

Should We Be Puzzled by Forward Guidance? Technical Appendix*

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A Data Appendix

This appendix details how we use interest rate futures contracts to derive market-based measures of expected future interest rates. In our baseline specification in the main text, we use a Gurkaynak, Sack and Swanson (2005)-style path factor as our measure of forward guidance shocks, which is extracted from a combination of federal funds and eurodollar futures contracts. Below, we provide additional information on these futures contracts and detail how we construct our measure of forward guidance shocks.

A.1 Federal Funds Futures Contracts

To capture unexpected changes in expectations about interest rates in the near term (one day to three months ahead), we use federal funds futures contracts. Federal funds futures contracts settle based on the average of the daily effective federal funds rate during the contract expiration month. We obtain daily data on the closing price of federal funds futures contracts from the Chicago Mercantile Exchange. Let p_t^j denote the price at time t of the federal funds futures contract expiring j -months ahead, where $j = 0$ corresponds to the spot-month contract expiring in the current month. Then $f_t^j = 100 - p_t^j$ is the time t expectation of the average effective federal funds rate j -months ahead.

Surprise Component of the Current Target Federal Funds Rate

The day before the current FOMC meeting, the spot-month federal funds future contract satisfies:

$$f_{t-1}^0 = \frac{d_0}{m_0} r_{-1} + \frac{m_0 - d_0}{m_0} E_{t-1}(r_0) + \mu_{t-1}^0, \quad (1)$$

where r_{-1} is the annualized target federal funds rate prevailing before the meeting (which is assumed to equal the effective federal funds rate each day of the month before the meeting) and r_0 is the annualized target federal funds rate after the meeting the next day. The term μ_{t-1}^0 is the term-premium for the spot-month federal funds futures contract which we assume is constant between the day before and the day of the FOMC meeting.

The next day, after the FOMC's rate decision, the spot-month federal funds future contract satisfies:

$$f_t^0 = \frac{d_0}{m_0} r_{-1} + \frac{m_0 - d_0}{m_0} r_0 + \mu_t^0. \quad (2)$$

Combining Equations 1 and 2, the unexpected policy surprise in the target federal funds

rate is, denoted by e_t^0 , is defined by:

$$e_t^0 \equiv r_0 - E_{t-1}(r_0) = [(f_t^0 - f_{t-1}^0) - (\mu_t^0 - \mu_{t-1}^0)] \frac{m_0}{m_0 - d_0} = (f_t^0 - f_{t-1}^0) \frac{m_0}{m_0 - d_0} \quad (3)$$

where the last equality follows from the assumption that the term-premium for the spot-month federal funds futures contract is constant between the day before and the day of the FOMC meeting. In practice, this assumption is reasonable except for meetings late in the month for which any small change in term-premia between the day-before and the day-of the FOMC meeting would be magnified by $m_0/(m_0 - d_0)$. To avoid this large scaling factor for events in the last week of the month, we use the following month's contract instead of the current month's contract. In this case, $e_t^0 = f_t^1 - f_{t-1}^1$ since for meetings late in a month there is no meeting the subsequent month.

Surprise Component of the Funds Rate Expected After 1st-Upcoming Meeting

While e_t^0 captures the monetary policy surprise generated by changes in the current target federal funds rate, forward guidance influences expectations about the future path of the federal funds rate. Therefore, to extract policy surprises in the future path of interest rates, we assume investors know the dates of future FOMC meetings and we also extract how their expectations change for rates at the upcoming FOMC meeting. The day before the current FOMC meeting, the federal funds future contract expiring $i(1)$ months ahead, which is the month of the 1st-upcoming FOMC meeting, satisfies:

$$f_{t-1}^{i(1)} = \frac{d_1}{m_1} E_{t-1}(r_0) + \frac{m_1 - d_1}{m_1} E_{t-1}(r_1) + \mu_{t-1}^1, \quad (4)$$

where r_0 is the annualized target federal funds rate rate set after the current meeting (which takes place the next day) and r_1 is the annualized target federal funds rate rate set after the next meeting (which takes place $i(1)$ months in the future). The term μ_{t-1}^1 is the term-premium for the federal funds futures contract expiring $i(1)$ months ahead, which is assumed to be constant between the day before and the day of the FOMC meeting.

The next day after the FOMC issues its statement at time t , the federal funds future contract expiring $i(1)$ months ahead, which is the month in which the 1st-upcoming FOMC meeting takes place, satisfies:

$$f_t^{i(1)} = \frac{d_1}{m_1} E_t(r_0) + \frac{m_1 - d_1}{m_1} E_t(r_1) + \mu_t^1. \quad (5)$$

Combining Equations 4 and 5, the unexpected policy surprise in the federal funds rate

expected to prevail after the 1st-upcoming FOMC meeting, denoted by e_t^1 , is defined by:

$$e_t^1 \equiv E_t(r_1) - E_{t-1}(r_1) = \left[\left(f_t^{i(1)} - f_{t-1}^{i(1)} \right) - \frac{d_1}{m_1} e_t^0 \right] \frac{m_1}{m_1 - d_1}. \quad (6)$$

For events in the last week of the month we use the next month's contract, which implies $e_t^1 = f_t^{i(1)+1} - f_{t-1}^{i(1)+1}$.

A.2 Eurodollar Futures Contracts

In order to capture investors' expectations about interest rates over horizons longer than a few months, we compute the changes in eurodollar futures contracts around FOMC announcements. We obtain daily data on the closing price of USD Eurodollar futures contracts from the CME Group with contracts maturing $i = 2, 3, 4, 5, 6, 7, 8$ quarters into the future.¹ Let r_i denote the annualized 3-month USD LIBOR interest rate at settlement i quarters in the future. Also, let $p_t^{x,i}$ denote the time t closing price of the Eurodollar contract expiring i quarters in the future and $f_t^{x,i} = 100 - p_t^{x,i}$ denote the implied rate. Then, the unexpected policy surprise, as implied by the Eurodollar contract maturing i quarters in the future, emanating from the FOMC meeting occurring on day t , is:

$$x_t^i \equiv E_t(r_i) - E_{t-1}(r_i) = f_t^{x,i} - f_{t-1}^{x,i}. \quad (7)$$

Unlike federal funds futures contracts, eurodollar futures don't settle based on the average of their underlying instrument during the settlement month. Therefore, there is no scaling necessary when using these interest rate futures contracts to extract expectations about the future path of monetary policy. For some robustness checks, we use the change in 4-, 8-, and 12-quarter ahead eurodollar futures contracts denoted by x_t^4 , x_t^8 , or x_t^{12} as our measure of forward guidance shocks during the zero lower bound period.

A.3 Computing Our Path Factor

We closely follow the appendix of Gurkaynak, Sack and Swanson (2005) to construct our path factor measure of forward guidance shocks. Their methodology uses principal component analysis to synthesize the information from numerous interest rate futures contracts into a single indicator of forward guidance surprises. We first standardize e^0 , e^1 , x_t^2 , x_t^3 , x_t^4 , x_t^5 , x_t^6 ,

¹Eurodollar futures settle in March, June, September, and December. Therefore, as an example, we define a 2-quarter ahead eurodollar future in January and February as the contract expiring in June and, beginning in March, we define the 2-quarter ahead eurodollar as the contract expiring in September.

x_t^7 , and x_t^8 such that each series has a mean of zero and a standard deviation of one. Then, we extract the first two principle components of these 9 time-series, denoted by f^1 and f^2 , over the sample of regularly scheduled FOMC meetings from January 1994 to December 2015. Next, we standardize f^1 and f^2 and then run the ordinary least squares (OLS) regressions $e_t^0 = \gamma_1 f_t^1 + \epsilon_t$ and $e_t^0 = \gamma_2 f_t^2 + \epsilon_t$. With γ_1 and γ_2 in hand, we transform f^1 and f^2 into z^1 (the unscaled target factor) and z^2 (the unscaled path factor) using the linear transformation:

$$\begin{bmatrix} z^1 & z^2 \end{bmatrix} = \begin{bmatrix} f^1 & f^2 \end{bmatrix} \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix}. \quad (8)$$

The matrix elements $\alpha_1, \alpha_2, \beta_1, \beta_2$ are identified from the four restrictions:

Restrictions 1 and 2: The columns of the transforming matrix have unit length (so that the target and path factors have a standard deviation of 1).

$$\begin{aligned} \alpha_1^2 + \alpha_2^2 &= 1 \\ \beta_1^2 + \beta_2^2 &= 1 \end{aligned}$$

Restriction 3: The target and path factors remain orthogonal after the transformation.

$$E(z^1 z^2) = \alpha_1 \beta_1 + \alpha_2 \beta_2 = 0$$

Restriction 4: The path factor has no influence on the current policy surprise e^0 . Since,

$$\begin{aligned} f^1 &= \frac{1}{\alpha_1 \beta_2 + \alpha_2 \beta_1} [\beta_2 z^1 - \alpha_2 z^2] \\ f^2 &= \frac{1}{\alpha_1 \beta_2 + \alpha_2 \beta_1} [\alpha_1 z^2 - \beta_2 z^1], \end{aligned}$$

then the effect of a change in z^2 on e^0 is defined by:

$$\frac{de^0}{dz^2} = \frac{de^0}{df^1} \frac{df^1}{dz^2} + \frac{de^0}{df^2} \frac{df^2}{dz^2} = -\gamma_1 \frac{\alpha_2}{\alpha_1 \beta_2 + \alpha_2 \beta_1} + \gamma_2 \frac{\alpha_1}{\alpha_1 \beta_2 + \alpha_2 \beta_1}$$

Hence, the restriction that $\frac{de^0}{dz^2} = 0$ implies the parameter restriction that: $\gamma_2 \alpha_1 = \gamma_1 \alpha_2$.

Finally, we scale the resulting z^1 and z^2 vectors. We scale the target factor so that e_t^0 has a one-for-one effect on it by regressing $e_t^0 = \beta_1 z_t^1 + \epsilon_t$ and then set $z^{target} = \beta_1 z^1$. We scale the path factor so that x_t^8 has a one-for-one effect on it by regressing $x_t^8 = \beta_2 z_t^2 + \epsilon_t$ and then set $z^{path} = \beta_2 z^2$.

B Model

In the symmetric equilibrium, the baseline model in Dynare notation is as follows:

```
model;

// Private Sector

y = pd^(-1)*n^(1 - alpha)*(u*k(-1))^alpha;
y = c + inv;
w = chi*(a/lambda)*n^eta;
lambda = a * (c - b*c(-1))^(1);
1 = beta * (lambda(+1)/lambda) * ( r/pie(+1) );
1 = beta * (lambda(+1)/lambda) * ( rr );

w = (1 - alpha)*(y*pd/n)/mu;
rrk*u = alpha*(y*pd/k(-1))/mu;
q*deltauprime*k(-1) = alpha*(y*pd/u)/mu;
k = (1 - deltau)*k(-1) + inv*( 1 - (phii/2)*(inv/inv(-1)-1)^(2) );
deltau = delta0 + delta1*(u-1) + (delta2/2)*(u-1)^(2);
deltauprime = delta1 + delta2*(u-1);

g1 = lambda*y/mu +
omega*beta*(pie(+1)/(piess^(1-gamma)*pie^gamma))^(theta)*g1(+1);

g2 = piestar*(lambda*y + omega*beta*((pie(+1)/(piess^(1-gamma)*pie^gamma))^(theta-1)
/piestar(+1))*g2(+1));

1 = omega*(pie/(piess^(1-gamma)*pie(-1)^gamma))^(theta-1)
+ (1-omega)*(piestar^(1-theta));
pd = omega*(pie/(piess^(1-gamma)*pie(-1)^gamma))^(theta)*pd(-1)
+ (1-omega)*piestar^(-theta);

theta*g1 = (theta-1)*psi*g2;

1 - q*(1 - (phii/2)*(inv/inv(-1)-1)^(2) - phii*(inv/inv(-1)-1)*(inv/inv(-1)) )
= beta * q(+1) * (lambda(+1)/lambda) * phii * (inv(+1)/inv - 1)*(inv(+1)/inv)^(2);
```

```
q = beta * (lambda(+1)/lambda) * ( rrk(+1)*u(+1) + q(+1) * (1 - deltau(+1)) );
```

```
a = (1-rhoa)*ass + rhoa * a(-1) + vola*ea;
```

```
nu = rhonu * nu(-1) + volnu*enu;
```

```
// Lagged Expectations
```

```
expy1 = y(+1);
```

```
lagey = expy1(-1);
```

```
expinv1 = inv(+1);
```

```
lageinv = expinv1(-1);
```

```
expu1 = u(+1);
```

```
lageu = expu1(-1);
```

```
exppie1 = pie(+1);
```

```
lagepie = exppie1(-1);
```

```
// Monetary Policy Rule
```

```
log(rd) = phir*log(rd(-1))
```

```
+ (1-phir) * (log(rss) + phipie*log(pie/piess) + phix*log(y/yss)) + nu;
```

```
r = rd;
```

```
// Blue Chip Survey Forecasts of 3-Month T-Bill Rate, 4-Quarters Ahead
```

```
expr1auxiliary = r(+1);
```

```
expr2auxiliary = expr1auxiliary(+1);
```

```
trate = 12*(1/3)*( log(r) + log(expr1auxiliary) + log(expr2auxiliary) );
```

```
exp1trate = trate(+1);
```

```
exp2trate = exp1trate(+1);
```

```
exp3trate = exp2trate(+1);
```

```
exp4trate = exp3trate(+1);
```

```
exp5trate = exp4trate(+1);
```

```
exp6trate = exp5trate(+1);
exp7trate = exp6trate(+1);
exp8trate = exp7trate(+1);
exp9trate = exp8trate(+1);
exp10trate = exp9trate(+1);
exp11trate = exp10trate(+1);
exp12trate = exp11trate(+1);
exp13trate = exp12trate(+1);
exp14trate = exp13trate(+1);
exp15trate = exp14trate(+1);

bcforecast = (1/3)*( exp12trate + exp13trate + exp14trate );

end;
```

Since the capital stock is predetermined, we lag the capital stock K variables by one period relative to the timing in the model derivation.

References

- Gurkaynak, Refet S, Brian Sack, and Eric Swanson.** 2005. “Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements.” *International Journal of Central Banking*, 1(1): 55–93.