement of the theoretical model, as well as necessary for the model's implied impulse responses to a monetary policy shock to be independent of the structural disturbances. Hence, in constructing $\{\hat{G}_t, \hat{Y}_t^w, \hat{Y}_t^p\}$, $\bar{e}_{1t}$ is set to zero at all times. This prevents the model from perfectly replicating $\{\bar{Z}_t\}$, but the structural disturbances still have the property that the variables' responses to $\bar{e}_{2t}, \bar{e}_{3t}$, and $\bar{e}_{4t}$ implied by the model perfectly match those from the VAR. Hence, $L_2(P_1, P_2) = 0$ for any value of P_1.

A variable that will figure prominently in the welfare analysis in the next section is the Pareto-efficient level of output, Y_t^e, the value of output that would obtain under completely flexible prices and wages. In our model, the Pareto-efficient level of output, or *potential* *output*, is given by

$$\hat{Y}_t^e = \frac{\omega + \sigma(1-a)}{\omega + a + \sigma(1-a)} \hat{Y}_t^w + \left(1 - \frac{\omega + \sigma(1-a)}{\omega + a + \sigma(1-a)}\right) \hat{Y}_t^p \tag{38}$$

where $\hat{Y}_t^e$ is expressed as a percent deviation from $\bar{Y}$. More precisely, due to the two-quarter lag in implementing consumption decisions, the variable that figures explicitly in the subsequent welfare analysis is the (two-quarter-ahead) expectation of the output gap, $E_{t-2}(\hat{Y}_t - \hat{Y}_t^e)$. The standard deviation of the expected output gap over our sample is 3.70, considerably higher than that of expected detrended output (2.17), but smaller than that of expected efficient output (4.90), due to a strong positive correlation (0.7) between expected detrended and efficient output. Whether these features of output are desirable attributes of monetary policy depends upon what affects the welfare of the representative household. To this issue we now turn.

4 Welfare

Earlier we emphasized the fact that deriving a structural model from individual optimizing behaviour has the advantage that the coefficients in the resulting model equations have a structural interpretation and, if the model is correctly specified, should remain invariant under alternative policies. A second advantage of an optimization-based model is the ability to perform welfare comparisons between alternative policy rules, in that the representative household's lifetime utility provides a model-consistent evaluation criterion. This section provides an approximation to the lifetime utility of the representative household, expressed in terms of a weighted sum of the variances of the endogenous variables. This approximation facilitates the evaluation of the welfare consequences of alternative policies, which is the subject of the remainder of this paper.

4.1 An Expression for the Representative Household's Welfare

The criterion to be used for evaluating alternative policies is the representative household's welfare, which can be expressed as

$$W = E\left[u(C_t;\xi_t) - \int_0^1 v(h_t^i;\zeta_t)di\right] \tag{39}$$

This objective is a simple transformation of the unconditional expectation of the household's lifetime utility (4), where the expectation is taken over all possible histories prior to date zero. Due to the assumption of perfect insurance among households, consumption is identical across households, and hence the first term inside brackets in (39) does not have a household index attached. The second term in brackets is understood as an average over possible histories of households' opportunity to change their wages.

In the Appendix we derive a second-order Taylor approximation of (39) around the same steady state considered in the log-linear approximations in section 2. This second-order approximation has the advantage that it can be evaluated in terms of the log-linear approximations to the model's exact equilibrium conditions derived in section 2. Specifically, the approximation can be expressed as

$$\begin{aligned} W &= -\Omega \Big[var(\pi_t) + (\psi^{p^{-1}} - 1) var(\pi_t - E_{t-2}\pi_t) + (E\pi_t)^2 \\ &+ c_1 var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e]) \\ &+ c_2 \{ var(\pi_t^w) + (\psi^{w^{-1}} - 1) var(\pi_t^w - E_{t-2}\pi_t^w) + (E\pi_t^w)^2 \} \Big] \quad (40) \\ &= -\Omega[L + (1 + c_2)\bar{\pi}^2] \quad (41) \end{aligned}$$

where Ω, c_1 and c_2 are combinations of the model's parameters, and

$$\begin{aligned} L &= var(\pi_t) + (\psi^{p^{-1}} - 1) var(\pi_t - E_{t-2}\pi_t) + c_1 var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e]) \\ &+ c_2 \Big[var(\pi_t^w) + (\psi^{w^{-1}} - 1) var(\pi_t^w - E_{t-2}\pi_t^w) \Big] \quad (42) \end{aligned}$$

is the welfare loss associated with variability of the output gap and price and wage inflation. In transforming (40) to (41) we made use of the fact that, because the real wage is assumed stationary, $E(\pi_t^w)$ has to equal $\bar{\pi} \equiv E(\pi_t)$.[17] The coefficients c_1 and c_2 express the weights of output gap and wage inflation variability relative to price inflation variability in (42). For our parameter estimates, $c_1 = .007$ and $c_2 = .89$. The small value of c_1 implies that avoiding output variability, apart from those fluctuations caused by variations in the efficient level of output, is mostly undesirable.

The presence of the first moment $\bar{\pi}^2$ in (41) is due to the fact that even a constant, perfectly anticipated rate of inflation different from zero forces households and firms to adjust their wages and prices whenever they have the opportunity to do so. The implied dispersion of relative prices is welfare reducing because at any point in time the condition that the real wage equal the marginal rate of substitution is violated for most households,

[17] Because the second-order approximation (41) is taken around a steady state of zero wage and price inflation, the term $\bar{\pi}^2$ has to be small for the approximation to remain valid.

and likewise the condition that price equal marginal cost is violated for most firms. The first moment term is important once it is taken into account that nominal interest rates cannot fall below zero in an economy where non-interest-bearing money is held. Suppose a given interest rate policy implies an unconditional standard deviation $\sigma(R)$ for the nominal interest rate, and that under such a policy all realizations of the interest rate are confined to an interval of size $k\sigma(R)$ on each side of the steady state value $\bar{R}$. For the zero lower bound on nominal interest rates to hold at all times, $\bar{R} \geq k\sigma(R)$ has to hold. Since $\bar{R} = \bar{\pi} + \rho$, i.e. the steady state nominal interest rate equals the steady state inflation rate plus the steady state real interest rate, we have that $\bar{\pi} \geq k\sigma(R) - \rho$. This last inequality shows that a more volatile interest rate policy can only be implemented at the cost of a higher steady state inflation rate, which reduces welfare. In the results reported below, we take this constraint into account by minimizing the objective

$$W^R = -\Omega[L + (1 + c_2)(\max\{k\sigma(R) - \rho, 0\})^2] \qquad (43)$$

The values of k and ρ are set to 2.46 and 3.04% respectively, which have been obtained from the estimated VAR.

5 Simple Rules

Interest-rate rules that implement the optimal plan for some given objective are generally very complicated. Rotemberg and Woodford (1999) show that, for their model, rules confined to a few terms closely approximate the welfare achieved by unrestricted optimal plans. Also, because simple rules are more transparent, they are more likely to be inferred by private agents, thereby increasing the chance that a committed policy will reap its benefits. The form of simple rule we use is a generalization of Taylor's (1993) rule that includes feedback from wage inflation and lagged interest rates:

$$\hat{R}_t = a\pi_t + b\hat{Y}_t + c\pi_t^w + d\hat{R}_{t-1} \qquad (44)$$

This form of rule facilitates direct comparison with most recent analyses of simple rules, e.g. many of the papers in Taylor (1999a). In the following subsections, we consider in turn the properties of different special cases of (44), which are distinguished by either imposing specific values for the parameters a, b, c, and d or finding the optimal values for those parameters under the constraints of our estimated model. First, as a direct comparison to some of the optimal simple rules recommended in the contributions to the Taylor volume, we compute performance statistics under the prescriptions of four of the rules focused upon by Taylor (1999b) in the robustness analysis he provides. We then compute the optimal feedback parameters in a rule of the form (44).

In their analysis of simple interest rate rules based on a calibrated version of a model similar to ours, Erceg et al. (1999) argue that the monetary authority can nearly achieve the optimal plan through the simple policy of responding to inflation and the output gap. The existence of decision lags in our version of their model makes the two-quarter-ahead expectation of the output gap the relevant output variable in the welfare criterion, as shown in (42). Therefore, we also consider rules of the form:

$$\hat{R}_t = a\pi_t + bE_t(\hat{Y}_{t+2} - \hat{Y}^e_{t+2}) + c\pi^w_t + d\hat{R}_{t-1} \tag{45}$$

for the purpose of assessing whether, in the presence of a variance tradeoff, allowing the interest rate to respond to both inflation and the output gap (properly measured) can approximate the welfare optimal plan. Since we estimate the current output gap to be much more volatile than the expected output gap and because monetary policy cannot affect current output, including the expected output gap in (45) affords the best opportunity for output to play a non-trivial role in the simple rules we consider. One advantage of considering rules of the form (45) is that, in the presence of variance tradeoffs among these variables, we can assess whether the optimal relative weights in the rule match the relative weights given to these variables in the welfare objective. However, it should be noted, that from a practical perspective, it may be undesirable to adopt a rule that involves a response to the expected output gap. In light of the difficulty of estimating potential output, especially for the most recent observations, which are precisely the terms that would appear in the reduced form expression for the expected output gap, rules that respond to the output gap might suffer lower credibility and reduce the benefits gained from commiting to a simple rule. We therefore also consider special cases of (45) that omit responses to the output gap.

5.1 "Fixed" Taylor Rules

Taylor (1999b) undertakes a robustness analysis of five rules that emerge in the collected papers in Taylor (1999a) as being optimal under some set of conditions (e.g. model structure, parameter estimates). Our purpose here is to further investigate robustness of these rules in the context of our optimizing model with both sticky prices and wages. Our robustness analysis is interesting for the following reason. Among the five rules that Taylor considers, four emerge as (nearly) optimal from one class of models, while the fifth is a product of an entirely different class of model. The distinguishing characteristic between the model classes is not forward-looking behaviour *per se*. Rather, the two key differences are whether the model builds in exogenous inflation persistence (e.g. the Fuhrer-Moore model) and whether it includes an output-inflation variance tradeoff that is binding from a welfare perspective. In contrast to the other models and welfare criteria, Rotemberg and Woodford's (1999) model and welfare function do not contain these features. Consequently, Taylor finds Rotemberg and Woodford's optimal rule not to be robust when tested with the other models. Since we *do not* build in exogenous inflation persistence, but our model and welfare function *do* exhibit variance tradeoffs, it is interesting to examine which features are mainly responsible for the comparative results provided by Taylor.

The second through fifth columns of Table 3a present performance measures for our model economy under the rules analyzed by Taylor (therefore labelled as T_1 through T_4).[18] For comparison, the first column of the table (H) presents statistics under the historical rule estimated in the VAR.[19] The last column (T_5) shows statistics under a rule with the same coefficients as rule T_2, but with the two-quarter-ahead expected output gap replacing detrended output, i.e. a rule of the form (45). For each case, the table first presents the coefficients for the interest-rate rule, followed by the unconditional variances of the model's endogenous variables, the output gap, the two-quarter-ahead expected output gap, two-quarter ahead unexpected price inflation, and two-quarter-ahead unexpected wage inflation. The last three rows present our welfare statistics: our measure of welfare that disregards the zero lower bound on nominal interest rates, (42); the level of steady-state inflation necessary to avoid the zero lower bound for nominal interest rates to be binding; and our modified measure of welfare, (43). The variances of price and wage inflation and the interest rate are

[18] Here we do not consider the rule offered by Rotemberg and Woodford (1999). In the next section, we calculate the optimal parameters in (44) under the restriction that $c = 0$, which has the same form as Rotemberg and Woodford's rule.

[19] The variance of $E_{-2}(\hat{Y} - \hat{Y}^e)$ reported in Table 3 differs from the value reported in section 3.4. The latter is calculated using the actual (estimated) data series implied by our model and observations on the four endogenous variables; whereas, the statistic reported under H in Table 3 is calculated solely using the model and the estimated historical policy rule.

expressed in annualized percentage points, while the variance of output and the output gap are measured in percentage deviations from trend. To facilitate comparison, all variances, including those under the historical rule, have been computed under the assumption that no monetary policy shocks are present, i.e. $\epsilon_t = 0$ at all times.

One notable result in Table 3a is that historical policy is about as good as any of the Taylor rules from a welfare perspective, which contrasts with the results in Rotemberg and Woodford (1999) that non-trivial welfare gains can be achieved under either T_1 or T_2. Part of the reason is that our estimate of the variance of interest rates is relatively smaller in our longer sample. For instance, comparing H to T_1, the reductions in the variances of price and wage inflation achieved under T_1 from a strong response to inflation are offset by the lack of sufficient smoothing behaviour which results in more volatile interest rates and higher steady-state inflation than what was observed historically. Of course, more striking are comparisons to the studies which promoted rules T_1 to T_4 in the first place. As in Rotemberg and Woodford (1999), another notable result is that policies that do not involve smoothing (i.e. T_3 and T_4) perform substantially worse than the others since the consequent volatility in interest rates (which of course is found to be optimal in these cases) requires a high steady-state inflation rate.

Comparing T_5 to the other rules, the monetary authority can achieve a better welfare outcome by responding to the expected output gap instead of detrended output. This outcome occurs even with a vigorous output response that is seemingly unwarranted in view of the small weight on this term in the welfare function. Naturally, part of the reason for the improved outcome under this rule is a much lower variance for the expected output gap. Lower variances for price and wage inflation also help, which are largely responsible for the better performance of this rule relative to historical practice.

5.2 Optimal Simple Rules

In this subsection we calculate numerically the optimal coefficients in rules restricted to the simple class of the form (44) and (45). The first column in Table 3b (O_1) reports results for the best rule in the class of rules given by (44), i.e. when all four coefficients, a, b, c, and d are chosen to minimize the welfare objective (43). The remaining five columns of the table

Table 3: Statistics for Policy Rules

a. Historical and "Fixed" Taylor Rules

	H	T_1	T_2	T_3	T_4	T_5
a	–	3.00	1.20	1.50	1.50	1.20
b	–	0.80	1.00	0.50	1.00	1.00
c	–	–	–	–	–	–
d	–	1.00	1.00	–	–	1.00
$\text{var}(\hat{R})$	6.14	6.49	6.12	14.93	14.46	6.19
$\text{var}(\pi)$	2.00	0.60	1.60	6.26	5.75	0.39
$\text{var}(\pi^w)$	3.94	2.93	3.88	7.91	7.63	2.07
$\text{var}(\hat{Y})$	4.12	5.97	2.40	3.84	1.83	10.18
$\text{var}(E_{-2}(\hat{Y} - \hat{Y}^e))$	10.76	9.65	12.46	12.38	13.45	7.75
$\text{var}(\pi - E_{-2}\pi)$	0.43	0.24	0.35	0.59	0.59	0.24
$\text{var}(\pi^w - E_{-2}\pi^w)$	2.02	1.76	1.75	1.82	1.87	1.72
L	9.26	6.33	8.60	17.20	16.60	5.13
$\bar{\pi}$	3.05	3.23	3.05	6.47	6.31	3.08
W^R	26.89	26.01	26.14	96.18	91.92	23.04

b. Optimal Rules

	O_1	O_2	O_3	O_4	O_5	O_6
a	1.16	1.27	0.89	0.66	1.19	0.70
b	0.01	0.07	0.03	0.14	–	–
c	0.42	0.51	0.31	–	0.41	–
d	1.15	1.19	1.00	1.04	1.15	1.02
$\text{var}(\hat{R})$	1.59	1.59	1.65	1.58	1.59	1.57
$\text{var}(\pi)$	0.48	0.49	0.46	0.39	0.48	0.37
$\text{var}(\pi^w)$	1.77	1.76	1.77	2.12	1.77	2.19
$\text{var}(\hat{Y})$	13.87	13.65	13.72	11.14	14.34	12.52
$\text{var}(E_{-2}(\hat{Y} - \hat{Y}^e))$	11.37	10.97	10.86	8.95	11.43	9.63
$\text{var}(\pi - E_{-2}\pi)$	0.23	0.23	0.25	0.23	0.23	0.23
$\text{var}(\pi^w - E_{-2}\pi^w)$	1.35	1.37	1.45	1.69	1.34	1.68
L	4.92	4.90	4.98	5.26	4.92	5.35
$\bar{\pi}$	0.06	0.06	0.12	0.05	0.06	0.04
W^R	4.93	4.91	5.01	5.26	4.93	5.36

report results for different cases of (45).[20]

Rule O_2 places no restrictions on the four response coefficients. Rules O_3, O_4, and O_5 consider optimal simple rules in three interesting restricted cases of (45). Rule O_3 is the special case of $d = 1$. The results in Taylor (1999b) suggest that rules with $d > 1$ can lead to extremely explosive results in many of the models analyzed in that volume, whereas for some of those same models, a smoothing coefficient of one actually performs very well. We thus consider the welfare consequences of restricting d equal to one, which monetary policymakers may find to be a more robust strategy in the face of model uncertainty. The rules O_4 and O_5 correspond to cases when c and b are set to zero, respectively. Rule O_4 has the same form as Rotemberg and Woodford's optimal simple rule and is the natural benchmark to compare against T_5 (i.e. it is the optimized version of rules in the class that T_5 falls into). We include rule O_5 to assess the effects of excluding the output gap altogether. Finally, rule O_6 allows for responses to inflation and the lagged interest rate only. Results under this rule will allow us to assess whether restricting the response of interest rates in this manner - which is nearly optimal in optimizing models without variance tradeoffs - entails a substantial welfare loss in the presence of a variance tradeoff.

Some common observations can be made about the group of rules O_1 to O_6. As in Rotemberg and Woodford, each of the rules are characterized by very low (compared to historical standards) interest rate variability. The low variability is attributable to the high degree of interest rate inertia under all of these rules, and the fact that in our rational expectations model this degree of inertia is both anticipated by agents and credible. Furthermore, the steady-state inflation rate $\bar{\pi}$ induced by interest-rate variability (as discussed in section 4.1) is very small, indicating that the welfare gains from further stabilization that could be achieved by a more variable interest-rate policy are too small to warrant the concomitant increase in $\bar{\pi}$.[21] Also, as in Rotemberg and Woodford, the coefficient on the output term - whether it is detrended output in O_1, which is directly comparable, or the

[20] We consider special cases of (45) instead of (44) since, as we will see, O_2 results in a better welfare outcome than O_1.

[21] It is interesting to observe that even when we restrict d to be one, the variance of interest rates is much smaller than under rules T_1 and T_2. The reason there is such a large difference under O_2 is that the reponse to inflation is restrained and the response to output is negligible. That is, if we restrict the smoothing behaviour of the monetary authority, in parallel we should tone-down the feedback response to inflation and output to avoid the higher steady-state inflation rate that would be necessary to support more volatile interest rates.

expected output gap - is close to zero (it is actually smaller under O_1, 0.01, versus their value of 0.06). The variance of detrended output is much larger under any of these rules than under the historical one precisely because it is not detrended output that matters either for welfare or for predicting the endogenous variables that do matter for welfare. Interestingly, the variance of the expected output gap is generally not much different than observed historically, partly as a consequence of the low weight on this term in the welfare objective. Finally, the gain in welfare that is achieved by any of these rules is substantial compared to any of the rules analyzed in the previous subsection, including historical policy. The five-fold increase in welfare over historical policy, however, is not nearly as large as the almost fifteen-fold increase in welfare that Rotemberg and Woodford report for their more compact welfare function and model.

Turning to the rules individually, the unrestricted simple rules of both forms (O_1 or O_2) attain the best welfare outcomes. The similar welfare outcomes are attributable to the fact that the optimal response coefficients are similar, and most importantly for comparing to each other, that the responses to output are near zero.[22] It is interesting to note that the relative size of the optimal coefficients in O_2 does not exactly correspond to the relative weights given these variables in the welfare criterion. The lack of exact correspondance between the variables that appear in the welfare objective and those that we include in the simple rule, along with the nature of the variance tradeoff, does not allow a simple mapping from policymaker objective to instrument rule. The results for O_3 suggest that imposing the restriction $d = 1$ leads to only a small reduction in welfare (approximately two percent), which is sufficiently small to likely be offset by concern for adopting a potentially explosive regime. Furthermore, there is a unique and stable equilibrium under this rule even though the response coefficient on inflation is less than one because of the large size of d. For rule O_4, the coefficients on inflation and the lagged interest rate are smaller than Rotemberg and Woodford's, although our coefficients have qualitatively similar implications to theirs; namely, a strong response to inflation, a negligible response to output, and significant smoothing ($d > 1$). Particularly interesting is the response coefficient on inflation being smaller than one. The reason for this is simple: aggressive stabilization of price inflation in neglect of wage inflation stabilization leads to large welfare costs from highly disperse

[22] Since higher welfare is obtained under O_2, the remainder of the discussion focuses upon special cases of rules of the form (45).

labour supply in the face of volatile wage inflation. It is apparent that neglecting to respond to wage inflation can be costly. The welfare loss from this restricted rule compared to the unrestricted simple rule is 7.1 percent. For rule O_5, as one may expect, excluding output effects only a marginal deterioration in welfare and the optimal response coefficients on the other variables are similar to the unrestricted case. Lastly, eliminating an interest rate response to wage inflation as well as output (O_6) results in a non-trivial deterioration of welfare (9.2 percent), a situation that can easily be avoided especially since various measures of wage data are readily available to monetary policymakers. Thus, our main result for simple rules is: having the monetary authority adopt a highly inertial, though not necessarily explosive, interest rate policy (i.e. $d \approx 1$), which includes responses to price and wage inflation, is nearly optimal from a welfare perspective in the class of simple rules we consider.

6 Conclusions

In this paper we use an estimated version of a small dynamic equilibrium model with nominal rigidities in both product and labour markets to analyze optimal interest rate rules for monetary policy. Our estimates of key parameters, such as the intertemporal elasticity of substitution in consumption and the elasticity of labour supply, are close to the ranges of estimates obtained using disaggregated data. Based on our estimated model, we find that simple rules that include feedback responses from both price and wage inflation and exhibit smoothing, i.e. a large response to the past level of the interest rate, are nearly optimal from a welfare perspective. Furthermore, even in the presence of a variance tradeoff between price and wage inflation and the output gap, we do not find any significant role for the output gap in any rules we consider.

Both our estimation results and our analysis of interest rate rules suggest a number of possible avenues for future research. First, it may be profitable to use data on more labour market series in light of the relatively large standard errors we obtain for our estimates of the elasticity of labour supply and the degree of market power of workers. Second, a model with other labour market frictions, e.g. downward nominal wage rigidity, may help determine with more precision parameters that are crucial for the welfare analysis, e.g., the elasticity of labour supply. Alternatively, using our estimated standard errors, it would be interesting to consider the effects of parameter uncertainty on optimal interest rate rules.

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A Log-linear Approximations

A.1 Wage and Price Inflation

In this Appendix we derive equations (23) and (28). The first step is to compute a log-linear approximation to equation (22)). Let $\hat{v}_t^1 \equiv \log(W_t^1/W_t)$. The ratio W_t^1/W_T can then be approximated as $\hat{v}_t^1 - \sum_{k=1}^{T-t} \pi_{t+k}^w$. Similarly, the ratio

$$\frac{W_t^1}{P_T} = \frac{W_t^1}{W_t}\frac{W_t}{P_t}\frac{P_t}{P_T}$$

is approximated by $\hat{v}_t^1 + \hat{w}_t - \sum_{k=1}^{T-t} \pi_{t+k}$. Finally, using the production function (6), the deviation of hours from steady state can be expressed as $\hat{H}_t = \frac{1}{1-a}(\hat{Y}_t - \eta_t)$.

With this notation, the log-linear approximation of (22) can be written as

$$E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\left\{\omega\left[\frac{\hat{Y}_T - \eta_T}{1-a} - \tilde{\zeta}_T - \phi(\hat{v}_t^1 - \sum_{k=1}^{T-t}\pi_{t+k}^w)\right] - \hat{\lambda}_T - (\hat{v}_t^1 + \hat{w}_t - \sum_{k=1}^{T-t}\pi_{t+k})\right\} = 0, \quad (46)$$

where $\tilde{\zeta}_t \equiv -(v_{h\zeta}(\bar{H};0)/v_{hh}(\bar{H};0)\bar{H})\zeta_t$ is the disturbance to the marginal disutility of labour supply. Combining (16) and (19) yields

$$E_{t-1}\hat{\lambda}_T = -\sigma E_{t-1}[\hat{Y}_T - \hat{G}_T] \; \forall T \geq t+1 \quad (47)$$

while taking expectations as of $t-1$ of (15) yields

$$\begin{aligned} E_{t-1}\hat{\lambda}_t &= E_{t-1}[\hat{R}_t - \pi_{t+1} + \hat{\lambda}_{t+1}] \\ &= E_{t-1}[\hat{R}_t - \pi_{t+1} - \sigma(\hat{Y}_{t+1} - \hat{G}_{t+1})] \\ &= -\sigma E_{t-1}[\hat{Y}_t - \hat{G}_t] + \nu_{t-1} \quad (48) \end{aligned}$$

where

$$\begin{aligned} \nu_{t-1} &\equiv E_{t-1}[R_t - \pi_{t+1} - \sigma(\hat{Y}_{t+1} - \hat{Y}_t - \hat{G}_{t+1} + \hat{G}_t)] \\ &= E_{t-1}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) - E_{t-2}\sum_{T=t}^{\infty}(\hat{R}_T - \pi_{T+1}) \end{aligned}$$

Substituting these expressions for $E_{t-1}\hat{\lambda}_T$ into (46) and collecting terms, (46) can be written as

$$\begin{aligned} E_{t-1}\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\Bigg\{&\left(\frac{\omega}{1-a} + \sigma\right)\hat{Y}_T - \frac{\omega}{1-a}\eta_T - \omega\tilde{\zeta}_T - \sigma\hat{G}_T \\ &-(1+\omega\phi)\hat{v}_t^1 + \omega\phi\sum_{k=1}^{T-t}\pi_{t+k}^w - \hat{w}_t + \sum_{k=1}^{T-t}\pi_{t+k})\Bigg\} - \nu_{t-1} = 0. \quad (49) \end{aligned}$$

Furthermore, we transform the double summation

$$\begin{aligned} \sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\sum_{k=1}^{T-t}\pi_{t+k} &= \sum_{T=t+1}^{\infty}(\lambda\beta)^{T-t}\sum_{k=0}^{\infty}(\lambda\beta)^k\pi_T \\ &= (1-\lambda\beta)^{-1}\left(\sum_{T=t}^{\infty}(\lambda\beta)^{T-t}\pi_T - \pi_t\right) \end{aligned}$$

The double sum involving π_t^w is transformed analogously.

We next wish to obtain an expression for $\hat{v}_t^1$ in terms of π_t^w. Dividing both sides of (20) by W_t and taking the logarithm yields

$$0 \simeq (1-\lambda)\gamma^w \hat{v}_t^1 + (1-\lambda)(1-\gamma^w)\hat{v}_t^2 - \lambda\pi_t^w \tag{50}$$

Since $W_t^2 = E_{t-2}W_t^1$,

$$\hat{v}_t^2 = E_{t-2}\hat{v}_t^1 - (\pi_t^w - E_{t-2}\pi_t^w) \tag{51}$$

Substituting this expression into (50) we obtain

$$\pi_t^w = \frac{1-\lambda}{\lambda}\left[\gamma^w \hat{v}_t^1 + (1-\gamma^w)(E_{t-2}\hat{v}_t^1 - (\pi_t^w - E_{t-2}\pi_t^w))\right] \tag{52}$$

Taking expectations as of $t-2$ on both sides, $E_{t-2}\pi_t^w = \frac{1-\lambda}{\lambda}E_{t-2}\hat{v}_t^1$ and hence

$$\frac{1-\lambda}{\lambda}\hat{v}_t^1 = \frac{1}{\psi^w}\pi_t^w - \frac{1-\psi^w}{\psi_w}E_{t-2}\pi_t^w \tag{53}$$

where $\psi^w \equiv \gamma^w\lambda/(1-\gamma^w(1-\lambda))$ is defined as in (23). Substituting (53) for $\hat{v}_t^1$ in (49) and using the transformation for the double sums and the fact that $E_{t-1}\nu_{t+j} = 0\ \forall j \geq 0$ we obtain (23).

The derivation of (28) involves the same steps as above. Let $\hat{p}_t^1 \equiv \log(p_t^1/P_t)$. Then (27) can be approximated as

$$E_{t-1}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left\{\frac{1-a+\theta a}{1-a}\hat{p}_t^1 - \frac{1}{1-a}(a\hat{Y}_T - \eta_T) - \hat{w}_t - \sum_{k=1}^{T-t}\pi_{t+k}^w - \frac{\theta a}{1-a}\sum_{k=1}^{T-t}\pi_{t+k}\right\} = 0. \tag{54}$$

The double sums in (54) are being transformed as before. Furthermore, dividing (25) by P_t and taking the logarithm, and using the fact that $p_t^2 = E_{t-2}p_t^1$, we can derive an expression for $\hat{p}_t^1$ in terms of π_t analogous to (53),

$$\frac{1-\alpha}{\alpha}\hat{p}_t^1 = \frac{1}{\psi^p}\pi_t - \frac{1-\psi^p}{\psi_p}E_{t-2}\pi_t \tag{55}$$

where $\psi^p \equiv \gamma^p\alpha/(1-\gamma^p(1-\alpha))$ is defined as in (28). Substituting (55) for $\hat{p}_t^1$ in (54) and using the transformation for the double sums we obtain (28).

A.2 The Representative Household's Welfare

In this Appendix we derive the second-order approximation (40) to the representative household's welfare (39), using some results of Rotemberg and Woodford's (1997) Appendix 3. Specifically, we form a second-order Taylor series expansion of (39) around the steady state characterized by the efficient output level $\bar{Y}$ defined in (12) and zero wage and price inflation. Hence, we form the approximation around the same steady state around which the model's exact equilibrium conditions have been log-linearized.

Since the demand side of our model is identical to Rotemberg and Woodford's, the second-order approximation of $u(C_t;\xi_t)$ is identical to their equation (9.10) as well, which we reproduce here:

$$u(C_t;\xi_t) = u_c\bar{Y}\hat{Y}_t + \frac{1}{2}(u_c\bar{Y} + u_{cc}\bar{Y}^2)\hat{Y}_t^2 - u_{cc}\bar{Y}^2\hat{G}_t\hat{Y}_t + unf + tip + \mathcal{O}(||\xi||^3) \tag{56}$$

where unf stands for terms that are unforecastable two periods ahead (since in our model monetary policy affects output only with a lag of two periods), and tip denotes terms that are independent of monetary policy. $||\xi||$ is a bound on the amplitude of fluctuations in the exogenous disturbances, which we take to be the same for ξ, ζ, and η. The term $\mathcal{O}(||\xi||^3)$ indicates that terms of third or higher order in the deviations of the various variables from their steady-state values are being neglected.

Similarly, a second-order approximation of household i's disutility of labour supply is given by

$$v(h_t^i;\zeta_t) = v_h\bar{H}\hat{h}_t^i + \frac{1}{2}(v_h\bar{H} + v_{hh}\bar{H}^2)\hat{h}_t^{i2} - v_{hh}\bar{H}^2\tilde{\zeta}_t\hat{h}_t^i + tip + \mathcal{O}(||\xi||^3). \tag{57}$$

Integrating this expression over i yields

$$\int_0^1 v(h_t^i;\zeta_t)di = v_h\bar{H}E_i[\hat{h}_t^i] + \frac{1}{2}(v_h\bar{H} + v_{hh}\bar{H}^2)\left(E_i[\hat{h}_t^i]^2 + var_i(\hat{h}_t^i)\right) - v_{hh}\bar{H}^2\tilde{\zeta}_t E_i[\hat{h}_t^i] + tip + \mathcal{O}(||\xi||^3). \tag{58}$$

By integrating (7) over z, we obtain

$$H_t = \left[\int_0^1 (h_t^i)^{\frac{\phi-1}{\phi}} di\right]^{\frac{\phi}{\phi-1}}. \tag{59}$$

Using this expression and the fact that for a random variable X, $\log E[X] = E[\log X] + \frac{1}{2}var(\log X)$, we obtain that

$$\hat{H}_t \equiv \log(H_t/\bar{H}) = E_i[\hat{h}_t^i] + \frac{\phi-1}{2\phi}var_i(\hat{h}_t^i). \tag{60}$$

Solving (60) for $E_i[\hat{h}_t^i]$ and substituting in (58) yields

$$\int_0^1 v(h_t^i;\zeta_t)di = v_h\bar{H}\hat{H}_t + \frac{v_h\bar{H}}{2}(1+\omega)\hat{H}_t^2 + \frac{v_h\bar{H}}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i) - v_{hh}\bar{H}^2\tilde{\zeta}_t\hat{H}_t + tip + \mathcal{O}(||\xi||^3) \tag{61}$$

where ω is defined as in (23).

We next wish to substitute for $\hat{H}_t$ in (61) in terms of output. To do so, note first that the definition of $H_t = \int_0^1 H_t(z)dz$ implies that

$$\hat{H}_t = E_z[\hat{H}_t(z)] + \frac{1}{2}var_z(\hat{H}_t(z)). \tag{62}$$

Firms' production function in turn implies that

$$E_z[\hat{H}_t(z)] = (1-a)^{-1}(E_z[\hat{y}_t(z)] - \eta_t),\ var_z(\hat{H}_t(z)) = (1-a)^{-2}var_z(\hat{y}_t(z)) \tag{63}$$

and therefore

$$\hat{H}_t = (1-a)^{-1}(E_z[\hat{y}_t(z)] - \eta_t) + \frac{1}{2(1-a)^2} var_z(\hat{y}_t(z)). \tag{64}$$

Finally, deriving an expression for $\hat{Y}_t$ analogous to (62), substituting from this expression for $E_z[\hat{y}_t(z)]$ in (64), and substituting the resulting expression for $\hat{H}_t$ into (61) yields

$$\int_0^1 v(h_t^i;\zeta_t)di = \frac{v_h \bar{H}}{1-a}\left[\hat{Y}_t + \frac{1+\omega}{2(1-a)}\hat{Y}_t^2\right] - \frac{v_h \bar{H}}{1-a}\left[\omega\tilde{\zeta}_t\hat{Y}_t + \frac{1+\omega}{1-a}\eta_t\hat{Y}_t\right]$$
$$+\frac{v_h \bar{H}}{1-a}\left[\frac{1}{2}\left(\frac{1}{1-a} - \frac{\theta-1}{\theta}\right) var_z(\hat{y}_t(z)) + \frac{1-a}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i)\right] + tip + \mathcal{O}(\|\xi\|^3). \tag{65}$$

Because the efficient steady-state level of output is characterized by (12), it follows that

$$\frac{v_h \bar{H}}{1-a} = u_c \bar{Y}.$$

Hence,

$$u(C_t;\xi_t) - \int_0^1 v(h_t^i;\zeta_t)di = u_c\bar{Y}\left[\frac{\omega + a + \sigma(1-a)}{1-a}\left(\hat{Y}_t\hat{Y}_t^e - \frac{1}{2}\hat{Y}_t^2\right)\right.$$
$$\left. -\frac{1}{2}\left(\frac{1}{1-a} - \frac{\theta-1}{\theta}\right) var_z(\hat{y}_t(z)) - \frac{1-a}{2}(\phi^{-1}+\omega)var_i(\hat{h}_t^i)\right] + tip + \mathcal{O}(\|\xi\|^3) \tag{66}$$

where $\hat{Y}_t^e$ is the efficient level of output defined in (38). Taking the unconditional expectation of (66) then leads to an expression for (39) of the form

$$W = -\frac{u_c\bar{Y}}{2}\left[\frac{\omega + a + \sigma(1-a)}{1-a}\left(E[\hat{Y}_t^2] - 2E[\hat{Y}_t\hat{Y}_t^e]\right)\right.$$
$$\left. + \left(\frac{1}{1-a} - \frac{\theta-1}{\theta}\right) E[var_z(\hat{y}_t(z))] + (1-a)(\phi^{-1}+\omega)E[var_i(\hat{h}_t^i)]\right] + tip + \mathcal{O}(\|\xi\|^3). \tag{67}$$

We now wish to substitute for each of the three terms involving unconditional expectations in (67). First, rearranging the definition of $var(\hat{Y}_t - \hat{Y}_t^e)$ yields

$$E[\hat{Y}_t^2] - 2E[\hat{Y}_t\hat{Y}_t^e] = var(\hat{Y}_t - \hat{Y}_t^e) + E[\hat{Y}_t]^2 - E[\hat{Y}_t^{e2}] + E[\hat{Y}_t^e]^2 - 2E[\hat{Y}_t]E[\hat{Y}_t^e] \tag{68}$$

The second and last terms on the right-hand side of (68) are zero because the unconditional expectation of output from its long-run trend is zero by definition. The third and fourth terms equal $-var(\hat{Y}_t^e)$, a term that is independent of policy. Hence, in (67) we can substitute $var(\hat{Y}_t - \hat{Y}_t^e)$ for the left-hand side of (68). Taking account of the fact that interest rates affect output only with two periods lag, we instead susbtitute $var(E_{t-2}[\hat{Y}_t - \hat{Y}_t^e])$ in (67).

Second, from the demand functions for households' labour services (9) and producers' goods (10) it follows that

$$E[var_i(\hat{h}_t^i)] = \phi^2 E[var_i(\log W_t^i)] \tag{69}$$

and

$$E[var_z(\hat{y}_t(z))] = \theta^2 E[var_z(\log p_t(z))]. \tag{70}$$

Following the argument in Rotemberg and Woodford's Appendix 3, these equations can be rewritten as

$$E[var_i(\hat{h}_t^i)] = \phi^2 \frac{\lambda}{(1-\lambda)^2} \left[var(\pi_t^w) + (\psi^{w^{-1}} - 1) var(\pi_t^w - E_{t-2}\pi_t^w) + (E\pi_t^w)^2 \right] \tag{71}$$

and

$$E[var_z(\hat{y}_t(z))] = \theta^2 \frac{\alpha}{(1-\alpha)^2} \left[var(\pi_t) + (\psi^{p^{-1}} - 1) var(\pi_t - E_{t-2}\pi_t) + (E\pi_t)^2 \right] \tag{72}$$

where ψ^w and ψ^p are defined as in (23) and (28) respectively. Substituting (68), (71), and (72) into (67) and noting that

$$(1-a)(\phi^{-1} + \omega)\phi^2 \frac{\lambda}{(1-\lambda)^2} = \frac{1-\lambda\beta}{(1-\lambda)\kappa^w} \phi((1-a)\sigma + \omega)$$

and

$$\left(\frac{1}{1-a} - \frac{\theta - 1}{\theta} \right) \theta^2 \frac{\alpha}{(1-\alpha)^2} = \frac{1-\alpha\beta}{(1-\alpha)\kappa^p} \frac{\theta a}{1-a}$$

we obtain (40), where

$$\Omega \equiv \frac{u_c \bar{Y}}{2} \frac{1-\alpha\beta}{(1-\alpha)\kappa^p} \frac{\theta a}{1-a}$$

$$c_1 \equiv \left[\frac{1-\alpha\beta}{(1-\alpha)\kappa^p} \frac{\theta a}{1-a} \right]^{-1} \frac{\omega + a + \sigma(1-a)}{1-a}$$

and

$$c_2 \equiv \left[\frac{1-\alpha\beta}{(1-\alpha)\kappa^p} \frac{\theta a}{1-a} \right]^{-1} \frac{1-\lambda\beta}{(1-\lambda)\kappa^w} \phi((1-a)\sigma + \omega).$$

Figure 1: VAR Impulse Responses to a Monetary Shock

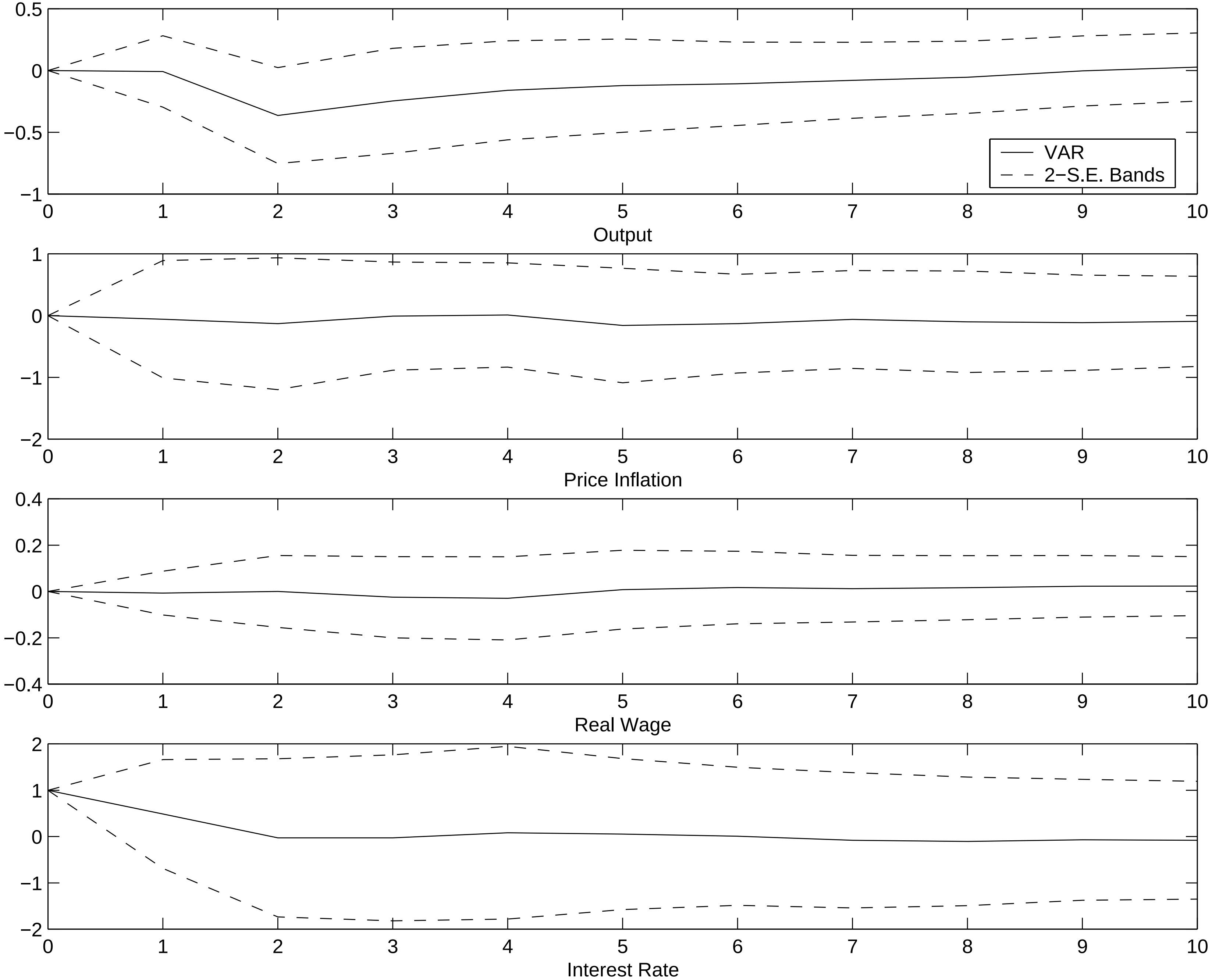

Figure 2: VAR and Model Impulse Responses to a Monetary Shock