PCE and CPI Inflation Differentials: Converting Inflation Forecasts
Model Specification

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This specification describes the models that are used to forecast the inflation differential. The 14 forecasts generated with these models provide different ways to allow the inflation differential to change over time.

The Average Differential Model

Forecasts 1–3 are the simplest since they are simply the average inflation differential estimated over three different time periods. Three sample periods—of varying length—are simple ways of allowing the average differentials to change over time. The sample periods are: 1995q1 – 2007q2, 2000q1 – 2007q2, and 2002q3 – 2007q2 (used in Chart 3). The three forecasts are called "Average, 1995q1," "Average, 2000q1," and "Average, 2002q3."

The estimated differential and RMSE for the three sample periods for the total and core inflation differential are:

	1995q1 – 2007q2	**2000q1 – 2007q2**	**2002q3 – 2007q2**
Total inflation differential	.525	.425	.341
RMSE	.541	.651	.620
Core inflation differential	.486	.278	.052
RMSE	.526	.553	.366

The Autoregressive Model

Forecasts 4–6 are based on an AR process for the inflation differential estimated over the same three sample periods. The AR model is slightly more sophisticated than the average differential model. The inflation differential is regressed against a constant and the lagged inflation differential. The lag length is chosen by AIC. The three forecasts are called "AR, 1995q1," "AR, 2000q1," and "AR, 2002q3."

The AR model can be written as:

$$\pi_t^c - \pi_t^P = \alpha + \sum_{t=1}^{K} \beta_k (\pi_{t-i}^c - \pi_{t-i}^P) + \varepsilon_t \qquad (3)$$

where the lag length is chosen optimally (using AIC).

The parameter estimates for the three sample periods for the total and core inflation differential are:

	1995q1 – 2007q2	**2000q1 – 2007q2**	**2002q3 – 2007q2**
Total inflation differential			
α	.385 (.143)	.326 (.135)	.454 (.150)
β_1	.278 (.143)	.255 (.190)	.098 (.232)
β_2			-.508 (.232)
RMSE	.527	.643	.588
Core inflation differential			
α	.195 (.081)	.144 (.097)	.033 (.070)
β_1	.594 (.114)	.488 (.160)	.498 (.203)
RMSE	.427	.492	.330

The Vector Autoregressive Model

Forecasts 7–9 are derived from a VAR process for CPI and PCE inflation estimated over the same three sample periods. The VAR is used to forecast future values for CPI and PCE inflation and then calculate the differential. The lag length is chosen by AIC. The three forecasts are called "VAR, 1995q1," "VAR, 2000q1," and "VAR, 2002q3."

The VAR model can be written as:

$$\pi_t^c = \alpha^c + \sum_{i=1}^{K} \left(\beta_i^c \pi_{t-i}^c + \beta_i^P \pi_{t-i}^P \right) + \varepsilon_t^c \qquad (4)$$

$$\pi_t^p = \alpha^p + \sum_{i=1}^{K} \left(\gamma_i^c \pi_{t-i}^c + \gamma_i^p \pi_{t-i}^p \right) + \varepsilon_t^p$$

The parameter estimates for the three sample periods for the total and core inflation differential are:

	1995q1 – 2007q2	2000q1 – 2007q2	2002q3 – 2007q2
Total inflation differential			
α^c	2.179 (.433)	2.439 (.681)	2.196 (1.146)
β_1^c	-.308 (.413)	-.209 (.589)	-1.457 (1.246)
β_1^p	.604 (.521)	.402 (.785)	1.905 (1.702)
α^p	1.761 (.325)	2.171 (.495)	2.029 (.787)
γ_1^c	-.600 (.310)	-.434 (.429)	-1.380 (.856)
γ_1^p	.916 (.392)	.596 (.570)	1.751 (1.169)
Maximum eigenvalue (modulus)	.413	.222	.282
RMSE	.532	.654	.651
Core inflation differential			
α^c	1.050 (.294)	1.050 (.389)	.748 (.511)
β_1^c	.638 (.130)	.581 (.179)	.170 (.344)
β_1^p	-.123 (.142)	-.070 (.194)	.468 (.442)
α^p	1.394 (.323)	1.680 (.429)	1.232 (.435)
γ_1^c	-.068 (.143)	-.065 (.197)	-.293 (.293)
γ_1^p	.316 (.155)	.199 (.213)	.671 (.378)
Maximum eige value (modulus)	.662	.593	.501
RMSE	.415	.463	.312

The Discounted Least Squares Model

While different sample periods is one procedure that can be used to account for a change in the mean, it is extreme in that it gives equal weight to all observations within the sample period and zero weight to observations outside the sample period. An alternative would be to use discounted least squares in which all data are used but older observations get less weight. Specifically, the weighting factor is λ^{t-j}, where $0< \lambda<1$. Branch and Evans (2006) suggest discounted least squares might work well for macroeconomic forecasting. In this paper, λ=.95, a value suggested by Branch and Evans and by Clark and McCracken (2006). With this weight, observations from 1985q1 get a weight of 1 percent, observations from 2000q1 get a weight of 23 percent, and observations from 2003q3 get a weight of 46 percent. Discounted least squares is used to estimate an AR model where the lag length is chosen by AIC (applied to a not discounted least squares model). The resulting forecast is called "DLS."

The parameter estimates for the total and core inflation differential are:

	1985q1 – 2007q2
Total inflation differential	
α	.378 (.079)
β_1	.162 (.115)
RMSE	.630
Core inflation differential	
α	.142 (.054)
β_1	.473 (.107)
β_2	.066 (.106)
RMSE	.427

The Exponential Smoothing Model

Forecasts 11–12 use exponential smoothing. Cogley (2002) uses exponential smoothing to estimate a time-varying inflation target of the central bank. Let μ_t be the unobserved time-varying mean inflation differential at time t and let π^{diff}_t be the actual inflation differential at time t. Exponential smoothing expresses the time-variation as follows:

$$\mu_t = \mu_{t-1} + \alpha\left[\pi_t^{diff} - \mu_{t-1}\right]$$

This procedure is simple to estimate and use to forecast. Forecast 11 is based on an estimated smoothing parameter, while forecast 12 is derived by setting the smoothing parameter = .125 (the value used by Cogley). The two exponential smoothing forecasts are called "Exponential, parameter estimated" and "Exponential, parameter set."

	1985q1 – 2007q2	
	α estimated	**α ≡ .1**
Total Inflation Differential		
α	.085	.1
RMSE	.634	.538
Core Inflation Differential		
α	.175	.1
RMSE	.635	.540

The Regime Switching Model

The regime switching model assumes the parameters can switch between two different regimes. The model can be written as follows:

$$\pi_t^c - \pi_t^p = \alpha(S_t) + \varepsilon_t$$

$$\varepsilon_t \sim N\left(0, \sigma(R_t)^2\right)$$

$$\alpha(S_t) = \begin{cases} \alpha_{low} \ if \ S_t = 1 \\ \alpha_{high} \ if \ S_t = 2 \end{cases}$$

$$Prob\left[S_t = j | S_t \, i\right] = p_{ij}$$

$$\sigma(R_t) = \begin{cases} \sigma_{low} \ if \ R_t = 1 \\ \sigma_{high} \ if \ R_t = 2 \end{cases}$$

$$Prob\left[R_t = j | R_{t-1} = i\right] = q_{ij}$$

The log-likelihood function is maximized numerically using the Nedler-Mean simplex algorithm over a large grid of starting values.

The parameter estimates for the total and core inflation differential are as follows.

	State 1 Low differential			State 2 High differential		
	α	σ	LR prob	α	σ	LR prob
Total inflation differential	.109 (.129)	.099 (.042)	.34	.632 (.090)	.998 (.457)	.66
Core inflation differential	.104 (.092)	.064 (.059)	.43	.780 (.069)	.265 (.1103)	.57

The RMSE is calculated as follows. Let $e_t(S_t=s)$ = residual conditional on regime s = 1, 2. The RMSE is then calculated using the following set of residuals: $e_t = p_1{}^*e_t(S_t=1) + p_2{}^*e_t(S_t=2)$. Thus, RMSE for the total inflation differential is .572, and the RMSE for the core inflation differential is .459.

The estimated matrices P = $[p_{ij}]$ and Q = $[q_{ij}]$ for the total inflation differential are:

$$P = \begin{bmatrix} .832 & .168 \\ .085 & .915 \end{bmatrix}$$

$$Q = \begin{bmatrix} .873 & .127 \\ .349 & .651 \end{bmatrix}$$

The estimated matrices P = $[p_{ij}]$ and Q = $[q_{ij}]$ for the core inflation differential are:

$$P = \begin{bmatrix} .9326 & .0674 \\ .0507 & .9493 \end{bmatrix}$$

$$Q = \begin{bmatrix} .8811 & .1182 \\ .0410 & .9590 \end{bmatrix}$$

Since $p_{11} \approx p_{22}$ and "large" for the total and core inflation differential, the long-run probability of being in state 1 is close to 50 percent. But, since $p_{11} < p_{22}$, there is a somewhat greater long-run probability of being in the high-mean state (state 2) than in the low-mean state (state 1). The long-run probability for the total inflation differential is 34 percent of being in the low-mean state and 66 percent of being in the high-mean state. The long-run probability for the core inflation differential is 43 percent of being in the low-mean state and 57 percent of being in the high-mean state.

The Time-Varying Parameter Model

In this model, the inflation differential is written as an AR(1) process, where the parameters follow a random walk process. The AR(1) model can be written as follows:

$$\pi_t^c - \pi_t^p = \alpha_t + \beta_t\left(\pi_{t-1}^c - \pi_{t-1}^p\right) + \varepsilon_t$$

$$\alpha_t = \alpha_{t-1} + \upsilon_t^a, \mathrm{var}\left(\upsilon_t^a\right) = \sigma^2\left[\upsilon^a\right]$$

$$\beta_t = \beta_{t-1} + \upsilon_t^\beta, \mathrm{var}\left(\upsilon_t^\beta\right) = \sigma^2\left[\upsilon^\beta\right]$$

ε_t and υ_t are independent

The parameter estimates for the AR(1) model are:

	Total inflation differential	Core inflation differential
$\sigma[\upsilon^\alpha]$	.026	.054
$\sigma[\upsilon^\beta]$	.00001	.062
α[2007q2]	.294	.089
β[2007q2]	.242	.352
RMSE	.680	.552

Forecast Average

The "Forecast average" is simply the average of the 14 forecasts generated with the previous models and sample periods.

CBO

While not included in the forecast average, the CBO provides forecasts for CPI inflation (overall and core) and PCE inflation (overall and PCE) for 2007–2017.

SPF

The Survey of Professional Forecasters (SPF) provides forecasts for both CPI inflation (core and overall) and PCE inflation (core and overall) for 2007–2009, and forecasts for overall inflation (CPI and PCE) for the next five years and for the next ten years.

Assessing the results

This section assesses the results of generating all 14 forecasts. Appendix Tables 1 and 2 show the total and core inflation differential forecasts for each model for several different horizons. In addition, to provide a sense of the long-run inflation differential, Appendix Charts 1 and 2 show the total and core inflation forecasts for 2017.

A few general observations can be made.

First, for the models estimated over the three different sample periods (Average, AR, and VAR), models 1–9, the forecasts of the overall and core inflation differentials get smaller the shorter the sample period.

Second, the long-run total inflation differential (2017) is broadly similar for all 14 forecasts, ranging between .3 and .5 percentage points. However, the long-run core inflation differential (2017) shows greater differences, ranging between .05 percentage points and .5 percent points.

Third, the forecasts of the overall inflation differential are quite similar across models and forecast horizons—once one excludes the forecasts estimated over the first sample period (1995q1 – 2007q2) and the third sample period (2002q3 – 2007q2). For these forecasts, the average forecast for 2008–2017 is .43 percentage point and ranges between .39 and .48 percentage point.

Finally, the forecasts for the core inflation differential using the regime switching model increases as the forecast horizon increases. For example, the forecasts are .315 percentage point (2008), .384 percentage point (2009), .426 percentage point (2010), and .488 percentage point (2017). This is not too surprising because even though the two regimes are relatively persistent, the long-run probability of being in the high inflation regime is 57 percent, and the high inflation mean differential is .78 percentage point, and the low inflation mean differential is .10 percentage point. The regime switching model recognizes that the inflation differential is high more than half of the time and is low less than half the time. Therefore, while the 2007q2 core inflation differential is likely in the low inflation differential regime (88 percent probability), the model expects the core inflation differential to spend more than half the time in the high differential regime.

The long-run total inflation differential estimated from the regime-switching model is close to the results from the other models. As with the core inflation differential, the long-run probability of being in the high inflation regime is 66 percent, and the high inflation differential is .63 percentage points and the low inflation differential is .11 percentage points. In addition, the results suggest that there is an 81 percent probability that the total inflation differential is currently in the high differential regime.

The other models, in contrast to the regime switching model, predict that the future inflation differential will be close to the current inflation differential. Since the current inflation differential is generally low, the other models generally predict a lower inflation differential than the regime switching model.

APPENDIX

Table 1

AVERAGE TOTAL INFLATION DIFFERENTIAL

Models	**2007**	**2008**	**2009**	**2010**	**2017**	**next10**
Average, 1995q1	.525	.525	.525	.525	.525	.525
Average, 2000q1	.425	.425	.425	.425	.425	.425
Average, 2002q3	.341	.341	.341	.341	.341	.341
AR, 1995q1	.873	.542	.534	.534	.534	.545
AR, 2000q1	.822	.444	.438	.438	.438	.448
AR, 2002q3	.527	.358	.326	.322	.322	.312
VAR, 1995q1	.865	.544	.534	.533	.533	.544
VAR, 2000q1	.825	.440	.438	.438	.438	.448
VAR, 2002q3	.659	.328	.336	.336	.336	.333
DLS	.787	.452	.451	.451	.451	.456
Exponential, parameter estimated	.364	.423	.423	.423	.423	.423
Exponential, parameter set	.343	.435	.435	.435	.435	.435
Regime Switch	.514	.477	.463	.458	.456	.462
TVP, AR1	.377	.394	.388	.388	.388	.399
Notes:						
Forecast average	.589	.438	.433	.432	.432	.435
Lower central tendency	.377	.394	.388	.388	.388	.399
Upper central tendency	.822	.477	.463	.458	.456	.462
Minimum	.341	.328	.326	.322	.322	.312
Maximum	.873	.544	.534	.534	.534	.545
CBO	.500	.300	.400	.400	.300	.320
SPF	.600	.300	.200			.300

Table 2

AVERAGE CORE INFLATION FORECAST

Models	**2007**	**2008**	**2009**	**2010**	**2017**	**next10**
Average, 1995q1	.486	.486	.486	.486	.486	.486
Average, 2000q1	.278	.278	.278	.278	.278	.278
Average, 2002q3	.052	.052	.052	.052	.052	.052
AR, 1995q1	.333	.483	.481	.481	.481	.482
AR, 2000q1	.269	.292	.281	.281	.281	.286
AR, 2002q3	.203	.091	.068	.066	.066	.077
VAR, 1995q1	.281	.428	.460	.467	.468	.459
VAR, 2000q1	.199	.244	.263	.266	.266	.261
VAR, 2002q3	.116	.015	.047	.046	.046	.044
DLS	.268	.320	.309	.307	.307	.311
Exponential, parameter esti-mated	.172	.191	.191	.191	.191	.191
Exponential, parameter set	.170	.182	.182	.182	.182	.182
Regime Switch	.196	.315	.384	.426	.488	.433
TVP, AR1	.148	.143	.137	.137	.137	.142
Notes:						
Forecast average	.254	.258	.276	.284	.269	.271
Lower central tendency	.170	.143	.137	.137	.137	.142
Upper central tendency	.278	.320	.384	.426	.468	.433
Minimum	.486	.486	.486	.486	.488	.486
Maximum	.052	.015	.047	.046	.046	.044
CBO	.6	.3	.5	.6	.3	.38
SPF	.3	.3	.3			

Chart 1

TOTAL INFLATION DIFFERENTIAL, 2017

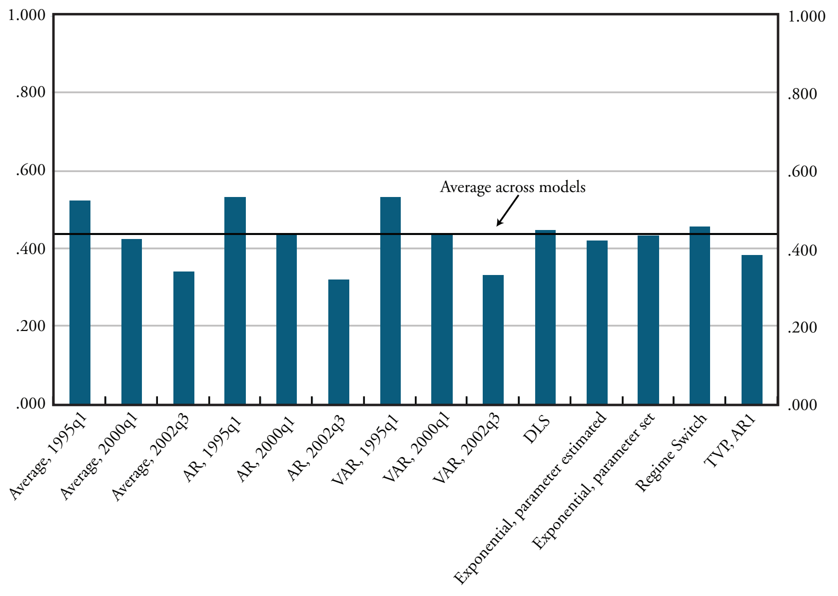

Chart 2

CORE INFLATION DIFFERENTIAL, 2017